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C.S. Liu
University of Maryland,East West Space Science Centre, U.S.A.

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Laser Acceleration of Monoenergetic Protons in Moving Double Layer from Thin Foil

V. K. Tripathi, C. S. Liu, and X. Shao
East West Space Science Center
University of Maryland
College Park, MD 20742

Abstract

We present analytic theory of laser acceleration of monoenergetic protons by irradiation on a thin foil, reported by Yan et al., 2008 in simulations. The ponderomotive force pushes the electrons forward, leaving ions behind until the space charge field balances the ponderomotive force at distance $\Delta_s \approx a_0(n_c/n_0)\lambda_L/\pi$, where a_0 and λ_L are the normalized laser amplitude and wavelength, n_c is the critical density and n_0 is the plasma density. For the target thickness $D = \Delta_s$, the electron sheath piled up at the rear surface is detached from the bulk ions and moves into vacuum, carrying with it the protons contained in the sheath width $\sim c/\omega_p$, where ω_p is the plasma frequency. These protons are trapped by the self field of the dense electron sheath and are collectively accelerated as a double layer by the laser ponderomotive force, giving proton energy ≈ 200 MeV at $a_0 = 5$, $n_c/n_0 = 10$ and pulse length 90 fs.

Laser acceleration of energetic protons by irradiation on solid thin foil target is an area of great interest and importance with potential applications ranging from the medical treatment of cancer to fast ignition laser fusion[1-10]. Experiments with a laser prepulse would produce a thin plasma layer at the foild surface. Main laser pulse would then accelerate electrons in the plasma to high energy either by p-polarized electric field or ponderomotive force and they penetrated into the rear surface of the target to create a space charge field which ionizes and accelerates ions to high energies. Generally the ions are accelerated to multi MeV energy with energy spectrum monotonically decreasing with energy and the nergy increasing with foil thickness.

Some experiments have reported quasi-monoenergetic ions[10-13] employing intricate and complex target designs. Recently, Yan et al.[14] have reported, using 1-D particle in cell simulations, a novel scheme for producing monoenergetic ions, in the hundreds of MeV range, with a specific foil thickness, equal to the distance of maximum charge separation at which the space charge force on electrons is balanced by the ponderomotive force. Nearly all the electrons of the foil are swept by the ponderomotive force and piled up at the rear surface of the foil. They are detached from the surface into the vacuum to form a moving double layer, trapping the ions in the sheath of width c/ω_p, the skin depth. The laser acceleration of double layer leads to monoenergetic ion (proton) generation.

We present, in this letter, an analytic theory of this collective acceleration of trapped protons in the moving double layer by laser ponderomotive force.

First, we derive the ponderomotive pressure of a relativistic intensity laser in an overdense plasma and demonstrate its relation to the usual radiation pressure. For simplicity, we consider a circularly polarized laser,

$$\vec{E}_i = i\vec{B}_i = (\hat{x} + i\hat{y})A_0 e^{-i(\omega t - \omega z/c)}, \tag{1}$$

normally incident on an overdense plasma (z > 0) of density n_0. The fields of the reflected wave (z < 0) are

$$\vec{E}_R = -i\vec{B}_R = (\hat{x} + i\hat{y})RA_0 e^{-i(\omega t + \omega z/c)}. \tag{2}$$

The transmitted wave has, for z >0,

$$\begin{aligned} \vec{E}_T &= (\hat{x} + i\hat{y})A_T(z)e^{-i\omega t}, \\ \vec{B}_T &= -(\hat{x} + i\hat{y})\frac{c}{\omega}\frac{\partial A_T}{\partial z}(z)e^{-i\omega t}. \end{aligned} \tag{3}$$

As long as the cold plasma permittivity $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}\frac{n_e}{n_0}\frac{1}{(1+a_T^2)^{1/2}} < 0$, where $a_T = \frac{e|A_T|}{m\omega c}$, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, -*e*, *m*, and n_e are the electronic charge, mass and density, the transmitted field is evanescent, i.e. $A_T = |A_T|\exp(i\phi_0)$, with ϕ_0 a constant determined by the continuity conditions at z = 0, and $|A_T|$ a monotonically decreasing function of z. The continuity of tangential components of fields at z=0 gives

$$\begin{aligned} (1+R)a_0 &= a_T(0)e^{i\phi_0}, \\ (1-R)a_0 &= \frac{c}{i\omega}\left.\frac{da_T}{dz}\right|_0 e^{i\phi_0}, \end{aligned} \tag{4}$$

leading to

$$a_T(0) + \frac{c}{i\omega}\left.\frac{da_T}{dz}\right|_0 = 2a_0 e^{-i\phi_0}$$

or $\quad a_T(0) = 2a_0\cos\phi_0, \quad \left.\frac{da_T}{dz}\right|_0 = 2a_0\sin\phi_0,$

$$a_T^2(0) + \frac{c^2}{\omega^2}\left(\left.\frac{da_T}{dz}\right|_0\right)^2 = 4a_0^2, \tag{5}$$

where $a_0 = \frac{eA_0}{m\omega c}$.

