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Nonlinear electromagnetic waves in electron-positron plasmas

By

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Outline

- A brief background of electron-positron plasmas? (Where do they occur? why are they of interest?)
- Propagation parallel to the magnetic field (Equations, conservation laws, etc)
- Propagation perpendicular to the magnetic field (Some specific results on Photon splitting.)
- Conclusions and outlook

Why are electron positron plasmas of interest?

- Their properties are different from ordinary electron-ion plasmas, for example they lack the slow time-scale connected with ion motion, Raman scattering is suppressed during certain conditions, etc.

- Electron-positron plasmas introduce new physics, e.g. creation and annihilation of particles.

- Sometimes the multiple time-scales of electron-ion plasmas are too difficult (for example numerically), and one can start by studying a simpler case.

- Studies of electron-positron plasmas are needed for an increased understanding of the early universe, pulsar atmospheres, and might be part also of future laboratory experiments.

Where can electron positron plasmas occur?

- The early universe (10^{-4} seconds until 15 seconds after big bang) is dominated by an electron positron plasma (temperature after 1 second 10^{10} K).

- Pulsar (and magnetar) environments: Pair-production by high-energy photons close to the surface lead to presence of an electron-positron pair plasma in pulsar and magnetar atmospheres (cf. the Goldreich-Julian expression for the pair-plasma density).

- Laboratory plasmas 1: Oppositely propagating high intensity laser beams hitting a thin gold foil may produce an ep-plasma with a positron density $5\times10^{22}\,\mathrm{cm}^{-3}$.
  (Shen and Meyer-ter-Vehn, Phys. Rev. E, **65**, 016405 (2001))

- Laboratory plasmas 2: Pair production from colliding pulses of the next generation of high intensity lasers (intensities need to approach the Schwinger limit).

Wave propagation parallel to the magnetic field

Important building blocks in a general theory for nonlinear EM wave propagation:

- A general expression for the ponderomotive force:

$$= \frac{e^2\omega}{m(\omega+\omega_c)c^2}\left[\frac{\partial}{\partial z}+\frac{k\omega_c}{\omega(\omega+\omega_c)}\frac{\partial}{\partial t}\right]|A|^2$$

V. I. Karpman and H Washimi, J. Plasma Phys. **18**, 173 (1977).

- Inclusion of relativistic nonlinearities, low-frequency electrostatic oscillations and second harmonics in a warm multi-component plasma

L. Stenflo, P. K. Shukla and M. Y. Yu, Astrophys. Space Sci, **117**, 303 (1985).

Wave propagation parallel to the magnetic field

Adaption of the general theory using that

- the plasma is a three component electron-positron-ion plasma
- a wave frequency much smaller than the electron cyclotron frequency (appropriate for pulsar and magnetar atmospheres where magnetic field is of the order $10^8 - 10^{10}$ T)
- the high-frequency pulse is moving with the group velocity (to first order in an amplitude expansion)

leads to a coupled set for the high-frequency vector potential amplitude A and the low-frequency electrostatic potential Φ:

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\omega_{ptot}^2}{c^2-v_t^2}\right)\phi=\frac{e|A|^2\omega^2}{2m_e}\left(\frac{\Omega_i^2}{\omega_c^2}+\frac{\omega_{ptot}^2\omega}{\omega_c^3}\right)$$

$$i\left(\partial_t+v_g\partial_z\right)A+\frac{1}{2}v_g'\partial_z^2A+\frac{\omega}{2\omega_{cp}^2}\left[\frac{n_{0i}e^3}{m_e^2c^2}\phi A-\sum_{e,p}\frac{e^2\omega^2\omega_p^2}{m_e^2c^2\omega_c^2}\left(\frac{v_t^2}{c^2}-\frac{2\omega}{\omega_c}\right)|A|^2A\right]=0.$$

Wave propagation parallel to the magnetic field

The equations can be put in the generic form

$$\mathrm{i}\frac{\partial A}{\partial \tau} + \frac{\partial^2 A}{\partial \xi^2} = -A\left(\Phi - \alpha\left|A\right|^2\right)$$

$$\frac{\partial^2 \Phi}{\partial \xi^2} + \Phi = \left|A\right|^2$$

where α is a constant that determines the relative importance of the self-nonlinearity. The system posses three conservations laws (Brodin and Lundberg, Phys. Rev. E , **57**, 7041 (1998).).

