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Miniworkshop on Strong Correlations in Materials and Atom Traps

4 - 15 August 2008

Shuttle for cold atoms: atomic quantum dot inside Bose Josephson junction.

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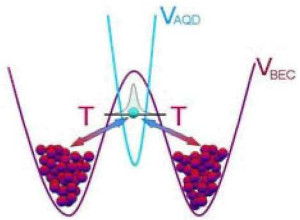

PRA 75, 053605 (2007)
PRA 77, 031602(R) (2008)

„Shuttle“ for cold atoms: atomic quantum dot inside Bose Josephson junction

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Miniworkshop on Strong Correlations in Materials and Atom Traps
Trieste, Italy
August, 13, 2008

Outline

- Motivation: mesoscopic physics
- Setup and formulation of the problem
- Josephson Effect: from SC to BEC
- Bose Josephson Junction with AQD
  (i) formalism
  (ii) results: strong coupling
  (iii) results: weak coupling
- Experimental suggestion
- Conclusions and perspectives

Motivation

Trapping cold atoms

Confining macroscopic nr of atoms vs single atom microtraps

How to trap single atoms?

Magneto-optical traps: 1994

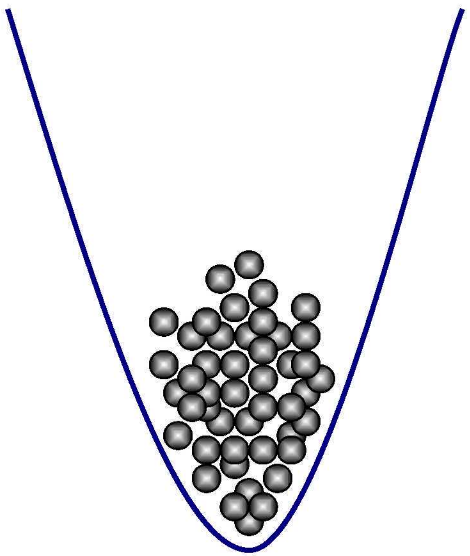
Dipole traps: controlled transport of a single atom

D. Meschede et al, Science 2001

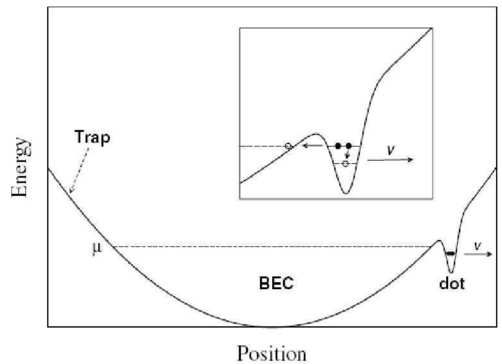
N. Schlosser et al, Nature 2001

Quantum tweezers:

Diener *et al.*, PRL **79**, (2002)



BEC

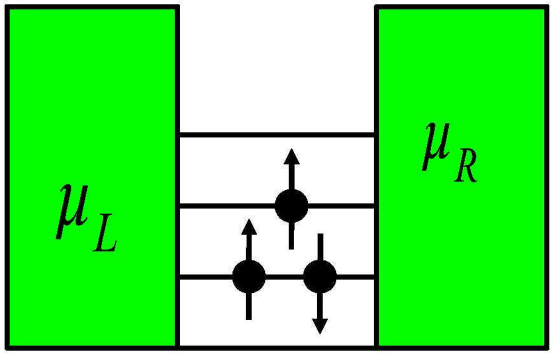


Application:Quantum information,
Single atom interferometry,
Single spin manipulation etc.

Motivation

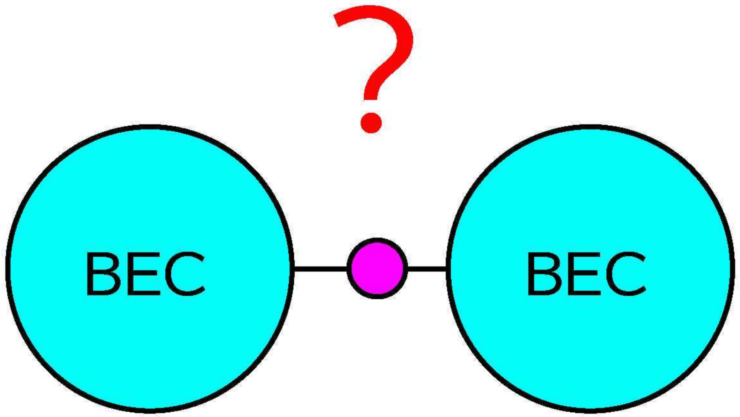
Nanoelectronics

Electronic quantum dots,
Quantum point contacts etc



Nanobosonics

Atomic quantum dot

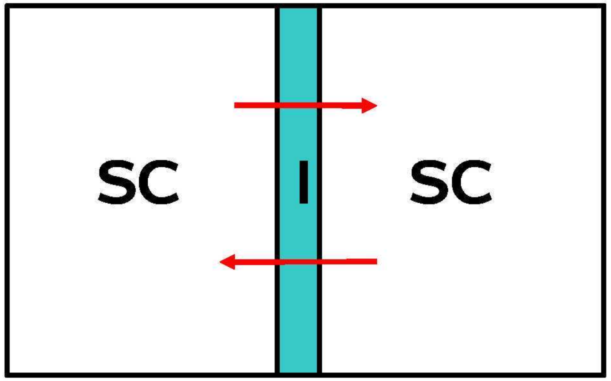


Questions:

2) **How does BJJ work?**
3) **How to couple AQD to SF reservoirs?**
4) **Is there any effect of AQD on the BEC and visa versa**

Josephson Effect

1) superconductors



Particle current oscillations between two weakly coupled macroscopic quantum systems

B. D. Josephson, 1962

w/o external voltage - dc Josephson effect → $I = I_c \sin(\theta)$

w external voltage – ac Josephson effect → $\frac{\partial \theta}{\partial t} = -\frac{\Delta\mu}{\hbar} = -\frac{2e\Delta V}{\hbar}$

$$\omega = \frac{\Delta\mu}{h}$$

<u>*Conditions:*</u>

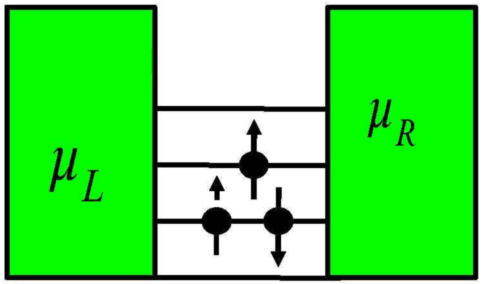
2) well defined quantum phase
3) different average energy per particle
4) weak coupling

Josephson Effect

1a) SC-X-SC structures

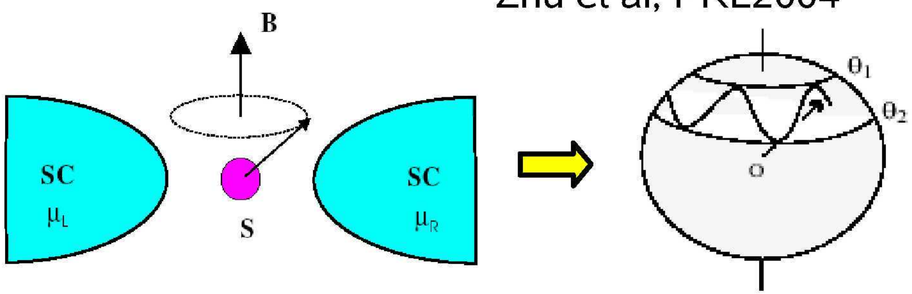
Nanoelectronics and Mesoscopic physics

Quantum dots,
quantum point contacts



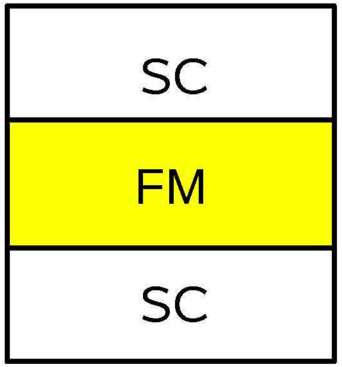
Supercurrent through a Kondo impurity
Glazman, Matveev, JETP Lett.1988

