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Anisotropic Temperature Instability in Non-Maxwellian Plasmas

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INTRODUCTION

Anisotropy of temperature is a feature of many plasma states.

For example,

In a magnetically confined plasma

Even in a field-free case, if the plasma has directional features

Temperature anisotropy in the equilibrium distribution function gives rise to instability called Weibel Instability.

It has been around for several decades and has wide range of applicability to astrophysical and space plasmas, as well as laboratory plasmas.

Weibel Instability has been extensively studied in both the relativistic and non-relativistic regimes.

The relativistic theory was developed to interpret the astrophysical data.

It can explain the generation of magnetic field in the vicinity of gamma ray burst sources, supernovae, and galactic cosmic rays.

Recently the Weibel instability has been analyzed for quantum plasmas described by the Wigner-Maxwell model. Quantum effects tend to suppress the instability for dense astrophysical objects such as white dwarfs and neutron stars.

For an anisotropic plasma

$$f_{\alpha 0} = F_{\alpha 0}(v_x^2) G_{\alpha 0}(v_y^2) H_{\alpha 0}(v_z^2)$$

the normal modes separate into one electrostatic mode and two electromagnetic modes.

$$\left[\omega^2 - \sum_\alpha \omega_{p\alpha}^2 \omega \int \frac{v_x(\partial f_{\alpha 0}/\partial v_x)}{k_x v_x - \omega} dv\right] E_x = 0$$

$$\left[k_x^2 c^2 - \omega^2 + \sum_\alpha \omega_{p\alpha}^2 + \sum_\alpha \omega_{p\alpha}^2 \int \frac{k_x v_y^2 (\partial f_{\alpha 0}/\partial v_x)}{k_x v_x - \omega} dv\right] E_y = 0$$

$$\left[k_x^2 c^2 - \omega^2 + \sum_\alpha \omega_{p\alpha}^2 + \sum_\alpha \omega_{p\alpha}^2 \int \frac{k_x v_z^2 (\partial f_{\alpha 0}/\partial v_x)}{k_x v_x - \omega} dv\right] E_z = 0$$

Electromagnetic instability of a Bi-Maxwellian Plasma

$$f_{\alpha 0} = \left(\frac{m}{2\pi\kappa T_x}\right)_\alpha^{1/2} \left(\frac{m}{2\pi\kappa T}\right)_\alpha \exp\left[-\left(\frac{m}{2\kappa T_x}\right)_\alpha v_x^2 - \left(\frac{m}{2\kappa T}\right)_\alpha (v_y^2 + v_z^2)\right], T_x \neq T_\perp$$

Dispersion relation

$$k_x^2 c^2 - \omega^2 + \sum_\alpha \omega_{p\alpha}^2 \left(1 - \frac{T_{\alpha\perp}}{T_{\alpha x}}\right) = \sum_\alpha \omega_{p\alpha}^2 \frac{T_{\alpha\perp}}{T_{\alpha x}} \xi_\alpha Z(\xi_\alpha)$$

Where

$$\xi_\alpha = \frac{\omega}{k_x\sqrt{2\kappa T_{\alpha x}/m_\alpha}}, \quad \text{and} \quad Z(\xi_\alpha) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} \frac{\exp(-x^2)\,dx}{x - \xi_\alpha}$$

$$Z(\xi_\alpha) = \begin{cases} -2\xi + \frac{4}{3}\xi^3 - \cdots + i\sqrt{\pi}\,\frac{k_x}{|k_x|} exp(-\xi^2) & |\xi| \ll 1 \\ -\frac{1}{\xi} - \frac{1}{2\xi^3} - \frac{3}{4\xi^5} - \cdots & |\xi| \gg 1 \end{cases}$$

In the limit $\xi_\alpha \ll 1$ (neglecting also $\omega^2 < k_x^2 c^2$)

$$k_x^2 c^2 + \sum \omega_{p\alpha}^2 \left(1 - \frac{T_{\alpha\perp}}{T_{\alpha x}}\right) = \sum_\alpha \omega_{p\alpha}^2 \frac{T_{\alpha\perp}}{T_{\alpha x}} \left(2\xi^2 + i\sqrt{\pi}\,\frac{k}{|k|}\xi\right)_\alpha \qquad |\xi_\alpha| \ll 1$$

with approximate solution (neglecting ion terms)

$$\omega = i\sqrt{\frac{2}{\pi}}\,|k_x|\sqrt{\frac{\kappa T_{ex}}{m_e}}\,\frac{T_{ex}}{T_{e\perp}}\left(\frac{T_{e\perp}}{T_{ex}} - 1 - \frac{k_x^2 c^2}{\omega_{pe}^2}\right)$$

This mode is either pure growing or damping. Growth occurs for wavelengths

$$k_x^2 < k_0^2 = \frac{\omega_{pe}^2}{c^2}\left(\frac{T_{e\perp}}{T_{ex}} - 1\right)$$

Computer Simulation experiment

The results are not restricted to a bi-Maxwellian distribution.

For example, an effective temperature anisotropy may occur due to two electron beams counter streaming with velocity u_0 and the plasma has temperature T_e in a direction perpendicular to the streaming direction

$$f_{e0} = \frac{1}{2}\frac{m_e}{2\pi\kappa T_e}\exp\left(-\frac{m_e}{2\kappa T_e}(v_x^2+v_z^2)\right)\left[\delta(v_y-u_0)+\delta(v_y+u_0)\right]$$

$$f_{i0} = \delta(\vec{v})$$

The instability condition is

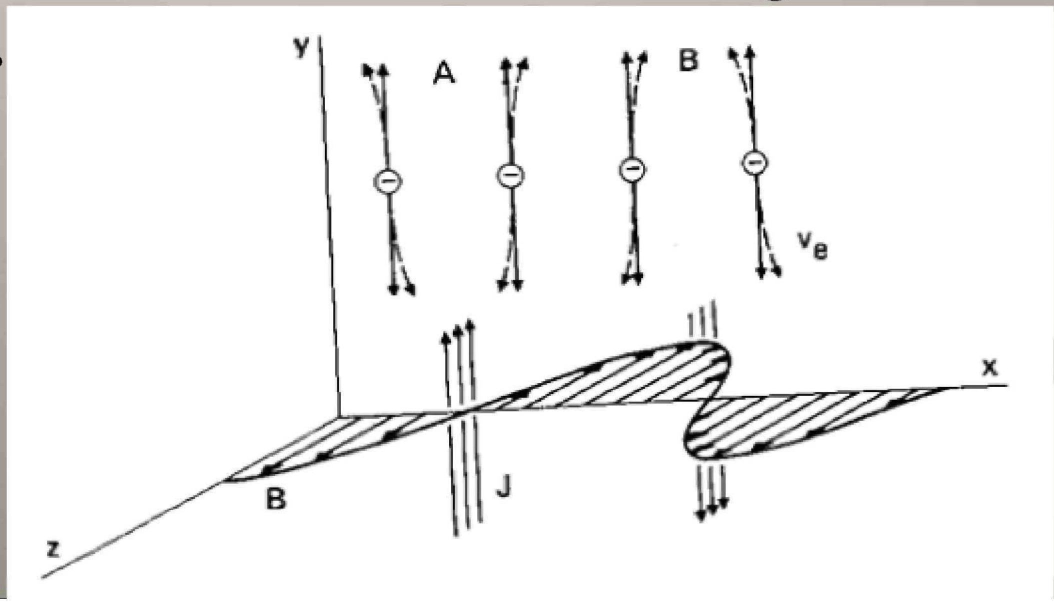
$$\frac{m_e {u_0}^2}{\kappa T_e} > \frac{k^2 c^2}{{\omega_{pe}}^2} + 1$$

