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Nonlinear Excitations in Dusty Plasma (Debye) Crystals.

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Nonlinear Excitations in Dusty Plasma (Debye) Crystals

What Plasma Physics Can Learn from Nonlinear Lattice Theories

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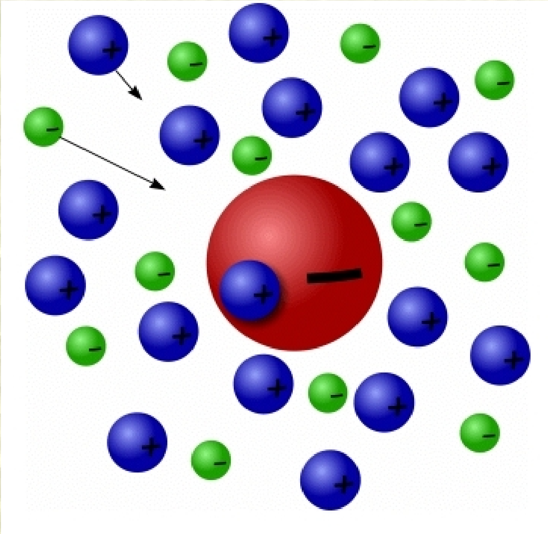
www.tp4.rub.de/~ioannis/conf/200807-ICTP-oral.pdf

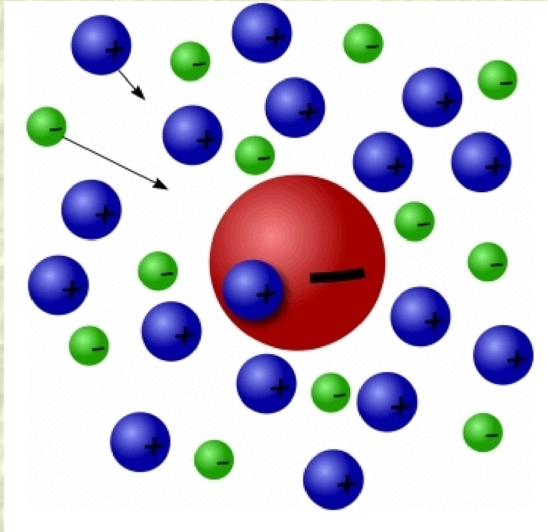
Dusty Plasmas (or *Complex Plasmas*): prerequisites

Plasma textbook model: 

→ *electrons* e^- (charge $q_e = -e$, mass m_e),

→ *ions* i^+ (charge $q_i = +Z_i e$, mass $m_i \gg m_e$); $q_i/m_i \ll q_e/m_e$

Dusty Plasmas (or *Complex Plasmas*): prerequisites



Dusty Plasmas (DP):

→ *electrons* e^- (charge $-e$, mass m_e),

→ *ions* i^+ (charge $+Z_i e$, mass $m_i \gg m_e$); $q_i/m_i \ll q_e/m_e$

→ charged particulates $\equiv$ *dust grains* $d^\pm$ (most often d^-):

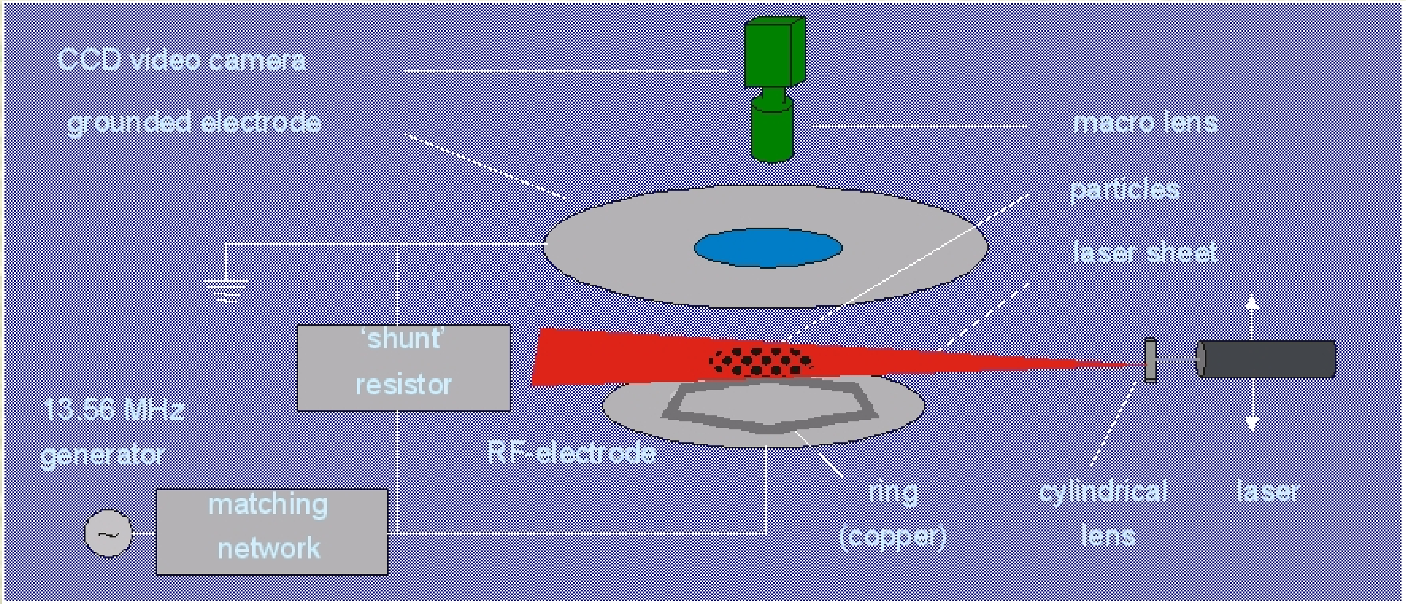
charge $Q = sZ_d e \sim \pm(10^3 - 10^4)\, e, \quad (s = \pm 1)$
mass $M \sim 10^9\, m_p \sim 10^{13}\, m_e$,
radius $r \sim 10^{-2}\, \mu m$ up to $10^2\, \mu\text{m}$.

Ultra-strongly coupled Dusty Plasmas (DPs)

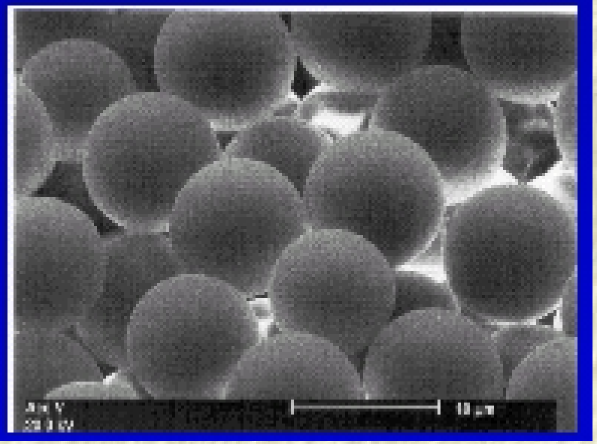
- *Low Q/M ratio* ($\omega_{p,d} \approx 10 - 100\,\mathrm{Hz}$)
-
$$\Gamma_{eff} = \frac{< E_{potential} >}{< E_{kinetic} >} \approx \frac{\frac{Q^2}{r}\, e^{-r/\lambda_D}}{k_B T}$$
- Contrary to "ordinary" (*weakly-coupled*) e-i plasmas ($\Gamma \ll 1$), *DPs* may be *strongly coupled*: *"liquid"*, *"crystalline"* states
- $\rightarrow$ *Dusty Plasma (Debye/Yukawa) Crystals!!! (DPCs)*
- Theoretical prediction: 1986 [H. Ikezi, Phys. Fluids **29**, 1764 (1986)]
- Experimental realization: 1994

  [H. Thomas, A. Melzer *et al.* PRL **73**, 652 (1994); Chu & Lin I J. Phys. D **27** 296 (1994), Hayashi & Tachibana, Jap. J. Appl. Phys. **33** L804 (1994)]

Dust Crystal experiments



Particles:
Melamine-Formaldehyde
diameter: few µm

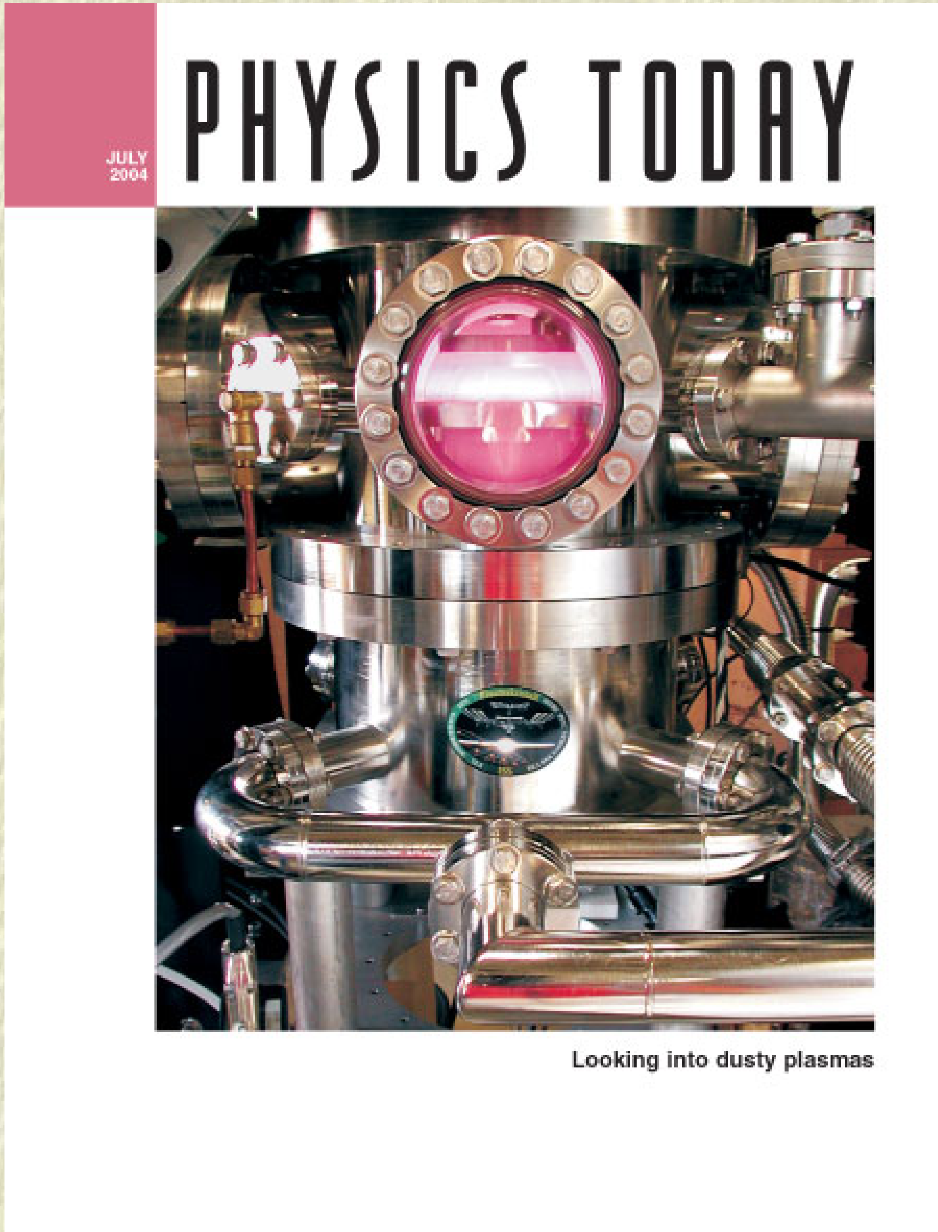


Gas:
noble gas (argon, krypton)
pressure: few Pa … 100 Pa (=1 mbar)

Ionisation fraction: 10^{-6} - 10^{-7}

Temperatures:
kT_e ~2-4 eV (electrons)
kT_i ~0.03 eV (ions)
kT_s ~0.025 eV (microparticles)
in crystalline state

[Source: H. Thomas, A. Melzer *et al.*, PRL 1994].