$$\frac{d}{d\tau}\int_{-\infty}^{\infty}\left|A\right|^2 d\xi = 0$$

(Number of high frequency quanta)

$$\frac{d}{d\tau}\int_{-\infty}^{\infty}\left(\left|\frac{\partial A}{\partial \xi}\right|^2 - \frac{\alpha}{2}\left|A\right|^4 | - \left|A\right|^2\Phi\right) d\xi = 0$$

(The Hamiltonian)

$$\frac{d}{d\tau}\int_{\xi_-}^{\xi_+}\left(\frac{\partial A}{\partial \xi}A^* - \frac{\partial A^*}{\partial \xi}A\right) = W_\Phi\Big|_{\xi_-}^{\xi_+}$$

(Energy)

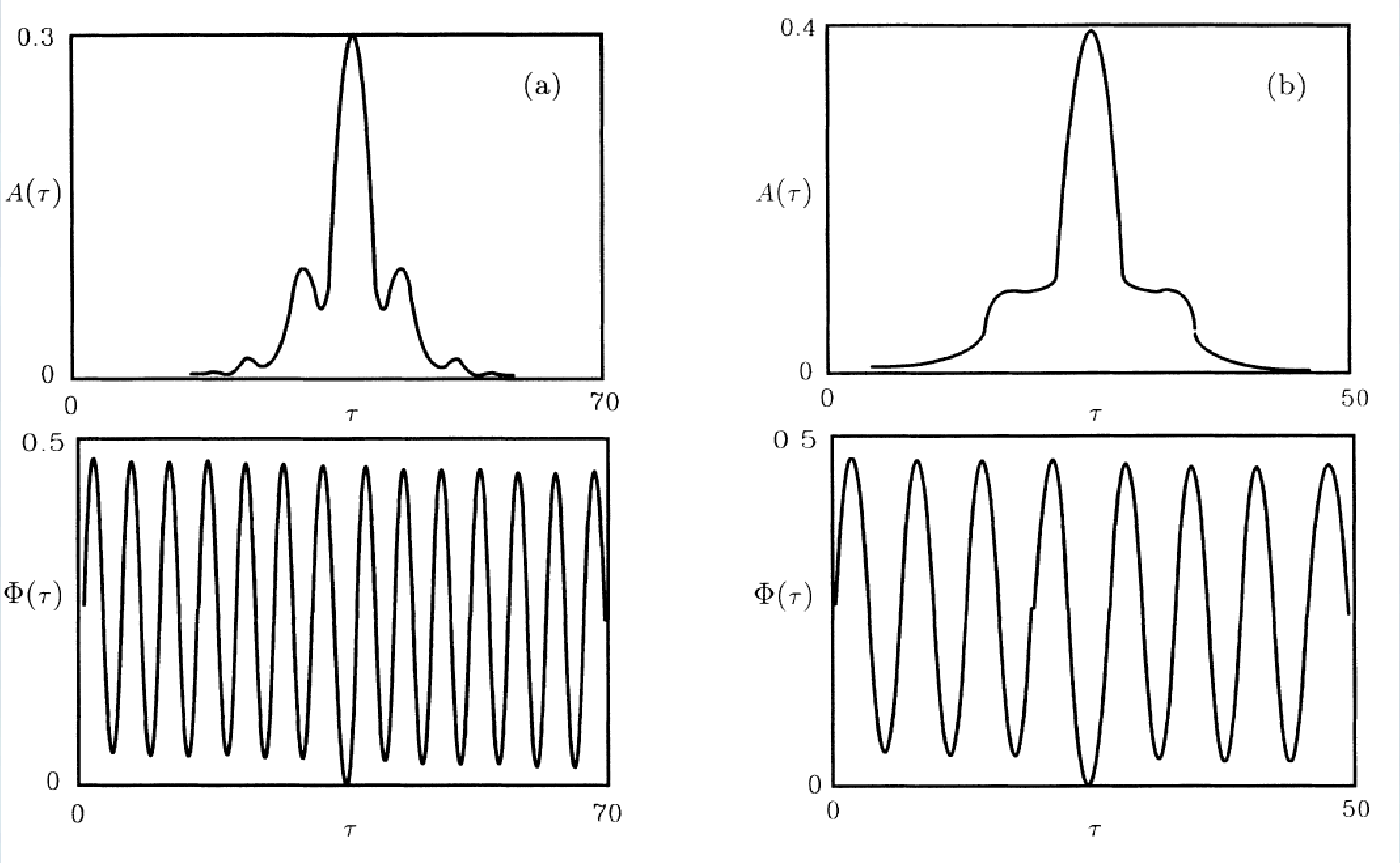
Some properties of the coupled system

When the self-nonlinearity is dominating

- Approximate soliton-formation of the high-frequency pulse (NLS-like)
- De-acceleration, and frequency down-conversion due to the wake-field generation

When the wake field (low-frequency) nonlinearity is dominating

- De-acceleration, and frequency down-conversion due to the wake-field generation
- The possibility to form “electromagnetic polarons” (localized nonlinear wave structures trapped by the wake field “lattice”) for suitable initial conditions.



Mironov *et al*, Phys. Rev. A. **42**, 4862 (1990).

Wave propagation perpendicular to the magnetic field

A complication for astrophysical applications (pulsar and magnetar atmospheres): The medium is nonlinear even without a plasma, as described by the QED vacuum polarization and magnetization:

$$\mathbf{P} = 2\varepsilon_0^2\kappa\left[2\left(E^2 - c^2B^2\right)\mathbf{E} + 7c^2(\mathbf{E}\cdot\mathbf{B})\mathbf{B}\right]$$

$$\mathbf{M} = -2\varepsilon_0^2\kappa\left[2\left(E^2 - c^2B^2\right)\mathbf{B} + 7(\mathbf{E}\cdot\mathbf{B})\mathbf{E}\right]$$

We note that:

- Vacuum polarization and magnetization tend to be important for the photon dynamics when the electromagnetic field strengths approach the Schwinger critical field $E_{cr} \approx 10^{18} V/m$.
