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Nonlinear dynamics of mirror instability.

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Nonlinear dynamics of mirror instability

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For small $\underline{p}$ (taking only the linear terms) we have

(if $k_z, k_y \neq 0$)

$$\frac{p}{k_z} = \sqrt{\frac{T_\parallel}{M}} \quad \frac{\dfrac{8\pi n_0 T_\perp}{H_0^{\,2}}\left(\dfrac{T_\perp}{T_\parallel} - 1\right) - \left(1 + \dfrac{8\pi n_0 T_\perp}{H_0^{\,2}} - \dfrac{8\pi n_0 T_\parallel}{H_0^{\,2}}\right)\dfrac{k_z^{\,2}}{k_y^{\,2}} - 1}{\sqrt{(\tfrac{1}{2}\pi)}\; 8\pi n_0 T_\perp^{\,2}/H_0^{\,2} T_\parallel} \, .$$

Accordingly, the criterion of instability ($p > 0$) is

$$\frac{8\pi n_0 T_\perp}{H_0^{\,2}} \; \frac{T_\perp}{T_\parallel} > 1 + \frac{8\pi n_0 T_\perp}{H_0^{\,2}} \, .$$

In the limit $H_0 \longrightarrow 0$, the condition (8) becomes $T_\perp > T_\parallel$.

If $k_y = 0$, the terms linear in p disappear from equation (5), terms give

$$\frac{p^2}{k_y^{\,2}} = - \frac{H_0^{\,2}}{4\pi n_0 M} \left\{ 1 + \frac{8\pi n_0 T_\perp}{H_0^{\,2}} - \frac{8\pi n_0 T_\parallel}{H_0^{\,2}} \right\} .$$

Equation (9) corresponds to the Alfvén magnetohydrodynamic branch.

MI linear growth rate for the arbitrary distribution function

Field line bending

FLR effect

Inverse response

$$\gamma_L = \frac{mp_\perp \left|k_\parallel\right|}{\pi^2 D}\left[K - \frac{k_\parallel^2}{\beta_\perp k_\perp^2}\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right) - \frac{3}{2\beta_\perp}k_\perp^2 \rho_i^2\right]$$

Here

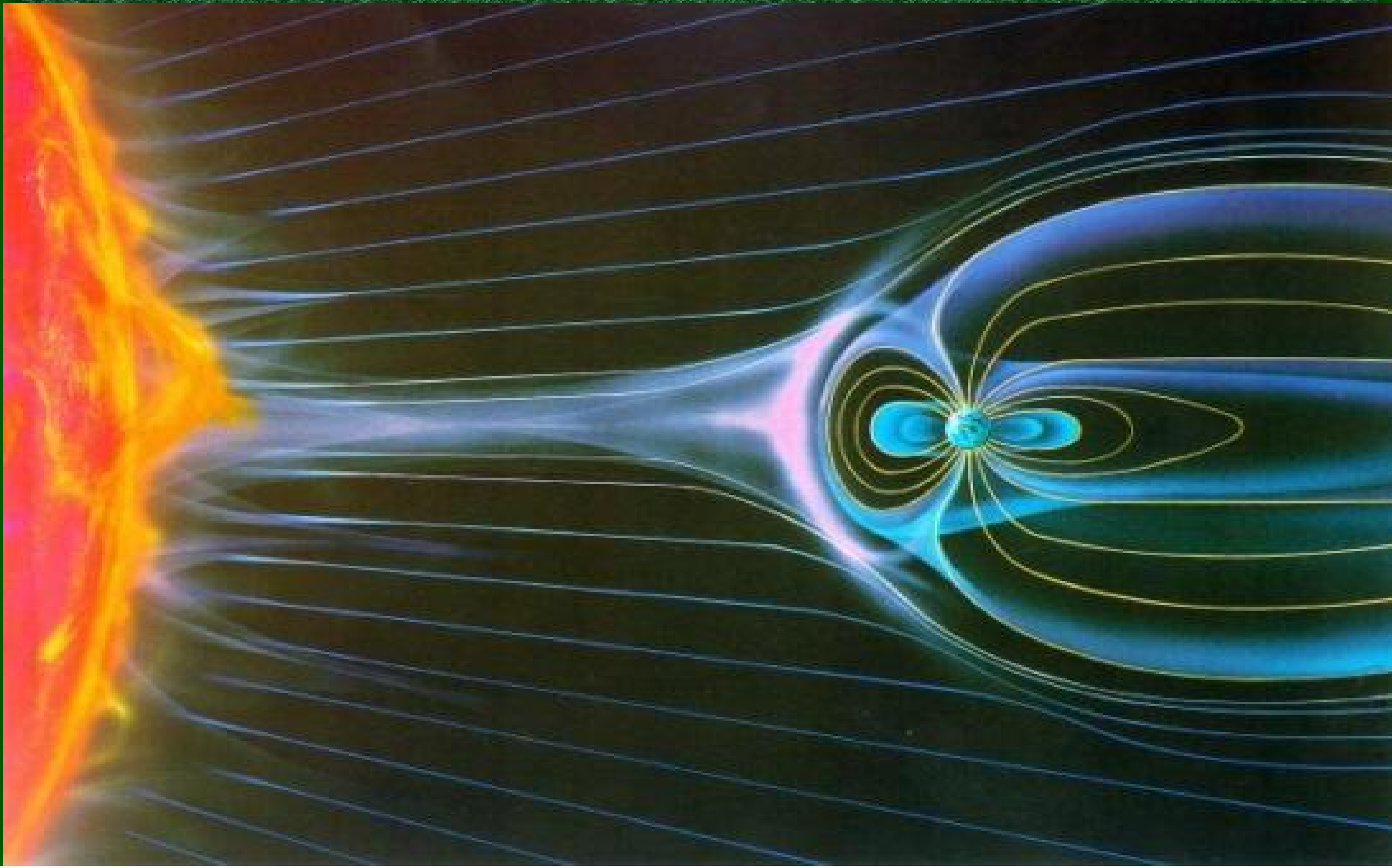
K=A-$\beta_\perp^{-1}$ and D=$-\int W^2 \frac{\partial F_{res}}{\partial W} dW \propto -\int \frac{v_\perp^4}{v_\parallel}\frac{\partial F_{res}}{\partial v_\parallel} d\mathbf{v}$

