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Miniworkshop on Strong Correlations in Materials and Atom Traps

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Superfluid phases of "magnetized" fermionic atomic gases.

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Superfluid Phases of “Magnetized” fermionic atomic gases

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DES & L. Radzihovsky, Phys. Rev. Lett. **96**, 060401 (2006);
Phys. Rev. B **75**, 136501 (2007); Annals of Physics **322**, 1790 (2007)

Recent preprint: arXiv:0807.0922

Open position for a postdoc!

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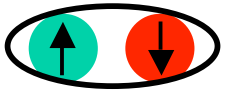
Funding: NSF; LA Board of Regents

Overview

- Bose-Einstein condensation (BEC) of dilute vapors of alkali atoms
  - All bosons in same quantum state
  - Superfluidity
  - Condensed matter physics in an atomic physics setting

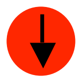
Anderson et al Science 95
Davis et al PRL 95

- Recent interest: condensation of two types of atomic fermion
  - Fermionic superfluid 

[e.g., Regal et al PRL 2004; Zwierlein et al PRL 2004,…]

- Relies on strong attraction between fermions: Feshbach resonance
  - Novel experimental knob: Tune interaction strength
  - Crossover from BEC to BCS superfluidity

Bardeen, Cooper, Schrieffer 1957

- Recent Work: Apply spin "magnetization" to fermion superfluid
  - Usual case: Equal numbers of  and 
  - Magnetized: More  than 

    also called "polarized" or "imbalanced"
  - Exotic phases, phase separation, tricritical points
  - Stability of superfluidity in strongly-correlated systems

Next: What is condensed matter?

Condensed matter physics

- Collective behavior of large numbers of particles

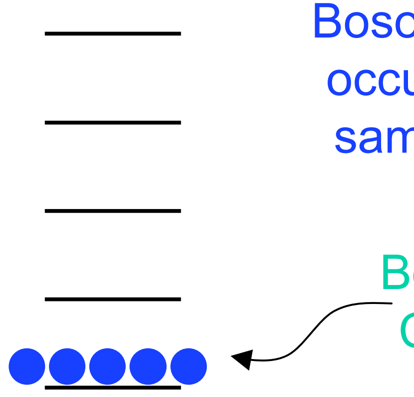
Pauli principle:

Fermions antisymmetric under exchange

$$\Psi_F(\mathbf{r}_1,\mathbf{r}_2) = -\Psi_F(\mathbf{r}_2,\mathbf{r}_1)$$

Bosons symmetric under exchange

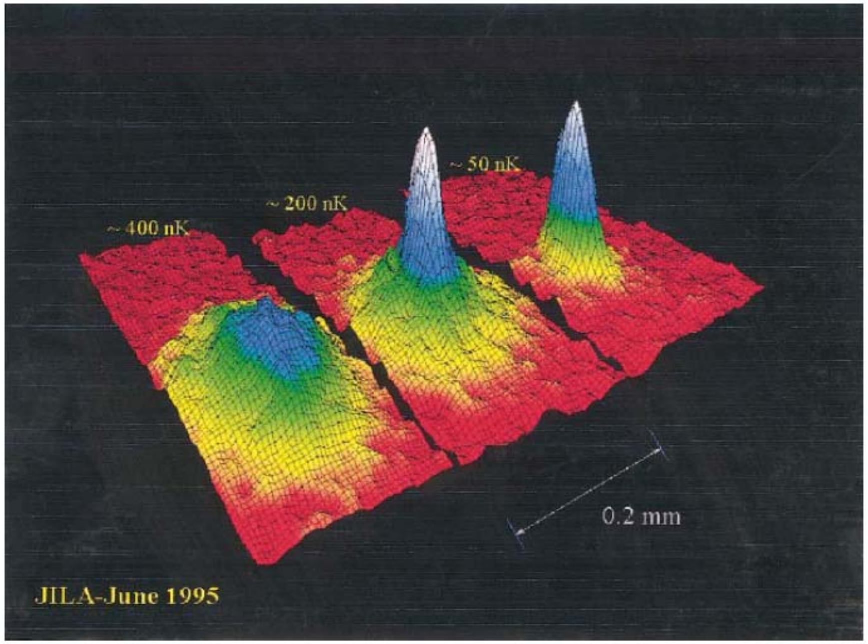
$$\Psi_B(\mathbf{r}_1,\mathbf{r}_2) = \Psi_B(\mathbf{r}_2,\mathbf{r}_1)$$



Bosons: Can occupy the same level

Bose-Einstein Condensate

BEC:



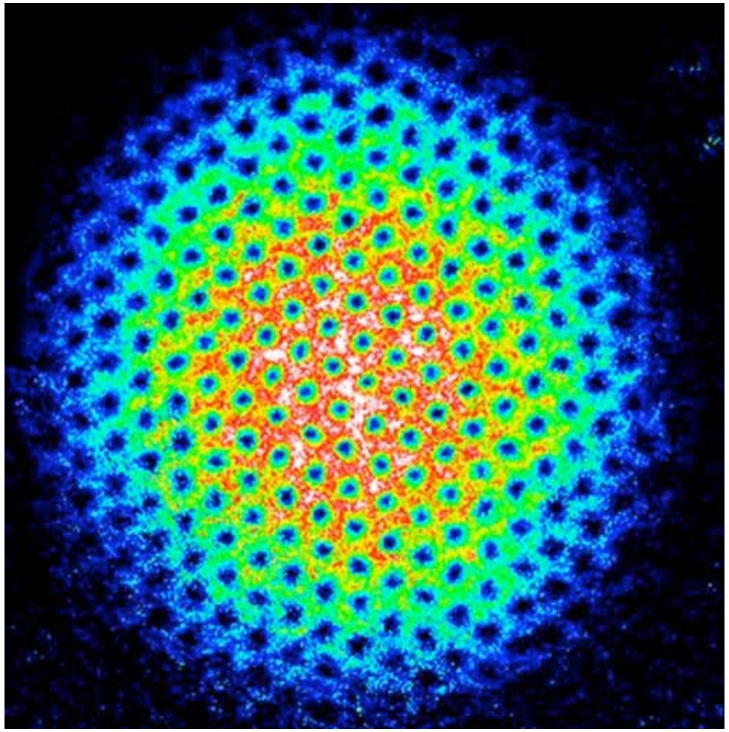
Anderson et al Science 95

Next: Properties of BEC's

BEC: Superfluidity

- Superfluid: No viscosity, irrotational flow

Rotating BEC nucleates vortices!



http://jilawww.colorado.edu/bec

- Density of Quantized Vortices: $n = \frac{m\Omega}{\pi\hbar}$

rotation rate

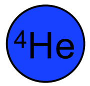
Onsager, Feynman, Tkachenko

Next: What about fermions?

How can fermions condense?

- Composite particles made of even number of fermions → Boson!

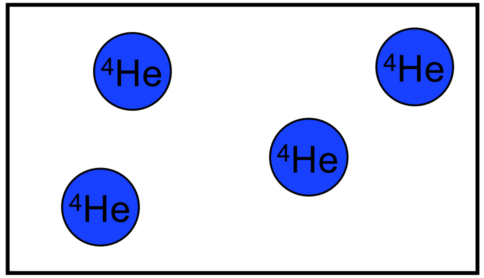
Fermion w.f.: $\Psi_F(\mathbf{r}_1, \mathbf{r}_2) = -\Psi_F(\mathbf{r}_2, \mathbf{r}_1)$

- Simple example: $2p + 2n + 2e$ → 

proton neutron electron

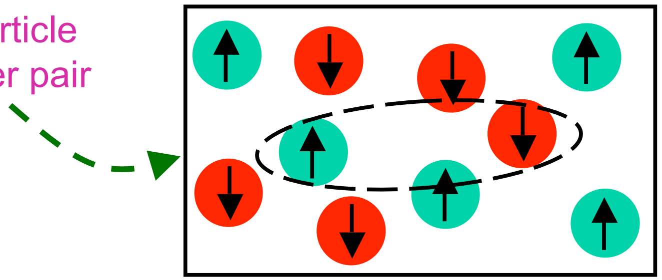
Helium

Superfluid helium



- Superconductivity: Cooper pairing of two fermion species

Size of “molecule” bigger than particle spacing: Cooper pair



Next: BCS

Superconductivity

- Electrons w/ attractive interactions: Instability!