The wave equation governing a_T is

$$\frac{d^2a_T}{dz^2} - \frac{\omega^2}{c^2}\left[\frac{\omega_p^2}{\omega^2}\frac{n_e}{n_0}\frac{1}{(1+a_T^2)^{1/2}} - 1\right]a_T = 0. \tag{6}$$

Initially, we may ignore the density modification ($n_e = n_0$) and integrate Eq. (6),

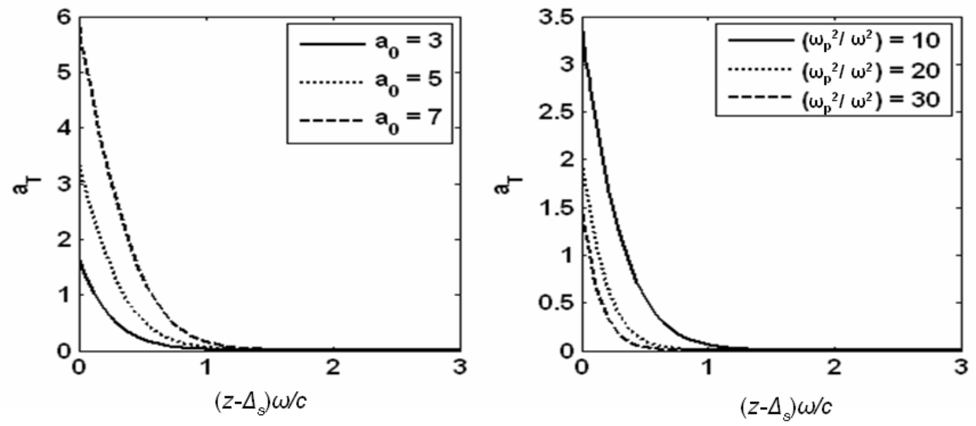
$$\frac{da_T}{dz} = -[\frac{4\omega_p^2}{c^2}[(1-a_T^2)^{1/2} - 1] - \frac{\omega^2}{c^2} a_T^2]^{1/2}, \tag{7}$$

with $z \to \infty$, $a_T \to 0$, $\frac{da_T}{dz} \to 0$.

From Eqs. (5) and (7), we have

$$a_T(0) = \frac{\omega}{\omega_p} a_0 (2 + \frac{\omega^2}{\omega_p^2} a_0^2)^{1/2}. \tag{8}$$

Solutions of Eqs. (7) and (8) for different cases of a_0 and ω_p^2/ω^2 are shown in Fig. 1. At higher initial amplitude, the transmitted amplitude is larger, however, its decay with distance is faster. On increasing the plasma density, the transmitted amplitude falls down.



(a) (b)

Figure 1: Distribution of normalized transmitted field amplitude vs z. (a) $a_0 = 3,5,7$ and $\omega_p^2/\omega^2 = 10$ (b) $\omega_p^2/\omega^2 = 10, 20, 30$ and $a_0 = 5$.

Now we can obtain the ponderomotive potential $\phi_P = -(mc^2/e)[(1+a_T^2)^{1/2} - 1]$ and ponderomotive force

$$F_P = e\frac{\partial \phi_P}{\partial z} = -\frac{mc^2 a_T}{(1+a_T^2)^{1/2}} \frac{\partial a_T}{\partial z}. \tag{9}$$

It is maximum at z = 0 and falls off over a scale length $\sim c/\omega_p$. If one integrates the ponderomotive force on all the electrons per unit x-y cross-section of the plasma and use Eq. (8), one obtains

$$F = \int_0^\infty F_P n_0 dz = n_0 mc^2 a_0^2 \frac{\omega^2}{\omega_p^2} = 2I_0/c, \tag{10}$$

where I_0 is the incident laser intensity. *F* exactly equals the momentum change per unit area of the incident laser, i.e., radiation pressure when reflectivity is 100%.