Spin dynamics in a Josephson junction
Zhu et al, PRL2004



Spin nutation due to supercurrent

Heterostructures



Triplet supercurrents in Half-metal

Keizer et al, Nature 2006,
Eschrig, Löfwander, Nature Physics, 2008

Josephson Effect

2) superfluids

Superfluid ^{3}He – **Pereverzev *et al*, Nature 1997**

Equations are the same, but $\Delta\mu = \frac{\Delta P m}{\rho}$

ΔP -pressure difference across the weak link,

m -mass of ^{4}He, ρ -liquid density

Problems:

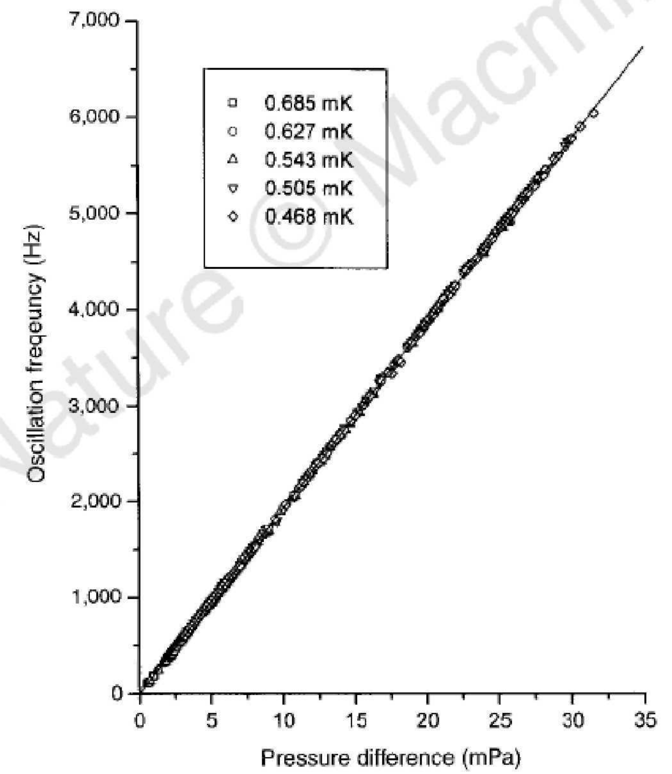
1) creation of a sufficiently small aperture, comparable in size to a healing length
2) detection of small mass currents

$\xi \propto$ ¿ 0.1 nm for ^{4}He

¿ 50 nm for ^{3}He

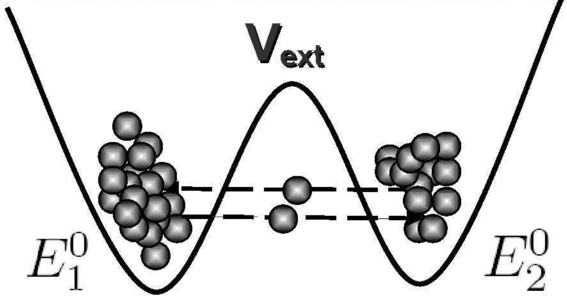
Josephson effect in superfluids can be heard

Superfluid ^{4}He – Sukhatme et al., Nature 2001



Josephson Effect

3) Bose-Einstein condensates of cold atoms



$$\hat{H} = \int dr\ \hat{\Psi}^\dagger(r,t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(r)\right]\hat{\Psi}(r,t) + \frac{g}{2}\int dr\ \hat{\Psi}^\dagger(r,t)\hat{\Psi}^\dagger(r,t)\hat{\Psi}(r,t)\hat{\Psi}(r,t)$$

dilute and cold systems: interaction is contact

$$g = \frac{4\pi\hbar^2 a_s}{m}$$

Because of the special role of the lowest energy state (condensation)

Description of a many-body bosonic system

Ground state is macroscopically occupied

$$|\phi_0(N)\rangle = |N\rangle \otimes |0,0,\ldots\rangle$$

and condensate operators can not annihilate it

$$\hat{a}_0|\phi_0(N)\rangle = \sqrt{N}|\phi_0(N-1)\rangle, \quad \hat{a}_0^\dagger|\phi_0(N)\rangle = \sqrt{N+1}|\phi_0(N+1)\rangle$$

⇓

Problem

Wick's theorem becomes extremely complicated as it is not possible to define normal ordered products with vanishing ground state expectation value

However: equations for this case are derived, but difficult to solve

S. Beliaev 1958 – diagrammatic technique for bosons

Description of a condensate: Bogoliubov prescription

$$\hat{a}_0,\ \hat{a}_0^\dagger \rightarrow \sqrt{N_0}$$

⇓

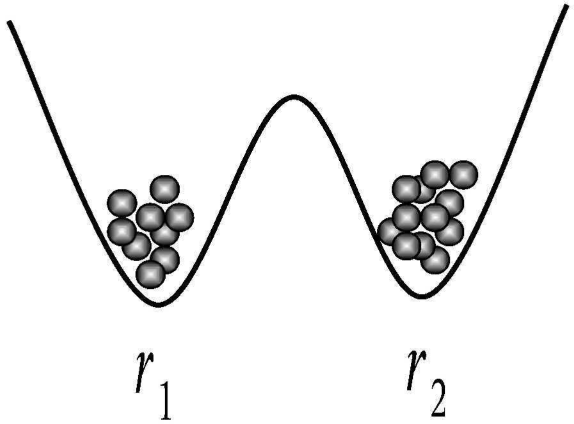
we can replace condensate operators by C-numbers

$$\Psi_0(x,t) \equiv \langle \hat{\Psi}(x,t) \rangle \neq 0$$

Mean-field ~~¿~~ ✗ ⟹ Gross-Pitaevskii

P.S.: in nonequilibrium ¿ play a crucial role
(Nonadiabatic BJJ: M. Trujillo Martinez, A. P., J. Kroha – see the poster)

Condensate in a double well: the two mode approximation



1) Expand

$$V(r)=V^{(2)}(r-r_1)+V^{(2)}(r-r_2)+\ldots$$

parabolic approx. around each min

2) Define a state $\varphi_0(r)$

single particle GS mode of $V^{(2)}(r)$

3) Define local mode solutions of individual wells

$$\varphi_{01}(r)=\varphi_0(r-r_1)$$

$$\varphi_{02}(r)=\varphi_0(r-r_2)$$

$$\int dr\varphi_{0i}\varphi_{0j}=\delta_{ij}+\varepsilon(1-\delta_{ij})$$

⇒ Energy states of the global double well potential are

$$\varphi_{1,2}=\frac{1}{\sqrt{2}}(\varphi_{01}\pm\varphi_{02})$$

Milburn *et al*, PRA1997

Bose Josephson junction

$$\langle\hat{\Psi}(r,t)\rangle = \Psi(r,t) = \Psi_1(t)\Phi_1(r) + \Psi_2(t)\Phi_2(r)$$

$$\Psi_1 \propto \sqrt{N_1}e^{i\theta_1} \qquad \Downarrow \qquad \Psi_2 \propto \sqrt{N_2}e^{i\theta_2}$$