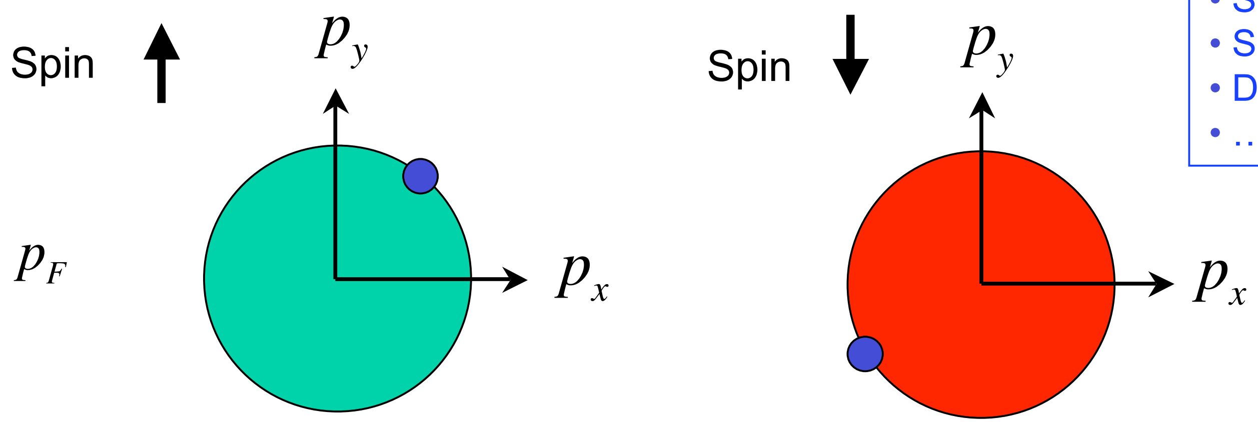
$(\sigma = \uparrow, \downarrow)$ $\quad g < 0$ BCS theory: due to phonon exchange

$$H = \sum_{p,\sigma} (\varepsilon_p - \mu) c^{\dagger}_{p\sigma} c_{p\sigma} + g \sum_{p,q,k} c^{\dagger}_{k\uparrow} c^{\dagger}_{p\downarrow} c_{k+q\downarrow} c_{p-q\uparrow}$$

- BCS ground state wavefunction: Pair opposite-p states

- Superconductivity
- Spectral Gap
- Diamagnetism
- …

Fermi momentum p_F



- Pair amplitude: $\Delta = \langle c_{\mathbf{p}\uparrow} c_{\mathbf{-p}\downarrow} \rangle$ $\dashrightarrow$ $\Delta \propto \exp[-\frac{2\pi^2}{|g| p_F}]$ Nonperturbative

Next: Cold atoms

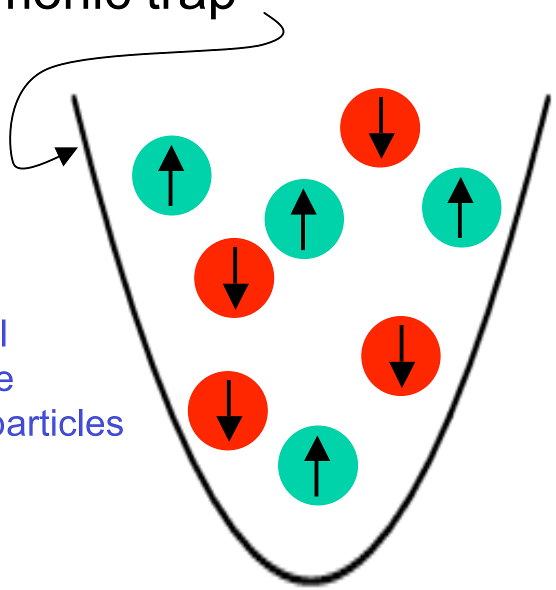
Fermionic pairing of cold atoms

Regal et al PRL 04; Zwierlein et al ibid; Kinast et al ibid, Bartenstein et al ibid, Bourdel et al ibid; Partridge et al ibid 05…

- Fermionic superfluidity: atomic fermions ^{40}K, ^{6}Li

Ultracold: ~10-100 nK   Dilute: ~ 10^{10}-10^{13} cm^{-3}

- Harmonic trap

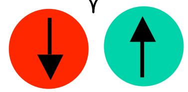


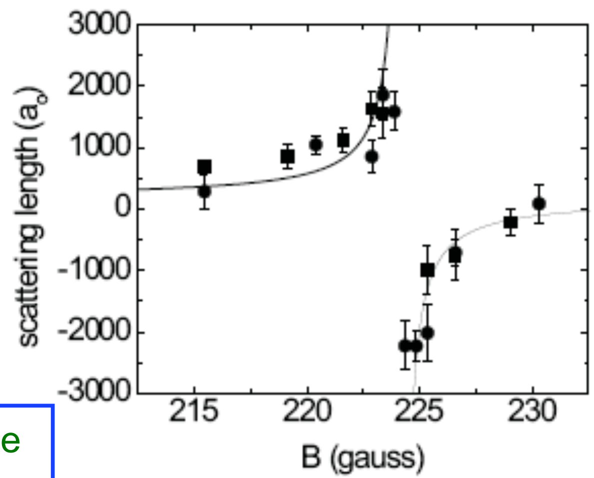
n.b.: actual exp'ts have ~10^5 -10^7 particles

– "Spin" -- different atomic hyperfine states

(e.g. ^{40}K: f=9/2, m_f = 7/2,9/2)

total atom spin





Regal & Jin PRL 90, 230404 (2003)

- Feshbach resonance: magnetic field B

s-wave scattering length $$a_s \propto -\frac{1}{B-B_0}$$

Formation of two-particle bound state

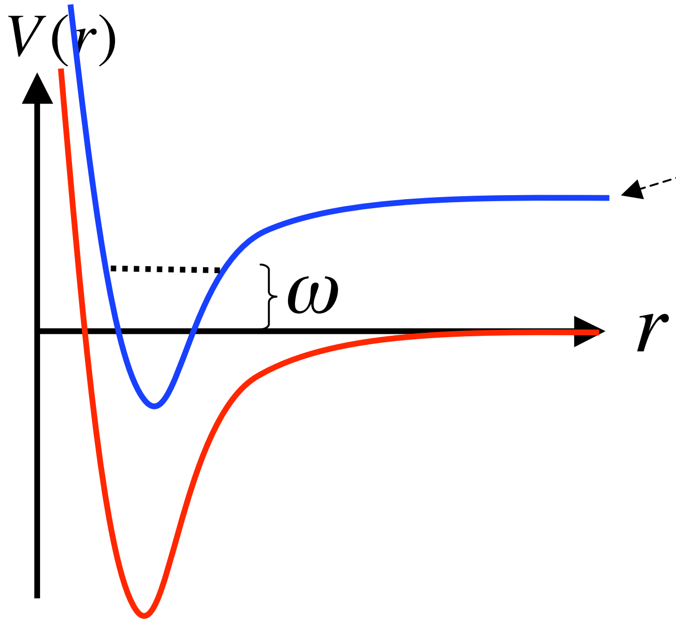
B_0 "resonance position"

Next: Feshbach resonance

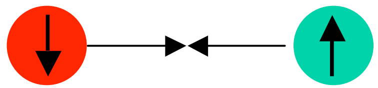
What is a Feshbach resonance?

e.g., Timmermans et al
Phys. Rep. 99

- Atomic interactions: depend on electron spin state



- – Incident atoms: electronic triplet "Open channel"
- – Bound state in singlet potential "Closed channel"
- – Singlet & triplet channels coupled via hyperfine



- External B field: Tunes relative energy of singlet & triplet
  - – Zeeman interaction
  - – Detuning $\omega \propto B - B_0$ ← Resonance position
- Resonance scattering: divergence of scattering length at $\omega \to 0$

Experimentally tunable scattering length

$$a_s \propto -\frac{1}{B - B_0}$$

Next: Model

One channel model of Feshbach Resonance

- One-channel model: (Same as before!!)

