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The Interplay between Magnetic Reconnection and the Kelvin-Helmholtz and Rayleigh-Taylor Instabilities in a Magnetized Inhomogeneous Plasma with Velocity Shear/1

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The interplay between Magnetic Reconnection and the Kelvin-Helmholtz and Rayleigh- Taylor instabilities in a magnetized inhomogeneous plasma with velocity shear

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Instabilities on the shoulders of instabilities

Fluids and plasmas in nature do not usually manifest themselves in quiescent stationary or quasistationary states (what we generally call "equilibria") but exhibit a rich dynamical behaviour consisting or linear and nonlinear waves, coherent nonlinear structures such as vortices and solitary perturbations, or fully developed turbulence.

From a conceptual point of view when we analyze such systems we first need to go through a semantic effort, i.e., we need to create a language that can be used to label and thus to identify the basic dynamical mechanism that we recognize to be at play in the system we are examining.

The creation of such a language is a first fundamental step that allows us to examine, describe and then communicate the results of a physical experiment or of a numerical simulation.

In the vocabulary created for describing fluids and plasmas we may use the concept of equilibrium state, although very rarely a real system is found in such idealized conditions, and then study the stability of this equilibrium configuration against linear and nonlinear instabilities and study how each instability grows (threshold conditions, linear growth rate, nonlinear evolution, saturation etc...).

This language is particularly useful when we are interested in understanding the specific physical mechanism that drives the system away from the idealized equilibrium condition, how to contrast this mechanism (stabilize the system) if this is required, or deal with situations where rather well defined initial conditions are physically present and the system evolution can be satisfactorily analyzed in terms of a restricted number of elementary excitations (instabilities).

On the contrary, in physical situations when no satisfactory knowledge of the initial conditions is available, or when this information becomes no longer relevant over the long time scale of the evolution of the system, or in general when a very large number of excitations are present, the statistically based language of turbulence appears to be more effective in conveying information over the time development of the system.

In this presentation I will mostly adopt the former type of language but will focus my attention not so much on the single instability mechanism and the single instability growth and saturation, but mainly on the competition between a restricted number of instabilities (excitations) of the system, and on how this competition may become experimentally observable in specific plasma regimes of interest.

Different instabilities can compete either because they are present from the start, as they can feed on the same energy source present from the start in the equilibrium configuration, or because they arise as secondary instabilities that feed on an energy source that was not present initially in the equilibrium configuration but that has been made available by the development of the primary instability. Since these instabilities interact with each other, which is the primary and which are the secondary instabilities depends on the equilibrium configuration we have chosen to start from. Clearly this choice is not arbitrary as it must reflect our physical modelling of the system.

In this presentation I will focus my attention on the interplay and on the competition between a Kelvin-Helmholtz primary instability, with its nonlinear evolution characterized by the vortex pairing process, and the onset of secondary instabilities, such as the Rayleigh Taylor instability in the presence of a density inhomogeneity, or magnetic reconnection in the case of a frozen magnetic field with filed lines stretched by the differential plasma motion.

Kelvin-Helmholtz instability in a fluid system

In fluid dynamics the Kelvin-Helmholtz instability represents a basic phenomenon[1] that can occur when a velocity shear is present within a fluid or when there is sufficiently large velocity difference across the interface between two fluids[2].

Principle of the Kelvin-Helmholtz instability

Let us consider the flow of an (incompressible inviscid) fluid[3] along two horizontal parallel infinite streams of different velocities $\mathbf{u}_1$ and $\mathbf{u}_2$.

[1] Cigarette smoke, sand dunes, clouds, water waves, flags...

[2] Helmholtz, Hermann Ludwig Ferdinand von, "On the discontinuous movements of fluids", Monthly Reports of the Royal Prussian Academy of Philosophy in Berlin, **23**, 215 (1868);
Kelvin, Lord (William Thomson), "Hydrokinetic solutions and observations," Philosophical Magazine, **42**, 362 (1871).

[3] Some of the figures shown in this presentation are not original and can be found on the web.

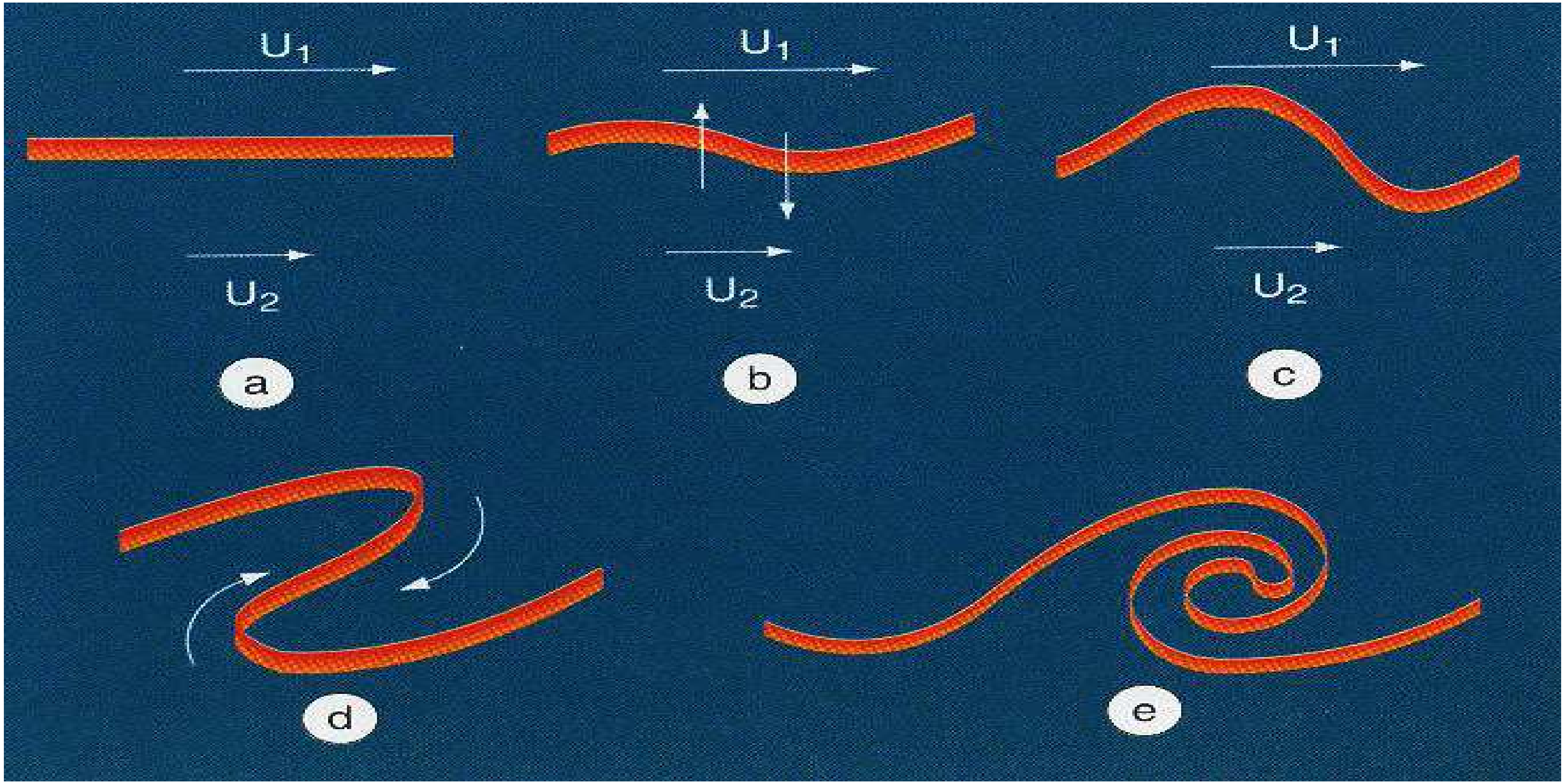

Figure 1: Folding of the vorticity layer

The horizontal boundary, corresponding to a sharp difference of velocity in the fluid is a "shearing layer". In this layer, vorticity ($\omega \equiv \nabla \times \mathbf{u}$) is approximatively uniform, while it is zero outside of the layer as velocities are uniform.

The fluid pressure in the concavities is higher than the pressure in the convexities (Bernoulli equation) so the initial perturbation is reinforced. Thus the vorticity sheets at the interface bends and the interface rolls up in the direction corresponding to the vorticity direction of the mixing layer.

The characteristic linear growth rate is given by

$$\gamma = k|\mathbf{u}_1 - \mathbf{u}_2|/2.$$

The K-H instability can be appear in different forms depending on the velocity profile, fluid viscosity, plasma magnetic field etc.

