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Nonlinear propagation of crossing electromagnetic waves in vacuum due to photon-photon scattering.

D. Tommasini
University of Vigo
Dept. of Applied Physics
Ourense
Spain
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Daniele Tommasini

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Figure 1: Feynman diagram for photon-photon scattering
Effective Lagrangian for the e.m. fields for $h\nu \ll m_e c^2$

(Euler - Heisenberg)

\[ \mathcal{L} = \mathcal{L}_0 + \xi \left[ \mathcal{L}_0^2 + \frac{7\varepsilon_0^2 c^2}{4} (\mathbf{E} \cdot \mathbf{B})^2 \right] , \]

\[ \mathcal{L}_0 = \frac{\varepsilon_0}{2} \left( \mathbf{E}^2 - c^2 \mathbf{B}^2 \right) \]

\[ \xi = \frac{8\alpha^2 \hbar^3}{45 m_e^4 c^5} = 6.7 \times 10^{-30} \frac{m^3}{J} . \]
$\mathbf{B} = \nabla \wedge \mathbf{A}$ and $\mathbf{E} = -c\nabla A^0 - \frac{\partial \mathbf{A}}{\partial t}$.

For $x$-polarized waves $A^\mu = (0, A(t, y, z), 0, 0)$

$\frac{\delta \int \mathcal{L}}{\delta A} = 0$ gives the NON-LINEAR WAVE EQUATION

$$0 = \partial_y^2 A + \partial_z^2 A - \partial_t^2 A + \xi \varepsilon_0 \left\{ \begin{array}{l}
\left[ (\partial_t A)^2 - 3(\partial_y A)^2 - (\partial_z A)^2 \right] \partial_y^2 A + \\
\left[ (\partial_t A)^2 - (\partial_y A)^2 - 3(\partial_z A)^2 \right] \partial_z^2 A - \\
\left[ 3(\partial_t A)^2 - (\partial_y A)^2 - (\partial_z A)^2 \right] \partial_t^2 A + \\
4 (\partial_z A \partial_t A \partial_z \partial_t A - \partial_z A \partial_y A \partial_z \partial_y A + \partial_y A \partial_t A \partial_y \partial_t A) \end{array} \right\}$$
LINEARITY OF QED $\rightarrow$ SUM OVER ALL VIRTUAL PATHS $\rightarrow$ NONLINEAR EQS. FOR THE REAL PARTICLES FIELDS

✓ DO ELECTROMAGNETIC WAVES PROPAGATE NONLINEARLY IN THE VACUUM? Yes if crossing (scattering).

✓ IS IT A PROBLEM FOR COMMUNICATIONS? Effect smaller than atmosphere.

✓ CAN WE DETECT SUCH NONLINEARITY? Need very high power to compensate $\xi = 6.7 \times 10^{-30} m^3/J$
ULTRAHIGH POWER LASER BEAMS
Peak intensities and energy density of LASER pulses

✓ CURRENT LIMIT: \( I \sim 2 \times 10^{22} \text{W cm}^{-2} \), or \( \xi \rho \sim 4 \times 10^{-12} \) (HERCULES, V. Yanovsky et al., Optics Express 16, 2109 (2008)).

✓ NEXT (European Extreme Light Infrastructure, ELI): \( I \sim 10^{25} \text{W cm}^{-2} \), or \( \xi \rho \sim 2 \times 10^{-9} \)

✓ NEXT TO NEXT: \( I \sim 10^{28} \text{W cm}^{-2} \), or \( \xi \rho \sim 2 \times 10^{-6} \)

EVEN AT NEXT TO NEXT GENERATION: $\xi \rho \lesssim 10^{-6}$

✔ EXPLAIN DIFFICULTY FOR OPTICAL OBSERVATION OF $\gamma - \gamma$ SCATTERING

HERCULES $\rightarrow$ Power per pulse $\simeq 3 \times 10^{14} \text{W}$, an order of magnitude larger than the total power used by Mankind! $(\Delta t \sim 3 \times 10^{-14} \text{s} \rightarrow \text{Energy per pulse} \sim 10 \text{J})$

ELI $\rightarrow$ pulses of $10^{18} \text{W}$, an order of magnitude larger than the total power that the earth receives from the sun. $(\Delta t \sim 10^{-18} - 10^{-15} \text{s} \rightarrow \text{Energy per pulse} \sim 1 - 10^3 \text{J})$

USES:

✓ 1) ACCELERATING CHARGED PARTICLES;

✓ 2) ULTRASHORT TIME MICROSCOPY;

✓ 3) EXPLORE THE NONLINEARITY OF THE VACUUM (search of $\gamma - \gamma$ scattering).
PREVIOUS PROPOSALS (Reference: M. Marklund and P.K. Shukla, Rev. Mod. Phys. 78, 591 (2006))


✓ Three beams scattering: Adler, 1971; Moulin and Berndard, 1999; Lundstrom et al., 2006.