- For pulsar field strengths $B_p \approx 10^8 T$ we have $cB_p / E_{cr} \approx 0.03$ whereas for magnetar field strengths $B_m \approx 10^{10} T$ the critical field ratio can even exceed unity.
- The strong magnetar fields is believed to cause QED photon-splitting, which can be the cause of the radio-silence of magnetars. (Baring and Harding, Astrophys. J., **507**, 55 (1998).)
- Pair-production by high-energy photons close to the surface lead to presence of an electron-positron pair plasma in pulsar and magnetar atmospheres (cf. the Goldreich-Julian expression for the pair-plasma density).

Photon splitting in vacuum

Prerequisite: Assume linear wave propagation perpendicular (for the sake of simplicity) to an external magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$.

The QED vacuum polarization and magnetization then lead to the dispersion relations

$$\omega_\perp^2 = k_\perp^2 c^2 \left(1 - 8\xi\right)$$

and

$$\omega_\parallel^2 = k_\parallel^2 c^2 \left(1 - 14\xi\right)$$

where the indices $\perp$ and $\parallel$ denote polarizations with the electric field perpendicular and parallel to the external magnetic field, respectively, and $\xi = \kappa\varepsilon_0 c^2 B_0^2$

Photon splitting in vacuum

The photon splitting: A nonlinear parametric interaction of three waves, satisfying the energy and momentum conservation

$$\omega_{\perp} = \omega_{1\|} + \omega_{2\|}$$
$$k_{\perp} = k_{1\|} + k_{2\|}$$

Note that:

1) We need a reasonably strong pump field $E_{\perp}$ for the process to take place.
2) One of the modes must be backscattered (we will assume $k_{2\|} < 0$)
3) For $\xi \ll 1$ the backscattered mode has a much smaller frequency $\omega_{2\|} \ll \omega_{\perp}$

Accumulated effect of splitting: The central frequency of the spectra decreases, and the radiation become linearly polarized with parallel polarization.