JULY
2004
PHYSICS TODAY
Looking into dusty plasmas

Overview of current activity on NL crystals: theory *vs* experiment and community *vs* community

- Experimental DP Research *"Community A"*:

  G E Morfill (MPIeP Garching, Germany), *A Piel* (Kiel, Germany), *A Melzer* (Greifswald, Germany), *D. Samsonov* (Manchester, UK), *J Goree* (Iowa, US), *V Fortov* (Moscow, Russia), *Lin I* (Taiwan), *A Samarian* (Sydney, AUS), *S Takamura* (Nagoya, Japan), ...

- Focus on: statistical mechanics, kinetic phenomena, linear dust-lattice waves (DLWs), *nonlinear (NL)* DLWs, ...

- Theoretical NL DPC Research *"Community B"*:

  F. Melandsø (1996); M Amin, P K Shukla *et al* (1998+); B Farokhi *et al* (1999+); S. Zhdanov *et al.* (2002); K. Avinash *et al.* (2003); V. Fortov (2004); I Kourakis *et al* (2004+).

- The Nonlinear Science *"Community C"*
  lattice theories, nonlinear optics, photonic crystals, granular "phononic" crystals, periodic metamaterials, complex systems

Outline – Menu

1. *Dusty plasmas (DP) & DP crystals (DPCs)*: Prerequisites
   Nonlinearity in 1D DP crystals: Origin and modelling

2. *Transverse* dust-lattice (*TDL*) excitations:
   amplitude modulation, transverse envelope structures

3. *Longitudinal* dust-lattice (*LDL*) excitations:
   amplitude modulation, envelope structures, solitons

4. *1D Discrete Breathers (DBs)*

5. *2D DBs* in hexagonal crystals

6. Conclusions

Preliminaries – 1D DP crystals (*undamped*): Model Hamiltonian

$$H = \sum_n \frac{1}{2} M \left(\frac{d\mathbf{r}_n}{dt} \right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(\mathbf{r}_n)$$

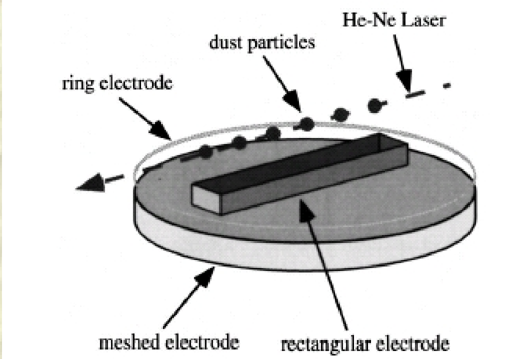


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode.

Terms include:

– *Kinetic energy*;

– $\Phi_{ext}(\mathbf{r}_n)$ accounts for *'external' force fields*:
may account for *confinement potentials*
and/or *sheath electric* forces, i.e. $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$.

– Coupling: $U_{int}(r_{nm})$ is the *interaction potential energy*;

Q.: Nonlinearity: *Origin: where from ?*
Effect: which consequence(s) ?

Nonlinearity in DPCs: Origin & modelling (1)

→ *Sheath* environment (*anharmonic* vertical potential):

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2}M\omega_g^2(\delta z_n)^2 + \frac{1}{3}M\alpha\,(\delta z_n)^3 + \frac{1}{4}M\beta\,(\delta z_n)^4 + ...$$

cf. experiments [Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)];

$\delta z_n = z_n - z_{(0)}$; α, β, ω_g are defined experimentally

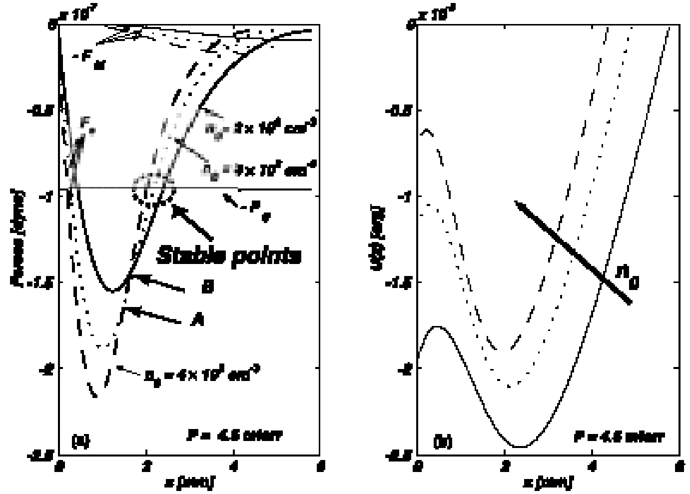


Figure 3: (a) Forces and (b) trapping potential profiles $U(z)$ as function of distance from the electrode for: $n_0 = 2\times10^8 cm^{-3}$ (solid line), $n_0 = 3\times10^8 cm^{-3}$ (dashed line), $n_0 = 4\times10^8 cm^{-3}$ (dotted line). The parameters are: P = 4.6 mtorr, $T_e = 1\ eV$, $T_i = T_n = 0.05\ eV$, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\,cm^{-3}$, $\phi_w = 6\ V$.

Source: Sorasio *et al.* (2002).

Nonlinearity in DPCs: Origin & modelling (2)

→ *Interactions* between grains: *anharmonic*, screened ES:

$$U_{Debye}(r) = \frac{q^2}{r} e^{(-r/\lambda_D)}$$

Expanding $U_{int}(r_{nm}) = U_{int}(\sqrt{(\Delta x_{nm})^2 + (\Delta z_{nm})^2})$ near:

$\Delta x_{nm} = x_n - x_{n-m} \approx m r_0, \qquad \Delta z_{nm} = z_n - z_{n-m} \approx 0$,

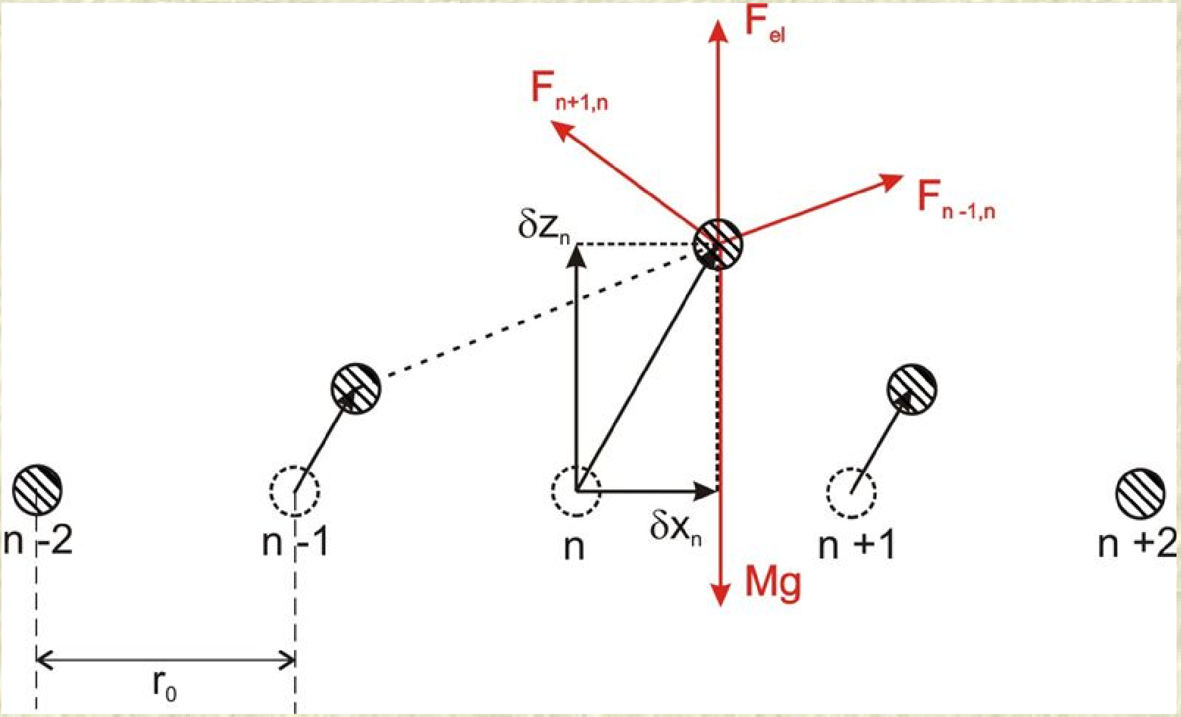
one obtains:

$$\begin{aligned} U_{nm}(r) \approx\ & \frac{1}{2} M \omega_{L,0}^2 (\Delta x_{nm})^2 + \frac{1}{2} M \omega_{T,0}^2 (\Delta z_{nm})^2 \\ & + \frac{1}{3} u_{30} (\Delta x_{nm})^3 + \frac{1}{4} u_{40} (\Delta x_{nm})^4 + \ldots + \frac{1}{4} u_{04} (\Delta z_{nm})^4 + \\ & + \frac{1}{2} u_{12} (\Delta x_{nm})(\Delta z_{nm})^2 + \frac{1}{4} u_{22} (\Delta x_{nm})^2 (\Delta z_{nm})^2 + \ldots \end{aligned}$$

Details in: I Kourakis and P. K. Shukla, *Int. J. Bif. Chaos* **16**, 1711 (2006).

Nonlinearity in DPCs: Origin & modelling (3)

→ *Coupling* among degrees of freedom induces nonlinearity: anisotropic motion, *not* confined along principal axes ($\sim \hat{x}$, $\hat{z}$).



