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Non-Newtonian mechanics and manipulation of oscillation centers

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Non-Newtonian mechanics and manipulation of oscillation centers

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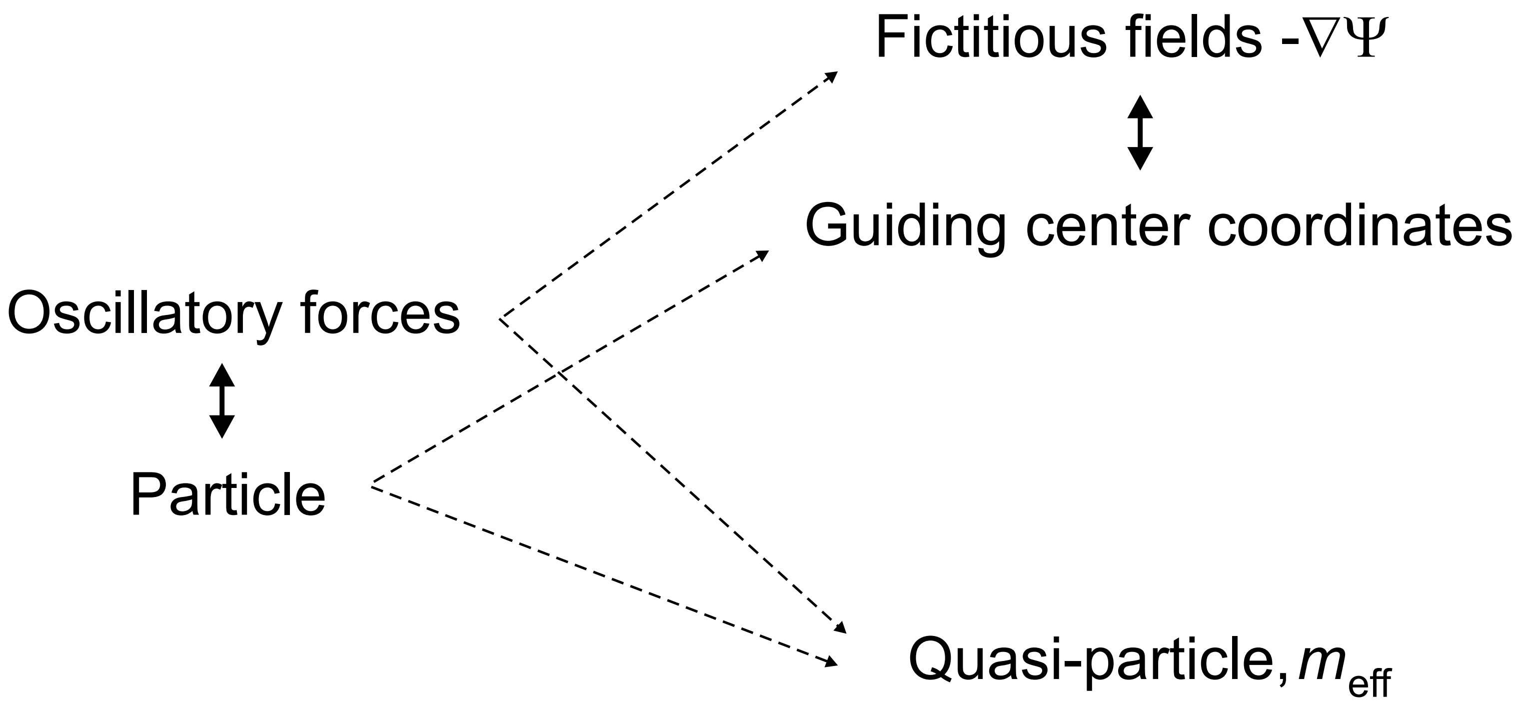
Guding Centers as Quasi-particles



Fully relativistic object-oriented approach:
all ponderomotive, diamagnetic etc. effects
are embedded in the effective mass

Examples

$$\boxed{\mathcal{H} = \sqrt{m_{\text{eff}}^2 c^4 + \bar{p}^2 c^2}, \quad m_{\text{eff}} = c^{-2}\big(\mathbf{J}\cdot\boldsymbol{\Omega} - \langle L\rangle\big)_{\langle\bar{\mathbf{v}}\rangle=0}}$$

- Particle in a magnetic field:

$$\mathcal{H} = \sqrt{m^2c^4 + 2\mu B mc^2 + p_{\parallel}^2 c^2} \approx$$

$$\approx p_{\parallel}^2/2m + \mu B$$

$$\boxed{m_{\text{eff}} = m\sqrt{1 + 2\mu B/mc^2}}$$

e.g., Grebogi and Littlejohn, Phys. Fluids, 1984 ; Boozer, Phys. Plasmas, 1996

- Particle in a vacuum laser pulse:

$$\mathbf{A} = (mc^2/e)\,\mathbf{a}(\mathbf{r},t) \propto e^{i\xi}$$

$$\xi = \omega t - \mathbf{k}\cdot\mathbf{r}, \quad \mathbf{k} \approx \hat{\mathbf{z}}\,\omega/c$$

