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Role of resonant vs non-resonant wave-particle interactions in electromagnetic turbulence

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*Acknowledgments: S. Briguglio, L. Chen, G. Fogaccia, G. Vlad, R. White

Outline

- Numerical simulations challenge: what is the relationship between fast ion induced collective effects and turbulent transport ? – if there's any!
- Using asymptotic techniques for space-time scale separation: initial value radial envelope problem
- Extension to non-linear problems
- Applications:
  - Drift and Drift Alfvén turbulence (zonal flow)
  - Energetic Particle Modes (zonal flow/changes)
- Discussions

3D Hybrid MHD-GK simulation of EPM

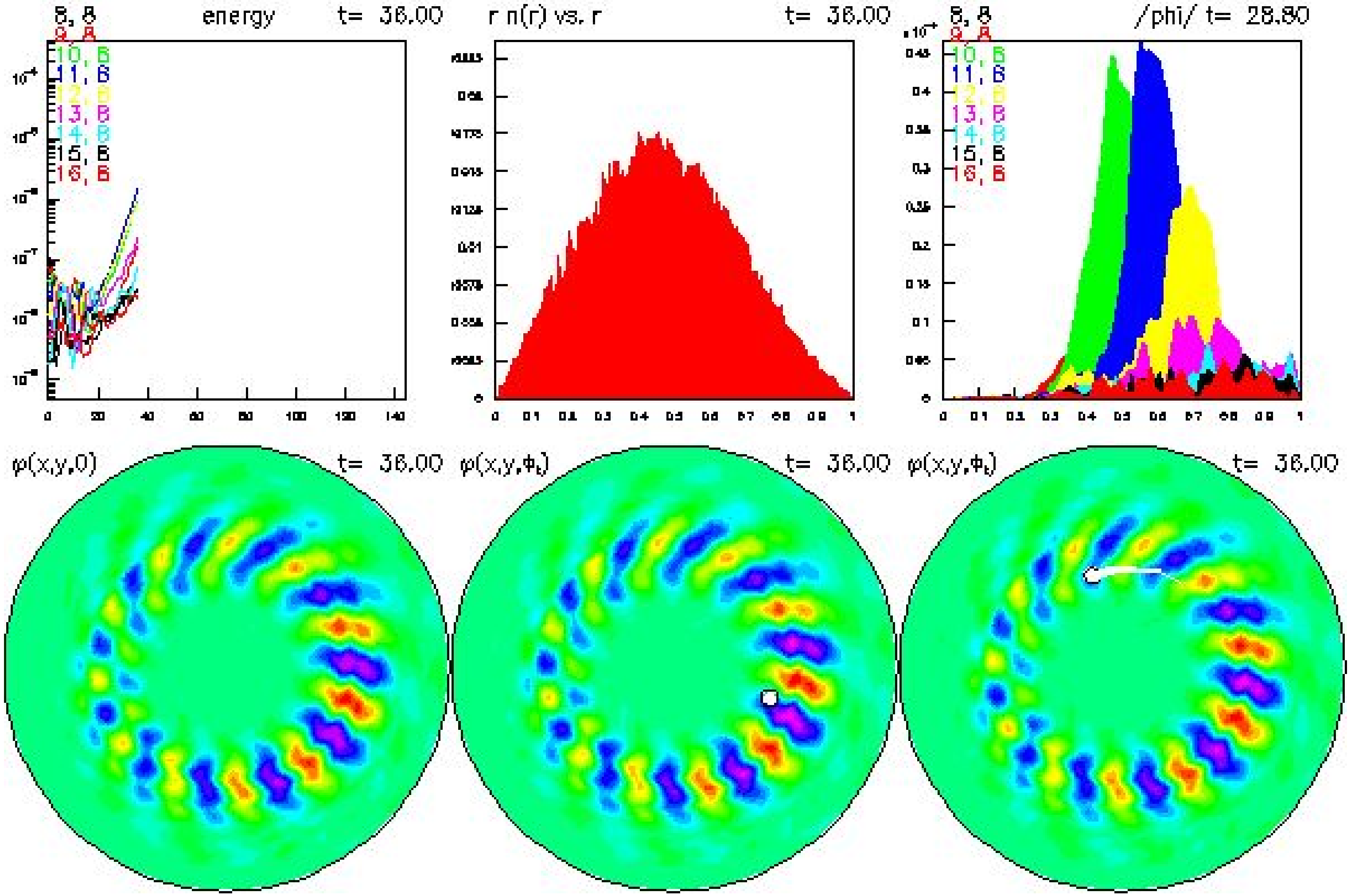
Briguglio et al., PoP **2**, 3711 , (1995); and PoP **5**, 3287, (1998)

- Use of a *nonperturbative* 3D Hybrid MHD Gyrokinetic code confirms previous findings that strong radial redistributions in the energetic particle source take place when the EPM excitation threshold is exceeded, yielding potentially large particle losses and, eventually, mode saturation

- Such a threshold may occur at experimentally accessible values of β_E, *e.g.*, as low as $\beta_{E0}^{th} = 0.75\%$ (on axis value) for $n = 8$ EPM excitation by Maxwellian energetic ions with $\rho_{LE}/a = 0.01$ and a pressure profile, $\beta_E = \beta_{E0} \exp(-r^2/L_{pE}^2)$, with $L_{pE}/R_0 \simeq 0.075$ and $a/R_0 = 0.1$.

- With the same parameters and profiles, new simulation results of an $n = 8$ EPM indicate that evident radial fragmentation of the EPM coherent eddies $(k_\theta = k_\parallel = 0,\ k_r \neq 0)$ is present and it is visible both in the contour-plots and in the radial variation of the various poloidal harmonics in which the eigenmode is decomposed. This fragmentation, meanwhile, is associated with a diffusive transport of fast ions, as it may be inferred from modifications in the fast particle density profile.