The electron energy is the effective temperature in one direction & T_e in its perpendicular direction.

Physical Mechanism for the Weibel Instability

Consider an electron-ion plasma in which the ions are fixed and the electrons are hotter in the y-direction than in x or z-direction.

To see how magnetic field perturbation would grow, suppose a field $\mathbf{B} = B_z\hat{\mathbf{z}}\ \cos kx$ spontaneously arises from noise. The Lorentz force then bends the electron trajectories with the result that upward-moving $-e\mathbf{v}\times\mathbf{B}$ electrons congregate at B and downward-moving ones at A. The resulting current $\mathbf{j} = -en_0\mathbf{v}_e$ sheets generate magnetic field that enhances the original field and thus perturbation grows.



Effective Anisotropy created from outside the plasma (Gillani & Tsintsadze)

The velocity distribution in plasma is initially isotropic but when a relativistic, intense electromagnetic wave interacts with the plasma, it generates an additional momentum and thus temperature anisotropy is created.

Consider an intense circularly polarized electromagnetic wave propagating along z-axis E $(E_X, E_Y, 0)$ and B $(B_X, B_Y, 0)$. It interacts with the electron plasma (while the ions are immobile) and creates additional momentum. Thus the total perpendicular momentum becomes

$$\vec{p}_\perp = \vec{p}_\perp^{\,th} + \frac{e}{c}\vec{A}_\perp$$

Here $\vec{p}_\perp^{\,th}$ is thermal momentum of the electrons

and $\frac{e}{c}\vec{A}_\perp$ is the field momentum.

For a sufficiently strong pulse, the thermal momentum may be ignored compared to the field momentum i.e., the electrons move with a constant perpendicular momentum $\hat{p}_\perp = \frac{e}{c}\vec{A}_\perp$ so that the distribution function may be expressed as

$$F(p_\perp^2, p_z) = \frac{1}{2\pi p_\perp}\delta(p_\perp - \hat{p}_\perp)F_1(p_z)$$

where $F_1(p_z)$ is the parallel momentum distribution function, which is chosen as

$$F_1(p_z) = D\ \exp\left[\frac{-c\sqrt{m^2c^2 + \hat{p}_\perp^2 + p_z^2}}{T_z}\right]$$

By substituting the distribution function and after performing integration, we get

$$0 = 1 - \frac{c^2 k_z^2}{\omega^2} - \frac{\omega_p^2}{\omega^2} \int_{-\infty}^{+\infty} \frac{F_1(p_z) dp_z}{\gamma} \left[1 - \frac{\hat{p}_\perp^2 (\omega^2 - c^2 k_z^2)}{2m^2 c^2 \left(\gamma\omega - \frac{k_z p_z}{m} \right)^2} \right]$$

where

$$\gamma = \sqrt{1 + \frac{\hat{p}_\perp^2}{m^2 c^2} + \frac{p_z^2}{m^2 c^2}} \quad ; \quad \hat{\gamma}_\perp = \sqrt{1 + \frac{\hat{p}_\perp^2}{m^2 c^2}}$$

Now we define the effective perpendicular temperature

$$T_\perp = \int d^3p\left(\frac{p_\perp^2}{2\gamma m}\right)F(p_\perp^2,p_z) = \int d^3p\left(\frac{p_\perp^2}{2\gamma m}\right)\frac{1}{2\pi p_\perp}\delta(p_\perp - \hat{p}_\perp)F_1(p_z)$$

$$= \frac{K_0\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)}{K_1\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)}\frac{\hat{p}_\perp^2}{2\hat{\gamma}_\perp m}$$

The dispersion relation now becomes

$$0 = 1 - \frac{c^2k_z^2}{\omega^2} - \frac{\omega_p^2}{\omega^2\hat{\gamma}_\perp}\left[\frac{K_0\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)}{K_1\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)} - \frac{T_\perp K_1\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)}{T_z K_0\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)} - \left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)^{1/2}\frac{T_\perp\omega_i\sqrt{\left(\frac{\pi}{2}\right)}}{T_z(c^2k_z^2-\omega^2)}\frac{K_{1/2}\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)}{K_0\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)}\right]$$

To simplify further, we take the asymptotic limit of the MacDonald function for

$$\frac{m^2c^2}{T_z}\hat{\gamma}_\perp \gg 1$$

and obtain

$$0 = 1 - \frac{c^2k_z^2}{\omega^2} - \frac{\omega_p^2}{\omega^2\hat{\gamma}_\perp}\left[1 - \frac{T_\perp}{T_z} - \frac{T_\perp}{T_z}\left(\frac{m^2c^2}{T_z}\hat{\gamma}_\perp\right)^{1/2}\frac{\omega_i\sqrt{\left(\frac{\pi}{2}\right)}}{(c^2k_z^2-\omega^2)}\right]$$

Low frequency case $\omega \ll ck_z$

$$\omega = -\frac{ick_z}{\omega_p^2}\left(\frac{2}{\pi}\right)^{1/2}\frac{T_z}{T_\perp}\sqrt{\left(\frac{T_z}{mc^2\hat{\gamma}_\perp}\right)}\left(\hat{\gamma}_\perp c^2k_z^2 + \omega_p^2\left(1-\frac{T_\perp}{T_z}\right)\right)$$

