The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
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1953-37

International Workshop on the Frontiers of Modern Plasma Physics

14 - 25 July 2008

Laser wakefield acceleration in the Peta-Watt regime.

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Laser wakefield acceleration in the Peta-Watt regime

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Outline

- Short and long laser pulses
- Modified wakefield acceleration
- Stochastic wakefield acceleration
- Unstable betatron oscillations
- Stochastic two-wave acceleration
- Snow-plow acceleration
- Photon mirror effect
- Imperfect relativistic mirror
- Conclusions

Motivations

Laser wakefield acceleration concept:
-T. Tajima and J.M. Dawson, PRL (1979)

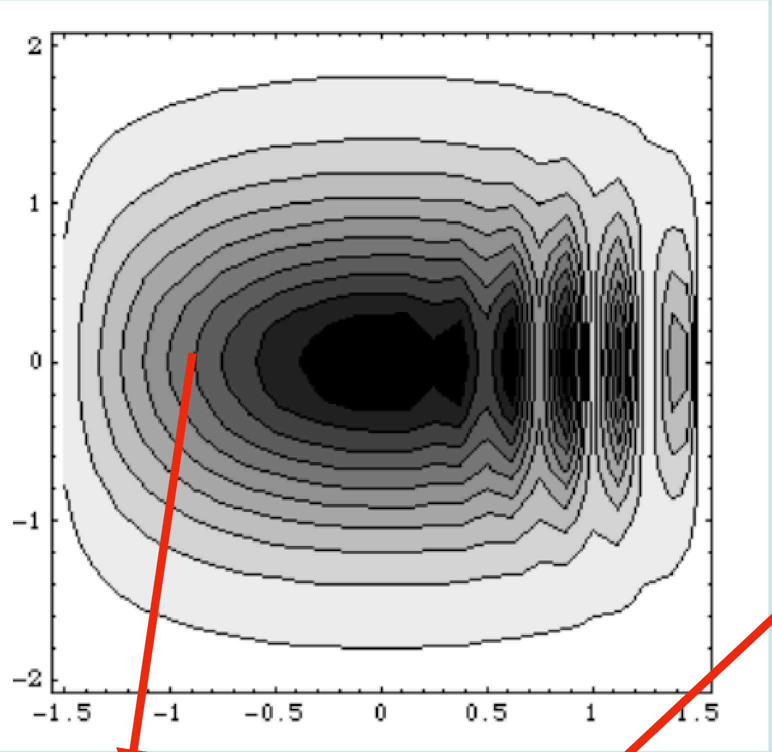
Bubble regime:
- A. Pukhov and J. Meyer-ter-Vehn, APB (2002)

Experimental results:
-J. Faure et al; S. Mangles et al.; C. Geddes et al, N (2004)
- W. Leemans et al, NP (2006)

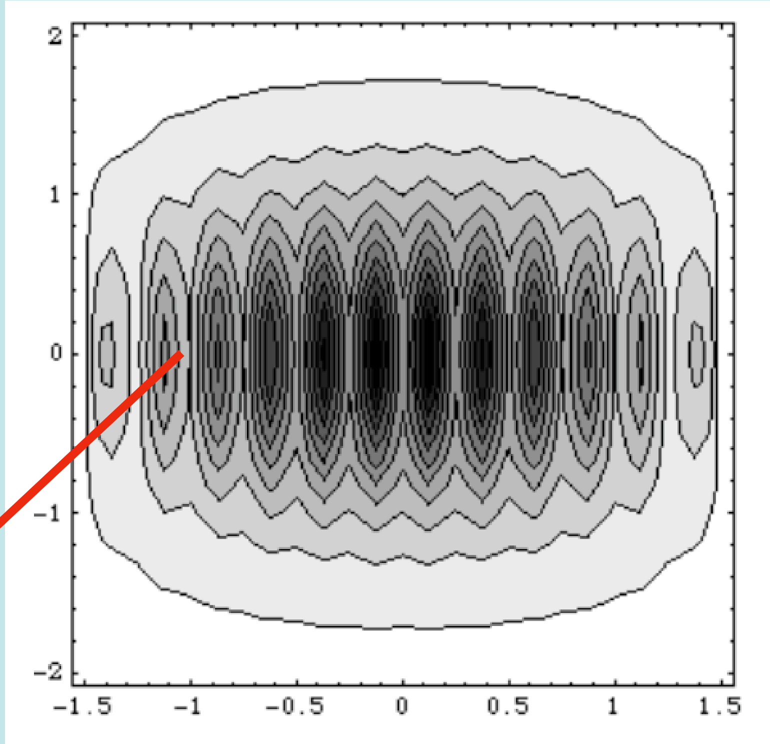
Peta-Watt experiments:
- RAL; HIPER - European experiment

Laser wakefield regimes

Short laser pulse



Long laser pulse



Acceleration region

$$\Delta t \ \omega_p \geq 1$$

Modified wakefield acceleration

Particle trajectories in parallel phase space

$$p(\eta) = \gamma_g^2 \left\{ [h_0 - \varphi(\eta)]\beta_g \pm \sqrt{[h_0 - \varphi(\eta)]^2 - \nu^2(\eta)/\gamma_g^2} \right\}$$