- The diffusive nature of both particle and energy fluxes associated with fast ions is confirmed by analytical studies, which yield explicit expressions for fast ion transports [see later].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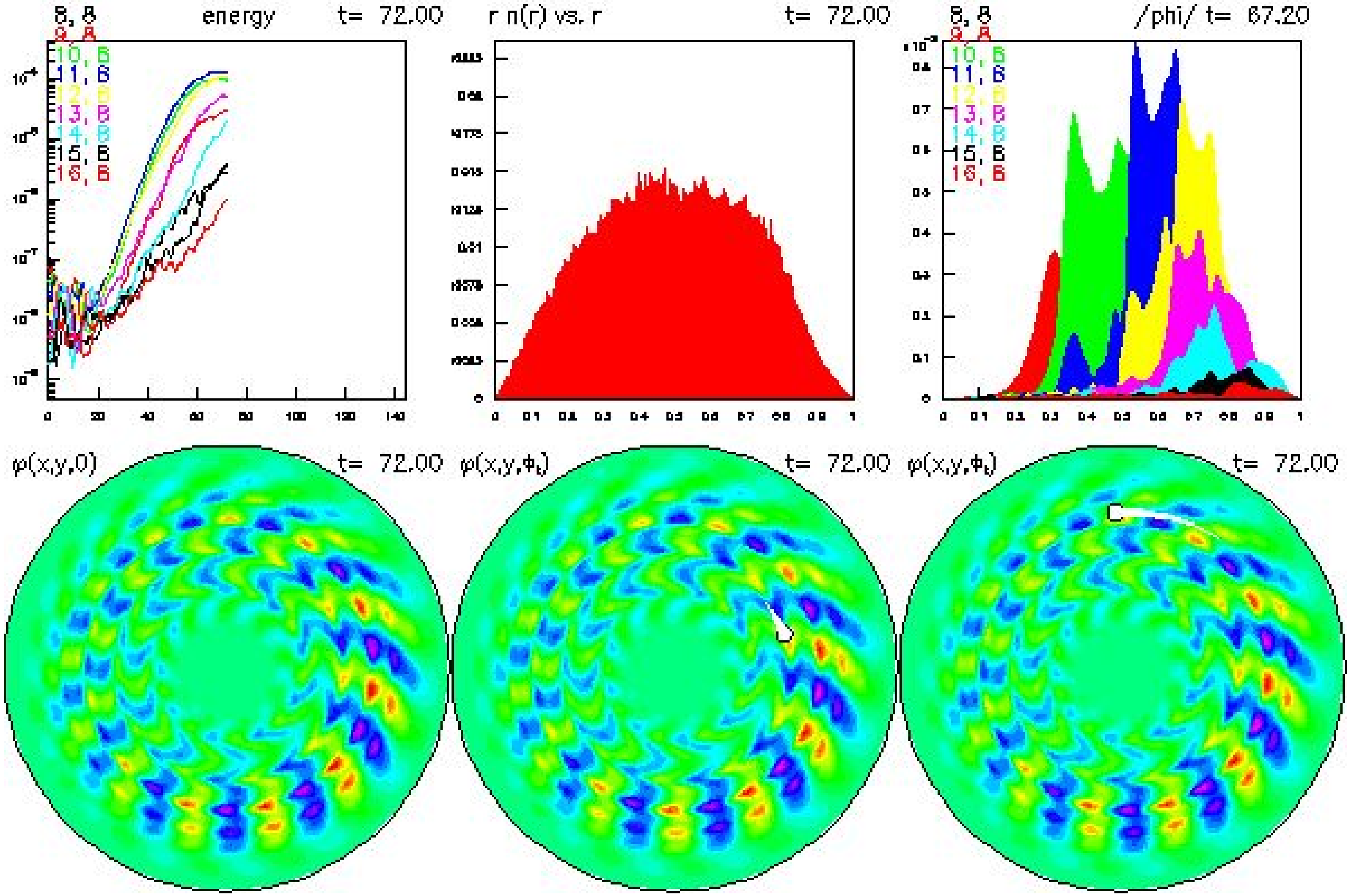
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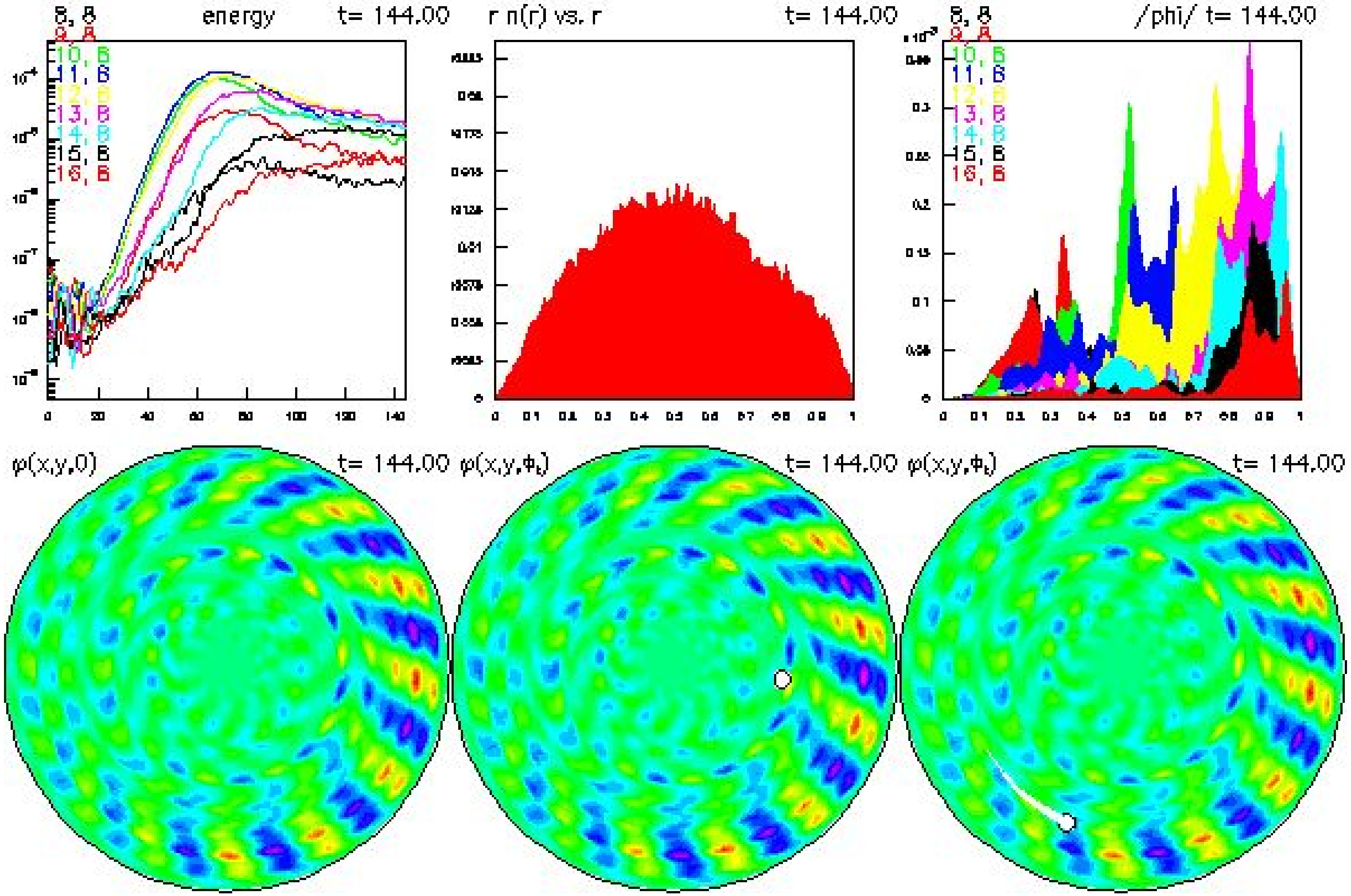

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φ(x,y,0)
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Modulational Instability of DAW - EPM

- 3D Hybrid MHD-Gyrokinetic simulations show evidence of radial fragmentation of a single coherent toroidal mode ⇒ analogy with modulational instability of a single toroidal drift wave (Chen, Lin, White, 1999; PoP **7**, 3129, (2000)
- Radial fragmentation: excitation of low frequency axisymmetric perturbation ⇒ NL mechanism is necessary
- Generalization of NL theory of zonal flows to e.m. fluctuation (ref. also Das, Diamond *et al.* 2000; Smoliakov *et al.* 2000) allows us:
  - to explore implication of zonal flows/currents generation by waves of the shear Alfvén branch
  - to discuss relationship between plasma turbulence and fast particle driven modes (Chen *et al.* IAEA Sorrento, paper TH4/5; Chen *et al.*, Nucl. Fus. **41**, 747, (2001); Zonca *et al.*, Varenna 2000)

Using asymptotic techniques based on scale separation

- Fourier decomposition of scalar potential fluctuations:

$$\delta\phi = \mathrm{e}^{in\xi} \sum_m \mathrm{e}^{\mathrm{i}(nq-m)\chi} \delta\phi_m(r,t)$$

- Fourier harmonics $\delta\phi_m(r,t)$ have two scale structures:
  - $\approx (nq')^{-1}$ due to $-1 \lesssim k_\parallel qR = (nq-m) \lesssim 1$
  - $\approx \epsilon L_p \ll L_p$ due to equilibrium variation; $\epsilon \approx n^{-2/3}\,;\ n^{-1/2}\,;\ L_p/R$
- Fast radial scale $x \equiv (nq/r)(r-r_0)$ formally treated in Fourier space: χ is the dual variable of x w.r.t. Fourier Transform:
  - $k_\parallel qR = (nq-m) = \mathrm{i}\partial_\chi = sx \quad , \quad s \equiv rq'/q$

- Multiple scale structure of Fourier harmonics:

$$\delta\phi_m(r,t) = \underbrace{A(r,t)}_{\text{envelope}} \underbrace{\int_{-\infty}^{\infty} e^{-ix\kappa}\delta\Phi(\kappa,r,t)d\kappa}_{\text{parallel mode structure}}$$

$$= \exp i \int nq'\theta_k dr \int_{-\infty}^{\infty} e^{-ix\kappa}\delta\Phi(\kappa,r,t)d\kappa$$

$$\theta_k = -i\frac{1}{nq'}\frac{\partial}{\partial r}$$

- Mapping (r,χ) into (r,κ):

$$\partial_\chi \Rightarrow s\partial_\kappa \quad , \quad F(\chi) \Rightarrow F(\kappa/s) \quad , \quad -\frac{i}{nq'}\frac{\partial}{\partial r} = -\frac{i}{s}\frac{\partial}{\partial x} \Rightarrow \theta_k - \frac{\kappa}{s}$$