$$\boxed{m_{\text{eff}} = m\sqrt{1 + \langle a^2\rangle_{\xi}}}$$

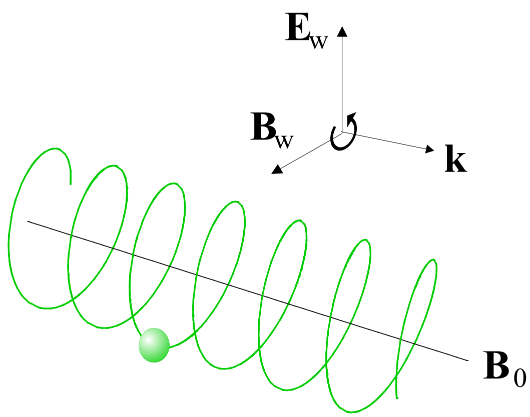
e.g., Kibble, Phys. Rev., 1966 ; Bauer et al.*, Phys. Rev. Lett., 1995 ; Mora and Jr., Phys. Plasmas, 1997*

- m_{eff} can also be a negative or even nonanalytic function of the particle average velocity and the oscillation energy; hence non-Newtonian dynamics.

Outline

- Example: laser field over a dc magnetic field
- Particles with anisotropic dispersion $\bar{p}(\bar{v})$:
  - Negative-mass plasma maser
  - Hamiltonian ratchet and current drive
  - Frequency doubling
- Selective barriers:
  - Nonrelativistic hybrid potentials
  - One-way walls and cooling effects
- Quantum-like effects

Example: Laser Field + DC Magnetic field

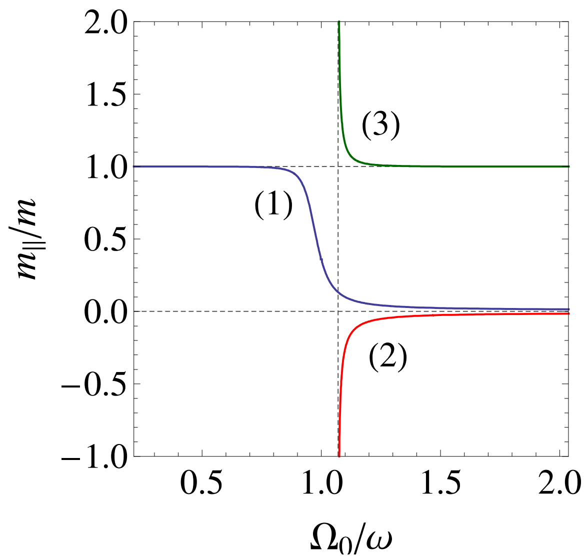


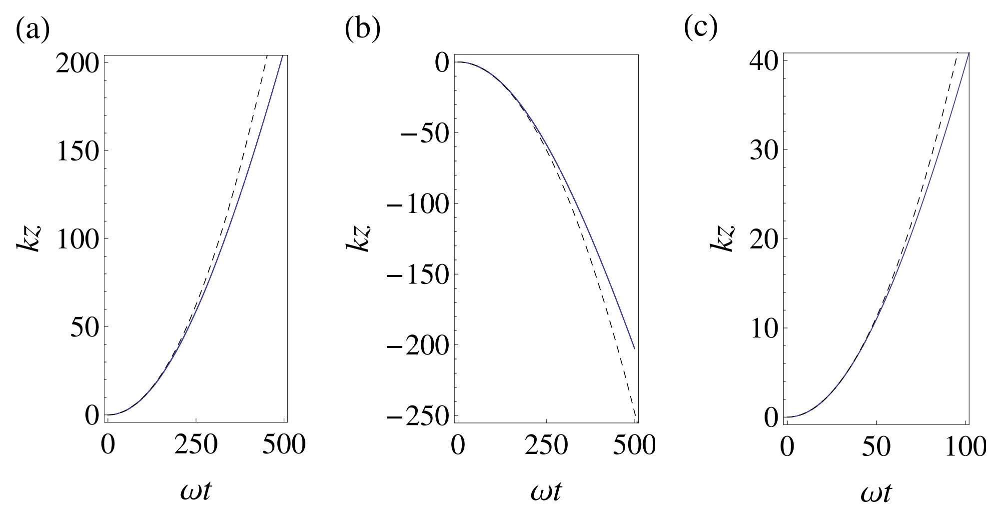
$$m_{\|}(\mathbf{E}_\mathrm{w}, \mathbf{B}_0)\, \frac{d^2 \bar{z}}{dt^2} = F_{\|}$$

$$m_{\|} = \frac{\partial}{\partial \bar{v}} \left(\bar{\gamma} m_\mathrm{eff} \bar{v} - \frac{c^2}{\bar{\gamma}} \frac{\partial m_\mathrm{eff}}{\partial \bar{v}} \right)$$