The ponderomotive force pushes the electrons forward leaving behind a positive ion space charge and piling up electrons at the laser front into a sheath of width $l_s \sim c/\omega_p$. As the laser front moves a distance Δ, the electron density in the sheath is $n_e \cong n_0(1+\Delta/l_s)$ and the space charge field at z = Δ is $\vec{E}_s = \hat{z}4\pi n_0 e\Delta$, $\vec{E}_s(\Delta)$ increases with Δ. At $\Delta = \Delta_s$ where space charge force at $z = \Delta_s$ balances the ponderomotive force $eE_s = F_P(\Delta_s)$ on electrons, the acceleration of the electron sheath nearly vanishes, Δ_s turns out to be

$$\begin{aligned}\Delta_s &= -\frac{c^2}{\omega_p^2}\frac{a_T(\Delta_s)}{(1+a_T^2(\Delta_s))^{1/2}}\left(\frac{\partial a_T}{\partial z}\right)_{\Delta_s} \\ &= \frac{c}{\omega_p}\frac{\omega}{\omega_p}\frac{a_T(\Delta_s)}{(1+a_T^2(\Delta_s))^{1/2}}(4a_0^2 - a_T^2(\Delta_s))^{1/2}.\end{aligned} \tag{11}$$

$a_T(\Delta_s)$ and $(\frac{\partial a_T}{\partial z})_{\Delta_s}$ are the same as $a_T(0)$ and $(\frac{\partial a_T}{\partial z})_0$ given by Eqs. (7) and (8) with ω_p replaced by $\omega_p(n_e/n_0)^{1/2} = \omega_p(1+\omega_p\Delta_s/c)^{1/2}$. Defining $\Delta_s^{'} = \omega_p\Delta_s/c$, Eq. (11) can be written as

$$\begin{aligned}\Delta_s^{'} &= \frac{a_0^2}{a_0^2+\omega_p^2(\Delta_s^{'}+1)/\omega^2}\left(2(\Delta_s^{'}+1)+\frac{\omega^2 a_0^2}{\omega_p^2}\right)^{1/2}\left[4-\frac{\omega^2}{\omega_p^2(\Delta_s^{'}+1)}\left(2+\frac{\omega^2 a_0^2}{\omega_p^2(\Delta_s^{'}+1)}\right)\right]^{1/2} \\ &\cong \frac{2a_0^2}{a_0^2+\omega_p^2(\Delta_s^{'}+1)/\omega^2}\left(2(\Delta_s^{'}+1)+\frac{\omega^2 a_0^2}{\omega_p^2}\right)^{1/2}\end{aligned} \tag{12}$$

If the thickness of the thin foil, D, is equal to Δ_s, the electron sheath is simply detached from the bulk ions and moves out in vacuum, trapping the ions within its width ~ skin depth. This double layer is accelerated by the laser ponderomotive force or radiation pressure as we shall demonstrate. We have plotted in Fig. 2 the variation of optimum foil thickness normalized to electron skin depth as a function of normalized laser amplitude for $\omega_p^2/\omega^2 = 10, 20, 30$. D scales almost linearly with a_0. For $a_0 = 5$, $\omega_p^2/\omega^2 = 10$, $D = 0.13\lambda_L$, which compares well with the value of $D = 0.2\lambda_L$ considered by Yan et al. in their simulation.

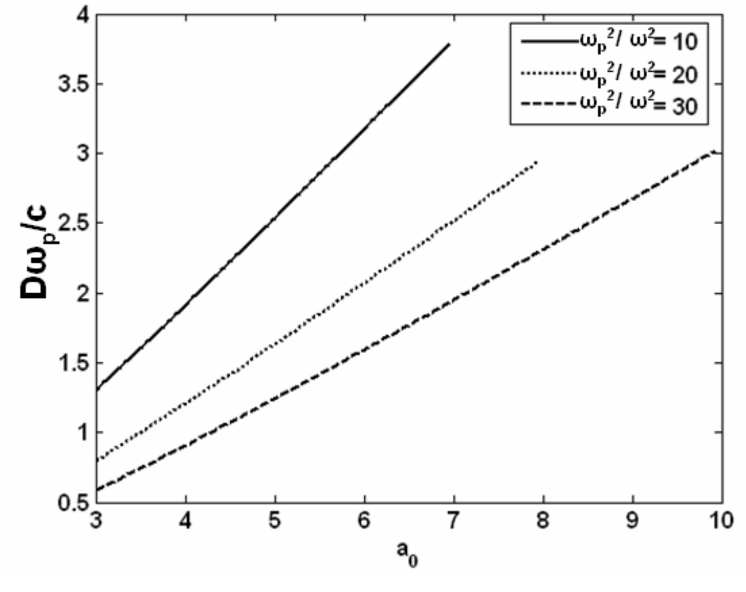


Figure 2: Variation of optimum foil thickness normalized to electron skin depth as a function of normalized incident laser amplitude for $\omega_p^2/\omega^2 = 10, 20, 30$. Solution is obtained with uniform electron density.