Instabilty threshold

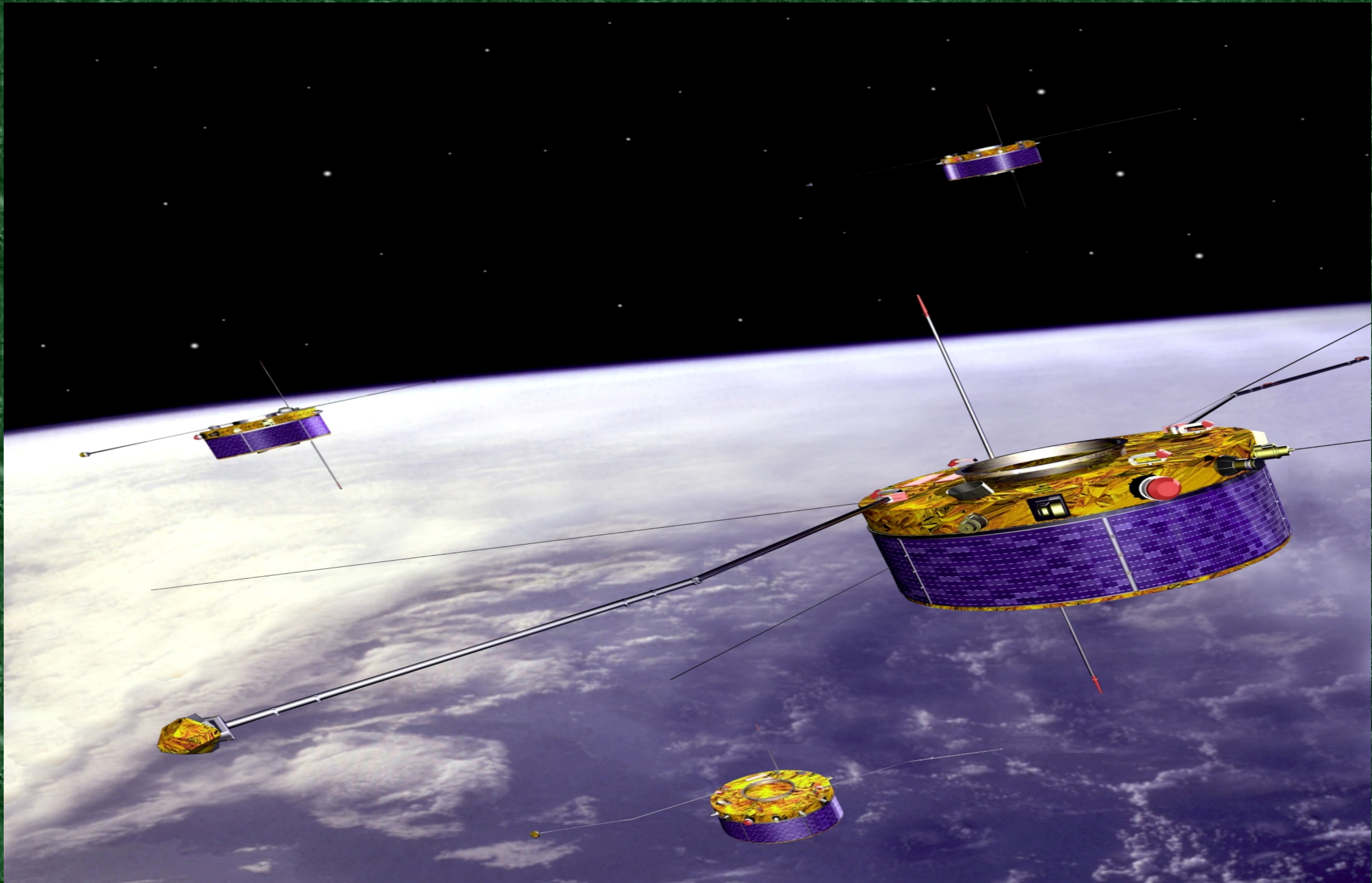
Mirror waves are found in:

- Ring-current plasma
- Earth’s magnetosheath
- Planetary magnetosheaths
- Cometary comas
- Wake of Io
- Solar wind
- Laboratory plasmas (teta-pinches)

