Cold Atoms and Molecules:
Condensed Matter Physics & Quantum Information

“Quantum Simulators”

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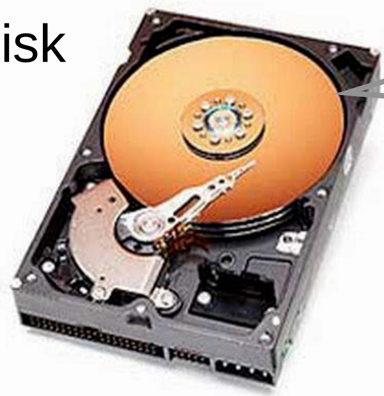
IQOQI
AUSTRIAN ACADEMY OF SCIENCES

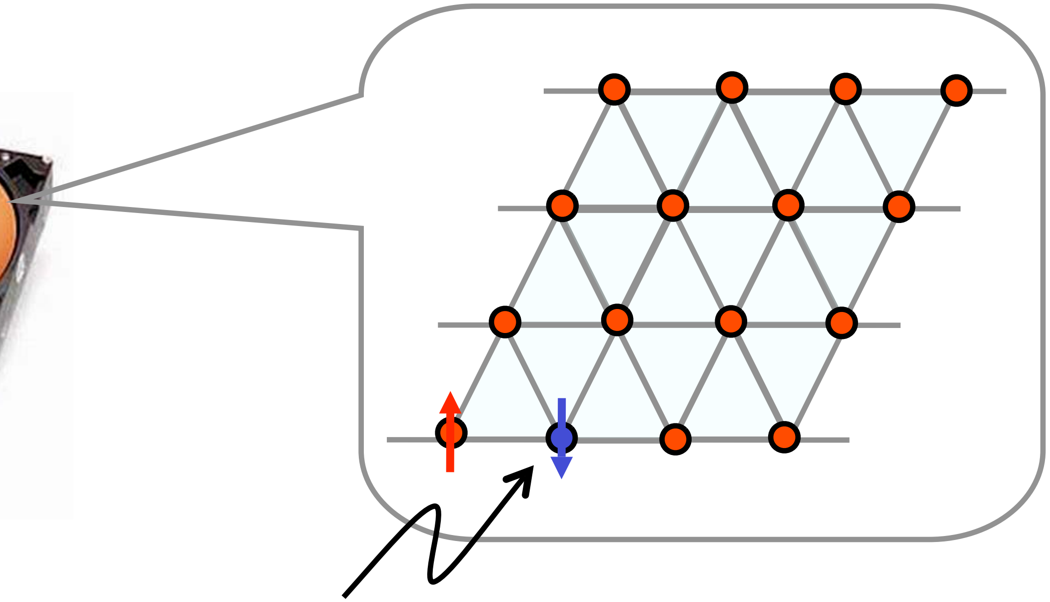
SFB
Coherent Control of Quantum Systems

€U networks

Magnetism … a simple (quantum) physics problem (?)

computer hard disk



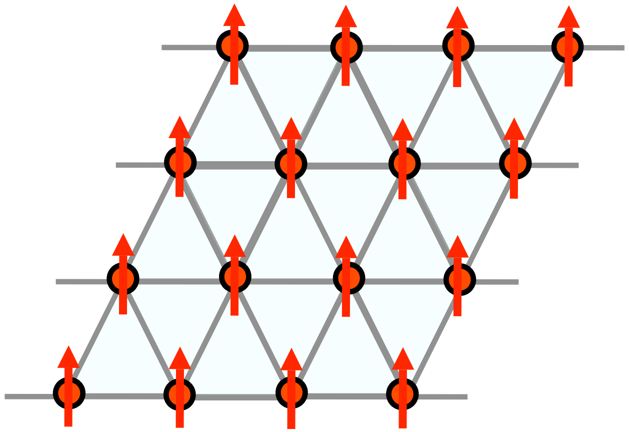


✓atoms as small magnets
✓store “0” and “1”
✓many body: spin-1/2 system

Magnetism … a simple (quantum) physics problem (?)

- **classical physics:**

  The system is in one of the possible configurations ...



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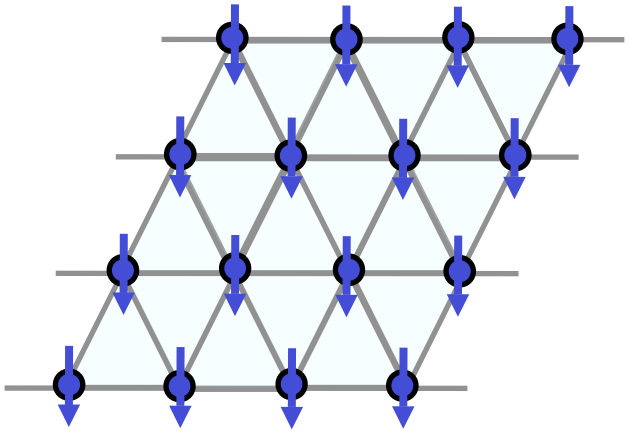

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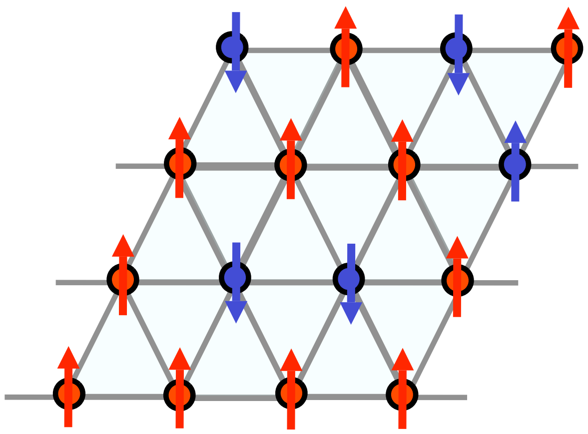
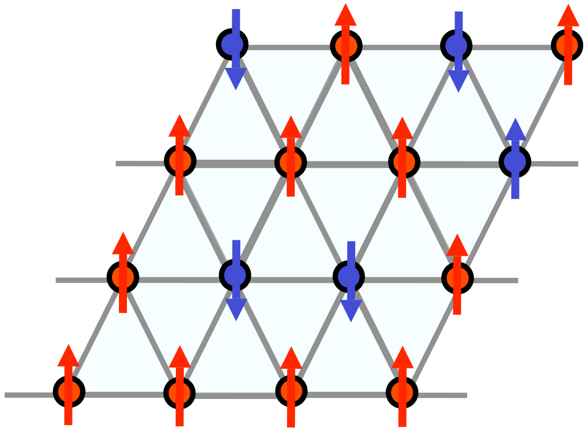


Magnetism … a *hard* quantum physics problem

- **QUANTUM physics:**

  The system is in a *superposition* state of all possible configurations …

  "**Entanglement**"



$$|\Psi\rangle = c_1|\uparrow\uparrow\uparrow\ldots\uparrow\uparrow\rangle + c_2|\uparrow\uparrow\uparrow\ldots\uparrow\downarrow\rangle + \ldots + c_{2^N}|\downarrow\downarrow\downarrow\ldots\downarrow\downarrow\rangle$$

wave function

Example: N=300 atoms

corresponds to 2^{300} complex numbers ~ # atoms in visible universe

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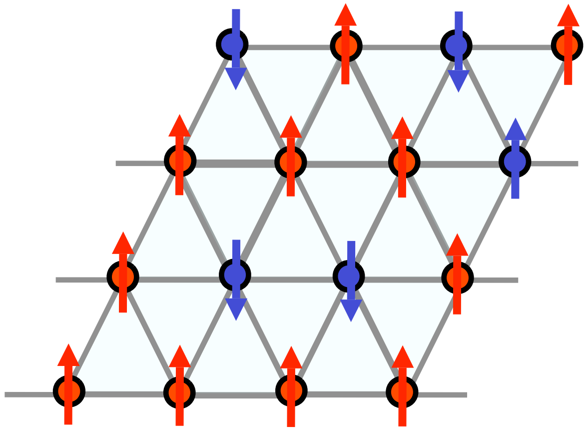

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wave function

Remarks:
- 1D / DMRG
- Quantum Monte Carlo (fermions?)
- 2D (?)