Canonical Josephson Hamiltonian

$$H = \frac{\Lambda}{2}n^2 + \Delta E n - \sqrt{1-n^2}\cos(\theta)$$

fractional population imbalance **phase difference**

$$n = \frac{N_1 - N_2}{N_{tot}}$$

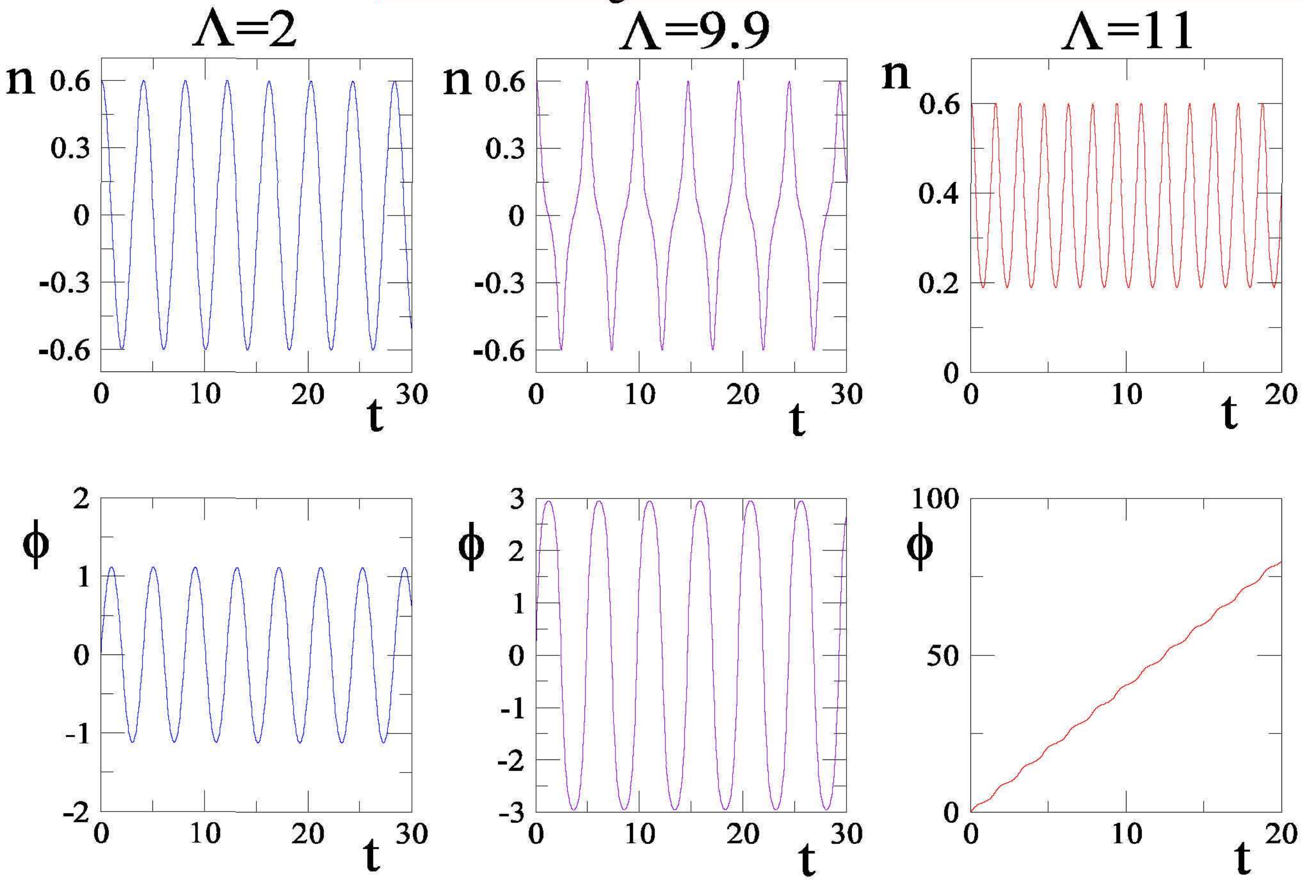
Interaction parameter: $\Lambda = \dfrac{UN_{tot}}{2\kappa}$

Bose Josephson Junction

A. Smerzi et al.,PRL 79 (1997)

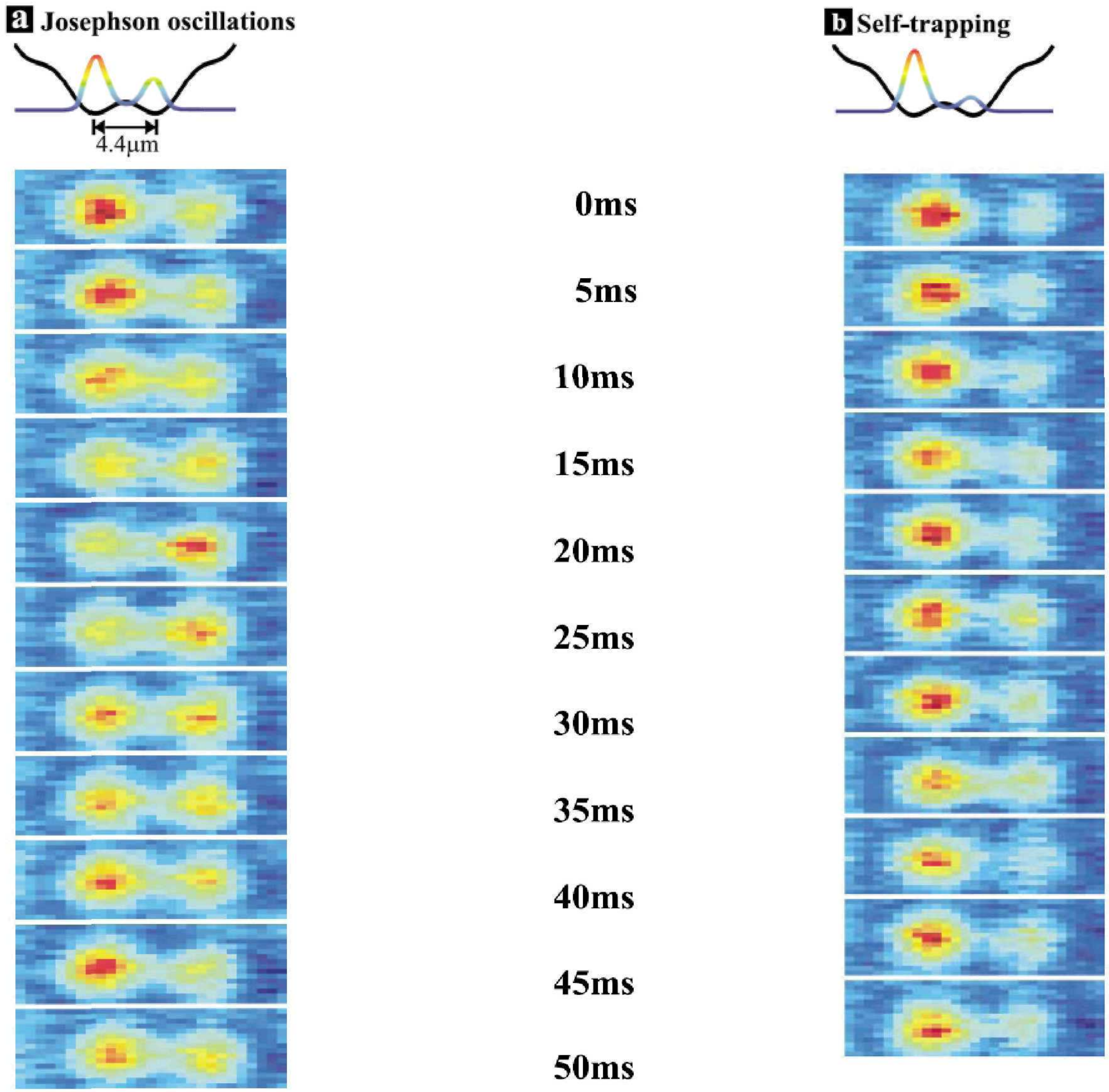
increase interaction parameter $\Lambda = \frac{UN_{tot}}{2\kappa}$ →