- Eikonal Ansatz for the radial envelope make it possible to solve the 2D problem of plasma wave propagation in the form of two nested 1D wave equations: provided

$$\left|\frac{nq'\theta_k'}{(nq'\theta_k)^2}\right| \ll 1$$

$$\begin{aligned}
\text{2D ODE} \qquad & L(\partial_t, \partial_r, \partial_\chi; r, \chi)\delta\phi &=& \; 0 \\
& \overbrace{\text{symmetric}} & \Downarrow & \\
\text{1D ODE} \qquad & \mathcal{L}(\partial_t, \partial_\kappa, \theta_k; r, \kappa)A(r,t)\delta\Phi(\kappa, r, t) &=& \; 0 \\
& \underbrace{\overbrace{\text{symmetric}}} & \Downarrow & \\
& \int_{-\infty}^{\infty} d\kappa\delta\Phi(\kappa, r, t)\mathcal{L}(\partial_t, \partial_\kappa, \theta_k; r, \kappa)A(r,t)\delta\Phi(\kappa, r, t) &=& \; 0 \\
& & \Downarrow & \\
\text{1D }\Psi\text{DE} \qquad & D(\partial_t, \theta_k; r)A(r,t) &=& \; 0
\end{aligned}$$

Initial value radial envelope problem

- Time scale separation: assume that wave structures are characterized by slow time variations around a given frequency ω; *i.e.* $\partial_t = -\mathrm{i}\omega + \partial_t$, with $|\omega^{-1}\partial_t| \ll 1$.

- All relevant spatial scales of the radial envelope are shorter than equilibrium scales. This implies that the functional form of the local dispersion function $D(r, \omega, \theta_k)$ allows us to reconstruct the radial wave operator using $\theta_k \Rightarrow (-\mathrm{i}/nq')\partial_r$ with ∂_r acting on $A(r,t)$ only.

- For a given linear dispersion function $D(r, \omega, \theta_k)$, $\theta_k \equiv (-i/nq')\partial/\partial r$, the linear propagator generates a ΨDE.

$$D\left(\omega + \mathrm{i}\partial_t, (-\mathrm{i}/nq')\partial_r; r\right) A(r,t) = 0$$

- The ΨDE is locally solvable via eikonal approach. This possibility is guaranteed by separation of space-time scales.

- This generates equations for the slow space-time evolution of the radial envelope. Initial value problem.

$$\left\{ \overbrace{\omega^{-1}\partial_t - \frac{\gamma}{\omega}}^{\text{drive/damping}} - \underbrace{\frac{\xi}{nq'\theta_k}\partial_r}_{\text{group vel.}} + \overbrace{i(\lambda+\xi)}^{\text{potential well}} + \underbrace{\mathrm{i}\frac{\lambda}{(nq'\theta_k)^2}\partial_r^2}_{\text{(de)focusing}} \right\} A(r,t) = 0$$

$$\lambda = \left(\frac{\theta_k^2}{2}\right)\frac{\partial^2 D_R/\partial\theta_k^2}{\omega\partial D_R/\partial\omega} \; ; \; \xi = \frac{\theta_k(\partial D_R/\partial\theta_k) - \theta_k^2(\partial^2 D_R/\partial\theta_k^2)}{\omega\partial D_R/\partial\omega} \; ,$$

$$\gamma = \frac{-D_I}{\partial D_R/\partial\omega} \; ; \; \theta_k \text{ solution of } D_R(r,\omega,\theta_k) = 0 \; .$$

Extension to Non-Linear Problems

- Non-linear interactions naturally enter on a time scale which is comparable with the inverse linear growth rate. Assume $|\gamma_L/\omega| \ll 1$.
- Parallel mode structure will be essentially unaltered. Intimately connected only with linear wave dispersive properties.
- Non-linear dynamics enters in the initial value radial envelope problem.

$$\left\{ \omega^{-1}\partial_t - \frac{\gamma}{\omega} - \frac{\xi}{nq'\theta_k}\partial_r + i(\lambda + \xi) + \mathrm{i}\frac{\lambda}{(nq'\theta_k)^2}\partial_r^2 \right\} A(r,t) = \text{Nonlinear Terms}$$

- Non-linear Terms hierarchy: dominant contribution from zonal response(L. Chen *et al.*, PoP2000; F. Zonca *et al.*, Varenna 2000):

  - Tokamak turbulent transport: Zonal Flows $(c/B)\mathbf{b} \times \nabla\delta\phi_z(r)$
  - Fast Ion collective effects: Zonal Distribution Function $\delta F_z(r)$: wave-particle resonances

- NL coupling scheme: Chen *et al.* PoP2000

$$\delta\phi_{\mathrm{DAW-EPM}} = \delta\phi_0 + \delta\phi_+ + \delta\phi_- + c.c.$$

$$\delta\phi_0(\mathrm{pump\ DAW-EPM}) = e^{i\int n\theta_k dq + in\varphi}\sum_m e^{-im\vartheta}\delta\phi_m + c.c.$$

$$\delta\phi_\pm(\mathrm{sidebands}) = \begin{pmatrix} e^{i\int n\theta_k dq} \\ e^{-i\int n\theta_k^* dq} \end{pmatrix} e^{\pm in\varphi + i\int K_z dr}\sum_m e^{\mp im\vartheta}\delta\phi_m^{(\pm)} + c.c.$$

$$\delta\phi_z(\mathrm{zon.\ flow}) = e^{i\int K_z dr}\delta\phi_z + c.c.$$

- Therefore: (and similarly for vec. potential)

$$\delta\phi_+ \Leftrightarrow \delta\phi_0\delta\phi_z$$
$$\delta\phi_- \Leftrightarrow \delta\phi_0^*\delta\phi_z$$

□ Generalization to e.m. fluctuations: Chen *et al.* NF2001

- Strong zonal current screening due to thermal electrons

$$\frac{4\pi}{c}\delta j_{\parallel e,z} \simeq -\frac{\delta A_{\parallel,z}}{\lambda_e^2} \qquad \lambda_e = \frac{c}{\omega_{pe}}$$

- Neglecting terms $\approx k_\perp^2 \lambda_e^2$, NL equation for $\delta A_{\parallel,z}$ is

$$\delta j_{\parallel i,z} + \delta j_{\parallel e,z} \simeq \delta j_{\parallel e,z} \simeq 0$$

- $\delta A_{\parallel,z}$ can be considered a passive field since

$$\frac{\omega}{k_\parallel c}\frac{\delta A_{\parallel,z}}{\delta \phi_z} \approx k_\perp^2 \lambda_e^2 \ll 1$$

- E.M. equation for zonal fields: ... ref. Rosenbluth and Hinton 1999 for the e.s. case $(\chi_i \simeq 1.6 q^2 K_z^2 \rho_{Li}^2 / \epsilon^{1/2})$

$$\frac{ne^2}{T_i}\partial_t \chi_i \delta\phi_z = -\frac{c}{B} k_\theta K_z \left[\langle e \Delta_{kk'} \rangle + \right.$$

$$\left. \frac{1}{4\pi}\left(\delta A_{\parallel k'}^{(+)} \nabla_\perp^2 \delta A_{\parallel k}^* - \delta A_{\parallel k}^* \nabla_\perp^2 A_{\parallel k'}^{(+)} - A_{\parallel k'}^{(-)} \nabla_\perp^2 \delta A_{\parallel k} + \delta A_{\parallel k} \nabla_\perp^2 A_{\parallel k'}^{(-)} \right) \right]$$

- Reynolds Stress is imbedded in $\Delta_{kk'}$; Full Finite Larmor Radius; let $\lambda_z \equiv K_z q (v_\perp^2/2 + v_\parallel^2)/(\omega_{ci} v_\parallel)$

$$\begin{aligned}\Delta_{kk'} &= \left[J_0^2(\lambda_z) J_0(\gamma_z) J_0(\gamma_k'^{(+)}) - J_0^*(\gamma_k) \right] \delta L_{k'}^{(+)} \delta \bar{H}_k^* - \left[k \leftrightarrow k'^{(+)} \right] \\ &\quad - \left[J_0^2(\lambda_z) J_0(\gamma_z) J_0^*(\gamma_k'^{(+)}) - J_0(\gamma_k) \right] \delta L_{k'}^{(-)*} \delta \bar{H}_k + \left[k \leftrightarrow k'^{(-)} \right]\end{aligned}$$

- In the large FLR (Finite Larmor Radius) Reynolds stress is $O(\gamma_k^{-1})$ w.r.t. Maxwell stress ... to be noted for ETG (el. Reynolds stress?)