$$H = \sum_{p,\sigma} (\varepsilon_p - \mu) c^\dagger_{p\sigma} c_{p\sigma} + g \sum_{p,q,k} c^\dagger_{k\uparrow} c^\dagger_{p\downarrow} c_{k+q\downarrow} c_{p-q\uparrow}$$

two species of fermion $(\sigma = \uparrow, \downarrow)$

Attractive interactions $g < 0$

- Correct two-body physics

  – Vacuum scattering length: $a_s \propto \dfrac{1}{|g| - 2\pi^2 / m\Lambda}$ (Λ UV cutoff)

Holland et al PRL 01
Ohashi & Griffin PRL 02
Andreev et al PRL 04

Scattering length diverges → Bound state in two-particle potential

- Many-body physics: BCS-BEC crossover

Small $|g|$ ↔ BCS; Large $|g|$ ↔ BEC

Next: Mean-field theory

BEC-BCS crossover: Mean-field theory

Leggett 1980
Nozieres & Schmitt Rink 1985
Sa De Melo et al 1993

- Theory: smooth crossover between BEC and BCS limits
  - Mean-field theory: $\Delta = \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$

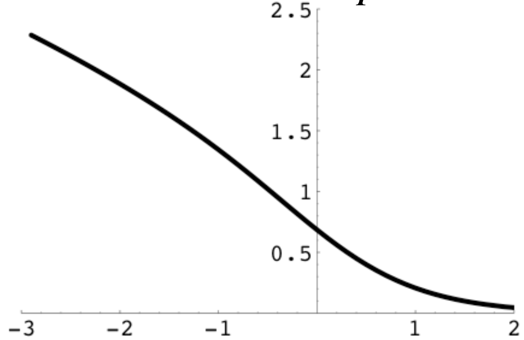
BEC ←

→ BCS: positive detuning

- $\Delta > \varepsilon_F, \mu < 0$

Strong pairing — No F.S.

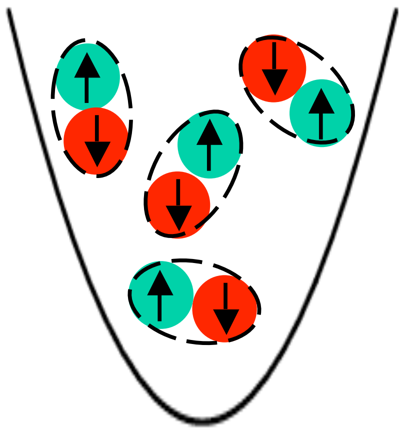
Pairing Δ / ε_F



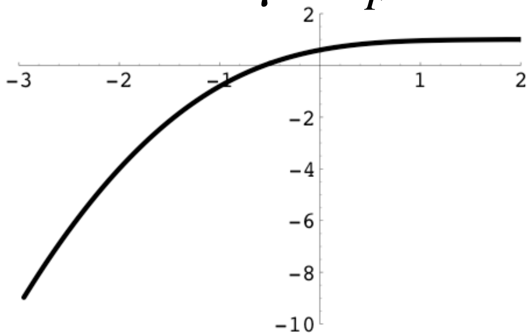
$-\dfrac{1}{k_F a_s} \propto B - B_0$

- $\Delta << \varepsilon_F, \mu > 0$

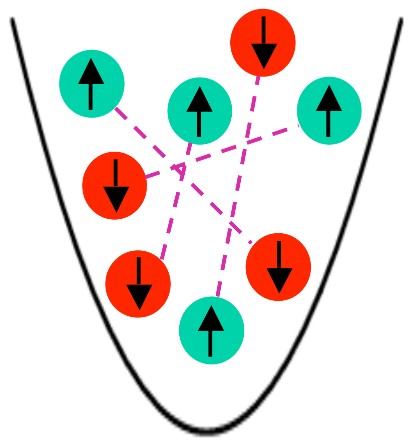
Weak pairing — Fermi surface



Chemical potential μ / ε_F

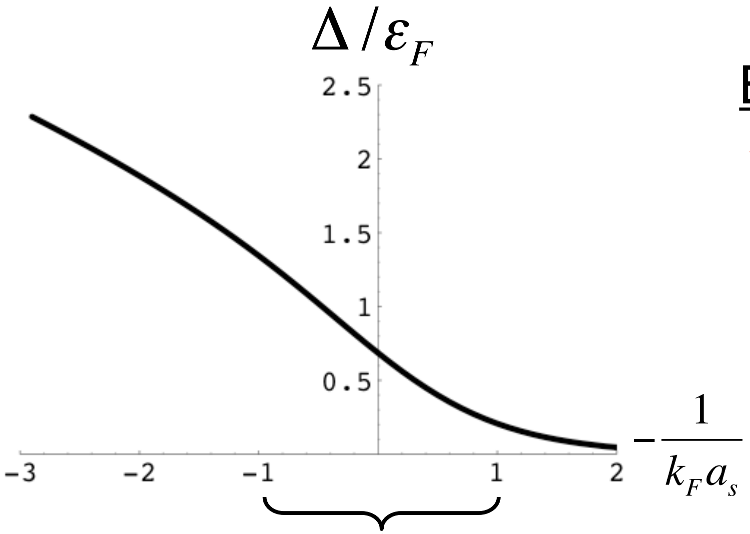


$-\dfrac{1}{k_F a_s}$



Next: Validity

Validity of BEC-BCS mean-field theory



BEC regime

$a_s > 0, |a_s| << k_F^{-1}$

Gas of repulsive bosons

BCS wavef'n: $a_m = 2a_s$

Sa de Melo PRL 93

Exact analysis: $a_m = 0.6a_s$

Petrov et al PRL 04

BCS regime

Valid at large positive detuning (Weak-coupling BCS)

$a_s < 0, \ |a_s| << k_F^{-1}$

- Unitary regime: $|a_s| >> k_F^{-1}$ Universal: Only energy scale is $\varepsilon_F = \frac{k_F^2}{2m}$
  - Experimental regime: Strongly correlated
  - Introduce artificial small parameter or Monte Carlo

Carlson & Reddy PRL 05
Lobo et al PRL 06
Recati et al PRL 08

Narrow resonance - Andreev et al PRL 2004 (weakly-coupled FR)

Epsilon expansion - Nishida & Son PRL 2006

Large N limit - M.Y. Veillette, D.S., L. Radzihovsky 2007

also Nikolic & Sachdev 2007

Next: Experiments

Experiments: Smooth crossover & superfluidity

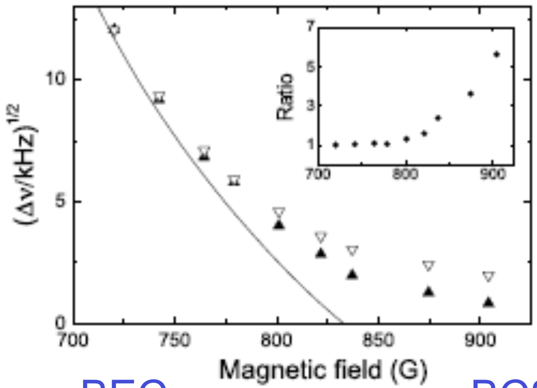
- Measure condensation directly: Occupation of lowest state