$$\gamma_g = (1 - \beta_g^2)^{-1/2}$$

$$\nu^2(\eta) = 1 + [\vec{p}_{\perp 0} - \vec{a}(\eta)]^2$$

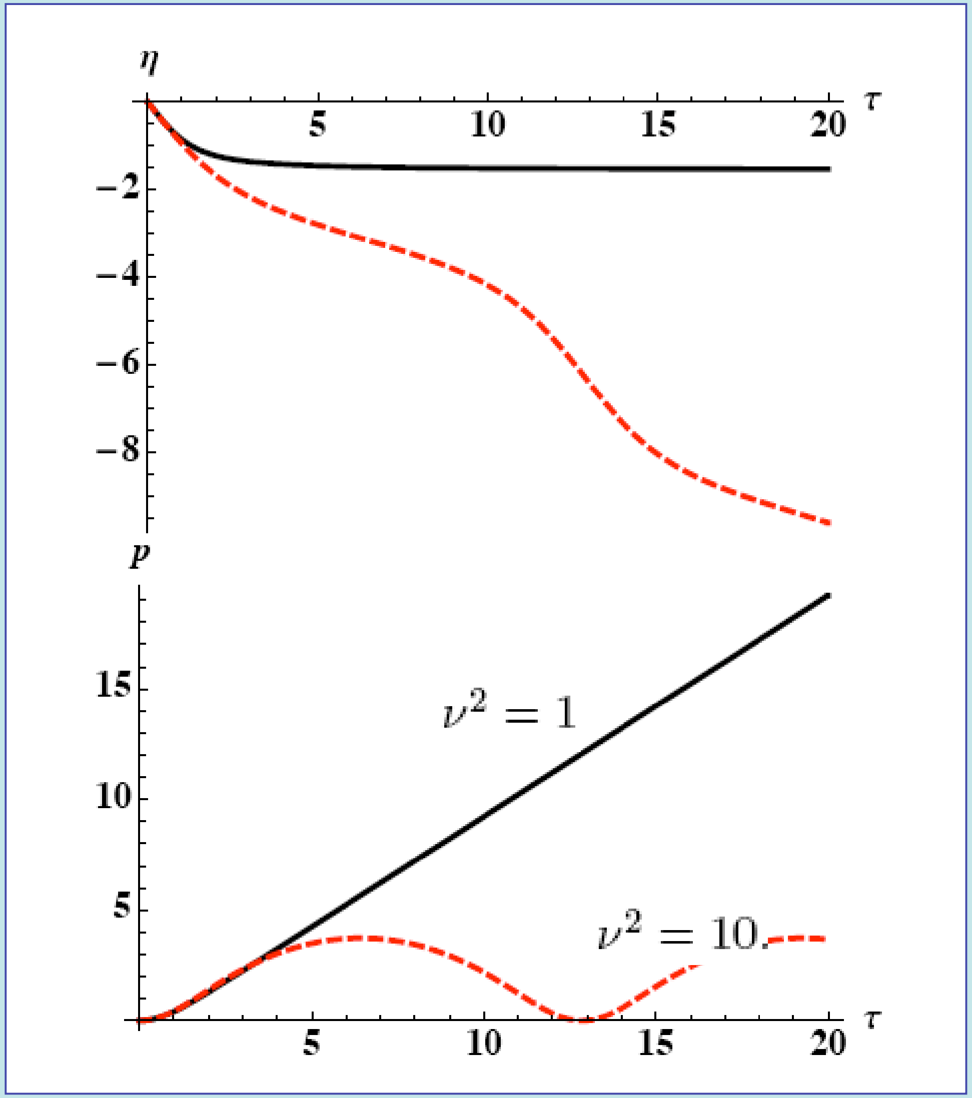
Maximum energy gain

$$\gamma_2 \simeq \frac{\gamma_1(1 - \beta_g) + (\varphi_{max} - \varphi_{min})}{(1 - \beta_g |a_0|)}$$

Pure wakefield acceleration

$$\Delta\gamma \equiv \gamma_2 - \gamma_1 \simeq 2\gamma_g^2(\varphi_{max} - \varphi_{min})$$

Reduction of acceleration efficiency for the modified wakefield scenario



Mendonça, PPCF (2008)

Stochastic wakefield acceleration

Coupled nonlinear oscillations (vector and scalar potentials)

$$\frac{d^2\eta}{d\tau^2} = G_0 \sin\theta + G_1 \sin(k_p\eta)$$

$$G_0 = \left(-\frac{G_1}{k_p\varphi_0} + \frac{\Omega p}{k\gamma^2}\right)\frac{a_0 k}{\gamma} p_{\perp 0} \quad , \quad G_1 = \frac{k_p\varphi_0}{\gamma^2}\left[1 + (\vec{p}_{\perp 0} - \vec{a})^2\right]$$

$$\theta = (k\eta - \Omega\tau)$$

$$\Omega = \frac{\omega}{c} - k\beta_g$$

Purely parallel motion

$$p_{\perp 0} = 0.$$

$$\frac{d^2\eta}{d\tau^2} = G \sin(k_p\eta)$$

$$G = \frac{k_p\varphi_0(1 + a_0^2)}{(1 + p^2 + a_0^2)^{3/2}}$$

No stochastic behavior is expected
(circular polarization)

Linear polarization case

$$\frac{d^2\eta}{d\tau^2} = G_1 \sin(k_p \eta) + \frac{1}{2} G_2 \sin\left[2k\left(\eta - \frac{\Omega}{k}\tau\right)\right]$$

$$p_{\perp 0} = 0.$$

Similar to the perturbed pendulum

Two main resonances

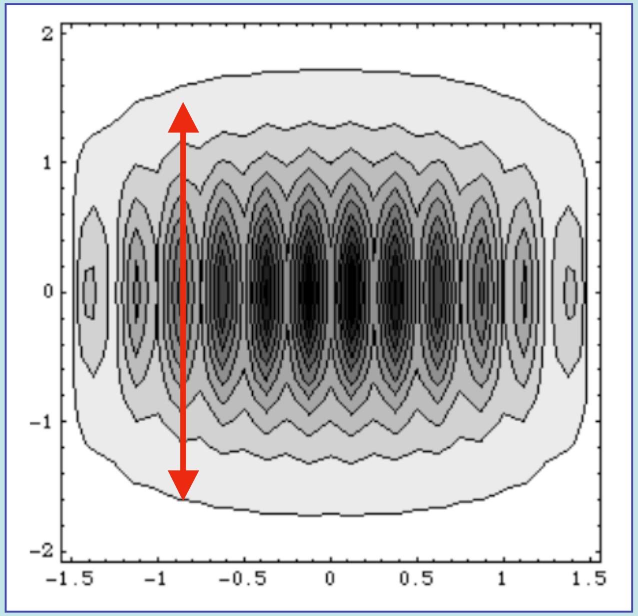
$$u_1 = \left(\frac{d\eta}{d\tau}\right)_1 = 0 \quad , \quad p_1 = \gamma_g \beta_g$$

$$u_2 = \left(\frac{d\eta}{d\tau}\right)_1 = \frac{\Omega}{k} \quad , \quad p_2 = \gamma_2 u_2 = \frac{1 + a_0(0)}{\sqrt{(1 - \Omega^2/k^2)}}$$

