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Entanglement Skyrmions in multicomponent quantum Hall systems.

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Entanglement Skyrmions in multicomponent quantum Hall systems

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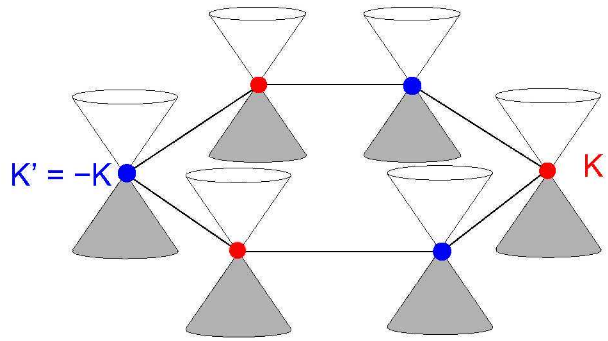


arXiv:0806.0229

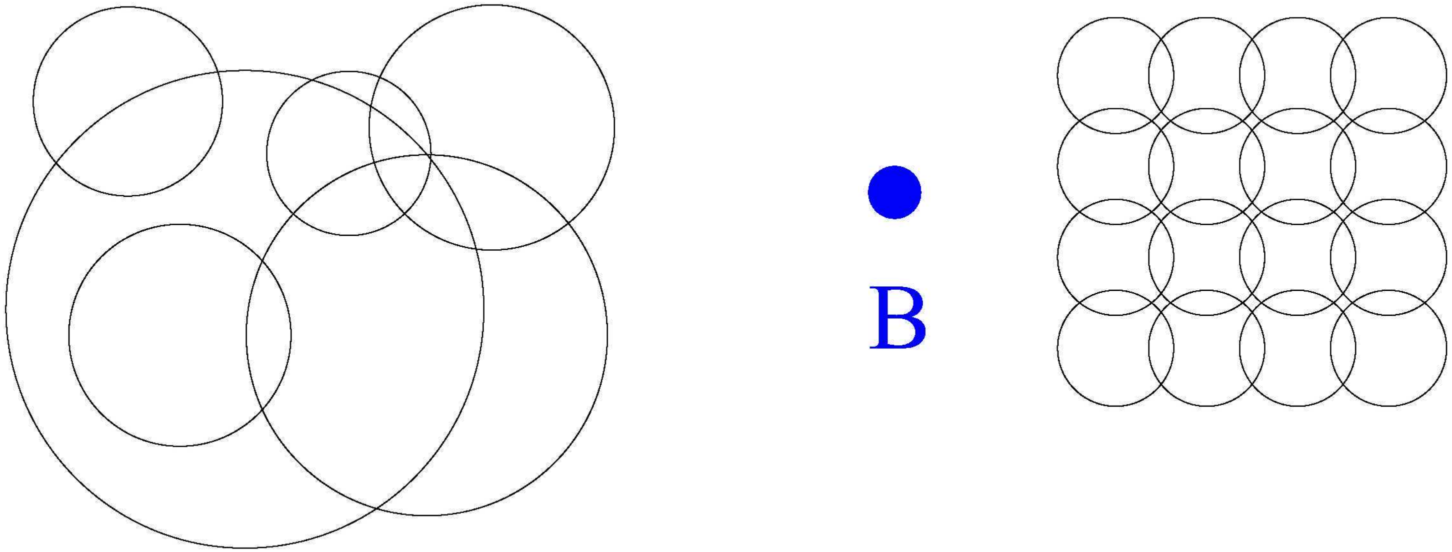
Outline

Fate of SU(4) Skyrmions in presence of realistic anisotropies

- Quantum Hall ferromagnetism and Skyrmions
- Motivation
  - Multi-component quantum Hall systems
  - Experiments on quantum Hall bilayers Eisenstein group
- SU(4) Skyrmions Arovas, Karlhede, Lilliehook
  - parametrisation
  - topological density
  - entanglement properties
- NMR experiments and Skyrmion dynamics
- Energy scales in bilayers and graphene

Quantum Hall physics

Electrons moving in $d = 2$ in a strong magnetic field



- classical: circular motion of fixed cyclotron frequency ω_c
- quantum: quantised energies $E_n = \hbar\omega_c(n + 1/2)$

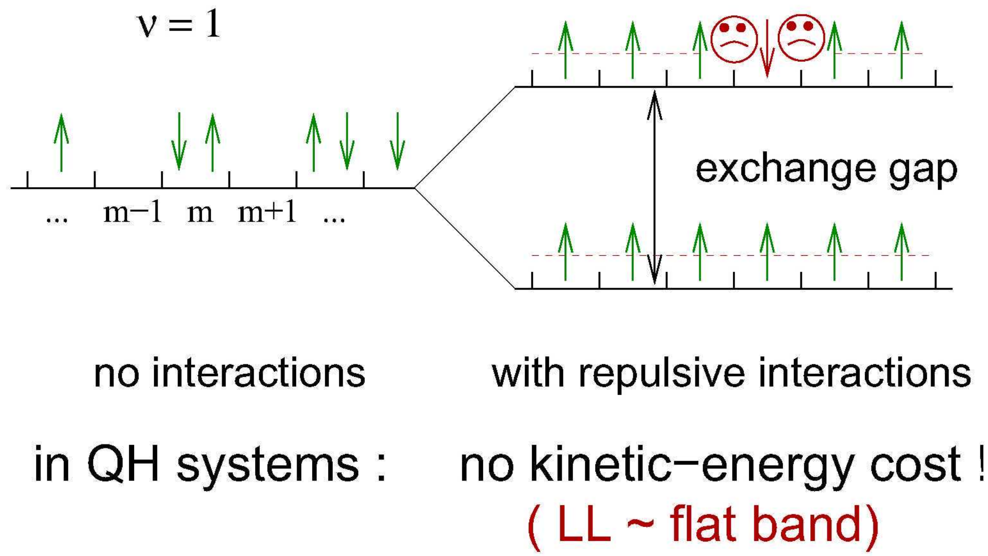
  $\Rightarrow N_\phi$-degenerate Landau levels (# of flux quanta in system)
- filling frac. $\nu = N_{e^-}/N_\phi$ commensurate $\Rightarrow$ quant. Hall effects
- new twist: relativistic Fermions in graphene, $E_n \propto \sqrt{nB}$

Flat-band ferromagnetism at $\nu = 1$

- Degenerate Landau levels have ‘quenched’ kinetic energy
- Splin-splitting due to Zeeman energy but in addition:

Coulomb repulsion favours anti-symmetric orbital wavefunction

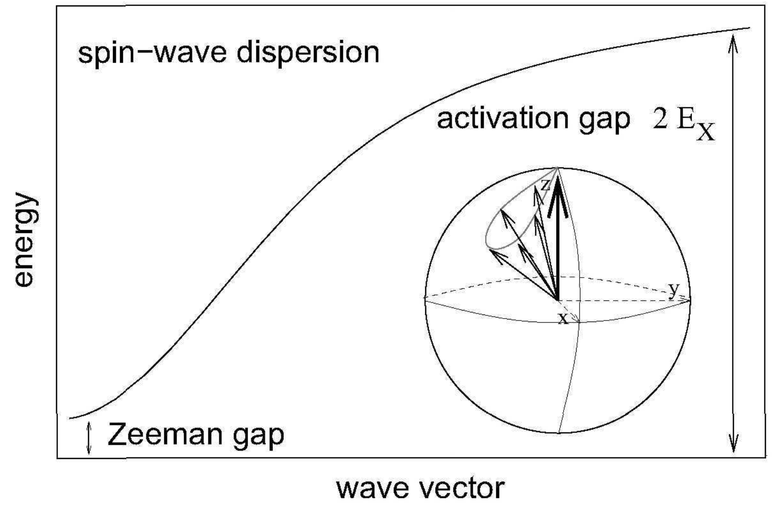
$\rightarrow$ spin wavefunction: symmetric (ferromagnet)



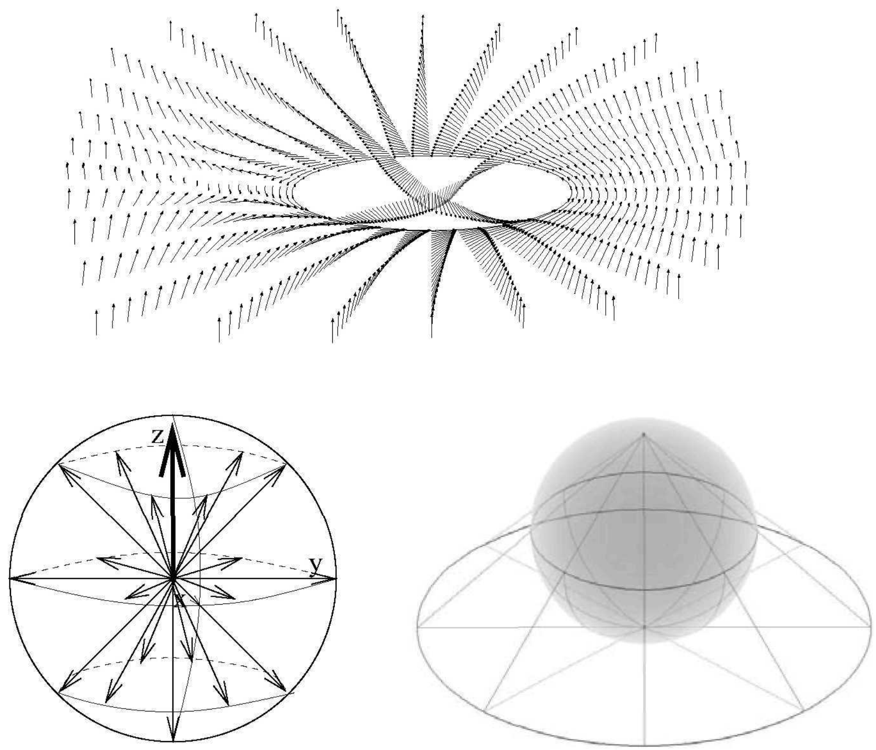
- Collective effect (‘exchange enhanced spin splitting’)
- Zeeman splitting not necessary

Topological spin excitations: Skyrmions

Spin waves:



Skyrmions:



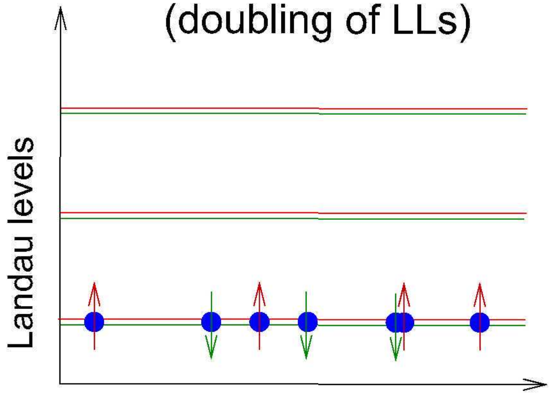
- non-topological
- charge-neutral

- topological
- quantised charge

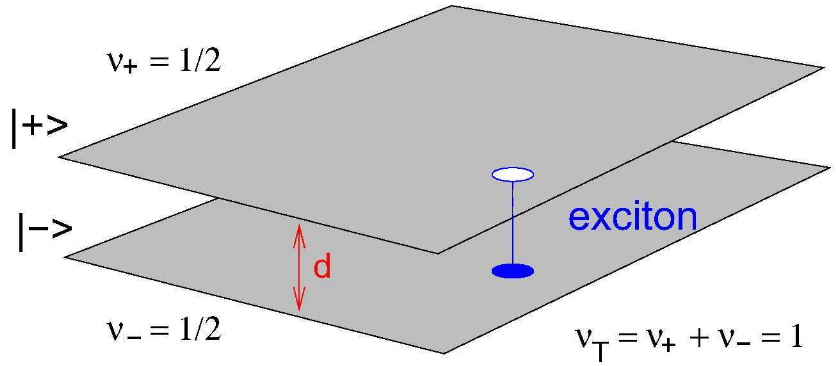
Multi-Component Systems (Internal Degrees of Freedom)

A physical spin: SU(2)

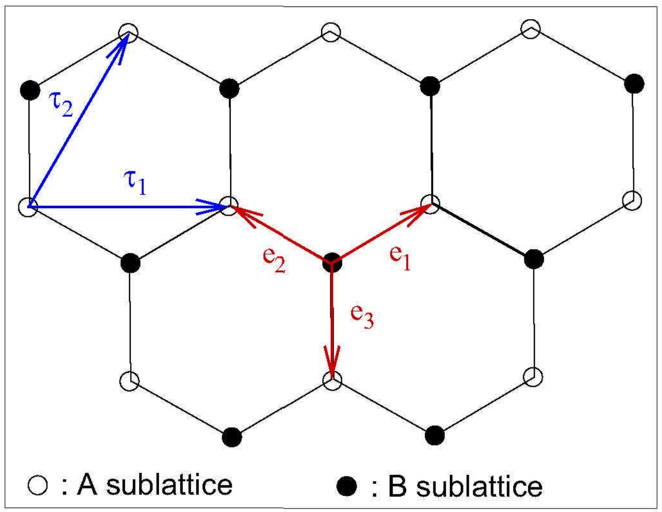
(doubling of LLs)



B bilayer: SU(2) isospin



C graphene (2D graphite)