Regal et al PRL 2004
Zwierlein et al PRL 2004

- Measure binding energy of pairs/molecules

Increases with reduced detuning

Chin et al Science 2004
Partridge et al PRL 2005



BEC BCS

- Collective oscillations: consistent with superfluidity

Indirect measure of superfluidity

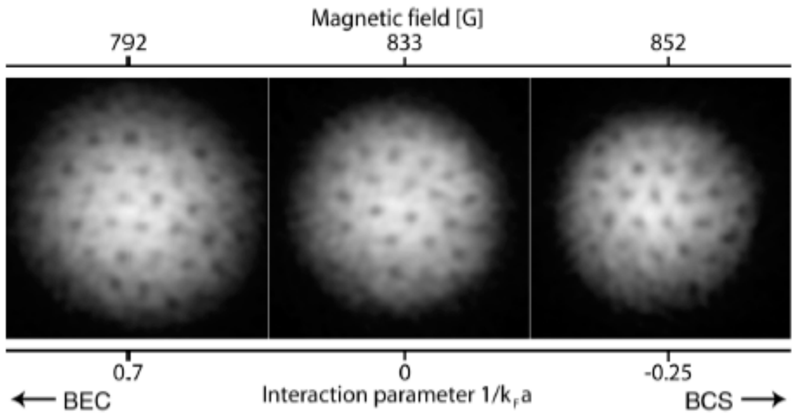
Kinast et al PRL 2004

- Rotation of cloud: Vortices across resonance

Zwierlein et al Nature 2005

Direct measure of superfluidity

Vortices in a Bose-Einstein condensate



Vortices in a neutral BCS superconductor

Next: Spin Polarization

Applied spin polarization

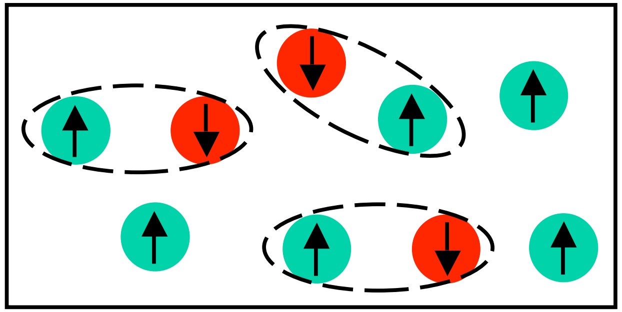
- Recent work*: Explore changing relative number of ↓ , ↑

Hard to do in condensed-matter settings!!

  - Experimental "knob"

Polarization: $P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

Aim: Extend phase diagram to polarized case $P \neq 0$



  - Zeeman magnetic field h: favors more ↑ than ↓
    - Superconductors: Clogston limit
    - FFLO state: $\Delta(\mathbf{r}) \propto \cos[\mathbf{Q}\cdot\mathbf{r}]$
    - Crystalline color superconductivity; "Breached pair" state; Deformed Fermi surface

Alford et al PRD 01 Liu & Wilczek PRL 03 Sedrakian PRA 05

  - Smooth crossover fractured: Phase transitions, polarized superfluidity, polarized Fermi liquid, phase separation, …

Theory: DS & L. Radzihovsky, PRL 06, Ann. Phys. 07, PRB 07, Bedaque et al PRL 03, Carlson & Reddy PRL 05, Cohen PRL 05 Pao et al PRB 06, Son & Stephanov PRA 06, Chien et al PRL 06, Gubbels et al PRL 06, Chevy PRL 06, Lobo et al PRL 07, Parish et al Nat. Phys. 07, Pilati PRL 08, ….

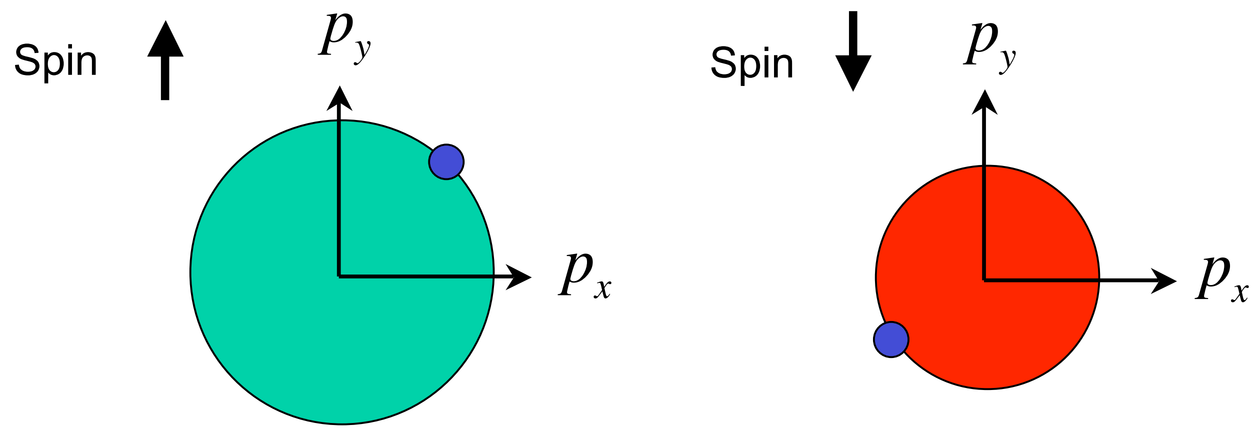
Exp't: Zwierlein et al Science 06, Partridge et al Science 06, Shin et al PRL 06, Partridge et al PRL 07, Schunck et al Science 07, Shin et al Nature 08, …

Next: FFLO

FFLO state

Fulde & Ferrell PR 1964;
Larkin & Ovchinnikov JETP 1965

- Excess spin ↑ : $p_{F\uparrow} > p_{F\uparrow}$



- Pairing near Fermi surface: $Q \equiv p_{F\uparrow} - p_{F\downarrow}$

Finite-momentum pairing! $\Delta(\mathbf{r}) \propto \exp[i\mathbf{Q}\cdot\mathbf{r}]$

$\Delta(\mathbf{r}) \propto \cos[\mathbf{Q}\cdot\mathbf{r}]$

LO state slightly more stable

- Evaded observation
  - Disorder
  - Physical magnetic field to orbital electron motion
  - Possibly observed in $CeCoIn_5$

Radovan et al Nature 2003
Bianchi et al PRL 2003

- Motivation: Observe FFLO in cold-atom experiment?
  - Perfectly clean; Purely Zeeman coupling; no lattice
  - Spontaneous crystalline order observable in time-of-flight exp'ts

Next: Pol. SF

Model of a polarized superfluid

$$H = \sum_{p,\sigma} (\varepsilon_p - \mu_\sigma) c^\dagger_{p\sigma} c_{p\sigma} + g \sum_{p,q,k} c^\dagger_{k\uparrow} c^\dagger_{p\downarrow} c_{k+q\downarrow} c_{p-q\uparrow}$$

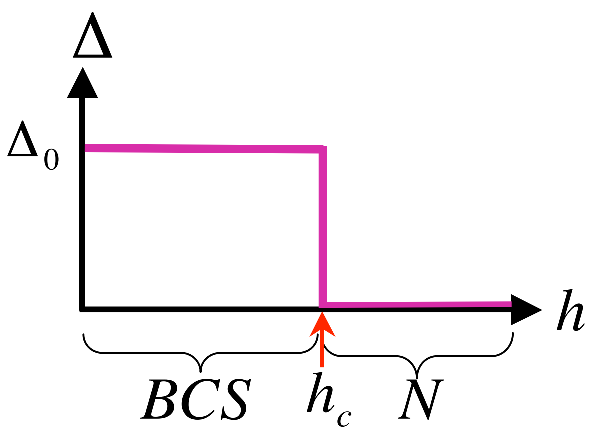
Let's start at T=0 in BCS regime

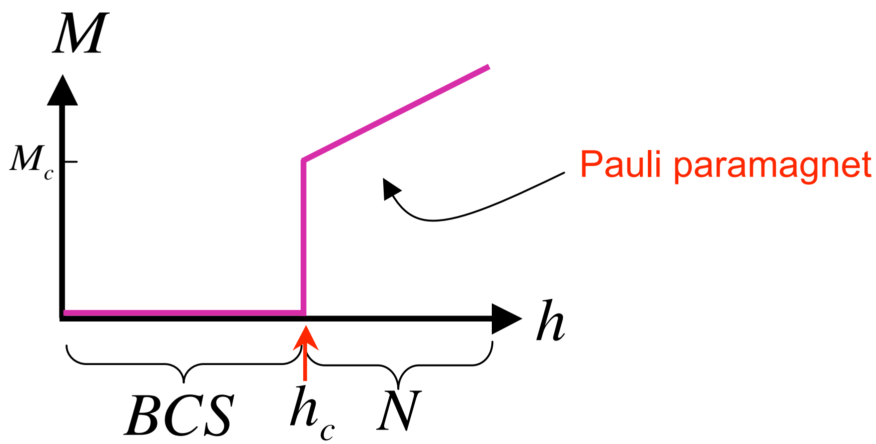
– Chemical pot. diff.: $h = \mu_\uparrow - \mu_\downarrow$