Figure 2: Artistic view of a vorticity field

Figure 3: Kelvin Helmholtz instability of a cloud

In the nonlinear phase the instability leads to rolled up vortices and to large-scale coalescence through subsequent pairing/merging events between adjacent vortices.

See animation

$http://www.moisie.math.uwaterloo.ca/\sim kglamb/$
$Hydrodynamic\_Instability\_animations.html$

Rayleigh-Taylor instability in a fluid system

In fluid dynamics the Rayleigh-Taylor instability represents another basic phenomenon. It occurs when a lighter fluid is accelerated into a heavier fluid.

For an inhomogeneous fluid system in a gravitational field this instability was first discovered by Lord Rayleigh in the 1880[4].

It is responsible for the fact that, if surface tension effects are neglected, it is not possible to keep water inside an inverted container (i.e., open at its bottom) balanced by atmospheric pressure.

[4] Rayleigh, Lord (John William Strutt), *"Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density,"* Proceedings of the London Mathematical Society, Vol. 14, pages 170 - 177 (1883)

Later the same concept was applied to accelerated fluids by Sir Geoffrey Taylor[5].

Understanding the rate of mixing caused by the Rayleigh-Taylor instability is important to a wide range of applications, that range from inertial confinement fusion, nuclear weapons explosions[6], supernova explosions and supernova remnants, to oceanography and atmospheric physics, to laboratory and space plasmas etc.

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[5]Taylor, Sir Geoffrey Ingram,*"The instability of liquid surfaces when accelerated in a direction perpendicular to their planes,"* Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. 201, No. 1065, pages 192 - 196 (1950)

[6]It is interesting to look at the dates of these papers: see e.g., *Fermi, E. 1951, "Taylor instability of an incompressible liquid", The Collected Papers of Enrico Fermi (ed. E. Segre), vol. 2, pp. 816, 821.*
$>>$ A discussion is presented in simplified form of the problem of the growth of an initial ripple on the surface of an incompressible liquid in the presence of an acceleration, g, directed from the outside into the liquid. The model is that of a heavy liquid occupying at $t = 0$ the half space above the plane $z = 0$, and a rectangular wave profile is assumed. The theory is found to represent correctly one feature of experimental results, namely the fact that the half wave of the heavy liquid into the vacuum becomes rapidly narrower while the half wave pushing into the heavy liquid becomes more and more blunt. ....

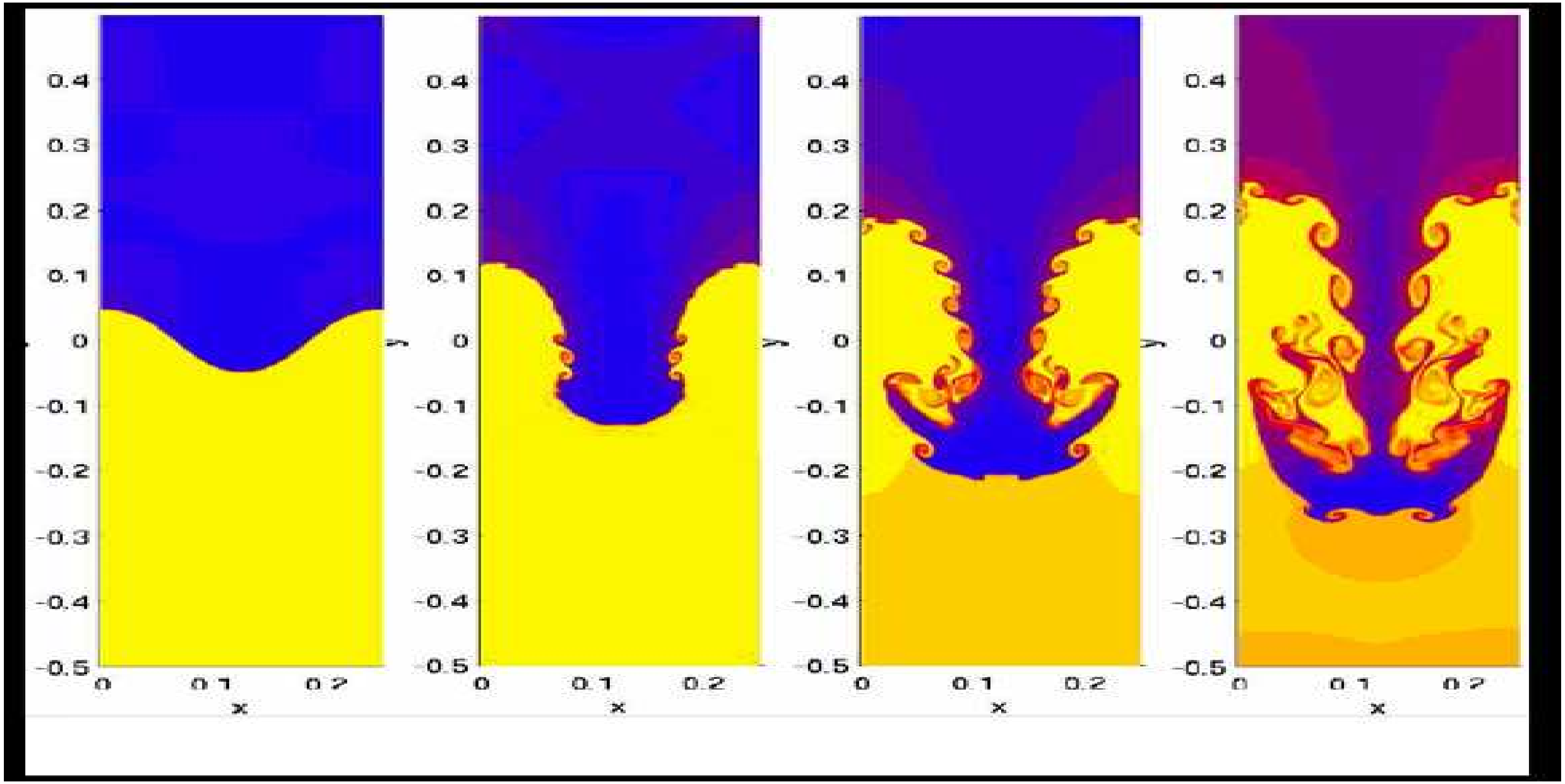

Figure 4: Nonlinear evolution of the Rayleigh-Taylor instability

A family of different instabilities can be grouped under the general name of Rayleigh-Taylor instabilities, with e.g., an inhomogeneous pressure playing the role of the inhomogeneous density, or electromagnetic radiation pressure taking the role of the lighter fluid or even, in the case of a magnetized confined plasma, magnetic field pressure[7] and magnetic field line curvature playing the role of the lighter fluid and of gravity, respectively.

In the simple case of two inmiscible incompressible fluids with densities ρ_1 and $\rho_2 < \rho_1$ respectively where the denser fluid 1 is initially on top of fluid 2, the instability linear growth rate γ can be written as

$$\gamma^2 = |k| g \, \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}, \qquad \gamma^2 \sim |k| g, \quad \text{for } \rho_2 \ll \rho, \tag{1}$$

with k the wavenumber in the interface plane and g the gravity acceleration.

[7] Note however that the tensor nature of the magnetic pressure, of the Maxwell stress tensor, makes a magnetic Rayleigh-Taylor instability evolve differently from a fluid Rayleigh-Taylor instability

The apparently unbound increase of the growth rate at smaller wavelengths is interrupted by effects, such as surface tension for a real fluid system, that are not included in Eq.(1). At small wavelengths these effects first reduce the mode growth rate and finally stabilize the mode.
Different stabilizing mechanisms have been shown to arise for different forms of Rayleigh-Taylor instabilities, ranging from matter ablation in the pellet compression in inertial fusion, to field line tension in the case of magnetically confined plasmas.

The Rayleigh-Taylor instability is of special importance in the case of inertial fusion where a fuel pellet is compressed by the reaction force exerted by the surface layers of the pellet ablated by the energy deposited by a high intensity laser pulse[8]

[8] S. Atzeni and J. Meyer-ter-Vehn, "The Physics of Inertial Fusion" (Clarendon Press, Oxford, 2004)

Magnetic field line reconnection

Magnetic fields represent a fundamental feature of laboratory and space plasmas. At low frequencies and long spatial scales, magnetic fields emerge as the dominant factor in the dynamics of a plasma as a consequence of the effective cancellation of the electric forces due to plasma quasi-neutrality.
A direct link between the particle and the magnetic field dynamics in a plasma, which can be applied to a variety of different plasma regimes, is obtained by combining Faraday's law $\nabla \times \mathbf{E} = -(1/c)\,\partial\mathbf{B}/\partial t$ and the mean electron momentum equation

$$m_e n\,[\partial \mathbf{u}_e/\partial t + (\mathbf{u}_e \cdot \nabla)\mathbf{u}_e] = -\nabla \cdot \mathbf{\Pi}_e - ne\,[\mathbf{E} + \mathbf{u}_e/c \times \mathbf{B}] + \mathcal{C},$$

where $\mathbf{\Pi}_e$ is the effective electron "pressure" tensor and $\mathcal{C}$ stands generically for collisional effects.