Instability criterion (resonance overlapping)

$$\sqrt{G_1} + \sqrt{G_2/2} \le |u_1 - u_2| = \frac{\Omega}{k}$$

Unstable betatron oscillations



Perturbed transverse oscillations

$$\frac{d^2 p_\perp}{d\tau^2} = -\frac{1}{\gamma}\frac{\partial^2 \varphi}{\partial r^2}(p_\perp - a_0 \cos\theta)$$

$$\gamma = \sqrt{\gamma_0^2 + p_\perp(p_\perp - 2a_0\cos\theta)}$$

Simple model for the transverse potential

$$\varphi(r) \simeq \frac{1}{2}\varphi_0 r^2$$

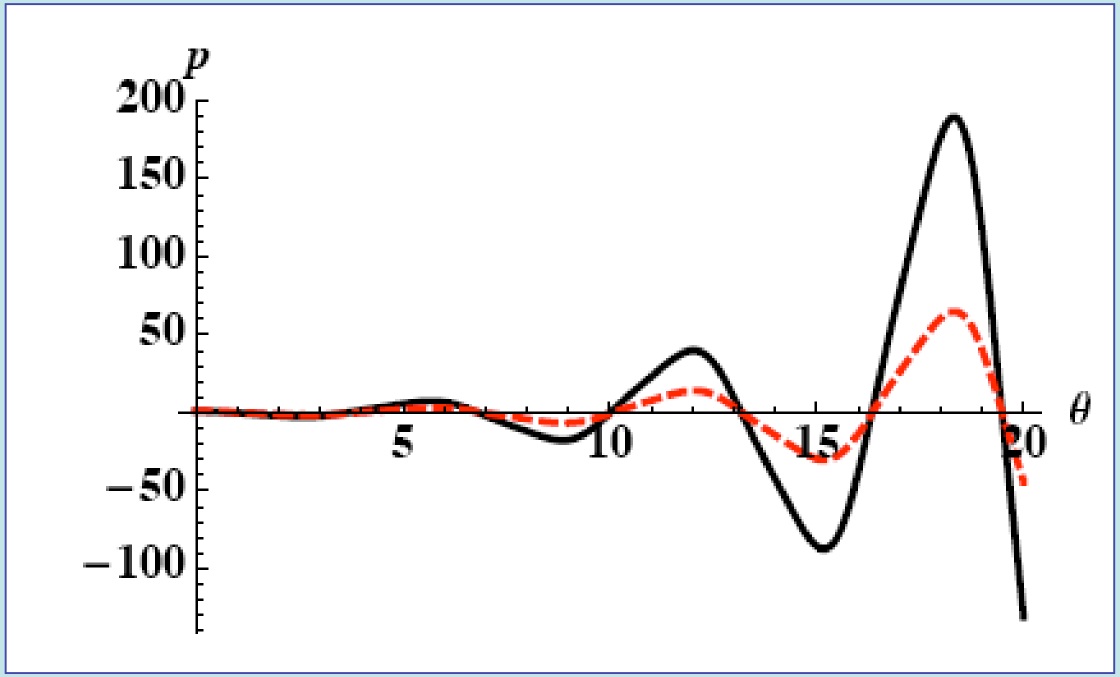
Forced Mathieu equation

$$\frac{d^2 p_\perp}{d\tau^2} + \Omega_B^2 \left(1 + \epsilon \cos 2\theta\right) p_\perp = -\epsilon_1 \cos\theta$$

$$\Omega_B^2 = \frac{\omega_B^2}{\Omega^2}\left(1 + \frac{a_0^2}{2\gamma_0^2}\right)$$

$$\epsilon = \frac{a_0^2}{a_0^2 + 2\gamma_0^2}$$

$$\epsilon_1 = a_0 \frac{\omega_B^2}{\Omega^2}$$



Particle deconfinement + emission of betatron radiation bursts

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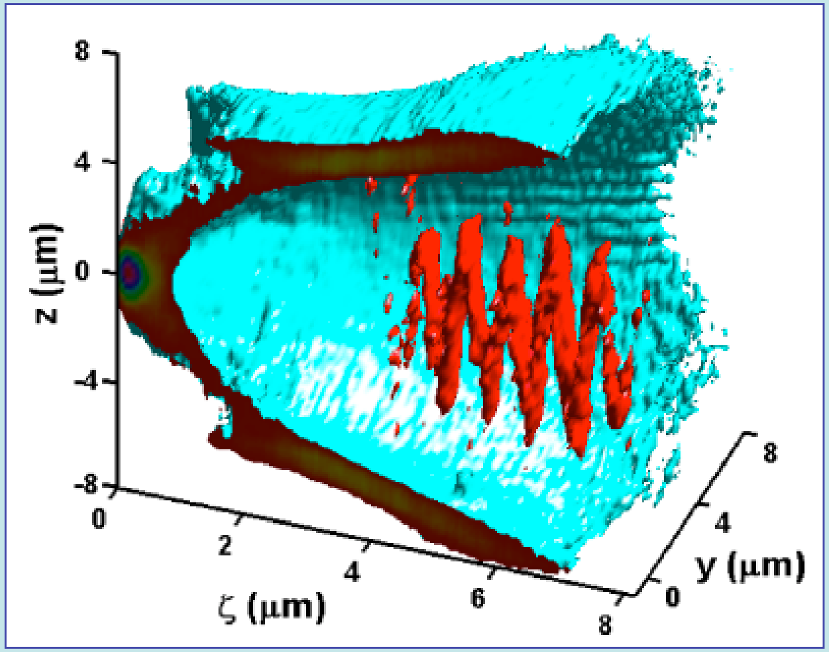
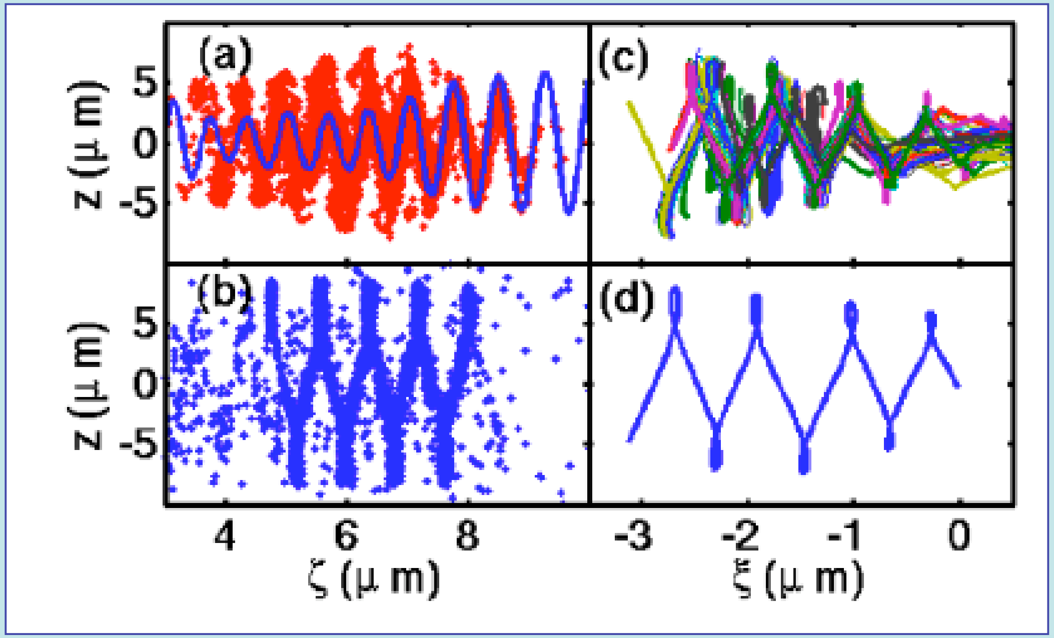