- BCS under Zeeman field h: 1st-order transition to unpaired $h = h_c \cong \frac{1}{\sqrt{2}} \Delta_0$

  Clogston PRL 1962; Sarma J. Phys. Chem. Sol. 1963

- Fixed μ, h: Pairing Δ, density n, magnetization M jump at transition





- Experiment: Fixed n and M
  - Polarization $P<P_c$: cannot be attained
  - Phase separation to achieve imposed polarization

BCS	Normal
$n_\uparrow = n_\downarrow$	$n_\uparrow > n_\downarrow$

Next: Phase Diagram

BCS regime ground-state phase diagram

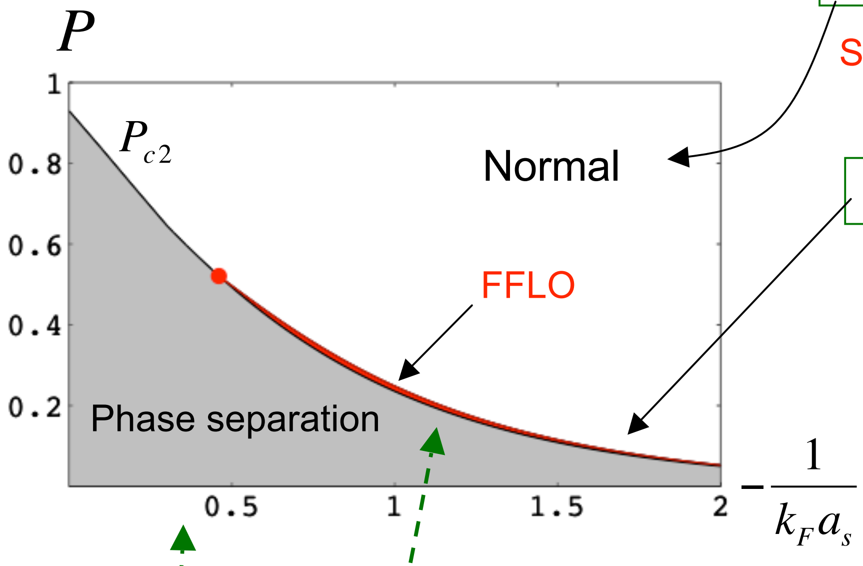
- Phase separation at fixed particle number

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

Correlated normal phase

Schunck et al Science 2007

Thin window of FFLO

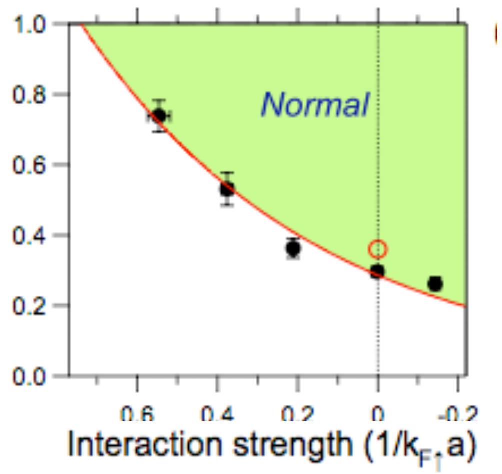


- Paired BCS phase: $P = 0$
- Cannot magnetize a BCS state

Sarma 1963

- First-order phase boundary: $P_{c2} \propto \Delta_0$

- Recent Data: Shin arXiv:0805:0623



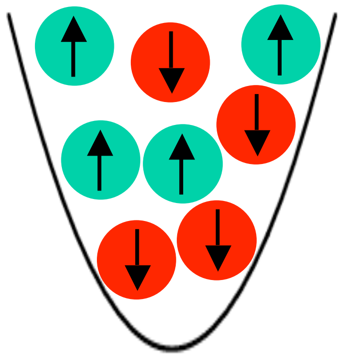
Theory qualitatively correct, quant. incorrect!

Next: Phase Separation

Phase separation in a polarized Fermi gas

- Pairing of minority spin, excess majority phase-separates

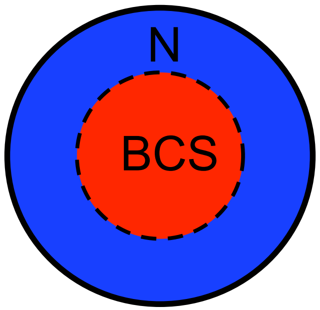
BCS	Normal
$n_\uparrow = n_\downarrow$	$n_\uparrow > n_\downarrow$



- Harmonic trap: higher density phase (superfluid) falls to the center

Inner core: { BCS, $n_\uparrow = n_\downarrow$ }

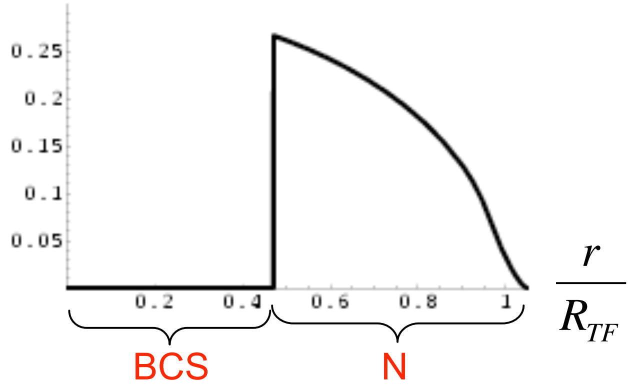
Outer shell: { Normal, $n_\uparrow > n_\downarrow$ }



Calculation: Local density approx.

"Magnetization"

$$M(r) = n_\uparrow(r) - n_\downarrow(r)$$



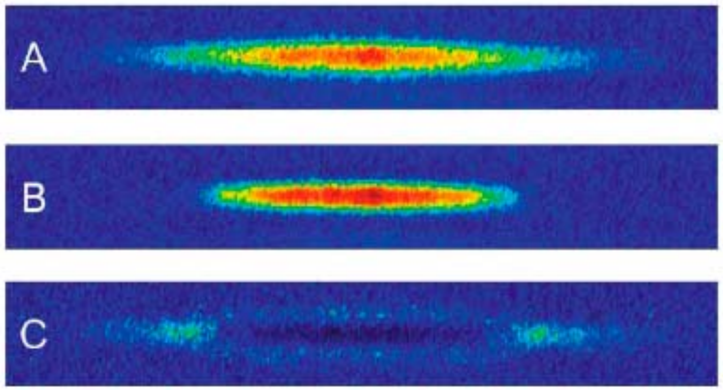
Shells: Imposed polarization goes to the edge, center paired!

Next: Experiments

Evidence for shell structure in phase separation regime

- Partridge et al Science 2006: Density data
  - Integrated in one direction
  - Highly prolate trapping potential

$$N_\uparrow = 8.6\times 10^4$$
$$N_\downarrow = 6.5\times 10^4$$



$n_\uparrow(r)$

$n_\downarrow(r)$

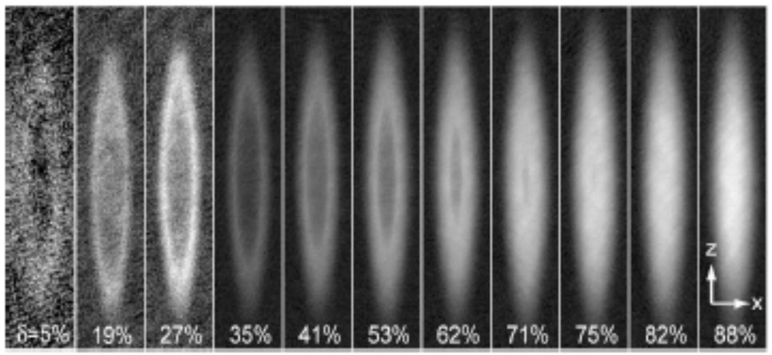
$M(r)$

Excess polarization pushed to boundary!