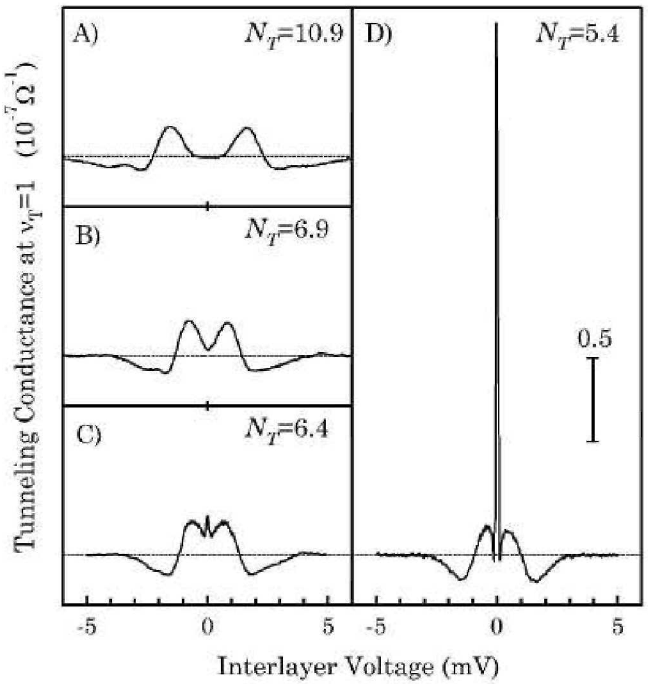
two−fold valley
degeneracy
$\longrightarrow$ SU(2) isospin

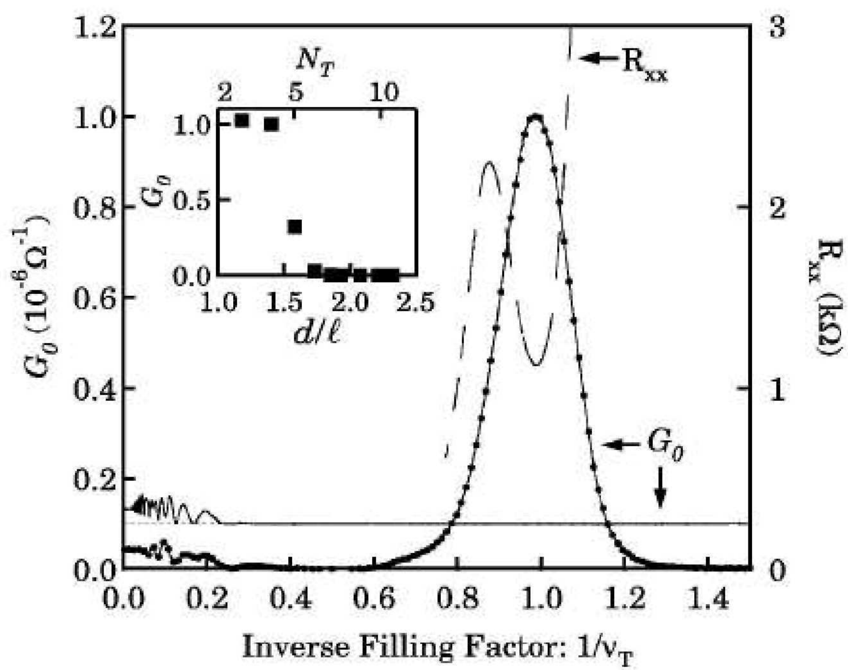
spin + isospin : SU(4)

Quantum coherence in quantum Hall bilayers at $\nu = 1$

Interlayer tunneling as a probe of interlayer ferromagnetism

- close layers: interlayer coherence $\Rightarrow$ ‘easy tunnelling’
- distant layers: separate ’composite Fermi liquids’



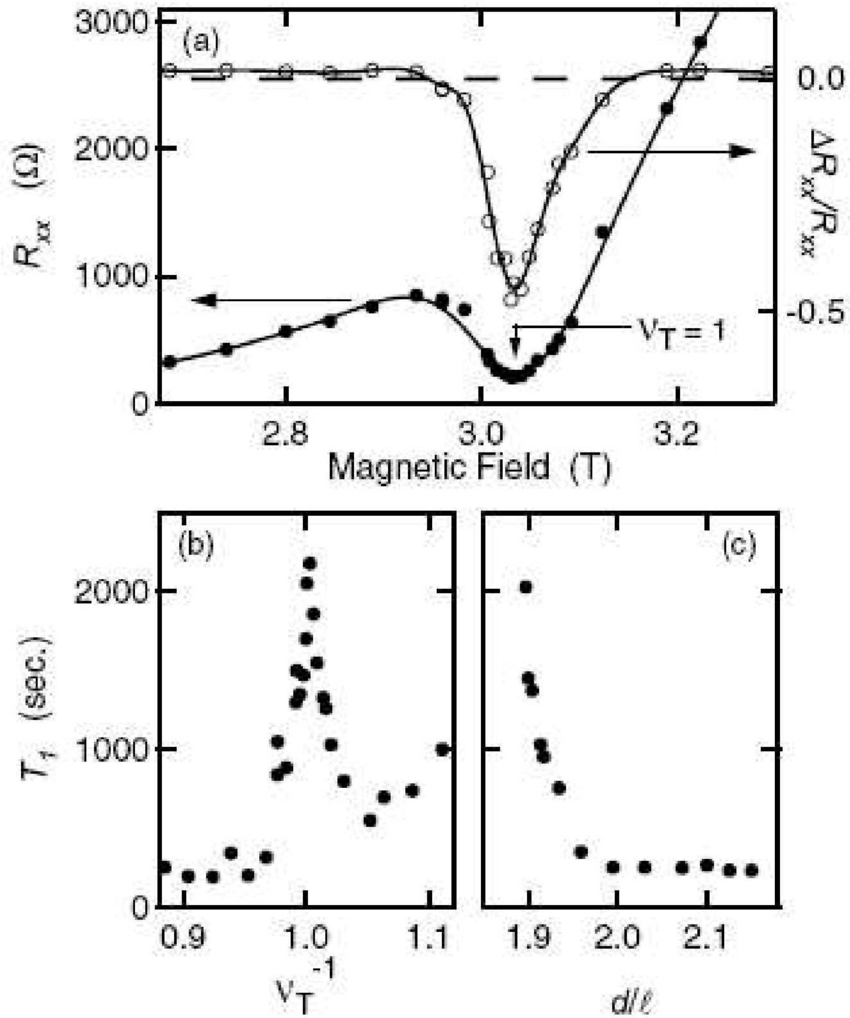


Spielman et al. Phys. Rev. Lett. **84**, 5808, (2000)