Dodin and Fisch, Phys. Rev. E, 2008

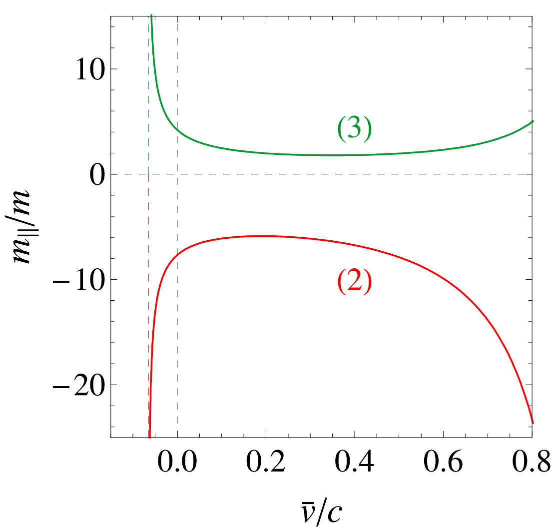
- Three different masses:

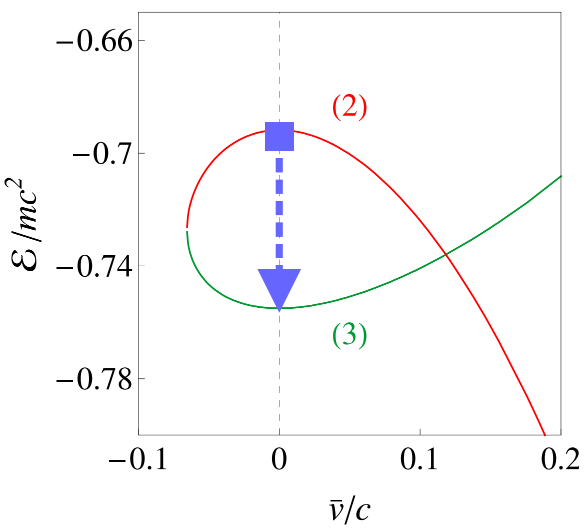


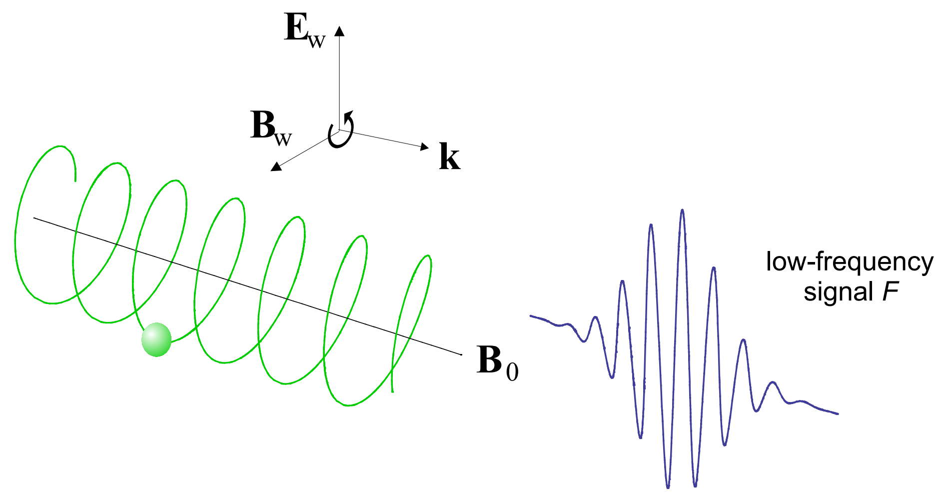


(a) $m_{\|1} > 0$, (b) $m_{\|2} < 0$, (c) $m_{\|3} > 0$.
Numerical (solid) vs $z(t) = F_{\|} t^2 / 2m_{\|}$ (dashed).

Anisotropic Dispersion. Negative-Mass Amplifier







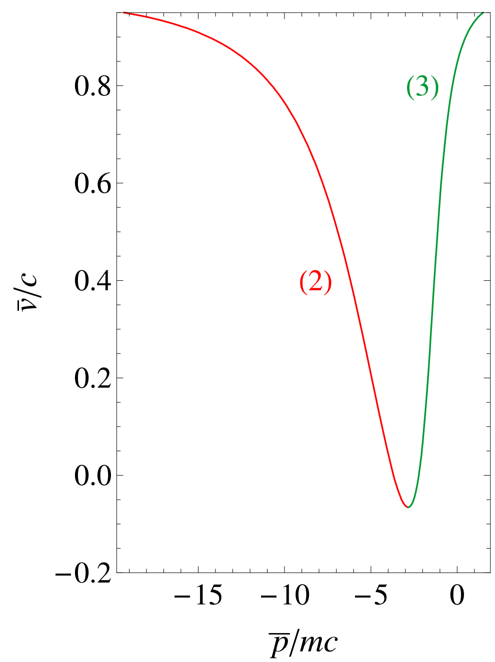
- Particles cannot absorb energy at branch 2; instead, their own energy goes to the pulse.

$$\boxed{\Delta\mathcal{E} = (m_{\mathrm{eff},2} - m_{\mathrm{eff},3})\, c^2 > 0}$$