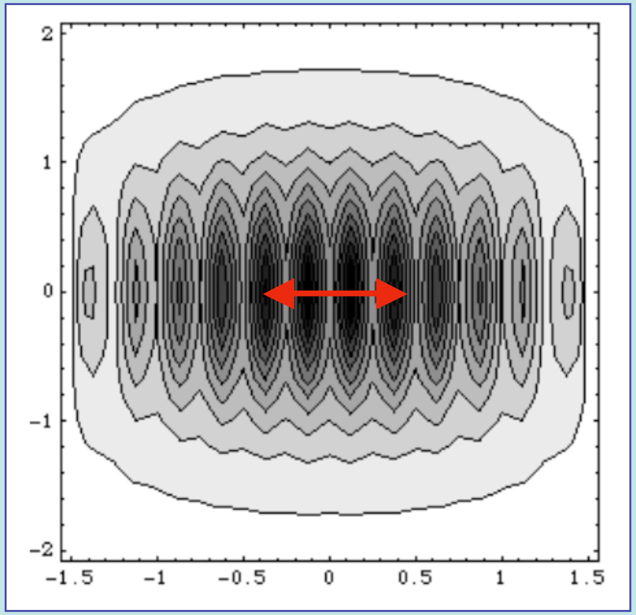
Short laser pulse wakefield

(pic simulations)



K. Németh et al., PRL (2008)

Stochastic beat-wave acceleration



Two counter-propagating waves

$$\vec{a}_1(\eta,\tau) = \vec{a}_1 \cos(k_1\eta)$$

$$\vec{a}_2(\eta,\tau) = \vec{a}_2 \cos[k_2(\eta+\beta_2\tau)]$$

$$\eta = z - \beta_1\tau \quad , \quad \beta_1 = \frac{\omega_1}{k_1 c}$$

$$\beta_2 = \beta_1\frac{k_2}{k_1} + \frac{\omega_2}{k_2 c}$$

Parallel equation of motion

$$\theta_1 = k_1\eta \qquad \theta_2 = k_2(\eta+\beta_2\tau)$$

$$\frac{dp}{d\tau} = \frac{1}{\gamma}(\vec{p}_{\perp 0} - \vec{a}) \cdot \frac{\partial \vec{a}}{\partial \eta}$$

$$\frac{\partial \vec{a}}{\partial \eta} = -\vec{a}_1 k_1 \sin\theta_1 - \vec{a}_2 k_2 \sin\theta_2$$

Main nonlinear resonances

First wave resonance

$$u_1 \equiv \left(\frac{d\eta}{d\tau}\right)_1 = 0$$

$$(\Delta u)_1 = (\sqrt{1+a_1^2} - \beta_1) - (1-\beta_1) = \sqrt{1+a_1^2} - 1 \simeq |a_1|^2$$

Second wave resonance

$$u_2 \equiv \left(\frac{d\eta}{d\tau}\right)_2 = -\beta_2 = -\beta_1 \frac{k_2}{k_1} + \frac{\omega_2}{k_2 c} \quad , \quad (\Delta u)_2 = \sqrt{1+a_2^2} - 1 \simeq |a_2|^2$$

Beat wave resonances

$$u_{3,4} \equiv \left(\frac{d\eta}{d\tau}\right)_{3,4} = \mp \frac{\beta_2}{1 \pm k_2/k_1} \quad , \quad (\Delta u)_{3,4} \simeq \sqrt{|\vec{a}_1 \cdot \vec{a}_2|}$$

Stochasticity criterion

$$|a_1| + \sqrt{|\vec{a}_1 \cdot \vec{a}_2|} \geq \left| \frac{\beta_2}{1 + k_2/k_1} \right|$$

