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Non-Newtonian mechanics of oscillation centers

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Abstract. Classical particles oscillating in high-frequency or static fields effectively exhibit a modified rest mass m_{eff} which determines the guiding center motion. Unlike the true mass, m_{eff} depends on the field parameters and can be a nonanalytic function of the particle average velocity and the oscillation energy; hence non-Newtonian "metaplasmas" that permit a new type of plasma maser, signal rectification, frequency doubling, and one-way walls.

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1. INTRODUCTION

Problems connected with multiscale adiabatic dynamics of classical particles in oscillatory and static fields enjoy simplification within the guiding-center approach, which allows separating fast oscillatory motion of the particles from their slow translational motion [1, 2, 3, 4, 5]. Often, the average forces on a guiding center are then written in terms of fictitious fields, such as ponderomotive [6, 7, 8], diamagnetic [9], or other potentials [5, 10]. However, within a more general relativistic approach [11], the average forces are *embedded* into the properties of the particle guiding center, so the latter is treated as a non-Newtonian quasi-particle with a variable effective mass m_{eff}.

In this paper, we study effects that flow from m_{eff} dependence on the guiding center velocity $\bar{\mathbf{v}}$ and the oscillation energy W. Particularly, we propose a number of derivative applications, including a new type of plasma maser, signal rectification, frequency doubling, and others possible due to specific properties of "metaplasmas" formed by quasi-particles with variable m_{eff}. To the extent that $m_{\text{eff}}(\bar{\mathbf{v}})$ can be controlled, these metaplasmas can exhibit solid-state-like phenomena, and devices utilizing these phenomena would operate at much higher powers as compared to their solid-state counterparts. Also, to the extent that $m_{\text{eff}}(W)$ can be varied, non-conservative Hamiltonian average forces are produced, resulting in one-way-wall effects [10, 12, 13, 14, 15] and can be employed, e.g., for current drive [16, 17, 18] and atomic cooling [14, 19].

The paper is organized as follows: In Sec. 2 we briefly restate the effective mass concept. In Sec. 3 we contemplate possible applications of non-Newtonian phenomena due to m_{eff} dependence on the guiding center velocity. In Sec. 4 we derive nonrelativistic potentials that are due to m_{eff} dependence on the particle oscillation energy and explain how one-way walls are produced. In Sec. 5 we summarize our main ideas.

2. EFFECTIVE MASS

Basic equations. — Consider a classical particle undergoing arbitrary quasi-periodic oscillations superimposed on the average motion. In the adiabatic regime, one can map out the quiver dynamics by changing variables [1, 2, 3, 4, 5]; hence the guiding center is treated as a "dressed", or quasi-particle. Suppose, for now, that the pump fields causing the oscillations do not vary along the trajectory. Then the associated field tensor $F_{\mu\nu}$ will not enter the averaged equations as a force. However, it will affect the motion such that, in response to *additional* perturbation fields $\tilde{F}_{\mu\nu}$, the guiding center will react as if it had a modified mass.

In Ref. [11] we show that the particle guiding-center Lagrangian in the field $F_{\mu\nu}$ can be written in the form equivalent to that for a free particle with an invariant mass $m_{\text{eff}}(F_{\mu\nu})$:

$$\mathscr{L} = -m_{\text{eff}}c^2/\overline{\gamma}, \quad m_{\text{eff}} = c^{-2}\big(\mathbf{J}\cdot\mathbf{\Omega} - \langle L\rangle\big)', \quad \overline{\gamma} = \sqrt{1-\overline{v}^2/c^2}, \tag{1}$$