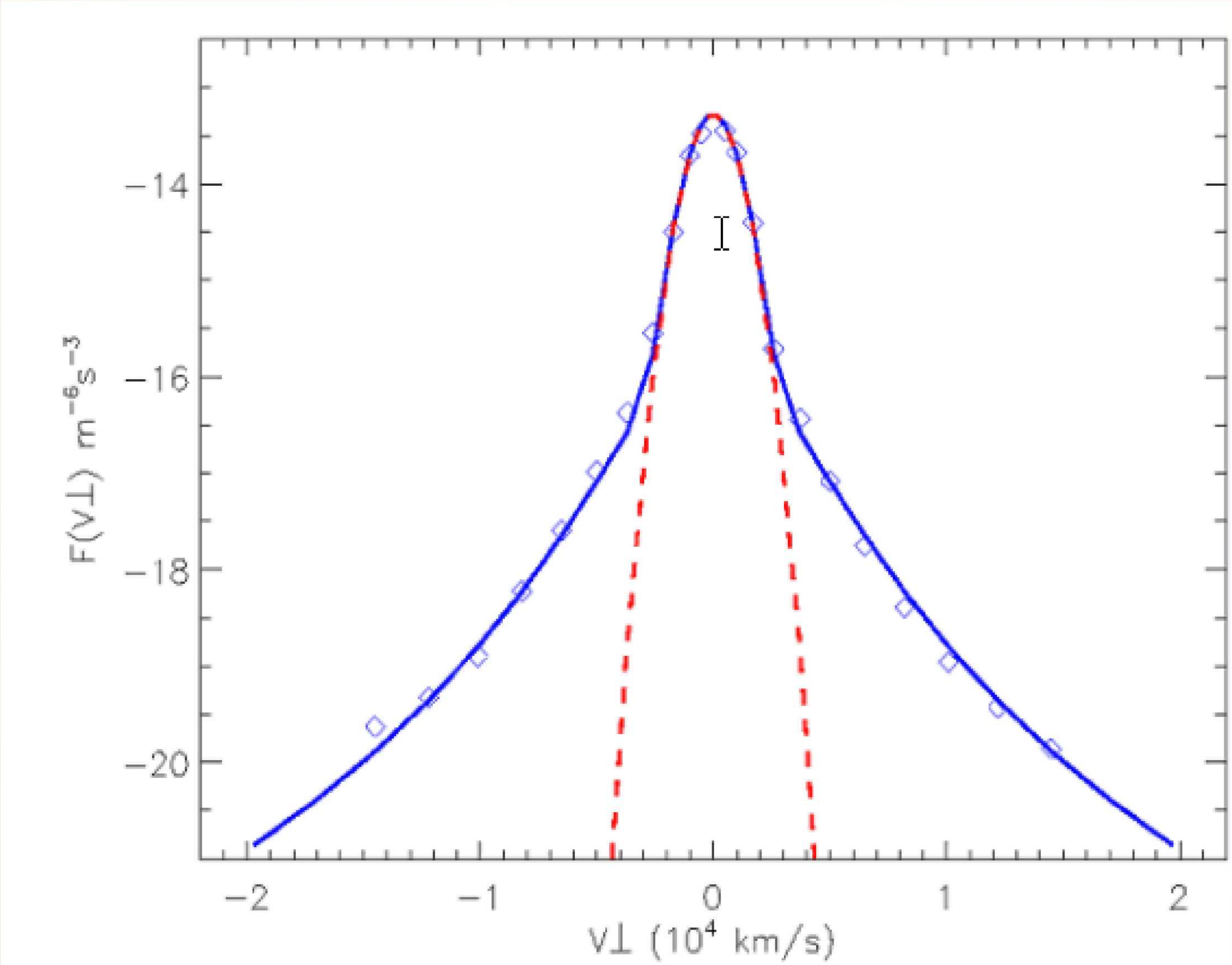
For $T_\perp = \dfrac{e^2A_\perp^2}{2\hat{\gamma}_\perp mc^2} >> T_z$, growth occurs for wave lengths

$$k_z^2 < k_o^2 = \frac{\omega_p^2}{c^2\hat{\gamma}_\perp}\left(\frac{T_\perp}{T_z} - 1\right)$$

Non-Maxwellian Velocity Distribution

So far we have discussed Maxwellian velocity distributions but such distributions may not be adequate to describe all the real situations. Indeed, there exists a variety of environments in space, lab and astrophysical plasmas where departure from Maxwellian distribution is observed. For example,

1. The solar wind acceleration (at supersonic speeds of about 700-800 km/s) is a long standing problem which still remains unexplained. The majority of the models used are generalizations of traditional hydrodynamic Parker's model and are Based on local thermodynamic equilibrium, but that is far from being true for the solar wind electrons. Indeed, the velocity distribution functions of the electrons measured are not Maxwellian.



[Adapted from Makismovic et al, JGR, 2005]

2. In tokamaks, there is evidence of non-Maxwellian electron distribution function.

It is well known that when intense electron heating or current drive technique is applied, high energy tails in the distribution function can develop. In many tokamak devices like TFTR, JET and Frascati Torus, non-Maxwellian distortions in the electron distribution function have been observed which are based on discrepancies in the electron temperature measurements with electron cyclotron emission (ECE) diagnostic and Thomson Scattering (TS) technique. TCV (Lausanne variable configuration tokamak) also has reported the role of non-Maxwellian electron distribution (in 2^{nd} & 3^{rd} harmonic electron cyclotron absorption).

3. In the study of Solar Transition Region (separating the upper Chromosphere from the Corona), the existing models assume nearly Maxwellian electron velocity distribution function and compute heat flux and the excitation, ionization and recombination rates. However, this region is characterized by steep temperature gradients which lead to distribution functions for electrons and protons to be non-Maxwellian. For this region, Spitzer-Harm theory is not applicable. Heat flux acquires non-Local character. Further, the rate of ionization, excitation and recombination differ from those computed assuming Maxwellian distribution functions for the electrons.