As long as the form and dependencies of the effective "pressure" tensor $\mathbf{\Pi}_e$ are not specified, the electron momentum equation is general (aside for the dissipative term $\mathcal{C}$, it corresponds to the first velocity moment of Vlasov's equation for the electron distribution function $f_e(\mathbf{x}, \mathbf{v}, t)$) and thus is not based on any fluid model. Kinetic effects enter the expression of the pressure tensor which is defined in terms of the electron distribution function.

In the case of low frequency, large scale phenomena in a magnetized plasma described by the MHD equations, we can identify the electron mean velocity $\mathbf{u}_e$ with the plasma fluid velocity $\mathbf{u}$ and we can assume that the pressure is isotropic, $\mathbf{\Pi}_e \to p_e\mathbf{I}$, and that it obeys a barotropic closure of the form $p_e = p_e(n)$.

For these low frequency phenomena, the effects of electron inertia and of electron viscosity in the collisional term $\mathcal{C}$ can be neglected in a number of plasma regimes of interest. Then, with η the electric resistivity of the plasma, we obtain

$$\nabla \times [\mathbf{E} + \mathbf{u}/c \times \mathbf{B}] = \nabla \times [(\eta c/4\pi)\nabla \times \mathbf{B}] \, .$$

In the ideal limit $\eta \rightarrow 0$, it reduces to the well known magnetic flux conservation theorem

$$d\Phi/dt = 0,$$

where Φ is the magnetic flux through a surface moving together with the plasma, i.e. with the plasma fluid velocity $\mathbf{u}$. This flux conservation is generally referred to as the "freezing" of the magnetic field in the plasma.
An alternative formulation of the freezing condition reads

$$d(\mathbf{dl} \times \mathbf{B})/dt = 0,$$

where $\mathbf{dl}$ is the vector connecting two close points in the plasma.
This latter formulation states that if two points $\mathbf{x}_1, \mathbf{x}_2$ are connected at $t = 0$ by a magnetic field line, then at all times t there exists a field line that connects $\mathbf{x}_1(t), \mathbf{x}_2(t)$, where $\mathbf{x}_1(t), \mathbf{x}_2(t)$ are the trajectories of the two initial points $\mathbf{x}_1, \mathbf{x}_2$ as determined by the plasma fluid velocity $\mathbf{u}$.

The conservation of the magnetic field correction is directly related to the formation of current and vorticity layers that lead eventually to the breaking of the field lines that get reconnected in a different global pattern .

Magnetic field line reconnection is indeed one of the most general phenomena in magnetized plasmas[9] and has been widely investigated in astrophysical and space plasmas, in laboratory magnetically confined plasmas and, more recently, in relativistic laser produced plasmas. The very name "reconnection" is in the negative as it implies that it is absent over most of the plasma so that we can give a meaning to the concept of time evolution of field lines. Only locally, around the so called critical points, field lines break and "reconnect" in a different pattern. This separation is valid only in plasma regimes where the processes leading to the breaking of the connections are *per se* weak, but are locally enhanced by the formation at the critical points of small spatial scales (singular perturbations).

[9] D. Biskamp, Magnetic reconnection in plasmas; Cambridge University Press; (2005).

Closely related to the topological nature of magnetic reconnection is its interpretation as a process that leads to magnetic energy conversion.
This is best understood in the case of ideally stable MHD plasma equilibria with inhomogeneous magnetic fields and current gradients. These equilibria can become unstable when the (infinite number of) constraints arising from the conservation of the magnetic connections between plasma elements are removed and lower magnetic energy states become available to the plasma.
This energy release can lead to the interpretation of the nonlinear development of magnetic reconnection as a transition, forbidden within the ideal MHD equations, between two MHD (equilibrium) states with different magnetic energies, the excess energy being eventually dissipated into heat (or transported away by accelerated particles). This relationship between dissipation and reconnection ceases to be valid in collisionless plasma regimes where we find that, even maintaining a fluid-like plasma description with a barothropic scalar pressure, magnetic connections are broken not by electron resistivity but by electron inertia.

