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Quantum Plasmas – Recent Developments

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Quantum Plasmas – Recent Developments
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Overview

- Why look at quantum plasma effects?
- Schrödinger’s description.
- Non-relativistic single electron dynamics.
- Paramagnetic electrons.
- From micro to macro physics.
- MHD regime.
- Conclusions - what the future might bring.
Quantum plasmas

Quantum plasmas
Quantum plasmas

- Manifold applications (Pines, 1961; Kremp et al. 2005)
  - Ultracold plasmas (Rydberg states) (Li et al., PRL, 2005).
  - Laser-plasmas (Glenzer et al., PRL, 2007).
  - Spintronics.

- Interesting fundamental aspects of matter dynamics.
- Quantum to classical transition?
- Collective quantum systems.
Schrödinger description

Electron properties described by complex scalar wavefunction $\psi$ ($|\psi|^2 = \text{probability}$)

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi$$

where we have the Hamiltonian operator

$$H = -\frac{\hbar^2}{2m_e} \nabla^2 - e\phi$$

and $\phi$ is the external electrostatic potential and $e$ being the magnitude of the electron charge.
Schrödinger description

- Nice approach: allows for easy generalizations, new interactions can be incorporated in Hamiltonian.
- Microscopic equations of motion

\[
\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{1}{i\hbar} [F, H]
\]

for some operator \(F\), \([F, H]\) Poisson brackets. Example:

\[
\mathbf{v} \equiv \frac{d\mathbf{x}}{dt} = [\mathbf{x}, H] = \frac{\mathbf{p}}{m_e}
\]

for previous scalar electron description.
Schrödinger description

Quantum statistical dynamics

\[
W(t, x, p) = \frac{1}{(2\pi \hbar)^3} \int d^3 y \exp(i p \cdot y / \hbar) \langle \psi^*(t, x + y/2) \psi(t, x - y/2) \rangle 
\]

\[
\left( \frac{\partial}{\partial t} + \frac{p}{m} \cdot \nabla \right) W + \frac{2e}{\hbar} \phi \sin \left( \frac{\hbar}{2} \nabla \cdot \nabla_p \right) W = 0
\]

Fluid moments from quantum kinetic equation, or from summing up particle contributions in Madelung picture.
Schrödinger description

▷ Quantum pressure (gradient of Bohm-de Broglie potential)

\[ m \frac{dv}{dt} \sim \frac{\hbar^2}{2m} \nabla \left( \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right) \]

▷ Gives higher order dispersion (spreading of electronic wave function).
Plasmonic devices

- Surface plasmon polaritons propagating on conductor surface.
- Classically, surface is sharp.
- Broadening of surface layer due to quantum dispersion.
- Finite width gives damping!
\[ \omega \approx \frac{\omega_p^{(0)}}{(1 + \epsilon_d^{(0)})^{1/2}} \left[ 1 + (0.6 + 2i) \left( \frac{\hbar k^2}{m\omega_p^{(0)}} \right)^{1/2} \right] \]

\[ \delta_{SP} \approx \left( \frac{\lambda}{100 \text{ nm}} \right)^4 \mu \text{m} \]

\[ \lambda \sim 30 \text{ nm} \]

\[ \delta_{SP} \sim 10 \text{ nm} \]
Spin contributions

Electron properties described by complex spinor wavefunction (spin degrees of freedom)

\[ \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \]

and the Pauli Hamiltonian operator

\[ H = \frac{1}{2m_e} \left( \frac{1}{i\hbar} \nabla + \frac{e}{c} A \right)^2 + \mu_B B \cdot \sigma - e\phi \]

Here \( A \) is the vector potential, \( \sigma \) is the spin operator, \( \mu_B = e\hbar/2m_e \) is the Bohr magneton.
Quantum equations of motion for electrons

\[ \mathbf{v} = \frac{1}{m_e} \left( \mathbf{p} + \frac{e}{c} \mathbf{A} \right) \]

\[ m_e \frac{d\mathbf{v}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{2\mu_B}{\hbar} \nabla (\mathbf{B} \cdot \mathbf{S}) \]

\[ \frac{d\mathbf{S}}{dt} = \frac{2\mu_B}{\hbar} \mathbf{B} \times \mathbf{S} \]

where \( \mathbf{S} = (\hbar/2)\mathbf{\sigma} \).
Multiparticle theory

> For spin systems: Decompose spinor wave function $\psi_\alpha = \sqrt{n_\alpha} \exp(iS_\alpha/\hbar)\varphi_\alpha$, $\varphi_\alpha$ unit spinor.