NMR experiments in quantum Hall bilayers I

Probe of spin dynamics at and near excitonic condensate

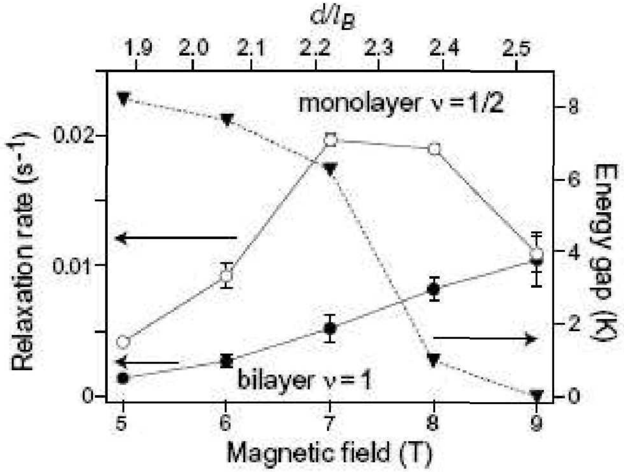
- Heat or NMR pulse → growth of effective electron Zeeman field → do spins respond?
- Nuclear spin relaxation is detected resistively
- $\nu = 1$ state not fully polarised
- T_1 drops away from $\nu = 1$

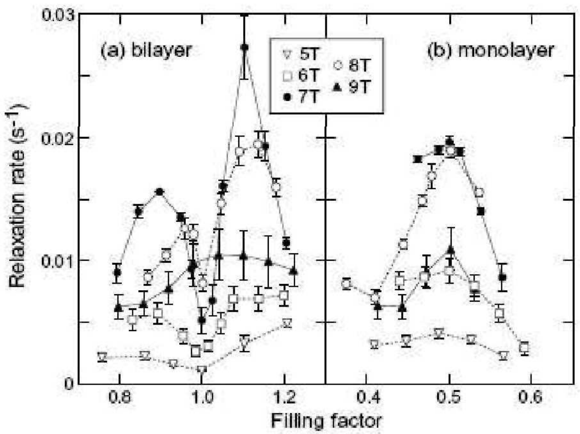


Spielman et al., Phys. Rev. Lett. **94**, 076803 (2005)

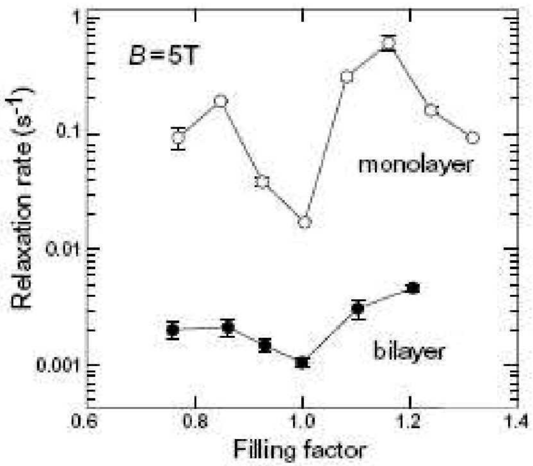
NMR experiments in quantum Hall bilayers II

Current-pump and resistive detection S. Kraus et al., PRL **89**, 266801 (2002)





Kumada at al., Phys. Rev. Lett. **94**, 096802

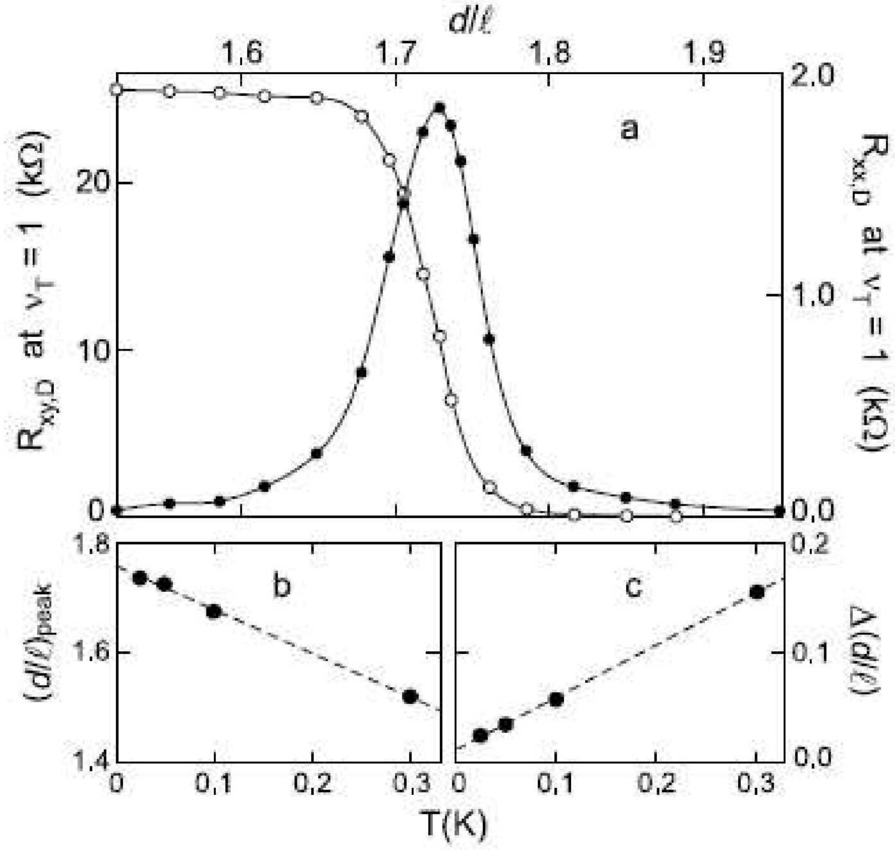


Phase coexistence scenario

- Theoretical proposal: first order trans. Schliemann, Girvin+MacDonald
- Smooth crossover due to inhomogeneities (puddle formation)? Stern, Halperin, 2002

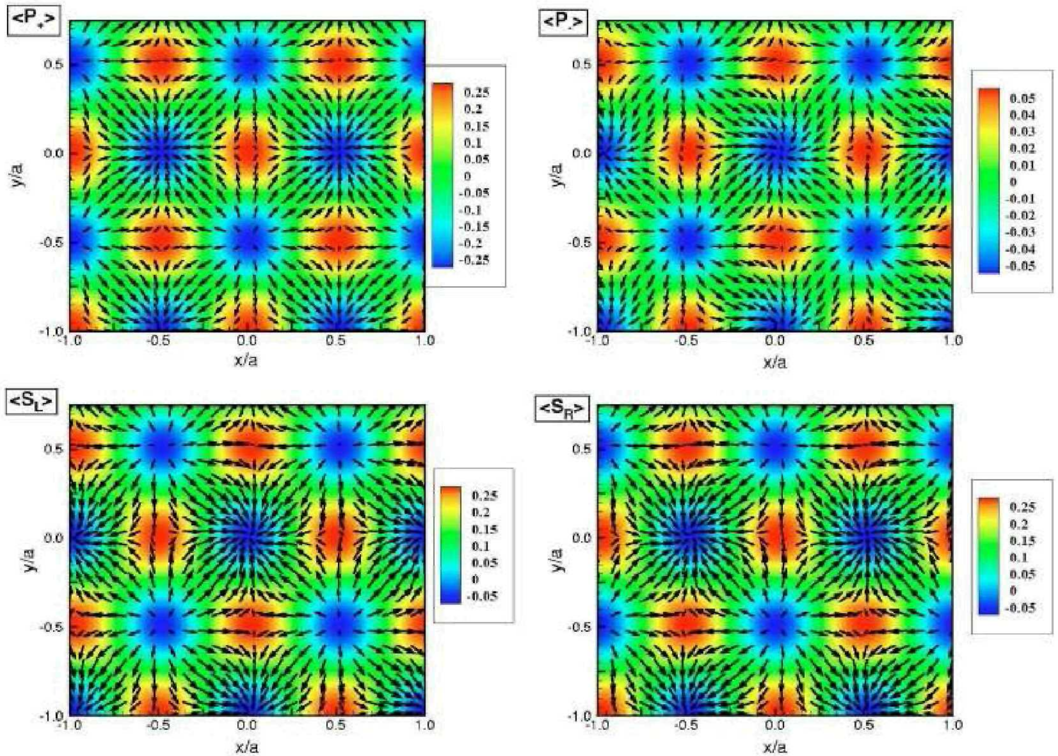
$\Rightarrow$ spin active only in part of system?