$\Lambda > \Lambda_c$ *- macroscopic self-trapping*



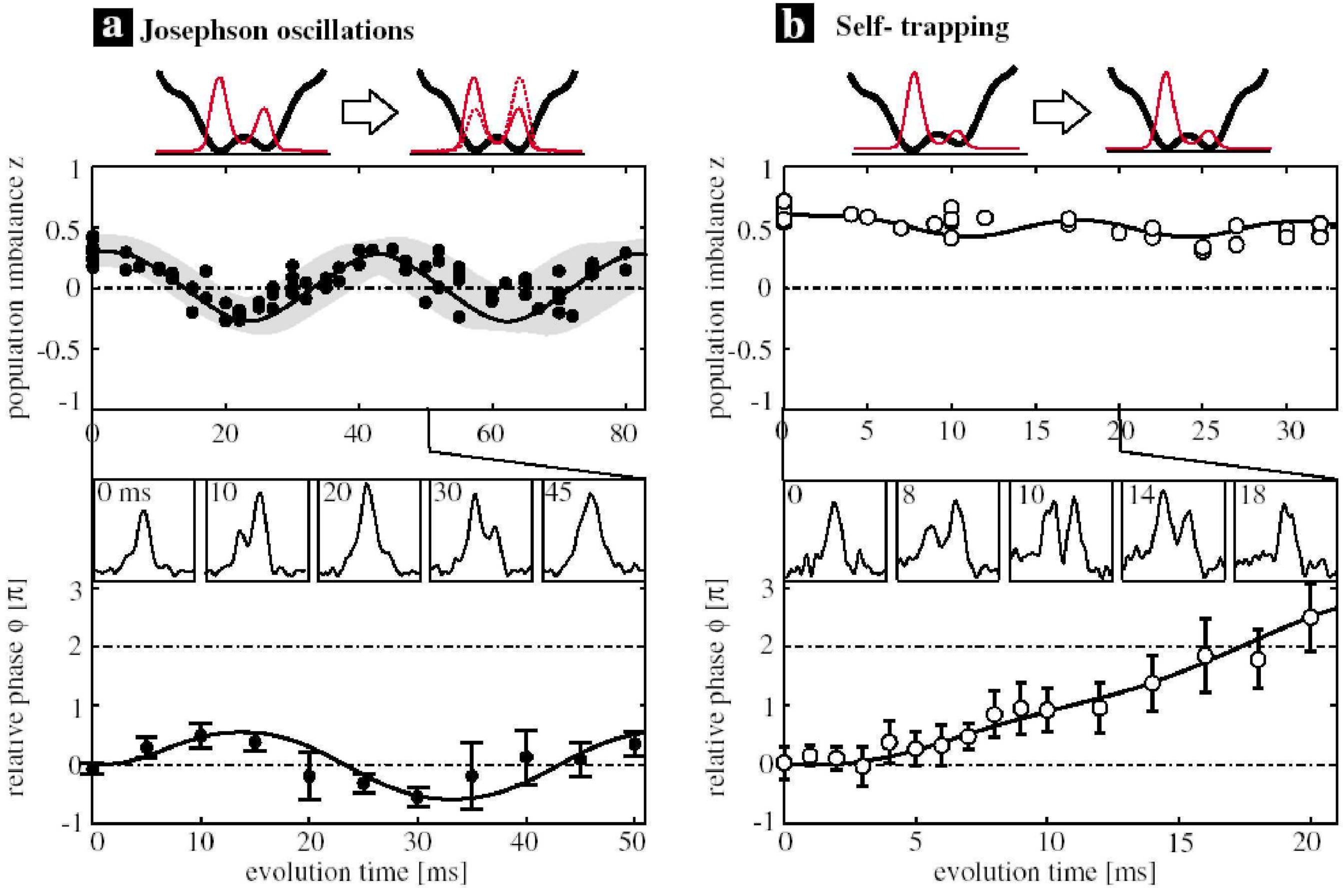
Experiment: M.Albiez *et al.*,PRL 95, 010402 (2005)

BJJ: experiment



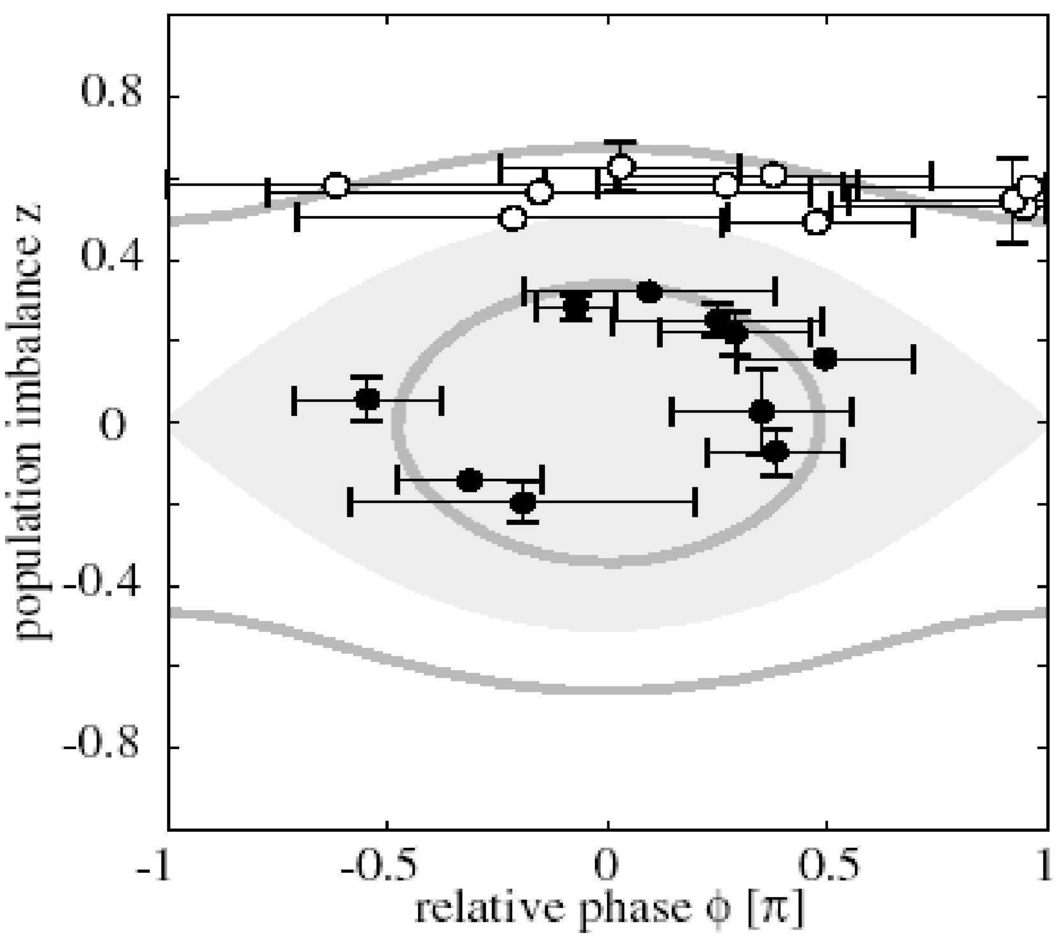
M.Albiez *et al.*,PRL 95, 010402 (2005)

BJJ: experiment



M.Albiez *et al*.,PRL 95, 010402 (2005)

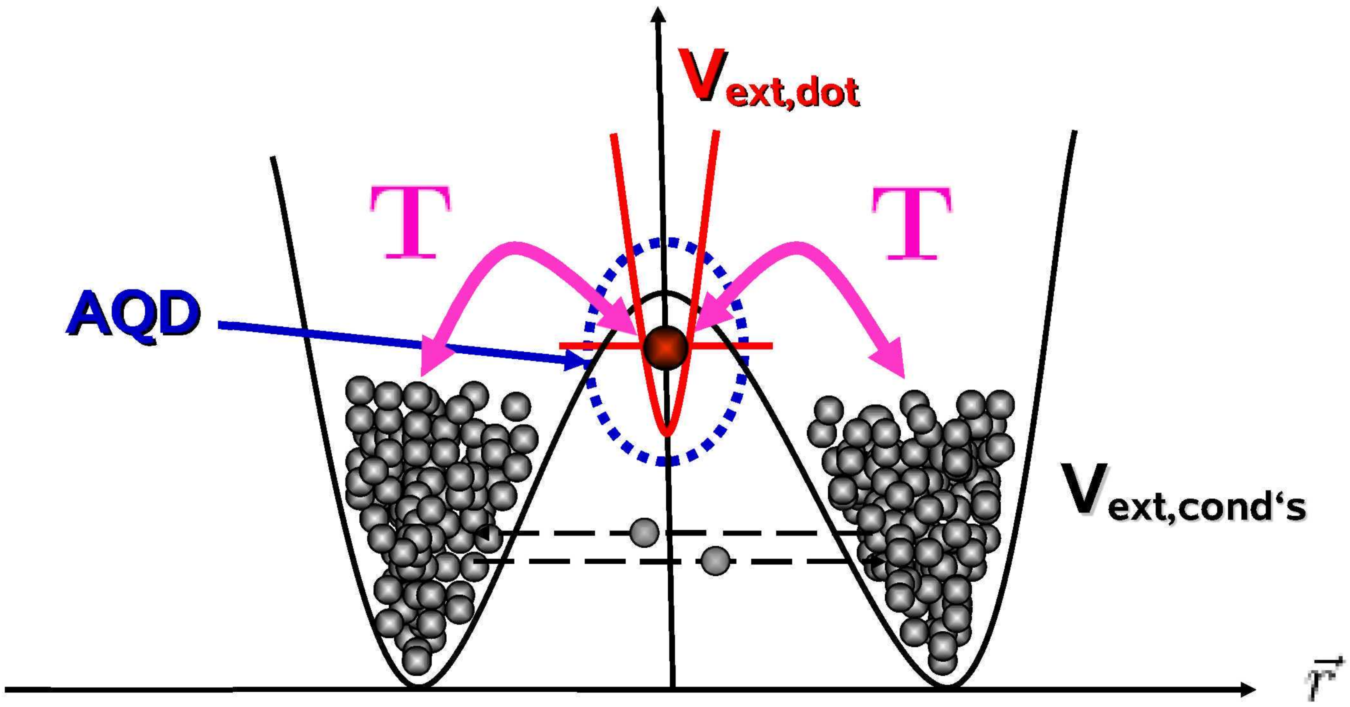
BJJ experiment: quantum phase plane portrait