- Lasing mechanism using $m_\parallel < 0$

Krömer, Phys. Rev., 1958 ; Andronov et al., *Pis'ma Zh. Eksp. Teor. Fiz., 1984*

- Dispersion law:

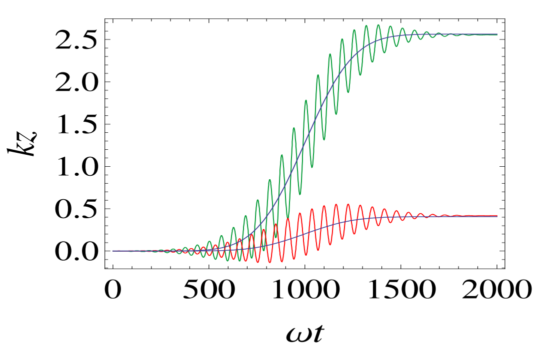


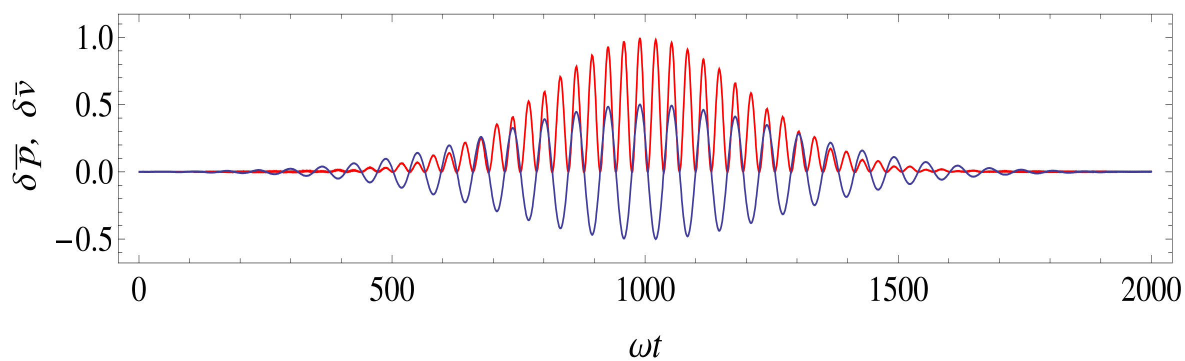
$$\bar{p} = \bar{\gamma} m_{\text{eff}} \bar{v} - (c^2/\bar{\gamma})\, \partial_{\bar{v}} m_{\text{eff}}$$

$$\dot{\bar{p}} = \mathcal{F}\, \sin \varpi t \quad \Rightarrow \quad \bar{p} = -(\mathcal{F}/\varpi) \cos \varpi t$$

$$\bar{v} \approx \bar{v}_0 + \Delta \bar{p}\ \partial_{\bar{p}} \bar{v} + (\Delta \bar{p}^2/2)\, \partial^2_{\bar{p}\bar{p}} \bar{v}$$

$$\boxed{\bar{v} \approx \bar{v}_0 \underbrace{- \frac{\mathcal{F}^2}{4\varpi^2 m_\parallel} \frac{\partial m_\parallel^{-1}}{\partial \bar{v}}}_{\bar{v}_{\text{drift}}} + \underbrace{v_1\ \cos \varpi t + v_2\ \cos 2\varpi t}_{\bar{v}_\sim}}$$





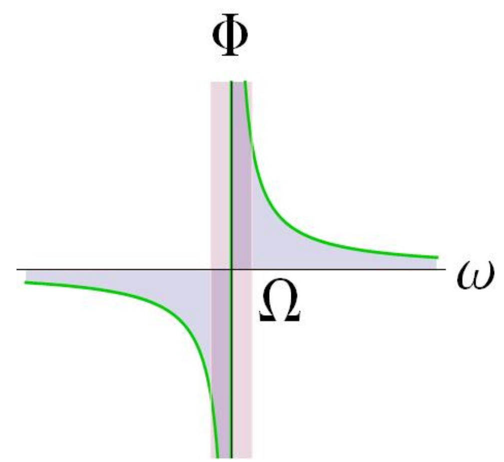
Nonrelativistic Effective Potential

- Nonrelativistic energies:
  - For small δm, Ψ can yet be a significant effective potential

$$\boxed{\mathcal{H} \approx \frac{\bar{p}^2}{2m} + \Psi, \quad \Psi = \delta m\, c^2}$$

$$\delta m \equiv m_{\text{eff}} - m \ll m$$

- Natural oscillators:
  - Atoms and molecules – quantum oscillators
  - Clusters (ion core + electron cloud), $\Omega = \omega_p/\sqrt{3}$
  - Charged particles in a magnetic field, $\Omega = eB/mc$