- In the small FLR (Finite Larmor Radius) limit this reduces to ($\alpha_0 \equiv 1 + \delta P_{\perp \mathrm{i}0}/(ne\delta\phi_0)$); $\mathbf{b}\cdot\nabla\delta\psi_{k,k'} \equiv -(1/c)\partial_t \delta A_{\| k,k'}$

$$\partial_t \chi_{iz}\delta\phi_{\mathrm{z}} = \frac{c}{B} k_\vartheta k_{\mathrm{z}} k_{\mathrm{z}}^2 \rho_{Li}^2 \left[\left(\alpha_0 - \left| \frac{k_\| v_A}{\omega_0} \right|^2 \right) \langle\langle |\Psi_0|^2 \rangle\rangle + 2\alpha_0 \mathbb{R}\mathrm{e} \langle\langle (\Phi_0 - \Psi_0)^* \Psi_0 \rangle\rangle \right.$$
$$\left. + \alpha_0 \langle\langle |\Phi_0 - \Psi_0|^2 \rangle\rangle \right] (A_0^* A_+ - A_0 A_-)$$

- Definitions: $[\delta\phi_k, \delta\psi_k] \Rightarrow A_0[\Phi_0, \Psi_0]$; $[\delta\phi_{k'}^{(\pm)}, \delta\psi_{k'}^{(\pm)}] \Rightarrow A_\pm[\Phi_0, \Psi_0]^{(\cdot,*)}$
- Exact cancellation of zonal flows and currents for a pure shear Alfvén wave: Alfvénic state (Hasegawa 1975)
- Similarly to e.s. case, we have spontaneous excitation of zonal flow via EPM (fast particles) and DAW

- Zonal field $\delta\phi_z$ equation is closed by NL-vorticity and NL-quasineutrality for $k, k'^{(\pm)}$ modes.
- Non-adiabatic particle response δH is obtained via NL-GKE (Frieman and Chen 1982)

$$\delta F = \frac{e}{m}\delta\phi\frac{\partial}{\partial v^2/2}F_0 + \sum_{\mathbf{k}_\perp}\exp\left(-\mathrm{i}\mathbf{k}_\perp\cdot\mathbf{v}\times\hat{\mathbf{b}}/\omega_c\right)\overline{\delta F}_k$$

$$\left(\partial_t + v_\parallel\partial_\parallel + \mathrm{i}\omega_d\right)_k\overline{\delta H}_k = \mathrm{i}\frac{e}{m}QF_0J_0(\gamma)\delta L_k - \frac{c}{B}\mathbf{b}\cdot\left(\mathbf{k}''_\perp\times\mathbf{k}'_\perp\right)J_0(\gamma')\delta L_{k'}\overline{\delta H_{k''}}$$

$$QF_0 = \omega_k\frac{\partial F_0}{\partial v^2/2} + \mathbf{k}\cdot\frac{\hat{\mathbf{b}}\times\nabla}{\omega_c}F_0 \quad ; \quad \mathbf{k}'_\perp + \mathbf{k}''_\perp = \mathbf{k}_\perp$$

$$\delta L_k = \left(\delta\phi - \frac{v_\parallel}{c}\delta A_\parallel\right)_k \quad ; \quad \gamma = k_\perp v_\perp/\omega_c$$

- NL-quasineutrality:

$$\frac{ne^2}{T_i}\left(1+\frac{T_i}{T_e}\right)\delta\phi_k = \left\langle eJ_0(\gamma)\overline{\delta H}_i\right\rangle_k - \left\langle e\overline{\delta H}_e\right\rangle_k$$

- NL-vorticity,

$$B\partial_\ell\left(-\nabla_\perp^2\frac{\partial_\ell\delta\psi}{B}\right)+\frac{\omega^2}{v_A^2}\frac{k_\perp^2}{b_i}\left[\left(1-\frac{\omega_{*ni}}{\omega}\right)(1-\Gamma_0)-\right.$$

$$\left.\frac{\omega_{*Ti}}{\omega}b_i(\Gamma_0-\Gamma_1)\right]\delta\phi = \frac{4\pi}{c^2}\sum_s\langle e\omega\omega_d J_0\delta H\rangle +$$

$$\frac{4\pi}{c^2}\partial_t\sum_s\langle e\frac{c}{B}\mathbf{b}\cdot(\mathbf{k}''_\perp\times\mathbf{k}'_\perp)(J_0(\gamma)J_0(\gamma')-J_0(\gamma''))\delta L_{k'}\delta H_{k''}\rangle +$$

$$\frac{\mathbf{b}\cdot(\mathbf{k}''_\perp\times\mathbf{k}'_\perp)}{cB}\partial_t\left(\delta A_{\parallel,k'}\nabla_\perp^2\delta A_{\parallel k''}\right)\;;\quad \delta L_k\equiv\delta\phi-\frac{v_\parallel}{c}\delta A_{\parallel k}$$

- Thermal particle nonlinearities enter both NL-quasineutrality and NL-vorticity
- In the NL-vorticity, Thermal particle nonlinearities are dominated by the formally nonlinear term

$$\frac{\text{ballooning} - \text{interchange NL}}{\text{formally NL}} \approx \frac{v_{thi}/R\omega}{k_\perp \rho_{Li}}$$

- Fast particle NLties enter via the ballooning-interchange term only: they carry pressure but not inertia

$$\left(\frac{\text{thermal}}{\text{fast}}\right)_{\text{NLties}} \approx \frac{\epsilon^{1/2}}{q^2}\frac{n_0}{n_E}\frac{L_{PE}}{R}\epsilon_0 \approx \frac{\epsilon^{1/2}}{q^2}\epsilon_0\left(\beta_E\frac{R}{L_{PE}}\right)^{-1} \approx \frac{\epsilon^{1/2}}{q^2}\epsilon_0$$

for $n_E/n_0 \approx \beta_E$ $(v_E \approx v_A)$ and $\beta_E R/L_{PE} \approx 1$; $\epsilon_0 \equiv 2(r/R_0 + \Delta')$.

- For thermal particles particles: dominant response in δH_z is due to $\delta \phi_{\rm z}$; larger by $(q^2 K_z^2 \rho_{Li}^2/\epsilon^{1/2})^{-1}$. $\delta\phi_z$ feeds back on $\delta\phi_k$

$$\delta\phi_{\rm z} \sim \epsilon_0 \frac{\epsilon^{1/2}}{q^2}\frac{c}{B}\frac{k_\theta K_z}{\omega_z}\delta\phi_k\delta\phi_{k'}$$

- Peculiarities of fast particles

  - dominant response in δH_z is due to "formally nonlinear" term; larger by $(q^2/\epsilon_0\epsilon^{1/2}) \Leftarrow$ resonant response
  - Particle NLties play a role via NL modification of $\delta H_k \sim \delta L_{k'} \delta H_z$: formally(only) analogous to quasi-linear diffusion
  - $\delta\phi_Z$ does not enter directly in NL-vorticity ant it may be viewed as evolving in a given NL-EPM background: explains why 3D Hybrid simulations are consistent!