M.Albiez *et al.*,PRL 95, 010402 (2005)

Problem: Physics of AQD coupled to Time-Dependent Condensates

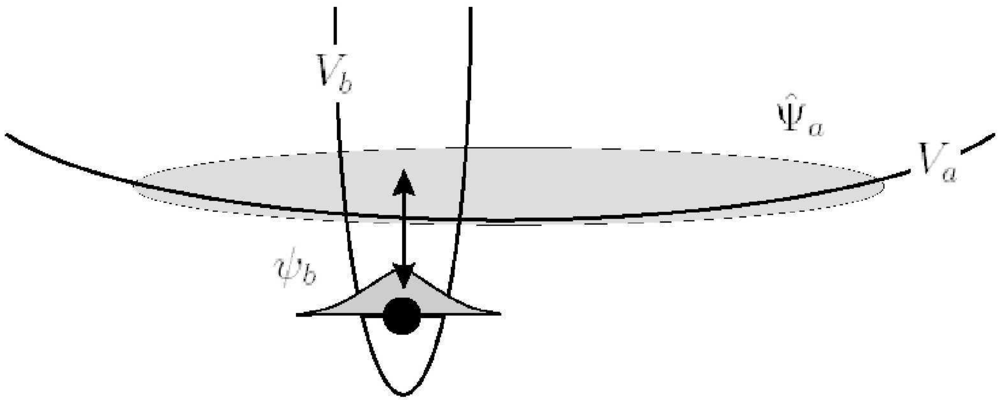
$$H = H_{cond's} + H_{dot} + H_{coupling}$$



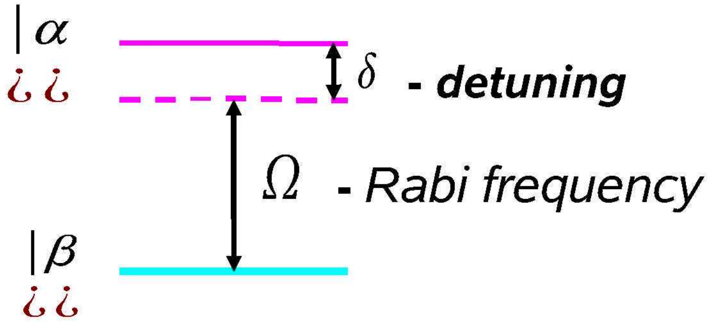
$$\Psi_1 \propto \sqrt{N_1} e^{i\theta_1} \qquad \Psi_2 \propto \sqrt{N_2} e^{i\theta_2}$$

BJJ+AQD

Coupling of the atomic quantum dot to a superfluid reservoir



via a Raman transition



A.Recati *et al*.,PRL 94, (2005) : mapping of the AQD+ uniform SF reservoir onto a spin – boson model

⬇

Dissipative phase transition which allows to directly measure Luttinger parameters

Hamiltonian

$$\hat{H} = U\left[|\Psi_1(t)|^4 + |\Psi_2(t)|^4\right] - \kappa\left[\Psi_1^*(t)\Psi_2(t) + \text{h.c.}\right]$$
$$+T\sum_{i=1,2}\{\Psi_i(t)\hat{\sigma}_+ + \text{h.c.}\} - \hbar\delta\left(\frac{1}{2} + \hat{\sigma}_z\right)$$

We assumed:

- **symmetric trap:** $E_1 = E_2$, $U_1 = U_2$
- **dot wave-function:** $|\,\Psi_{dot}\rangle = \alpha_0(t)\,|\,0\rangle + \alpha_1(t)\,|\,1\rangle$
- **pseudospin-1/2 formalism:** $|\,1\rangle \equiv \text{"}\uparrow\text{"} \quad \& \quad |\,0\rangle \equiv \text{"}\downarrow\text{"}$

  $\hat{d} \equiv \hat{\sigma}_- \quad \& \quad \hat{d}^\dagger \equiv \hat{\sigma}_+$
- **dot state vector:** $\vec{s}(t) = \langle\Psi_{dot}\,|\,(\sigma_x, \sigma_y, \sigma_z)\,|\,\Psi_{dot}\rangle$

BJJ+AQD: parameters and regimes

Main parameters:

interaction	detuning
$\alpha = \frac{UN_0}{T}$	$\beta = \frac{\delta}{T}$

(T is dot-condensate coupling)

Regimes:

Strong coupling	Weak coupling
$T >> \kappa$	$T \leq \kappa$
AQD ⇨ BEC	AQD ⇦ BEC

Equations for the noninteracting case

$$i\partial_t \Psi_1 = U|\Psi_1|^2\Psi_1 - \kappa\Psi_2 + Ts_-,$$
$$i\partial_t \Psi_2 = U|\Psi_2|^2\Psi_2 - \kappa\Psi_1 + Ts_-,$$

$$i\partial_t s_- = -\delta s_- - T(\Psi_1 + \Psi_2)s_z,$$
$$i\partial_t s_z = -2T(\Psi_1^* + \Psi_2^*)s_- + 2T(\Psi_1 + \Psi_2)s_+.$$

$\Updownarrow$

$$\hbar\partial_t \boldsymbol{s} = \boldsymbol{\omega}(t) \times \boldsymbol{s}$$

The only assumption

$$\frac{|s_-|^2}{|\psi_1 + \psi_2|^2} << 1$$

Analytical results for the noninteracting case

Particle imbalance

$$n(t) \;=\; \frac{\sqrt{C - \bar{s}_z - A_0 \cos(\omega_1 t)}}{N_0}\, \mathrm{Re}\left[e^{i\Omega t} \bar{\psi}(0)^*\right]$$

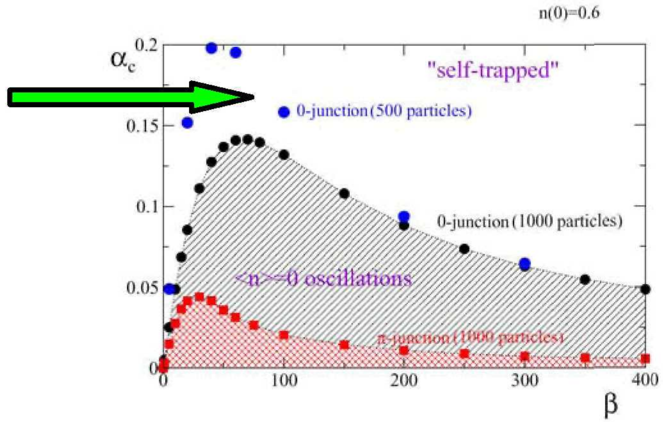
$$C = |\psi(0)|^2 + s_z(0)$$

two frequencies: $\omega_1 = 2T\sqrt{C}$ $\quad \Omega = 2\kappa - 2T^2 \frac{(\kappa - \delta)s_z(0) + 2\omega_1 s_-(0)}{(\kappa - \delta)^2 + \omega_1^2}$