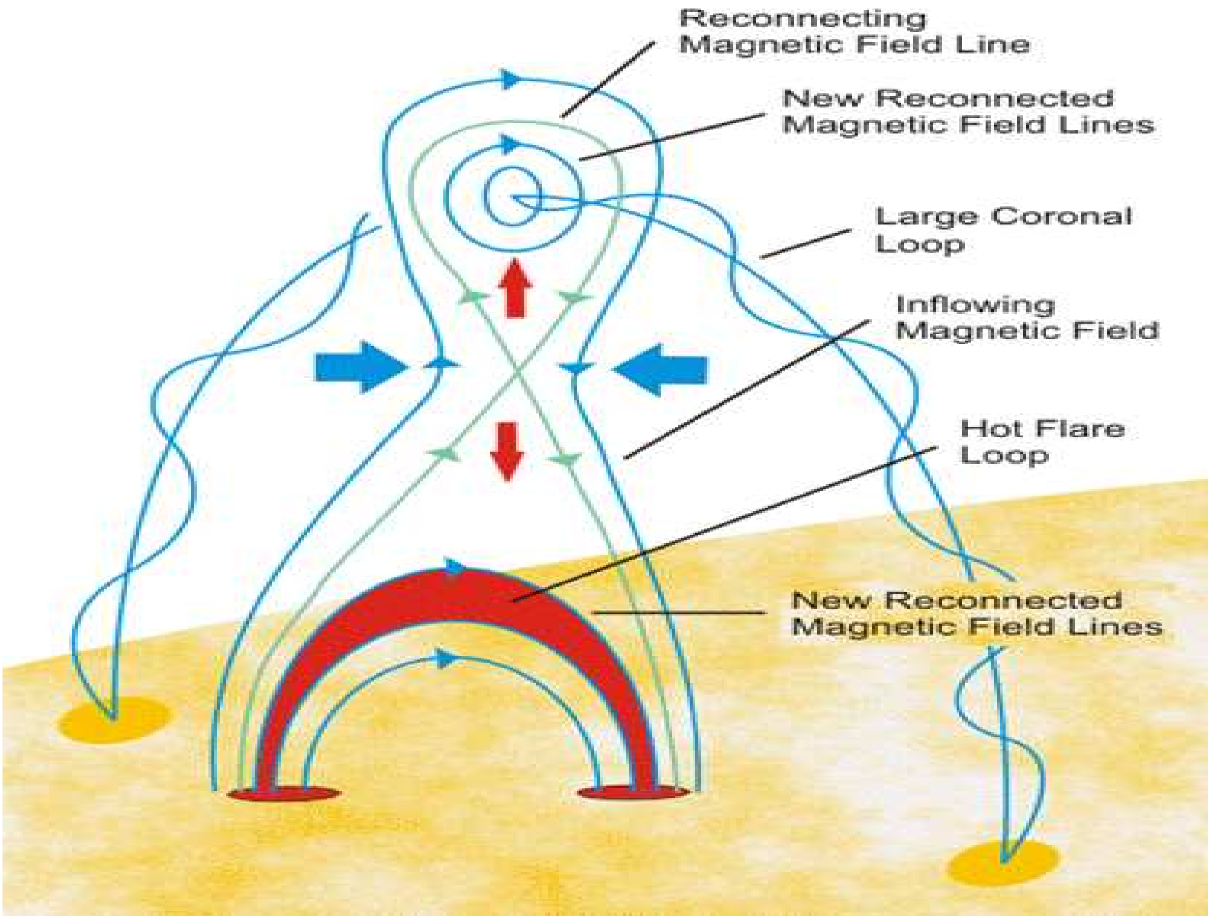

Figure 5: Magnetic reconnection in a coronal loop

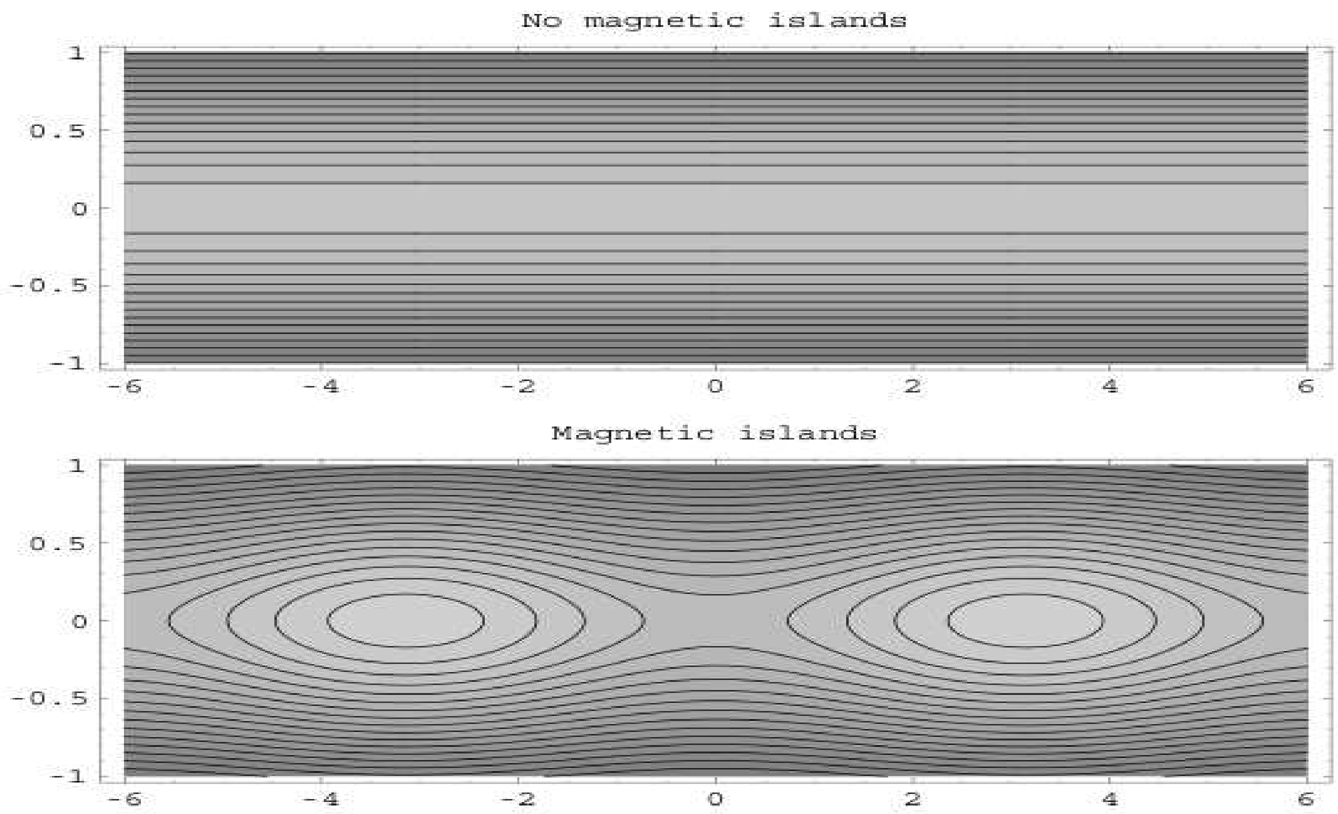

Figure 6: Magnetic island formation

"Opportunistic" development of the Kelvin-Helmoltz instability in a reconnection process and its timing

Let us consider the case of a plasma with a homogeneous density distribution in an initially static (no velocity field) configuration with a sheared magnetic field and an inhomogeneous current distribution such that magnetic reconnection modes can be unstable (positive Δ'). The development of the reconnection instabilities leads to the formation of convective cells and of vorticity layers where the Kelvin-Helmholtz instability finds the "opportunity" to develop.
This process was investigated[10] in a 2-D magnetic configuration with a shear field with a null line and a large guide field within a dissipationless fluid model.

[10] Cafaro E *et al.* 1998 *Phys. Rev. Lett.* **80** 4430 : Grasso D *et al.* 2001 *Phys. Rev. Lett.* **86** 505 ; Del Sarto D *et al.* 2003 *Phys. Rev. Lett.* **91** 235001; Pegoraro F *et al.* 2004 *Nonlinear Processes in Geophysics* **11** 567; Del Sarto D *et al.* 2005 *Phys. Plasmas* **12** 2317.

These investigations showed that electron compressibility slows down the development of the Kelvin-Helmholtz instability with the result that in the case of hot electrons, $\beta_e > m_e/M_i$, the establishment of a new macroscopic "equilibrium" corresponding to the saturation of the island growth, is laminar and produces a redistribution of the current and vorticity into increasingly thinner filaments.

On the contrary in the cold electron limit the Kelvin-Helmholtz instability has time to develop before a macroscopic magnetic "equilibrium" is reached and it disrupts the formation of regular current and vorticity filaments.

Indeed in this latter case there is a transition from an MHD regime to a HD regime, with the transfer of magnetic energy into plasma kinetic energy in the form of long lived fluid vortices.

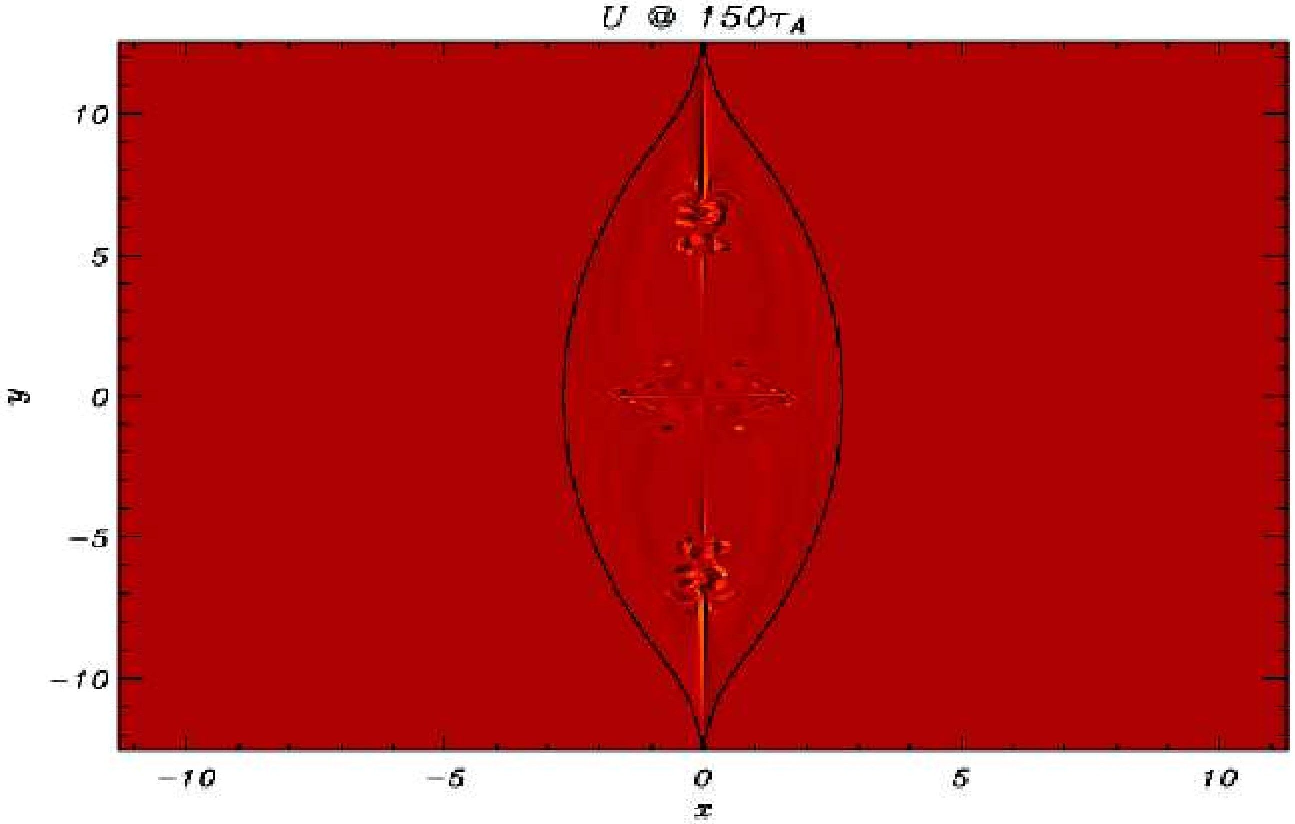

Figure 7: Vorticity structures inside a magnetic island

In the complex evolving velocity and magnetic field configuration that is produced by the development of the reconnection instability it is not easy to decide whether the Kelvin-Helmholtz instability is fully stabilized as the effects of electron compressibility become dominant.

The important point is that when the Kelvin-Helmholtz instability does not occur on a time scale that is as fast as reconnection it is not capable of forming, for example, well developed fluid vortices inside the reconnected magnetic island.