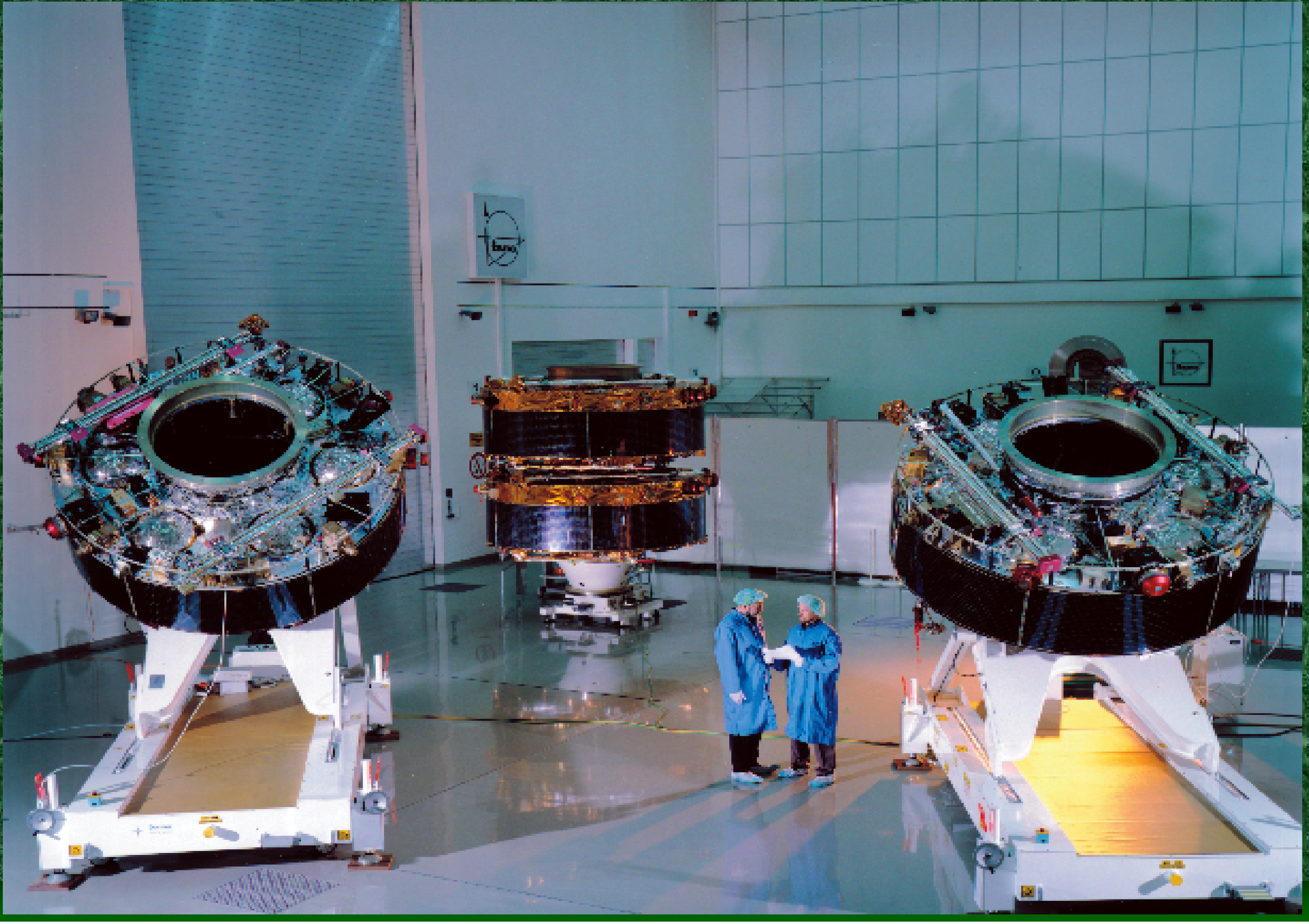
Solar-Terrestrial Interactions



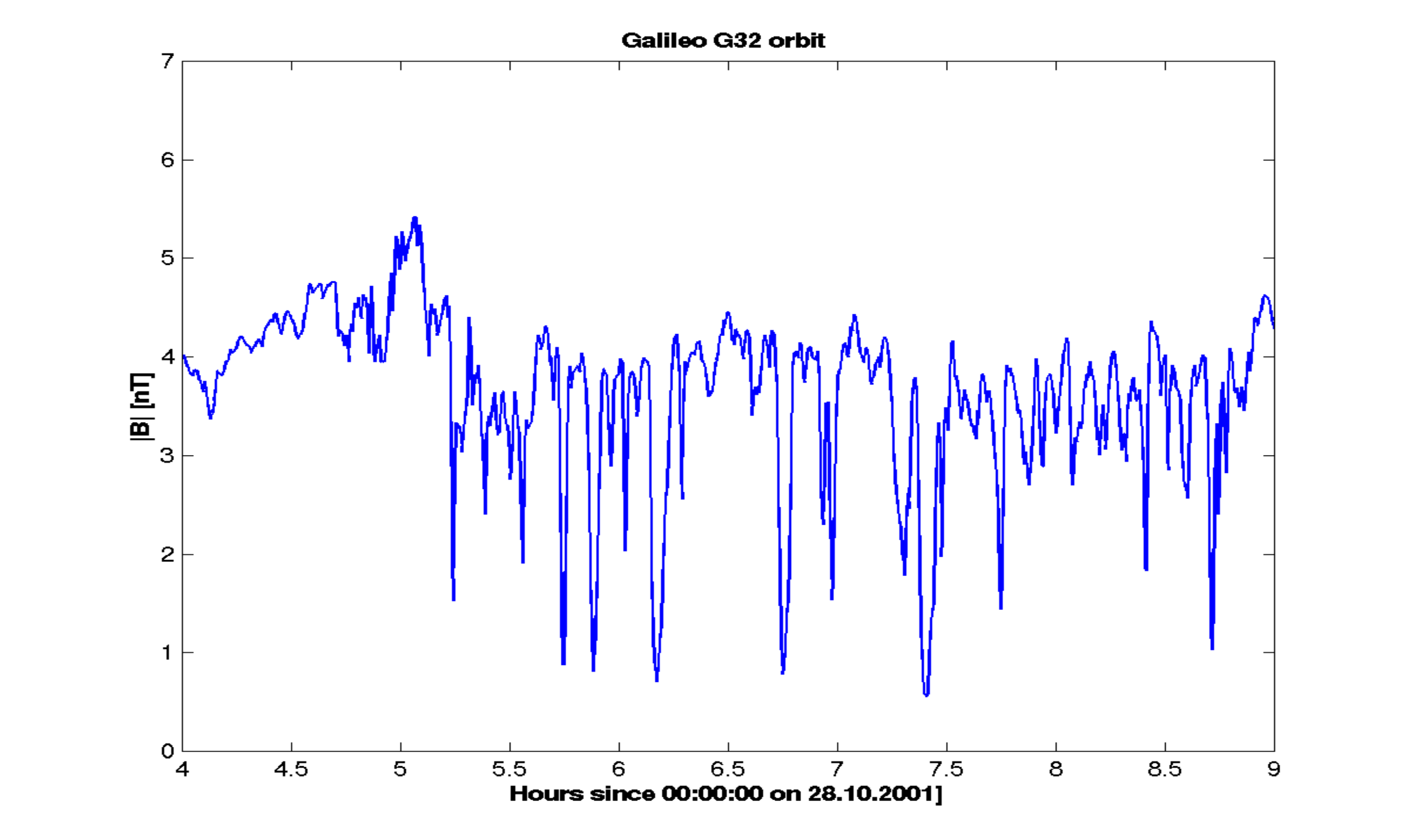
CLUSTER MISSION



Cluster satellites

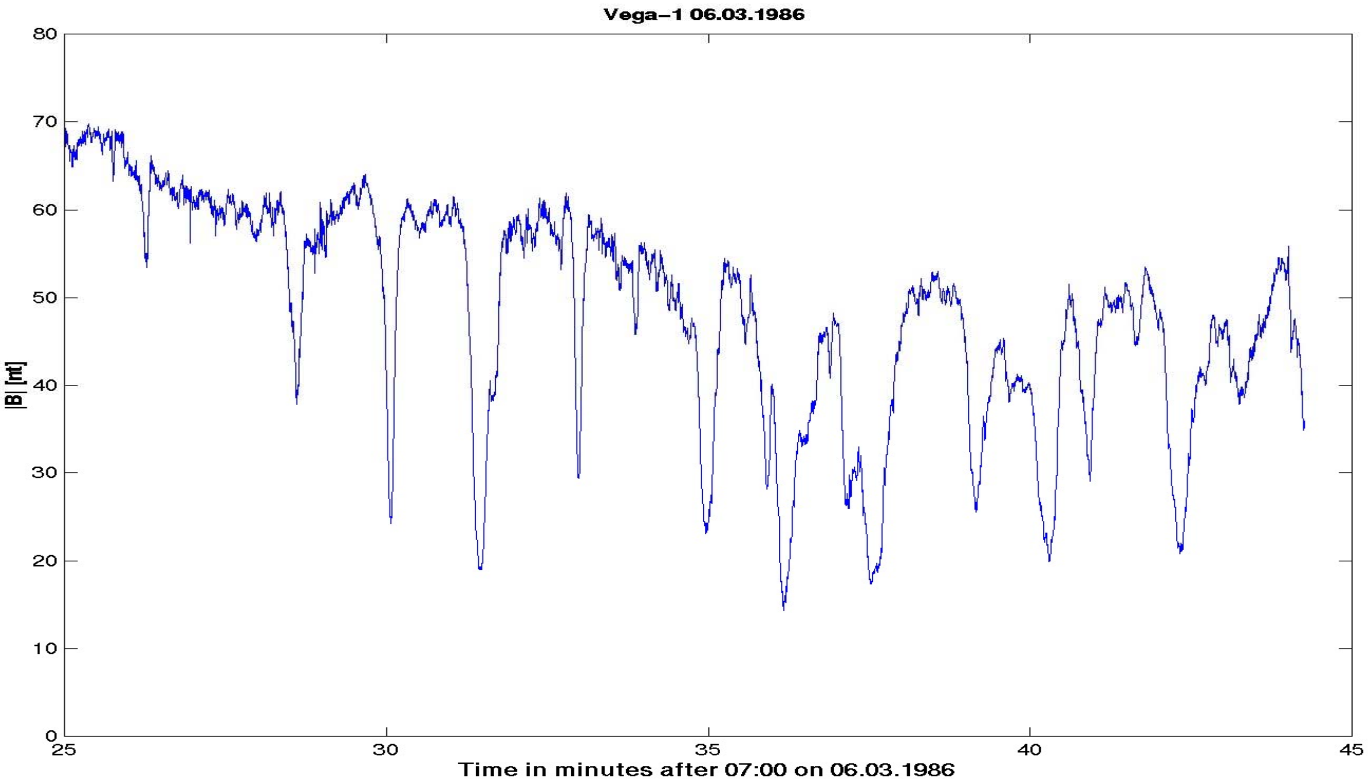