with $\overline{\mathbf{v}} = \langle \mathbf{v}\rangle$ and c being the average velocity and the speed of light, correspondingly. In the definition of m_{eff}, the right-hand side is to be evaluated in the guiding center rest frame (hence the prime), thus m_{eff} may depend on $\overline{\mathbf{v}}$; $\langle L\rangle$ is the time-average particle Lagrangian determined by $F_{\mu\nu}$ (for explanation on the gauge invariance see Ref. [11]); $\mathbf{J}$ are the actions and $\mathbf{\Omega}$ are the frequencies of oscillations in canonical angles, if any, to average over. For instance, for a particle in a dc magnetic field B, $\mathbf{J}\cdot\mathbf{\Omega} = \mu B/\gamma'$, where μ is the magnetic moment, and γ' is the Lorentz factor associated with the Larmor rotation, so $m_{\text{eff}} = m\sqrt{1+2\mu B/mc^2}$, m being the true mass. Similarly, for a nonrelativistic particle in a wave field, one gets $m_{\text{eff}} = m + \Phi/c^2$, where Φ is the ponderomotive potential [6, 7, 8, 20]. More examples of m_{eff} are found in Ref. [11].

Should $F_{\mu\nu}$ slowly vary with the guiding center coordinate $\overline{\mathbf{r}}$ or time t, the variations of m_{eff} will determine the average forces on the particle (e.g., $\mu\nabla B$-force and the average ponderomotive force $-\nabla\Phi$) derived from the Euler-Lagrange equations

$$d_t\,(\partial_{\overline{\mathbf{v}}}\mathscr{L}) = \partial_{\overline{\mathbf{r}}}\mathscr{L}. \tag{2}$$

Suppose now that the particle, with charge e, interacts also with a perturbation field $\tilde{F}_{\mu\nu}$ governed by $\tilde{A}_\mu = (\tilde{\mathbf{A}}, \tilde{\varphi})$, which is imposed over $F_{\mu\nu}$ [21, 22]. In the adiabatic regime, the orbit is not altered on the oscillation time scale; thus,

$$\mathscr{L} = -m_{\text{eff}}c^2/\overline{\gamma} + (e/c)(\overline{\mathbf{v}}\cdot\tilde{\mathbf{A}}) - e\tilde{\varphi}, \tag{3}$$

and a nonelectromagnetic potential can be added similarly. Then, the canonical momentum equals $\overline{\mathbf{P}} = \overline{\mathbf{p}} + (e/c)\tilde{\mathbf{A}}$, and the kinetic momentum $\overline{\mathbf{p}}$ is given by

$$\overline{\mathbf{p}} = \overline{\gamma} m_{\text{eff}}\overline{\mathbf{v}} - (c^2/\overline{\gamma})\,\partial_{\overline{\mathbf{v}}}\, m_{\text{eff}}. \tag{4}$$

Correspondingly, the Hamiltonian $\mathscr{H} = \overline{\mathbf{P}}\cdot\overline{\mathbf{v}} - \mathscr{L}$ reads

$$\mathscr{H} = \overline{\gamma} m_{\text{eff}}c^2 - (c^2/\overline{\gamma})\,(\overline{\mathbf{v}}\cdot\partial_{\overline{\mathbf{v}}}\, m_{\text{eff}}) + e\tilde{\varphi}, \tag{5}$$

so the motion is non-Newtonian at nonzero $\partial_{\overline{\mathbf{v}}} m_{\mathrm{eff}}$; yet $\mathscr{E} = \mathscr{H}(\overline{\mathbf{r}}, \overline{\mathbf{P}}, t)$ is conserved when $\mathscr{H}$ is independent of time, and m_{eff} can be viewed as the normalized quasi-energy of an unperturbed ($\tilde{F}_{\mu\nu} = 0$) particle in the guiding-center rest frame, $m_{\mathrm{eff}} = \mathscr{E}'/c^2$.

The unified effective mass formulation yields the known relativistic ponderomotive and diamagnetic forces, as well as magnetic drifts obtained by analyzing the m_{eff} dependence on the guiding center location and velocity [11]. Below, we consider another example of the classical particle nonlinear dynamics causing specific properties of m_{eff} and, therefore, of the guiding-center motion. In Sec. 3, we will employ these properties to suggest how the effective mass variability could be applied.