Modulational Instability of Drift and Drift-Alfvén Waves

- Derivation of 2D PDE for the slow radial and time evolution of the pump wave $\oplus$ sideband envelopes. Follow Chen *et al.* NF 2001. NL-terms $\Rightarrow$ Generalization of the linear formalism developed previously (Zonca and Chen, PFB **5**, 3668, 1993). See also Chen *et al.* submitted to PRL 2003; Zonca *et al.* submitted to PoP 2003.

- Convenient to separate the massless response to $k_\parallel \neq 0$ induction field (Chen - Hasegawa JGR 1991)

$$\overline{\delta H}^{LIN} = -\frac{e}{m} J_0(\gamma) \frac{QF_0}{\omega} \delta\psi + \delta K$$

- The NL-quasineutrality can be cast into the form

$$\frac{ne^2}{T_i}\left\{\left(1+\frac{T_i}{T_e}\right)(\delta\phi-\delta\psi)_k+\left[\left(1-\frac{\omega_{*ni}}{\omega}\right)(1-\Gamma_0(b_i))-\frac{\omega_{*Ti}}{\omega}b_i(\Gamma_0(b_i)-\Gamma_1(b_i))\right]\right\}\delta\psi_k$$

$$-\sum_{e,i}\langle eJ_0(\gamma)\delta K\rangle_k=-\frac{\mathrm{i}}{\omega_k}\left\langle e\frac{c}{B}\mathbf{b}\cdot(\mathbf{k}''_\perp\times\mathbf{k}'_\perp)(J_0(\gamma)J_0(\gamma')-J_0(\gamma''))\delta L_{k'}\overline{\delta H}_{ik''}\right\rangle_k$$

$$-\frac{\mathrm{i}}{\omega_k}\left\langle e\frac{c}{B}\mathbf{b}\cdot(\mathbf{k}''_\perp\times\mathbf{k}'_\perp)\delta\phi_{k'}\overline{\delta H}_{ek''}\right\rangle_k-\left\langle e\overline{\delta H}^{NL}_e\right\rangle_k$$

- Assume $k_\perp^2\rho_{Li}^2\ll 1$, and let

$$\delta K=\widehat{\delta K}_\phi(\delta\phi-\delta\psi)+\widehat{\delta K}_\psi\delta\psi$$

- NL-quasineutrality

$$\left(1+\frac{T_i}{T_e}-\sum_{e,i}\left\langle eJ_0(\gamma)\widehat{\delta K}_\phi\right\rangle_\pm\right)A_\pm\begin{pmatrix}\Phi_0-\Psi_0\\ \Phi_0^*-\Psi_0^*\end{pmatrix}$$

$$+\left[\left(1-\frac{\omega_{*pi}}{\omega}\right)b_{i\pm}-\sum_{e,i}\left\langle eJ_0(\gamma)\widehat{\delta K}_\psi\right\rangle_\pm\right]A_\pm\begin{pmatrix}\Psi_0\\ \Psi_0^*\end{pmatrix}$$

$$=-\frac{\mathrm{i}}{\omega_0}\frac{c}{B}\frac{T_i}{T_e}k_\vartheta k_z\delta\phi_z\left[\left(1+\frac{\omega_{*ni}}{\omega_0}\frac{T_e}{T_i}\right)\begin{pmatrix}A_0\Psi_0\\ A_0^*\Psi_0^*\end{pmatrix}-\begin{pmatrix}A_0(\Phi_0-\Psi_0)\\ A_0^*(\Phi_0^*-\Psi_0^*)\end{pmatrix}\right]$$

- E.S. limit $(\Psi_0\to 0)$

$$D_{S\pm}A_\pm=\frac{\mathrm{i}}{\omega_0}\frac{c}{B}\frac{T_i}{T_e}k_\vartheta k_z\delta\phi_z\begin{pmatrix}A_0\\ A_0^*\end{pmatrix}\qquad \text{(2D NL PDE)}\Rightarrow\text{(1D NL }\Psi\text{DE)}$$

- Estimate growth rate of zonal flow modulational instability in the local limit (Chen *et al.* 1999). Also ref. P. Diamond *et al.*

$$D_{S\pm} = \left\langle \left\langle \left(1 + \frac{T_i}{T_e} - \sum_{e,i} \langle eJ_0(\gamma) \widehat{\delta K}_\phi \rangle_\pm \right) \begin{pmatrix} \Phi_0^2 \\ \Phi_0^{*2} \end{pmatrix} \right\rangle \right\rangle \left\langle \left\langle \begin{pmatrix} \Phi_0^2 \\ \Phi_0^{*2} \end{pmatrix} \right\rangle \right\rangle^{-1}$$

$$D_{S\pm} \simeq \mathrm{i}(\partial D_{S0r}/\partial \omega_0)(-\mathrm{i}\Delta \pm \Gamma_z \pm \gamma_d)$$

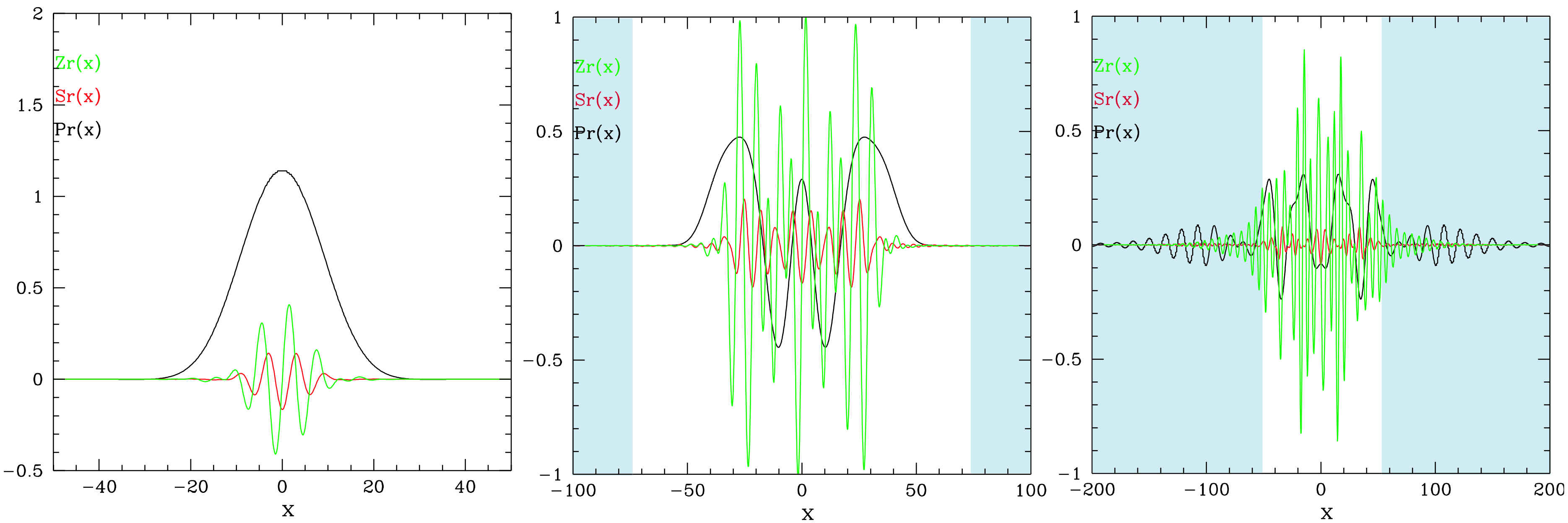
- Definitions: $\Delta = (k_z^2/2)(\partial^2 D_{S0r}/\partial k_r^2)/(\partial D_{S0r}/\partial \omega_0)$ is the frequency mismatch, $k_r = nq'\theta_k$, $\Gamma_z = -\mathrm{i}\omega_z$ and γ_d is the sideband damping.
- Take the limit $\gamma_d \ll |\Delta|, \gamma_M$; then:

$$\Gamma_z = \gamma_M \left(1 - \Delta^2/\gamma_M^2\right)^{1/2}$$

$$\gamma_M^2 = (2\alpha_0 \epsilon^{1/2}/1.6q^2)(T_i/T_e)(\omega_0 \partial D_{S0r}/\partial \omega_0)^{-1} k_z^2 \rho_{Li}^2 k_\vartheta^2 v_{thi}^2 \langle\langle |eA_0\Phi_0/T_i|^2 \rangle\rangle$$

- Scaling of modulational instability growth rate above threshold is linear and not quadratic with the mode amplitude.