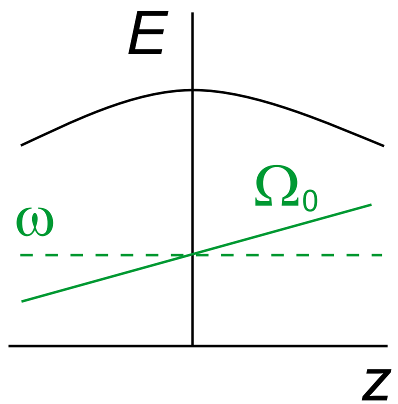


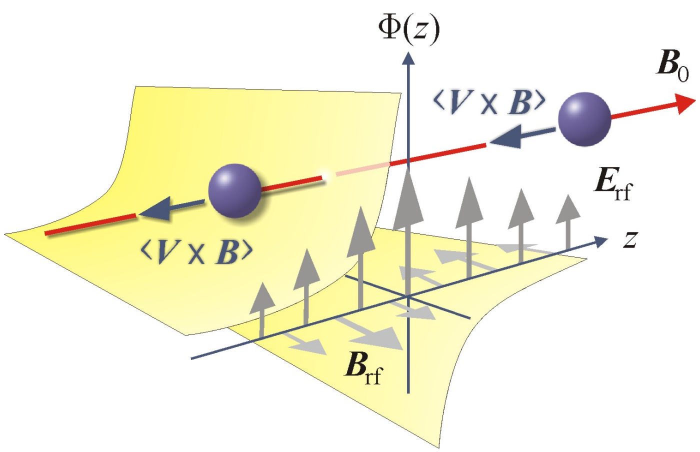
$$\boxed{\Psi \approx \Phi + J\Omega, \quad \Phi \approx \frac{\kappa_1 |E|^2}{\omega - \Omega}}$$

Dodin and Fisch, Phys. Lett. A, 2006

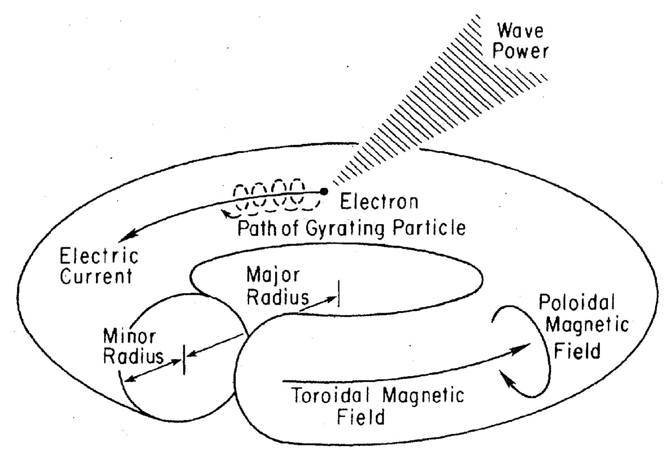
$\kappa_1 > 0$; J is the action, Ω is the natural oscillation frequency, e.g., $J\Omega = \mu B$

Singular Asymmetric Ponderomotive Potential





- Asymmetric potential $\Phi \propto \frac{|E|^2}{\omega - \Omega(z)}$:
  - Repulsive at $\Omega(z) < \omega$
  - Attractive at $\Omega(z) > \omega$
- "Maxwell demon" effect
- Possible current drive mechanism



Fisch et al., *Phys. Rev. Lett., 2003*; *Dodin* et al., *Phys. Plasmas, 2004*; *Suvorov and Tokman, Fiz. Plazmy, 1988*

One-way Walls for Nonlinear Oscillators. Translational Cooling

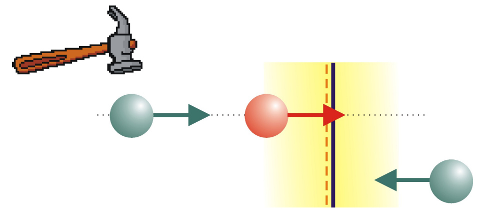
- Energy-dependent ponderomotive potentials:
  - Nonlinear frequency, at $\omega \approx \Omega$:

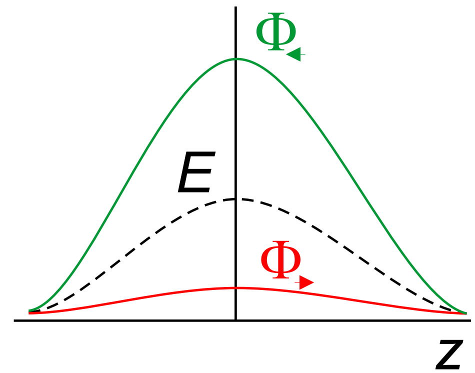
$$\boxed{\Psi = \frac{\kappa_1 |E|^2}{\omega - \Omega(J)}}$$

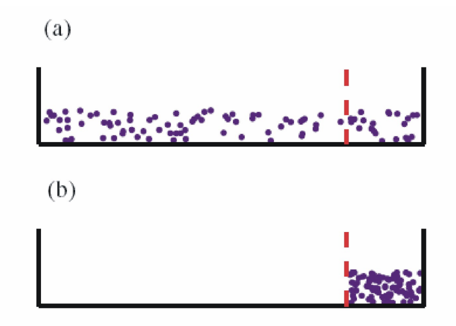
  - Coupled oscillations, at $\omega \approx \Omega_1 - \Omega_2$:

$$\boxed{\Psi = \kappa_2 |E|^2 \, \frac{J_1 - J_2}{\Omega_1 - \Omega_2 - \omega}}$$

