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Nonlinear propagation of crossing electromagnetic waves in vacuum due to photon-photon scattering.

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Nonlinear propagation of crossing electromagnetic waves in vacuum due to photon-photon scattering

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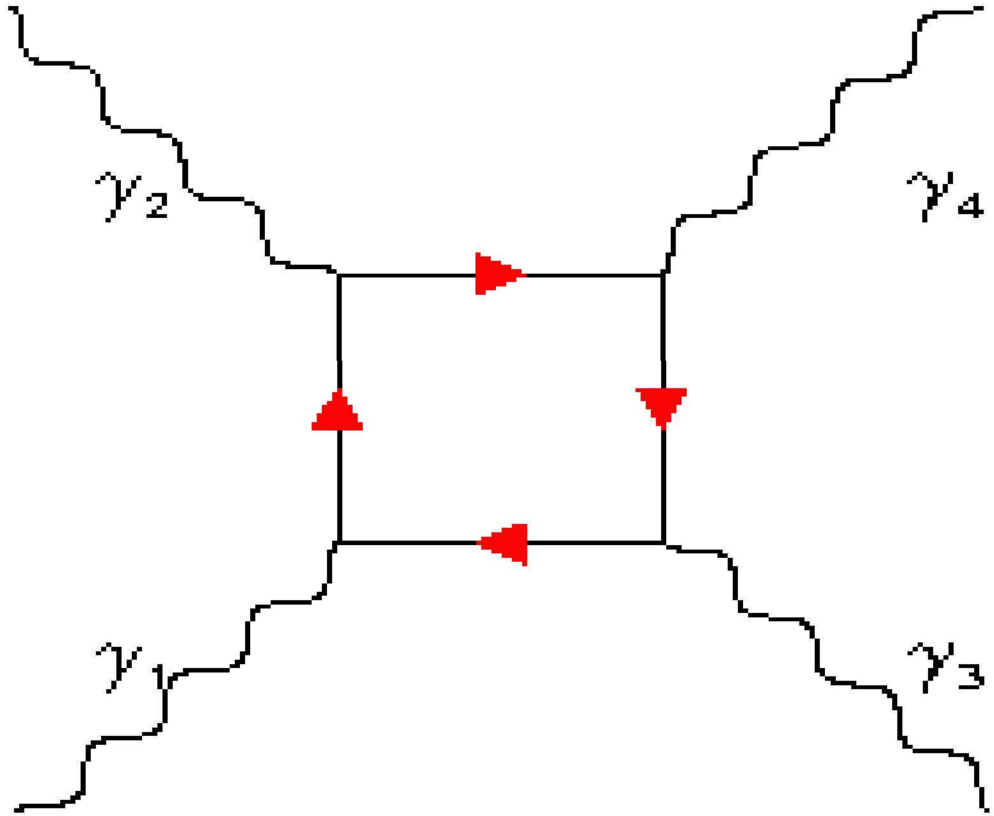

Figure 1: Feynman diagram for photon-photon scattering

Effective Lagrangian for the e.m. fields for $h\nu \ll m_e c^2$ (Euler - Heisenberg)

$$\mathcal{L} = \mathcal{L}_0 + \xi \left[\mathcal{L}_0^2 + \frac{7\epsilon_0^2 c^2}{4} (\mathbf{E} \cdot \mathbf{B})^2 \right],$$

$$\mathcal{L}_0 = \frac{\epsilon_0}{2} \left(\mathbf{E}^2 - c^2 \mathbf{B}^2 \right)$$

$$\xi = \frac{8\alpha^2 \hbar^3}{45 m_e^4 c^5} = 6.7 \times 10^{-30} \frac{m^3}{J}.$$

$\mathbf{B} = \nabla \wedge \mathbf{A}$ and $\mathbf{E} = -c\nabla A^0 - \frac{\partial \mathbf{A}}{\partial t}$.
For x-polarized waves $A^\mu = (0, A(t, y, z), 0, 0)$
$\frac{\delta \int \mathcal{L}}{\delta A} = 0$ gives the **NON-LINEAR WAVE EQUATION**

$$
\begin{gathered}
0 = \partial_y^2 A + \partial_z^2 A - \partial_t^2 A + \xi\epsilon_0 \{ \\
[\ (\partial_t A)^2 - 3(\partial_y A)^2 - (\partial_z A)^2]\, \partial_y^2 A + \\
[\ (\partial_t A)^2 - (\partial_y A)^2 - 3(\partial_z A)^2]\, \partial_z^2 A - \\
[\ 3(\partial_t A)^2 - (\partial_y A)^2 - (\partial_z A)^2]\, \partial_t^2 A + \\
4\,(\partial_z A \partial_t A \partial_z \partial_t A - \partial_z A \partial_y A \partial_z \partial_y A + \partial_y A \partial_t A \partial_y \partial_t A)\ \}
\end{gathered}
$$