Example. — Consider a relativistic particle in a wave propagating along a static magnetic field [11]. Assume a smooth magnetic field $\mathbf{B} = \nabla \times \mathbf{A}_{\mathrm{dc}}$, approximately in the $\hat{\mathbf{z}}$ direction; then the vector potential $\mathbf{A}_{\mathrm{dc}}$ can be considered a linear function of the particle displacement $\mathbf{r}_\perp$ from the guiding center location. We will also assume a vacuum pump wave, for simplicity having a circular polarization in the plane transverse to $\mathbf{B}$, so the total vector potential reads $\mathbf{A} = \mathbf{A}_{\mathrm{dc}} + \mathbf{A}_{\mathrm{w}}$,

$$\mathbf{A}_{\mathrm{dc}} = B(z)\left(\hat{\mathbf{z}} \times \mathbf{r}_\perp\right)/2, \quad \mathbf{A}_{\mathrm{w}} = (mc^2/e)(a_0/\sqrt{2})\left(\hat{\mathbf{x}}\cos\xi - \hat{\mathbf{y}}\sin\xi\right), \tag{6}$$

where $a_0 \equiv eE_0/mc\omega = \mathrm{inv}$ is allowed to slowly vary in space and time, E_0 is the amplitude of the electric field $\mathbf{E} = -(1/c)\,\partial_t \mathbf{A}_{\mathrm{w}}$, $\xi = \omega t - kz$ is the phase, and $k = \omega/c$.

The corresponding particle motion is integrable, and m_{eff} is found analytically [11]:

$$m_{\mathrm{eff}} = m\left[1 + s^2 + \frac{a_0^2(2-\sigma)}{4(1-\sigma)^2}\right]\left[1 + s^2 + \frac{a_0^2}{2(1-\sigma)^2}\right]^{-\frac{1}{2}}. \tag{7}$$

Here $s^2 = 2\mu B/mc^2$ is the normalized Lorenz-invariant magnetic moment of the particle;

$$\sigma = \sigma_0/u = (\Omega_0\gamma^{-1})/(\omega - kv_z) \tag{8}$$

is the ratio of the particle relativistic gyrofrequency and the Doppler-shifted wave frequency (here $\sigma_0 = \Omega_0/\omega$, and $\Omega_0 = eB/mc$); $u = \gamma - p_z/mc$ is an integral given by

$$u = h\sqrt{1 + s^2 + \frac{a_0^2}{2(1-\sigma)^2}}, \quad h = \sqrt{\frac{c - \overline{v}}{c + \overline{v}}}; \tag{9}$$

γ and p_z are the particle Lorentz factor and momentum component; $mc^2a_0^2/4$ equals the zero-B nonrelativistic ponderomotive potential $\Phi = e^2E_0^2/4m\omega^2$.

By definition, Eq. (7) only refers to the particle motion along z. It yields the magnetized particle relativistic Hamiltonian [23, 24] and drifts [1, 23, 25, 26, 27, 28] at $a_0 = 0$ [11], the relativistic ponderomotive Hamiltonian [22, 29, 30, 31, 32, 33, 34, 35, 36, 37] at $B = 0$, the nonrelativistic ponderomotive potential in a magnetic field [6, 7, 38] at $v/c \ll 1$, and the "classic" ponderomotive potential Φ [6, 7, 8, 20] at $v/c \ll 1$ together with $B = 0$. From Eq. (7), it also follows that $m_{\mathrm{eff}}(a_0 < 4) > 0$, yet $m_{\mathrm{eff}}(a_0 > 4) < 0$ at least for some $\sigma > 1$.

With relativistic effects present, the cyclotron resonance is nonlinear and permits multiple energy states at given $\bar{v}$ and μ, as follows from Eq. (9), which can be rewritten as a fourth-order algebraic equation for u:

$$2(u-\sigma_0)^2(h^{-2}u^2-s^2-1)-a_0^2u^2=0. \tag{10}$$

In Ref. [11], we show that possible are, in fact, one or three energy states, *all being stable* to the variation of initial conditions, unlike for a one-dimensional (1D) nonlinear oscillator [39]. Thus, three different values of m_{eff} are possible, and a guiding center behaves differently in response to perturbation forces depending on which m_{eff} is selected.