Galileo

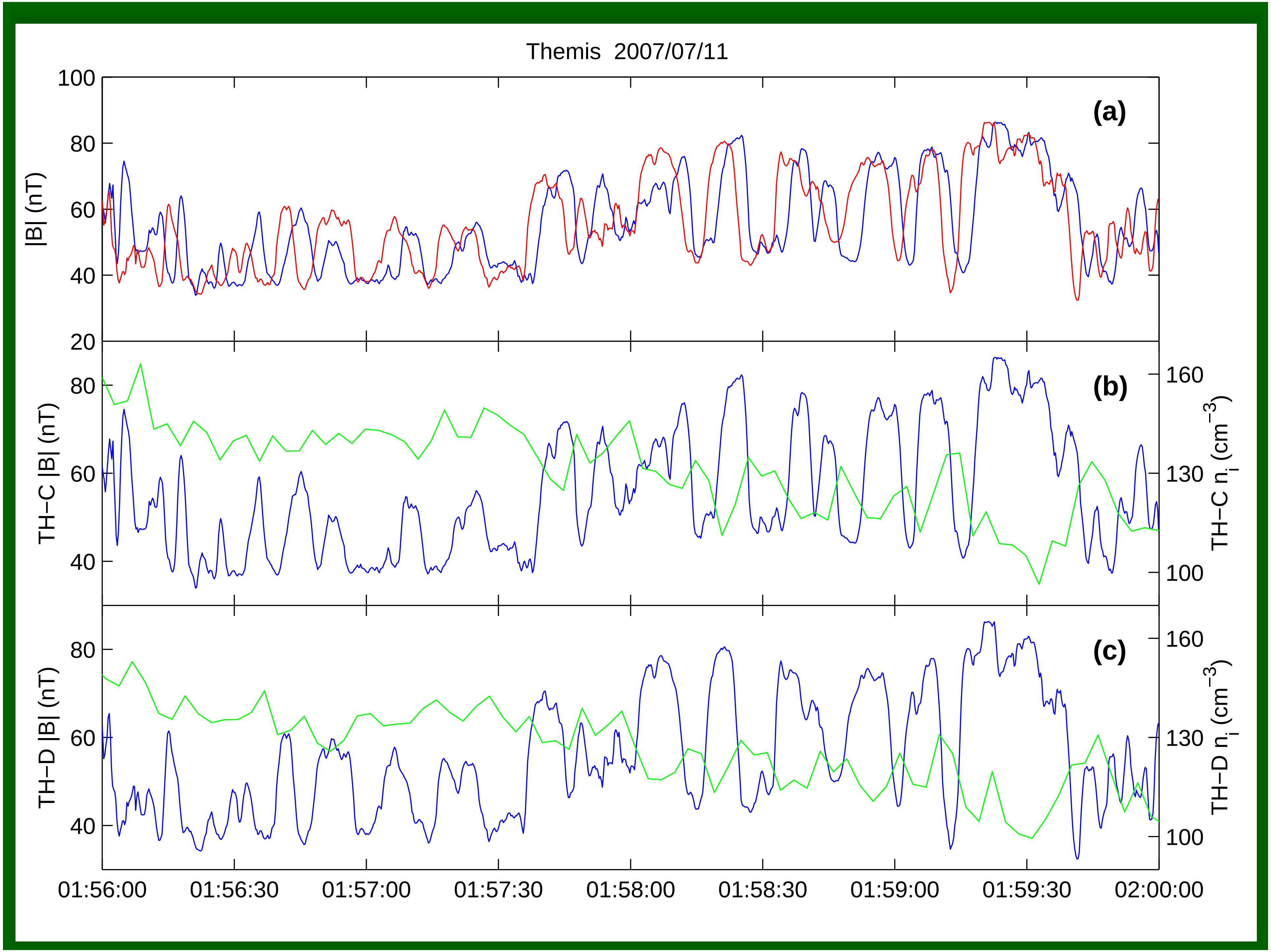


Galileo G32 orbit
|B| [nT]
0
1
2
3
4
5
6
7
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
Hours since 00:00:00 on 28.10.2001]