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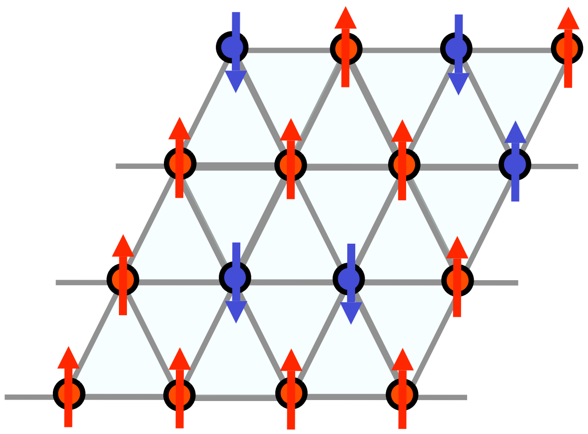

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- **Feynman**: it is difficult to simulate quantum physics on a classical computer
- **Feynman’s Universal Quantum Simulator:**

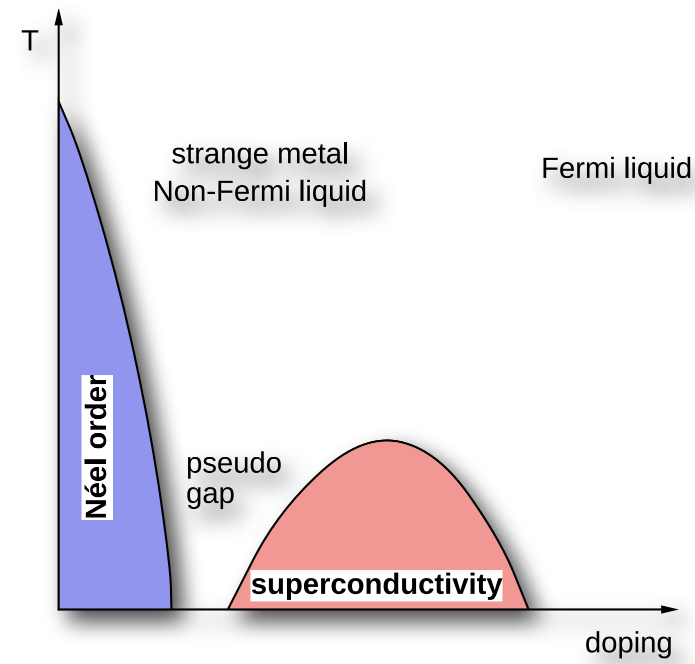
  UQS = controlled *quantum* device which efficiently reproduces the dynamics of any other many-particle quantum system (with short range interactions)

A relevant example: High-Tc Hubbard Hamiltonian

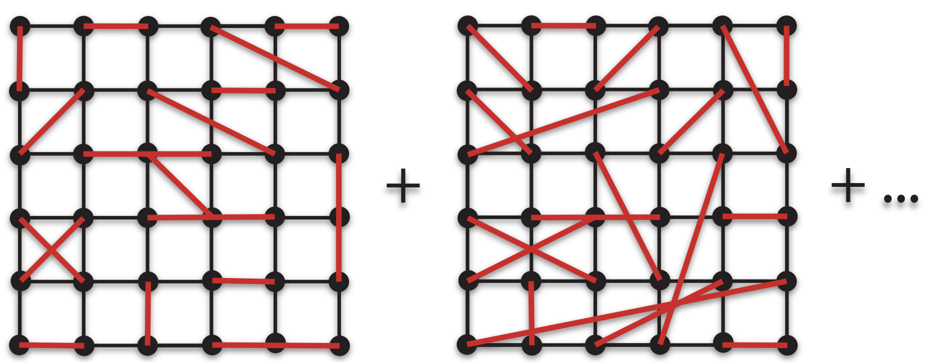
- Fermi Hubbard model in 2D

$$H = -t \sum_{i,j,\sigma} (c_{i\sigma}^{+} c_{j\sigma} + c_{j\sigma}^{+} c_{i\sigma}) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

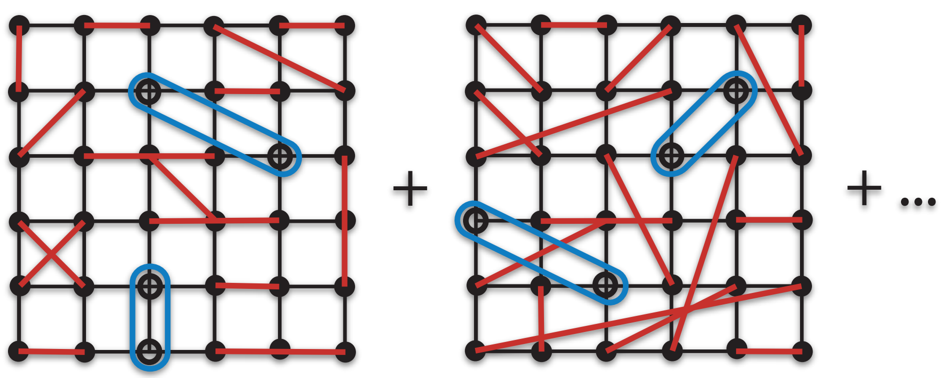
- Anderson conjectured that the high-temperature superconductor might be a doped resonant valence bond (RVB) state



half-filling (parent compounds):
superposition of singlet coverings



hole-doping:
hole pairs condense (BCS)



- … on a fundamental level unsolved problem: insight (not solution) via UQS (?)

… in Europe:

SCALA

CONQUEST

… in the US:

OLE
OPTICAL
LATTICE EMULATOR

Quantum Simulators
(special purpose quantum computers)