Rewrite the average motion equation (2) as $d_t\bar{v}=\tilde{F}$, where $m_{||}=\partial_{\bar{v}}\bar{p}$, or

$$m_{||}=\partial_{\bar{v}}\left[\bar{\gamma}m_{\text{eff}}\bar{v}-(c^2/\bar{\gamma})\,\partial_{\bar{v}}m_{\text{eff}}\right],\quad \tilde{F}=-\partial_{\bar{z}}\left[\bar{\gamma}m_{\text{eff}}c^2-(c^2/\bar{\gamma})\,(\bar{v}\,\partial_{\bar{v}}m_{\text{eff}})+e\tilde{\varphi}\right]-\partial_t\bar{p},$$

where $\bar{p}=\bar{p}(\bar{z},\bar{v},t)$ [Eq. (4)]; hence $m_{||}$ is the effective longitudinal mass [40], and $\tilde{F}$ is the perturbation force. A straightforward derivation yields

$$m_{||}=m\bar{\gamma}^3\Gamma_2^{3/2}\Gamma_3^{-1}\qquad \Gamma_n=1+s^2+(a_0^2/2)(1-\sigma)^{-n}, \tag{11}$$

Γ_2 coinciding with $u^2(\bar{v}=0)$. In the absence of the laser field ($a_0=0$), Eq. (11) reads $m_{||}=m_{\text{eff}}\bar{\gamma}^3>0$, as one would expect for a particle with m_{eff} independent of $\bar{v}$ [40]. However, for nonzero a_0, one can show that $m_{||1,3}>0$, yet $m_{||2}<0$ for any $\bar{v}$ and s. Thus a particle residing at the second branch will exhibit unusual behavior in response to the force $\tilde{F}$, such as that, e.g., due to a gravitational or an electrostatic potential. Unlike a "normal" particle with a positive mass, a particle with $m_{||}<0$ will accelerate adiabatically in the direction *opposite* to $\tilde{F}$. Also, Eq. (11) predicts that $m_{||}$ will be an asymmetric function of $\bar{v}$; hence non-Newtonian effects, which we discuss in Sec. 3.

3. ANISOTROPIC DISPERSION

Consider the dispersion relation $\bar{p}(\bar{v})$ flowing from Eq. (7) as an example illustrating non-Newtonian effects for guiding centers subjected to a perturbation force $\tilde{F}$, assuming uniform and stationary background fields **E** and **B**. The branches 2 and 3 merge at $m_{||}(\bar{v})\to\infty$ so as to yield continuous yet double-valued $\bar{p}(\bar{v})$ [Figs. 1(a), (b)]; thus a particle can be adiabatically transferred between these branches as $\bar{p}$ changes, and the dependence of the particle quasi-energy $\mathscr{E}$ on $\bar{v}$ is α-shaped [Fig. 1(c)]. The local maximum corresponds to $m_{||}<0$ (branch 2) at rest ($\bar{v}=0$), whereas the minimum corresponds to $m_{||}>0$ (branch 3), also at rest; hence the possible applications.

Negative-mass plasma maser. — The presence of an energy zone with $m_{||}<0$ allows for a lasing mechanism known from solid state physics as the Negative Effective Mass Amplifier and Generator, or NEMAG [41, 42, 43] and is explained as follows [44]. Suppose a cold plasma is first created in a dc magnetic field, after which the wave is turned on slowly with initial $a_0=0$ and $\omega<\Omega_0$, so particles occupy the positive-mass branch 3. Suppose that the particles are then adiabatically pushed in the $-z$ direction. As a result, their guiding-center momentum $\bar{p}$ is reduced, causing the particle transfer