Photon splitting in a pair plasma

Maxwell's equations with the current

$$\mathbf{j} = \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t} + \sum\nolimits_{e,p} qn\mathbf{v},$$

together with the equation of motion for the particles

$$\frac{du^{\mu}}{d\tau} = qF^{\mu\nu}u_{\nu},$$

result in the three coupled equations

$$\frac{\partial E_{\perp}}{\partial t} + v_{g\perp}\frac{\partial E_{\perp}}{\partial x} = \omega_{\perp}\left(C_{pl} + C_{QED}\right)\frac{E_{1\|}E_{2\|}}{E_{cr}}$$

$$\frac{\partial E_{1\|}}{\partial t} + v_{1\|}\frac{\partial E_{1\|}}{\partial x} = \omega_{1\|}\left(C_{pl} + C_{QED}\right)\frac{E_{\perp}E_{2\|}^{*}}{E_{cr}}$$

$$\frac{\partial E_{2\|}}{\partial t} + v_{2\|}\frac{\partial E_{2\|}}{\partial x} = \omega_{2\|}\left(C_{pl} + C_{QED}\right)\frac{E_{\perp}E_{1\|}^{*}}{E_{cr}}$$

Photon splitting in a pair plasma

Where the plasma and QED contributions to the nonlinear coupling are