Generalized Non-Maxwellian Distribution Function

A generalized Non-Maxwellian distribution is the so called (*r*, *q*) distribution represented by

$$f(r,q) = \left[\frac{3}{4\pi\Psi_{\perp}^{2}\Psi_{\parallel}}\right]\left[\frac{\Gamma(q)}{(q-1)^{\frac{3}{2+2r}}\,\Gamma\left[q-\frac{3}{2+2r}\right]\Gamma\left[1+\frac{3}{2+2r}\right]}\right]\times\left[1+\frac{1}{q-1}\left(\frac{v_{\parallel}^{2}}{\Psi_{\parallel}^{2}}+\frac{v_{\perp}^{2}}{\Psi_{\perp}^{2}}\right)^{r+1}\right]^{-q}$$

where *r* and *q* are the spectral indices, Ψ is related to the particle thermal velocity v_T in the following manner

$$\Psi_{\parallel}^{2} = \left[\frac{3(q-1)^{\frac{-1}{r+1}}\,\Gamma\left[q-\frac{3}{2+2r}\right]\Gamma\left[\frac{3}{2+2r}\right]}{\Gamma\left[\frac{5}{2+2r}\right]\Gamma\left[q-\frac{5}{2+2r}\right]}\right]\left(v_{T_{\parallel}}^{2}\right)$$

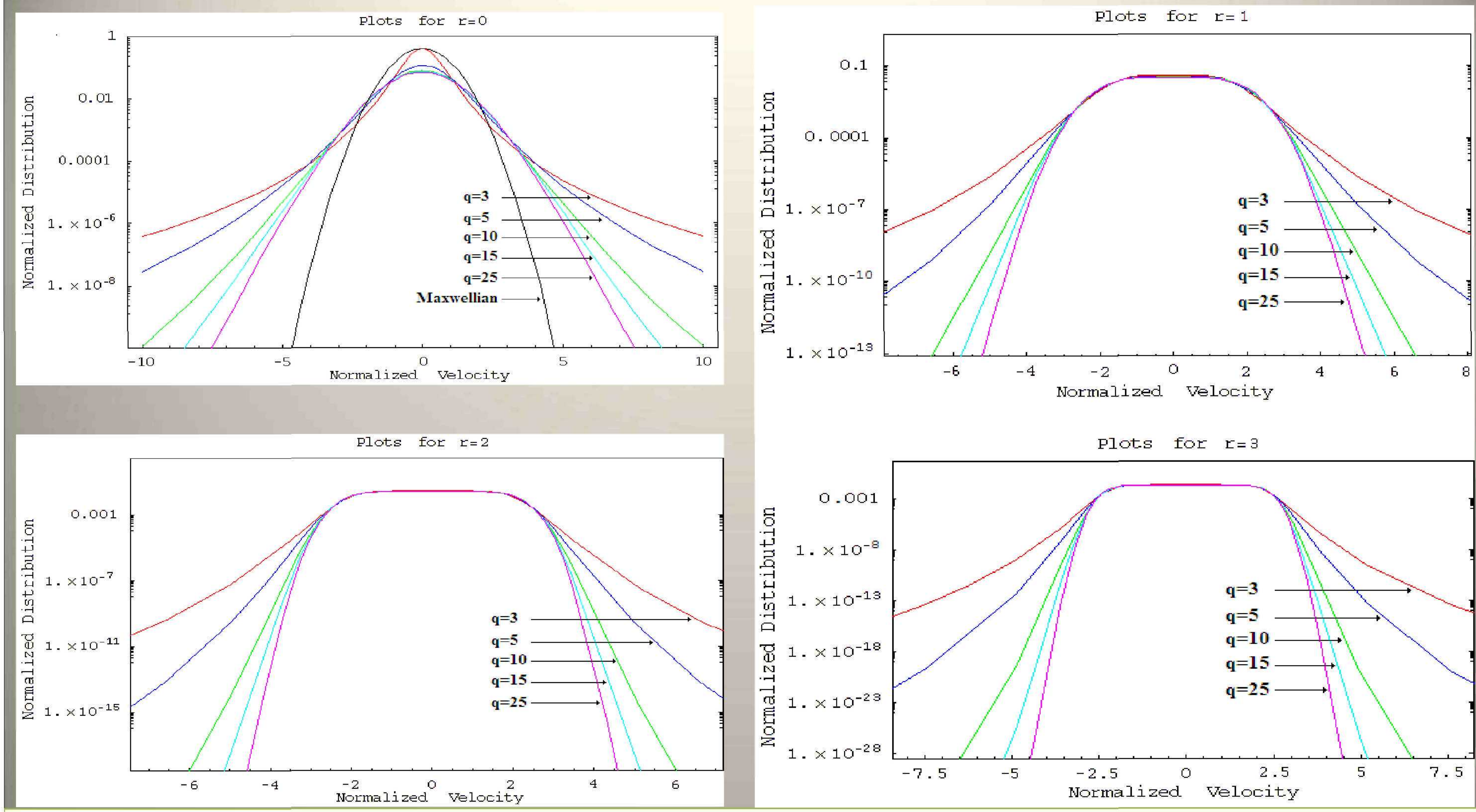
and

$$\Psi_{\perp}^{2} = \left[\frac{3(q-1)^{\frac{-1}{r+1}}\,\Gamma\left[q-\frac{3}{2+2r}\right]\Gamma\left[\frac{3}{2+2r}\right]}{\Gamma\left[\frac{5}{2+2r}\right]\Gamma\left[q-\frac{5}{2+2r}\right]}\right]\left(v_{T_{\perp}}^{2}\right)$$

Here we note that $q > 1$ and $q(1 + r) > \frac{5}{2}$ are conditions which emerge from the normalization and the definition of the temperature.

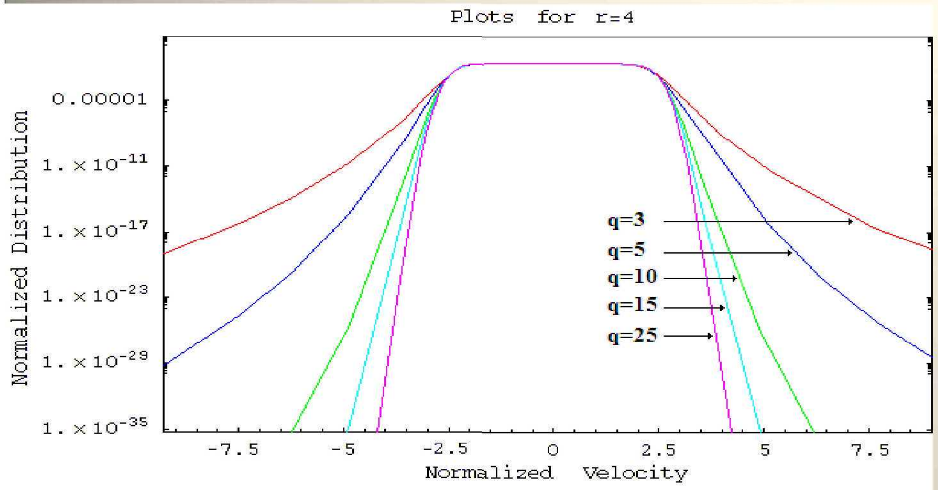
The choice of (*r* , *q*) yields a wide variety of velocity distributions which might be adequate to describe different physical plasma environments in lab, space and astrophysical phenomena.