- Possible to manipulate particles with fixed Ω
- Translational cooling is possible

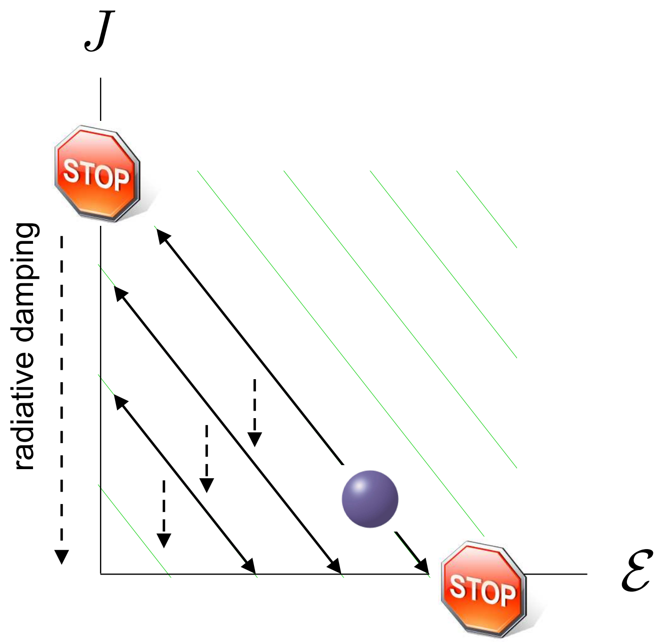






Dodin and Fisch, Phys. Lett. A, 2006; *Raizen* et al., *Phys. Rev. Lett., 2005*; *Ruschhaupt and Muga, Phys. Rev. A, 2004*; *Price* et al., *Phys. Rev. Lett., 2008*; *Thorn* et al., *Phys. Rev. Lett., 2008*

Sisyphus-like Translational Cooling



- Translational motion is coupled with internal oscillations:

$$\mathcal{E} + (\Omega - \omega)J = \text{const}, \quad \Omega > \omega$$

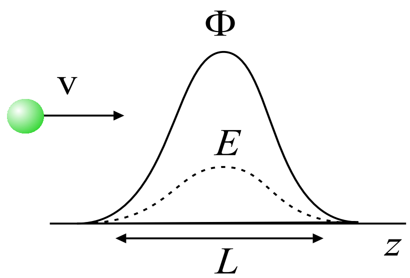
- Radiative damping reduces the free oscillation energy; thus the oscillation action J is decreased.

- "Dressed" particle energy $\mathcal{E} \equiv mv^2/2 + \Phi$ is reduced accordingly.

Assumptions: no collisions, negligible photon recoil

Dodin and Fisch, Phys. Lett. A, 2006; *Dodin and Fisch, Phys. Plasmas, 2007*

Quantum Analogy



$$E_{\sim}(z,t) = E(z)\cos\omega t$$

$$\mathbf{d} = \hat{\alpha}\mathbf{E}, \quad \text{if } \epsilon \ll 1$$

$$\epsilon = \lambda_{\text{eff}}/L, \quad \lambda_{\text{eff}} = \bar{v}T$$

λ_{eff} is the average displacement on the oscillation period T

$$\bar{z} = z - \epsilon^0 z_{\sim}^{(0)}(z,t) - \epsilon^1 z_{\sim}^{(1)}(z,t) + \ldots$$

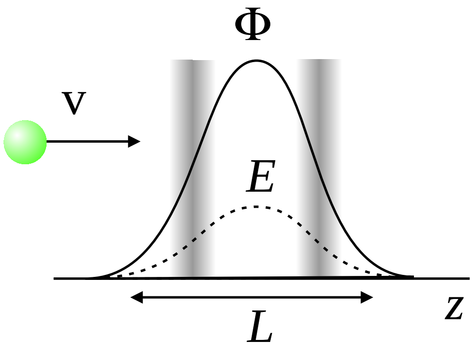
$$m\ddot{\bar{z}} = -\Phi'(\bar{z})$$

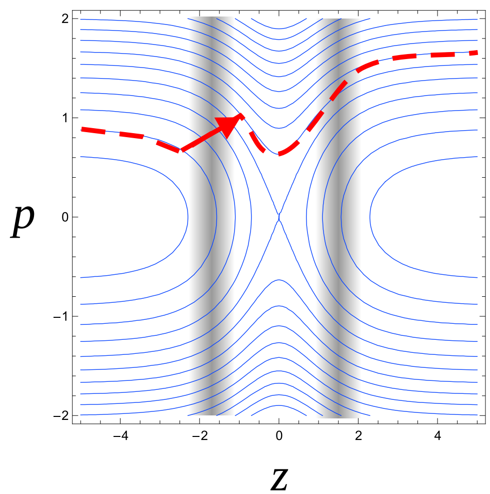
$$\bar{z} = z - \epsilon^0 z_{\sim}^{(0)}(z,t) + \delta\bar{z}, \qquad \delta\bar{z} \sim \lambda_{\text{eff}}$$