weak coupling: $\omega_1 = 0$ $\quad \Omega = 2\kappa$

strong coupling: $\frac{\Omega}{T} = \frac{2\beta}{\beta^2 + 4C}$

Strong coupling results

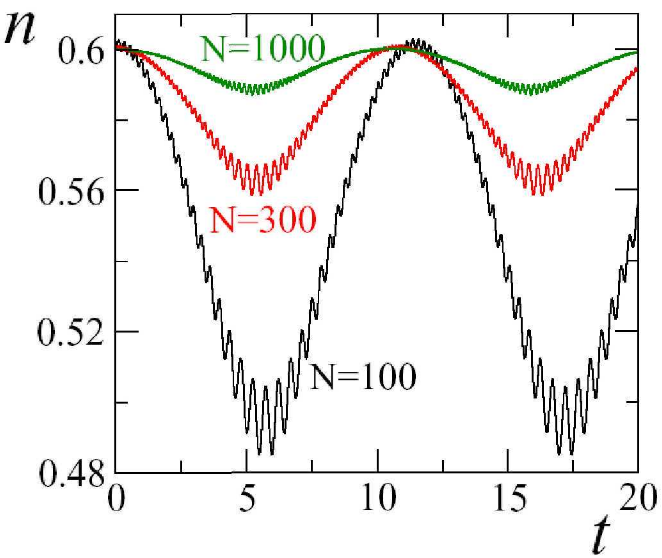


Particle imbalance

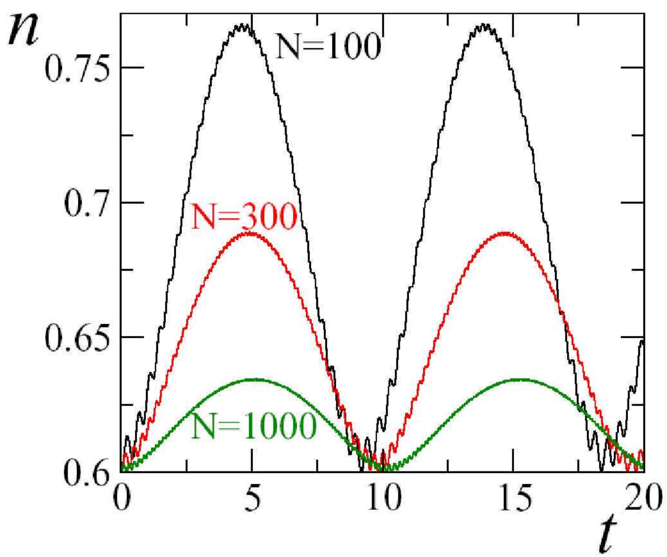
(i) "self-trapping"

$n(0)=0.6, \ \kappa=0$

$\phi(0)=0$

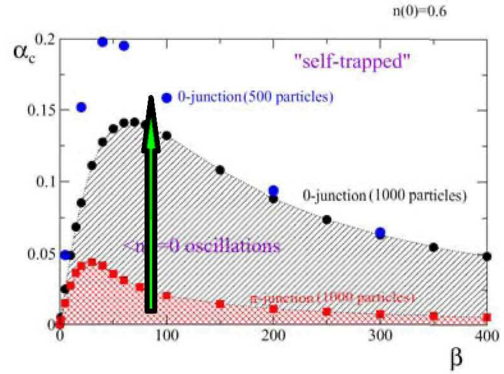


$\phi(0)=\pi$

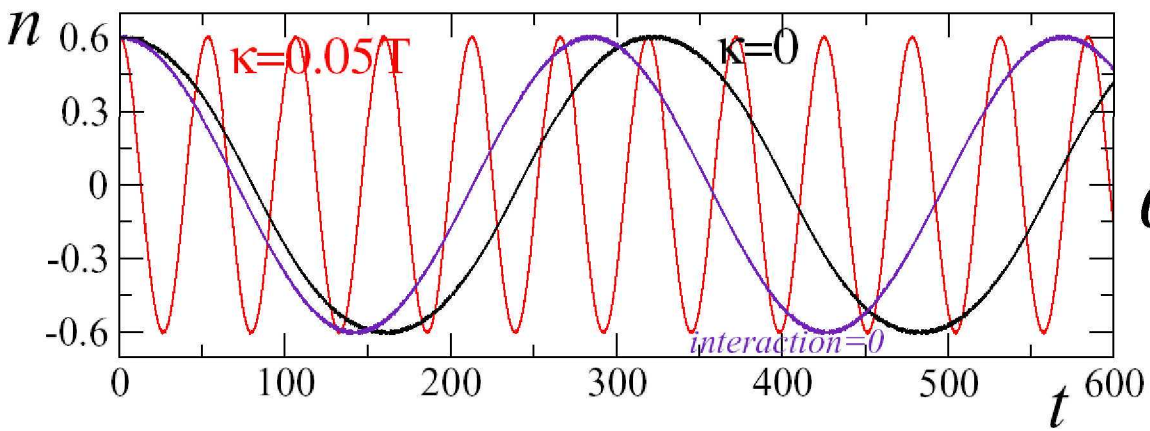


Strong coupling

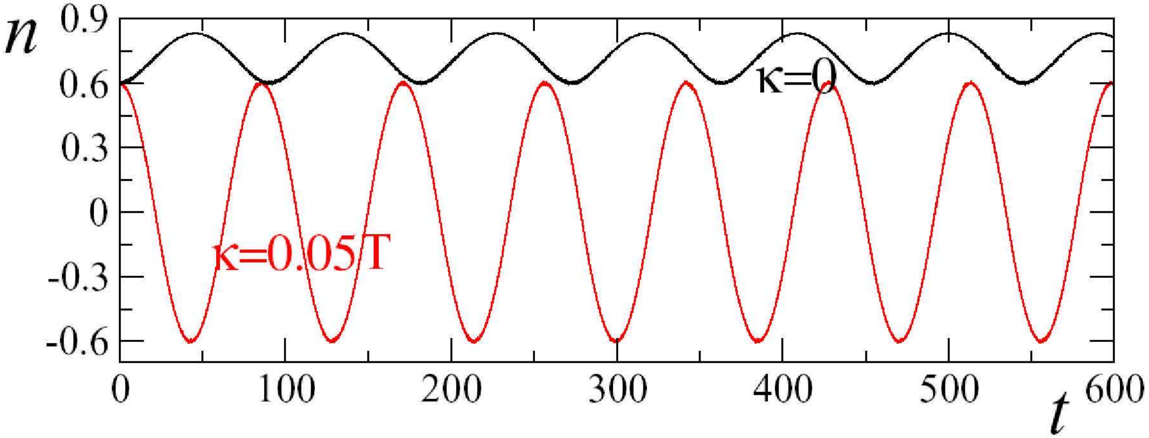
Results for particle imbalance
(ii) large amplitude oscillations