LINEARITY OF QED $\longrightarrow$ SUM OVER ALL *VIRTUAL* PATHS $\longrightarrow$ NONLINEAR EQS. FOR THE *REAL* PARTICLES FIELDS

- ✔ DO ELECTROMAGNETIC WAVES PROPAGATE NONLINEARLY IN THE VACUUM? Yes if crossing (scattering).
- ✔ IS IT A PROBLEM FOR COMMUNICATIONS? Effect smaller than atmosphere.
- ✔ CAN WE DETECT SUCH NONLINEARITY? Need very high power to compensate $\xi = 6.7 \times 10^{-30} m^3/J$

ULTRAHIGH POWER LASER BEAMS
Peak intensities and energy density of LASER pulses

- ✔ CURRENT LIMIT: $I \sim 2 \times 10^{22} W cm^{-2}$, or $\xi\rho \sim 4 \times 10^{-12}$ (HERCULES, V. Yanovsky et al., Optics Express 16, 2109 (2008)).

- ✔ NEXT (European Extreme Light Infrastructure, ELI): $I \sim 10^{25} W cm^{-2}$, or $\xi\rho \sim 2 \times 10^{-9}$

- ✔ NEXT TO NEXT: $I \sim 10^{28} W cm^{-2}$, or $\xi\rho \sim 2 \times 10^{-6}$

(Reference: G.A.Mourou, T.Tajima and S.V.Bulanov, Rev. Mod. Phys. 78 (2006) 310)

EVEN AT NEXT TO NEXT GENERATION: $\xi\rho \lesssim 10^{-6}$

- ✔ EXPLAIN DIFFICULTY FOR OPTICAL OBSERVATION OF $\gamma - \gamma$ SCATTERING
- ✔ JUSTIFY PERTURBATIVE TREATMENT (A. Ferrando, H. Michinel, M. Seco, and D. T., Phys. Rev. Lett., **99**, 150404 (2007).

HERCULES $\longrightarrow$ Power per pulse $\simeq 3 \times 10^{14} W$, an order of magnitude larger than the total power used by Mankind!
($\Delta t \sim 3 \times 10^{-14} s \longrightarrow$ Energy per pulse $\sim 10 J$)

ELI $\longrightarrow$ pulses of $10^{18} W$, an order of magnitude larger then the total power that the earth receives from the sun.
($\Delta t \sim 10^{-18} - 10^{-15} s \longrightarrow$ Energy per pulse $\sim 1 - 10^{3} J$)

USES:

- ✔ 1) ACCELERATING CHARGED PARTICLES;
- ✔ 2) ULTRASHORT TIME MICROSCOPY;
- ✔ 3) EXPLORE THE NONLINEARITY OF THE VACUUM (search of $\gamma - \gamma$ scattering).

PREVIUOS PROPOSALS (Reference: M. Marklund and P.K. Shukla, Rev. Mod. Phys. **78**, 591 (2006))

- ✔ QED Vacuum shows DC Kerr effect: Klein and Nigam, 1964. Observation of laser-induced birefringence requires extra free electron laser: Aleksandrov, Anselm and Moskalyov, 1985; Heinzl *et al.*, 2006; Di Piazza *et al.*, 2006.

- ✔ Three beams scattering: Adler, 1971; Moulin and Berndard, 1999; Lundstrom *et al.*, 2006.

- ✔ Harmonic generation in inhomogeneous magnetic field: Ding and Kaplan, 1992.

- ✔ Resonant interactions in MW cavities: Brodin, Marklund and Stenflo, 2001.