[cf. A. Ivlev *et al.*, PRE **68**, 066402 (2003); I. Kourakis & P. K. Shukla, Phys. Scr. (2004)]

Discrete coupled equations of motion

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2 \left(\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n\right)$$

$$-a_{20}\left[\left(\delta x_{n+1} - \delta x_n\right)^2 - \left(\delta x_n - \delta x_{n-1}\right)^2\right] + a_{30}\left[\left(\delta x_{n+1} - \delta x_n\right)^3 - \left(\delta x_n - \delta x_{n-1}\right)^3\right]$$

$$+ a_{02}\left[\left(\delta z_{n+1} - \delta z_n\right)^2 - \left(\delta z_n - \delta z_{n-1}\right)^2\right]$$

$$-a_{12}\left[\left(\delta x_{n+1} - \delta x_n\right)\left(\delta z_{n+1} - \delta z_n\right)^2 - \left(\delta x_n - \delta x_{n-1}\right)\left(\delta z_n - \delta z_{n-1}\right)^2\right],$$

$$\frac{d^2(\delta z_n)}{dt^2} = \omega_{0,T}^2 \left(2\delta z_n - \delta z_{n+1} - \delta z_{n-1}\right) - \omega_g^2\, \delta z_n$$

$$- K_1 \left(\delta z_n\right)^2 - K_2 \left(\delta z_n\right)^3 + \frac{a_{02}}{r_0}\left[\left(\delta z_{n+1} - \delta z_n\right)^3 - \left(\delta z_n - \delta z_{n-1}\right)^3\right]$$

$$+ 2\, a_{02}\left[\left(\delta x_{n+1} - \delta x_n\right)\left(\delta z_{n+1} - \delta z_n\right) - \left(\delta x_n - \delta x_{n-1}\right)\left(\delta z_n - \delta z_{n-1}\right)\right]$$

$$- a_{12}\left[\left(\delta x_{n+1} - \delta x_n\right)^2\left(\delta z_{n+1} - \delta z_n\right) - \left(\delta x_n - \delta x_{n-1}\right)^2\left(\delta z_n - \delta z_{n-1}\right)\right].$$

Continuum coupled equations of motion

$$\ddot{u} - c_L^2\, u_{xx} - \frac{c_L^2}{12} r_0^2\, u_{xxxx} =$$

$$- 2\, a_{20}\, r_0^3\, u_x\, u_{xx} + 3\, a_{30}\, r_0^4\, (u_x)^2\, u_{xx}$$

$$- a_{12}\, r_0^4 \left[(w_x)^2\, u_{xx} + 2 w_x w_{xx} u_x \right] + 2\, a_{02}\, r_0^3\, w_x\, w_{xx}\,,$$

$$\ddot{w} + c_T^2\, w_{xx} + \frac{c_T^2}{12} r_0^2\, w_{xxxx} + \omega_g^2\, w =$$

$$- K_1\, w^2 - K_2\, w^3 + 3\, a_{02}\, r_0^3\, (w_x)^2\, w_{xx}$$

$$+ 2\, a_{02}\, r_0^3\, (u_x\, w_{xx} + w_x\, u_{xx}) - a_{12}\, r_0^4 \left[(u_x)^2\, w_{xx} + 2 u_x u_{xx} w_x \right],$$

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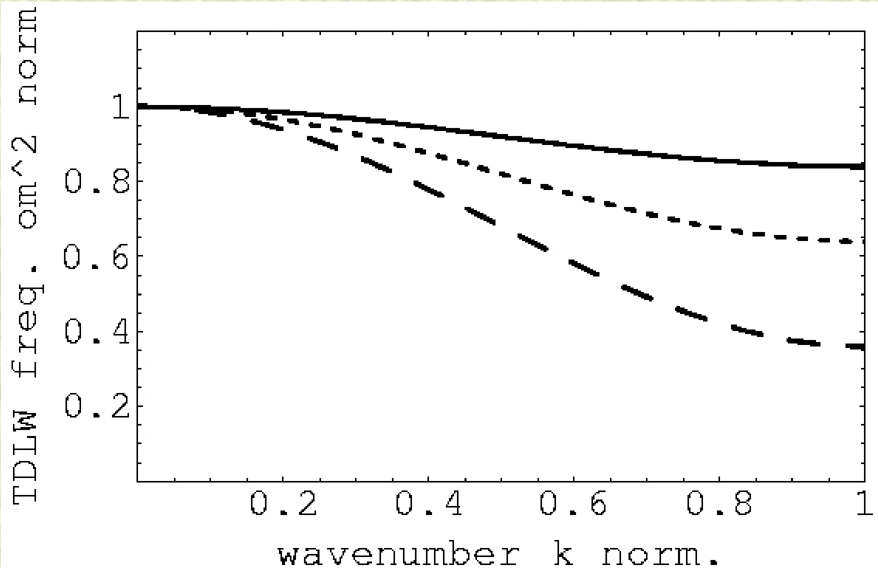
2. *Transverse* DP-lattice excitations

The vertical $n-$th grain displacement $\delta z_n = z_n - z_{(0)}$ obeys

$$\frac{d^2(\delta z_n)}{dt^2} + \omega_{T,0}^2 \left(\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n \right) + \omega_g^2\,\delta z_n = 0$$

* $\omega_{T,0} = \left[-qU'(r_0)/(Mr_0)\right]^{1/2} = \omega_{DL}^2\,\exp(-\kappa)\,(1+\kappa)/\kappa^3 \quad (\dagger)$
* $\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$; λ_D is the *Debye length*;
* *Optical* dispersion relation (*backward* wave, $v_g < 0$):

$$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2\,\sin^2(kr_0/2)$$



2. *Transverse* DP-lattice excitations

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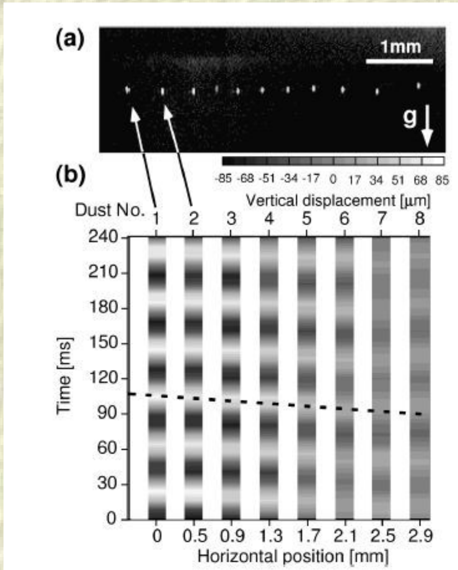
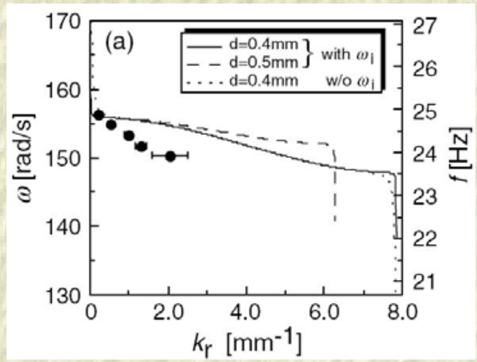
$$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2\,\sin^2(kr_0/2)$$

† Cf. experiment:
T. Misawa *et al.*, *PRL* **86**, 1219 (2001)
(Nagoya, Japan).





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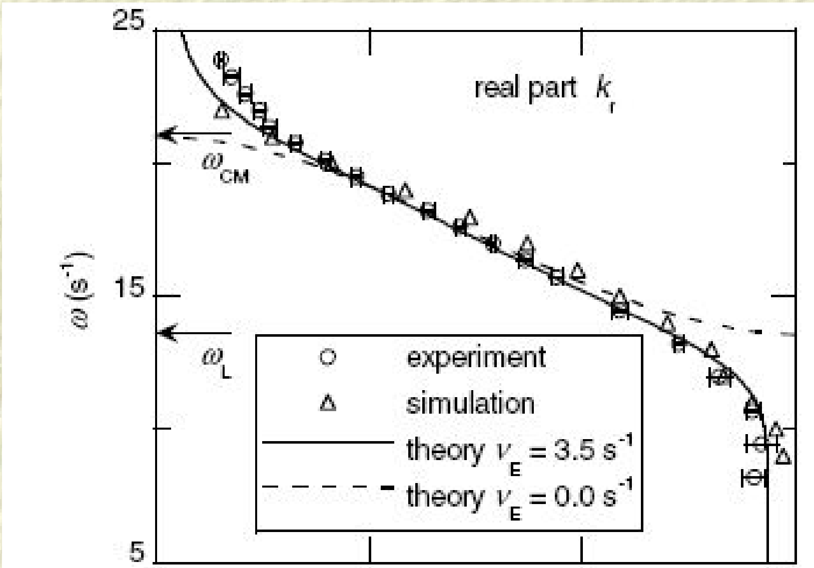
† Cf. experiment:
B. Liu *et al.*, *PRL* **91**, 255003 (2003)

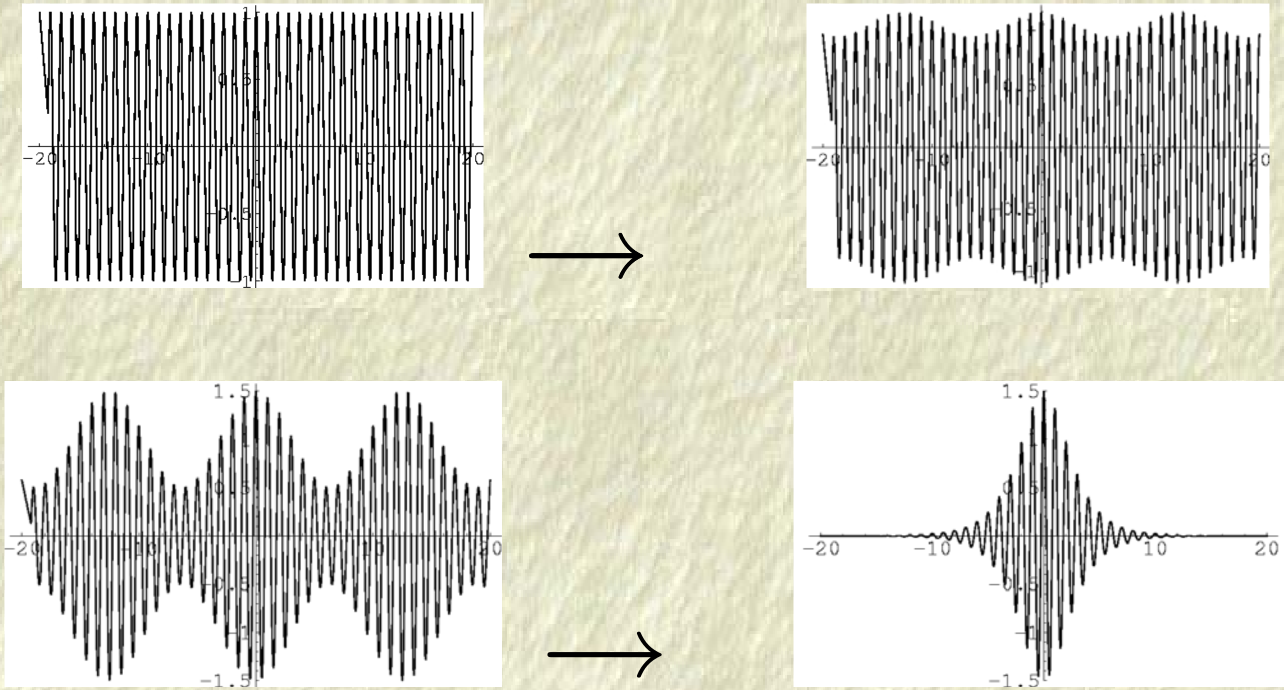
(Iowa, USA).



