Collective effects and self-consistent energetic particle dynamics in burning plasmas

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International Centre for Theoretical Physics, Trieste, 13 Oct. - 7 Nov. 2003

*Acknowledgments: S. Briguglio, L. Chen, G. Fogaccia, G. Vlad
Outline

- Historic Background.
- Overview of shear Alfvén Spectra.
- Eigenmodes vs. Resonant Modes.
- Nonlinear Dynamics Aspects.
- 3D Hybrid MHD-Gyrokinetic Simulations.
- Transition from weak to strong energetic particle transport: Avalanches.
- Discussions.
Historic Background

- Possible detrimental effect of Shear Alfvén (SA) instabilities on energetic ions recognized theoretically before experimental evidence was clear.

  Rosenbluth and Rutherford, PRL 34, 1428, (1975)
  ⇒ resonant wave particle interaction of ≈ MeV ions with SA inst. due to $v_E \approx v_A$ ($k_\parallel v_A \approx \omega_E$)

- Experimental observation of fishbones on PDX – McGuire et al., PRL 50, 891, (1983) – fast $\perp$ injected ion losses . . .
  . . . and by theoretical explanation of internal kink excitation –
  Coppi, Porcelli, PRL 57, 2272, (1986)
Existence of gaps in the SA continuous spectrum (due to lattice symmetry breaking) \( \omega \approx \frac{v_A}{2qR} \) – Kieras and Tataronis, J. Pl. Phy. 28, 395, (1982)


Possible excitations of TAE by energetic particles …


Experimental evidence …
Wong et al., PRL 66, 1874, (1991)
Heidbrink et al, PRL 71, 855, (1993) \( \Rightarrow \) BAE \( \omega \approx \omega_{ti} \approx \omega_{*pi} \)


Overview of shear Alfvén spectra

Energetic Particle Modes (EPM): forced oscillations

\[ \text{TAE} - \text{KTAE} \Rightarrow \text{Transition to EPM} \]

Energetic Particle Modes (EPM): forced oscillations

Beta induced Alfvén Eigenmodes (BAE)
Kinetic Ballooning Modes (KBM)

\[ \text{KBM} \oplus \text{BAE} \Rightarrow \text{Alfvén ITG (AITG)} \]

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**Eigenmodes vs. Resonant Modes**

- Fundamental difference in mode dynamics and particle transports is to be attributed to mode excitation and particle phase space motion.
- Use Secular Perturbation Theory in nonlinear Hamiltonian dynamics ...
- Extended Phase Space to treat explicit time dependencies: $2N \Rightarrow (2N+2)$-dim.; for low frequency modes ($\omega \ll \omega_{ci}$) the resulting 8-dim phase space reduces ($\mu$ and $\mathcal{H} \equiv H(\mu, P_\phi, J_\parallel) - H$ are conserved) to 4-dim phase space, i.e. the general problem is equivalent to an autonomous Hamiltonian with 2 degrees of freedom.
- Use Secular Perturbation Theory is a method for locally removing a single resonance: what happens in the multiple resonance case ??

$$H = H_0(J) + \epsilon H_1(J, \theta)$$

$$\omega_1 = \frac{\partial H_0}{\partial J_1}, \quad \omega_2 = \frac{\partial H_0}{\partial J_2}, \quad \frac{\omega_2}{\omega_1} = \frac{h}{k}, \quad h, k \in \mathbb{Z}$$

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Canonical transformation with generating function \( F_2 = (h\theta_1 - k\theta_2)\hat{J}_1 + \theta_2\hat{J}_2 \)

\[
J_1 = \frac{\partial F_2}{\partial \theta_1} = h\hat{J}_1 \quad J_2 = \frac{\partial F_2}{\partial \theta_2} = \hat{J}_2 - k\hat{J}_1
\]

\[
\hat{\theta}_1 = \frac{\partial F_2}{\partial \hat{J}_1} = h\theta_1 - k\theta_2 \quad \hat{\theta}_2 = \frac{\partial F_2}{\partial \hat{J}_2} = \theta_2
\]