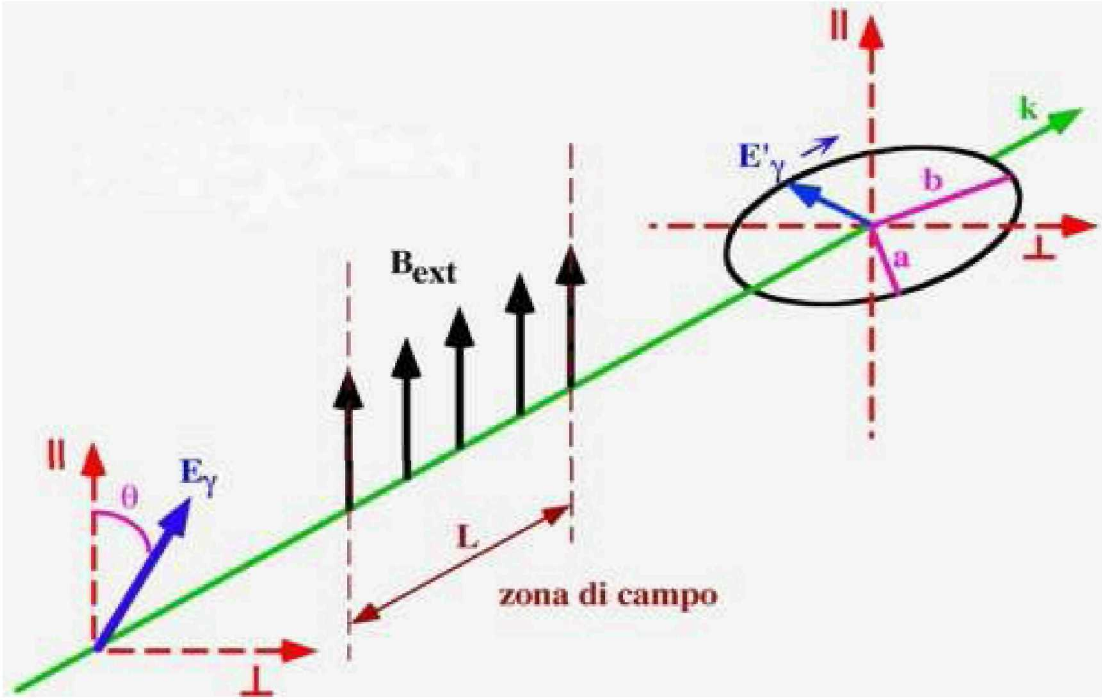

Figure 2: The search for the *magnetic* birefringence of the vacuum (R.Baier and P.Breitenlohner, 1967; S.L.Adler 1971; Z and I Bialynicka-Birula, 1970; E.Iacopini and E.Zavattini, 1979) used to set THE CURRENT LIMIT on $\sigma_{\gamma-\gamma} \lesssim 10^{-60} m^2$, for optical wavelengths, or $\xi_{\rm exp} < 3.1 \times 10^{-26} m^3/J$, 4.6×10^3 times higher than the QED value! M.Bregant et al., PVLAS, arXiv:0805.3036 (20 May 2008).