What if *nonlinearity* is taken into account?

$$\frac{d^2 \delta z_n}{dt^2} + \omega_{T,0}^2 \left(\delta z_{n+1} + \delta z_{n-1} - 2\, \delta z_n \right) + \omega_g^2\, \delta z_n$$
$$+ \alpha \, (\delta z_n)^2 + \beta \, (\delta z_n)^3 = 0 \, .$$

* *Intermezzo: NL wave amplitude modulation → localisation!*



Large amplitude oscillations - envelope structures

A reductive multiple scales technique, yields

$$\delta z_n \approx \epsilon\,(A\,e^{i\phi_n} + \text{c.c.}) \,+\, \epsilon^2\,\alpha\left[-\frac{2|A|^2}{\omega_g^2} + \left(\frac{A^2}{3\omega_g^2}\,e^{2i\phi_n} + \text{c.c.}\right)\right] + \ldots$$

- *Harmonic Generation* (multiples of $\phi_n = nkr_0 - \omega t$)

- The harmonic amplitude $A(X, T)$:
  – depends on the *slow* variables $\{X, T\} = \{\epsilon(x - v_g t), \epsilon^2 t\}$;
  – obeys the *nonlinear Schrödinger equation* (NLSE):

$$i\,\frac{\partial A}{\partial T} + P\,\frac{\partial^2 A}{\partial X^2} + Q\,|A|^2\,A = 0\,, \qquad (1)$$

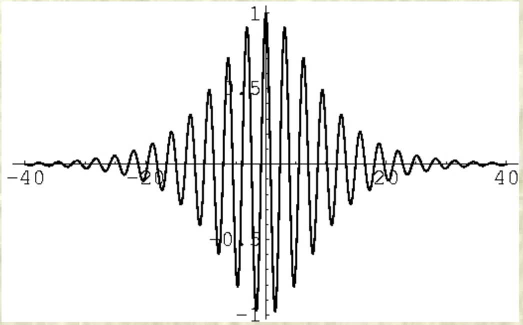
  – *Dispersion coefficient*: $P = \omega''(k)/2$ $\rightarrow$ see DR;

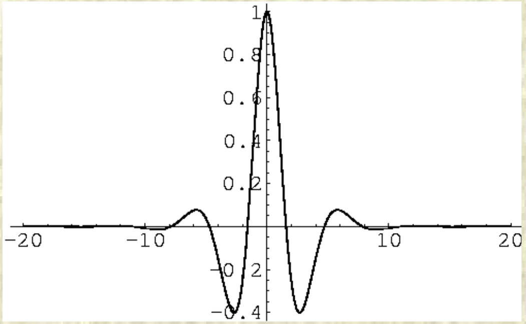
  – *Nonlinearity coefficient*: $Q = \left[10\alpha^2/(3\omega_g^2) - 3\,\beta\right]/2\omega$.

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 2322 (2004); **11**, 3665 (2004).]

Modulational stability analysis & envelope structures

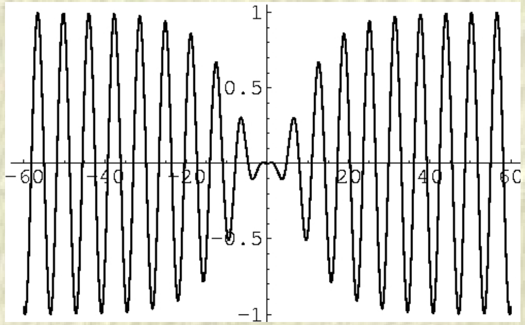
- $PQ > 0$: *Modulational instability*, *bright solitons*:

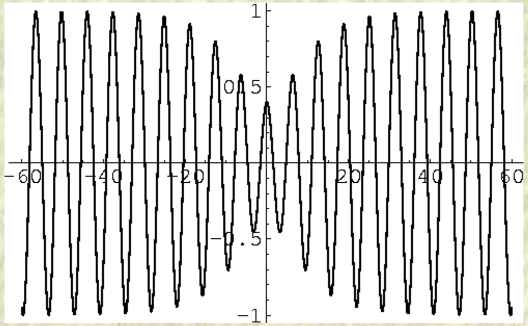




  → *TDLW*s: possible for *short* wavelengths i.e. $k_{cr} < k < \pi/r_0$.

- $PQ < 0$: Carrier wave is *stable*, *dark/grey solitons*:





  → *TDLW*s: possible for *long* wavelengths i.e. $k < k_{cr}$.

Rem.: $Q > 0$ for *all* known experimental values of α, β

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)].

Experimental confirmation (2005)

PHYSICAL REVIEW E **71**, 026410 (2005)

Vertical wave packets observed in a crystallized hexagonal monolayer complex plasma

D. Samsonov, S. Zhdanov,* and G. Morfill
Max-Planck-Institut für Extraterrestrische Physik, D-85740 Garching, Germany
(Received 30 January 2004; revised manuscript received 27 May 2004; published 24 February 2005)

PHYSICAL REVIEW E **71**, 026410 (2005)

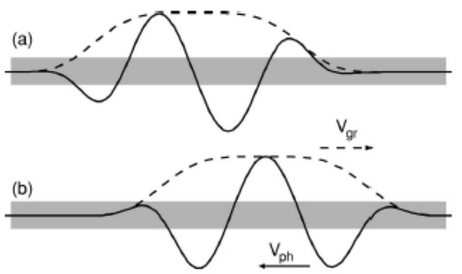


FIG. 2. Side view of the vertical wave packet propagating in a monolayer lattice. (a) Initial position of the packet. (b) Displaced packet. The group velocity V_{gr} (the velocity of the packet envelope marked by the dashed line) is directed right, while the phase velocity V_{ph} (the velocity of the modulating wave marked by the solid line) points left. The illuminating laser beam is indicated by the gray bar. We observe only the areas where it intersects the lattice (solid line), which appear as moving stripes in the experiment.

height than that used in Ref. [16], mostly vertical oscillations were excited. It was accompanied by a very weak compressional soliton (with compression ratio ≃1). A time interval of about 1 min was allowed for the lattice to come into equilibrium. Several experimental runs were recorded with very good reproducibility.

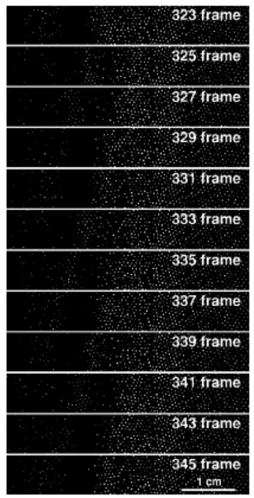


FIG. 3. Top view of the lattice at time 1.4–1.5 s after the excitation, when the wave packet was well formed. The wire positioned at the left edge of the field of view below the lattice excited the wave packet. The particles are visible only if they are in the plane of the illuminating laser sheet. The stripes of particles apparently move from right to left due to the vertical wave motion. They correspond to the lines of constant phase (phase motion). Individual particles do not move horizontally. The numbers on the images indicate the frame number (at 230.75 fps).

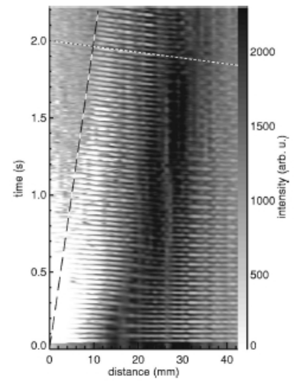


FIG. 4. Visualization of an experimentally observed vertical wave packet. The intensity of light scattered by the particles (which depends on their vertical positions) is plotted vs distance and time. Lighter regions indicate where particles are displaced from their equilibrium positions. Vertical oscillations with a frequency of about 16 Hz are excited by an electrostatic pulse applied to a wire placed at distance 0 mm. A wave packet, appearing as inclined stripes, is formed at time 1 s. The dashed line is drawn approximately through the ends of the stripes. Its angle indicates the packet group velocity of 4.5 mm/s. The dotted line is drawn parallel to the stripes. It shows the vertical wave phase velocity of −290 mm/s, which is negative because the dispersion relation is inverse optical like.

to inverse optical-like dispersion of the vertical wave mode.

(Qualitative study; modulational instability not investigated.)

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3a. (*Nonlinear*) *longitudinal* excitations.

The *nonlinear* equation of longitudinal motion reads:

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2 \left(\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n\right)$$
$$-a_{20} \left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2\right]$$
$$+\, a_{30} \left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3\right]$$

– $\delta x_n = x_n - nr_0$: longitudinal dust grain displacements

– Cf. *Fermi-Pasta-Ulam (FPU) problem*:
anharmonic spring chain model.

Coefficients (for Debye interactions)

In general:

$$a_{20} = -\frac{1}{2M} U'''(r_0) \,, \qquad a_{30} = \frac{1}{6M} U''''(r_0) \,,$$

$$\omega_{0,L}^2 = U''(r_0)/M \,,$$

For a Debye (Yukawa) potential $U_D(r) = Q^2 \, e^{-r/\lambda_D}/r$:

$$\omega_{0,L}^2 = \frac{2Q^2}{M\lambda_D^3} e^{-\kappa} \frac{1 + \kappa + \kappa^2/2}{\kappa^3} \equiv c_L^2/(\kappa^2 \lambda_D^2) \,,$$

$$p_0 \equiv 2a_{20}\kappa^3\lambda_D^3 = \frac{6Q^2}{M\lambda_D} e^{-\kappa} \left(\frac{1}{\kappa} + 1 + \frac{\kappa}{2} + \frac{\kappa^2}{6} \right) ,$$

$$q_0 \equiv 3a_{30}\kappa^4\lambda_D^4 = \frac{Q^2}{2M\lambda_D} e^{-\kappa} \frac{1}{\kappa} \Big(\kappa^4 + 4\kappa^3 + 12\kappa^2 + 24\kappa + 24 \Big) .$$

Longitudinal Dust-Lattice wave (LDLW) modulation

The reductive perturbation technique (cf. above) now yields:

$$\delta x_n \approx \epsilon \left[u_0^{(1)} + (u_1^{(1)}\, e^{i\phi_n} + \text{c.c.})\right] + \epsilon^2\, (u_2^{(2)}\, e^{2i\phi_n} + \text{c.c.}) + \ldots\,,$$

[Harmonic generation; cf. K. Avinash PoP 2004].

where the amplitudes now obey the coupled equations:

$$i\frac{\partial u_1^{(1)}}{\partial T} + P_L\, \frac{\partial^2 u_1^{(1)}}{\partial X^2} + Q_0\, |u_1^{(1)}|^2 u_1^{(1)} + \frac{p_0 k^2}{2\omega_L}\, u_1^{(1)} \frac{\partial u_0^{(1)}}{\partial X} = 0\,,$$

$$\frac{\partial^2 u_0^{(1)}}{\partial X^2} = -\frac{p_0 k^2}{v_{g,L}^2 - c_L^2}\, \frac{\partial}{\partial X}\, |u_1^{(1)}|^2 \equiv R(k) \frac{\partial}{\partial X}\, |u_1^{(1)}|^2$$

– $Q_0 = -\frac{k^2}{2\omega}\left(q_0\, k^2 + \frac{2p_0^2}{c_L^2 r_0^2}\right)$; $P_L = \omega_L''(k)/2\,;$

– $v_{g,L} = \omega_L{}'(k)$; $\{X, T\}$ are *slow* variables;

– $p_0 = -r_0^3 U'''(r_0)/M \equiv 2a_{20} r_0^3$, $q_0 = U''''(r_0) r_0^4/(2M) \equiv 3a_{30} r_0^4$.

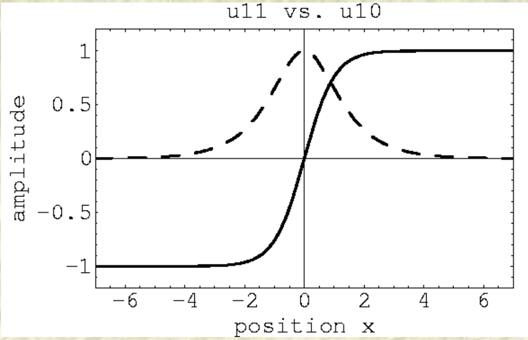
– $R(k) > 0$, since $\forall\, k$ $\quad v_{g,L} < \omega_{L,0}\, r_0 \equiv c_L$ *(sound velocity)*.