Vega



Vega-1 06.03.1986
|B| [nt]
0
10
20
30
40
50
60
70
80
25
30
35
40
45
Time in minutes after 07:00 on 06.03.1986



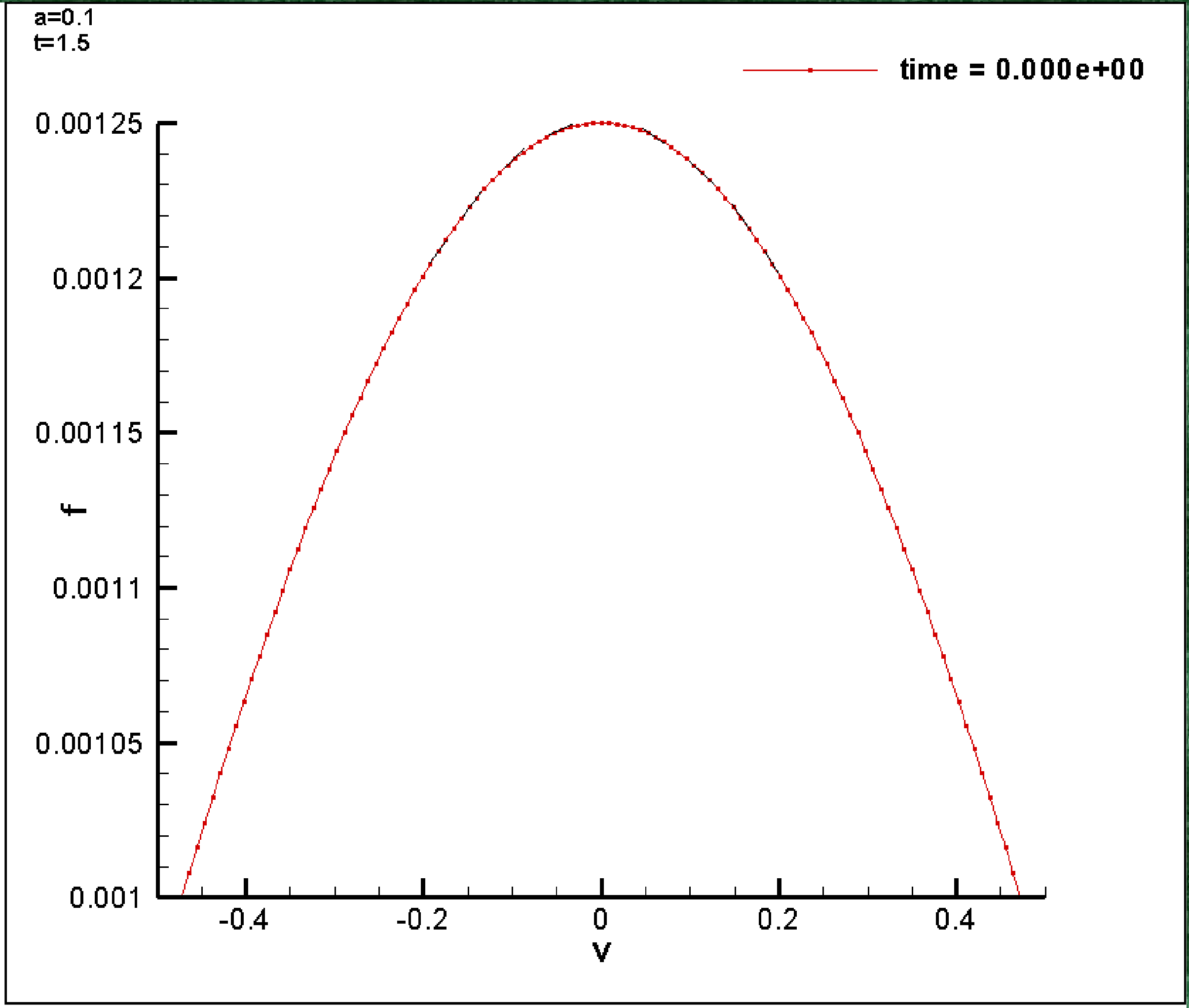
Themis 2007/07/11
(a)
(b)
(c)
|B| (nT)
TH−C |B| (nT)
TH−D |B| (nT)
TH−C n_i (cm^{-3})
TH−D n_i (cm^{-3})
100
80
60
40
20
160
130
100
01:56:00
01:56:30
01:57:00
01:57:30
01:58:00
01:58:30
01:59:00
01:59:30
02:00:00

Perpendicular plasma pressure balance

$$\delta p_\perp + \frac{B_0 \delta B_z}{\mu_0} - \frac{3}{2}\rho_i^2 \nabla_\perp^2 \frac{B_0 \delta B_z}{\mu_0} = -\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right) \nabla_\perp^{-2} \frac{\partial^2}{\partial z^2} \frac{B_0 \delta B_z}{\mu_0}$$

$$\delta p_\perp = \delta p_\perp^{res} + \delta p_\perp^{ad} + \delta p_\perp^{hd}$$