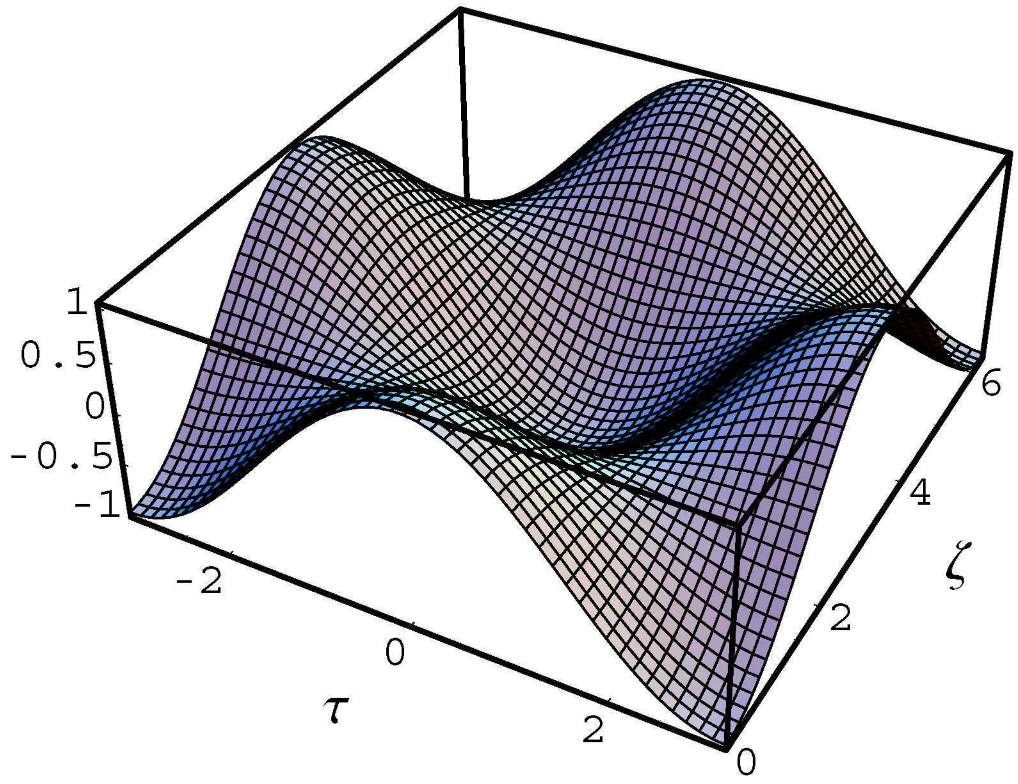

Figure 3: Numerical solution, $A_{\rm num}/\mathcal{A}$, for two scattering waves, for $\xi\bar{\rho} = 0.0025$. ($\tau \equiv \omega t$ and $\zeta \equiv kz$.) (D.T., A.Ferrando, H.Michinel, M.Seco, Phys. Rev. A **77**, 042101 (2008))