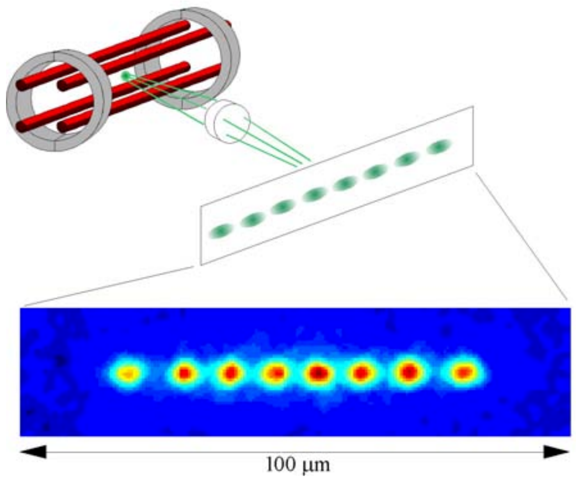
- *Analog* Quantum Simulation

- *Digital* Quantum Simulation

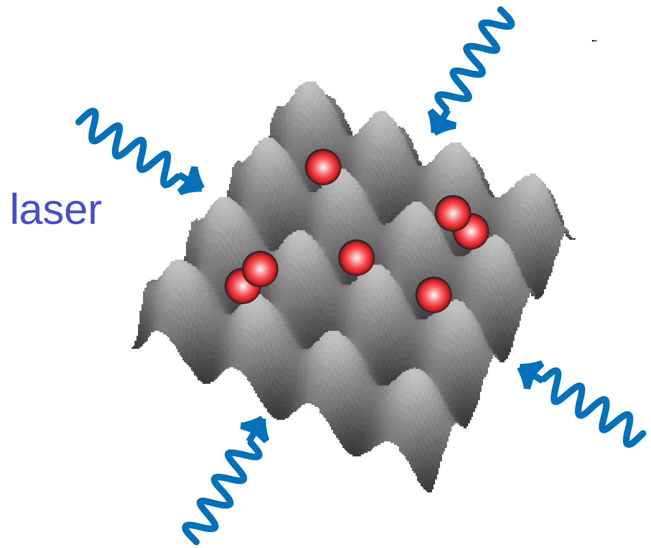
- *How to build* a quantum simulator?

Quantum optical systems

- Trapped ions

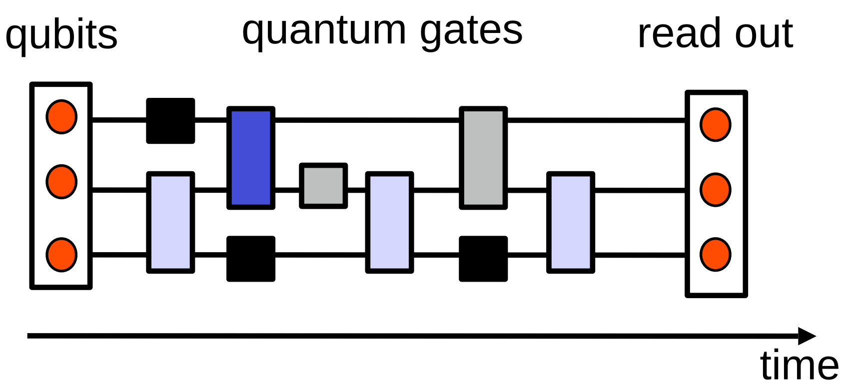


- Cold atoms in optical lattices

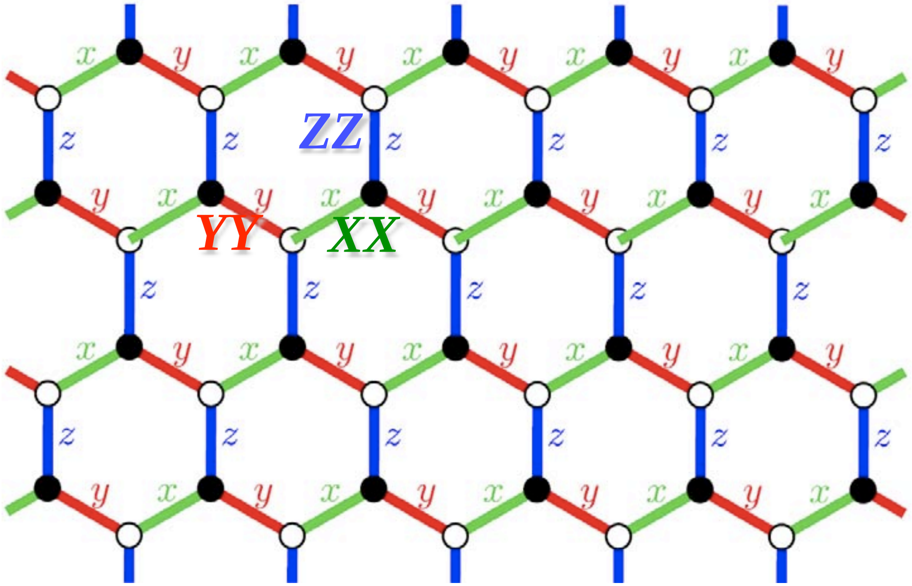


Quantum Info & Cond Mat

- Quantum Logic Network Models

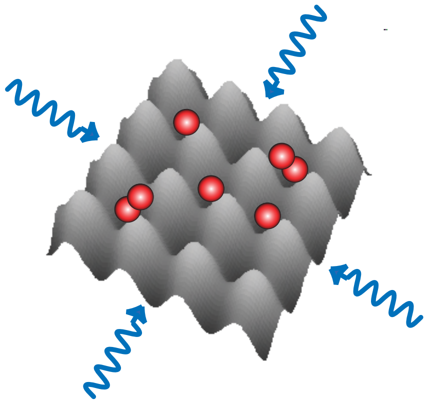


- "Quantum simulator" of cond mat models
  - Hubbard and spin models
  - analog [& digital: special purpose QC]

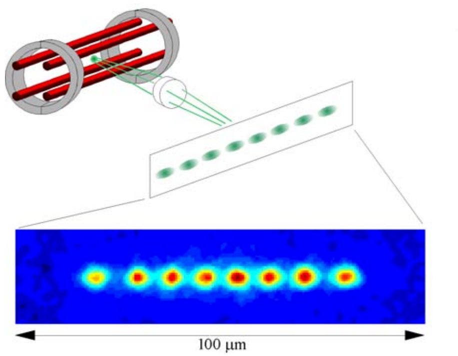


Atoms & Ions

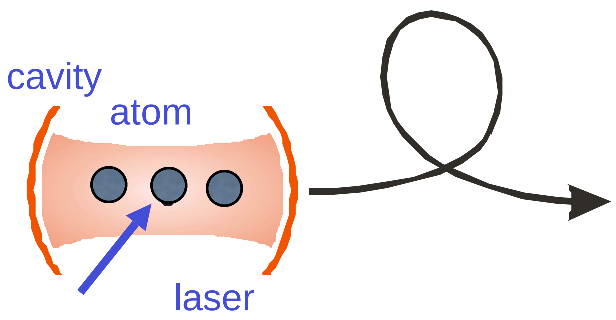
- cold atoms in optical lattices



- trapped ions / Wigner crystals

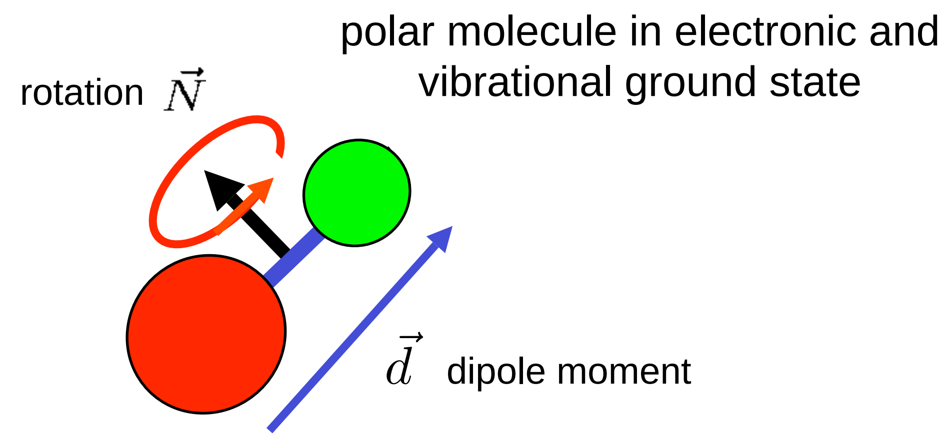


- CQED



- atomic ensembles

Polar Molecules



- what's new? ... electric dipole moment
  - couple rotation to DC / AC microwave fields
  - strong dipole-dipole / long range couplings
- ... in addition what we do with cold atoms