Flattening of the velocity distribution function



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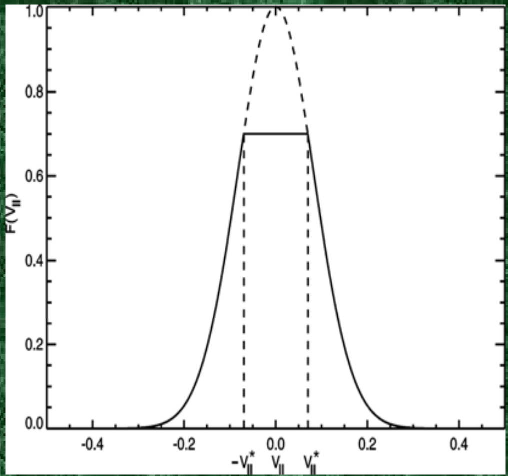
NL effects associated with trapped particles

$$\delta p_{\perp}^{res} = -\pi m \frac{\partial b}{\partial t}\int_0^{\infty} v_{\perp}^5 dv_{\perp} \lim_{\nu\to 0}\int_{v_{\parallel}^*}^{\infty} d v_{\parallel} \frac{\nu}{\nu^2 + k_{\parallel}^2 v_{\parallel}^2}\frac{\partial F}{v_{\parallel}\partial v_{\parallel}}$$

where $v_{\parallel}^* = v_{\perp} \mid b \mid^{1/2}$ and F is

$$\mathrm{F} = \frac{n}{\pi^{3/2} v_{T_{\perp}}^2 v_{T_{\parallel}}} e^{-\frac{v_{\perp}^2}{v_{T_{\perp}}^2} - \frac{v_{\parallel}^2}{v_{T_{\perp}}^2}}$$



When $v_{\parallel}^* << v_{T_{\parallel}}$, i.e. $|b| << 1$

$$\delta p_{\perp}^{res} = \frac{2\, nm}{\pi^{1/2} \mid k_{\parallel} \mid v_{T_{\perp}}^2 v_{T_{\parallel}}^3}\frac{\partial b}{\partial t}\int_0^{\infty} v_{\perp}^5 e^{-\frac{v_{\perp}^2}{v_{T_{\parallel}}^2}} dv_{\perp} \lim_{\nu\to 0}\left(\frac{\pi}{2} - \arctan\frac{v_{\perp} \mid k_{\parallel} \| b \mid^{1/2}}{\nu}\right)$$

when amplitude is small one has the standard linear (Landau) response

$$\delta p_{\perp}^{res} = p_{\perp}\frac{2\pi^{1/2}}{\mid k_{\parallel} \mid v_{T_{\parallel}}}\frac{T_{\perp}}{T_{\parallel}}\frac{\partial b}{\partial t}$$

when $\mid k_{\parallel} \mid v_{\parallel}^* >> \nu \qquad \delta p_{\perp}^{res} \to 0$

Adiabatic part of the pressure peturbation

$$\delta p_{\perp}^{ad} = \frac{3p_{\perp}}{k_{\parallel}^{2} v_{T_{\parallel}}^{2}} \left(\frac{T_{\perp}}{T_{\parallel}} \right)^{1/2} \frac{1}{|b|^{1/2}} \frac{\partial^{2} b}{\partial t^{2}}$$

Contrary to the linear limit
in the NL regime the expansion
parameter $\omega / k_{\parallel} v_{T_{\parallel}}$ is now replaced
by $\omega / k_{\parallel} v_{\parallel}^{*} = \omega / k_{\parallel} v_{T_{\perp}} |b|^{1/2}$

Quasi-Hydrodynamic part

$$\delta p_{\perp}^{hd} = -2Ap_{\perp}b + \frac{15}{8}Ap_{\perp}\left(\frac{T_{\perp}}{T_{\parallel}}\right)^{1/2}|b|^{1/2}b$$

Mirror force

Effect of flattening of the ion distribution function

NL MI dispersion relation near the saturated state

Near saturate state $\delta p_{\perp}^{res} \rightarrow 0$ and

$$[\beta_{\perp} A - 1 - \frac{k_{\parallel}^2}{k_{\perp}^2}(1 + \frac{\beta_{\perp} - \beta_{\parallel}}{2}) - \frac{3}{2} k_{\perp}^2 \rho_i^2$$

$$- \frac{15}{8}\left(\frac{T_{\perp}}{T_{\parallel}}\right)^{1/2} |b|^{1/2}]b - \frac{(3/2)}{k_{\parallel}^2 v_{T\parallel}^2}\left(\frac{T_{\perp}}{T_{\parallel}}\right)^{1/2} \frac{\beta_{\perp}}{|b|^{1/2}} \frac{\partial^2 b}{\partial t^2} = 0$$

The model equation (in the dimensionless form)

$$\frac{\partial^2}{\partial \xi^2}\left(1+\frac{\partial^2}{\partial \xi^2}-|h|^{1/2}\right)h+\frac{\partial^2 h}{\partial \tau^2}=0$$

NL growth rate

$$\gamma_{NL} = \frac{2^{1/2}\left|b\right|^{1/4}\left|k_{\|}\right| v_{T_{\|}}}{3^{1/2}\beta_{\perp}^{1/2}}\left(\frac{T_{\|}}{T_{\perp}}\right)^{1/4}[\beta_{\perp}A - 1 - \frac{3}{2}k_{\perp}^{2}\rho_{i}^{2}$$

$$-(1+\frac{\beta_{\perp}-\beta_{\|}}{2}) - \frac{15}{8}\left(\frac{T_{\perp}}{T_{\|}}\right)^{1/2}\left|b\right|^{1/2}]^{1/2}$$

Maximum growth rate

The maximum growth is attained at

$$k_\perp^2 \rho_i^2 = \frac{2}{9}\left(\beta_\perp A - 1 - \frac{15}{8}\left(\frac{T_\perp}{T_\parallel}\right)^{1/2} |b|^{1/2}\right)$$

$$k_\parallel^2 \rho_i^2 = \frac{2}{27}\frac{\beta_\perp}{(1+\frac{\beta_\perp - \beta_\parallel}{2})}\left(\beta_\perp A - 1 - \frac{15}{8}\left(\frac{T_\perp}{T_\parallel}\right)^{1/2} |b|^{1/2}\right)^2$$