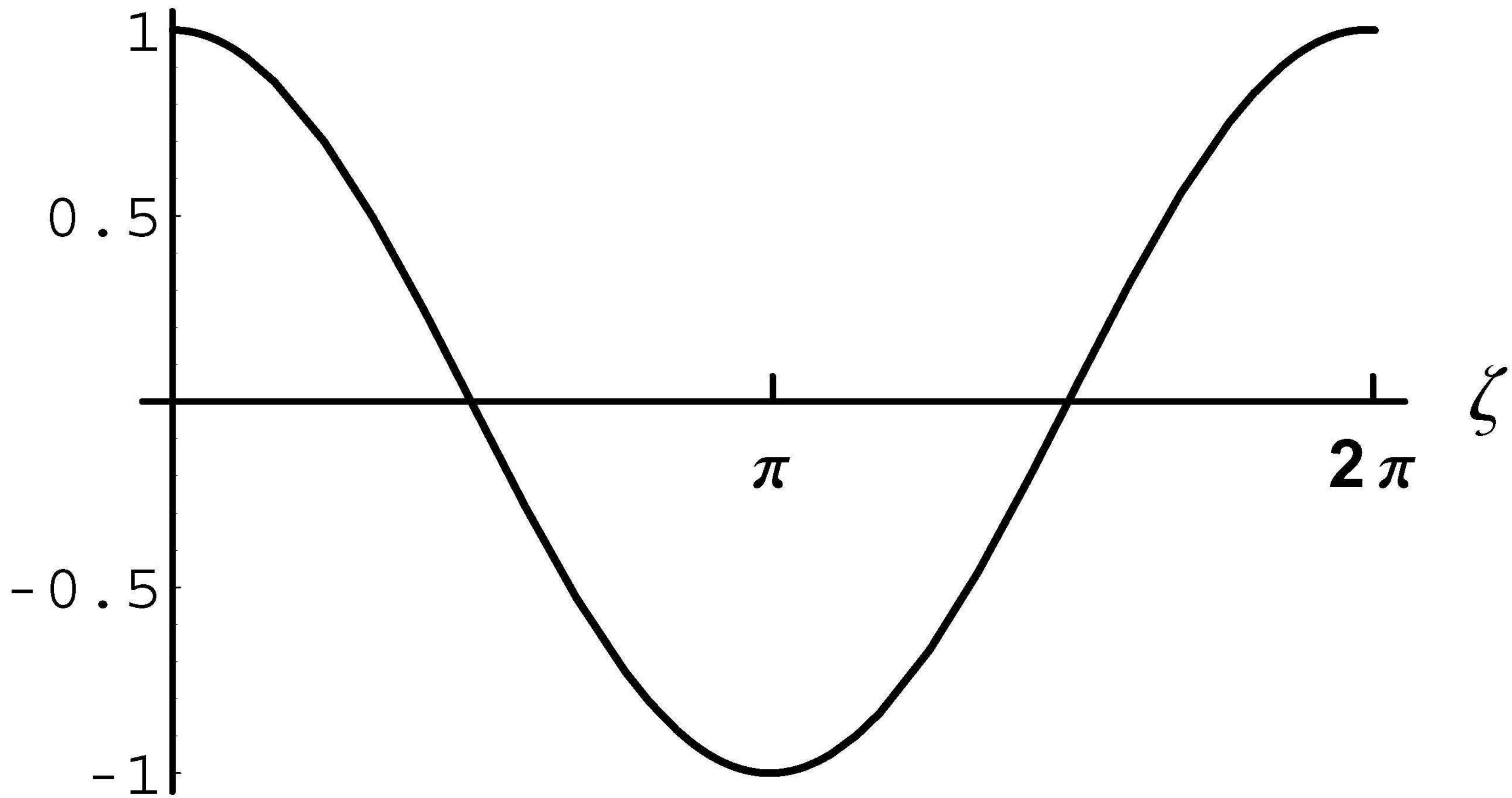

Figure 4: Zero-time plot of the numerical solution, as a function of the adimensional space coordinate $\zeta \equiv kz$.