Adiabatic interaction	$\lambda_{\text{eff}} \ll L$	*"Quasi-classical" interaction*
Nonadiabatic interaction	$\lambda_{\text{eff}} \gtrsim L$	*Quantum-like interaction*
Location uncertainty	$\lambda_{\text{eff}} = \bar{v}T$	*Effective de Broglie wavelength*
Guiding center	$\leftrightarrow$	*Quantum object*

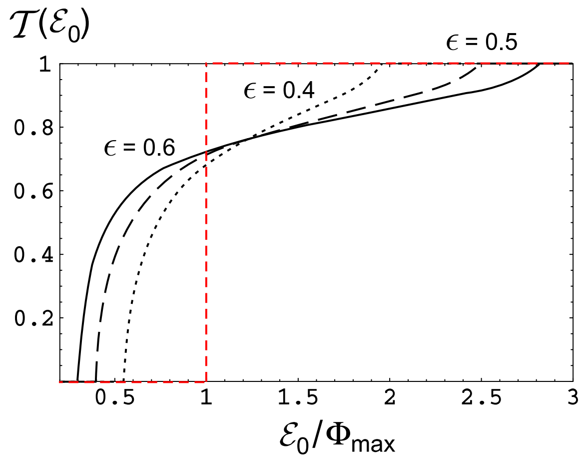
Dodin and Fisch, Phys. Rev. Lett., 2005

Quantum-like Tunneling and Reflection

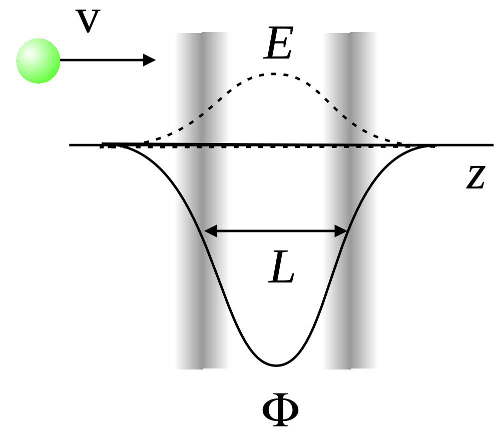


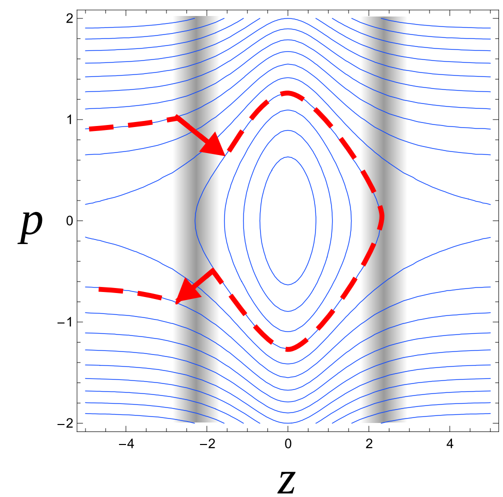


$$\epsilon \sim \bar{v}/\omega L(z), \qquad L = E/|E'|$$



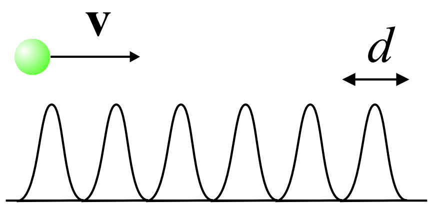
$$\boxed{\mathcal{T} \approx \frac{1}{2} - \frac{1}{\pi}\ \arcsin \frac{\Phi_{\max} - \mathcal{E}_0}{\delta\mathcal{E}(\epsilon)}}$$