$$C_{pl} = i\left(\frac{\alpha}{90\pi\xi}\right)^{1/2} \frac{k_\perp c}{\omega_\perp} \frac{\omega_p^2}{\omega_{1\|}\omega_{2\|}}$$

and

$$C_{QED} = i\left(\frac{\alpha\xi}{90\pi}\right)^{1/2} \left[20\frac{k_\perp c}{\omega_\perp} + 14\left(\frac{k_{1\|}c}{\omega_{1\|}} + \frac{k_{2\|}c}{\omega_{2\|}}\right)\right]$$

respectively, and the group velocities for the parallel and perpendicular modes are determined from the dispersion relations

$$\omega_\perp^2 = k_\perp^2 c^2 \left(1 - 8\xi\right)$$

and

$$\omega_\|^2 = k_\|^2 c^2 \left(1 - 14\xi\right) + \omega_p^2$$

Analysis of the coupled equations

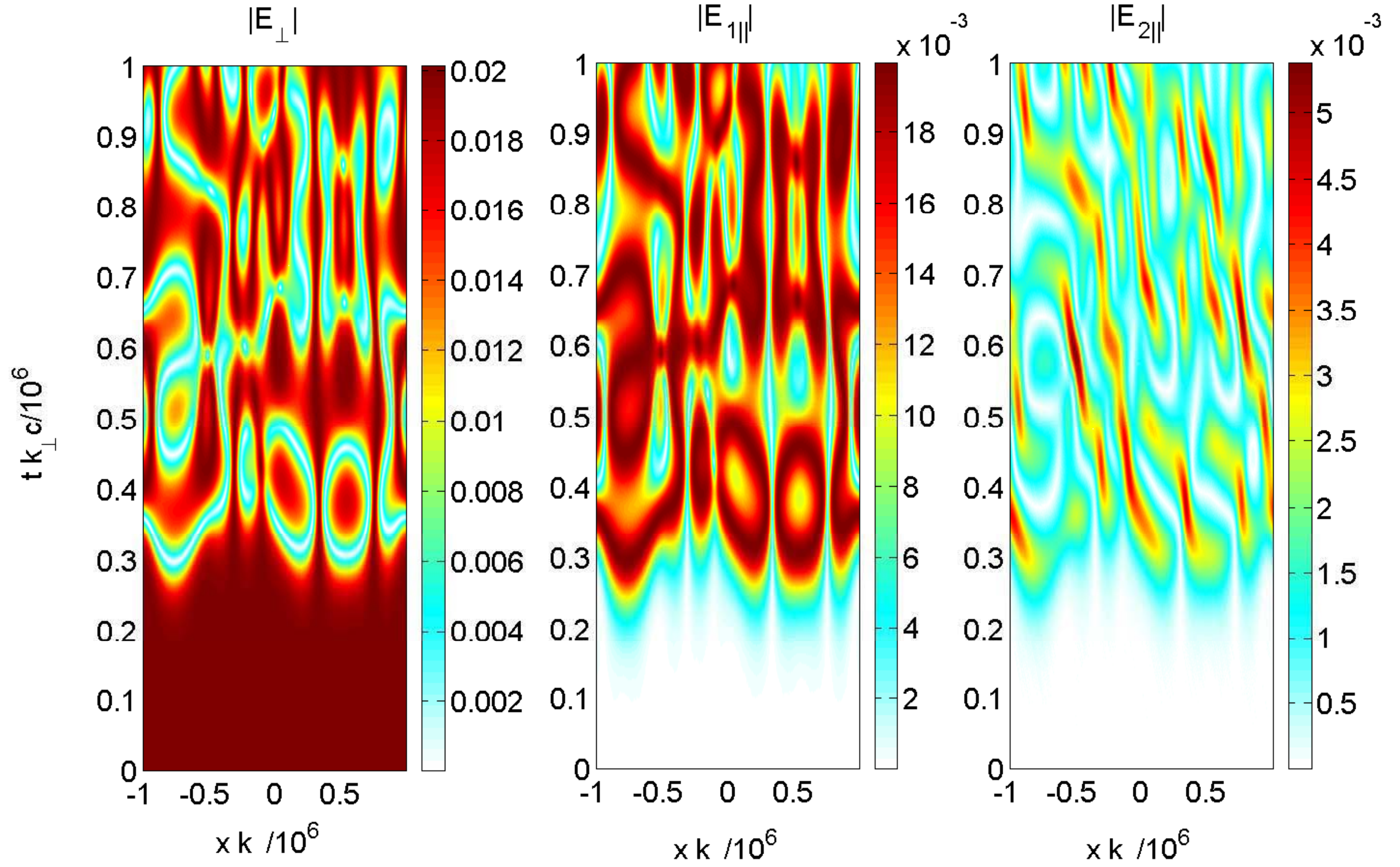
From the three coupled equations we first note that a homogeneous pump wave amplitude $E_{\perp 0}$ subject to small perturbations (of the amplitudes) $E_{1\parallel} \propto \exp[i(Kx - \Omega t)]$ and $E_{2\parallel} \propto \exp[-i(Kx - \Omega t)]$ is unstable with the growth rate