Inhomogeneous magnetized plasmas with velocity shear

The Kelvin-Helmholtz, the Rayleigh-Taylor and the Magnetic Reconnection instabilities are the main actors on the stage of the nonlinear dynamics of a magnetized plasma, as they can be driven directly or indirectly, i.e., in the form of "secondary instabilities", by inhomogeneities in the plasma velocity, in the plasma density (or pressure) and in the plasma currents.
In general, as investigated extensively in the literature, these instabilities do not occur separately and the final state of the system depends on their nonlinear interaction.

This complex interaction is governed by the time scales of the different processes at play, which involve both large spatial scales from which the initial drive of the instabilities originates, and small spatial scales where e.g., magnetic field line reconnection can occur.

Again, the timing between the two instabilities will determine the structure of the final configuration that the system can reach.

The qualitative differences (not simply quantitative differences) between the possible final states are evident and can in principle be used as a diagnostic tool in order to determine experimentally how fast reconnection evolves in the different plasma regimes on the clock set by the evolution of the Kelvin-Helmholtz instability.

Configurations with a velocity shear and a density gradient

Here I shall focus my attention on the interplay between the Kelvin-Helmholtz instability and an opportunistic Rayleigh-Taylor instability driven by the acceleration inside the rotating vortices in the presence of a plasma density gradient. Such a configuration is of interest for the case of the interaction of the solar wind with the Earth magnetosphere in regions where magnetic reconnection is not expected to occur.

Two-dimensional simulations of the Kelvin-Helmholtz instability[11] show that in an inhomogeneous compressible plasma with a density gradient (in a transverse magnetic field configuration) the vortex pairing process and the Rayleigh-Taylor secondary instability compete during the non-linear evolution of the vortices.

[11] M. Faganello, F. Califano, F. Pegoraro, *Phys. Rev. Lett.*, **100**, 015001, (2008)

Two different regimes exist depending on the value of the density jump across the velocity shear layer.
These regimes have different physical signatures that can be crucial for the interpretation of satellite data of the interaction between the solar wind and the magnetospheric plasma.

The magnetospheric case

The Kelvin-Helmholtz instability has been shown[12] to play a crucial role in the interaction between the solar wind and the Earth's magnetosphere and to provide a mechanism by which the solar wind can enter the Earth's magnetosphere.

[12] H. Hasegawa *et al.*, Nature **430**, 755 (2004).

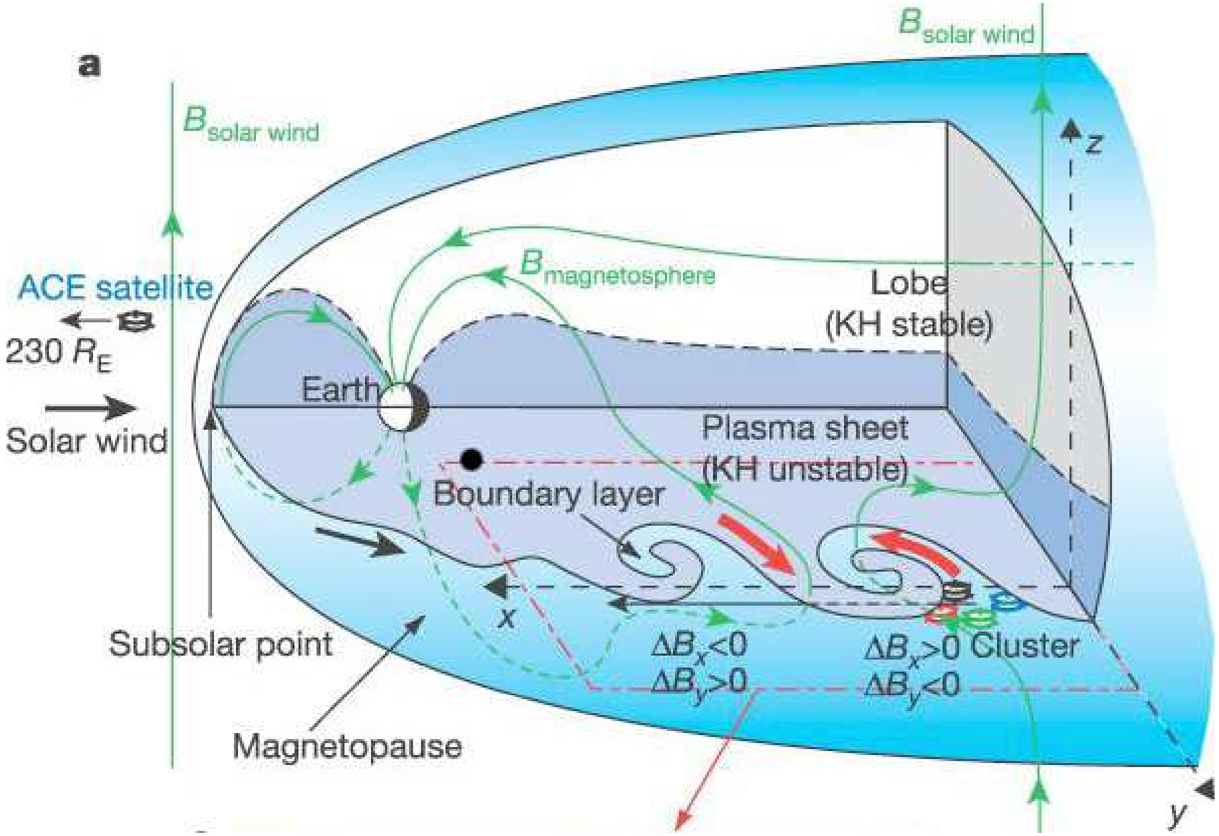

Figure 8: Solar wind interaction with the Earth magnetosphere, from H. Hasegawa *et al.*, Nature **430**, 755 (2004).

Magnetic reconnection is believed to dominate the transport properties at the low latitude magnetopause when the field in the solar wind and the geomagnetic field are antiparallel (southward solar wind magnetic field). If magnetic reconnection were the only mixing mechanism in the magnetotail, one would expect that mixing between the solar wind and the magnetospheric plasma would not occur during northward magnetic field periods.

Actually, an increase of the plasma content in the outer magnetosphere during northward magnetic field periods is not only observed but is even larger than during southward configurations[13].

For these reasons, the Kelvin-Helmholtz instability has been invoked as a possible mechanism in order to account for the increase of the plasma transport. In particular, the Kelvin-Helmholtz instability can grow along the flank magnetopause at low latitude, where a velocity shear exists and where the nearly perpendicular magnetic field does not inhibit the development of the instability[14].

[13] D. G. Mitchell, J. Geophys. Res. **92**, 7394 (1987); H. Hasegawa *et al.*, Geophys. Res.Lett. **431**, L06802 (2004); H. Hasegawa *et al.*, Nature **430**, 755 (2004).

[14] A. Miura, Phys. Rev. Lett. **16**, 779 (1982); J.U. Brackbill *et al.*, Phys. Rev. Lett.**86**, 2329 (2001); C.

The Kelvin-Helmholtz instability provides an efficient mechanism for the formation of a mixing layer and for the entry of solar plasma into the magnetosphere, explaining the efficient transport during northward solar wind periods.

Several observations support this explanation and show that the physical quantities observed along the flank magnetopause at low latitude are compatible with a Kelvin-Helmholtz vortex[15].

Hashimoto *et al.*, Adv. Space. Res **37**, 527 (2006)

[15] H. Hasegawa *et al.*, Nature **430**, 755 (2004); D.H. Fairfield *et al.*, J. Geophys. Res. **105**, 21159 (2000); A. Otto *et al.*, J. Geophys. Res. **105**, 21175 (2000).

Vortex pairing and density inhomogeneity

Vortex pairing is believed to be the major process causing the increase in the thickness of the mixing layer in the downstream region of the magnetotail. This inverse cascade is a well known phenomenon in two-dimensional HD[16], and can be expected to be an efficient process in the nearly two-dimensional external region of the magnetopause at low latitude[17]. However, the density inhomogeneity in the layer between the solar wind and the magnetosphere strongly modifies the non-linear evolution of the Kelvin-Helmholtz instability and makes the Kelvin-Helmholtz vortices produce a different type of secondary instability which quickly leads to the onset of turbulence[18] in the system.

[16] C.D. Winant *et al.*, J. Fluid Mech. **63**, 237 (1974); F.K. Browand *et al.*, J. Fluid Mech.. **76**, 127 (1976).
[17] A. Miura, Phys. Plasmas **4**, 2871 (1997) ; A. Otto *et al.*, J. Geophys. Res. **105**, 21175 (2000).
[18] W.D. Smyth, J. Fluid Mech. **497**, 67 (2003); Y. Matsumoto *et al.*, Geophys. Res. Lett. **31**, 2807 (2004).

Indeed the centrifugal acceleration of the rotating Kelvin-Helmholtz vortex acts as an "effective" gravity force on the plasma. If the density variation is large enough, the Rayleigh-Taylor instability can grow effectively along the vortex arms.