Figure 2: The search for the magnetic birefringence of the vacuum (R.Baier and P.Breitenlohner, 1967; S.L.Adler 1971; Z and I Bialynicka-Birula, 1970; E.Iacopini and E.Zavattini, 1979) used to set THE CURRENT LIMIT on $\sigma_{\gamma-\gamma} \lesssim 10^{-60} m^2$, for optical wavelengths, or $\xi_{\text{exp}} < 3.1 \times 10^{-26} m^3/J$, 4.6 $\times$ 10$^3$ times higher than the QED value! M.Bregant et al., PVLAS, arXiv:0805.3036 (20 May 2008).
Figure 3: Numerical solution, $A_{\text{num}}/A$, for two scattering waves, for $\xi \bar{\rho} = 0.0025$. ($\tau \equiv \omega t$ and $\zeta \equiv kz$.) (D.T., A.Ferrando, H.Michinel, M.Seco, Phys. Rev. A 77, 042101 (2008))
Figure 4: Zero-time plot of the numerical solution, as a function of the adimensional space coordinate $\zeta \equiv k z$. 
Figure 5: Detail of the zero-time functions $A_{\text{num}}/A$ (upper curve) and $A_{\text{lin}}/A$ (lower curve), for values of the adimensional space coordinate $\zeta \equiv kz$ close to the second zero.
Figure 6: Relative error \( \frac{A_{\text{num}} - A_{\text{lin}}}{A} \) of the linear approximation, as a function of the adimensional time and space coordinates \( \tau \equiv \omega t \) and \( \zeta \equiv k z \).
VARIATIONAL METHOD
It gives an ANALYTICAL approximation, valid also for LARGER DISTANCES and applicable to NON-SYMMETRIC configurations.

AN \( \times \)-POLARIZED SOLUTION OF THE LINEAR MAXWELL EQUATIONS:

\[
A = \frac{A}{2} \left[ \cos(kz - \omega t) + \cos(kz + \omega t) \right] = A \cos(kz) \cos(\omega t),
\]

ANSATZ FOR AN (APPROXIMATE) SOLUTION OF THE NON-LINEAR EQUATIONS:

\[
A = A [\alpha(z) \cos(kz) + \beta(z) \sin(kz)] \cos(\omega t).
\]
Minimize:

\[ \Gamma = \int_{-\infty}^{\infty} dz \left( \frac{\omega}{2\pi} \int_{0}^{2\pi/\omega} dt \mathcal{L} \right) \]

(with additional average over fast variation in \( z \))

APPROXIMATE RESULT OF THE MINIMIZATION:

\[ A = A_0 \cos(\omega t) \cos[(k + \chi)z], \]

where

\[ \chi = 2\xi \rho k. \]

RESULTING PHASE SHIFT:

\[ \Delta \phi = \chi z. \]
Figure 7: Relative error \((A_{\text{num}} - A_{\text{var}})/A\) of the variational approximation, as a function of the adimensional time and space coordinates \(\tau \equiv \omega t\) and \(\zeta \equiv kz\).
GENERALIZATION: ’low’ power wave scattering with high power wave

\[ A = A \cos(kz + \omega t + \phi) + \alpha(z) \cos(kz - \omega t) + \beta(z) \sin(kz - \omega t) \]
Approximate Variational solution:
\[ \alpha(z) = \alpha_0 \cos(\eta z) \text{ and } \beta(z) = -\alpha_0 \sin(\eta z). \]
Valid for any \( \alpha_0 \), provided that \( |\alpha_0| \ll |A| \).

As a result, the low power beam becomes
\[ A_l(t, z) = \alpha_0 \cos[(k + \eta)z - \omega t], \]

Therefore
\[ \Delta \Phi = \eta \Delta z \simeq 4\xi \rho k \Delta z. \]
PROPOSAL OF EXPERIMENT AT ELI
(first stage=TOMORROW):

\[ \lambda = 800\text{nm}, \ I = 10^{29}\text{Wm}^{-2}, \ \tau = 10\text{fs}, \ d \approx 10\mu\text{m}. \]

Use three beams (can be two ‘normal´ beams that are in phase between each other and a contrapropagating High Power Laser beam). Thus \( \Delta \Phi \approx 2 \times 10^{-7}\text{rad} \).

\( \Delta \phi \) as small as \( 10^{-7}\text{rad} \) can be measured (Kang et al., 1997) comparing with the reference beam which propagated alone (no effect of QED vacuum).
PROPOSAL OF EXPERIMENT AT HERCULES (=TODAY):

\[ \lambda = 810\text{nm}, \ I = 2 \times 10^{26} \text{Wm}^{-2}, \ \tau = 30\text{fs}, \ d \approx 0.8\mu m \]

Same configuration as above \[ \longrightarrow \ \Delta \Phi \approx 10^{-9} \] (from QED),
two order of magnitude smaller than the possibility of detection.

HOWEVER, THIS WILL ALLOW FOR AN IMPROVEMENT OF THE LIMIT on \( \gamma - \gamma \) SCATTERING, i.e. on \( \xi_{\text{exp}} \), DOWN TO THE RANGE \( 10^{-27}m^3/J \), AN ORDER OF MAGNITUDE BETTER THAN (PVLAS) CURRENT LIMIT.
CONCLUSIONS

✓ Crossing electromagnetic waves in vacuum get a phase shift from QED $\gamma - \gamma$ scattering

✓ At HERCULES, our proposed experiment can improve the limit on $\gamma - \gamma$ scattering at least by an order of magnitude

✓ At ELI the QED phase shift will be detectable, possibly providing the first evidence of $\gamma - \gamma$ scattering