new system

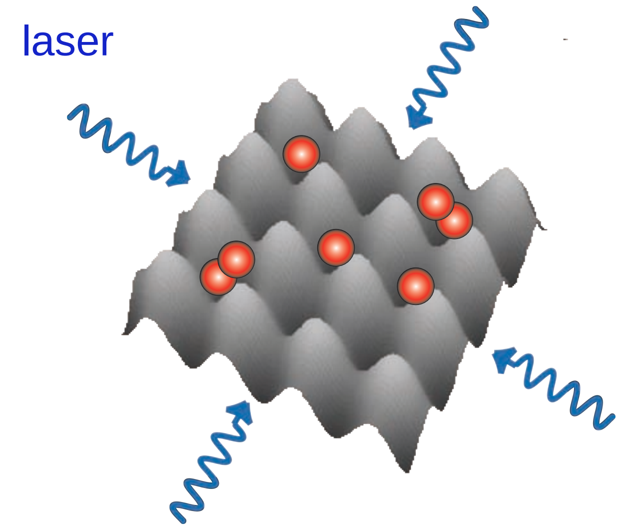
Overview:

Cold atoms ($\rightarrow$molecules) in optical lattices

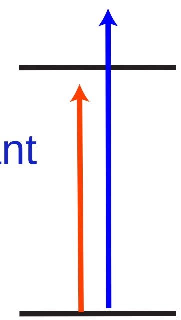
- the AMO Hubbard toolbox
- quantum info & cond mat perspective
- ... and how to make cold molecules

Cold atoms in optical lattices: Hubbard models

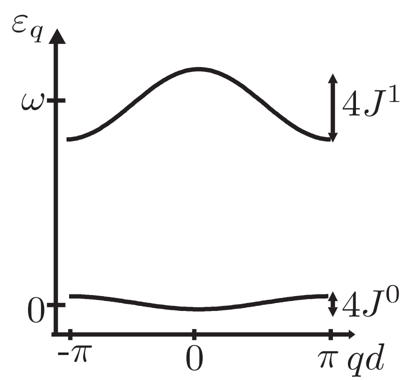
- Loading bosonic or fermionic atoms into optical lattices
- Atomic Hubbard models with controllable parameters
  - bose / fermi in 1,2&3D
  - spin models



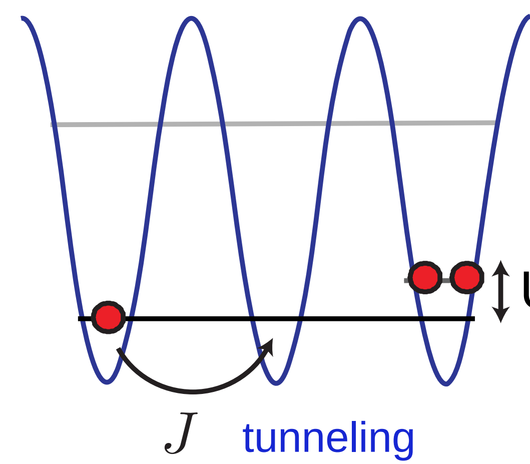
optical lattice as array of microtraps



AC Stark shift



band structure (1D)



"engineering Hubbard Hamiltonians"

= AMO toolbox

$$\hat{H} = -\sum_{\alpha\neq\beta} J_{\alpha\beta}\hat{a}^\dagger_\alpha a_\beta + \frac{1}{2}U\sum_\alpha \hat{a}^\dagger_\alpha\hat{a}^\dagger_\alpha\hat{a}_\alpha\hat{a}_\alpha$$

single band Hubbard model

kinetic energy: hopping

interaction: onsite repulsion

Cold atoms in optical lattices: Hubbard models

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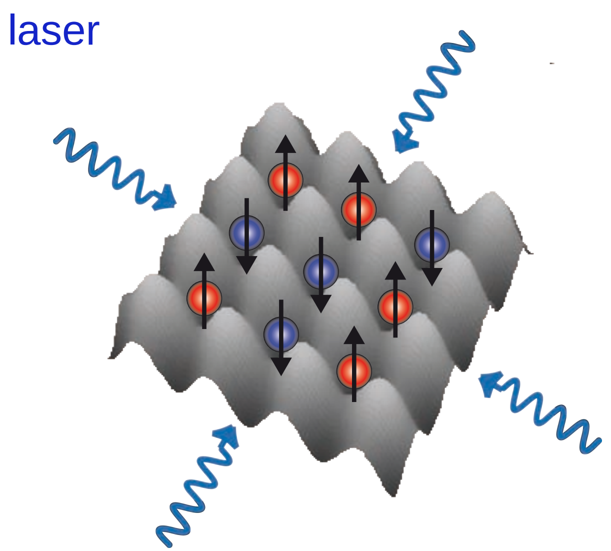

optical lattice as array of microtraps

$$\alpha \, |\uparrow\rangle + \beta \, |\downarrow\rangle$$

filling the lattice with “qubits”

Cold atoms in optical lattices: Hubbard models

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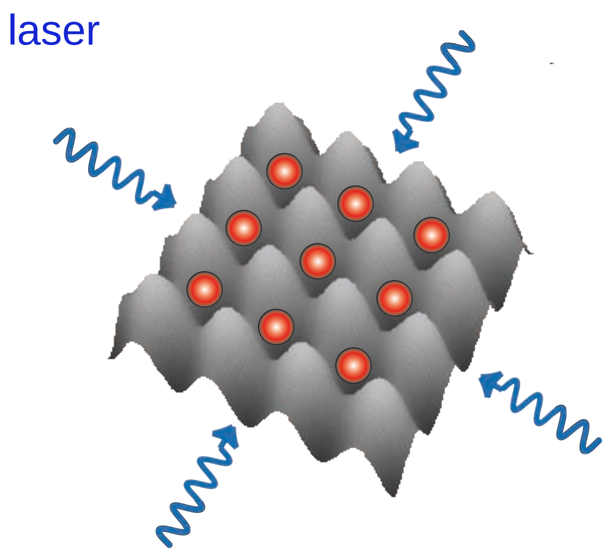

optical lattice as array of microtraps

regular filling with atoms!?