After averaging on \( \hat{\theta}_2 \) (near resonance)

\[
\bar{H} = \bar{H}_0(\hat{J}) + \epsilon\bar{H}_1(\hat{J}, \hat{\theta}_1) = \bar{H}_0(\hat{J}_0) + \Delta\bar{H}
\]
Standard Hamiltonian

\[ \Delta \bar{H} \simeq \frac{1}{2} F(\Delta \hat{J}_1)^2 - G \cos \theta_1 \]

\[ F = \frac{\partial^2 \bar{H}_0}{\partial \hat{J}^2_1} \]

\[ G \cos \theta_1 \simeq -\epsilon \bar{H}_1 \]
Further complications: mode frequency often sweeps: fast vs. slow sweeping
Qualitative description in terms of frequency sweeping,

\[
\frac{\dot{\omega}}{\omega} \sim \gamma_L \ll \omega_B^2/\omega; \quad \text{adiabatic (TAE)}
\]

\[
\frac{\dot{\omega}}{\omega} \sim \gamma_L \gg \omega_B^2/\omega; \quad \text{fast (EPM)}
\]

Adiabatic (TAE) case: quasilinear flattening is dominant and, in the absence of externally imposed adiabatic frequency chirping (e.g., via equilibrium changes), saturation is either at \( \omega_B \approx \gamma_L \) or it occurs via other mode-mode coupling mechanisms.

Fast (EPM) case: there no time for the distribution to flatten and the mode should freely grow \( \Rightarrow \) particle convection/mode particle pumping ???

Saturation should occur at \( \omega_B \approx (\omega \gamma_L)^{1/2} \); consistent with F. Zonca and L. Chen BAPS 43, 1921, (1999).
(Weak) Modes in the Toroidal gap: linear

- High-$n$ modes $n \gtrsim \epsilon a/\rho_{LE}$ are most unstable
- Kinetic $(n \gg 1)$ analyses refer to simple and/or model equilibria
  - TAE modes: weakly damped. But equilibrium effects (e.g., finite-$\beta$ enhance continuum damping

- Low shear (core region) is characterized by modes localized in a single gap

- Are there possible tokamak operation regimes free of TAE modes??? (in the case of model equilibria yes: $\alpha = -R_0 q^2 \beta' > \alpha_{crit}$)
(Weak) Modes in the GAP: nonlinear

- NL Saturation via mode-mode coupling (also Thyagaraja et al., Proc. EPS-97, vol 1, p 277, 1997):
  - Saturation via “ion Compton scattering” at $\delta B_r/B \approx \epsilon^2(\gamma_L/\omega)^{1/2}$ (Hahm and Chen, PRL 74, 266, (1995))
  - Saturation via $\delta E^* \times \delta B$ at $\delta B_r/B \approx \epsilon^{5/2}/nq$ (Zonca et al., PRL 74, 698, (1995))
  - Saturation via $\delta n/n$ at $\delta B_r/B \approx \epsilon^{3/2}\beta^{1/2}$ (Chen et al., PPCF 40, 1823, (1998))

- NL Saturation via phase-space nonlinearities (wave-particle trapping):
  - Steady-state: $(\delta B_r/B)^{1/2} \approx \omega_B \approx \gamma_L(\nu_{eff}/\gamma_d)$ for $\gamma_d \ll \nu_{eff} \sim \nu(\omega/\omega_b)^2$ (Berk, Breizman, Phys. Fluids B 2, 2246, (1990))
• TAE pulsations: \((\delta B_r/B)^{1/2} \approx \omega_B \approx \gamma_L\) for \(\gamma_d \gg \nu_{eff} \sim \nu(\omega/\gamma_L)^2\) (Berk et al, PRL 68, 3563, (1992))