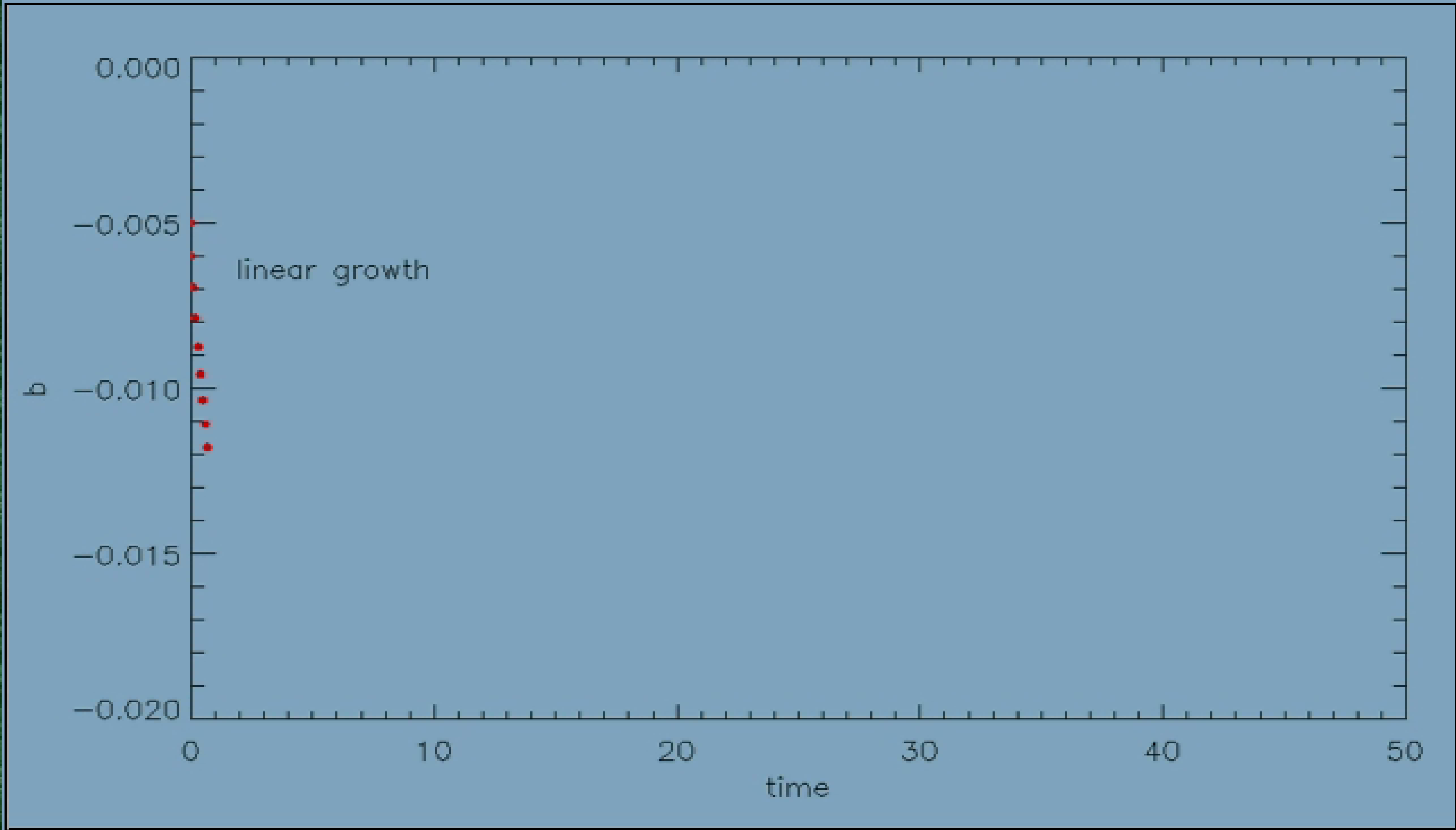
$$\frac{\gamma_{NL}}{\omega_{ci}} = \frac{|b|^{1/4}}{9\sqrt{3}\beta_\perp^{1/2}(1+\frac{\beta_\perp - \beta_\parallel}{2})^{1/2}}\left(\frac{T_\parallel}{T_\perp}\right)^{3/4}\left(\beta_\perp A - 1 - \frac{15}{8}\left(\frac{T_\perp}{T_\parallel}\right)^{1/2} |b|^{1/2}\right)^{3/2}$$

General case

$$\frac{1}{|b|^{1/2}}\frac{\partial^2 b}{\partial t^2}+\left(1-\frac{2}{\pi}\arctan\frac{|b|^{1/2}}{\lambda}\right)\frac{\partial b}{\partial t}=\left(K-|b|^{1/2}\right)b$$

MI growth rate

(K=0.1, h=-0.005)



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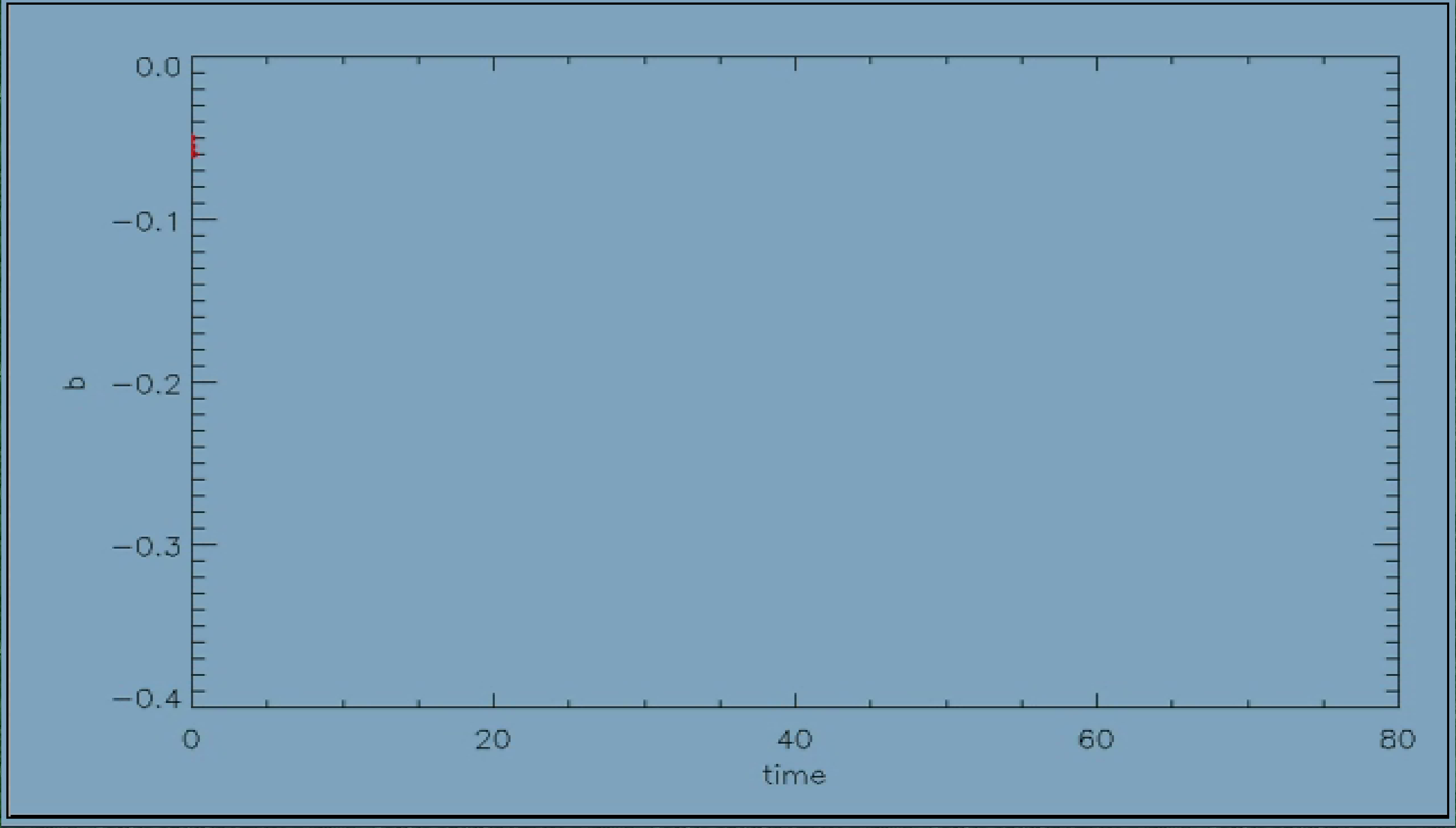
MI growth rate
(K=0.5, h=0.05)