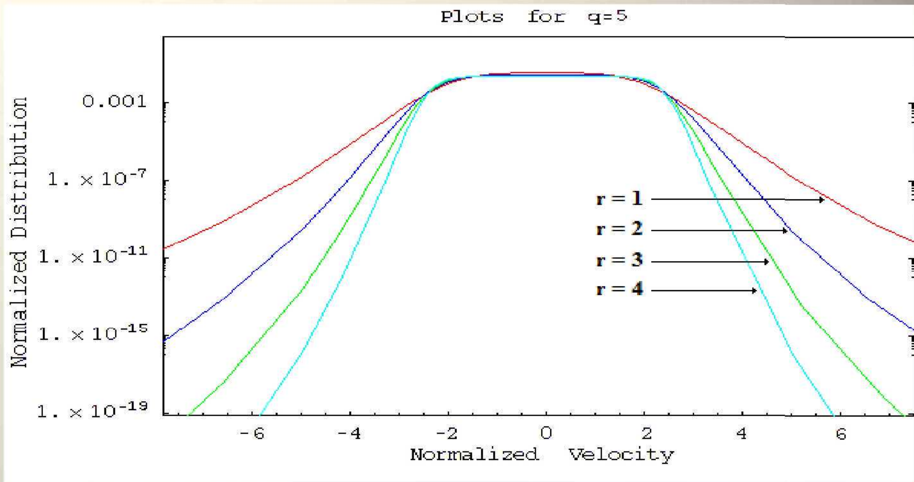
Depending on the values of *r* and *q* , *t*hese distributions acquire broad shoulders or high energy tails.
The following graphs illustrate this point:

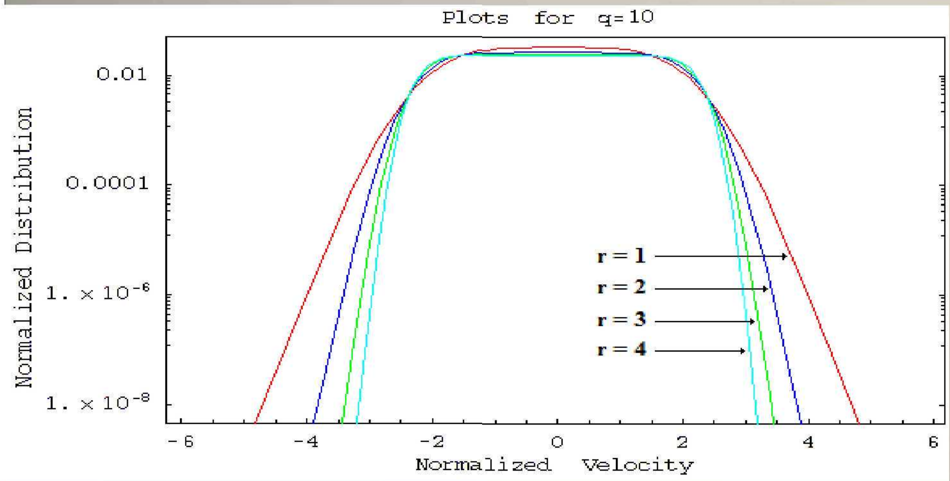


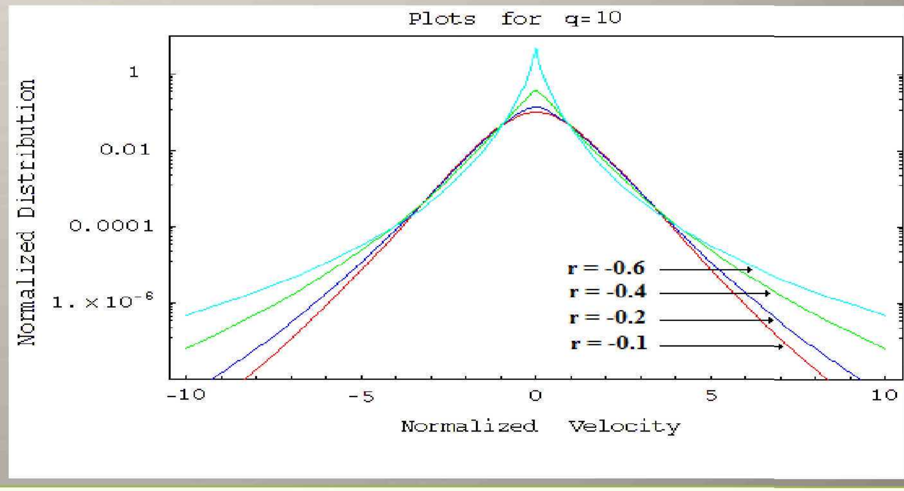
a. **For r = 0, there is no shoulder in the distribution function and as we increase q, the high energy tail is reduced.**

b. **For r $\geq$1 , the distribution function develops a broad shoulder and with increase in r, the height of shoulder reduces by an order of magnitude.**









c. **For a given value of *q*, as we increase *r* the high energy tail is curtailed. For increase in *q*, the height of the shoulder increases.**

d. **For *r* negative, the shoulder vanishes and instead a peak is formed.**

Weibel Instability with non-Maxwellian velocity distribution

We shall now describe how the Weibel Instability is affected due to non-Maxwellian velocity distribution. First, we write down the well known linear dispersion relation for an electromagnetic wave propagating along z – direction $\boldsymbol{k} = \hat{e}_z k$ in cylindrical coordinates:

$$\omega^2 - c^2k^2 - \omega_{pe}^2 + \pi\omega_{pe}^2\left(\frac{k}{m}\right)\int_{-\infty}^{\infty}\frac{dp_{\parallel}}{(\omega - k_{\parallel}v_{\parallel})}\int_{0}^{\infty}p_{\perp}^3\,dp_{\perp}\times\left(\frac{\partial f_0}{\partial p_{\parallel}}\right) = 0$$

Maxwellian Distribution Function

Using Maxwellian distribution , we obtain

$$\omega^2 - c^2k^2 - \omega_{pe}^2 =$$

$$\frac{2}{m^5\sqrt{\pi}}\frac{\omega_{pe}^2}{\theta_\perp^2\theta_\parallel^3}\left(\frac{k}{m}\right)\int_{-\infty}^{\infty}\frac{p_\parallel dp_\parallel}{(\omega - k_\parallel v_\parallel)}\int_0^{\infty} p_\perp^3 \exp\left[-\frac{1}{m^2}\left(\frac{p_\parallel^2}{\theta_\parallel^2}+\frac{p_\perp^2}{\theta_\perp^2}\right)\right]dp_\perp$$

$$\omega^2 - c^2k^2 - \omega_{pe}^2 + \omega_{pe}^2\left(\frac{T_\perp}{T_\parallel}\right) + \omega_{pe}^2\left(\frac{T_\perp}{T_\parallel}\right)(\xi)Z(\xi) = 0$$

where

$$\xi = \frac{\omega}{k_\parallel}\Big/\sqrt{\frac{2\kappa T_\parallel}{m}} \quad and \quad Z(\xi) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp[-x^2]}{(x-\xi)}dx$$