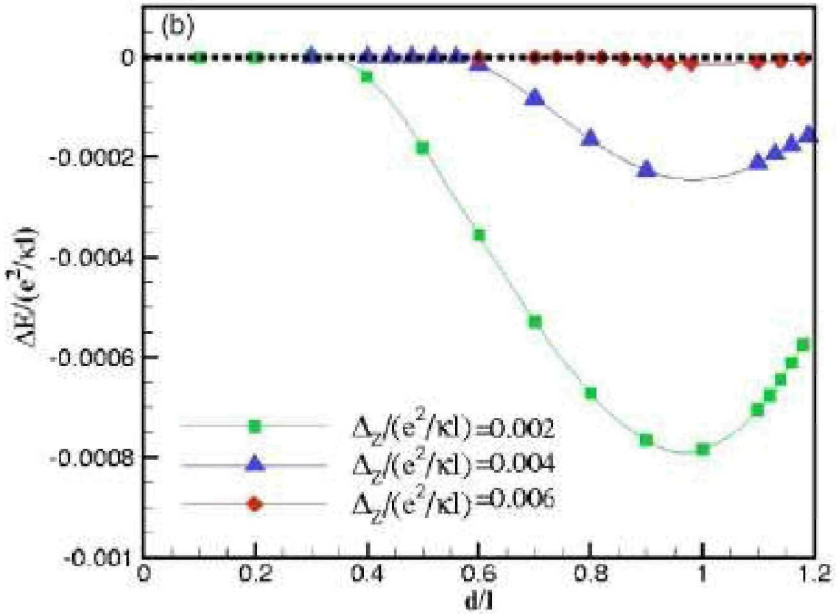
Question: 'intrinsic' spin dynamics in incompressible excitonic phase also possible?



Kellog et al. PRL **90**, 246801 (2003)

The case for entangled textures I





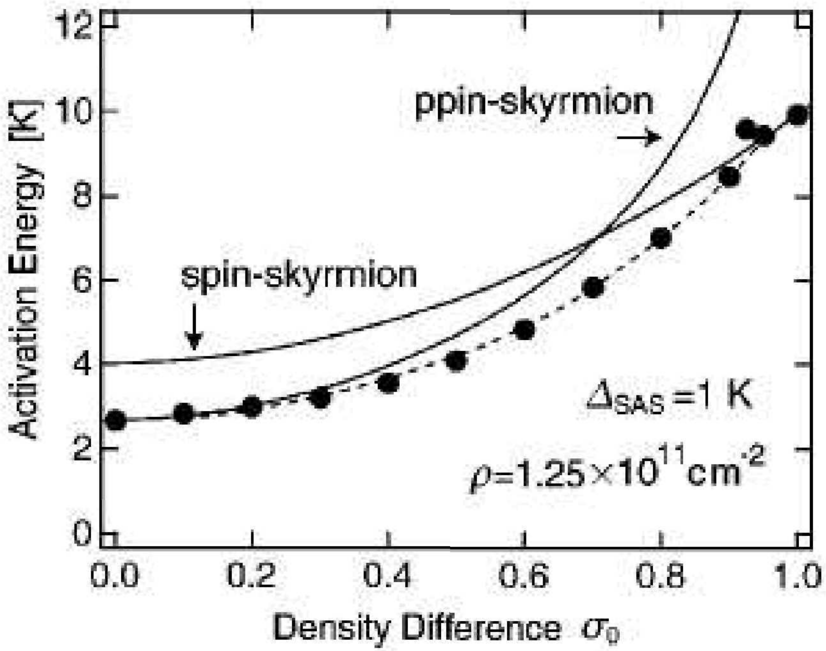
Combined spin-pseudospin textures energetically competitive

Bourassa et al, PRB **74**, 195320 (2006)

The case for entangled textures II

Bilayer with charge imbalance

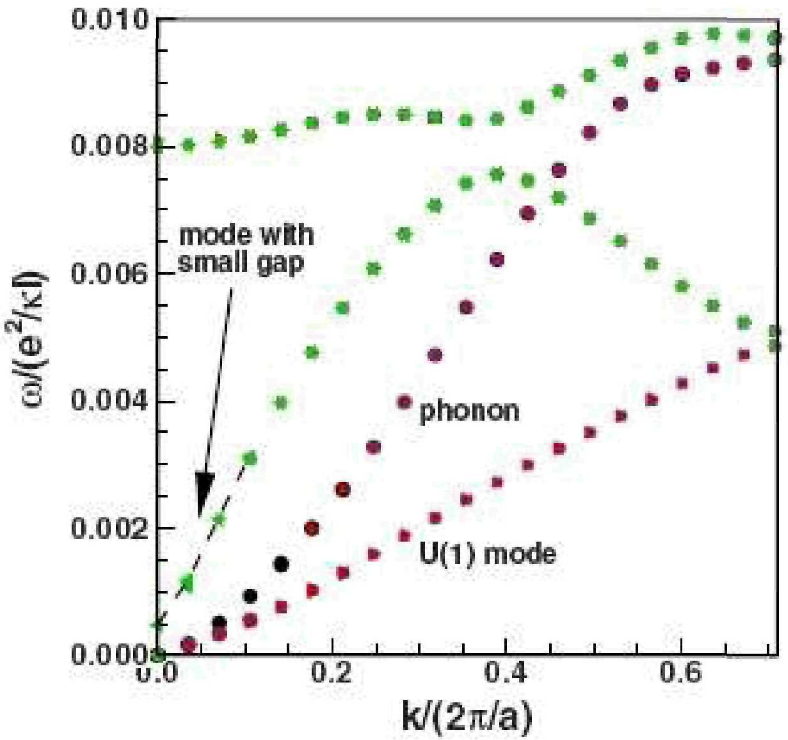
- poor variational energy of pure Skyrmions