As the ion-electron double layer is accelerated by the radiation force the laser reflectivity $|R|^2$ becomes less than 1, and the radiation force on the double layer per unit area becomes $F = (I_0/c)(1+|R|^2)$. If the velocity of the double layer is V_f, the work done by the ponderomotive force per unit area per second is FV_f, hence the reflected power per unit area $I_0|R|^2 = I_0 - FV_f$. This gives

$$|R|^2 = \frac{1-V_f/c}{1+V_f/c}, F = \frac{2I_0/c}{1+V_f/c}.$$

Further, the intensity of incident radiation on the moving front is reduced by a factor $(1-V_f/c)$ due to the stretching of the pulse, hence

$$F = (2I_0/c)(1-V_f/c)/(1+V_f/c). \tag{13}$$

The mass per unit area of the double layer is $m_i n_0 l_s$ where m_i is the ion mass. Solving the equation of motion

$$\frac{d(\gamma_f V_f)}{dt} = \frac{2I_0}{m_i n_0 l_s c}\frac{1-V_f/c}{1+V_f/c}, \tag{14}$$

where $\gamma_f = (1-V_f^2/c^2)^{-1/2}$, one obtains

$$\left(\frac{1+V_f/c}{1-V_f/c}\right)^{3/2} + 3\left(\frac{1+V_f/c}{1-V_f/c}\right)^{1/2} = 6R\frac{t}{T_L} + 4, \tag{15}$$

where $R = 2I_0T_L/(m_i c^2 n_0 l_s) = 4\pi(m/m_i)a_0^2\omega/\omega_p$ and $T_L = 2\pi/\omega$ is the laser period. For $m_i/m = 1836$, $a_0 = 5$, $\omega_p^2/\omega^2 = 10$, $R \cong 0.05$. The characteristic time for ion acceleration is $\sim 30T_L$. We have plotted on Fig. 3 the variation of ion energy

$$\varepsilon_i = (\gamma_f - 1)m_i c^2$$

as a function of time for $a_0 = 5$, $\omega_p^2/\omega^2 = 10$.

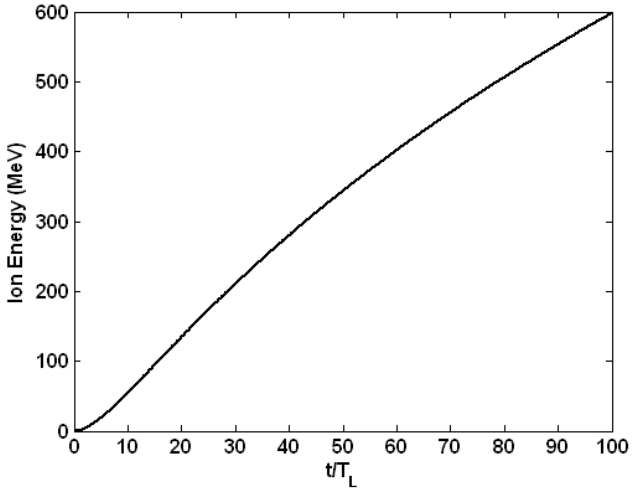


Figure 3: Variation of ion energy as a function of time for $a_0 = 5$, $\omega_p^2/\omega^2 = 10$.

The ion energy varies pretty linearly with time upto $t/T_L \sim 50$ and then varies bit slowly. The behavior is similar to the one observed in 1-D PIC simulations by Yan et al. for the same parameters. Typically in time ~ 90 fs at an intensity of 7.5□10^{19} w/cm^2 in a ten times critical density plasma one obtains ion energy gain ~ 200 MeV. With higher a_0 energy gain is faster. However, with increasing plasma density the energy gain decreases as the double layer becomes heavier.

The critical parameter for collective double layer ion acceleration is the thickness of the thin foil. If it is less than the optimal length $D < \Delta_s$, the space charge field is not enough to carry the ions with electron sheath. For $D > \Delta_s$, the space charge force stops the electron sheath, hence no double layer acceleration occurs. The optimum width of the foil $D = \Delta_s$. This width increases with the intensity of the laser and decreases with the density of the plasma.