($\xi > 1$)

$$\omega^4 - (c^2k^2 + \omega_{pe}^2)\omega^2 - \omega_{pe}^2(k_\parallel^2 v_{T_\parallel}^2)\left(\frac{T_\perp}{T_\parallel}\right) = 0$$

(> 1)

$$\mathrm{Im}\,\omega = \sqrt{\frac{2}{\pi}}k_\parallel v_{T_\parallel}\left[1 - \left(1 + \frac{c^2k^2}{\omega_{pe}^2}\right)\Big/\frac{T_\perp}{T_\parallel}\right]$$

(r,q) Distribution Function

$$f_{(r,q)} = \frac{3}{4\pi\Psi_\perp^2\Psi_\parallel}\left(\frac{\Gamma(q)}{(q-1)^{\frac{3}{2+2r}}\,\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times$$
$$\times\left[1+\frac{1}{q-1}\left(\frac{v_\parallel^2}{\Psi_\parallel^2}+\frac{v_\perp^2}{\Psi_\perp^2}\right)^{r+1}\right]^{-q}$$

$(\xi > 1)$

$$\omega^4-\left(c^2k^2+\omega_{pe}^2\right)\omega^2-\left(\frac{\omega_{pe}^2}{2}\right)\left(\frac{\Psi_\perp^2}{\Psi_\parallel^2}\right)\left(1+(-1)^{(2q+2r)}\right)\times$$
$$\times\left(\frac{(q-1)^{\frac{1}{r+1}}\Gamma\left(q-\frac{5}{2}\right)\Gamma\left(\frac{5}{2+2r}\right)}{\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(\frac{3}{2+2r}\right)}\right)\left(\frac{k_\parallel^2\Psi_\parallel^2}{3}\right)=0$$

$(\xi < 1)$

(i) r=0:

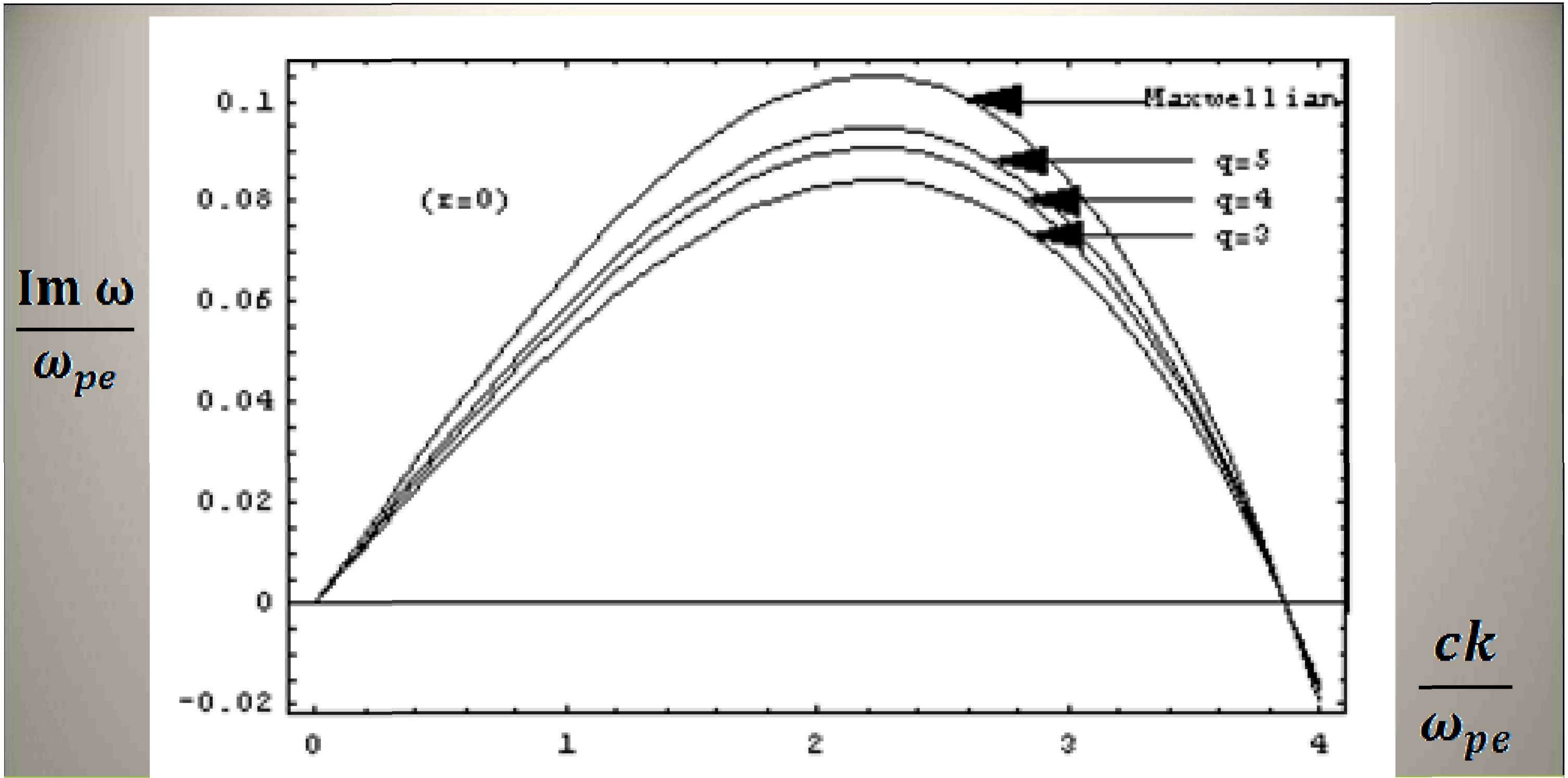
$$Z^{**}_{0,q}(\xi) = \frac{1}{\sqrt{\pi}}\left(\frac{\Gamma(q)}{(q-1)^{\frac{3}{2}}\,\Gamma\left(q-\frac{3}{2}\right)}\right)\int_{-\infty}^{\infty}\frac{\left(1+\frac{x^2}{q-1}\right)^{-q-1}}{(x-\xi)}dx$$