• Both steady-state and TAE pulsations yield negligible losses unless phase-space stochasticity is reached, possibly via domino effect (Berk et al, Nuc. Fus. 35, 1661, (1995))

• In the case of a single mode, spontaneous formation near threshold of hole-clump pair in phase space (Berk et al., Phys. Lett. A, 234, 213, (1997)) may yield to frequency chirping and/or pitchfork splitting of mode-frequency

• Theory seems to explain pitchfork splitting of TAE lines observed in JET (Fasoli, IAEA-TCM-97); however, \(\delta \omega \sim \gamma_L(\gamma_d/\gamma_L)^{1/2}(\gamma_L/\nu_{eff})^{3/2}\); thus, large chirping requires very small \(\nu_{eff}\).
Pitchfork splitting of TAE in JET

Energetic Particle Modes (EPM)


- KTAE are only moderately unstable

- If strongly driven, KTAE merge into an EPM (forced oscillation ⇔ resonant excitation) Zonca, Chen, Phys. Plasmas 3, 323, (1996)
• TAE to EPM transition as $\alpha_{\text{core}} = - R_0 q^2 \beta'$ increases (Santoro, Chen, Phys. Plasmas 3, 2349, (1996))

$$\omega_A = \frac{v_A}{q R_0}$$

$1$ TAE
$$\omega = \frac{v_A}{2 q R_0}$$

$1$ EPM
$$\omega = k_{\|} v_A$$

$\alpha_{\text{core}} = - R_0 q^2 \beta'$
ICRF Experiments on TFTR

- EPM and TAE excitations, by ICRF induced fast minority ion tails on TFTR. From Bernabei et al., Phys. Plasmas 6,1880, (1999).
□ Energetic Particle Modes*, appear to be excited in the plasma core, due to strong energy source due to ICRF tail ions

□ Energetic Particle Modes*, are forced oscillations at $\omega \simeq \bar{\omega}_{dE}$. They chirp downward in frequency because $\bar{\omega}_{dE} = k_\theta \rho_{LE} v_{thE}/R_0$ (cf. later).

□ TAE Modes, are eventually excited at the plasma edge, due to ICRF tail ions transported outward by EPM.

□ Fast Ion Losses, seem associated with TAE’s, and appear to be related to EPM frequency chirping.

ICRF Experiments on JET


Alfvén Cascades in reversed-q equilibria (advanced tokamak)
EPMs are excited at different radial locations

- Strong resonant excitation of EPMs should occur at $r/a \approx 0.2$, where $\alpha_H$ is maximum. L. Chen, Phys. Plasmas 1, 1519, (1994).


Energetic Particle Modes, appear to be excited in the plasma core, due to strong energy source due to ICRF tail ions

Energetic Particle Modes, are resonant at $\omega \simeq \bar{\omega}_dE$, $\omega \simeq \bar{\omega}_dE + \ell \omega_{BE}$. $\Rightarrow$ frequency chirping (preferentially downward) (cf. later).

Alfvén Cascades, alone cause only weak (local) particle redistributions

Alfvén Cascades in combination with EPMs, are eventually excited at $q - min$, due to ICRF tail ions transported outward by EPM.

NNBI Experiments on JT-60U

- EPM and TAE excitations, by negative neutral beam injection induced fast minority ion tails on JT-60U. From Shinohara et al., Nucl. Fusion 41, 603, (2001).
Losses due to Abrupt Large amplitude Events (ALE), are consistent with resonant mode excitation. From Shinohara et al., Nucl. Fusion 41, 603, (2001).
3D Hybrid MHD-GK simulation of EPM

EPM: Energetic Particle Transports

\[ \frac{\Delta N}{N} > 0.7_a \]

\( n = 4 \)

\( n = 1 \)

\[ \frac{n_f}{n_i} \]

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EPM: Threshold depends on $n$