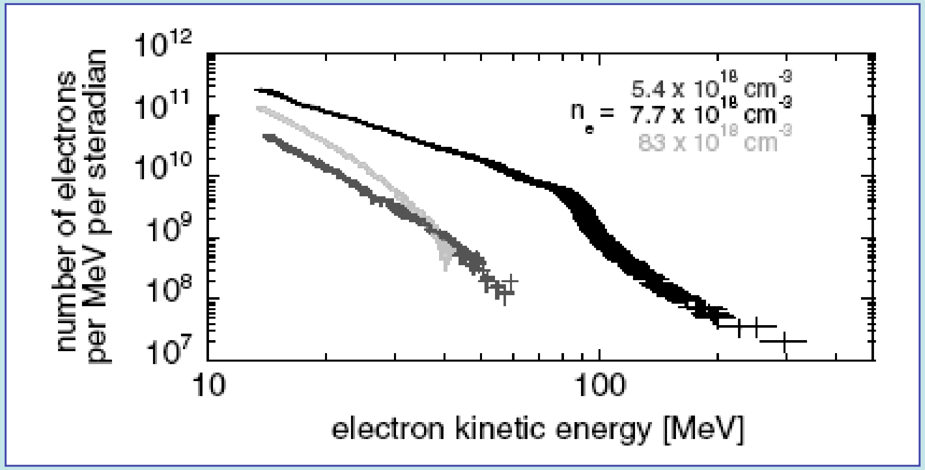
$\longrightarrow$

Standing wave

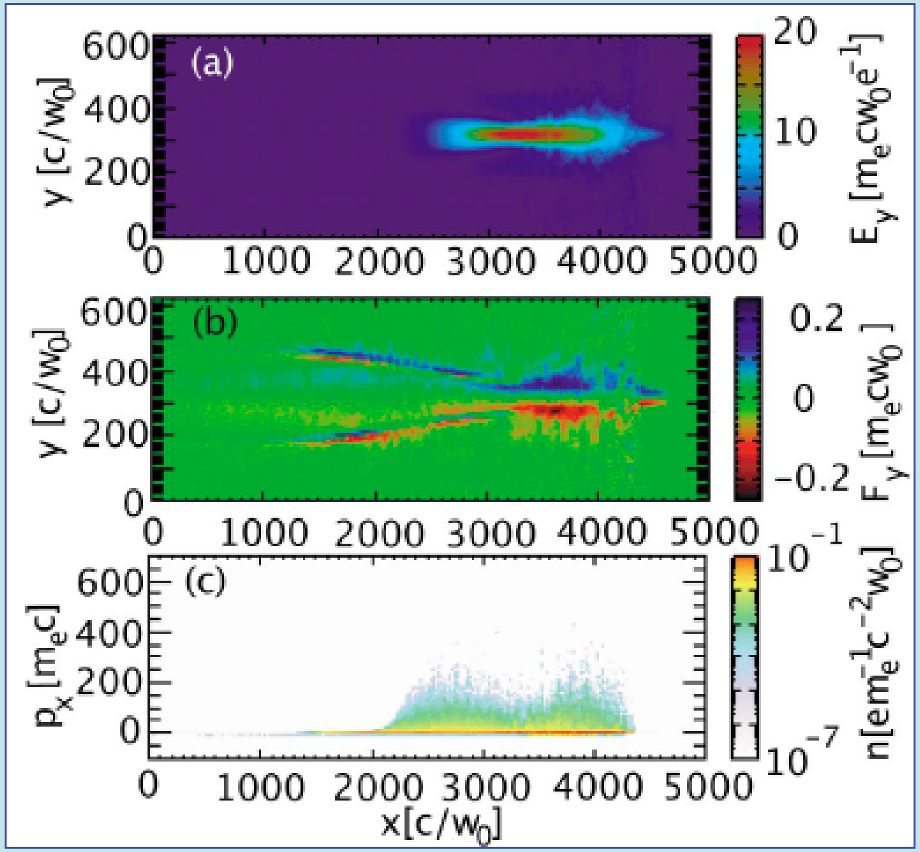
$$|a| \geq \frac{\omega_1}{2k_1 c} \simeq \frac{1}{2}$$

$$|a_1| = |a_2| = |a|,$$

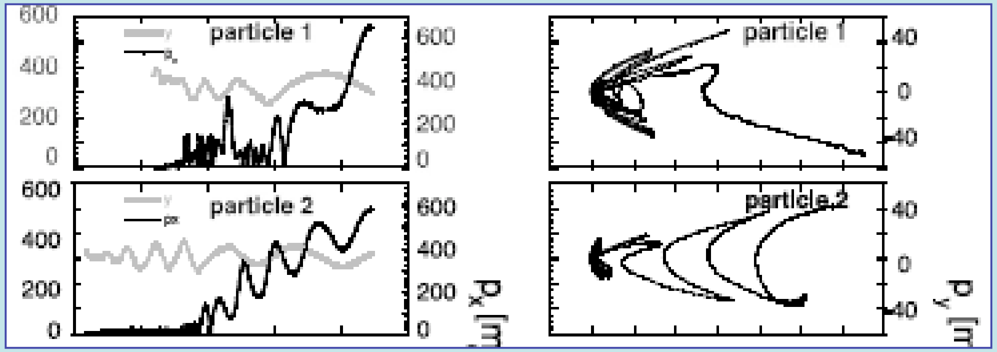
Peta-Watt laser experiments



S. Mangles et al., PRL (2005)



Pic simulations



Question:
Source of stochastic behavior

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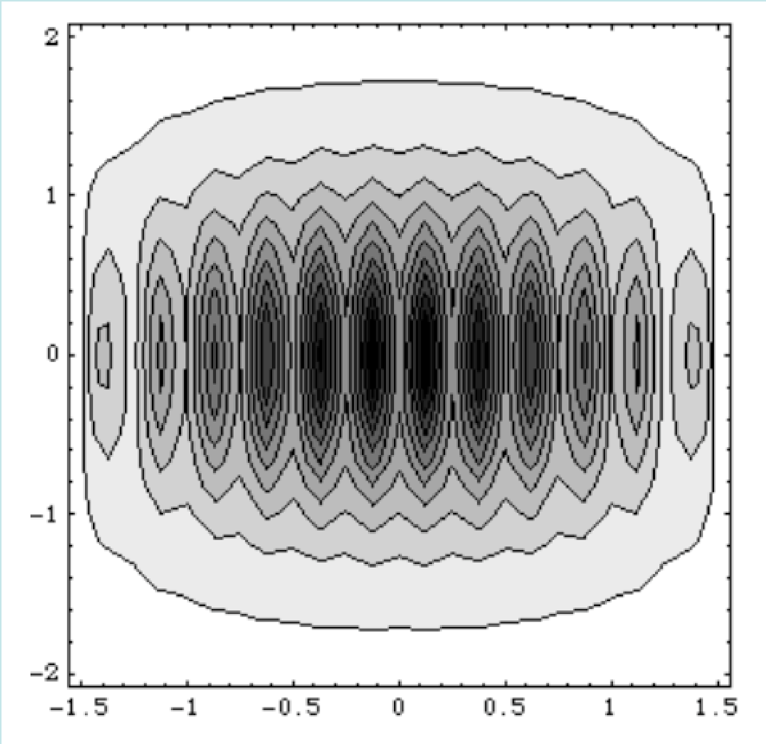
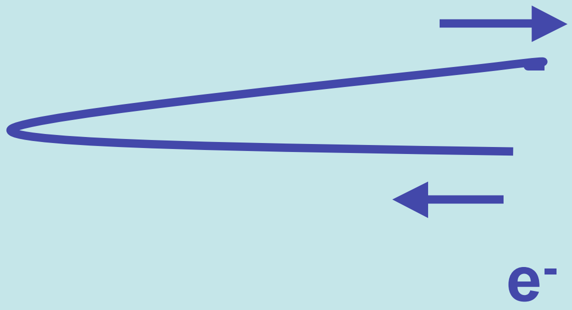
Snow-plow acceleration