Cold atoms in optical lattices: Hubbard models

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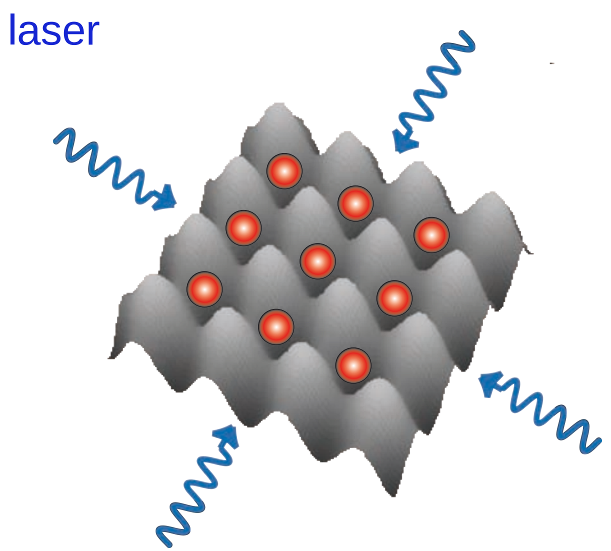
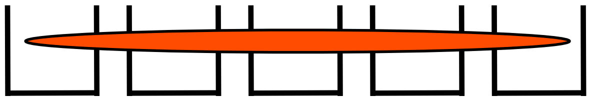


optical lattice as array of microtraps

- shallow lattice: superfluid J>>U
- deep lattice: Mott insulator J<<U

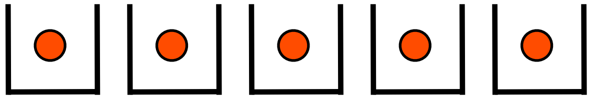


$$\left(b_1^\dagger + \ldots + b_M^\dagger\right)^N |\mathrm{vac}\rangle$$

delocalized atoms: BEC

(weakly interacting)

quantum phase transition



$$b_1^\dagger b_2^\dagger \ldots b_M^\dagger |\mathrm{vac}\rangle$$

"Fock states"

(strongly interacting)

Cold atoms in optical lattices: Hubbard models

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- Atomic Hubbard models with controllable parameters
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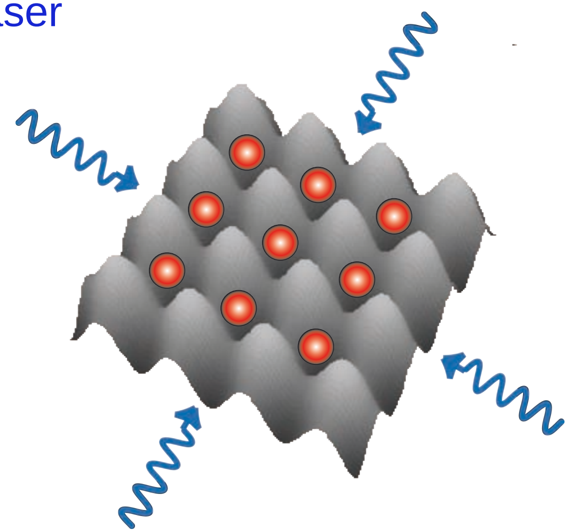
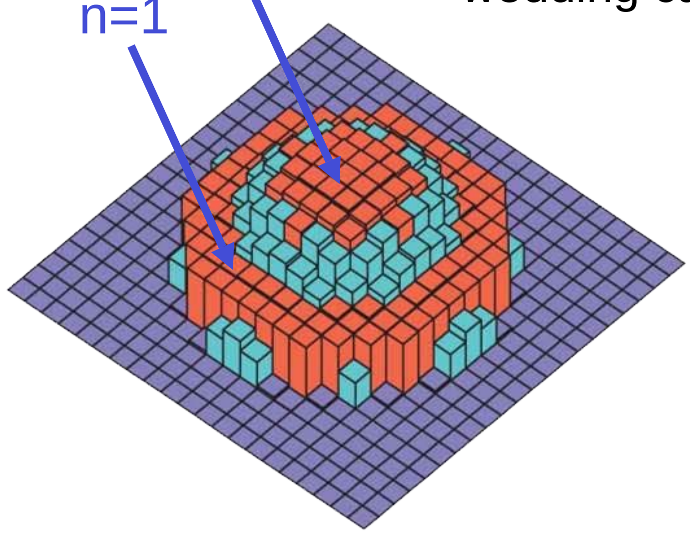
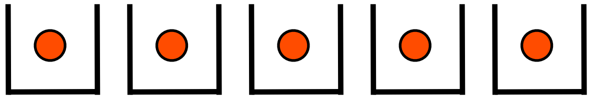


optical lattice as array of microtraps



density in harmonic trap: "wedding cake"

- deep lattice: Mott insulator J<<U



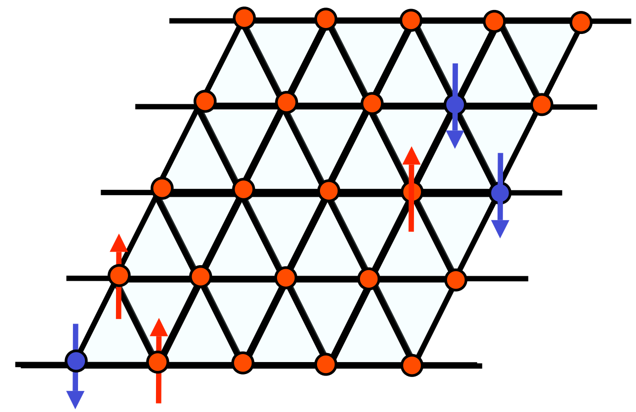
$$b_1^\dagger b_2^\dagger \ldots b_M^\dagger |\mathrm{vac}\rangle$$

"Fock states"

(strongly interacting)

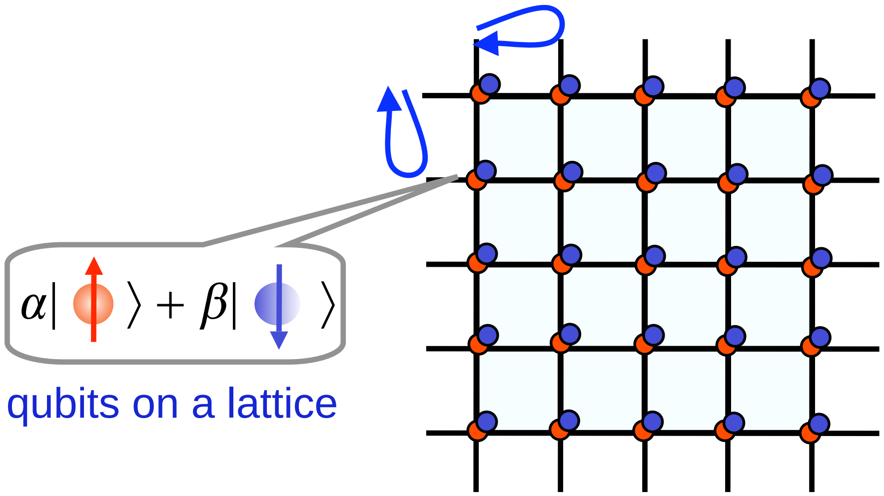
Why? … condensed matter physics & quantum information

- condensed matter physics
  - strongly correlated systems
  - time dependent, e.g. quantum phase transitions
  - …
  - exotic quantum phases (?)