$$\gamma = \sqrt{\omega_{1\parallel}\omega_{2\parallel}\left|C_{pl} + C_{QED}\right|^{2} \frac{\left|E_{\perp 0}^{2}\right|}{E_{cr}^{2}} - \frac{\left(v_{g1} + v_{g2}\right)^{2} K^{2}}{4}}$$

for long-wavelength perturbations.

Analysis of the coupled equations

Case 1: Numerical study of an initially homogeneous pump wave $E_{\perp 0}$ together with initial thermal fluctuations for the parallel polarized modes. No plasma is present.


|E⊥|
|E1|||
|E2|||
x 10^-3
x 10^-3
t k⊥ c/10^6
x k /10^6
x k /10^6
x k /10^6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-1
-0.5
0
0.5
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
2
4
6
8
10
12
14
16
18
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

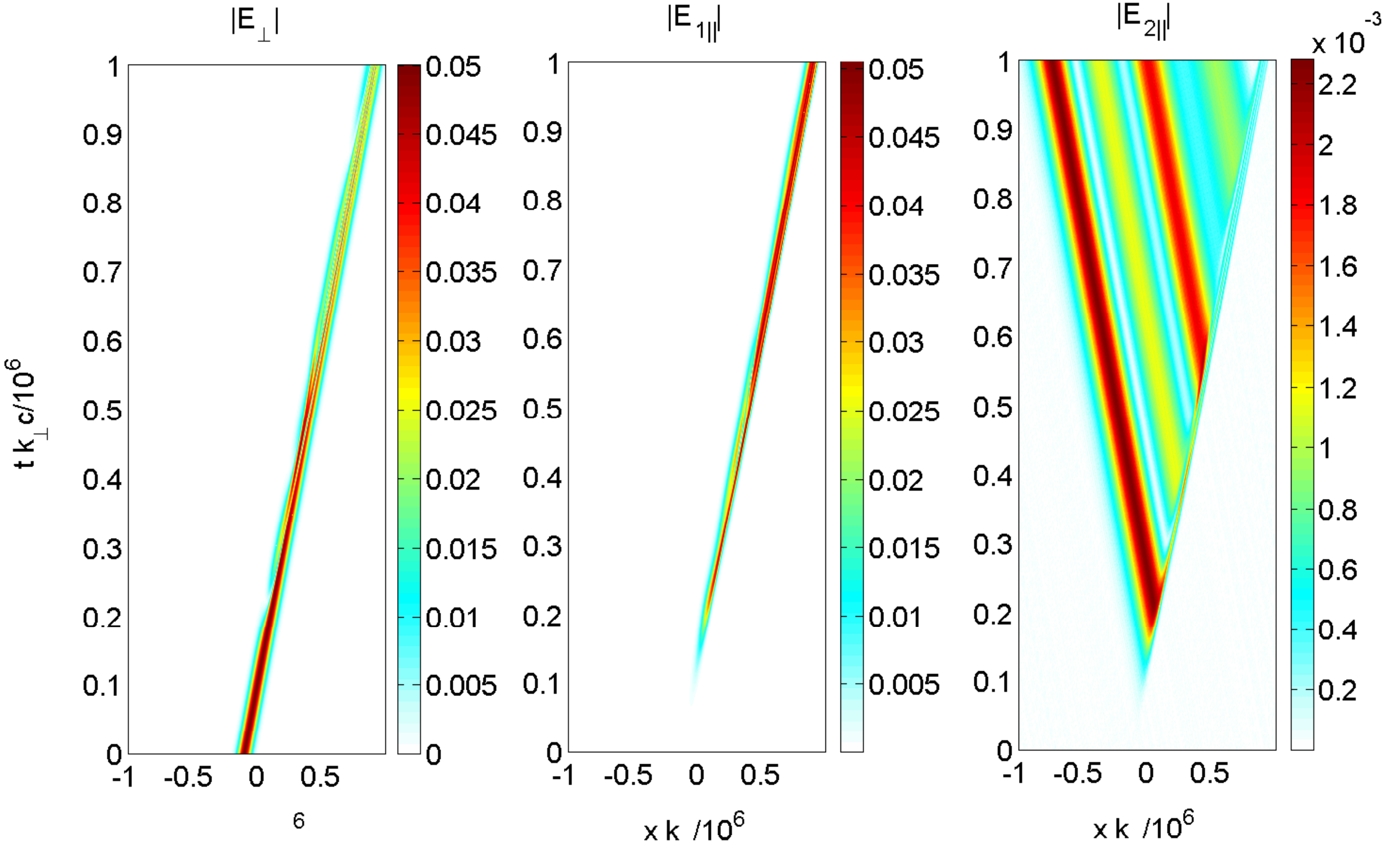
Analysis of the coupled equations

Note that:

- We have linear instability followed by nonlinear chaotic behavior.
- Most of the energy oscillates between the pump and the forward scattered mode.

Analysis of the coupled equations

Case 2: Numerical study of an initially localized pump pulse $E_{\perp 0} \propto \exp[-(x - x_p)^2 / L^2]$ together with initial thermal fluctuations for the parallel polarized modes. No plasma is present.


|E⊥|
|E1||
|E2||
t k⊥ c/10^6
x k /10^6
x 10^-3
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
-1
-0.5
0
0.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
6