Laser pulse





Startsev and McKinstrie, PoP (2003)
Mendonca, Silva and Bingham, JPP (2007)

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Electron trajectories

$$p(\eta) = \frac{h_0\beta_g}{1-\beta_g^2}\left\{1 \pm \sqrt{1 - \frac{1-\beta_g^2}{h_0^2\beta_g^2}[\nu^2 + g(\eta) - h_0^2]}\right\}.$$

$$\nu^2 = 1 + p_{\perp 0}^2$$

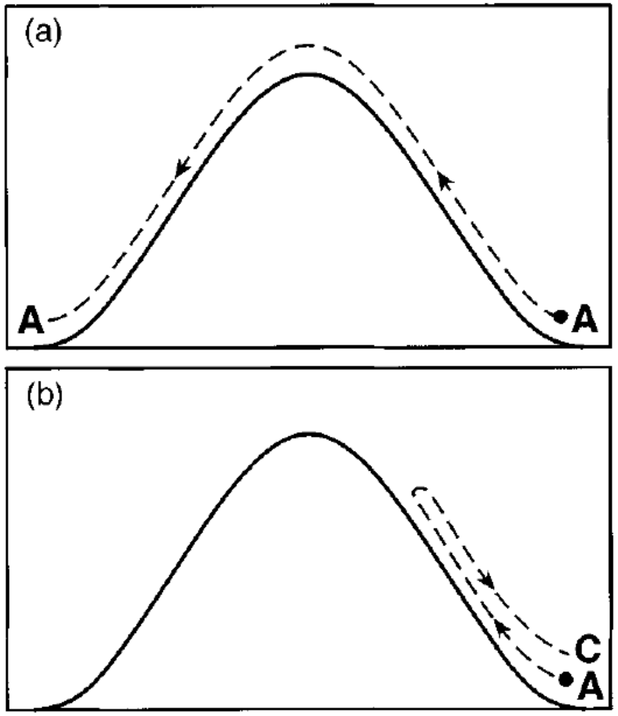
Gaussian laser beam

Trajectory reversal in (p,η) phase space

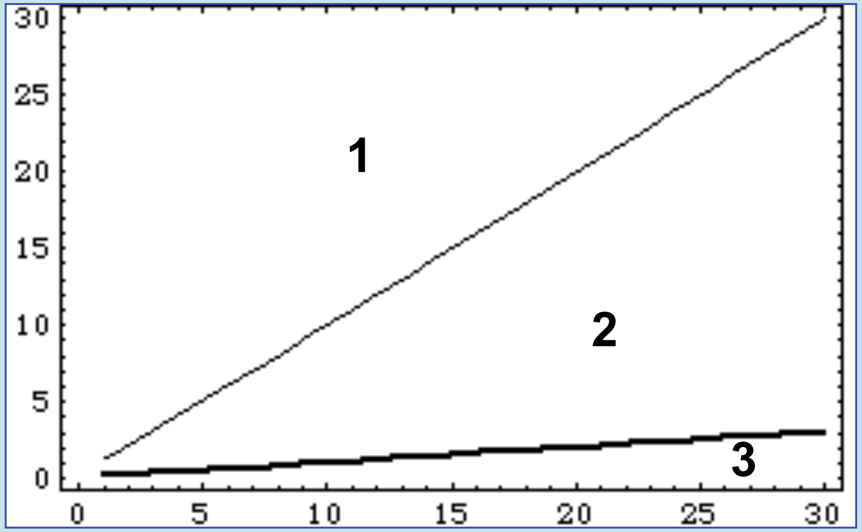
$$\frac{dp}{d\eta} \to \pm\infty. \longrightarrow a_0^2(1-\alpha) \geqslant h_0^2\gamma_g^2 - \nu^2$$

Particle reflection

$$p = 0 \longrightarrow a_0^2(1-\alpha) \geqslant h_0^2 - \nu^2 = 1.$$



Types of trajectories



h_0

$h_x > h_r$

1 - Particle transmission

$$h_x = h_r \gamma_g = \sqrt{\nu^2 + a_0^2(1-\alpha)}.$$

$h_r < h_0 < h_x$

2 - Particle reflection but no trajectory reversal (reflected electrons overtaken by laser pulse)

$$h_r = \gamma_g^{-1} \sqrt{\nu^2 + a_0^2(1-\alpha)}.$$

$h_0 < h_r$

3 - Particle reflection and trajectory reversal (Snow-plow effect)