- quantum information processing
  - new quantum computing scenarios, e.g. "one way quantum computer"

"quantum simulator"



qubits on a lattice

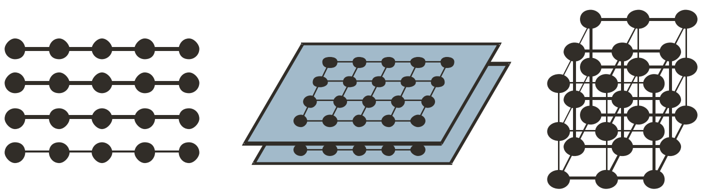
entangling qubits via "Ising" (cluster state)

- experiments [Bloch et al. 2001, Esslinger, Denschlag / Grimm, Porto, Ketterle, ...]
  - Superfluid-Mott insulator quantum phase transition
  - spin dependent lattice & entanglement
  - molecules …

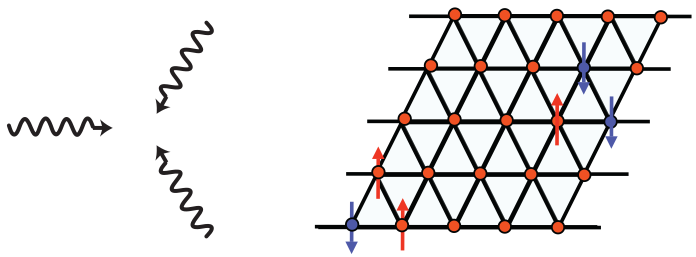
AMO Hubbard toolbox

D. Jaksch & PZ,
Annals of Physics 2005

- time dependence
- 1D, 2D & 3D

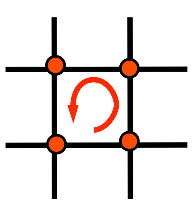


- various lattice configurations



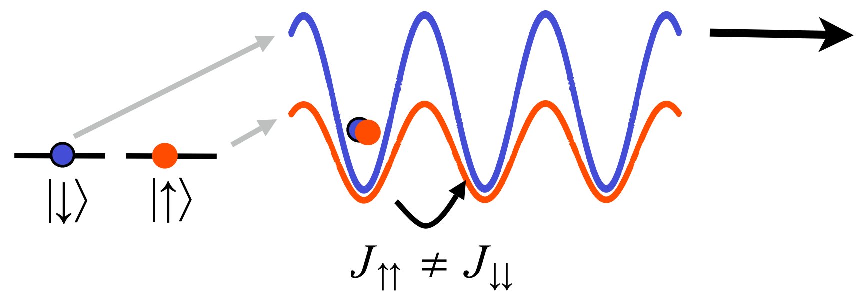
laser   triangular lattice

- create effective magnetic fields



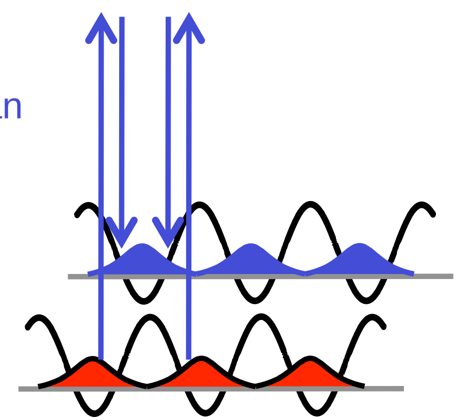
$$J_{\alpha\beta} \longrightarrow J_{\alpha\beta}\, e^{ie \int_\alpha^\beta \vec{A}\cdot d\vec{l}}$$

- spin-dependent lattices



- laser induced hoppings

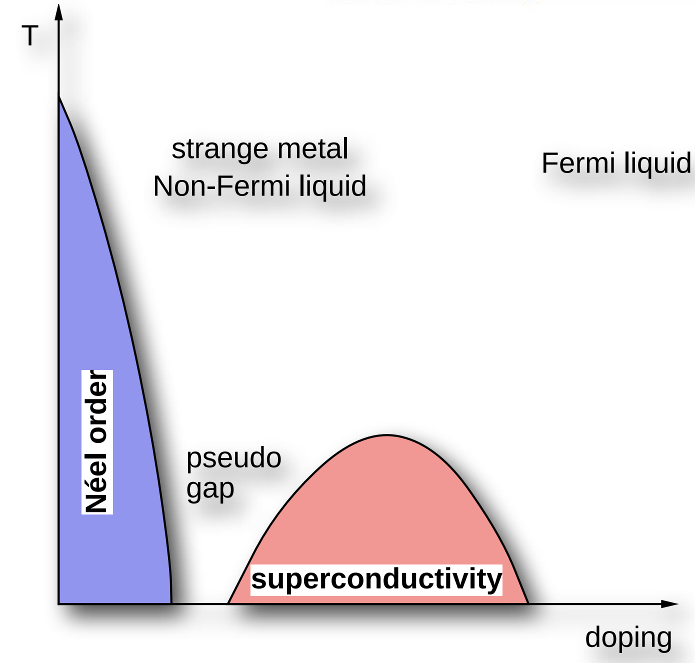
Raman



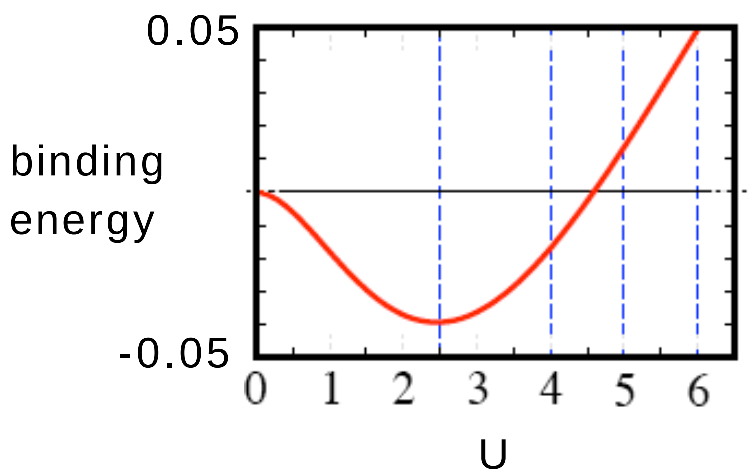
1. High-Tc Hubbard Hamiltonian

- Fermi Hubbard model in 2D

$$H = -t \sum_{i,j,\sigma} (c_{i\sigma}^{+} c_{j\sigma} + c_{j\sigma}^{+} c_{i\sigma}) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



- atomic physics challenges: cooling (!?), measurement, ...



binding energy 4% of width of Bloch band

(units of hopping t)