Ezawa, Tsitsishvili,
PRB **70**, 125304 (2004)

Collective mode spectrum

- extra soft mode



Côté et al., PRB **76**, 125320 (2007)

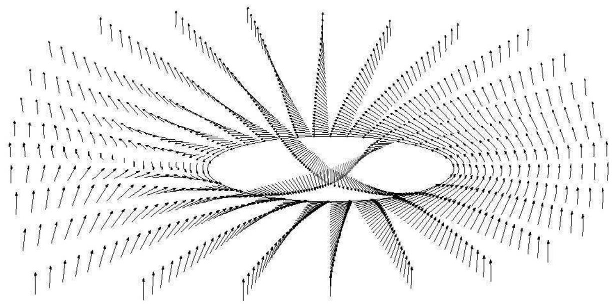
Skyrmion parametrisation

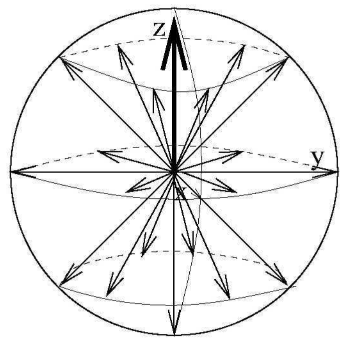
Slowly spatially varying spinor for Skyrmion of 'size' λ:

$$|\sigma(\mathbf{r})\rangle = \begin{pmatrix} \cos\frac{\theta(\mathbf{r})}{2} \\ \sin\frac{\theta(\mathbf{r})}{2}\exp[i\phi(\mathbf{r})] \end{pmatrix}$$

with 'stereographic projection'

$$\tan\frac{\theta(\mathbf{r})}{2}\exp[i\phi(\mathbf{r})] = \frac{x+iy}{\lambda} \equiv \frac{z}{\lambda}$$





Skyrmions for multicomponent systems I

Four-component spinors

$$\begin{aligned} w_1 &\rightarrow \uparrow, \mathrm{top} \\ w_2 &\rightarrow \uparrow, \mathrm{bottom} \\ w_3 &\rightarrow \downarrow, \mathrm{top} \\ w_3 &\rightarrow \downarrow, \mathrm{bottom} \end{aligned}$$

For an isotropic system, w_1,...,w_4 are analytic functions of spatial coordinate z.

Topological density

- Berry connection

$$\begin{aligned} \mathcal{A} &= \frac{1}{i}\langle\Psi|\nabla\Psi\rangle \\ \oint \mathcal{A}.d\mathbf{r} &= 2\pi Q_{\mathrm{top}} \end{aligned}$$

- $Q_{\mathrm{top}} = \pm 1$ for a skyrmion.
- ‘restores’ commensurability in presence of hole

Skyrmions for multicomponent systems II

General texture (spin down, pseudospin along x at $|z| = \infty$):

$$\begin{aligned} |\mathcal{G}\rangle &= (w_1, w_2, w_3, w_4) \\ &= (\lambda_1, \lambda_2, z - b, z + b)/(|\lambda_1|^2 + |\lambda_2|^2 + 2|z|^2 + 2|b|^2) \end{aligned}$$

Special cases:

- $\lambda_1 = \lambda_2,\ b = 0 \ldots$ spin Skyrmion $|\mathcal{S}\rangle$
- $\lambda_1 = \lambda_2 = 0 \ldots$ 'bi-Meron' $|\mathcal{M}\rangle$
- $\lambda_1 = -\lambda_2,\ b = 0 \ldots$ entangled texture $|\mathcal{E}\rangle$

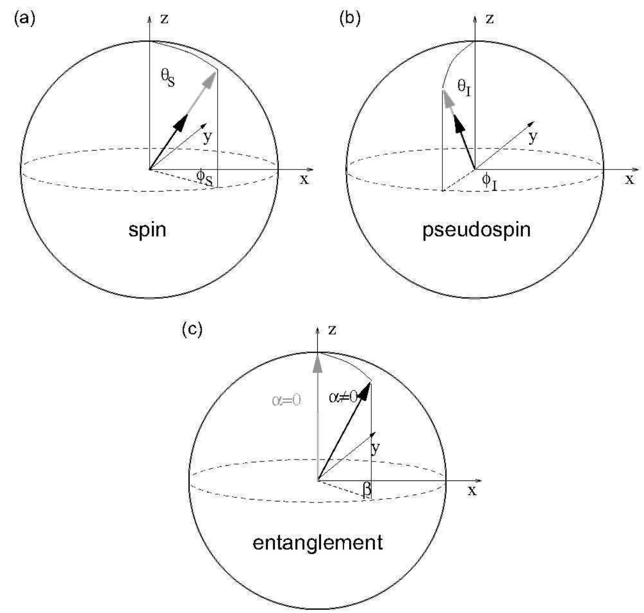
SU(2)×SU(2) parametrisation by Schmidt decomp.

Two SU(2) copies $\Rightarrow$ four-spinor w (slowly varying)

- define local spinors $|\psi_{S,I}\rangle\ ,\ |\chi_{S,I}\rangle$ for spin S and ps.spin I.
- Schmidt decomposition for full SU(4) wavefunction:

$$\Psi(z) = \cos\left(\frac{\alpha}{2}\right)|\psi_S\rangle\otimes|\psi_I\rangle + \sin\left(\frac{\alpha}{2}\right)\exp\left(i\beta\right)|\chi_S\rangle\otimes|\chi_I\rangle$$

- S, I manifestly on equal footing
- third Bloch sphere appears



Topological density

Berry connection $\mathcal{A} = \frac{1}{i}\langle\Psi|\nabla\Psi\rangle$ and top. density $\mathcal{B} = \nabla \times \mathcal{A}$

$$\mathcal{A}(\mathbf{r}) = \sin^2\frac{\alpha}{2}\nabla\beta + \cos\alpha\left(\sin^2\frac{\theta_S}{2}\nabla\phi_S + \sin^2\frac{\theta_I}{2}\nabla\phi_I\right)$$

$$\begin{aligned}\mathcal{B}(\mathbf{r}) \quad = \quad & \cos\alpha\{\rho_{\text{top}}\left[\mathbf{n}\left(\theta_S,\phi_S\right)\right] + \rho_{\text{top}}\left[\mathbf{n}\left(\theta_I,\phi_I\right)\right]\} \\ & + \rho_{\text{top}}\left[\mathbf{n}(\alpha,\beta)\right] \\ & + \ \sin^2\frac{\theta_S}{2}\rho_{\text{top}}\left[\mathbf{n}(\alpha,\phi_S)\right] + \sin^2\frac{\theta_I}{2}\rho_{\text{top}}\left[\mathbf{n}(\alpha,\phi_I)\right]\end{aligned}$$

where $\rho_{top} = \frac{\epsilon^{ij}}{8\pi}\mathbf{n}(\theta,\phi)\cdot\left[\partial_i\mathbf{n}(\theta,\phi)\times\partial_j\mathbf{n}(\theta,\phi)\right]$

- one term depends on α,β *only*: $\rho_{top}(\alpha,\beta)$

$\Rightarrow$ corresponding Skyrmion is $|\mathcal{E}\rangle$!