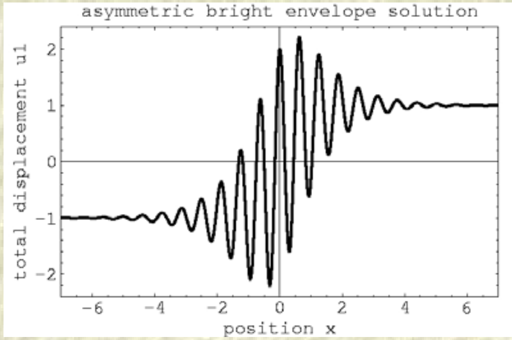
Asymmetric *longitudinal* envelope structures.

– The system of Eqs. for $u_1^{(1)}$, $u_0^{(1)}$ may be solved exactly:
$\rightarrow$ *asymmetric* envelope solutions.

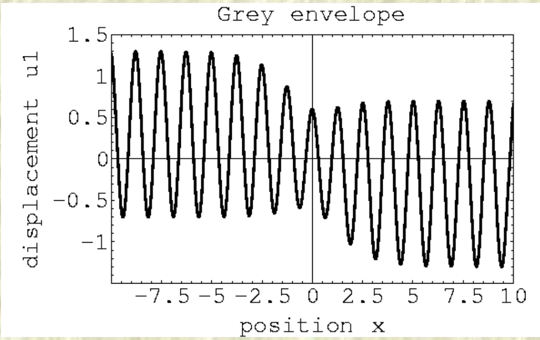
– $P = P_L = \omega_L''(k)/2 < 0$;

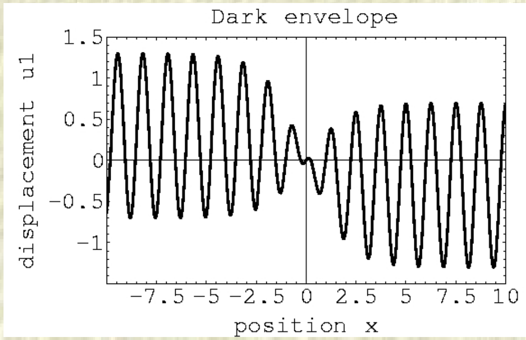
– $Q > 0$ (< 0) prescribes *stability* (instability) at *low* (high) k.





(at high k)





(at low k)

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 1384 (2004).].

3b. *Longitudinal* soliton formalism.

A link to soliton theories:

– *Continuum approximation*, viz. $\delta x_n(t) \to u(x,t)$.

– *"Standard"* description: keeping *lowest order* nonlinearity,

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} = -\,p_0\,u_x\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$ and p_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), slow profile evolution in time τ and defining the *relative displacement* $w = u_\zeta$, one obtains (for $\nu = 0$) the Korteweg-deVries Equation:

$$w_\tau - a\,w\,w_\zeta + b\,w_{\zeta\zeta\zeta} = 0$$

Defs.: $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$.

The KdV description: The Melandsø (1996) theory

The Korteweg-deVries (KdV) Equation

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, here) solutions, in the form (here):

$$w_1(\zeta, \tau) = -w_{1,m} sech^2 \Big[(\zeta - v\tau - \zeta_0)/L_0 \Big]$$

– Pulse amplitude: $w_{1,m} = 3v/a = 6vv_0/|p_0|$;

– Pulse width: $L_0 = (4b/v)^{1/2} = [2v_1^2 r_0^2/(vv_0)]^{1/2}$;

– Note that: $w_{1,m} L_0^2 = \text{constant}$ (cf. experiments)†.

– This solution is a negative pulse for $w = u_x$,
describing a *compressive* excitation for the *displacement* $\delta x = u$,
i.e. a localized increase of *density* $n \sim -u_x$.

F Melandsø 1996; S Zhdanov *et al.* 2002; K Avinash *et al.* 2003; V Fortov *et al.* 2004.

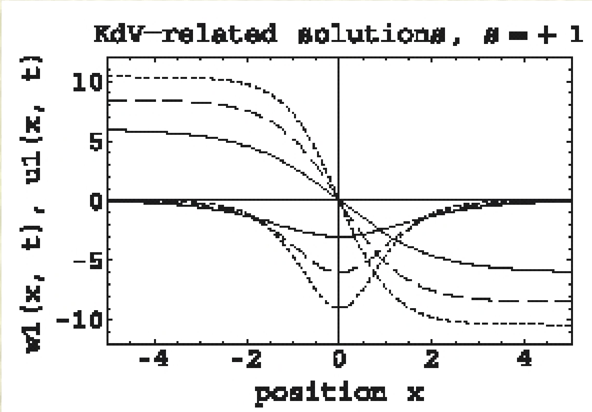
The Korteweg-deVries (KdV) Equation

$$w_\tau - a\,w\,w_\zeta + b\,w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, since $a > 0$) solutions, in the form:

$$w_1(\zeta, \tau) = -\frac{3v}{a}\mathrm{sech}^2\left[(v/4b)^{1/2}(\zeta - v\tau - \zeta_0)\right]$$

– This solution is a negative pulse for $w = u_x$,
describing a *compressive* excitation for the *displacement* $\delta x = u$,
i.e. a localized increase of *density* $n \sim -u_x$ †.



† F. Melandsø 1996; S. Zhdanov *et al.* 2002; K. Avinash *et al.* 2003; V. Fortov *et al.* 2004.

Characteristics of the KdV theory

The *Korteweg - deVries* theory:

– provides a *qualitative description* of *compressive* excitations;

– benefits from the KdV *“artillery”* of analytical know-how: *integrability*, *multi-soliton* solutions, *conservation laws*, ... ;

but possesses certain drawbacks:

– *approximate* derivation:
– propagation velocity v near (longitudinal) sound velocity c_L,
– time evolution terms omitted *‘by hand’*,
– higher order nonlinear contributions omitted;

– *only* accounts for *compressive* solitary excitations (for Debye interactions).

Experimental observation of *rarefactive* LDL solitons

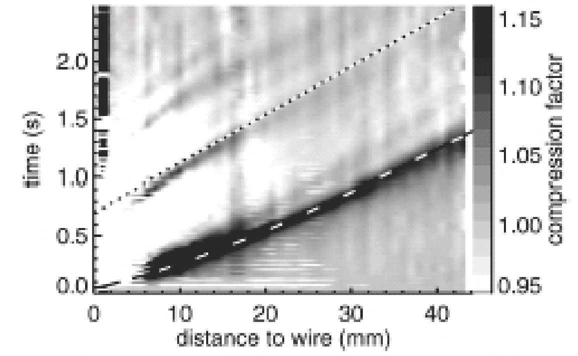


FIG. 2. Compression factor n/n_0 as a function of time and distance to the wire. Darker regions correspond to higher compression. The lower dashed curve is a fit to the trajectory of the soliton. The upper dotted line was drawn above a weak secondary pulse with Mach number $M \simeq 1$. Its slope is determined by the dust lattice wave speed $C_{DL} = 23$ mm/s in the middle of the lattice.

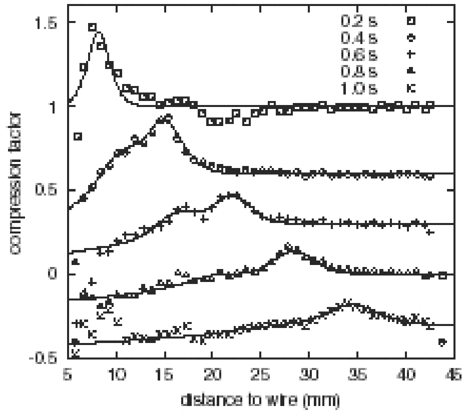


FIG. 3. Compression factor n/n_0 versus distance to the wire at different times. The solid lines show the theoretical fits to the experimental data. Two solitons are present. The fits and experimental points at later times are offset down (by 0.4, 0.7, 1.0, 1.3, respectively).

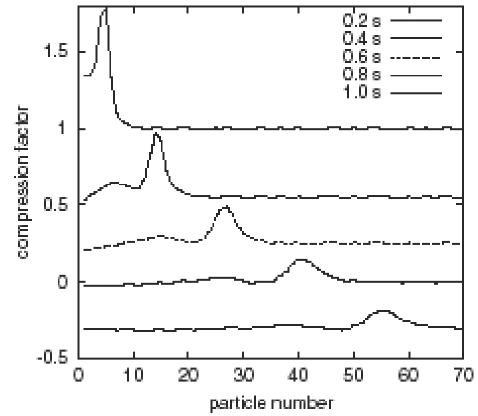


FIG. 5. Compression factor versus particle number for a simulated linear chain model. It describes the formation of two (or more) solitons by a single excitation pulse and qualitatively agrees with the experiment. The curves at later times are offset down (by 0.45, 0.75, 1.0, and 1.3, respectively).

[Samsonov *et al.*, PRL 2002].

- *Only compressive* solitons predicted by KdV theory;
- *Only compressive* solitons experimentally anticipated and, hence, reported in 2002;
- What about *rarefactive longitudinal solitons*?

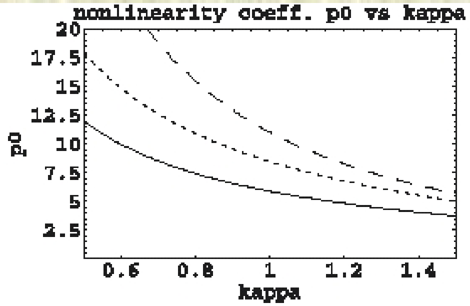
Extended longitudinal soliton formalism

Q.: *What if we also kept the next order in nonlinearity ?*

– *“Extended”* description: :

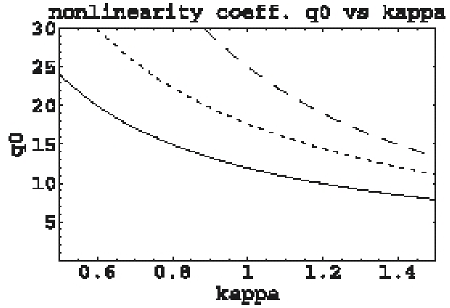
$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} = -\,p_0\,u_x\,u_{xx} + q_0\,(u_x)^2\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$, $p_0 \sim -U'''(r)$ and $q_0 \sim U''''(r)$ (cf. above).



(b)

Fig. 4. (a) The nonlinearity coefficient p_0 (normalized over $Q^2/(M\lambda_D)$) is depicted against the lattice constant κ for $N = 1$ (first-neighbor interactions: —), $N = 2$ (second-neighbor interactions: - - -), $N = \infty$ (infinite-neighbors: – – –), from bottom to top. (b) Detail near $\kappa \approx 1$.



(b)

Fig. 5. (a) The nonlinearity coefficient q_0 (normalized over $Q^2/(M\lambda_D)$) is depicted against the lattice constant κ for $N = 1$ (first-neighbor interactions: —), $N = 2$ (second-neighbor interactions: - - -), $N = \infty$ (infinite-neighbors: – – –), from bottom to top. (b) Detail near $\kappa \approx 1$.

Rq.: q_0 is *not* negligible, compared to p_0! (instead, $q_0 \approx 2p_0$ practically!)