- Fully non-local case (1D ΨDE): local transport becomes non-local due to dependencies on drift-wave intensity ⇒ Size-scaling of turbulent transport. Chen *et al.* sub. to PRL2003; Zonca *et al.* APS-DPP2003.



☐ Consider the E.M. limit $|\Phi_0 - \Psi_0| \ll |\Psi_0|$ in the NL-vorticity

$$\left\{ \partial_\theta \left(\frac{k_\perp^2}{k_\vartheta^2} \partial_\theta \right) + \frac{\omega^2}{\omega_A^2} \frac{k_\perp^2}{k_\vartheta^2} \left[\left(1 - \frac{\omega_{*pi}}{\omega} \right) - \frac{3}{4} b_i \left(1 - \frac{\omega_{*pi}}{\omega} - \frac{\omega_{*Ti}}{\omega} \right) \right] \right.$$

$$\left. - \frac{4\pi q^2 R_0^2}{k_\vartheta^2 c^2} \sum_{e,i} \langle e \omega \omega_d J_0 \widehat{\delta K}_\psi \rangle \right\}_\pm A_\pm \left(\begin{array}{c} \Psi_0 \\ \Psi_0^* \end{array} \right) + \left\{ \frac{\omega^2}{\omega_A^2} \frac{k_\perp^2}{k_\vartheta^2} \left[\left(1 - \frac{\omega_{*pi}}{\omega} \right) \right. \right.$$

$$\left. \left. - \frac{3}{4} b_i \left(1 - \frac{\omega_{*pi}}{\omega} - \frac{\omega_{*Ti}}{\omega} \right) \right] - \frac{4\pi q^2 R_0^2}{k_\vartheta^2 c^2} \sum_{e,i} \langle e \omega \omega_d J_0 \widehat{\delta K}_\phi \rangle \right\}_\pm A_\pm \left(\begin{array}{c} \Phi_0 - \Psi_0 \\ \Phi_0^* - \Psi_0^* \end{array} \right)$$

$$= \frac{4\pi \mathrm{i} \omega_0}{k_\vartheta^2 c^2} \frac{c}{B} \frac{n e^2}{T_i} q^2 R_0^2 k_\vartheta k_z \delta\phi_z b_i \left(\begin{array}{c} A_0 \Phi_0 \\ A_0^* \Phi_0^* \end{array} \right)$$

- Assume $k_\parallel^2 q^2 R_0^2 \ll 1$ to determine $E_\parallel$ from quasineutrality

$$\begin{pmatrix} \Phi_0 - \Psi_0 \\ \Phi_0^* - \Psi_0^* \end{pmatrix} A_\pm \simeq -\left(\frac{(k_\parallel^2 v_A^2/\omega^2) b_i}{T_i/T_e + \omega_{*ni}/\omega} \right)_\pm \begin{pmatrix} \Psi_0 \\ \Psi_0^* \end{pmatrix} A_\pm - \mathrm{i} \frac{c}{B} \frac{k_\vartheta k_z}{\omega_0} \delta\phi_z \begin{pmatrix} A_0 \Psi_0 \\ A_0^* \Psi_0^* \end{pmatrix}$$

- Use this to eliminate $E_\parallel$ from NL vorticity

$$D_{M\pm} A_\pm = \mathrm{i} \frac{\omega_0}{\omega_A^2} \frac{c}{B} k_\vartheta k_z \delta\phi_z \left(1 + \frac{K_\parallel^2 v_A^2}{\omega^2} \right)_\pm \begin{pmatrix} A_0 \\ A_0^* \end{pmatrix} \quad , \qquad \text{(2D NL PDE)}$$

$$D_{M\pm} \equiv \left\langle\left\langle \begin{pmatrix} \Psi_0 \\ \Psi_0^* \end{pmatrix} \mathcal{L}_{M\pm} \begin{pmatrix} \Psi_0 \\ \Psi_0^* \end{pmatrix} \right\rangle\right\rangle \left\langle\left\langle \frac{k_{\perp\pm}^2}{k_\vartheta^2} \begin{pmatrix} \Psi_0^2 \\ \Psi_0^{2*} \end{pmatrix} \right\rangle\right\rangle^{-1} ,$$

$$K_{\parallel\pm}^2 \equiv \left\langle\left\langle \begin{pmatrix} \Psi_0 \\ \Psi_0^* \end{pmatrix} \frac{k_{\perp\pm}^2}{k_\vartheta^2} k_{\parallel\pm}^2 \begin{pmatrix} \Psi_0 \\ \Psi_0^* \end{pmatrix} \right\rangle\right\rangle \left\langle\left\langle \frac{k_{\perp\pm}^2}{k_\vartheta^2} \begin{pmatrix} \Psi_0^2 \\ \Psi_0^{2*} \end{pmatrix} \right\rangle\right\rangle^{-1}$$

- Estimate growth rate of zonal flow modulational instability in the local limit (Chen *et al.* 2000). Also ref. Das, Diamond *et al.* 2000; Smoliakov *et al.* 2000.

$$\Gamma_z = 2k_\vartheta^2\rho_{Li}^2 \frac{k_z^2 v_{thi}^2}{\omega_0}\frac{\omega_0^2}{\omega_A^2}\frac{\epsilon^{1/2}}{1.6q^2}\left\langle\left\langle\left|\frac{eA_0\Psi_0}{T_i}\right|^2\right\rangle\right\rangle \frac{\mathbb{I}\mathrm{m}\left[D_{M+}\left(1+K_\parallel^2 v_A^2/\omega^2\right)_-\right]}{|D_{M+}|^2}$$

$$\times\left[\left(\alpha_0 - \left|\frac{K_\parallel^2 v_A^2}{\omega_0^2}\right|\right) - 2\alpha_0 \mathbb{R}\mathrm{e}\left(\frac{(K_\parallel^2 v_A^2/\omega^2)K_\perp^2\rho_{Li}^2}{T_i/T_e + \omega_{*ni}/\omega}\right)_\pm\right] \quad ,$$

$$K_{\parallel +}^2 K_{\perp +}^2 \equiv \left\langle\left\langle \Psi_0^* k_{\perp +}^2 k_{\parallel +}^2 \Psi_0 \right\rangle\right\rangle \left\langle\left\langle |\Psi_0|^2 \right\rangle\right\rangle^{-1}$$

- Specialize to cases of KAW and AITG

- Ref: Chen *et al.* NF 2001
- KAW: $D_M = -q^2 R_0^2 K_\parallel^2 + (\omega^2/\omega_A^2)[1 - K_\perp^2 \rho_{Li}^2 (3/4 + T_e/T_i)]$

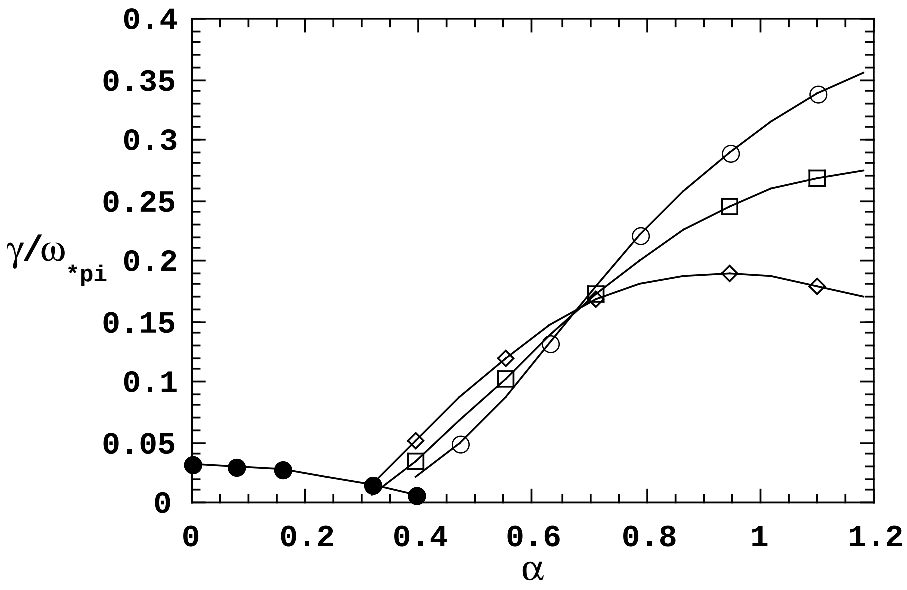
$$\hat{\gamma}_M^2 = 2 k_\vartheta^2 \rho_{Li}^2 k_z^2 v_{thi}^2 \frac{\epsilon^{1/2}}{1.6 q^2} \left(\frac{3}{4} - \frac{T_e}{T_i} \right) K_\perp^2 \rho_{Li}^2 \left\langle \left\langle \left| \frac{e A_0 \Psi_0}{T_i} \right|^2 \right\rangle \right\rangle \quad ,$$