2. Lattice Spin Models: "virtual quantum materials"

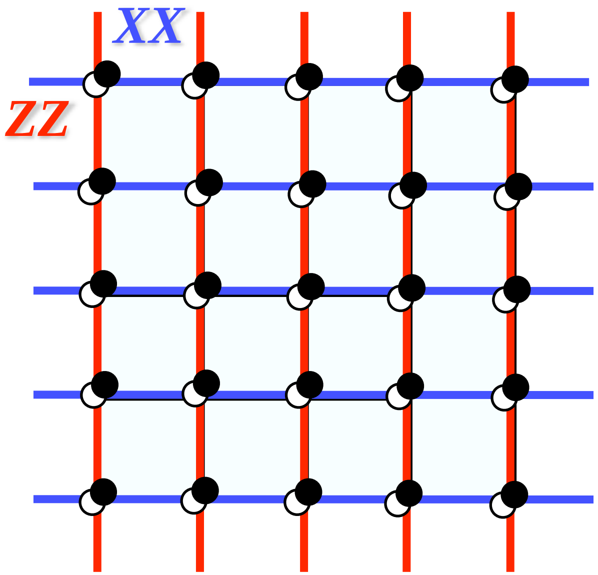
- Protected Quantum Memory*

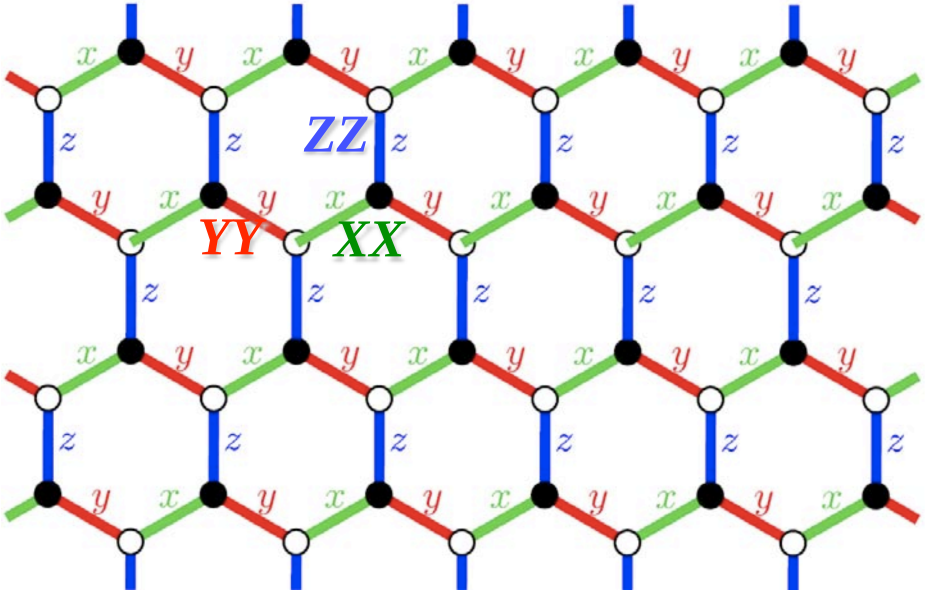
* B.Douçot, M.V.Feigel'man, L.B.Ioffe, A.S. Ioselevich, Phys.Rev.B **71**, 024505 (2005)

- The Kitaev Model**

* E.Dennis, A.Yu.Kitaev, J.Preskill, JMP **43,**4452
** A.Yu.Kitaev, *Ann. Phys.* **321**, 2 (2006)

• topologically ordered groundstates





$$H_{\rm spin} = J \sum_{i=1}^{\ell} \sum_{j=1}^{\ell} \left(\sigma^z_{i,j}\sigma^z_{i,j+1} + \cos\zeta\, \sigma^x_{i,j}\sigma^x_{i+1,j} \right)$$

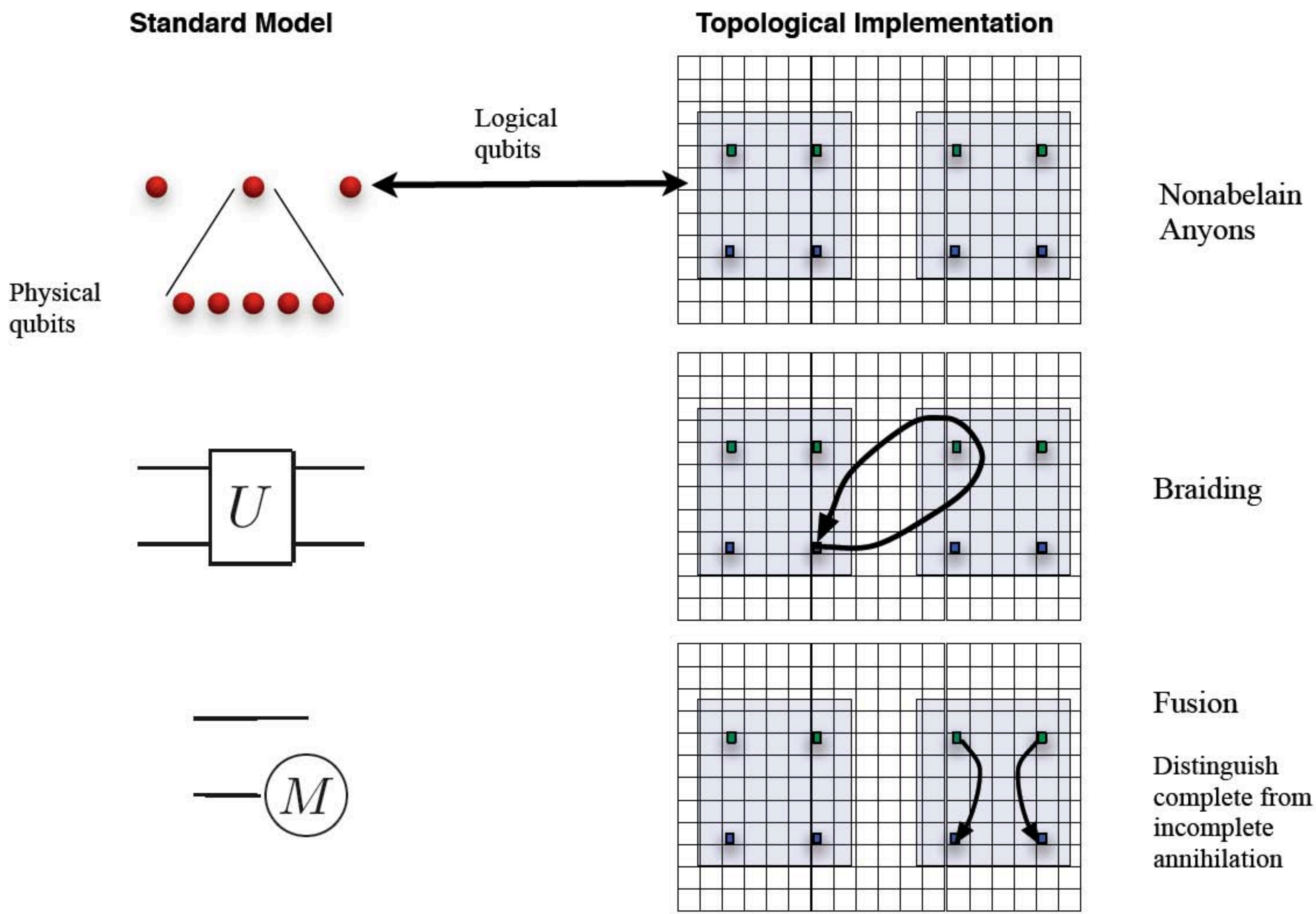
$$H_{\rm spin} = J_\perp \sum_{\rm x-lks} \sigma^x_i \sigma^x_j + J_\perp \sum_{\rm y-lks} \sigma^y_i \sigma^y_j + J_z \sum_{\rm z-lks} \sigma^z_i \sigma^z_j$$

protected quantum memory:
degenerate ground states as qubits

A. Micheli, G.K. Brennen, P. Zoller, Nature Physics 2, 341 (2006).

polar molecules

Motivation: [babysteps] towards manipulating anyons (?)