How quickly the vortex becomes turbulent is crucial since the turbulence caused by the onset of the Kelvin-Helmholtz secondary instability may destroy the structure of the vortices before they coalesce and may thus be the major cause of the increase in the width of the layer with increasing velocity and density inhomogeneity.

Rolled-up vortices, generated by the Fast Growing Mode and entering in the non-linear stage, could then evolve following an inverse cascade process, or by developing secondary instabilities, as for example the Rayleigh-Taylor instability or Vortex Induced Reconnection[19].

[19] A. Otto *et al.*, J. Geophys. Res. **105**, 21175 (2000); Z.X. Liu *et al.*, , Geophys. Res. Lett. **15**, 752 (1988).

The role of Vortex Induced Reconnection will be discussed at the end of this presentation. Here I will consider the initial magnetic field to be perpendicular to the plane where the Kelvin-Helmholtz instability develops and to have no inversion points, i.e. we do not include magnetic reconnection effects.

For a relatively moderate initial density jump, $\Delta n = 0.5$, Fig.9, top row, the formation and the evolution of two HD vortices. We see (left frame) that two vortices start to interact (middle frame) following an inverse cascade. Eventually, the vortices merge generating a single vortex (right frame).
In the case of a large initial density jump, $\Delta n = 0.8$, the evolution of the system is strongly affected by the development of secondary Rayleigh-Taylor instabilities inside the vortex arms. After the generation of the two Kelvin-Helmholtz vortices, bottom row of Fig.9 left frame, the Rayleigh-Taylor instability starts to develop inside the arms of the vortices, leading to the formation of a turbulent layer along the y-direction with typical transverse width of the order of the vortex size. The vortex pairing interaction is depressed by the onset of the secondary instability.

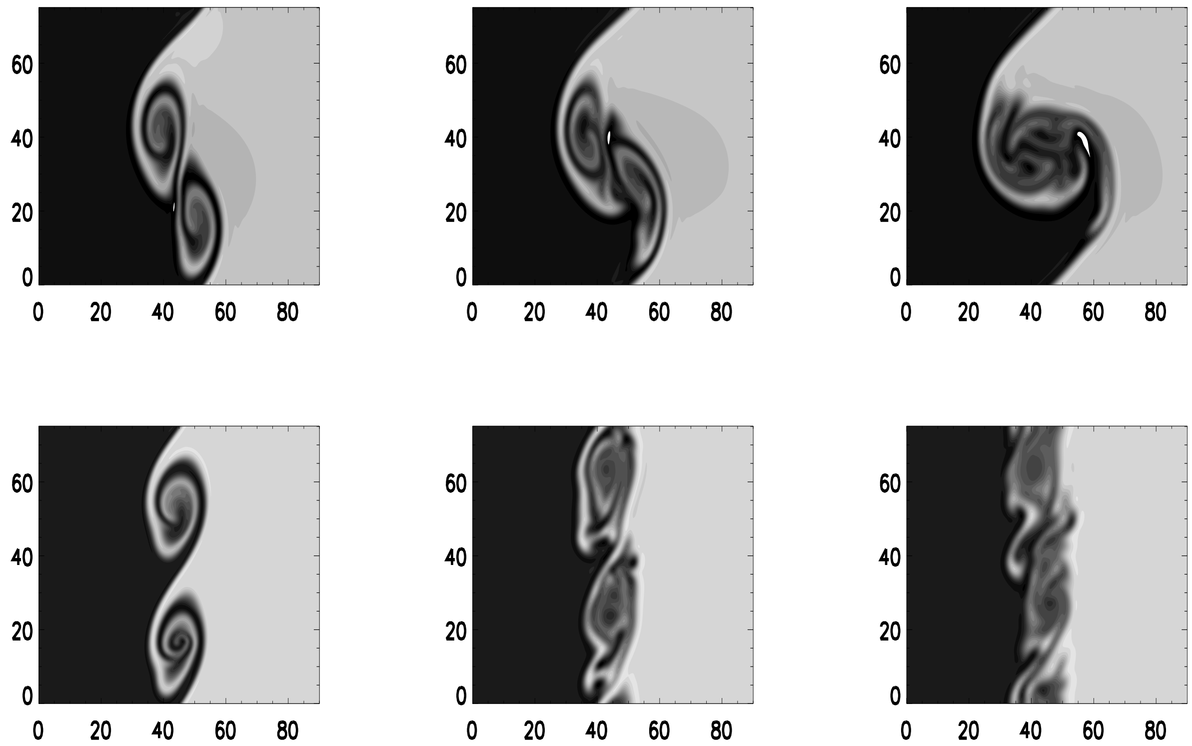

Figure 9: Shaded isocontours in the (x, y) plane of the plasma density for an initial density jump $\Delta n = 0.5$ (first row) at $t = 370, 395, 425$ and for $\Delta n =$ 0.8 (second row) at $t = 350, 392, 425$.

Time scale of the secondary instability and final state

In order to estimate the growth time of this secondary instability, we may consider each of the two vortices generated by the Kelvin-Helmholtz instability separately and assume such a structure to be stationary in the time interval $300 \leq t \leq 350$. During this period, the vortex propagates along the y-direction with a constant phase velocity and nearly constant amplitude. We may model the vortex at $t = 300$ as an "equilibrium" configuration and take the values n_1, u_1 and n_2, u_2 inside two nearby vortex arms connected to the more and to the less dense parts of the plasma respectively, as the density and velocity values of two superposed inhomogeneous fluid plasmas in a slab geometry. In this model, the two plasma slabs are subjected to an "effective" gravity which corresponds to the centripetal acceleration arising from the arms curvature.
Taking into account the profiles of the velocity shear and of the density of the vortex, we find that the estimate of the growth rate of the Rayleigh-Taylor instability is in agreement with that observed in the numerical simulations. The competition between the vortex pairing process and the development of the Rayleigh-Taylor instability has important consequences from an observational point of view and can affect the transport properties of the system.

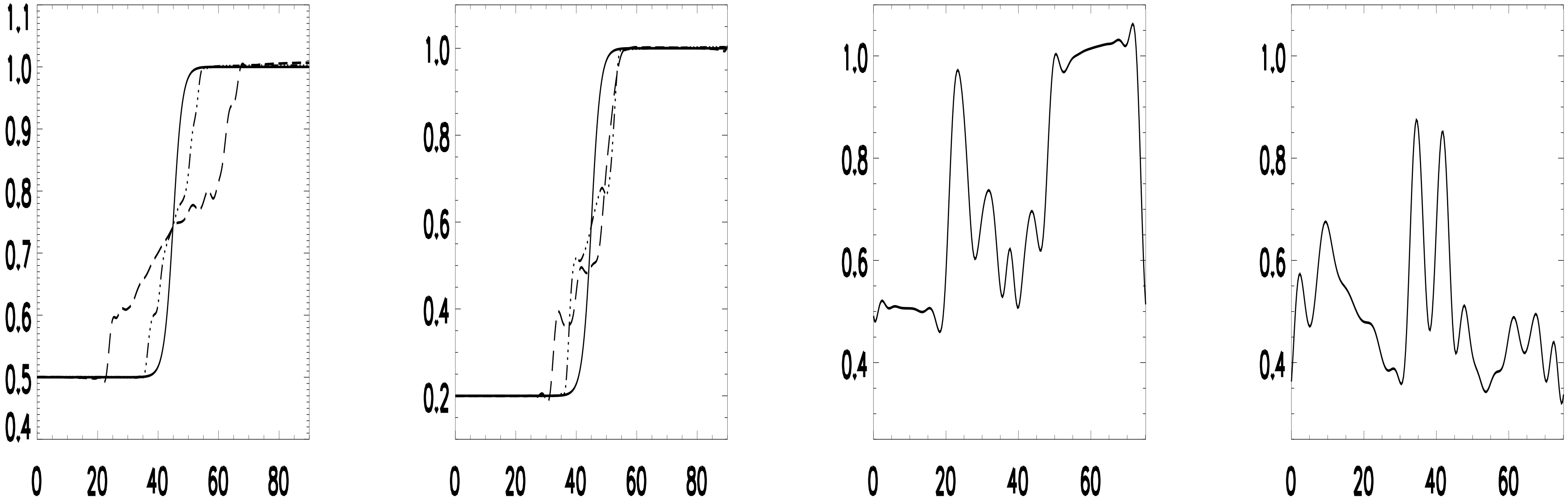

Figure 10: Plasma average density profile along the inhomogeneity direction at $t = 0$ continuous, $t = 310$ dash - three dotted, $t = 425$ dashed line, and along the solar wind direction at $t = 425$, for an initial density jump $\Delta n = 0.5$ and for $\Delta n = 0.8$.