Extended longitudinal soliton formalism *(continued)*

Q.: *What if we also kept the next order in nonlinearity ?*

– *"Extended"* description: :

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} = -\,p_0\,u_x\,u_{xx} + q_0\,(u_x)^2\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$, p_0 and q_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), and defining the *relative displacement* $w = u_\zeta$, one has

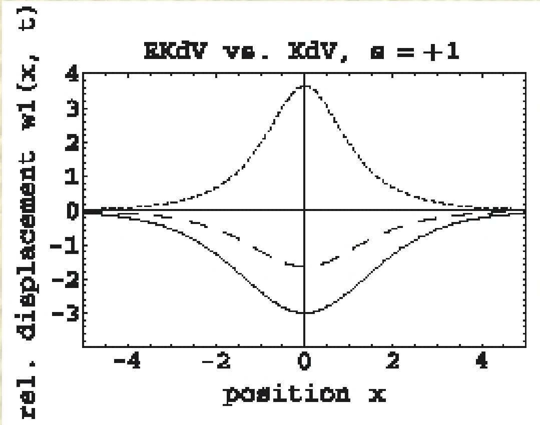
$$w_\tau - a\,w\,w_\zeta + \hat{a}\,w^2\,w_\zeta + b\,w_{\zeta\zeta\zeta} = 0 \qquad (2)$$

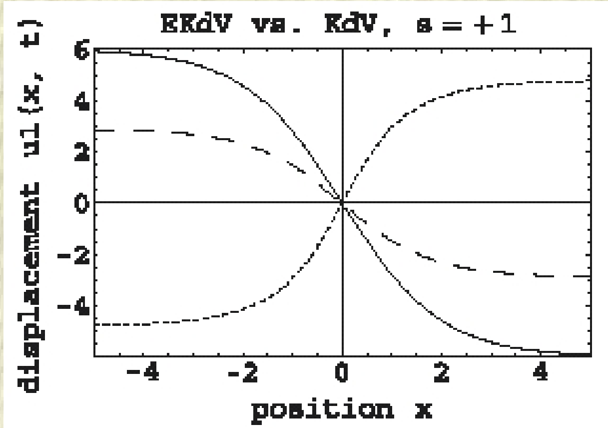
(for $\nu = 0$); $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$; $\hat{a} = q_0/(2c_L) > 0$.

Characteristics of the KdV/mKdV/EKdV theory

The *extended Korteweg - deVries* Equation:

– accounts for *both compressive and rarefactive* excitations;





(*horizontal grain displacement* $u(x,t)$)

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– is previously widely studied, in literature;

Still, ...

– It was derived under the *assumption*: $v \approx c_L$.

One more alternative: the Boussinesq theory

The *Generalized* *Boussinesq* (Bq) Equation (for $w = u_x$):

$$\ddot{w} - c_L^2 \, w_{xx} = \frac{c_L^2 r_0^2}{12} \, w_{xxxx} - \frac{p_0}{2} \, (w^2)_{xx} + \frac{q_0}{2} \, (w^3)_{xx}$$

– predicts *both compressive and rarefactive* excitations;

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– has been widely studied in literature;

and, ...

One more alternative: the Boussinesq theory

The *Generalized Boussinesq* (Bq) Equation (for $w = u_x$):

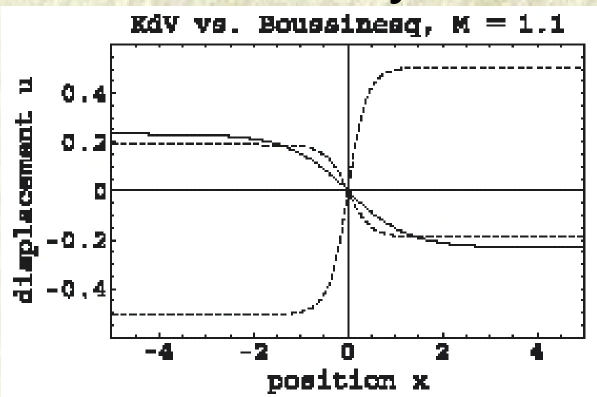
$$\ddot{w} - c_L^2 \, w_{xx} = \frac{c_L^2 r_0^2}{12} \, w_{xxxx} - \frac{p_0}{2} \, (w^2)_{xx} + \frac{q_0}{2} \, (w^3)_{xx}$$

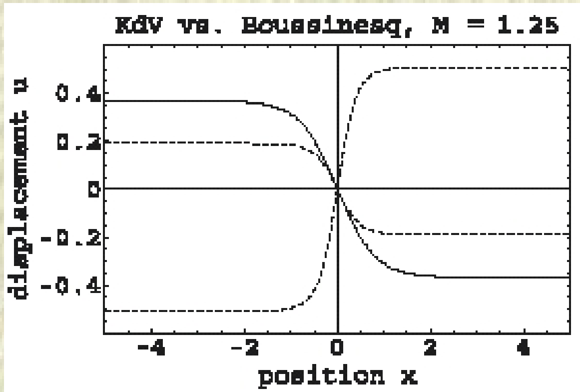
– predicts *both compressive and rarefactive* excitations;

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

and, ...

– relaxes the velocity assumption, i.e. is valid $\forall \quad v > c_L$.





[I Kourakis & P K Shukla, *European Phys. J. D*, **29**, 247 (2004)] .

Added *“in proof”*:
Experimental observation of rarefactive LDL solitons (2008)

Solitary Rarefaction Wave in a Three Dimensional Complex Plasma

Ralf Heidemann, Max-Planck-Institut für extraterrestrische Physik

Abstract
Observation of a solitary rarefaction wave in a three dimensional complex plasma containing monodisperse microparticles will be presented.
The experiments were performed in a capacitively coupled, symmetrically driven RF discharge. The discharge chamber is a slightly modified version of the PK3plus design currently flying on board the ISS.
A gas temperature gradient of 400K/m was applied to balance gravity and to levitate the particles in the plasma bulk. The wave was exited by a short voltage pulse on the electrodes of the rf discharge chamber. It was found that the wave propagates with constant speed and that the amplitude damping factor is significantly lower then Epstein damping.

Poster by Ralf Heidemann et al, ICPDP5 conf, Acores May 2008

Outline – Menu

1. *Dusty plasmas (DP) & DP crystals (DPCs)*: Prerequisites
   Nonlinearity in 1D DP crystals: Origin and modelling

2. *Transverse* dust-lattice (*TDL*) excitations:
   amplitude modulation, transverse envelope structures

3. *Longitudinal* dust-lattice (*LDL*) excitations:
   amplitude modulation, envelope structures, solitons

4. **1D *Discrete Breather excitations* (*intrinsic localized modes*)**

5. 2D *Discrete Breather excitations* in hexagonal crystals

6. Conclusions

4. Transverse Discrete Breathers (DBs)

VOLUME 84, NUMBER 4 PHYSICAL REVIEW LETTERS 24 JANUARY 2000

Discrete Breathers in Nonlinear Lattices: Experimental Detection in a Josephson Array

E. Trías,[1] J. J. Mazo,[1,2] and T. P. Orlando[1]

[1]*Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

[2]*Departamento de Física de la Materia Condensada and ICMA, CSIC-Universidad de Zaragoza, E-50009 Zaragoza, Spain*

(Received 7 April 1999)

We present the experimental detection of discrete breathers in an underdamped Josephson-junction array. Breathers exist under a range of dc current biases and temperatures, and are detected by measuring dc voltages. We find that the maximum allowable bias current for the breather is proportional to the array depinning current, while the minimum current seems to be related to a junction retrapping mechanism. We have observed that this latter instability leads to the formation of multisite breather states in the array. We have also studied the domain of existence of the breather at different values of the array parameters by varying the temperature.

PACS numbers: 74.50.+r, 45.05.+x, 63.20.Pw, 85.25.Cp

Discrete breathers have been mathematically proven to be generic solutions for the dynamics of nonlinear coupled oscillators [1,2] and have been theoretically studied in depth in the last few years [3,4]. These solutions are characterized by an exponential localization of the energy. Their existence results from the nonlinearity and discreteness of the system. Discreteness is essential to preventing resonances between the breather and the system characteristic frequencies. Surprisingly, this localization occurs in perfectly regular systems, so that it is intrinsic and different from Anderson localization or any other localization due to the presence of imperfections or impurities in the lattice. Thus, discrete breathers are also known as intrinsic localized modes. They have been proposed to exist in diverse systems such as in spin wave modes of antiferromagnets [5], DNA denaturation [6], and the dynamics of Josephson-junction networks [7–9]. Also, they have been shown to be important in the dynamics of mechanical engineering systems [10,11].

tions that govern the motion of a driven pendulum [14]: $i = \ddot{\varphi} + \Gamma\dot{\varphi} + \sin\varphi$. The response of the junction to a current is measured by the voltage of the junction which is given by $v = (\Phi_0/2\pi)d\varphi/dt$. By coupling junctions it is possible to construct solid-state physical realizations of nonlinear oscillator systems. Moreover, since the parameters, such as Γ, vary with temperature, a range of parameter space can be studied easily with each sample.

The inset of Fig. 1 shows a schematic of the anisotropic ladder array. The junctions are fabricated using a Nb-Al_2O_x-Nb trilayer technology with a critical current density of 1 kA/cm^2. The current is injected and extracted through bias resistors in order to distribute the external current as uniformly as possible through the array. These resistors are large enough so as to minimize any deleterious effects on the dynamics. The anisotropy of the array h is the ratio of areas of the horizontal to vertical junctions. In our arrays $h = 1/4$ and $h = I_{ch}/I_{cv} = R_v/R_h = C_h/C_v$, and $\Gamma_v = \Gamma_h = \Gamma$

4. Transverse Discrete Breathers (DBs)

Localizing Energy Through Nonlinearity and Discreteness

Intrinsic localized modes have been theoretical constructs for more than a decade. Only recently have they been observed in physical systems as distinct as charge-transfer solids, Josephson junctions, photonic structures, and micromechanical oscillator arrays.

David K. Campbell, Sergej Flach, and Yuri S. Kivshar

In solid-state physics, the phenomenon of localization is usually perceived as arising from *extrinsic* disorder that breaks the discrete translational invariance of the perfect crystal lattice. Familiar examples include the localized vibrational phonon modes around impurities or defects (such as atomic vacancies or interstitial atoms) in crystals and Anderson localization of electrons in disordered media.[1] The usual perception among solid-state researchers is that, in perfect lattices—those free of extrinsic defects—phonons and electrons exist only in extended, plane wave states. That notion extends to any periodic structure, such as a photonic crystal or a periodic array of optical waveguides. Such firmly entrenched perceptions were severely jolted in the late 1980s by the discovery that *intrinsic* localized modes[2] (ILMs), also known as discrete breathers[3] (DBs), are, in fact, typical excitations in perfectly periodic but strongly nonlinear systems.

The past several years have seen this prediction confirmed by a flood of experimental observations of ILMs in physical systems ranging from electronic and magnetic solids, through microengineered structures including Josephson junctions and optical waveguide arrays, to laser-induced photonic crystals. Experimentalists are currently hot on the trail of ILMs in Bose–Einstein condensates (BECs) and biopolymers. Hopes are high that these exotic excitations will be useful in all-optical logic and switching devices and in targeted breaking of chemical bonds, and will prove helpful to the understanding of melting processes in solids and conformational changes in biomolecules.

discusses this path to DBs in more detail. An ILM is an excitation that is localized in space by the intrinsic nonlinearity of the medium, rather than by a defect or impurity. Box 2 reviews this path to ILMs.