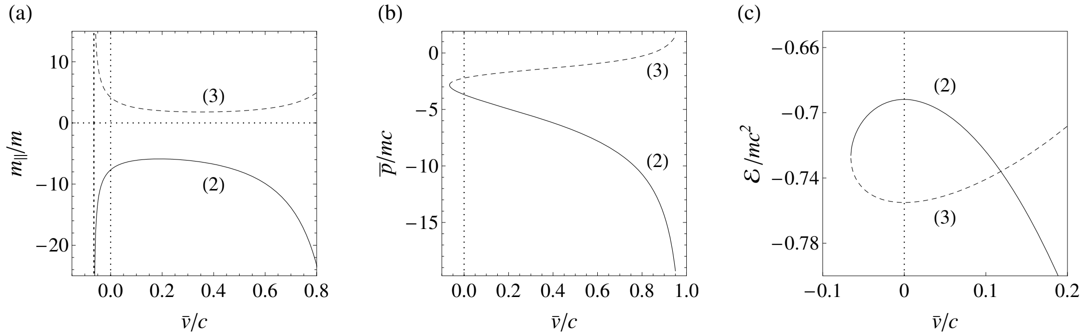

FIGURE 1. The guiding center effective longitudinal mass $m_{||}$, the canonical momentum $\overline{p}$, and the quasi-energy $\mathscr{E}$ for the branches 2 (solid) and 3 (dashed) as in Ref. [11] ($a_0 = 5\sqrt{2}$, $\sigma_0 = 8.3$, $s = 0$). The guiding center velocity $\overline{v}$ is measured in units c; $\overline{p}$ is measured in units mc; $\mathscr{E}$ is measured in units mc^2. The dotted lines mark zeros and, in Fig. (a), also the vertical asymptote at $v = v_*$ for $m_{||} \to \pm\infty$.

to the higher-energy branch 2 [Fig. 1(b)]; hence the population inversion. Consider the interaction of the inverted population with a low-frequency pulse $\tilde{F}$, which will transfer the momentum $\Delta\overline{p} = \int_{-\infty}^{+\infty} \tilde{F}[\overline{\mathbf{r}}(t), t]\, dt$ to individual particles. Since the particles get trapped and untrapped by the pulse [45, 46] (or when its envelope has a time scale comparable to $2\pi/\varpi$ or acts over a nonuniform background [20]), $\Delta\overline{p}$ will generally be nonzero; thus, the pulse will exchange energy with the plasma. The energy $\Delta\mathscr{E}$ transferred to the pulse per particle will depend on $\Delta\overline{v}(\Delta\overline{p})$. Yet, since the negative-mass rest state corresponds to the energy maximum, $\Delta\mathscr{E}$ will be positive for all $\Delta\overline{p}$; particularly, the transition back to the positive-mass rest state will yield the largest gain

$$\Delta\mathscr{E} = (m_{\text{eff},2} - m_{\text{eff},3})\, c^2 > 0, \tag{12}$$

where the right-hand side is evaluated at $\overline{v} = 0$. The pulse can therefore be amplified or generated from noise, assuming an appropriate resonant feedback.

Like in solid-state or gas lasers, it is the pump wave energy that is being channeled through particles to the amplified signal in the proposed scheme. Thus, in contrast to free electron lasers and similar devices [47, 48] or a related cyclotron instability used for α channeling [49], the particle translational energy is not at stake here, and neither collective effects are essential, unlike in the known plasma masers [50, 51]. These features make the new mechanism unique in its class and promising in marrying the advantages of solid-state oscillators with high powers, which plasma naturally tolerates.

Hamiltonian ratchet. — Suppose that a weak oscillating field is imposed over a uniform stationary pump, so the guiding center canonical momentum is governed by

$$d_t\overline{p} = \tilde{F}, \quad \tilde{F} = \mathscr{F} \sin\varpi t. \tag{13}$$

If $\mathscr{F}$ is fixed or evolves adiabatically in time, Eq. (13) yields $\overline{p} = \overline{p}_0 + \delta\overline{p}$, where $\overline{p}_0$ is a constant, and $\delta p = -(\mathscr{F}/\varpi)\cos\varpi t$ has a zero average. Assuming that $\mathscr{F}$ is small, one can Taylor-expand the guiding center velocity $\overline{v}(\overline{p})$ around $\overline{v}_0 \equiv \overline{v}(\mathscr{F} = 0)$ so as to get

$$\overline{v} \approx \overline{v}_0 + \delta\overline{p}\; \partial_{\overline{p}}\overline{v} + (\delta\overline{p}^2/2)\, \partial^2_{\overline{p}\overline{p}}\overline{v}, \tag{14}$$