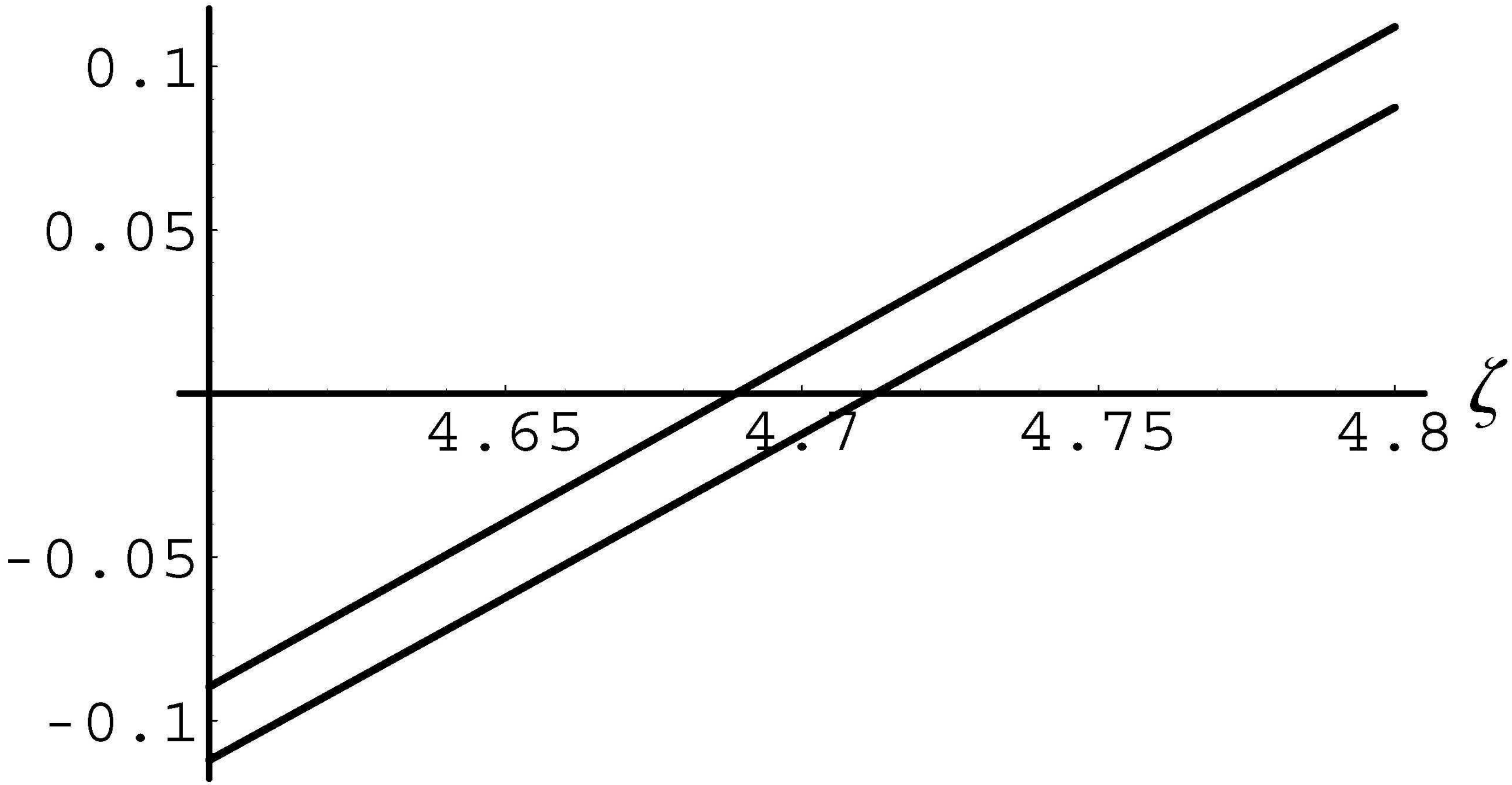

Figure 5: Detail of the zero-time functions $A_{\rm num}/\mathcal{A}$ (upper curve) and $A_{\rm lin}/\mathcal{A}$ (lower curve), for values of the adimensional space coordinate $\zeta \equiv kz$ close to the second zero.