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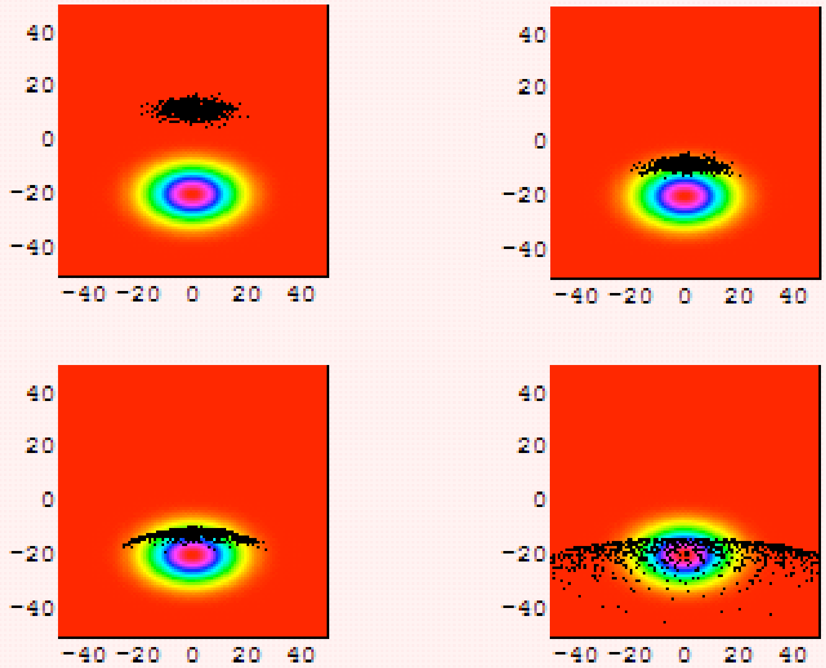
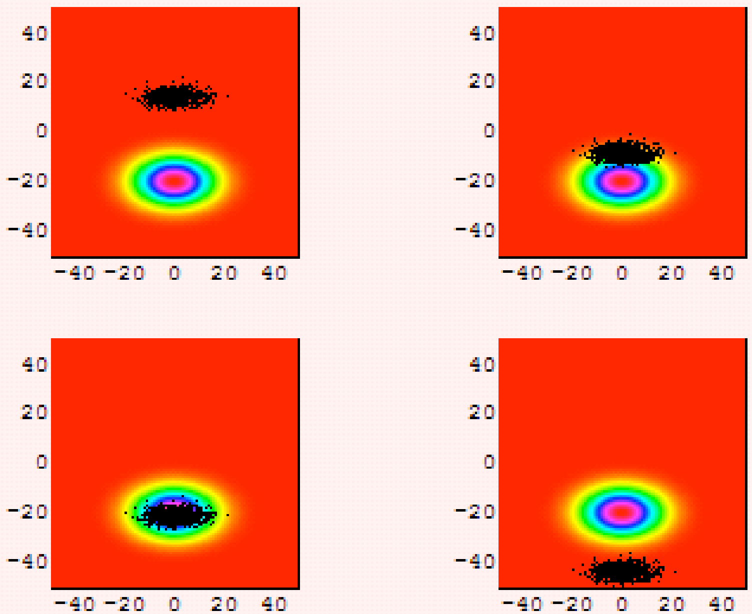
Reflected electron beam
(with side-scattering)



E-beam: 100MeV. Laser pulse: 1 mm, r-gauss 15mm, 30fs, 10^{20}W/cm^2

Transmitted electron beam



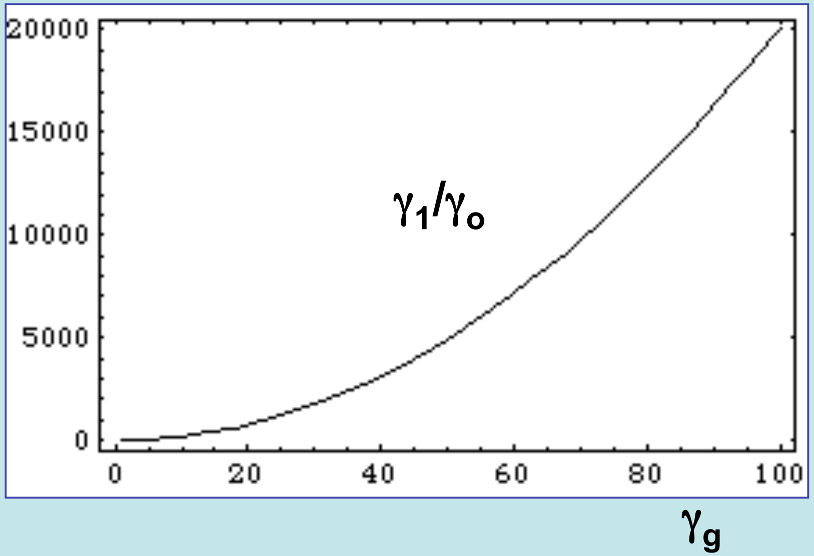
E-beam: 100MeV. Laser pulse: 1 mm, r-gauss 15mm, 30fs, 10^{16}W/cm^2

A. Guerreiro and M. Eloy (2006)

Photon mirror

Energy gain of reflected electrons

$$\gamma_1 = \gamma_g^2 \left[\gamma_0 (1+\beta_g^2) - 2\beta_g \sqrt{\gamma_0^2 - \nu^2}\right]$$



For highly energetic particles

$\gamma_0, \gamma_1 \gg \nu^2$

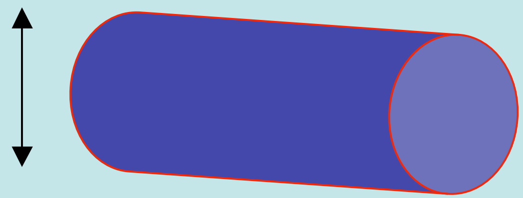
$$\boxed{\gamma_1 = \gamma_0 \frac{1+\beta_g}{1-\beta_g} \simeq 4\gamma_0\gamma_g^2}$$

Identical to photon energy shift by double Doppler shift
(Laser beam = photon mirror, from the e$^-$ point of view)

Example: for initial energy E_i = 50 MeV, and γ_g~ 200 (60), final energy would be E_f ~ 10TeV (1.4 TeV).

Laser group velocity in vacuum

Beam waist, w



Uncertainty principle

$$\Delta r_{\perp} . \Delta p_{\perp} \geq h,$$

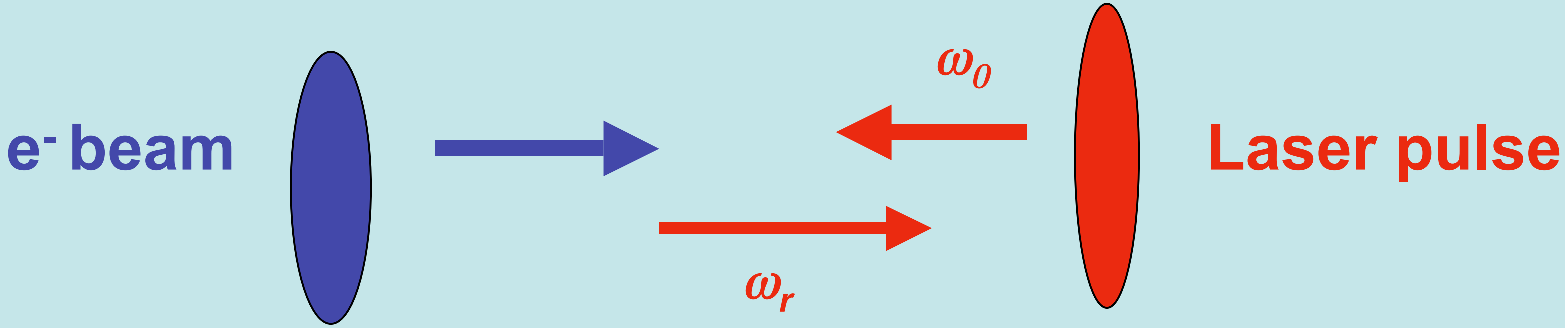
$$wk.\sin\theta = wk\sqrt{1 - \frac{v_g^2}{c^2}} \geq 1$$