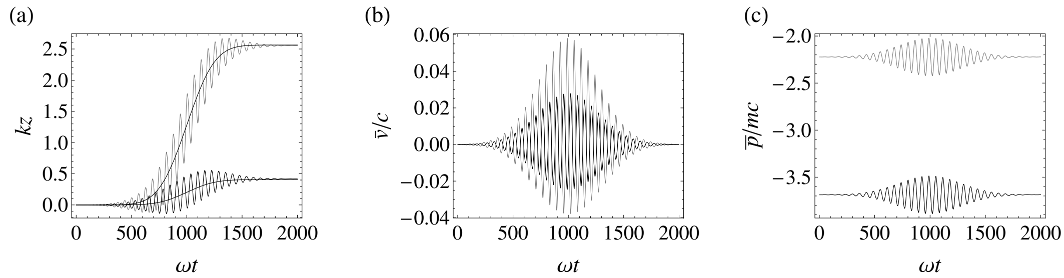

FIGURE 2. The particle drift induced by the perturbation force (13), with $\mathscr{F} = \mathscr{F}_0 \exp[-(t-t_0)^2/T^2]$. Here $\mathscr{F}_0 = 0.01$, $\varpi = 0.1\omega$, $\omega T = 400$, $\omega t_0 = 1000$, $a_0 = 5\sqrt{2}$, $\sigma_0 = 8.3$, $s = 0$; the initial guiding center velocity is $\bar{v}_0 = 0$; the particle is at the branch 2 (black) or 3 (gray). (a) The guiding center coordinate $\bar{z}$ vs. time (here $\bar{z}$ matches z); the nonoscillatory curves are analytical results derived from Eq. (16). (b) The guiding center velocity $\bar{v}$ vs. time. (c) The guiding center canonical momentum $\bar{p}$ vs. time. Same units as in Fig. 1; also, z is measured in units $k^{-1} = c/\omega$, and t is measured in units ω^{-1}.

or, using $m_{||} = \partial_{\bar{v}}\bar{p}$,

$$\bar{v} \approx \bar{v}_0 - [\mathscr{F}/(\varpi m_{||})] \cos\varpi t + \tilde{\Phi}\,(\cos 2\varpi t + 1)\,\partial_{\bar{v}}(m_{||}^{-1}), \tag{15}$$

where $\tilde{\Phi} = \mathscr{F}^2/4\varpi^2 m_{||}$ is the perturbation ponderomotive potential.

Eq. (15) predicts that, in average over the oscillation period $2\pi/\varpi$, the particle will exhibit a drift velocity (in addition to $\bar{v}_0$):

$$\bar{v}_{\mathrm{d}} = \tilde{\Phi}\,\partial_{\bar{v}}(m_{||}^{-1}). \tag{16}$$

In contrast to an isotropic dispersion law $m_{||} = m_{||}(\bar{v}^2)$ precluding the drift at $\bar{v}_0 = 0$, an anisotropic $m_{||}(\bar{v})$ allows acceleration of initially resting particles with an oscillating $\tilde{F}$ having no bias. (This mechanism can be understood as a variation on the Hamiltonian ratchet; however, unlike those considered elsewhere [52, 53], the new ratchet relies on entirely regular dynamics.) For a magnetized wave-driven particle (Sec. 2) with $\bar{v}_0 = 0$, one has $\partial^2_{\bar{p}\bar{p}}\bar{v} > 0$ [Fig. 1(b)]; hence a drift with $\bar{v}_{\mathrm{d}} > 0$ [Fig. 2(a)].