- Shin et al PRL 2006: Shrinking BCS core with increasing polarization

Plots: Integrated Magnetization

$$M(r) = n_\uparrow(r) - n_\downarrow(r)$$



- Quantitative understanding: Go beyond local density approximation to handle trap

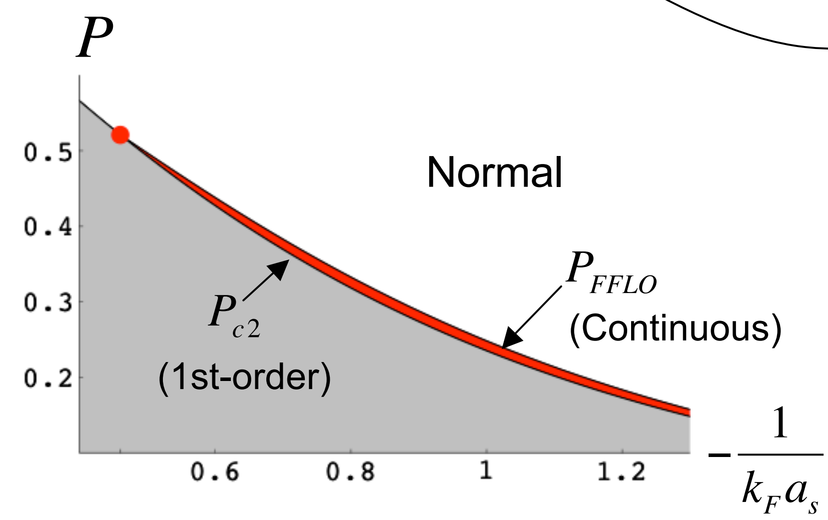
Kinnunen et al, Yi & Duan, Chevy, De Silva & Mueller, Imambekov et al, …

Next: FFLO

Predictions for FFLO regime

- Simplest FFLO-type state: $\Delta(\mathbf{r}) = \Delta_{\mathbf{Q}} \exp[i\mathbf{Q}\cdot\mathbf{r}]$
  - More generally: $\Delta(\mathbf{r}) = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} \exp[i\mathbf{Q}\cdot\mathbf{r}]$ Bowers & Rajagopal PRD 02

Actual FFLO window wider?



Weak-coupling BCS

- Critical polarization $P_{FFLO} \approx \frac{3}{2}\eta\frac{\Delta}{\varepsilon_F}$

weakly detuning-dependent

- FFLO wavevector: $Q \cong 2\eta\lambda\frac{\Delta}{\hbar v_F}$

$P_{FFLO} \cong .05$

$Q^{-1} \cong 5\mu m$

Chin et al Science 2004

- Phase separation: SF-FFLO coexistence underneath P_{FFLO}

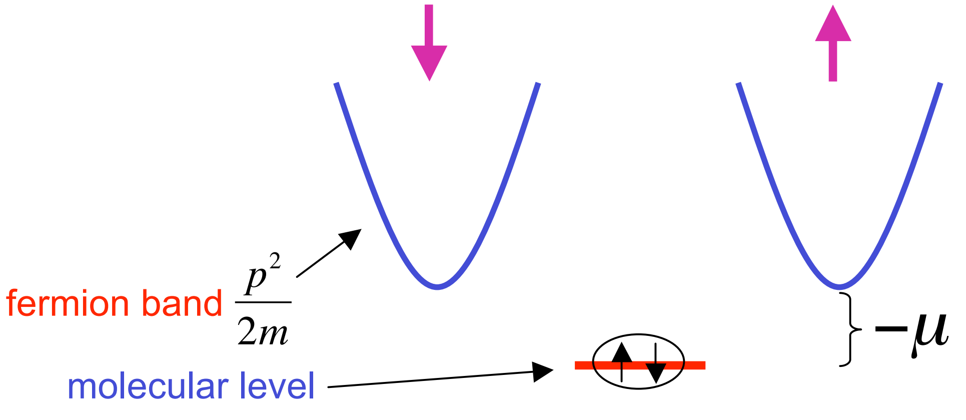
still observable in time of flight

Next: BEC regime

Polarize a BEC superfluid?

Imagine applying chemical potential difference h

- $h = 0$: BEC superfluid
  - Paired molecular bosons; Fermions gapped: $\mu < 0$



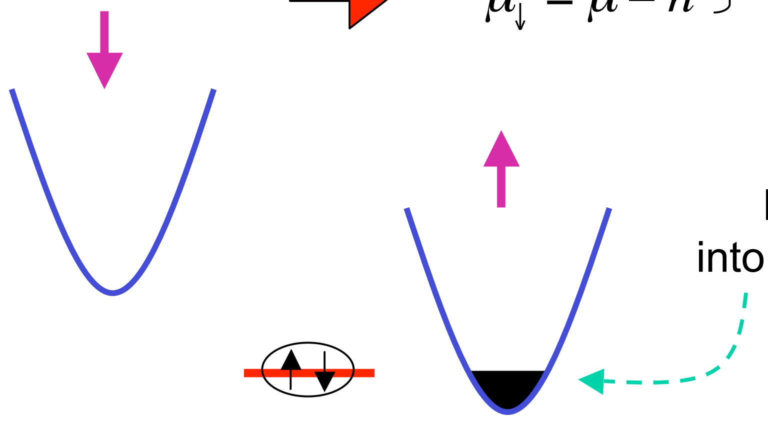
$$\mu = -\frac{\hbar^2}{2ma_s^2}$$

Mol. binding energy

- Apply h to induce polarization P ⇒

$$\left.\begin{aligned} \mu_\uparrow &= \mu + h \\ \mu_\downarrow &= \mu - h \end{aligned}\right\}$$

Tilt Fermion bands!



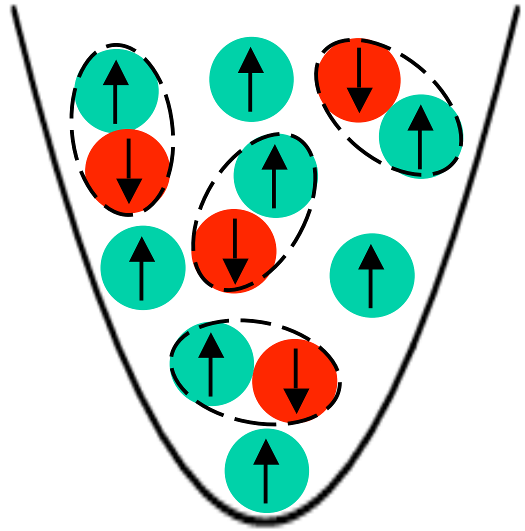
Pairs break up into spin-↑ fermions

- BEC & single-spin Fermi gas: Magnetic superfluid (SF_M)

Next: Even simpler cartoon picture

BEC limit: "Magnetic" Superfluid

- Negative detuning: BEC tolerates small polarization!
  - – Unlike BCS regime
  - – Minority spins pair; excess majority form Fermi sea



- Spin-up fermion & BEC are miscible fluids
  - – Analogous to ^3He-^4He mixtures!

fermion → boson

- First-order transition to phase sep. with increasing P
  - – Stability: $a_m > 0$

Mol. scattering length

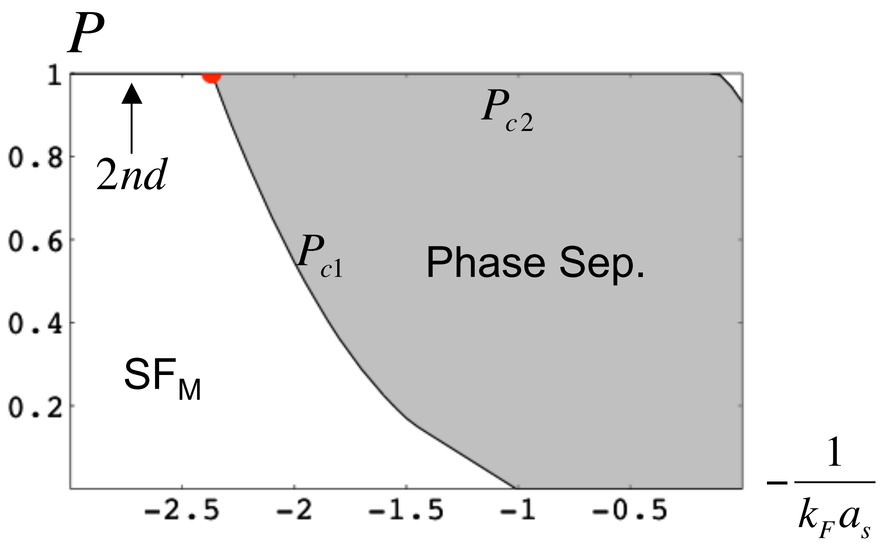
  - – a_m vanishes along P_{c1}

Instability to phase sep.