In Fig.10, first left, by taking the average density value in the y direction, we observe in the pairing case a density profile inclined in the x direction with a thickness directly related to the size of the vortex which corresponds to the the $m = 2$ mode at $t = 310$ and to the the $m = 1$ at $t = 425$.
In the "turbulent" case instead, (see Fig.10 second left), a "plateau" is formed in the central region of the initial sheared layer. We also note an asymmetric evolution of the average density profile, indicating a diffusion of the plasma from the dense to the tenuous region, and a typical thickness of this mixing layer comparable with the Fast Growing Mode vortex size.
In Fig.10, first right, we show the density profile along the y direction at $x = 45.0$ which corresponds approximately to the center of the layer.

We see at $t = 425$ that the density profile is characterized by well defined structures with typical length of the order of $L_y/3$ consisting of the well known step-like configuration that is directly related to the two vortex arms connected to the more and to the less dense parts of the plasma, and of a filament-like configuration that is related to the more complex central region of the vortex. This profile exhibits a well-defined periodicity, given by the wave length of the vortex, and is the signature of a rolled-up vortex. It corresponds either to the Fast Growing Mode or to its sub-harmonics generated by the inverse cascade.

On the other hand, in the case where the secondary instability develops, a sequence of alternating high and low density filaments that do not exhibit a well defined wave length is observed (see Fig.10, second right).
This fact is related to the transition of the system to a turbulent state with the formation of a mixing layer via the development of smaller and smaller structures.

Configurations with a velocity shear and an in plane magnetic field. How fast is the reconnection process?

In a magnetized plasma streaming with a nonuniform velocity, the Kelvin-Helmholtz (K-H) instability plays a major role not only by mixing the plasma but also by stretching the magnetic field lines leading to the formation of layers with a sheared magnetic field where magnetic field line reconnection can take place.

We refer again to the formation of a mixing layer between the Earth's magnetosphere and the solar wind at low latitudes during northward periods when the solar wind and the geomagnetic magnetic fields are parallel. As already discussed velocity shear, in the presence of a magnetic field nearly perpendicular to the plane defined by the velocity field and its inhomogeneity directions, makes the K-H instability grow along the flank magnetosphere.

If the Alfvén velocity associated to the in-plane magnetic field is sufficiently weak with respect to the fluid velocity jump, the K-H instability forms fully rolled-up vortices which advect the magnetic field lines into a complex magnetic configuration. This advection causes the in-plane magnetic field component to develop inversion points where magnetic reconnection can take place. Since the plasma dynamics is essentially driven by the vortex motion, these reconnection events are usually denoted as Vortex Induced Reconnection (VIR)[20],
Although magnetic reconnection occurs only locally, this process changes the global topology of magnetic field, which is a necessary condition for plasma mixing, and thus the evolution of the vortices themselves.

[20]A. Otto *et al.*, J. Geophys. Res. **105**, 21175 (2000);
H. Baty *et al.*, Phys. of Plasmas **10**, 4661 (2003);
T.K.M. Nakamura *et al.*, Geophys. Res. Lett. **32**, 21102 (2005);
T.K.M. Nakamura *et al.*, Adv. Space. Res. **37**, 522 (2006);
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In the last part of my lecture we will investigate the development of magnetic reconnection during the vortex pairing process and show that completely different magnetic structures are produced depending on how fast the reconnection process develops on the time scale set by the pairing process.

We consider a configuration with a value of the plasma β parameter (defined as the ratio of the plasma pressure over the total magnetic field pressure) of order unity and show that in this regime the Hall term in Ohm's law, which arises from the decoupling of electrons and ions inside the current layers, allows magnetic reconnection to occur on time scales fast enough to compete with the pairing process.
In these numerical simulations the conditions for magnetic reconnection are naturally provided, in an initially uniform in-plane magnetic field, by the motion of the K-H vortices that grow and pair in the initially imposed shear velocity field.

We find that if the Hall term is removed from Ohm's law, the development of reconnection, and thus eventually of the K-H vortices, is qualitatively, not only quantitatively, different.
This result provides a clear cut example of the feedback between large and small scale physics, as the necessary conditions for reconnection to occur are produced by the large scale vortices motion, but the specific physical processes that make reconnection act faster or slower determine eventually the evolution of the entire system and the final magnetic field structure.

We consider a 2D description of the system, with the inhomogeneity direction along x, the periodic direction along y, and z an ignorable coordinate. We adopt a two-fluid, quasineutral plasma model. The electric field $\mathbf{E}$ is calculated by means of the following generalized Ohm's law[21] ($\mathbf{u_e}$ electron velocity, $\mathbf{u_i}$ ion velocity, $\mathbf{j} = ne(\mathbf{u_i} - \mathbf{u_e})$)

$$\left(1 - d_e^2\,\nabla^2\right)\mathbf{E} = -\mathbf{u}_i \times \mathbf{B} + \frac{1}{n}\,\mathbf{j} \times \mathbf{B} - \frac{1}{n}\nabla P_e$$

[21] see e.g., F. Valentini *et al.*, J.Comp. Phys. **225**, 753 (2007)

where we use the following normalization quantites: $\bar{u} = u_A$; $\bar{\omega} = \Omega_i$; $\bar{l} = u_A/\Omega_i = c/\omega_i = d_i$; $\bar{n}$; $\bar{P}_{p/e} = \bar{n} m_i u_A^2$; $\bar{E} = m_i u_A \Omega_i/e$; $\bar{B} = m_i c \Omega_i/e$;
We consider an initial large-scale, sheared velocity field given by $\mathbf{U}_{eq} = (U_0/2)\,\tanh\left[(x - L_x/2)/L_u\right]\hat{\mathbf{y}}$.
Since we are primarily interested in the reconnection process, we consider a homogeneous density field in order to eliminate other secondary fluid instabilities.
The equilibrium magnetic field at $t = 0$ is homogeneous and given by $\mathbf{B}_{eq}(x, y) = B_{y,eq}\mathbf{e}_y + B_{z,eq}\mathbf{e}_z$. We take $L_u = 3.0$ and the box-length in the x direction $L_x = 90$. The box length in the periodic y-direction is $L_y = 30\pi$ in order to have well separated linear growth rates for the modes $m = 1, 2, 3$, where $m = 2$ corresponds to the Fast Growing Mode (FGM) of the K-H instability. We take $d_e^2 = m_e/m_i = 1/64$. The values of the dimensionless sound and Alfvén Mach numbers are set as $M_s = U_0/C_s = 1.0$, $M_{A,\perp} = U_0/U_{A,\perp} = 1.0$, $M_{A,\parallel} = U_0/U_{A,\parallel} = 20.0$, with $U_0 = 1.0$ and $U_{A,\perp}$, $U_{A,\parallel}$ the z and y component of the equilibrium Alfvén velocity, respectively.
This choice allows the K-H instability to develop into highly rolled-up vortices.