$\frac{L_{PE}}{R_0} = 0.07$

Minimum $\beta^{th}_{E_0} = 0.7\%$ at $n=8$
3D Hybrid MHD-GK simulations: JET rev. \( q^1 \)

- Confirm expectation of resonant EPM excitation at the radial position of max drive
- EPM gap modes are also excited near \( q_{min} \) with weaker growth rate
- Depending on \( \beta_H \), rapid non-linear fast particle transport may or may not establish a link between modes excited at different radial locations

Isotropic Maxwellian Distribution
Circular Shifted Flux Surfaces
Const. Fast Ion Temperature Profile
Peaking Factor \( \beta_{H0}/\langle \beta_H \rangle = 9.2 \)
\( v_H/v_A = 1.0, \rho_{LH}/a = 0.02, n = 4 \)

\(^1\)F. Zonca, et. al. Paper TH/4-4. 19.th IAEA FEC, Lyon, France,(2002)

\[ \beta_H/\beta_{H0} \]

\[ v_H/v_A = 1.0, \rho_{LH}/a = 0.02, n = 4 \]

\[ \beta_H/\beta_{H0} q \]

\[ a) b) \]
Autumn College on Plasma Physics

Weak Drive, $\beta_{H0} = 0.01$, and hollow $q$ profile (a)

![Graphs and diagrams showing plasma physics data.](image)
Moderate Drive, $\beta_{H0} = 0.025$, and hollow $q$ profile (a)
Fast ion Transport...

\[ \frac{\tau}{\tau_0} = 88.80 \] \( \frac{\tau}{\tau_0} = 204.00 \)

\( \beta_{H0} = 0.05 \), hollow \( q \)-profile

\( \beta_{H0} = 0.025 \), weakly hollow \( q \)-profile
Energetic Ion Avalanches and EPMs

Vlad et. al, IAEA-TCM, San Diego, Oct. 6-8 (2003); Zonca et. al, ibid.

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Relay-runner model for NL EPM dynamics
Propagation of the unstable front

\[ r_{\max} \frac{d(r_n)}{dr} \]

Linear phase, Convective phase, Diffusive phase


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Conclusions from EPM simulations with 3D Hybrid MHD-Gyrokinetic code

- Significant particle redistributions take place above linear excitation threshold of EPM: Transition from weak (diffusive) to strong (ballistic) energetic ion transport in avalanches at values that are relevant for burning plasmas

- Nonlinear saturation is due to (convective) displacement of the source (rather than wave-particle trapping)

- At present: single $n$ (but multiple $m$) simulations $\Rightarrow$ generalize to multiple $n$ !!!!

- Long time scale phase space dynamics (Fokker-Planck collision operator) missing $\Rightarrow$ difficult to generalize to $10^4 - 10^5$ Alfvén times
Discussions

- **Linear Theory**: sound and well understood. However, most codes still do not include nonperturbative particle dynamics

- **Nonlinear Theory**: Partially understood

  - Theory of Non-linear phase space dynamics (single mode) seems to explain a number of experimentally observed phenomena: saturation levels, pitch-fork splitting of spectral lines, chirping . . . (possibly)

  - NL GK-MHD simulations of EPM’s indicate saturation via source redistribution rather than $\omega_b \approx \gamma_L$; fast ion radial convection

  - What happens in the multiple $(m, n)$ case?? . . . and for a strong source???

  - Chirping is a very complex phenomenon, observed in most tokamaks with intense hot particle tails: due to equilibrium variations?? and/or Nonlinear
dynamics???

- Prediction and interpretation of particle losses is still lacking: domino effect (phase space stochasticity) ... and/or mode-particle pumping (particle convection)???

- Nonlinear Hamiltonian Dynamics: Strong mathematical methods exist ... but what about the plasma physicist original sin, ... solving the self-consistent problem???

- Experimental investigations: Understanding local transport, using ... high power density sources seems the key for a crucial progress and physics insights (... similar to thermal plasma transport problem ...)

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