By the early 1990s, researchers following these two paths had converged on the insight that stable localized periodic modes, whether called ILMs or DBs, were generic excitations in discrete nonlinear systems, and that to study them systematically, one should start with a system of uncoupled nonlinear oscillators—the "anti-continuum limit"—and treat the coupling as a weak perturbation.

To pursue this insight, consider the simple problem of a diatomic molecule, or dimer, modeled initially by a classical system of two coupled anharmonic oscillators. First, imagine that the interoscillator coupling is switched off; that leaves two independent nonlinear oscillators. The nonlinearity of the oscillators means that the frequency of their motion depends on the amplitude or, equivalently, the input energy. In the case of the familiar simple but nonlinear plane pendulum, for example, the period varies from the small oscillation harmonic limit of $2\pi\sqrt{l/g}$, where l is the length of the pendulum and g is the acceleration due to gravity, to infinitely long as the amplitude of the pendulum's angle approaches π. When the oscillators are completely uncoupled, we can form a localized mode by exciting only one of the oscillators, and the resulting frequency can fall anywhere in the range allowed by the form of the anharmonicity of the individual oscillator.

Now consider exciting one oscillator strongly but the second one only weakly so that most of the energy is initially localized at the first oscillator. Because the frequencies depend on the amplitudes, we can, in principle, choose amplitudes such that the frequencies of each oscillator are irrationally related. For strictly incommensurate frequencies, no possible resonances exist between any of the os-

4. Transverse Discrete Breathers (DBs) *(cont.)*

- Eq. of motion in the *transverse* direction:

$$\frac{d^2 u_n}{dt^2} + \omega_{T,0}^2 \,(\, u_{n+1} + \, u_{n-1} - 2\, u_n) + \omega_g^2 \, u_n + \alpha \, u_n^2 + \beta \, u_n^3 = 0$$

- Damping may be neglected (for low plasma density and/or pressure): $\nu/\omega_g \simeq 0.00154$ ([Misawa et al., PRL 2001]).

- 1D DPCs are *highly discrete* lattice configurations: $\epsilon = \omega_0^2/\omega_g^2 \simeq 0.016$ ([Misawa et al., 2001]); $\epsilon \simeq 0.181$ ([Liu et al., 2003]).

- One may seek *discrete breather* solutions (localized modes):

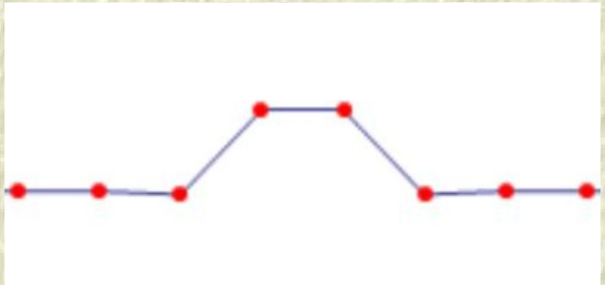
$$u_n(t) = \sum_m A_n(m) \exp(im\omega_B t)$$

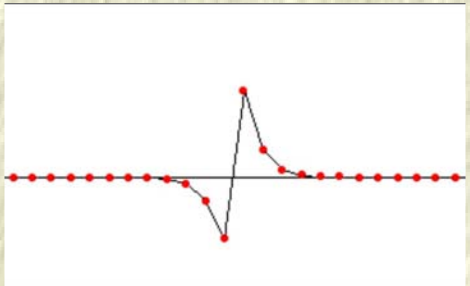
where only few ($m \simeq 1 - 5$) sites are excited.

4. 1D Transverse Discrete Breathers (DBs) *(cont.)*

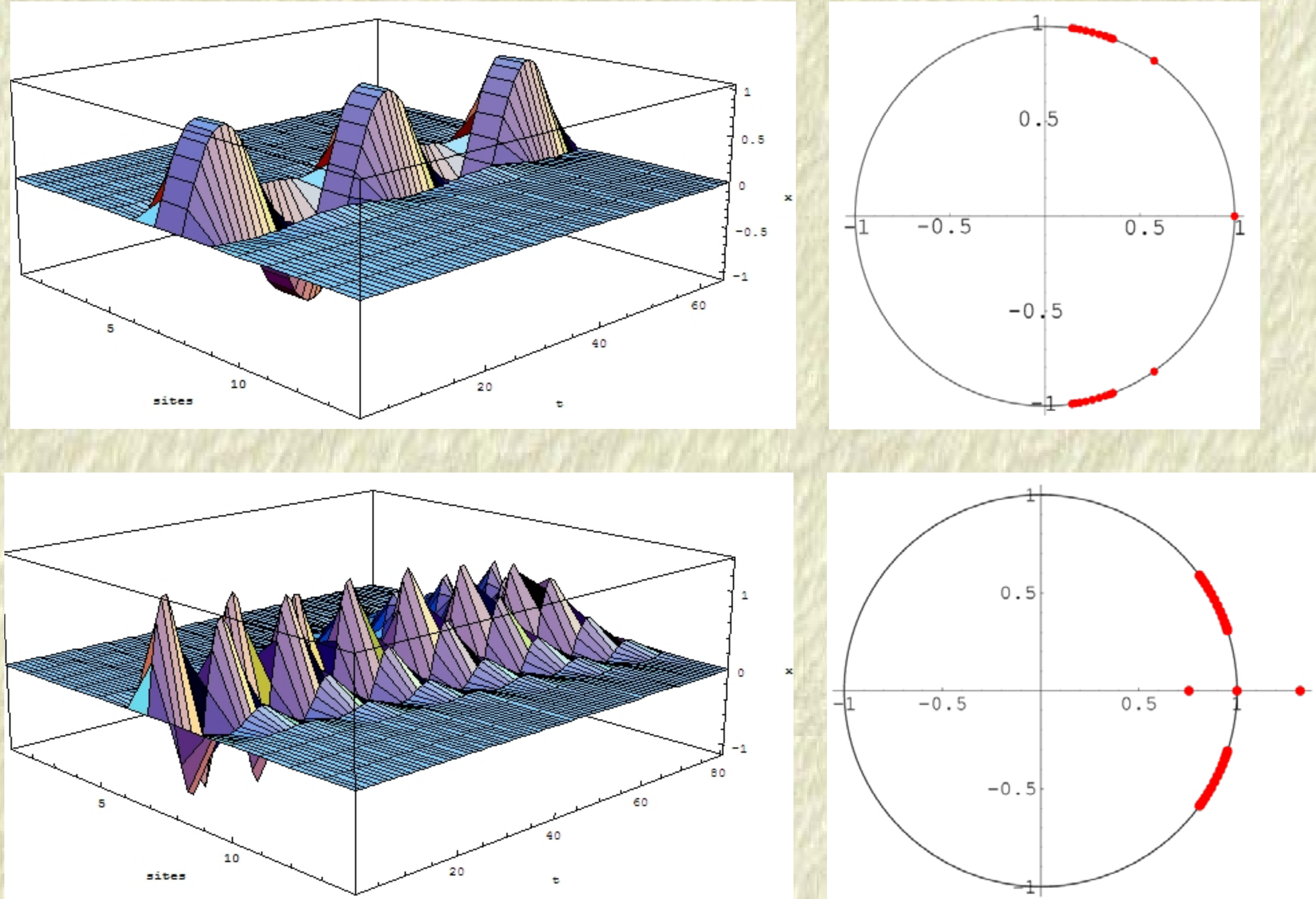
* *Non - resonance condition*: $m\omega_B \neq \omega_T(k) \qquad \forall k,\, m = 1, 2, ...$

* *In-phase* and *out-of-phase* DBs may occur in DPCs:





Stable vs. unstable Transverse DBs in DPCs



Ab-initio study, adopting values from the Kiel experiment

[Zafiu *et al.*, PRE **63** 066403 (2001).]

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6. Conclusions

5. 2D Transverse Discrete Breathers (DBs)

letters to nature

Observation of two-dimensional discrete solitons in optically induced nonlinear photonic lattices