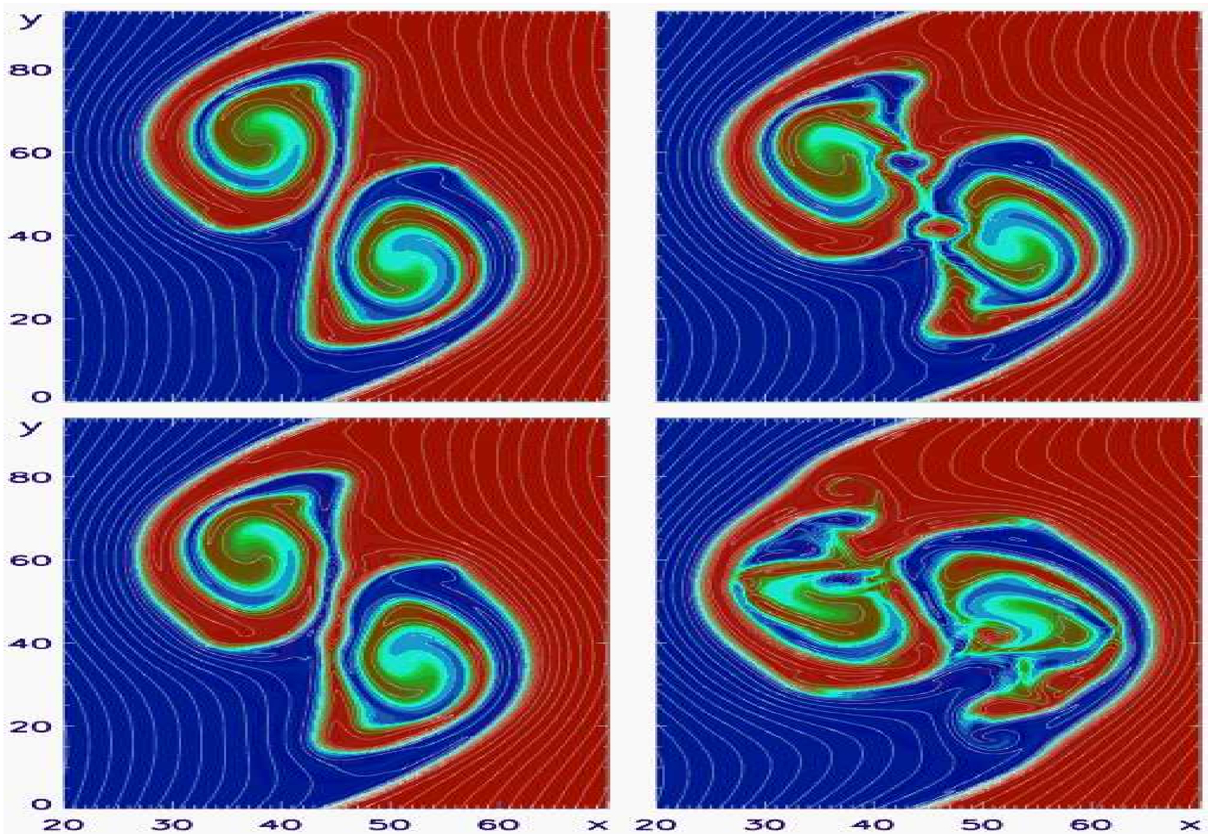

Figure 11: Plasma passive tracer and magnetic field lines in the (x, y) plane at $t = 426,\ 435$ (left), and $t = 460,\ 480$ (right).

In Fig.11 we show the nearly *frozen-in* magnetic field lines and a plasma passive tracer, advected by the velocity field, which is used in order to label the plasma domains initially on the right and on the left of the velocity null line. In the first frame (upper-left), we see the two vortices generated by the K-H instability, corresponding to the FGM wave number $m = 2$. As soon as the system enters in the non-linear phase of the K-H instability, the vortices start to pair following an inverse cascade process typical of 2D fluid systems. The good correlation between the plasma and the magnetic structures indicates that the magnetic field is still advected by the fluid velocity. In the first frame we see that the blue and the red domain are well separated by a $ribbon$ (white in the figure) of nearly parallel, compressed magnetic lines. This ribbon is rolled-up by the rotation of the two vortices, and, forms inside the folds between the vortices two current layers corresponding to two local magnetic inversion lines.

We also see the formation of a first couple of X-points (one for vortex) in this region at $x_1 = 44$, $y_1 = 65$ and at $x_2 = 45$, $y_2 = 35$, respectively. This is the first reconnection event observed in our simulation. At t = 440 we show in the second frame (left-bottom) the formation of a second pair of X-points (one for vortex) in the same inversion region ($x_3 = 44$, $y_3 = 50$ and $x_4 = 45$, $y_4 = 47$). Magnetic reconnection develops at these X-points and forms magnetic islands with typical size $\sim d_i$, the maximum value compatible with the dimension of the current sheet. At the same time, the field line ribbon between the second pair of X-points shrinks and finally opens up. A new ribbon of field lines appears at $t = 450$, right top frame. This new ribbon no longer separates the red and the blue plasma regions. Indeed, during this process, significant portions of the red plasma have been engulfed in the form of "blobs" into the blue plasma region and viceversa (right-bottom).

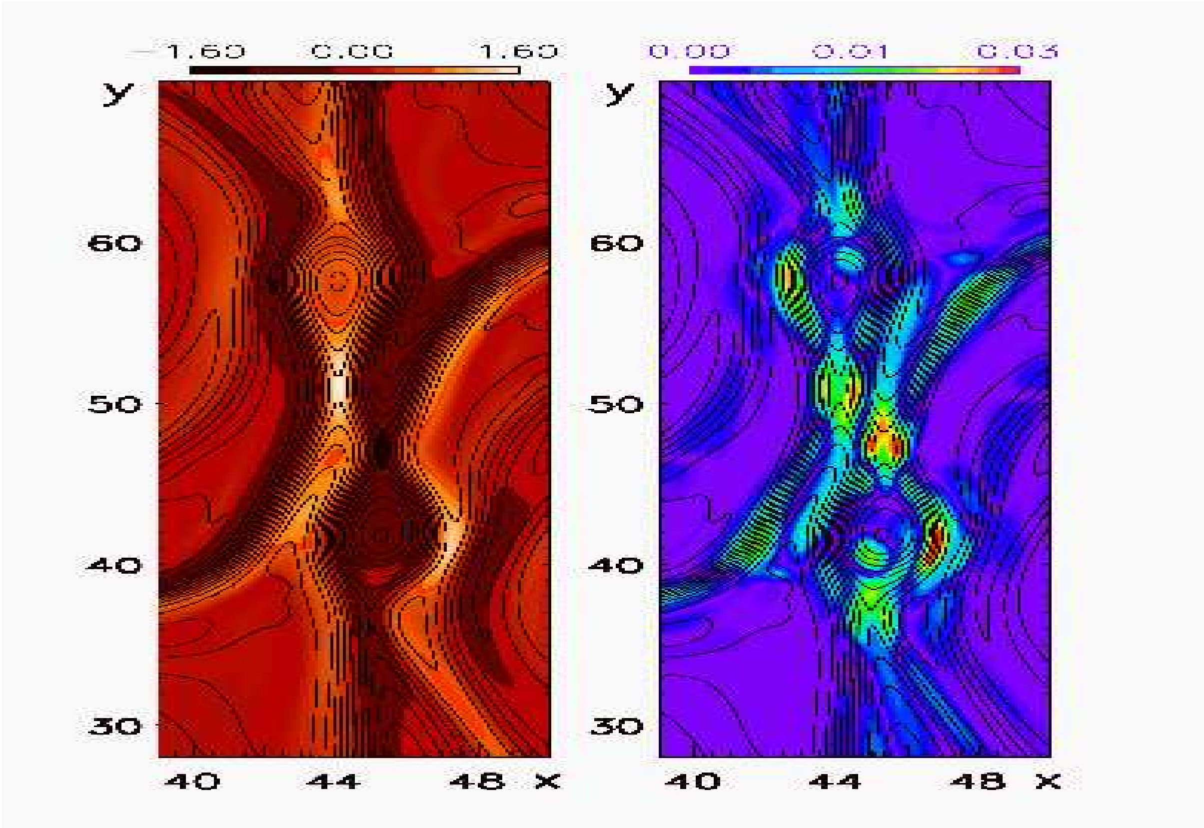

Figure 12: Left frame: shaded isocontours of the perpendicular current density and magnetic field lines in the region between the two pairing vortices at $t = 440$. Right frame: Ion decoupling region and magnetic field lines in the same region at $t = 440$.

The inflow plasma velocity at the second pair of X-points is approximately 0.1 times the value of the local Alfvén velocity U_a in the x-y plane, in agreement with the values of the inflow velocity expected in the case of fast magnetic reconnection.
Note that in the time interval given by a few growth times (a few times γ^{-1}) of the reconnection instability, the two vortices can only rotate by a few degrees so that the plasma displacement and the current rearrangement caused by the vortex rolling up are not sufficient to interfere with the development of the reconnection process.

In Fig.12, right frame, we show a wide ion decoupling region.
This region extends across the X-points over few d_i lengths. Inside the ion decoupling region, of width roughly equal to $1.5d_i$, the magnetic field is essentially frozen in the electron motion but the MHD frozen-in law is not satisfied and the two terms $\mathbf{U} \times \mathbf{B}$ and $\mathbf{J} \times \mathbf{B}$ have approximately the same absolute value.

Inside the thinner electron region the electrons are also decoupled from the magnetic field and magnetic reconnection can take place.

The role of the Hall term has also been demonstrated by running again the same simulation parameters simply omitting the Hall term in the generalized Ohm's law. Although the large scale motion of the vortices is still able to generate current sheets of comparable intensity and width, the process of magnetic reconnection is slower. In particular it does not succeed in forming the second pair of X-points in Fig.11 quickly enough.

The two vortices continue to roll-up while pairing and develop into a different magnetic pattern where magnetic islands are eventually generated all over the vortices leading to the disruption of the vortices.

The rolling up and pairing of the vortices develops on a time scale comparable with the reconnection time and affects its evolution.

This competition between the development of the large scale magnetic configuration and the evolution and the reconnection instability determine the development of the entire system. If magnetic reconnection is not fast enough, the rolling up of the vortices destroys the favorable conditions for the reconnection instability to grow.

Conclusions

The competition between primary and secondary instabilities can lead to qualitatively different physical states.

These differences can be observed experimentally.

The outcome of this competition can give us important pieces of information on the time development of the instabilities as we can use one to clock the development of the other.