Frequency doubling. — The velocity (15) also has an oscillatory component

$$\bar{v}_{\sim} = -[\mathscr{F}/(\varpi m_{||})] \cos\varpi t + \tilde{\Phi}\,\cos 2\varpi t\,\partial_{\bar{v}}(m_{||}^{-1}), \tag{17}$$

which contains both the first and the second harmonics of the signal $\tilde{F}$. Therefore, a linear [in terms of Eq. (13)] interaction with an oscillatory perturbation force allows one also to generate an output wave at a doubled frequency. Fig. 2(b) shows the asymmetry in $\bar{v}(t)$ oscillations for the system considered in Sec. 2, hence proving the presence of higher harmonics despite $\bar{p}(t)$ remains monochromatic [Fig. 2(c)]. If $\bar{v}_0 = \bar{v}_*$, such that $m_{||}(\bar{v}_*) \to \pm\infty$, the first harmonic in $\bar{v}(t)$ vanishes ($\partial_{\bar{p}}\bar{v} \to 0$), so it is *only* the second harmonic that contributes to the electric current (Fig. 3) and thus determines the particle radiation spectrum.

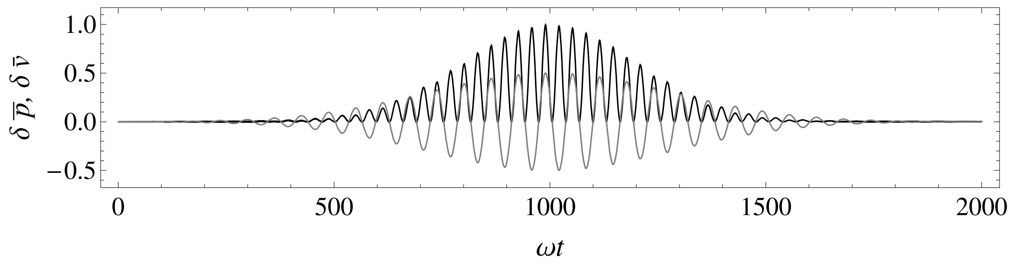

FIGURE 3. The particle response at $\overline{v}_0 = \overline{v}_*$ to the perturbation force (13) same as in Fig. 2. Variations of the guiding center canonical momentum (gray) and velocity $\overline{v}$ (black) show that, although $\overline{p}$ oscillates at the first harmonic of $\tilde{F}$, $\overline{v}$ oscillates at the second harmonic. Arbitrary units for $\delta\overline{p}$ and $\delta\overline{v}$; the time is measured in units ω^{-1}.

4. HYBRID POTENTIALS AND ONE-WAY WALLS

Substantial variations of the particle effective mass in relativistic fields yield "metaplasmas" with adjustable properties; hence parametric effects suitable for continuous conversion of electromagnetic signals [54] or static control over the plasma low-frequency dielectric constant $\varepsilon(m_{||})$, e.g., for inducing plasma transparency ($\varepsilon \to 1$) or sensitive detection due to abrupt acceleration of particles with $m_{||} \to 0$. Yet at lower energies guiding centers can also behave differently from Newtonian particles and thus permit complementary manipulation techniques, as discussed below.

Rewrite Eq. (1) as

$$\mathscr{L} = -mc^2 + m\overline{v}^2/2 - \Psi, \quad \Psi = \delta m c^2, \tag{18}$$

where we used $m_{\text{eff}} = m + \delta m$, $\delta m \ll m$, and $\overline{v}/c \ll 1$. For simplicity, assume that δm is independent of $\overline{\mathbf{v}}$; hence $\Psi \propto \delta m$ plays a role of an effective potential, which can be further expanded in the oscillation actions: $\Psi = \sum_i \Psi_i$. The linear expansion yields [55]

$$\Psi \approx \Psi_1 = W + \Phi, \quad W = \mathbf{J}\cdot\mathbf{\Omega}, \quad \Phi = -\mathbf{E}_0^* \cdot \hat{\boldsymbol{\alpha}} \cdot \mathbf{E}_0/4, \tag{19}$$