π*-junction*



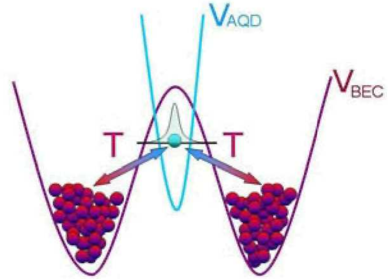
$\alpha = 0.01$



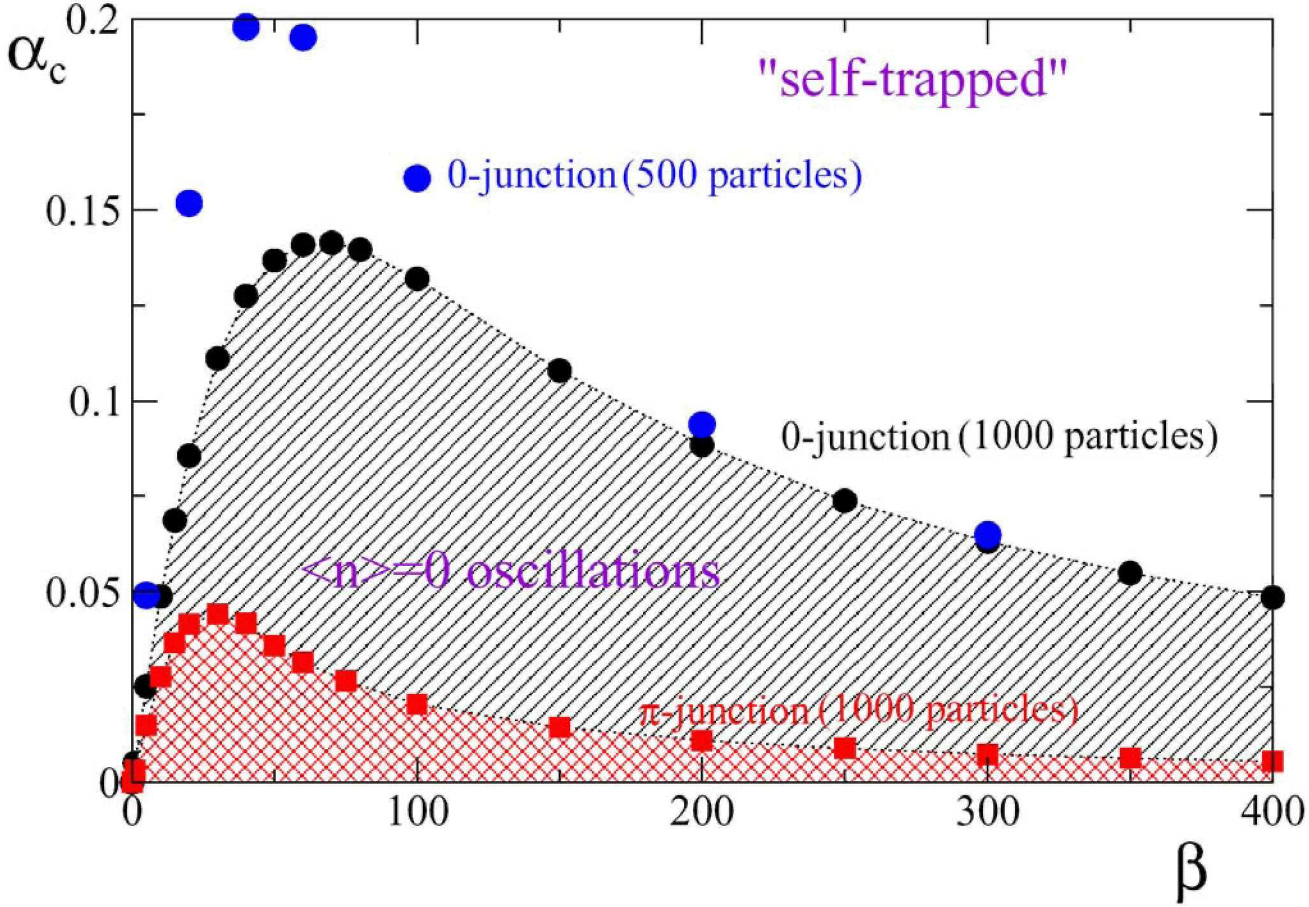
$\alpha = 0.1$

Strong coupling results

Phase diagram



n(0)=0.6



$$\alpha = \frac{UN_0}{T}$$

$$\beta = \frac{\delta}{T}$$

Strong coupling results

Estimate for the energies magnitudes in the NSTstate

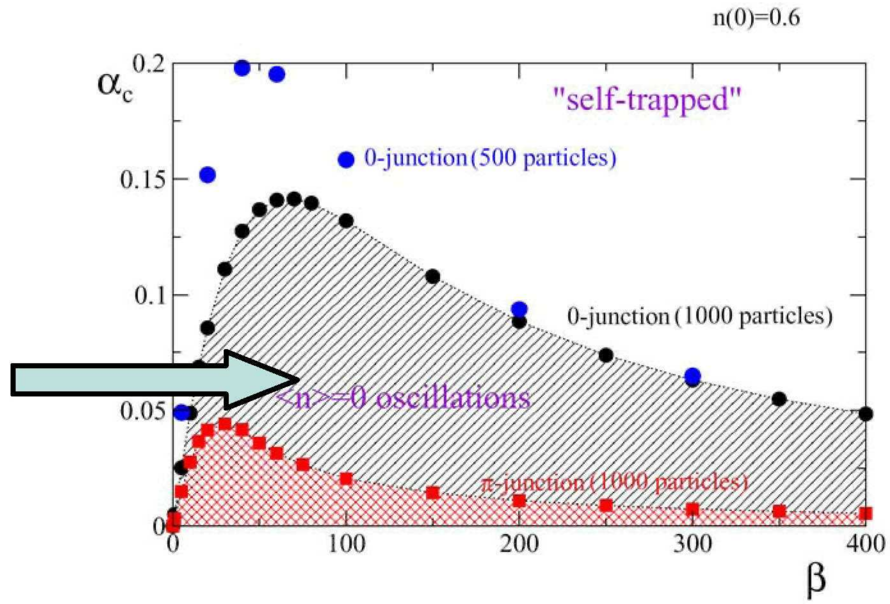
$$\frac{\beta}{\alpha} = \frac{\delta}{UN}$$

$$UN \propto 10\, nk$$

OK!

$$\delta \propto \frac{\beta}{\alpha} \times kHz$$

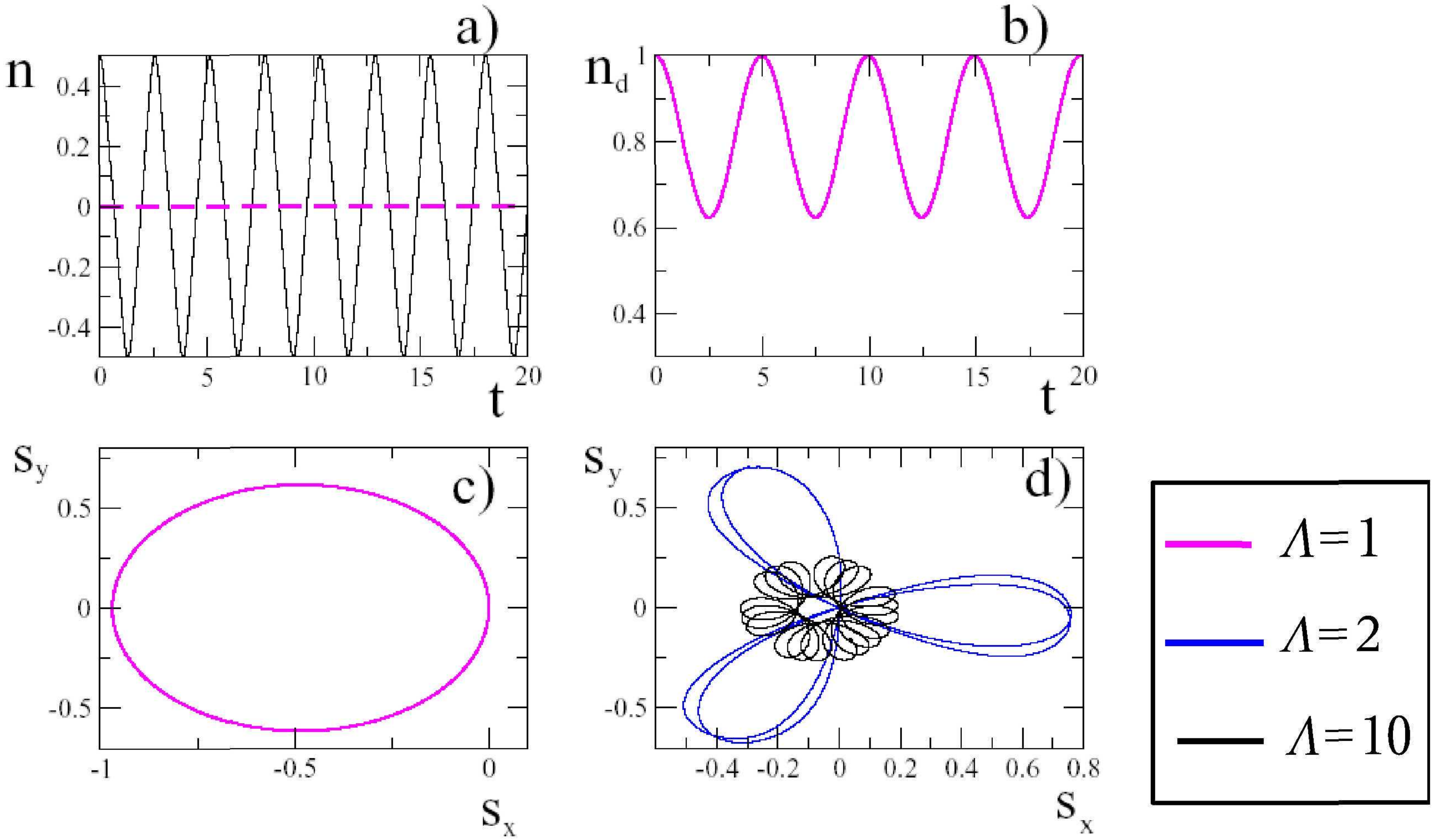
$$\frac{\beta}{\alpha} \propto 10^1 \ldots 10^4$$