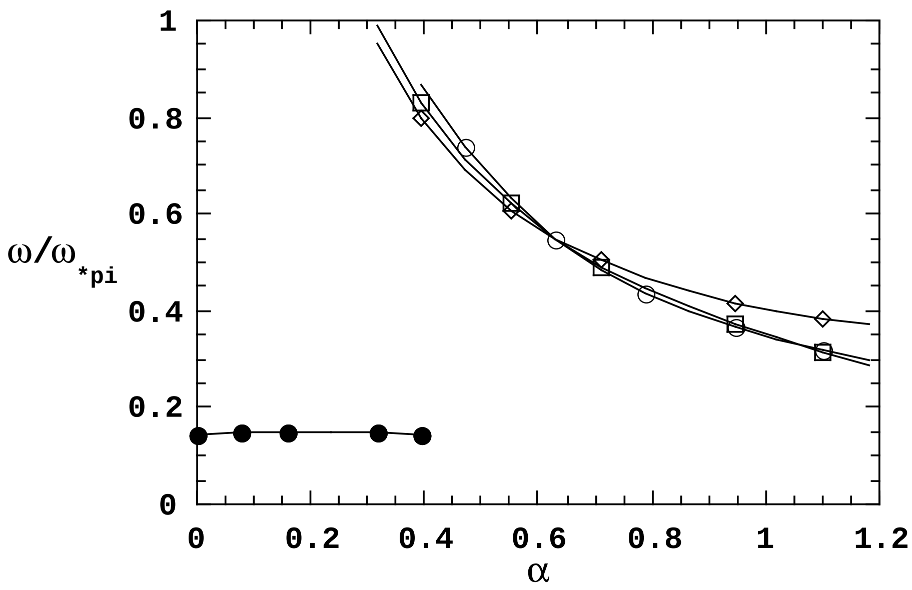
$$\hat{\Delta} = \left(\frac{3}{4} + \frac{T_e}{T_i} \right) k_z^2 \rho_{Li}^2 \frac{\omega_0}{2} \quad ,$$

$$\Gamma_{z,KAW} \simeq \hat{\gamma}_M \sqrt{1 - \hat{\Delta}^2/\hat{\gamma}_M^2} \quad .$$

- KAW's spontaneously generate zonal flows in their propagating region for $T_e < (3/4)T_i$ and in their cut-off region for $T_e > (3/4)T_i$ (Chen *et al.* NF 2001).

□ Alfvén ITG: Dong *et al.* Nuclear Fusion 39, 1917, (1999)





- Ref: Chen *et al.* NF 2001
- AITG: $D_M = \Lambda^2 + i\Lambda\delta W_f$ and Λ^2 is a generalized inertia

$$\Lambda^2 = \frac{\omega^2}{\omega_A^2}\left(1-\frac{\omega_{*pi}}{\omega}\right) + q^2\frac{\omega\omega_{ti}}{\omega_A^2}\left[\left(1-\frac{\omega_{*ni}}{\omega}\right)F(\omega/\omega_{ti}) - \frac{\omega_{*Ti}}{\omega}G(\omega/\omega_{ti}) - \frac{N^2(\omega/\omega_{ti})}{D(\omega/\omega_{ti})}\right] ,$$

$$\tilde{\gamma}_M^2 = 2k_\vartheta^2\rho_{Li}^2\left(\frac{k_z^2 v_{thi}^2}{\omega_A^2 \partial \mathbb{R}\mathrm{e}\Lambda^2/\partial\omega_0^2}\right)\frac{\epsilon^{1/2}}{1.6q^2}\left(1-\frac{\omega_{*pi}}{\omega_0}-\frac{\omega_A^2}{\omega_0^2}\mathbb{R}\mathrm{e}\Lambda^2\right)\left(1+\frac{\omega_A^2}{\omega_0^2}\mathbb{R}\mathrm{e}\Lambda^2\right)\left\langle\left\langle\left|\frac{eA_0\Psi_0}{T_i}\right|^2\right\rangle\right\rangle$$

$$\tilde{\Delta} = \frac{k_z^2}{2}\frac{\partial^2\delta W_f^2}{\partial k_r^2}\Big/\frac{\partial}{\partial\omega_0}\mathbb{R}\mathrm{e}\Lambda^2$$

$$\Gamma_{z,AITG} = \tilde{\gamma}_M\sqrt{1-\tilde{\Delta}^2/\tilde{\gamma}_M^2} \ .$$

- AITG spontaneously generate zonal flows for $\omega_0 > \omega_{*pi}$, which is the typical case for slightly unstable AITG.

Modulational instability of EPM

- Typical EPM mode structure in ballooning space

$$\Psi_0 = E_0(\theta)\exp\left(\mathrm{i}/2 + \mathrm{i}\Lambda_0\right)\theta + F_0(\theta)\exp\left(-\mathrm{i}/2 + \mathrm{i}\Lambda_0\right)\theta$$

- E_0, F_0 are the amplitudes of $k_\parallel q R_0 = \pm 1/2$ Alfvén waves; $\Lambda_0^2 = (\Omega_0^2 - 1/4)^2 - \epsilon_0^2\Omega_0^4$ is the (square of) continuum damping, with $\Omega_0 = \omega_0/\omega_A$.
- Recall: Particle NLties play a role via NL modification of $\delta H_k \sim \delta L_{k'} \delta H_z$: formally(only) analogous to quasi-linear diffusion

$$\overline{\delta H}_{\mathrm{z}} \simeq \frac{k_\theta c}{B}\frac{k_{\mathrm{z}}}{\omega_{\mathrm{z}}} J_0^2(\lambda_{\mathrm{z}}) \sum_{\ell}\sum_{(\pm)} \;\; M_\ell^{(\pm)} \;\; \left[A_0^* A_+ \mathrm{Im}\left(\frac{\delta \tilde{H}_{d0\ell}^{(\pm)*}}{A_0^*} - \frac{\delta \tilde{H}_{d+\ell}^{(\pm)}}{A_+} \right) \right.$$
$$\left. - A_0 A_- \mathrm{Im}\left(\frac{\delta \tilde{H}_{d0\ell}^{(\pm)}}{A_0} - \frac{\delta \tilde{H}_{d-(-\ell)}^{(\mp)}}{A_-} \right) \right] \;\; ,$$