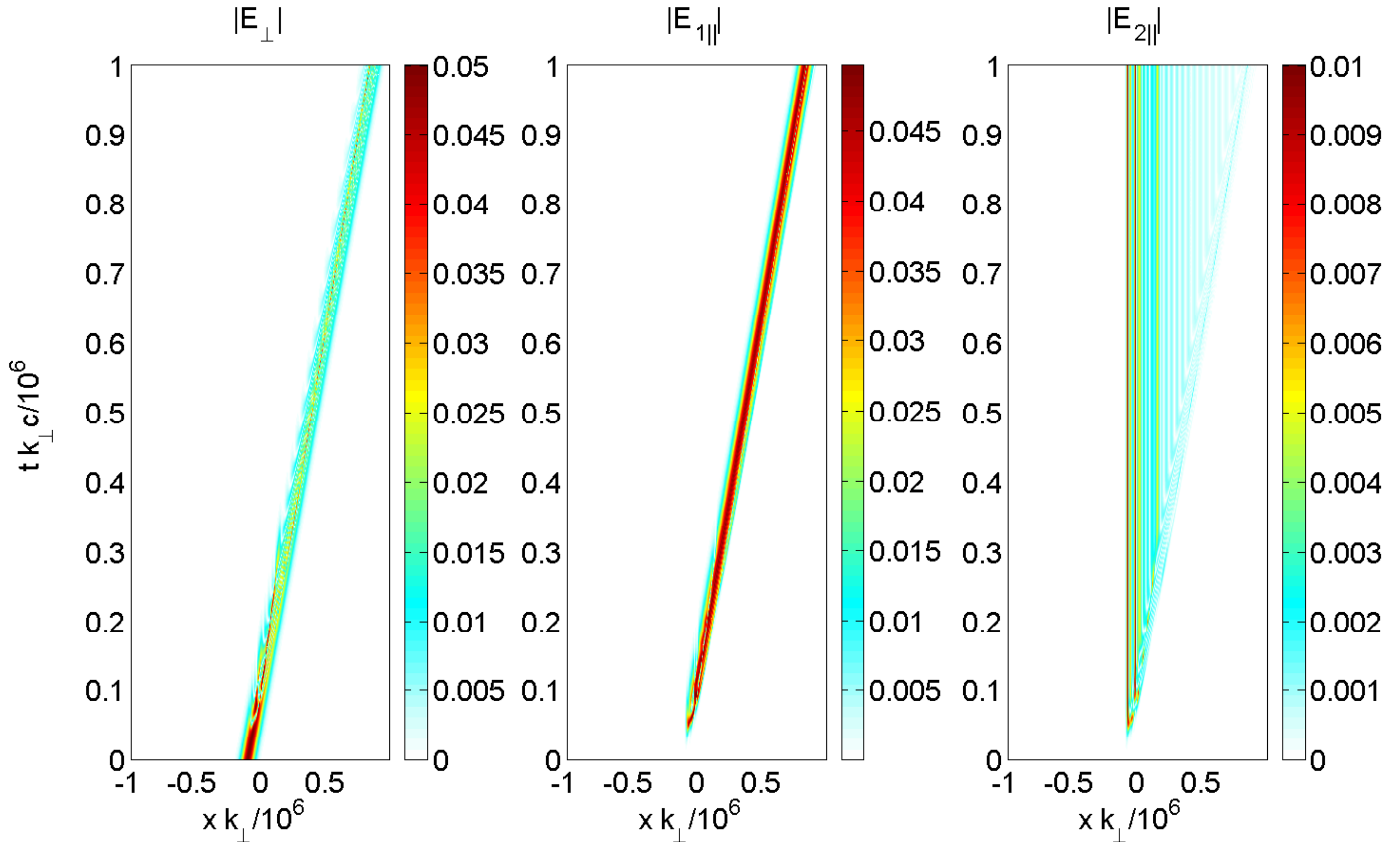
Analysis of the coupled equations

Note that:

- The behavior is somewhat more ordered as compared to the case of a homogeneous pump. In essence, we have filamentation of the pump and an effective damping, as the energy of the backscattered wave propagates out of the interaction region.
- As the pump wave energy decreases, the strength of the coupling gradually diminishes.

Analysis of the coupled equations

Case 3: Numerical study of an initially localized pump pulse $E_{\perp 0} \propto \exp[-(x-x_p)^2/L^2]$ together with thermal fluctuations for the parallel polarized modes. Plasma present, with a density that fulfills $\omega_p \approx 3\xi\omega_\perp$


$|E_\perp|$
$|E_{1||}|$
$|E_{2||}|$
$t k_\perp c/10^6$
$x k_\perp/10^6$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-1
-0.5
0
0.5
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001

Analysis of the coupled equations

Note that:

- The wave that was backscattered in the vaccum case is now more or less non-propagating ($\omega_{2\parallel} \approx \omega_p$).
- The time scale of the energy conversion is considerably shorter with the plasma present.
- We have more or less complete conversion from perpendicular to parallel polarization.

Conclusions

- There is rich physics associated with pair plasmas, somewhat different from ordinary ion-electron plasmas.
- Studies of pair plasmas are needed to understand the early universe, certain aspects of astrophysics, and possibly future laboratory plasmas.
- The presence of a small amount of ions in pair plasmas can significantly increase wake field generation of nonlinear EM-pulses
- The combined influence of a plasma, together with QED vacuum polarization and magnetization, is likely to be of importance for electromagnetic wave phenomena in the pair plasma of pulsar and magnetar atmospheres.
- The plasma effects are more important in the low-frequency regimes (radio waves), whereas the QED effects dominate in the high-frequency regimes (x-rays and above). In the intermediate regime, from infrared to UV-light, both effects should be included.