Weak coupling results

Results: 0-junction, “usual” Josephson oscillations

$$\Lambda = \frac{(U_1+U_2)N_T}{4\kappa} \qquad T << \kappa$$

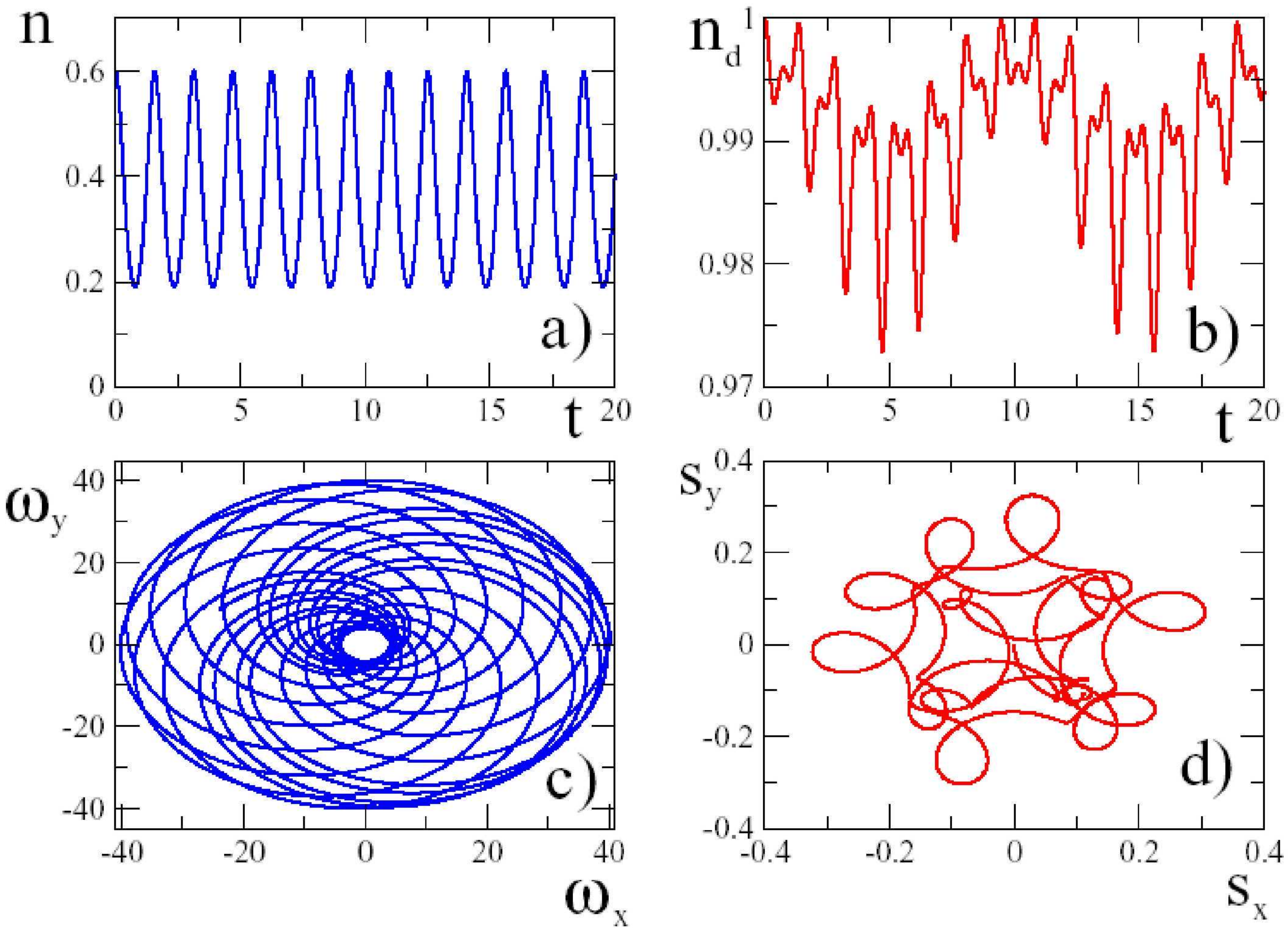


Weak coupling results

$$\partial_t \vec{s} = \vec{\omega}(t) \times \vec{s}(t)$$

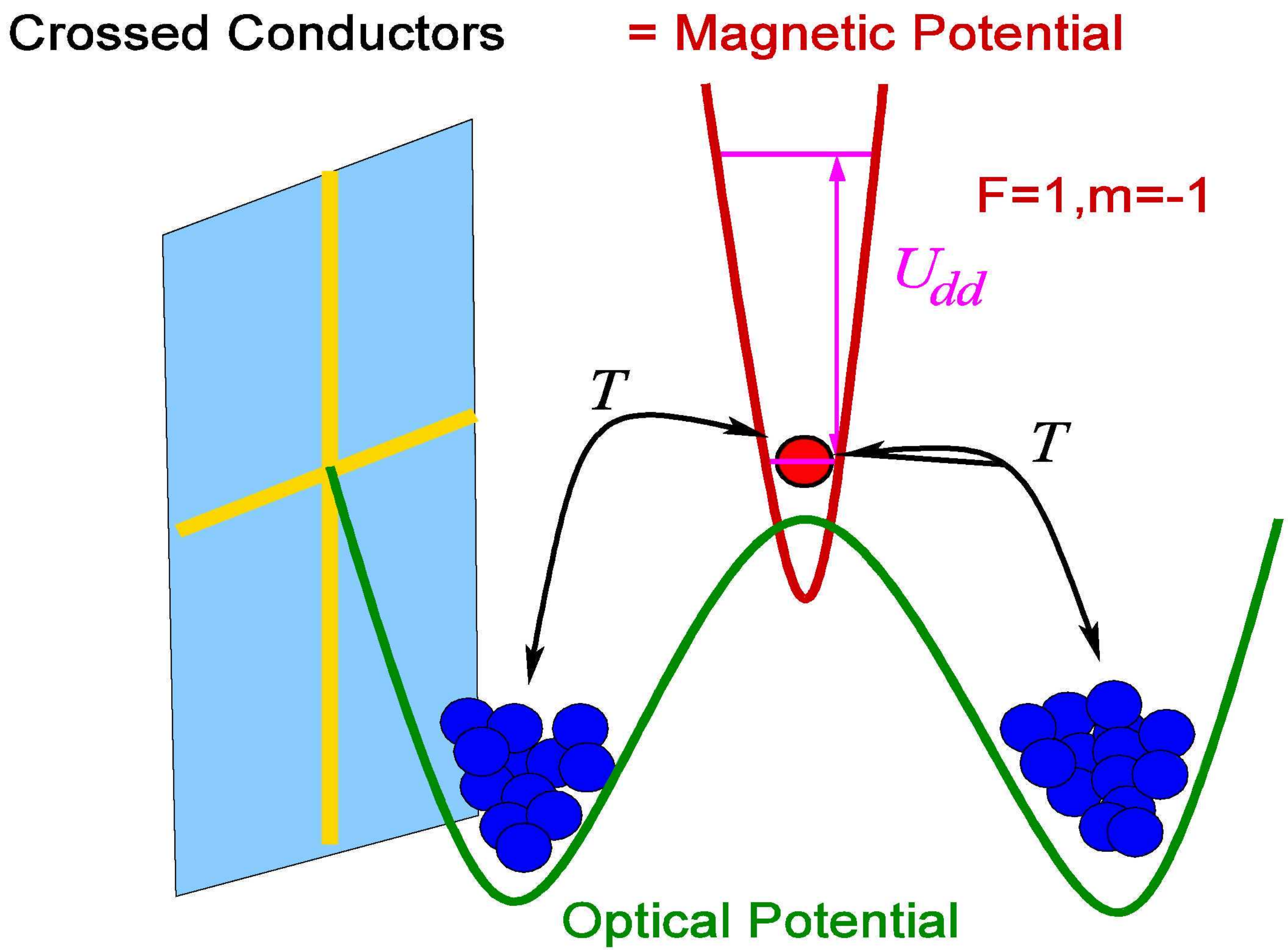
Results: 0-junction, macroscopic self-trapping

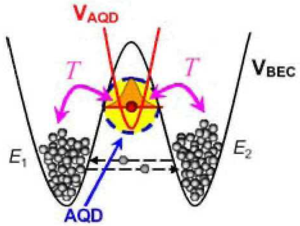
$n(0)=0.6$ $\Lambda=11$ $T<<\kappa$



experimental suggestion

Experimental setup for AQD coupled to BECs on a microchip





Conclusions

✓<u>Weak coupling regime</u>: AQD behaves as a quantum top, very sensitive to tunneling regimes between BECs:

- Usual Josephson oscillations: Quantum Top nutates
- Macroscopic Self-Trapping – multi-frequency rotations of Quantum Top

✓<u>Strong coupling regime</u>: AQD can induce strong oscillations between the condensates

✓Inportant: these are finite nr of particles effects

Future projects:

- *inclusion of quantum corrections*
- *periodic model*