Bogoliubov sound velocity vanishes

  - – tricritical point $P_{c1} \to 1$

Continuous transition



Next: Global phase diagram

T=0 phase diagram

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

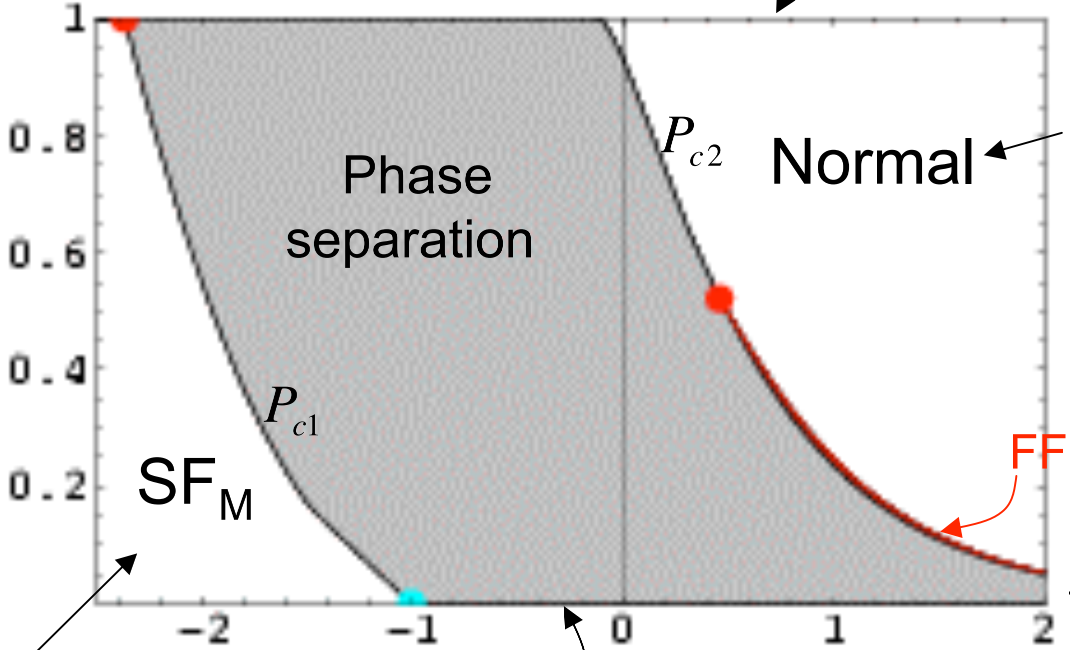
P_{c1}, P_{c2} 1st order

$FFLO - N$ 2nd

Top axis: $N_\downarrow = 0$

(Pure Normal)

Polarized Fermi gas



$-\frac{1}{k_F a_s} \propto B - B_0$

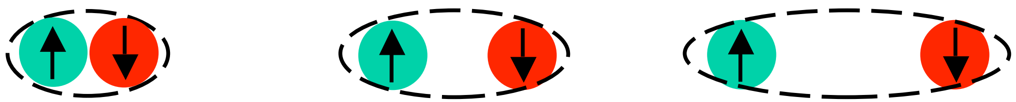
(detuning)

"Magnetic superfluid": Tightly-bound molecules and spin-polarized Fermi sea

BEC limit

BCS limit

Bottom axis: Smooth BEC-BCS crossover

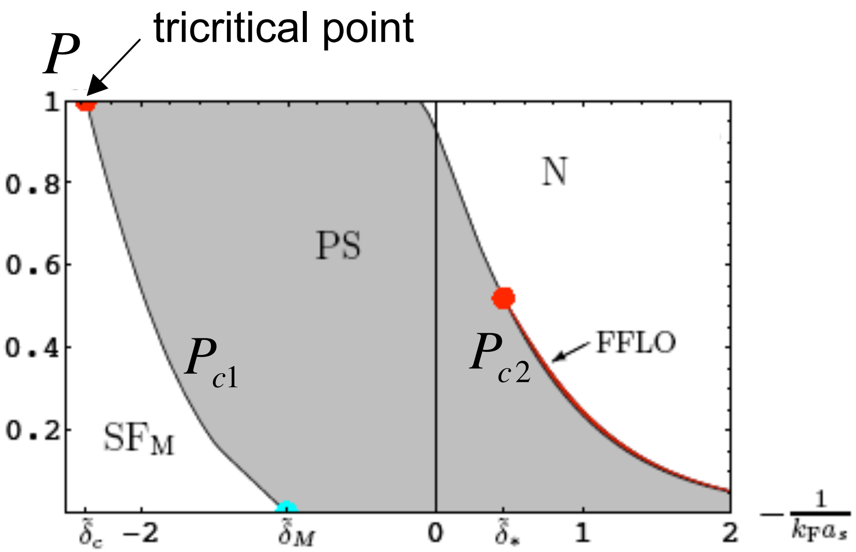


See also: Gu et al cond-mat 06, Parish et al Nat. Phys. 07

Next: Nonzero T

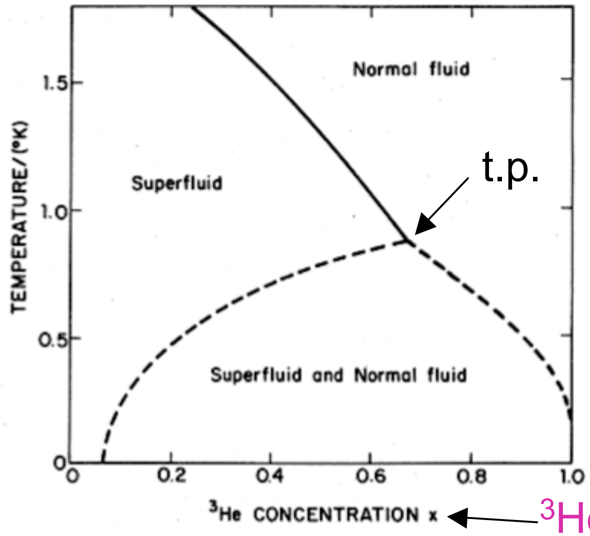
Tricritical point at nonzero T

Extension to finite T: (Parish et al, Chien et al)



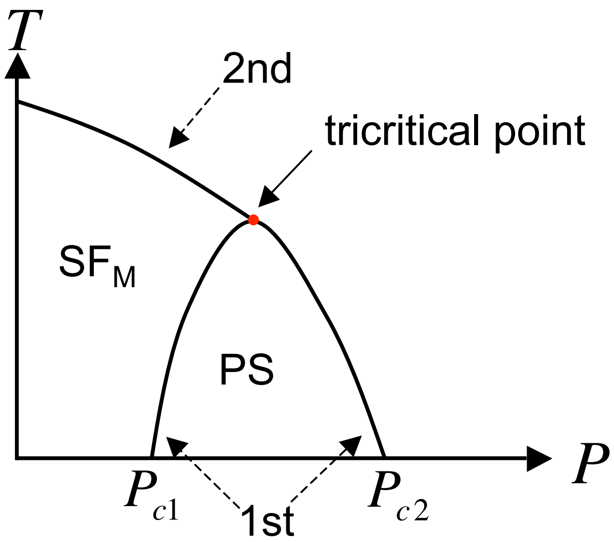
^3He-^4He mixtures

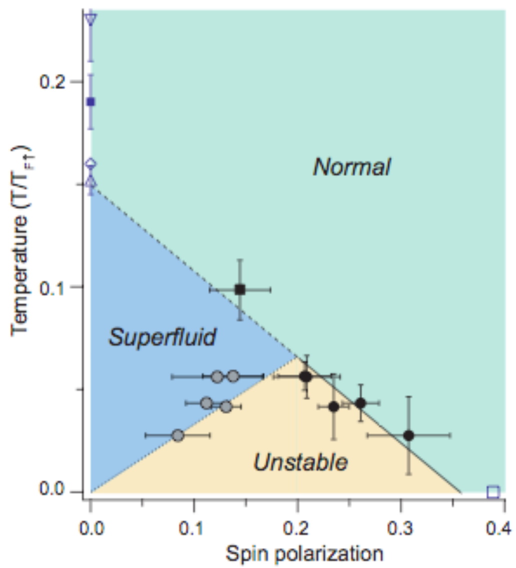
Graf, Lee, Reppy 67, et al.