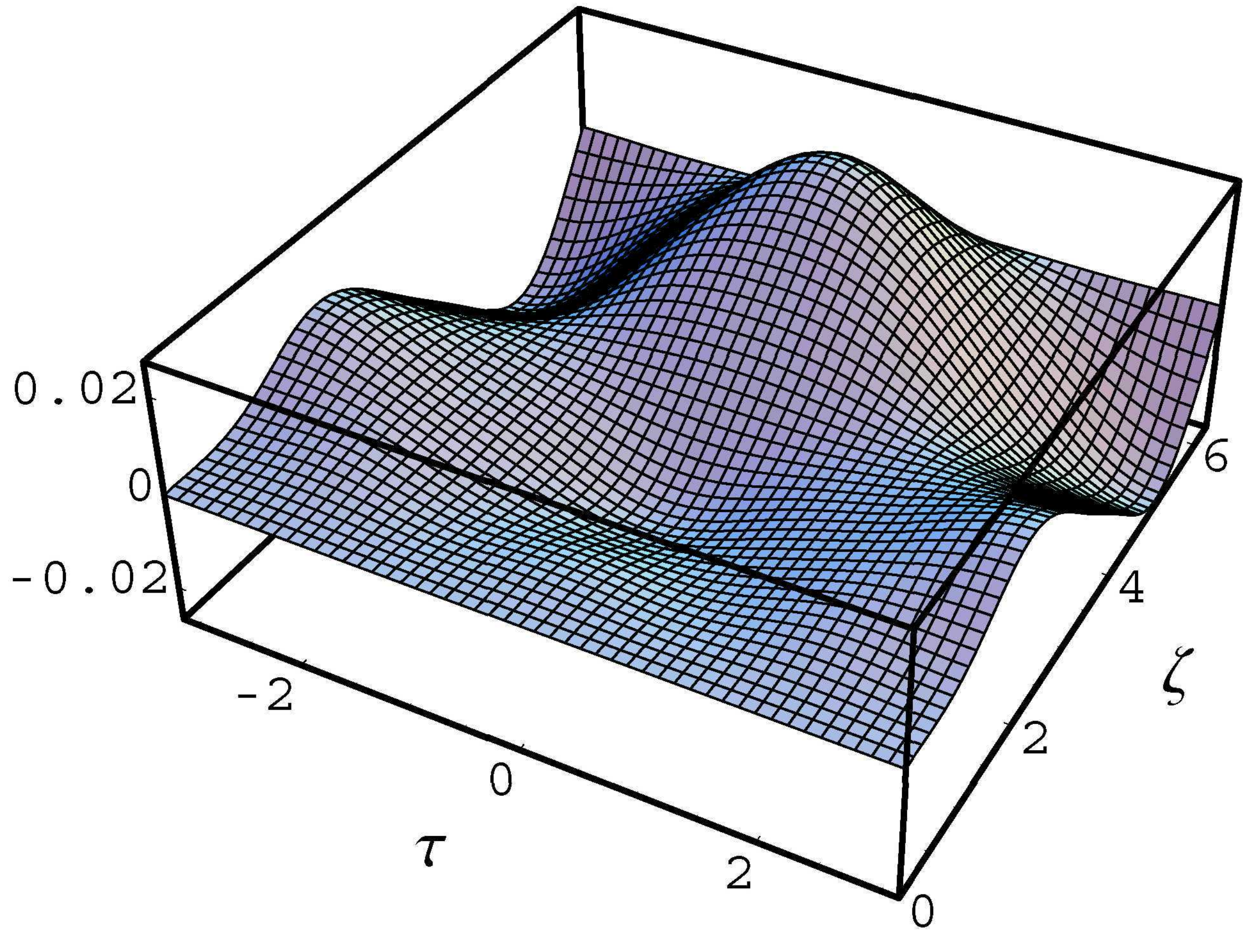

Figure 6: Relative error $(A_{\rm num} - A_{\rm lin})\,/\mathcal{A}$ of the linear approximation, as a function of the adimensional time and space coordinates $\tau \equiv \omega t$ and $\zeta \equiv kz$.

VARIATIONAL METHOD

It gives an ANALYTICAL approximation, valid also for LARGER DISTANCES and applicable to NON-SYMMETRIC configurations.

AN x-POLARIZED SOLUTION OF THE LINEAR MAXWELL EQUATIONS:

$$A = \frac{\mathcal{A}}{2}\left[\cos(kz - \omega t) + \cos(kz + \omega t)\right] = \mathcal{A}\cos(kz)\cos(\omega t),$$

ANSATZ FOR AN (APPROXIMATE) SOLUTION OF THE NON-LINEAR EQUATIONS:

$$A = \mathcal{A}\left[\alpha(z)\cos(kz) + \beta(z)\sin(kz)\right]\cos(\omega t).$$

Minimize:

$$\Gamma = \int_{-\infty}^{\infty} dz \left(\frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \mathcal{L} \right)$$

(with additional average over fast variation in z)
APPROXIMATE RESULT OF THE MINIMIZATION:

$$A = \mathcal{A} \cos(\omega t) \cos[(k + \chi)z],$$

where

$$\chi = 2\xi\rho k.$$

RESULTING PHASE SHIFT:

$$\Delta\phi = \chi z.$$

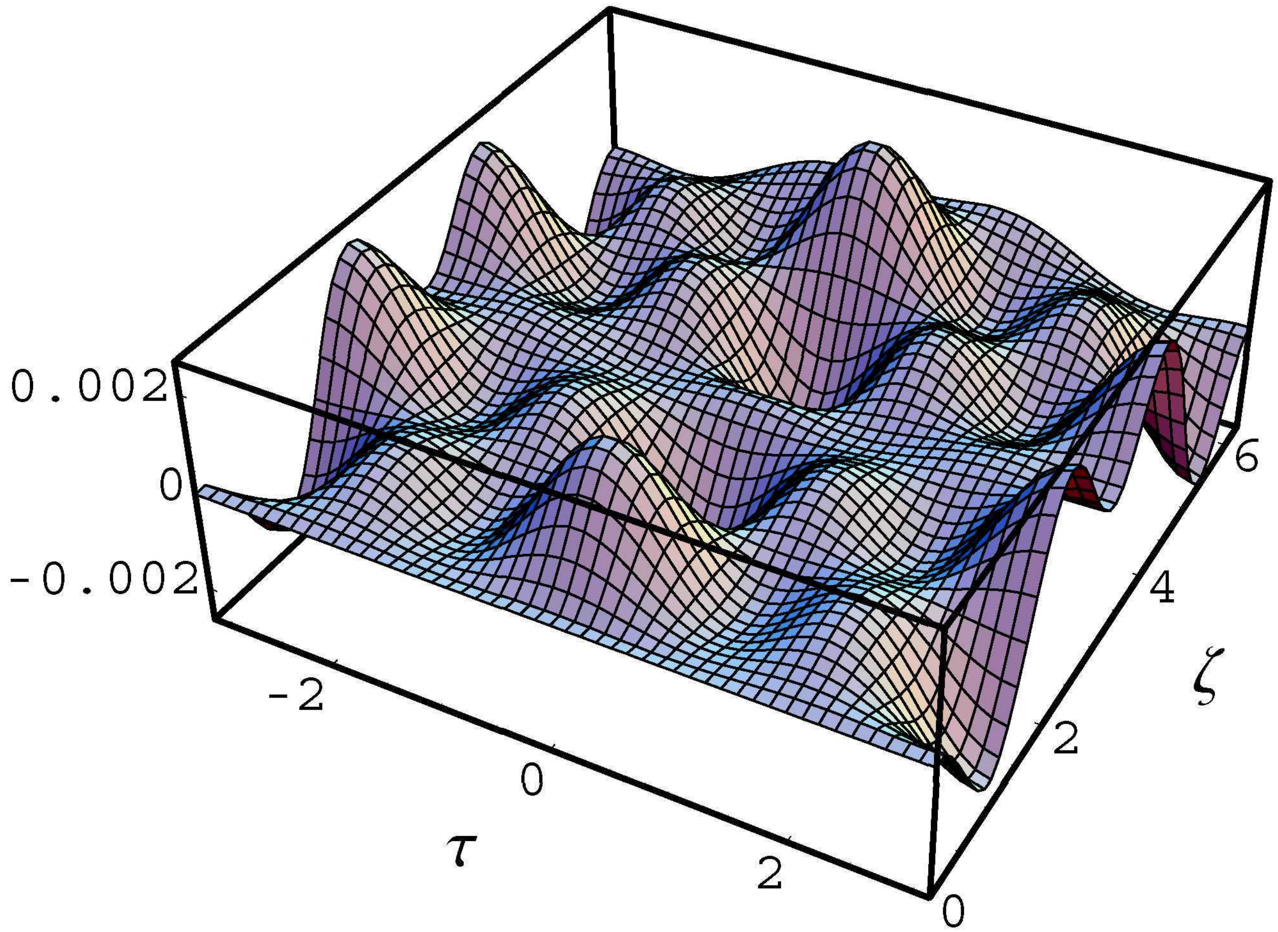

Figure 7: Relative error $(A_{\rm num} - A_{\rm var})/\mathcal{A}$ of the variational approximation, as a function of the adimensional time and space coordinates $\tau \equiv \omega t$ and $\zeta \equiv kz$.

GENERALIZATION: 'low' power wave scattering with high power wave

$$A = \mathcal{A}\cos(kz+\omega t+\varphi)+\alpha(z)\cos(kz-\omega t)+\beta(z)\sin(kz-\omega t)$$

Approximate Variational solution:
$\alpha(z) = \alpha_0 \cos(\eta z)$ and $\beta(z) = -\alpha_0 \sin(\eta z)$.
Valid for any α_0, provided that $|\alpha_0| \ll |\mathcal{A}|$.

As a result, the low power beam becomes

$$A_l(t, z) = \alpha_0 \cos[(k + \eta)z - \omega t],$$

Therefore

$$\Delta\Phi = \eta \Delta z \simeq 4\xi\rho k \Delta z.$$

PROPOSAL OF EXPERIMENT AT ELI
(first stage=TOMORROW):

$\lambda = 800nm$, $I = 10^{29}Wm^{-2}$, $\tau = 10fs$, $d \approx 10\mu m$.
Use three beams (can be two ‘normal´ beams that are in phase between each other and a contrapropagating High Power Laser beam). Thus $\Delta\Phi \approx 2 \times 10^{-7}rad$.

$\Delta\phi$ as small as $10^{-7}rad$ can be measured (Kang *et al.*, 1997) comparing with the reference beam which propagated alone (no effect of QED vacuum).

PROPOSAL OF EXPERIMENT AT HERCULES (=TODAY):

$\lambda = 810nm$, $I = 2 \times 10^{26} Wm^{-2}$, $\tau = 30fs$, $d \approx 0.8\mu m$
Same configuration as above $\longrightarrow \Delta\Phi \simeq 10^{-9}$ (from QED), two order of magnitude smaller than the possibility of detection.

HOWEVER, THIS WILL ALLOW FOR AN IMPROVEMENT OF THE LIMIT on $\gamma - \gamma$ SCATTERING, i.e. on $\xi_{\rm exp}$, DOWN TO THE RANGE $10^{-27} m^3/J$, AN ORDER OF MAGNITUDE BETTER THAN (PVLAS) CURRENT LIMIT.

CONCLUSIONS

- ✔ Crossing electromagnetic waves in vacuum get a phase shift from QED $\gamma - \gamma$ scattering
- ✔ At HERCULES, our proposed experiment can improve the limit on $\gamma - \gamma$ scattering at least by an order of magnitude
- ✔ At ELI the QED phase shift will be detectable, possibly providing the first evidence of $\gamma - \gamma$ scattering