In calculating $a_T(\Delta_s)$ we have assumed the electron density in the sheath, n_e, to be uniform. If one takes $eE_s = \partial/\partial z(e\phi_P)$ in the entire sheath region and use the Poisson's equation $\partial E_s/\partial z = 4\pi e(n_0 - n_e)$, one obtains

$$\frac{n_e}{n_0} = 1 + \frac{c^2}{\omega_p^2}\frac{\partial^2}{\partial z^2}(1 + a_T^2)^{1/2},$$

which on using Eq. (6) gives

$$\frac{n_e}{n_0} = 1 + a_T^2 + \frac{c^2}{\omega_p^2}\frac{a_T^2}{1 + a_T^2}\left(\frac{\partial a_T}{\partial z}\right)^2 - \frac{\omega^2}{\omega_p^2} a_T^2 (1 + a_T^2)^{1/2}. \tag{16}$$

We have solved Eq. (6) with this density variation and obtained Δ_s using Eq. (11). The variation of Δ_s for this non-uniform density sheath with a_0 is plotted in Fig. 4. The behavior is similar to Fig. 4. The new Δ_s is within a factor of 1.5 of the one obtained earlier.

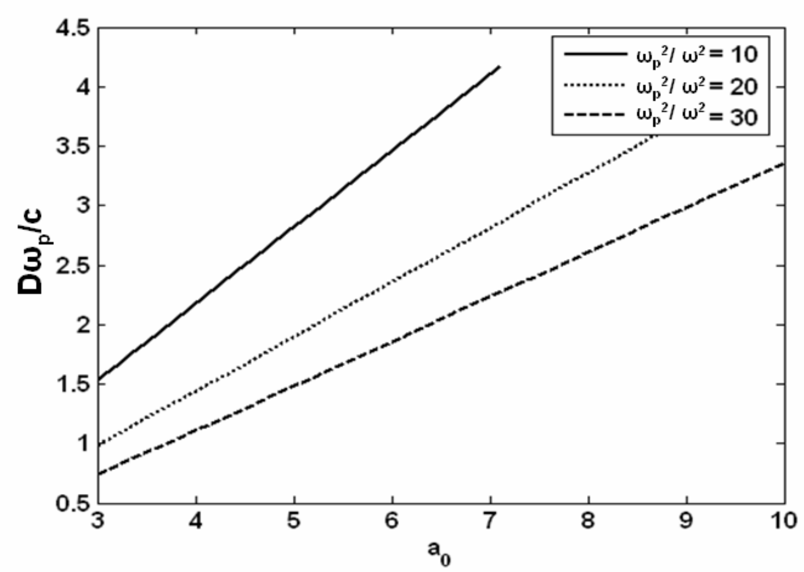
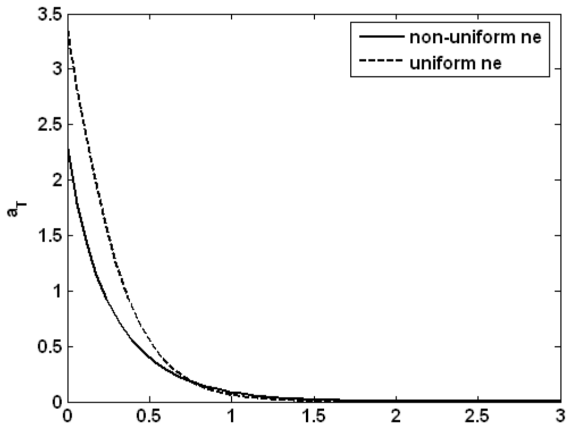


Figure 4: Variation of optimum foil thickness normalized to electron skin depth as a function of normalized incident laser amplitude for $\omega_p^2/\omega^2 = 10, 20, 30$. Solution is obtained with non-uniform electron density.

The bulk ions left behind the double layer, in the region $0 < z < \Delta_s$, could be accelerated by the space charge self field. An ion originated at position z_0 will experience a constant force $4\pi n_0 e^2 z_0$ until the 2D effects weaken it. In traveling upto $z = d$, it will gain energy $\varepsilon_i = 4\pi n_0 e^2 z_0 (d - z_0)$. Once d becomes comparable to the laser spot size r_0, the ion energy will saturate. The number of ions per unit cross-sectional area contained in the energy interval ε_i and $\varepsilon_i + d\varepsilon_i$ turn out to be $dN = f(\varepsilon_i)d\varepsilon_i$, with $f(\varepsilon_i) = n_0 d/(m\omega_p^2 d^2 - 2\varepsilon_i)$. However, 2-D effects could strongly modify it.

One more figure comparing the solution to Eq. (6) with uniform and non-uniform ne. (for your reference)



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