> Electron fluid equations (not complete!)

\[
m_e \left( \frac{\partial}{\partial t} + \mathbf{v}_e \cdot \nabla \right) \mathbf{v}_e = -e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \frac{\nabla p_e}{n_e} - \frac{2\mu_B}{\hbar} S_\alpha \nabla B^\alpha
\]

\[
\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{S} = \frac{2\mu_B}{\hbar} \mathbf{B} \times \mathbf{S} + \text{thermal and nonlinear spin terms}
\]
Maxwell’s equations

Due to the intrinsic magnetization, given by

\[ M = -\frac{2\mu_B n_e}{\hbar} S, \]

Ampère’s law is modified according to

\[ \nabla \times B = \mu_0(j + \nabla \times M) + \frac{1}{c^2} \frac{\partial E}{\partial t}. \]

Gives dynamic spin contribution to Maxwell’s equations.
MHD regime

Single fluid dynamics (for lowest order coherent spin)

\[ \rho \frac{dv}{dt} = -\nabla \left( \frac{B^2}{2\mu_0} - M \cdot B \right) + B \cdot \nabla \left( \frac{1}{\mu_0} B - M \right) - \nabla p + \frac{\hbar^2 \rho}{2m_e m_i} \left( \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right) \]

\[ \frac{\partial B}{\partial t} = \nabla \times \left\{ \mathbf{v} \times B - \frac{\nabla \times (B - \mu_0 M)}{e\mu_0 n_e} \times B - M_a \nabla B^a \right\} \]

Model magnetization using Brillouin function for spin-1/2 particles (for long enough time scales)

\[ M = \mu_B n_e \tanh x \hat{B} \]

where the Zeeman energy \( x = \mu_B B / k_B T_e \) gives the degree of alignment through the Brillouin function. For high temp., magnetization \( \rightarrow 0 \).
Instabilities and ferrofluids

Ferrofluids Nanostructured paramagnetic fluids. Formalism similar to the above applicable.

Normal field instability - saturated by gravity and surface tension (Cowley & Rosensweig 1967).

http://mrsec.wisc.edu/Edetc/cineplex/ff/text.html
Spin kinetics

Spin dependent distribution function: \( f = f(t, x, v, s) \)

\[
\frac{\partial f}{\partial t} + \frac{d\mathbf{x}}{dt} \cdot \nabla_x f + \frac{d\mathbf{v}}{dt} \cdot \nabla_v f + \frac{ds}{dt} \cdot \nabla_s f = 0
\]


Quantum equations of motion gives semiclassical dynamics.

See Gert Brodin’s talk.
Spin kinetics

Formal structure: Wigner matrix from density matrix

\[ W_{ab}(t, x, p) = \frac{1}{(2\pi\hbar)^3} \int d^3y \exp(i p \cdot y/\hbar) \rho_{ab}(t, x + y/2, x - y/2) \]

Distribution function defined by

\[ f(t, x, p, s) = \frac{1}{2} (1 + s \cdot \sigma)_{ab} W_{ab} \]

Issues in relativistic quantum plasmas

- Pair production: “Schwinger mechanism” for fields with spatial \textit{and} temporal variation.
  - Temporal compression: \textit{increased} production rate.
  - Spatial compression: \textit{lower} production rate.
  - Laser fields \xrightarrow{\text{production rate unknown}}

- Strong fields + relativistic plasma particles: can computational models be developed?

- Quantum field theoretical models (Melrose, 2008)?
Currently, gigagauss laboratory fields generated in solid-laser interactions.

Look for quantum plasma effects:
- Landau quantization.
- Spin effects.
- ...
Conclusions

- New important effects appear from collective quantum domain.

- Wide ranging possibilities for applications.
  - Nanomaterials.
  - Astroplasmas.
  - Ultracold plasmas.

- Interesting future possibilities:
  - Theoretical development: Dense, relativistic plasmas using computationally viable models.
  - Solitons and plasmonics.