where W is the oscillation energy [5], Φ is the ponderomotive potential [10], $\hat{\boldsymbol{\alpha}}$ is the particle polarizability tensor, and $\mathbf{E}_0$ is the oscillatory field amplitude [56]; e.g., $\hat{\boldsymbol{\alpha}} = -(e^2/m\omega^2)\hat{\mathbf{I}}$ for a free electron ($\hat{\mathbf{I}}$ being a unit tensor) yields $\Phi = e^2E_0^2/4m\omega^2$ [6, 7]. In this order, adiabatic motion [10, 20] is conservative, and, at fixed $\mathbf{\Omega}$, the average force on the particle is entirely determined by the $E_0(\overline{\mathbf{r}})$ profile. However, higher-order Ψ_i can depend on E_0 and $\mathbf{J}$ *simultaneously*, yielding so-called hybrid ponderomotive potentials.

For a particle with two natural frequencies satisfying $\Omega_1 - \Omega_2 \approx \omega$, the second-order hybrid potential reads [55]

$$\Psi_2 = \kappa_2 |E_0|^2\,(J_1 - J_2)/(\Omega_1 - \Omega_2 - \omega), \tag{20}$$

where $\kappa_2 > 0$ is a constant. Since Ψ_1 is nonresonant in this case [5], one also has $\Psi \approx \Psi_2$; thus the ponderomotive force essentially depends on the actions J_1 and J_2. Should particles incident on the oscillatory field from opposite directions receive different $\mathbf{J}$ via nonadiabatic interaction with another field, the barrier will operate asymmetrically and can be arranged to transmit particles from one side yet reflect them from another side. Should the particles later exhibit radiative thermalization *before* returning to the barrier, the oscillatory field will hence operate like a Maxwell demon, decreasing their translational entropy without collisions.

A similar effect is possible at the main yet nonlinear resonance. Suppose a single eigenfrequency $\Omega \approx \omega$; then the ponderomotive potential universally reads [5]

$$\Phi = \kappa_1 |E_0|^2/(\omega - \Omega), \tag{21}$$

where $\kappa_1 > 0$ is a constant. For Ω being a function of J, the maximum of Φ is sensitive to the particle internal energy; hence the one-way wall effect, as predicted for atoms in Refs. [14, 19, 15] and recently confirmed in experiments [57, 58].

Without a second field required to preheat particles, asymmetric barriers are also realized due to hysteresis [55]. Yet similar barriers are as well possible for linear oscillators – when the particle natural frequencies vary in space, either due to $\mathbf{J} \cdot \mathbf{\Omega}$ potential [59], or due to Φ being asymmetric at the main resonance. The latter effect is explained as follows: given a gradient of Ω, $\Phi(\bar{z})$ [Eq. (21)] exhibits a singularity at $\Omega = \omega$, and the effective potential is repulsive at $\Omega(\bar{z}) < \omega$ while being attractive at $\Omega(\bar{z}) > \omega$; hence a Hamiltonian ratchet is produced and can be employed, e.g., for current drive [16, 17, 18]. In Ref. [12], the anticipated dynamics was confirmed for cyclotron-resonant rf fields, and, in Ref. [13], an alternative scheme with abrupt $E_0(\mathbf{r})$ was proposed.

5. CONCLUSIONS

Classical particles oscillating in high-frequency or static fields effectively exhibit a modified rest mass m_{eff} which determines the guiding center motion. Unlike the true mass, m_{eff} depends on the field parameters, can take negative values, and is generally a nonanalytic function of the particle average velocity and the oscillation energy; hence guiding centers exhibit non-Newtonian effects, producing "metaplasmas" with unusual properties. In this paper, we propose a number of applications flowing from m_{eff} dependence on the guiding center velocity, including a new type of plasma maser, signal rectification, and frequency doubling. We also contemplate effects based on nonadiabatic variations of m_{eff}, particularly one-way walls, which could be applied for current drive and translational cooling. Like in Refs. [20, 60, 61] that cover the associated quantum-like effects, such variations of m_{eff} render additional flexibility in particle manipulation and allow techniques complementary to those for non-oscillatory objects.

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