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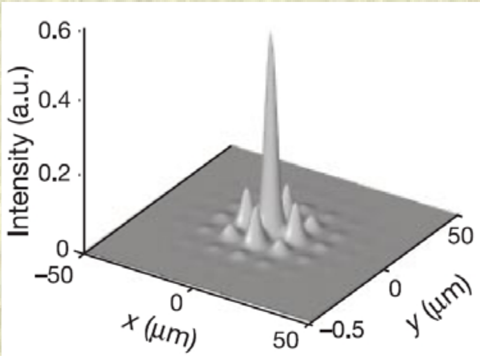
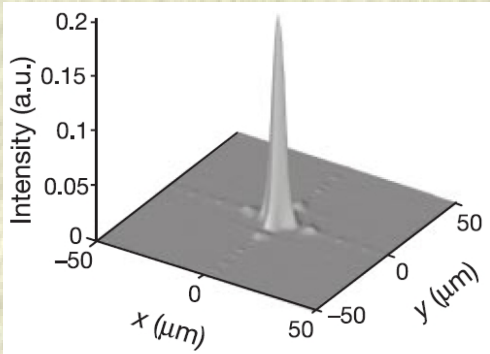
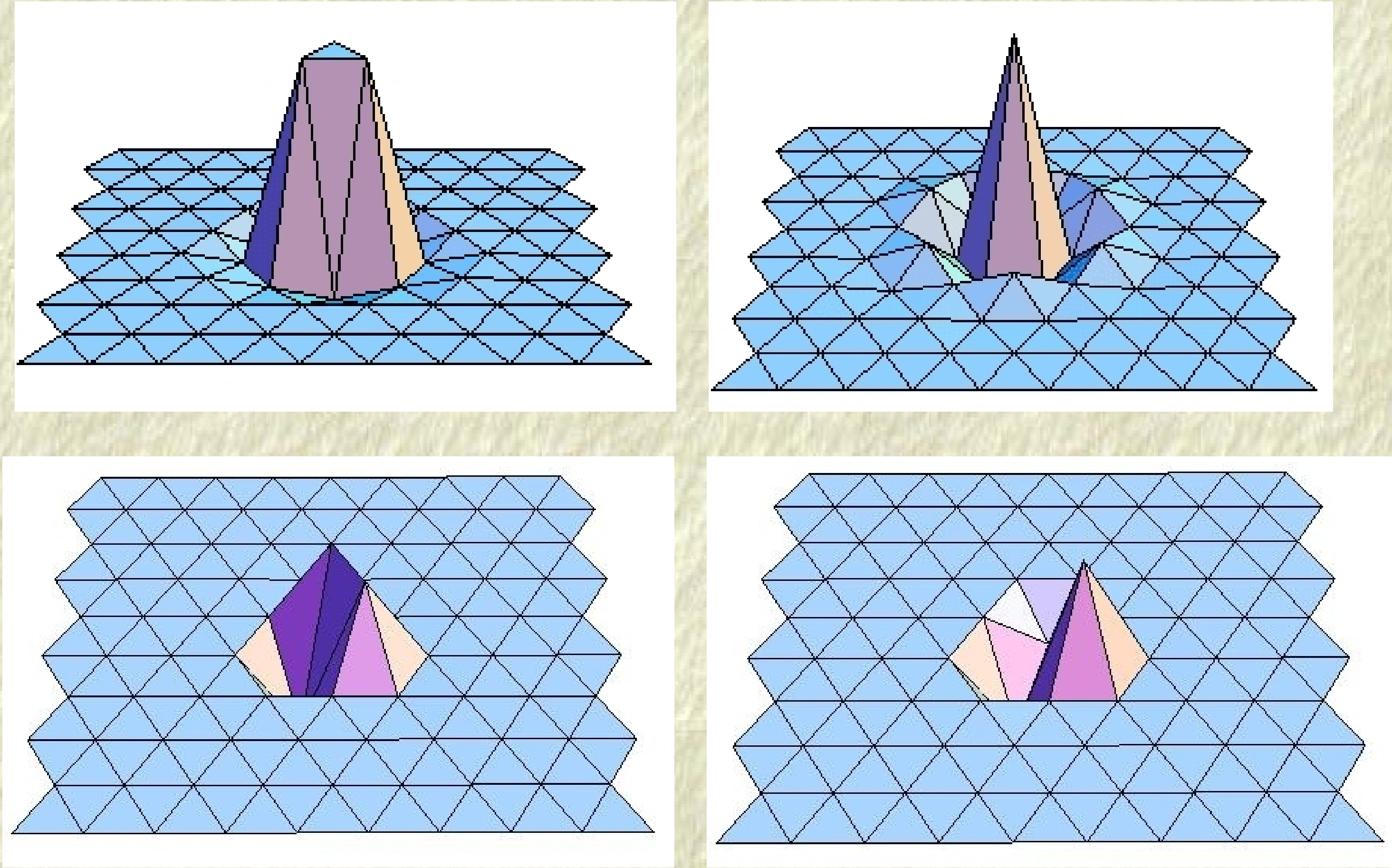
Nonlinear periodic lattices occur in a large variety of systems, such as biological molecules[1], nonlinear optical waveguides[2], solid-state systems[3] and Bose–Einstein condensates[4]. The underlying dynamics in these systems is dominated by the interplay between tunnelling between adjacent potential wells and nonlinearity[1–15]. A balance between these two effects can result in a self-localized state: a lattice or 'discrete' soliton[1,2]. Direct observation of lattice solitons has so far been limited to one-dimensional systems, namely in arrays of nonlinear optical waveguides[2,9–17]. However, many fundamental features are expected to occur in higher dimensions, such as vortex lattice solitons[18], bright lattice solitons that carry angular momentum, and three-dimensional collisions between lattice solitons. Here, we report the experimental observation of two-dimensional (2D) lattice solitons. We use optical induction, the interference of two can balance this effect, leading to a lattice (discrete) soliton[2,9,10]. For light propagating at an angle $\theta = k_x/k \approx k_x/k_z$ with respect to the array, the periodicity of the lattice becomes important, as the corresponding 'Bloch momentum' k_x can satisfy Bragg reflection conditions within the Brillouin zone (defined in the range $|k_xD| \leq \pi$, where D is the lattice spacing). Near the edge of this zone, diffraction becomes anomalous ('negative'), leading to such effects as diffraction management[13,14] and staggered (π out-of-phase) solitons[11,15,17]. These are some of the fundamental aspects of solitons in nonlinear periodic structures, which were originally discovered through the theoretical paradigm of the discrete nonlinear Schrödinger equation[1–4,16], hence the term 'discrete soliton'[1,2]. Experimentally, self-localized 'breathers' have been observed in various physical settings[5–8], but lattice (discrete) solitons have thus far been reported only in nonlinear optical systems and only in one-dimensional (1D) configurations[9–11,13,17]. In what follows we demonstrate bright 2D lattice solitons in their simplest realization: in-phase solitons at the base of the first Brillouin zone. In addition, we demonstrate bright self-trapped wave packets at the edge of the first Brillouin zone.

The formation of the 2D nonlinear photonic lattice relies on an optical induction technique[11,12] in which a 2D array of waveguides is induced in a nonlinear medium. We proposed this method theoretically[12] and recently demonstrated experimentally[11] 1D lattice solitons in a 1D waveguide array. The waveguide array is induced, in real time, in a photosensitive material by interfering two or more plane waves. A separate 'probe' beam is launched into the periodic waveguide array, where it exhibits discrete diffraction and, at a sufficiently high nonlinearity, forms a lattice soliton. For this system to work, it is essential that the waveguides are as uniform as possible, implying that the interference pattern (inducing the lattice) must





2D T-DBs: the anticontinuum limit method

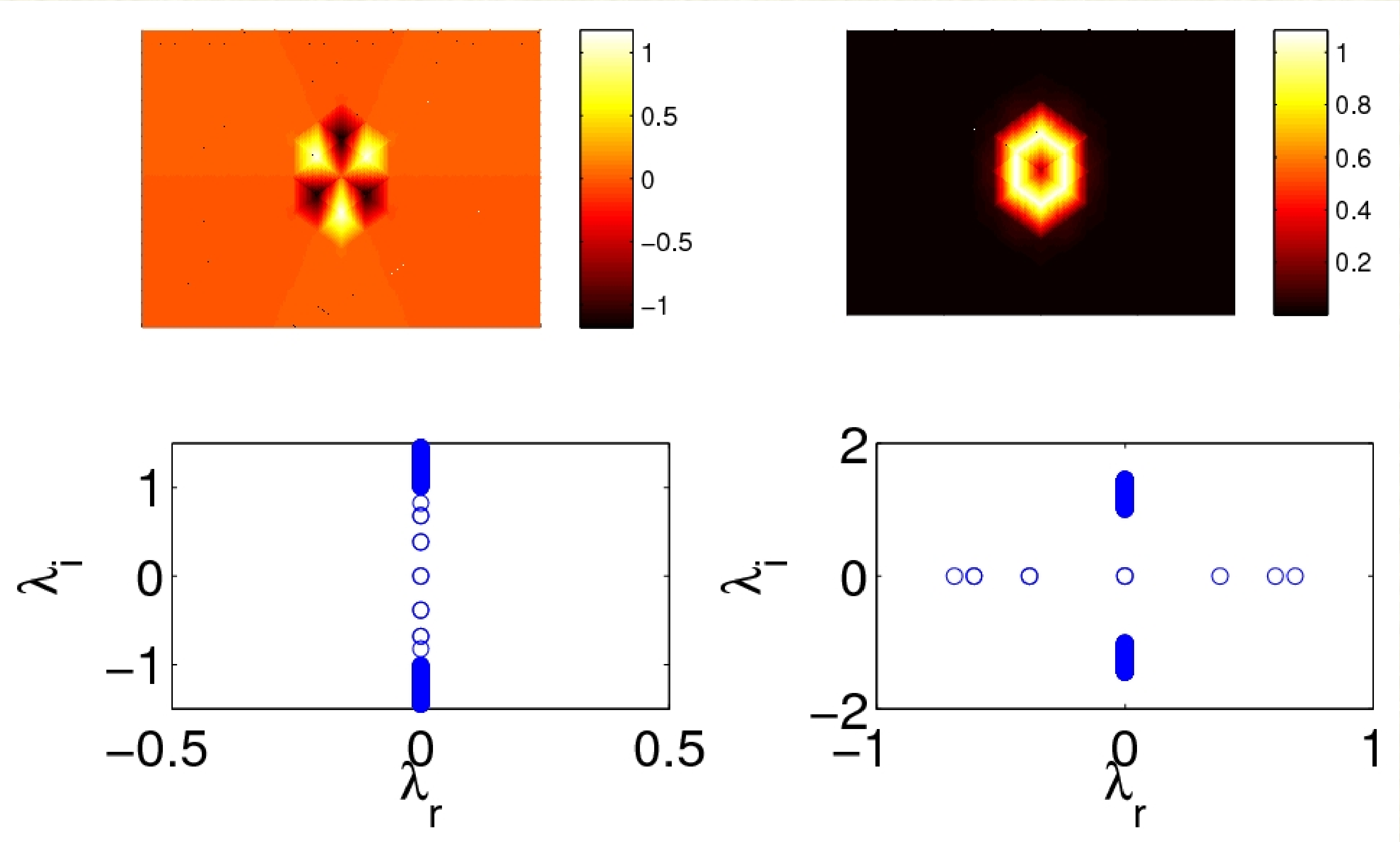


Ab-initio study, adopting values from the Kiel experiment

[Zafiu *et al.*, PRE **63** 066403 (2001).]; *see Videos*

V. Koukouloyannis & I. Kourakis, in preparation.

5. 2D T-DBs: the DNLS method



K Law, P Kevrekidis, V Koukouloyannis, I Kourakis, D Frantzeskakis & A R Bishop, sub PRE.

Conclusions - State of the Art

– *Dust crystals* provide an excellent test-bed for NL theories

– *Observations are possible at the kinetic level*: unique possibility for physical data processing & real-time analysis

– Link between *Plasma Phys., Solid State Physics, Stat. Mech.*

– *Theory (1D): Envelope solitons, (non-topological) solitons, Discrete Breathers*: predicted

– *Theory (2D): Discrete Breathers, vortices, ...*: predicted

– *Experiment:* Harmonic generation, density solitons, NL TDL oscillations, backward wave: observed (*Urge for more :-)*)

Overview of existing results

1. 1D: *Transverse* dust-lattice (*TDL*) motion ($\sim$ NL KG, inv. disp.):

   $\longrightarrow$ Envelope (NLS) solitons [IK & P K Shukla, Phys. Plasmas **11**, 1384 (2004)]

   $\longrightarrow$ DBs (ILMs) [V. Koukouloyannis & IK, PRE **76**, 016402 (2007)]

2. 1D: *Longitudinal* dust-lattice (*LDL*) motion) ($\sim$ FPU):

   $\longrightarrow$ *Asymmetric* envelope structures (coupled 0th/1st harmonics) [IK & P K Shukla, Phys. Plasmas **11**, 3665 (2004)]

   $\longrightarrow$ KdV vs. eKdV / Bq solitons [IK & PKS, Eur. Phys. J. D **29**, 247 (2004)]

   Rem.: experimentally observed (compressive case only)

3. 2D: *In-plane* ("LDL") motion in *hexagonal* DP crystals:

   $\longrightarrow$ Envelope structures [Farokhi, IK & PKS, Phys. Plasmas **13**, 122304 (2006)]

4. 2D: *Out-of-plane* (TDL) motion in *hexagonal* DP crystals:

   $\longrightarrow$ Envelope structures, DBs, vortices.

Future considerations & perspectives

1. *LDL-DBs ?* (∼ FPU);

2. *Damping* (dissipative system), ion drag, wake potentials, ...

3. Mixed *T-L Mode*: coupled FPU-NLKG Eqs. (ongoing work);

4. *2D hexagonal dust lattices*: vortices ? (seen experimentally);

5. ***Experimental feedback***:

   - establish & pursue contacts,
   - seek confirmation of results, motivate experiments ...

→ *A lot remaining to be done!*

Thank You !

Ioannis Kourakis

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Material from:

I Kourakis & P K Shukla, *Phys. Plasmas*, **11**, 2322 (2004); *ibid*, **11**, 3665 (2004)

idem, *Phys. Plasmas*, **11**, 1384 (2004); *idem*, *European Phys. J. D*, **29**, 247 (2004)

V Koukouloyannis & IK, PRE **76**, 016402 (2007)

F Farokhi, I Kourakis & P K Shukla, PoP **13** (12), 122304 (2006)

K Law, P Kevrekidis, V Koukouloyannis, I Kourakis, D Frantzeskakis & A R Bishop, sub PRE.

Slides available at: www.kourakis.eu