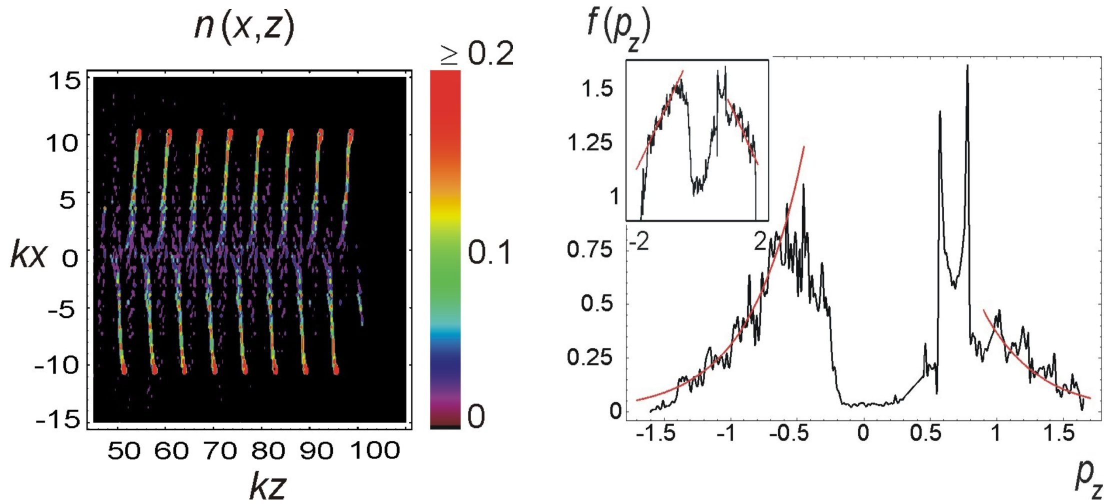


Dodin and Fisch, Phys. Rev. E, 2006

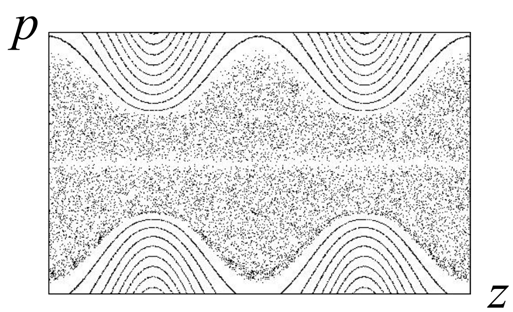
Particle Beam Slicing

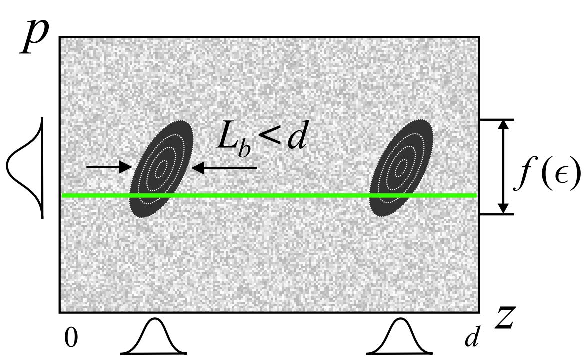


- Beam is sliced to bunches with $L_b < d$



Dodin and Fisch, Phys. Rev. Lett., 2007

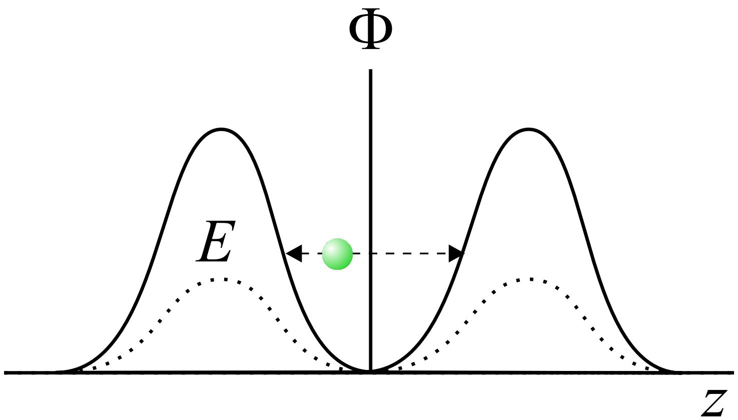


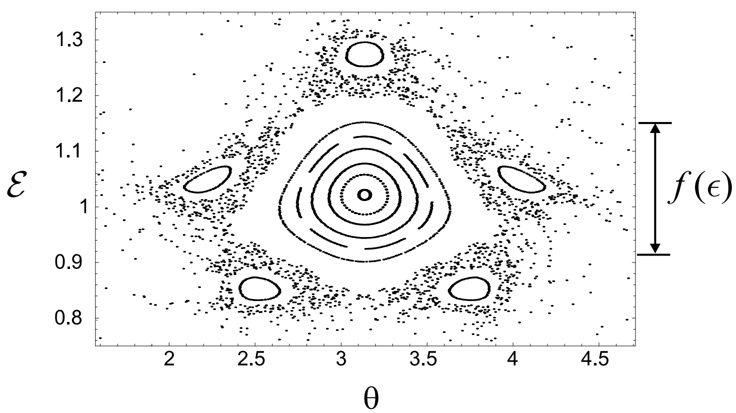


- Analogous technique is used to slice atomic beams using quantum accelerator modes.

d'Arcy et al., *Phys. Rev. A, 2003*

Quantized Energy States





- Only periodic orbits survive at $\epsilon \gtrsim 1$
- Guiding center phase: $\Delta\psi = 2\pi n$
- Bohr-Sommerfeld rule:

$$\boxed{\oint k\,dz = 2\pi n}$$

$$k = 2\pi/\lambda_{\text{eff}}, \quad \lambda_{\text{eff}} = \bar{v}T$$

Dodin and Fisch, Phys. Rev. Lett., 2005

Summary

- Guiding centers can be viewed as non-Newtonian quasi-particles with a rest mass m_{eff} different from the particle true mass.

- m_{eff} depends on the field parameters and can be a nonanalytic function of the particle average velocity and the oscillation energy; hence the applications.

- Adiabatic effects:
  - negative-mass plasma maser,
  - signal rectification,
  - frequency doubling.

- Nonadiabatic and quantum-like effects:
  - one-way walls for current drive and translational cooling,
  - phase-dependent transmission ("tunneling"),
  - particle beam slicing,
  - selective confinement using quantized energy levels.