For q=3,4,5

$$\mathrm{Im}\,\omega_{0,3} = -(0.5002)\frac{k_{\parallel}v_{T\parallel}}{\omega_{pe}^2}\left(\frac{T_{\parallel}}{T_{\perp}}\right)\left(c^2k^2+\omega_{pe}^2\left(1-\frac{T_{\perp}}{T_{\parallel}}\right)\right)$$

$$\mathrm{Im}\,\omega_{0,4} = -(0.649)\frac{k_{\parallel}v_{T\parallel}}{\omega_{pe}^2}\left(\frac{T_{\parallel}}{T_{\perp}}\right)\left(c^2k^2+\omega_{pe}^2\left(1-\frac{T_{\perp}}{T_{\parallel}}\right)\right)$$

$$\mathrm{Im}\,\omega_{0,5} = -(0.698)\frac{k_{\parallel}v_{T\parallel}}{\omega_{pe}^2}\left(\frac{T_{\parallel}}{T_{\perp}}\right)\left(c^2k^2+\omega_{pe}^2\left(1-\frac{T_{\perp}}{T_{\parallel}}\right)\right)$$



When we fix r , then for small values of q , the system deviates from its equilibrium position i.e., away from Maxwellian and the growth rate is suppressed. However, for larger q , the behavior approaches that of the Maxwellian.

(ii) r=1:

$$Z_{1,q}^{**}(\xi) = \frac{1}{\pi}\left(\frac{(q-1)^q\,\Gamma(q)}{2q(2q-1)\Gamma(q-1)}\right)\int_{-\infty}^{\infty}\frac{\left(x^2\right)^{1-2q}}{(x-\xi)}\times$$

$$\times\left[1-2q+2q\left(1+\frac{q-1}{(x^2)^2}\right)^q {}_2F_1\left[q+1, q-\frac{1}{2}, q+\frac{1}{2}, \frac{1-q}{(x^2)^2}\right]\right]dx$$

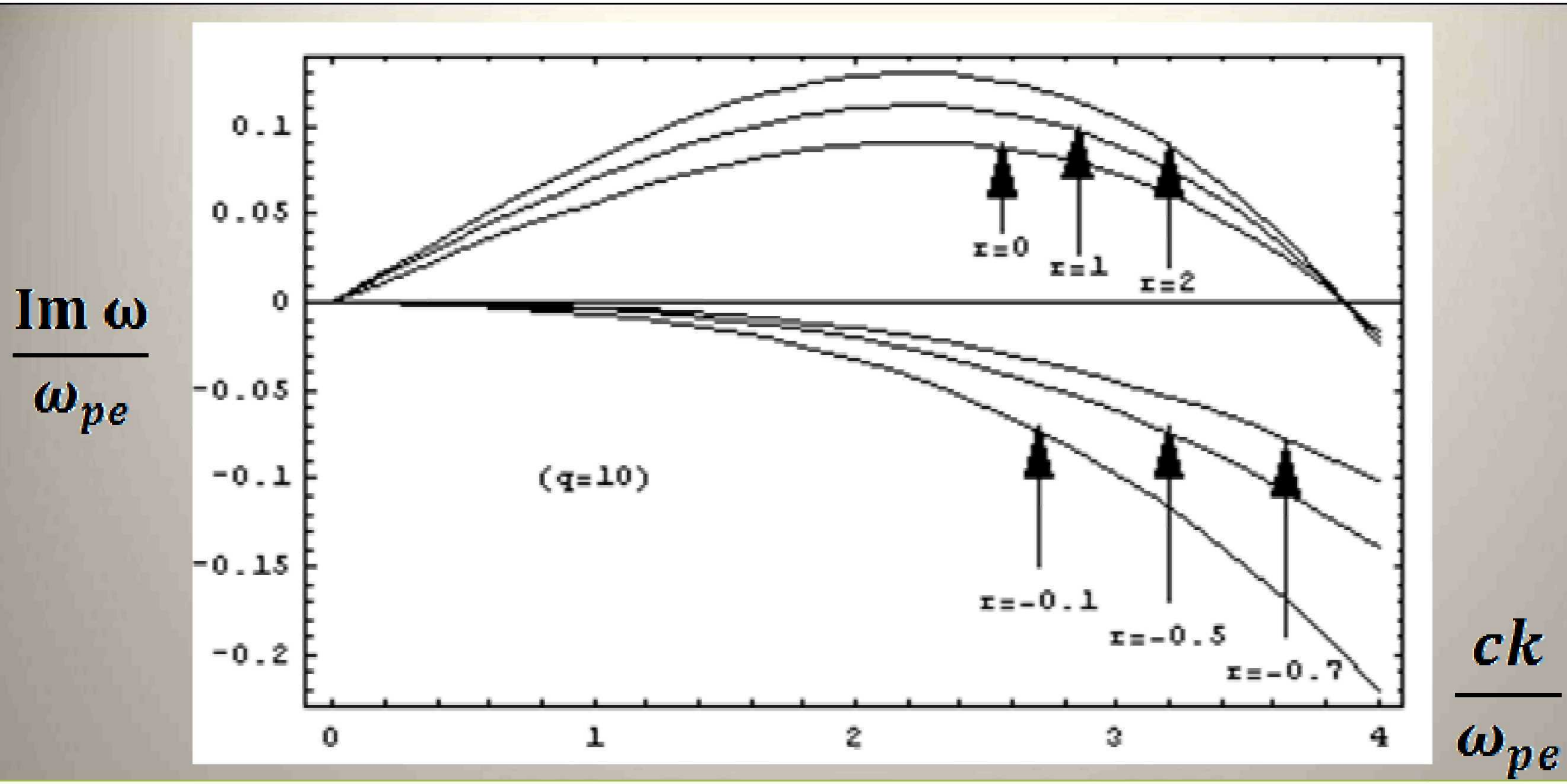
For q=3,4

$$\operatorname{Im}\omega_{(1,3)} = -(0.838)\frac{k_\parallel v_{T\parallel}}{\omega_{pe}^2}\left(\frac{T_\parallel}{T_\perp}\right)\left(c^2k^2+\omega_{pe}^2\left(1-\frac{T_\perp}{T_\parallel}\right)\right)$$

$$\operatorname{Im}\omega_{(1,4)} = -(0.856)\frac{k_\parallel v_{T\parallel}}{\omega_{pe}^2}\left(\frac{T_\parallel}{T_\perp}\right)\left(c^2k^2+\omega_{pe}^2\left(1-\frac{T_\perp}{T_\parallel}\right)\right)$$

(iii) r=-0.7 and q=10:

$$\operatorname{Im}\omega = -(0.152)\frac{k_\parallel v_{T\parallel}}{\omega_{pe}^2}\left(\frac{T_\parallel}{T_\perp}\right)\left(c^2k^2+\omega_{pe}^2\right)$$



If we fix q and vary r, e.g., take q=10 and r=–0.1, –0.5, –0.7, 0, 1, 2, we find that upon increasing the value of positive r, the growth rate increases with respect to the Maxwellian, but for the negative r values, it decreases and the instability disappears and instead damping occurs.