Entanglement of spin and pseudospin

For $\sin\alpha \neq 0$, S and I are entangled:

$$\begin{aligned} \rho_S &= Tr_I\left(|\psi\rangle\langle\psi|\right) = \cos^2\frac{\alpha}{2}|\psi_S\rangle\langle\psi_S| + \sin^2\frac{\alpha}{2}|\chi_S\rangle\langle\chi_S| \\ m_S^a &= Tr\left(\rho_S S^a\right) = \cos\alpha\langle\psi_S|S^a|\psi_S\rangle = \cos\alpha\; n^a\left(\theta_S, \phi_S\right) \end{aligned}$$

Measure of entanglement:

$$\Xi \;=\; 1 - \sum_i \langle m_{S,I}^i\rangle^2 = \sin^2\alpha = 4|w_1 w_4 - w_2 w_3|^2$$

Factorisable state $\Leftrightarrow$ $\Xi = 0$

Entanglement is spatially non-uniform

For general texture $(2\lambda = \lambda_1 + \lambda_2, 2\delta = \lambda_1 - \lambda_2, \phi = \arg zb^*\lambda^*\delta)$:

$$\Xi \;=\; \frac{16}{\mathcal{N}}\left[|\delta^2 z^2| + |\lambda^2 b^2| + 2|\lambda\delta z b|\cos\phi\right]$$

- Entanglement minimal: $\Xi = 0$ at $z = -b\lambda/\delta, \infty$
- Entanglement maximal: $\phi = 0$ and
  $|z_M| = -\frac{|\lambda b|}{|\delta|} + \sqrt{(|b|^2 + |\delta|^2 + |\lambda|^2) + \frac{|b|^2|\lambda|^2}{|\delta|^2}}$

Unentangled textures: *only* Skyrmion $\mathcal{S}$ and bi-Meron $\mathcal{M}$
Maximal entanglement $\Xi = 1$ only for $|\lambda| = |b|$

- at single point z_M unless $b = 0$: entanglement Skyrmion $\mathcal{E}$

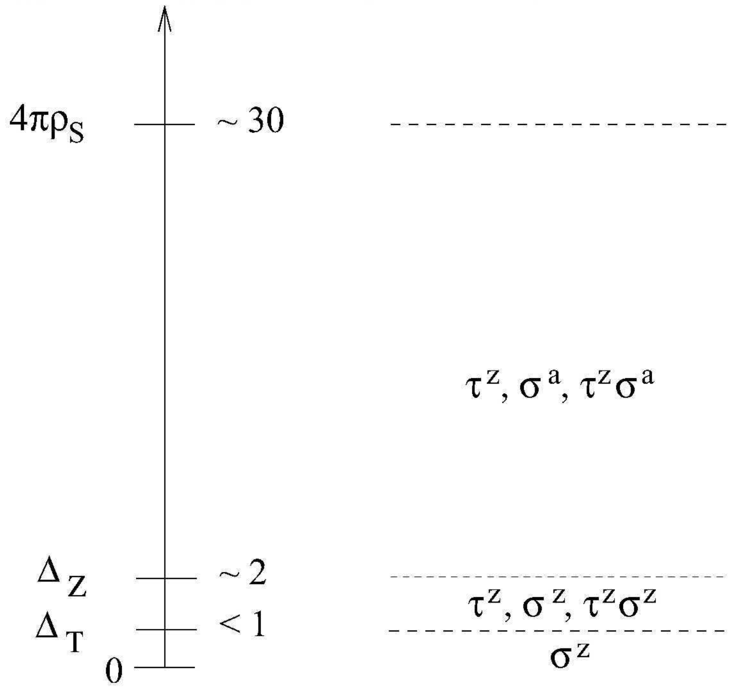
$$\langle\sigma^{x,y}\rangle = 0; \langle\tau^{y,z}\rangle = 0; \langle\sigma^z\rangle \text{ as for } \mathcal{S}$$

Realistic anisotropies

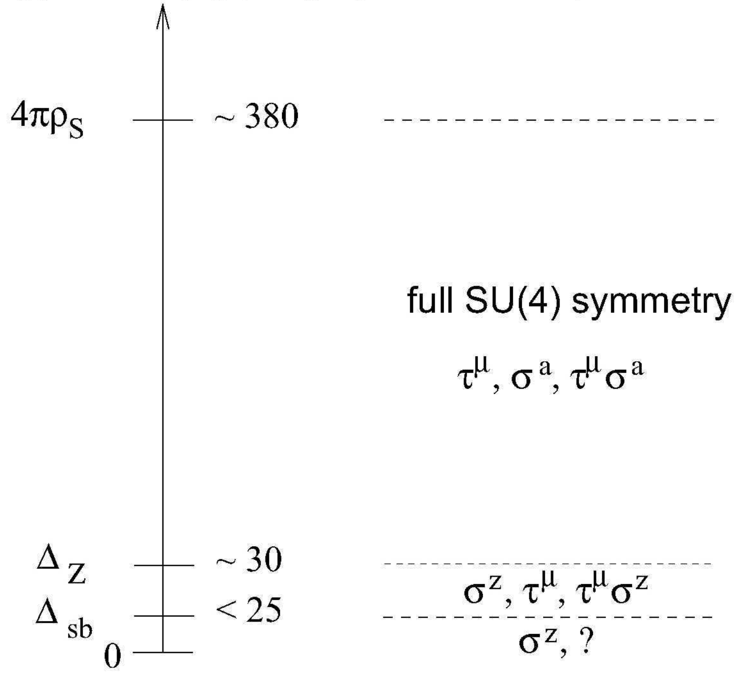
Hamiltonian can approximately have high $SU(4)$ symmetry

- Zeeman anisotropy: $SU(2) \to U(1)$
- Graphene: valley weakly split, $O(a/l_B)$
- Bilayers: charging energy: $SU(2) \to U(1)$; neglect tunnelling

Energy scales [K] (for bilayers at 6T)



Energy scales [K] (for graphene at 25T)



Degenerate texture families in anisotropic system

Still have U(1) generators: σ^z , τ^z and $\sigma^z \otimes \tau^z$

$\Rightarrow$ define entanglement operator $\mathcal{T}(\gamma) = \exp\left(i\gamma\sigma^z \otimes \tau^z\right)$

$\Rightarrow$ generates families of degenerate Skyrmions

- family members differ in entangelement

$$\Xi_{\substack{\min \\ \max}} = 4\left(|w_1 w_4| \mp |w_2 w_3|\right)^2$$

$$\begin{aligned} \Xi_{\max} &= 1 - \left[\left(|w_1| + |w_4|\right)^2 + \left(|w_2| + |w_3|\right)^2\right] \\ &\quad \times \left[\left(|w_1| - |w_4|\right)^2 + \left(|w_2| - |w_3|\right)^2\right] \end{aligned}$$