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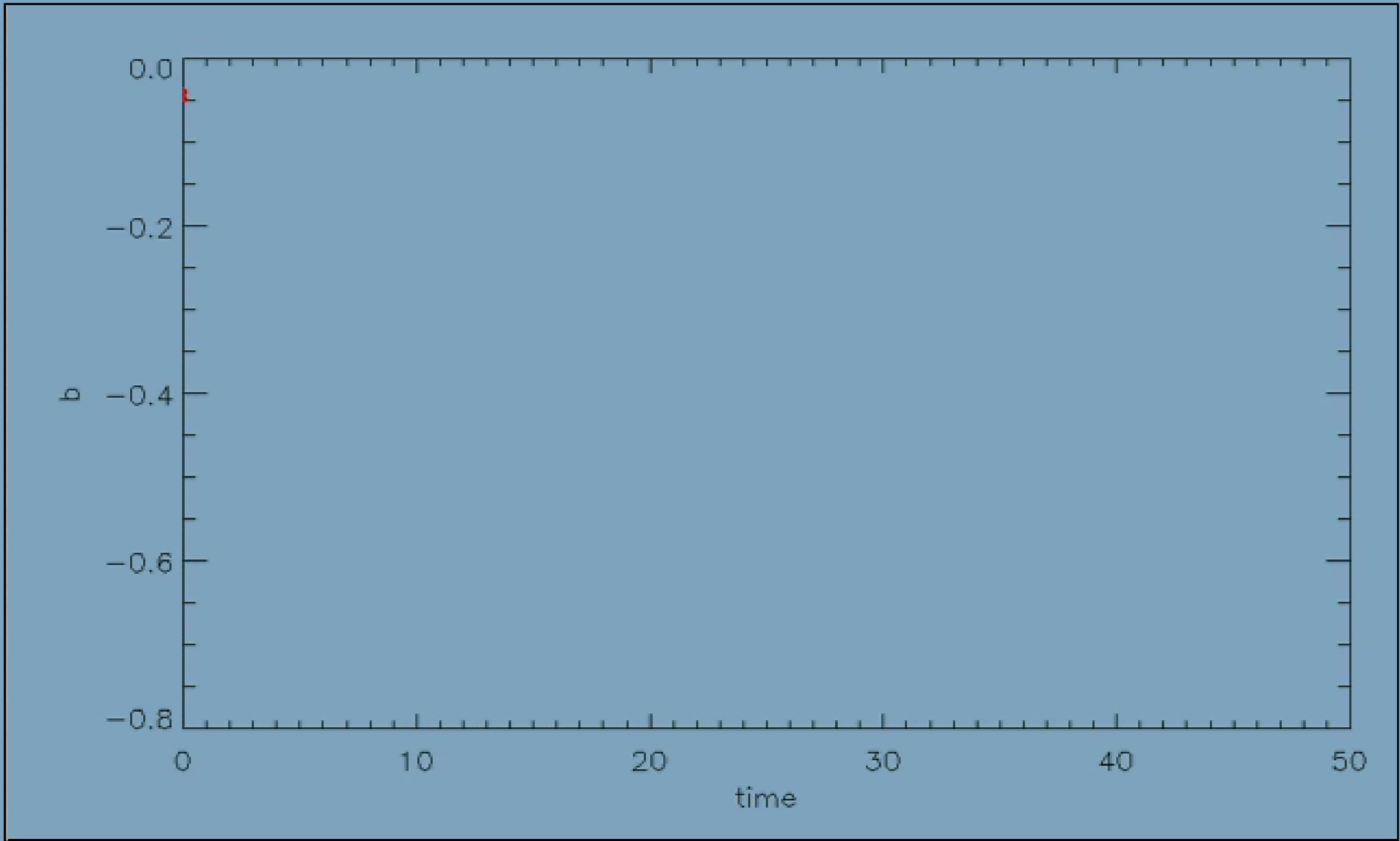
MI growth rate
(K=0.8, h=-0.05)



movie

Conclusions

- The main nonlinear mechanism responsible for mirror instability saturation is associated with modification (flattening) of the shape of the background ion distribution function in the region of small parallel particle velocities.
- The NL mode coupling effects are neglected here as corrections of the higher order of smallness.
- In the NL regime the MI dispersion relation becomes of the second order in frequency. One of its roots is found to be growing.
- In the course of the NL saturation the mirror mode spatial scales are cascading into the large spatial scales.

Thank you!