This is as expected because as r increases the shoulder in the distribution function broadens and the contribution of high energy particles is curtailed but for negative r with q fixed, the shoulder in the distribution function vanishes and instead a peak is formed in the distribution function which results in the suppression of the instability.

Semi-relativistic Distribution Function

$$f_0 = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(\frac{1}{mT_\perp}\right)\left(\frac{1}{mT_\parallel}\right)^{1/2}\left(1+\frac{mc^2}{T_\perp}\right)^{-1} \times$$

$$\times \exp\left[-\frac{p_\parallel^2}{2mT_\parallel} - \frac{mc^2}{T_\perp}\left(\sqrt{1+\frac{p_\perp^2}{m^2c^2}} - 1\right)\right]$$

$(\xi > 1)$

$$\omega^4 - \left(c^2k^2 + \omega_{pe}^2\right)\omega^2 - \omega_{pe}^2\left(k_\parallel^2 v_{T_\parallel}^2\right)\left(\frac{\chi T_\perp}{T_\parallel}\right) = 0$$

$(\xi < 1)$

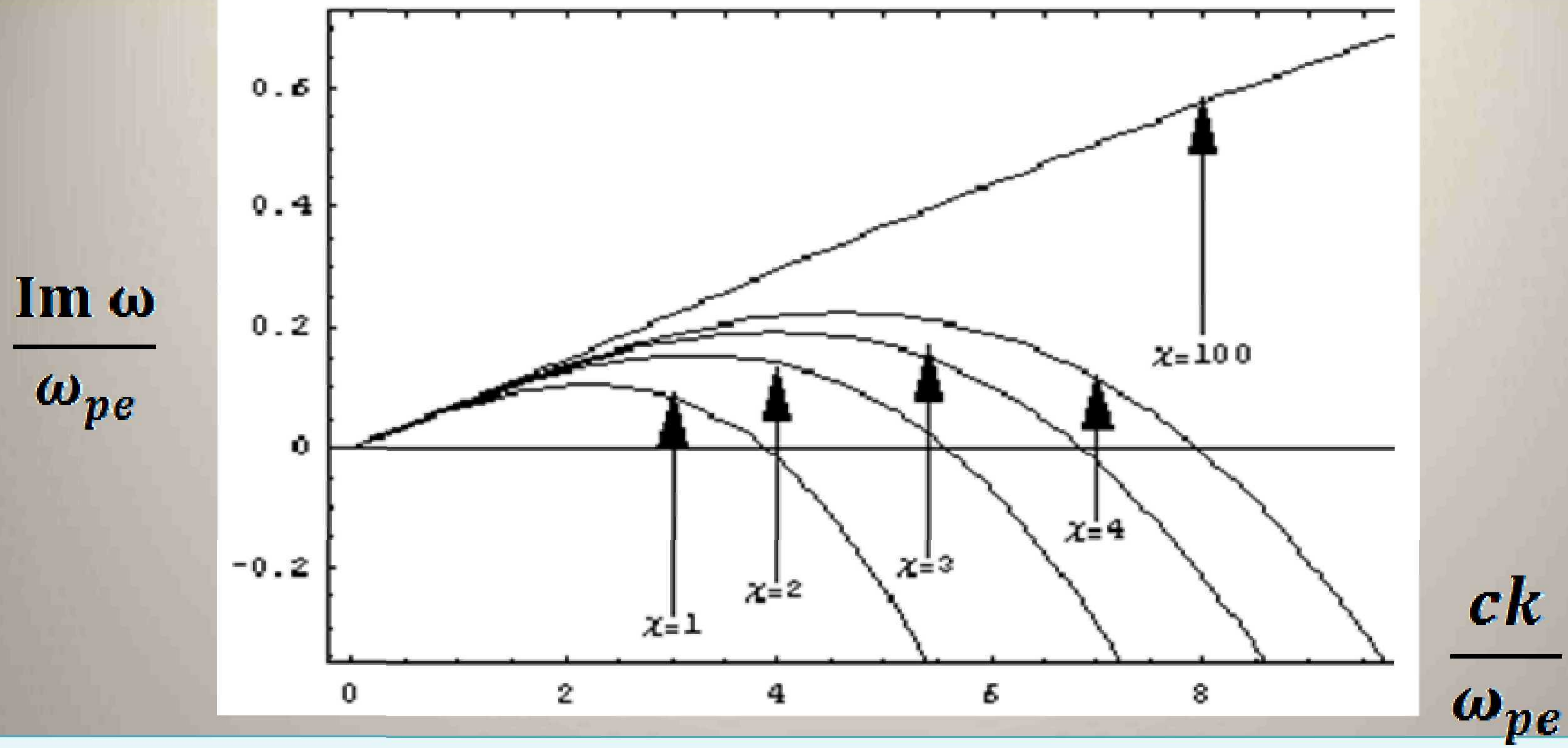
$$\mathrm{Im}\,\omega = \sqrt{\frac{2}{\pi}}\,k_\parallel v_{T_\parallel}\left[1 - \left(1 + \frac{c^2k^2}{\omega_{pe}^2}\right)\Big/\frac{\chi T_\perp}{T_\parallel}\right]$$

where

$$\chi = \left(3 + 3\frac{mc^2}{T_\perp} + \frac{m^2c^4}{T_\perp^2}\right)\Big/\left(1+\frac{mc^2}{T_\perp}\right)\frac{mc^2}{T_\perp}$$

χ symbolizes the relativistic contribution which becomes unity as we approach the non-relativistic Maxwellian.

χ ensures growth even for small temperature anisotropy.



As χ increases, (i) the growth rate increases (ii) the instability domain grows bigger and bigger and eventually the entire range of k becomes unstable.

Such an instability may occur in the spiraling arm of a galaxy where perpendicular temperature $T_{\perp}$ dominates over the parallel temperature $T_{\parallel}$

CONCLUSION

THE RESULTS OF THE PRESENT ANALYSIS OPENS A NEW WINDOW OF INVESTIGATING ELECTROMAGNETIC MODES IN DIFFERENT PLASMA SCENARIOS WHERE THE NON-MAXWELLIAN DISTRIBUTIONS BECOME RELEVANT.