^3He fraction

By analogy:





Shin et al
Nature 2008

Next: Tricritical point

What is a tricritical point? Griffiths PRL 1970

At t.p.:

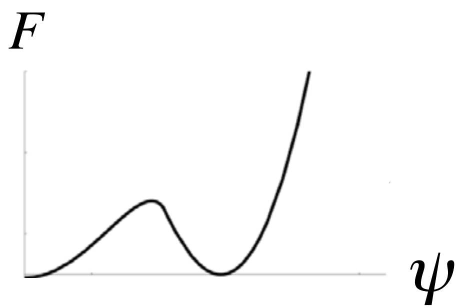
- Superfluid transition changes from 1st to 2nd order
- Molecular scattering length vanishes

ψ: Superfluid order parameter

General Ginzburg Landau theory

$$F = \int d^3x \left[\frac{\hbar^2 |\nabla \psi|^2}{2m_b} + \frac{1}{2} r |\psi|^2 + \frac{1}{4} u |\psi|^4 + \frac{1}{6} v |\psi|^6 \right]$$

$u > 0$: Continuous SF transition at $r \to 0$



$u < 0$: First-order transition

Tricritical point: $u = 0$

- Different scaling exponents $\alpha = \frac{1}{2}, \beta = \frac{1}{4}, \delta = 5$
- Logarithmic corrections to scaling*
  - –Instead of anomalous power laws!
  - –Marginal coupling v

Next: Predictions near T.P.

Wegner & Riedel PRB 1972; Stephen, Abrahams, & Straley PRB 1975

Behavior near the tricritical point

- Predictions for phenomena near tricritical point? Use RG to find log corrections

renormalization group

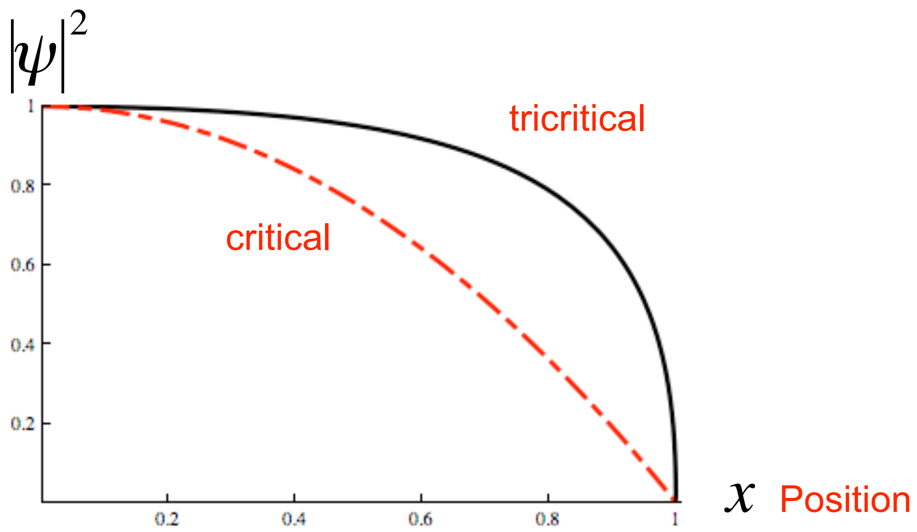
arXiv:0807.0922

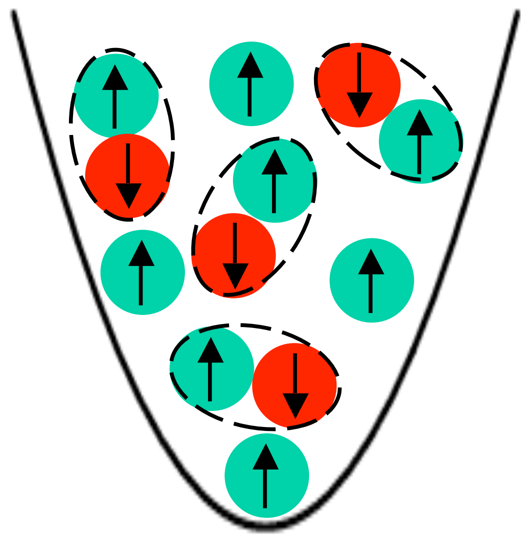
$$F = \int d^3x \left[\frac{\hbar^2 |\nabla \psi|^2}{2m_b} + \frac{1}{2} r|\psi|^2 + \frac{1}{4} u|\psi|^4 + \frac{1}{6} v|\psi|^6 \right]$$

- Onset of superfluidity: $|\psi|^2 \propto \sqrt{r \ln|r|}$

$\beta = \frac{1}{4}$

Shape of the cloud in a trap within LDA





Next: More

Behavior near the tricritical point, 2

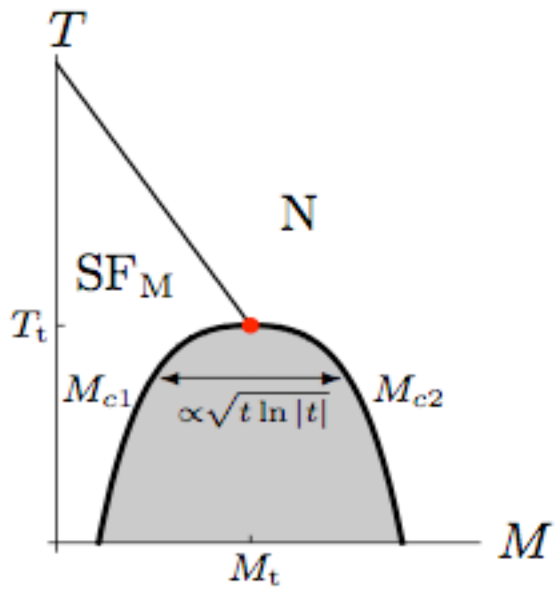
- Heat Capacity: $C \propto |t|^{-1/2} \ln^{1/2} |t|^{-1}$

Reduced temp $t = \frac{T - T_T}{T_T}$ $\alpha = \frac{1}{2}$,

Heat capacity measurable in cold atom exp'ts Kinast et al Science 2005

- Magnetization jump: $\delta M \propto \sqrt{t \ln |t|}$

Unique shape to coexistence curve



Next: Concluding remarks

Concluding remarks

- Cold-atom experiments studying superconductivity of paired fermions

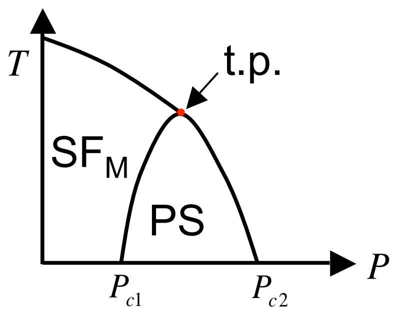
New context for strongly-correlated condensed-matter physics

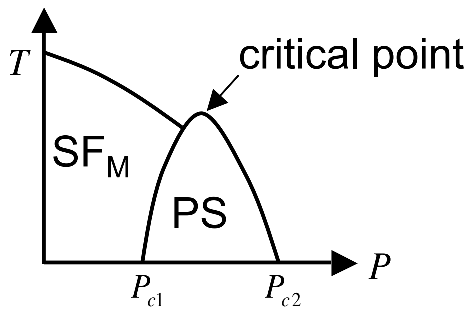
- Experiments already observed crossover between BEC and BCS states

- Different numbers of  ,  : Simple crossover “fractured”

  – Phase transitions
  – Phase separation (observed)
  – Magnetic superfluidity (observed, but not at $T \to 0$)
  – Fulde-Ferrell-Larkin-Ovchinnikov states (not observed)

- Future work:
  – Finite-T phase diagram near unitary point
  – Vicinity of the tricritical point
  – Strongly coupled normal state (recent MIT experiments)
  – Experimental signatures of FFLO and SF_M
  – Vortex state of rapidly-rotating superfluid Fermi gases





Blume Emery Griffiths 1970