Relativistic gamma factor

$$\boxed{\gamma_g \leq 2\pi \frac{w}{\lambda}}$$

For γ_g ~ 200 (60) we would need $\omega \sim 50\ (10)\ \lambda$

Electron beam as an imperfect mirror



Electron velocity, $\beta = \beta_0 + \beta'$

$$\boxed{\beta'(z,t) = \sum_l \beta_l \exp(ik_l z - i\omega_l t)}$$

$$\boxed{\beta_l = -i \frac{a_0^{(2l+1)}}{(2l+1)!!\gamma_0}}$$

Approximate solutions ($a_0 < 1$):

$a_0 = eE_0/mc\omega_0$

Radiated spectrum

$$\boxed{\omega \equiv \omega_r = (2l+1)\frac{1+\beta}{1-\beta}\omega_0}$$

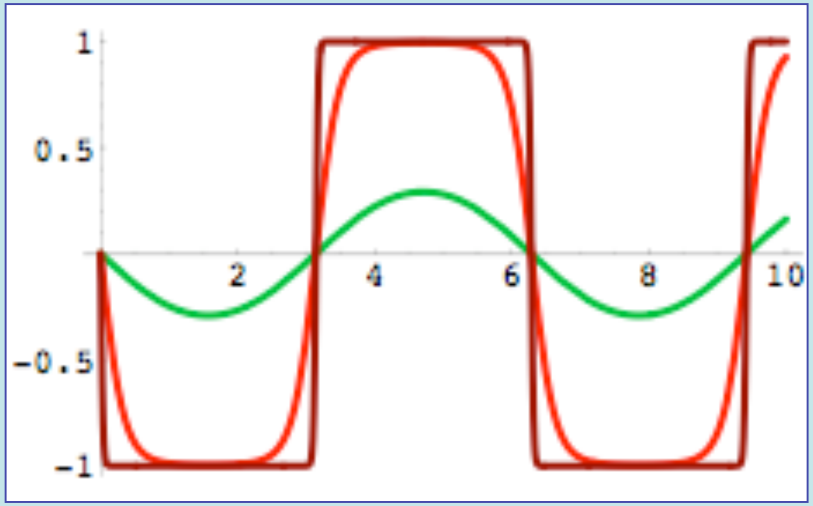
Radiation field

$$\boxed{\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A = e\mu_0 n_b c . f(z+ut).\beta'(z,t)}$$

Beam shape function

Analytical solutions, valid for $\gamma >> a_0 >> 1$

$$\tilde{\beta}(x,t) = \frac{1}{\gamma_0} \tanh\left[a_0 \sin(k_0 x - \omega_0 t)\right]$$



$a_0 = 30$

$a_0 = 3$

$a_0 = 0.3$

$$\tilde{\beta}_n \simeq \frac{2}{\pi \gamma_0} \frac{1}{(2n+1)}$$

Harmonic amplitudes for $a_0 >> 1$

(insensitive to the value of a_0)

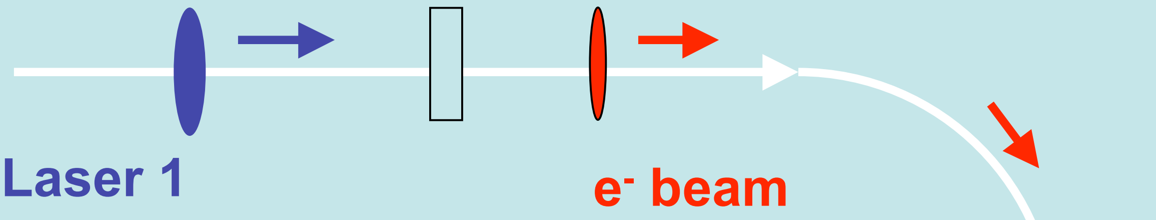
Radiated power

$$\frac{P_{\omega(n)}}{P_0} \propto \left(\frac{\omega_b}{\omega_0}\right)^4$$

(Modulated electron beam: power enhancement by a factor γ^2)

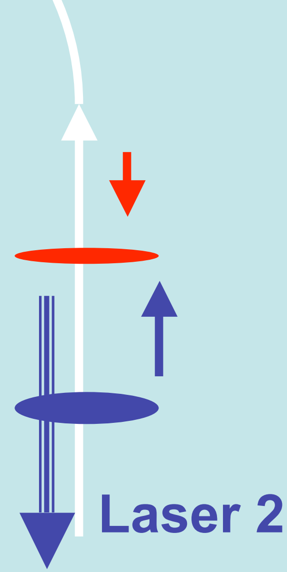
Mendonça and Serbeto (2008)

Sub-attosecond pulses



Double Doppler shifted odd harmonics

$$\omega_n = (2n+1)\gamma^2(1+\beta)^2\omega_0$$



Pulse duration

$$\tau_n = \frac{\tau_0}{\gamma^2(1+\beta)^2}$$

Expected pulse duration for 100 MeV e$^-$ beams

$$\tau_0 \cong 30\,fs \Rightarrow \tau_n \cong 2\times10^{-4}\,fs$$

Possible experimental configuration for Astra-Gemini (RAL)

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Conclusions

- Different acceleration processes relevant to Peta-Watt pulses were discussed;
- Modified wakefield reduces acceleration efficiency;
- Stochastic acceleration processes were identified;
- Betatron instability was demonstrates (bursts of radiation);
- Snow-plow acceleration was discussed;
- Electrons reflected by photon mirrors: single step acceleration;
- Electron beams act as imperfect mirrors for laser pulses;
- Double Doppler shifted odd harmonics can be produced;
- New experimental proposals can be made.