Fast particle diffusion equation

$$\frac{\partial}{\partial t}\delta n_E + \frac{\partial}{\partial r}\Gamma_E = 0$$

$$\frac{\partial}{\partial t}\left(\delta P_{\perp E} + \frac{\delta P_{\parallel E}}{2}\right) + \frac{\partial}{\partial r}Q_E = 0 \qquad \mathrm{Def:}\quad \mathcal{E} = \frac{m_E}{2}\left(v_\perp^2 + v_\parallel^2\right)$$

$$(\Gamma_E, Q_E) = k_\theta \frac{c}{B}\frac{e_E}{m_E}\left\langle J_0^2(\lambda_z)\sum_l \mathbb{I}\mathrm{m}\left\{\left(-\frac{QF_0}{\omega}\right)_k \left\langle\left\langle \frac{\Omega_{dk}^2}{\omega_k^*}\frac{k_\perp^2}{k_\theta^2}\frac{l^2 J_l^2(\lambda_k)}{\lambda_k^2}J_0^2(\gamma_k)\right\rangle\right\rangle(1,\mathcal{E})\right.\right.$$

$$\left.\left.\left[\frac{(|\Omega_r^2 - 1/4| + |\Lambda|)}{\omega_t(l + \Lambda_k + 1/2) - \omega_k} + \frac{(|\Omega_r^2 - 1/4| - |\Lambda|)}{\omega_t(l + \Lambda_k - 1/2) - \omega_k}\right]\right\}\right\rangle (A_+ A_0^* + A_- A_0)$$

$$\mathcal{L}_{0\ell}^{(\pm)} = \omega_t\left(\ell + \Lambda_0 \pm 1/2\right) - \omega_0 + \frac{k_\theta^2 c^2}{B^2}k_z^2 J_0^2(\lambda_z)\frac{M_\ell^{(\pm)}}{\omega_z}\left(|A_+|^2 + |A_-|^2\right) \quad ,$$

$$M_\ell^{(\pm)} = \frac{|\Omega_0^2 - 1/4| \pm |\Lambda_0|}{2}\left\langle\left\langle J_0^2(\gamma_0)J_\ell^2(\lambda_0)\frac{\ell^2\omega_t^2/\omega_0^2}{k_\perp^2/k_\theta^2}\right\rangle\right\rangle$$

□ Derivation of 2D PDE for the slow radial and time evolution of the EPM ⊕ sideband envelopes.

$$D_+ A_0^* A_+ = \mathrm{i}\frac{\gamma_M^2}{\omega_\mathrm{z}^2}\left(2A_0^* A_+ + A_0 A_-\right) \quad ,$$

$$D_- A_0 A_- = -\mathrm{i}\frac{\gamma_M^2}{\omega_\mathrm{z}^2}\left(A_0^* A_+ + 2A_0 A_-\right) \quad .$$

□ $D_\pm$ are the linear EPM sideband dispersion functions. With D_0 the EPM linear dispersion function:

$$D_+ \simeq \frac{\partial D_0}{\partial \omega_0}\omega_z - \frac{\partial \mathbb{R}\mathrm{e} D_0}{\partial \omega_0}\Delta_L \quad ,$$

$$D_- \simeq -\frac{\partial D_0^*}{\partial \omega_0}\omega_z - \frac{\partial \mathbb{R}\mathrm{e} D_0}{\partial \omega_0}\Delta_L \quad ,$$

- In the local limit, NL-vorticity reduce to: $(\Gamma_z \gg |\Delta_L|)$:

$$\Gamma_z = \left(\frac{9}{8|\partial D_I/\partial\omega_r|}\right)^{1/3} \gamma_M^{2/3} \quad , \qquad \Delta_L = (nq')^2 \frac{\partial^2 D_R/\partial k_r^2}{\partial D_R/\partial\omega_r}\left[1-\cos\left(\frac{k_z}{nq'}\right)\right]$$

$$\gamma_M^2 = \pi\epsilon_0 \frac{v_E^2}{v_A^2}\frac{\beta_E q^2}{8} q^2 k_\theta^4 \rho_{LE}^4 k_z^2 v_E^2 \left|\frac{e_E A_0}{T_E}\right|^2 \sum_\ell \sum_{\sigma=\pm}\left(|\Omega_r^2 - 1/4| + \sigma|\Lambda|\right)$$
$$\left\langle \frac{F_{0E}}{n_{0E}}\left(\frac{\hat\omega_{*E}}{\omega_r}-1\right)\delta(\mathcal{L}_{\ell,\sigma}/\omega_r)\left\langle\left\langle J_0^2(\lambda_L)\frac{\ell^2 J_\ell^2(\lambda_d)}{\lambda_d^2}\right\rangle\right\rangle^2 \right.$$
$$\left. J_0^2(\lambda_z)\left(\frac{v_\perp^2}{2v_E^2}+\frac{v_\parallel^2}{v_E^2}\right)^4\right\rangle \quad .$$

- Real NL frequency shift is produced: $\Delta_z = \pm\Gamma_z/\sqrt{3}$

- $\gamma_M^2 \sim \epsilon_0 \alpha_E |k_\theta \rho_{LE}| k_z^2 v_A^2 |\delta B_\theta/B|^2$.

- Solutions confirm exponential growth of the radial fragmentation, in analogy with modulational inst. of e.s. DW (Chen 1999).
- Analytic expressions give a $\Gamma_z \propto |\delta B_\theta / B|^{2/3}$ scaling, and a finite amplitude threshold for radial fragmentation, since they assume $\Gamma_z \gg |\Delta_L|$

$$\Delta_L = (nq')^2 \frac{\partial^2 D_R / \partial k_r^2}{\partial D_R / \partial \omega_r} \left[1 - \cos \left(\frac{k_z}{nq'} \right) \right]$$

- $\Gamma_z \propto |\delta B_\theta / B|$ scaling is obtained assuming $\Gamma_z \ll |\Delta_L|$

$$\omega_z = \Delta_z + i\Gamma_z = \pm \frac{3^{1/4} e^{\pm i\pi/4}}{|\Delta_L \partial \mathrm{Re} D_0 / \partial \omega_0|^{1/2}} \gamma_M \quad .$$

- Finite EPM threshold for the onset of the modulational instability would be brought into our model provided that dissipation effects are considered on the zonal field generation (Hinton and Rosenbluth, *op. cit.* 1999).

NL EPM induced zonal flow

- Given the NL EPM evolution and saturation via δH_z modulational instability, compute the non-self-consistent zonal flow; let $x \equiv r/a$ and $\tau \equiv v_A t/R$

$$\partial_\tau \partial_x \frac{e_E}{T_E} \delta\phi_z = \frac{1}{1.6q^2} \left(\frac{R_0}{a}\right)^{1/2} \left(\frac{v_E}{v_A}\right) \left(\frac{\rho_{LE}}{a}\right) x^{1/2}$$
$$\times \sum_m \frac{m}{x} \left(\frac{e_E}{T_E}\right)^2 \left[\delta\phi_m \nabla^2_{\perp,x} \delta\phi^*_m - \frac{v_A^2}{c^2} \delta A_{\parallel,m} \nabla^2_{\perp,x} \delta A^*_{\parallel,m} - \text{c.c.}\right]$$

- Computing the decorrelation time (Hahm-Burrel)
$\gamma_E \simeq -(r/q)\partial_r[(q/r)(c/b)\partial_r \delta\phi_z]$

$$\frac{R}{v_A}\gamma_E \simeq -\left(\frac{R}{a}\right)\left(\frac{\rho_{LE}}{a}\right)\left(\frac{v_E}{v_A}\right)\partial_x^2 \frac{e_E}{T_E}\delta\phi_z$$

Discussions

Drift and Drift Alfvén Waves

- □ Modulational instability of zonal flow due to a single coherent drift wave has been investigated for the e.s. drift and e.m. drift-Alfvén branch
- □ The general problem is formulated in terms of derivations of 2D PDE for the slow radial and time evolution of the waves $\oplus$ sideband envelopes
- □ Analytic expressions for the zonal flow growth rate are given in the local limit
- □ Studies of solutions of the actual 2D NL PDE system are in progress

Energetic Particle driven modes

- Significant particle redistributions take place above linear excitation threshold of EPM
- Peculiarity of the EPM NL dynamics is coming from the fundamental role played by wave-particle resonant interactions $\Rightarrow$ regulated via $[\delta\phi - (v_\parallel/c)\delta A_\parallel]_k \delta H_z$ response
- Modulational instability associated with radial EPM fragmentation ($K_Z \rho_{LE} \lesssim 1$) has been studied analytically and numerically.
- Radial EPM fragmentation is associated with fast particle diffusion and generation of zonal flow.
- May EPM, via zonal flow generation, have benign effects on thermal plasma transport????? ...; ref. T.S. Hahm *et al.* PoP **6**, 922, (1999); $\omega_{E,eff} \simeq \omega_{E,0} \Delta\omega_T \left(\Delta\omega_T^2 + 3\omega_f^2\right)^{-1/2}$