Family members

- $\mathcal{M}$ is not affected (eigenstate of σ^z)
- $\mathcal{S}$ becomes $\mathcal{E}$!!!
- $\Xi_{\min} = \Xi_{\max}$ for some $w_i = 0$
- NMR rate depends on γ

$$\begin{aligned}\langle S^+(t)S^-(0)\rangle_\gamma &= \cos^2(2\gamma)\langle S^+(t)S^-(0)\rangle_{\gamma=0} \\ &\quad + \sin^2(2\gamma)\langle S^+(t)\tau^z(t)S^-(0)\tau^z(0)\rangle_{\gamma=0}\end{aligned}$$

$\rightarrow$ NMR rate for Skyrmion crystal is open problem

Leading graphene Hamiltonian: SU(2) valley invariant

$$H^n_{SU(2)} = \frac{1}{2}\sum_{\mathbf{q}} v^G_n(q)\bar{\rho}(-\mathbf{q})\bar{\rho}(\mathbf{q});$$

with total projected density $\bar{\rho}(\mathbf{q}) = \bar{\rho}^{++}(\mathbf{q}) + \bar{\rho}^{--}(\mathbf{q})$ and

$$v^G_n(q) = \frac{2\pi e^2}{\epsilon q}\mathcal{F}^2_n(q)$$

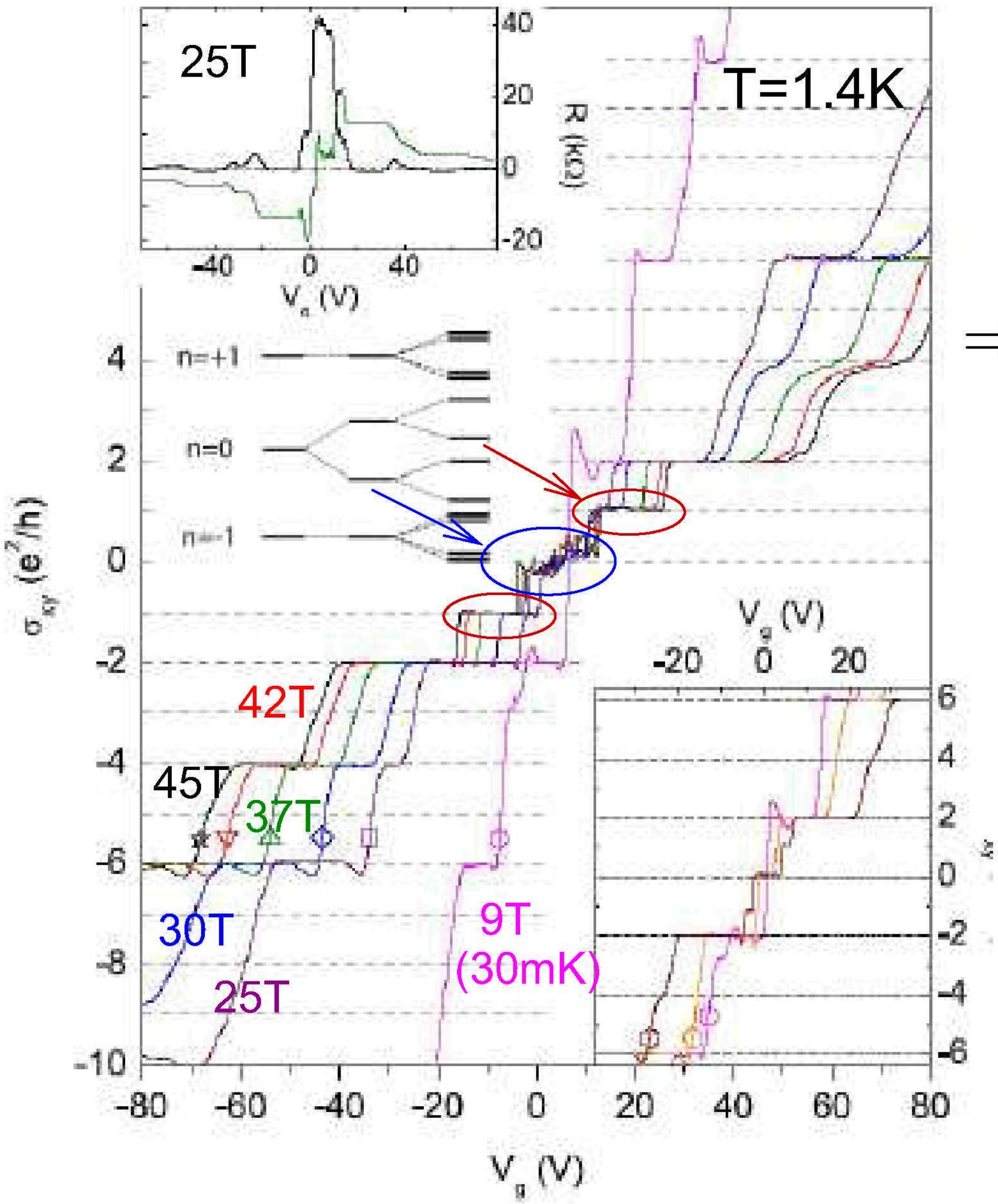
$$\mathcal{F}^2_0(q) = \exp(-q^2/2)$$

$$\mathcal{F}^2_n(q) = e^{-q^2/2}\left[L_{|n|}\left(\frac{q^2}{2}\right) + L_{|n|-1}\left(\frac{q^2}{2}\right)\right]^2$$

- Magnetic translation algebra for projected densities:

$$[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{q}')] = 2i\sin\left(\mathbf{q}\wedge\mathbf{q}'/2\right)\bar{\rho}(\mathbf{q}+\mathbf{q}')$$

Electron interactions in graphene → 'SU(4) Skyrmions



- Plateaux at $\nu = 0, \pm1, \pm4$
  Zhang et al. PRL 06

$\Rightarrow$ some addt'l Landau levels individually resolved

- Simplest consideration: gaps vs. broadening Γ
- $E_{sk}^{n=0} \approx 4$meV$> \Gamma >$ $E_{sk}^{n=1} \approx 1.8$meV
- $\nu = 4$ plateau only at strong fields$\Rightarrow$ need E_Z
- other scenarios exist. . .

Conclusions and open questions

- Generic textures exhibit entanglement
- Treatment of spin and pseudospin on equal footing
- Space-dependent entanglement as a source of topological charge ($\mathcal{E}$)
- Some symmetry operations generate entanglement
- Physical properties depend on the degree of entanglement

- Energetics of entangled textures ?
- Nature of low energy landscape ?
- Quantitative predictions for NMR ?
- Other physical signatures of entanglement ?