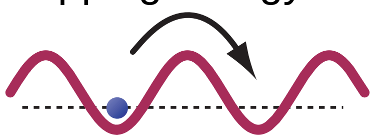
3. ... Extended Hubbard models: “exotic quantum phases”

- Extended Hubbard models in 1D and 2D:
  Example: three-body interactions with polar molecules

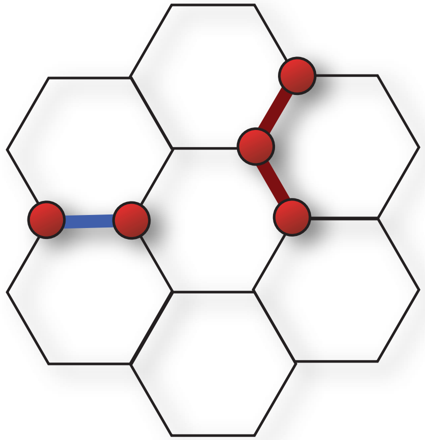
+ small next-nearest neighbor interactions

$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{1}{2} \sum_{i \neq j} U_{ij} n_i n_j + \frac{1}{6} \sum_{i \neq j \neq k} W_{ijk} n_i n_j n_k$$

hopping energy | two-body interaction | three-body interaction



compare: string net
Fidkowski et al.,
cond-mat/0610583



- strong three-body interaction

$$W/J \sim 0 \ldots 30$$
$$J \sim 0.1 E_r$$

- tunable two-body interaction

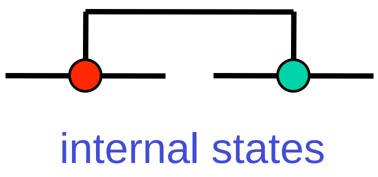
$$U/J \sim -300 \ldots 300$$

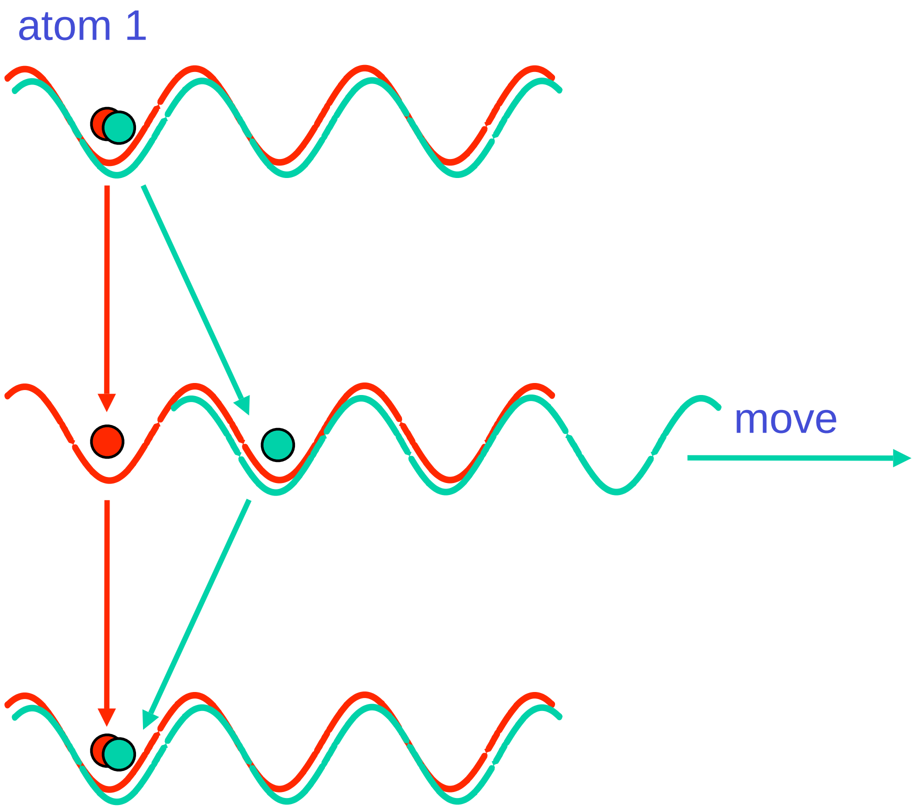
polar molecules

H.P. Büchler, A. Micheli, PZ, Nature Physics, July 2007 (article)

3. Digital Quantum Simulators

- interactions by moving the lattice: “colliding atoms by hand”





- Ising interaction as a building block for quantum simulators

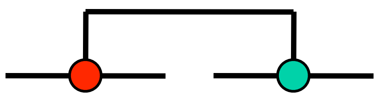
$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

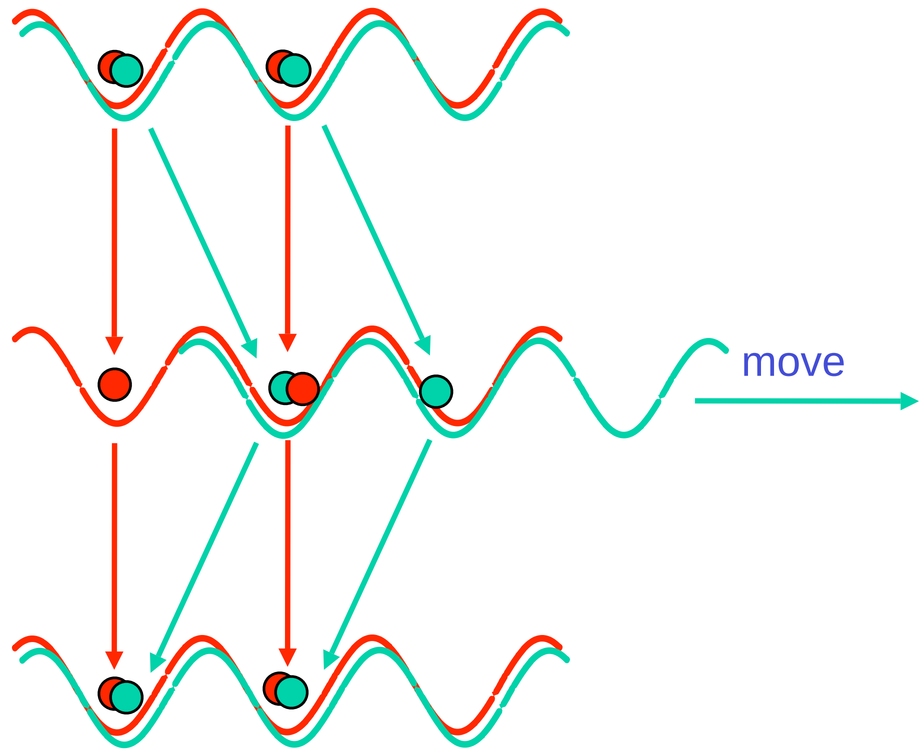
exp.: Bloch et al.

nearest neighbor, next to nearest neighbor ....

3. Digital Quantum Simulators

- interactions by moving the lattice: “colliding atoms by hand”





- Ising interaction as a building block for quantum simulators

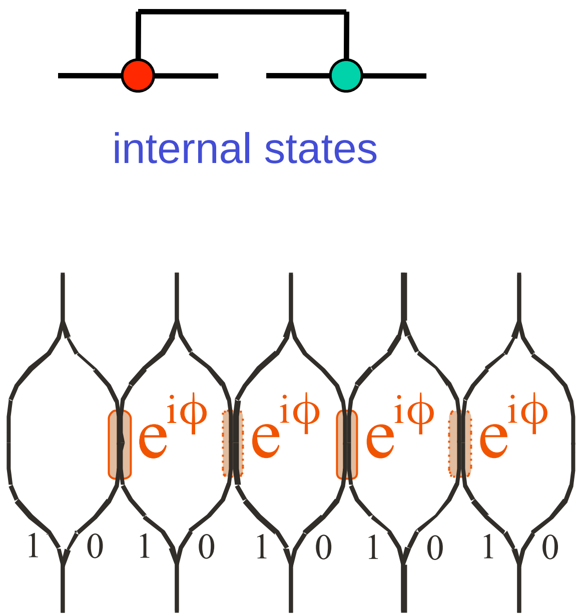
$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

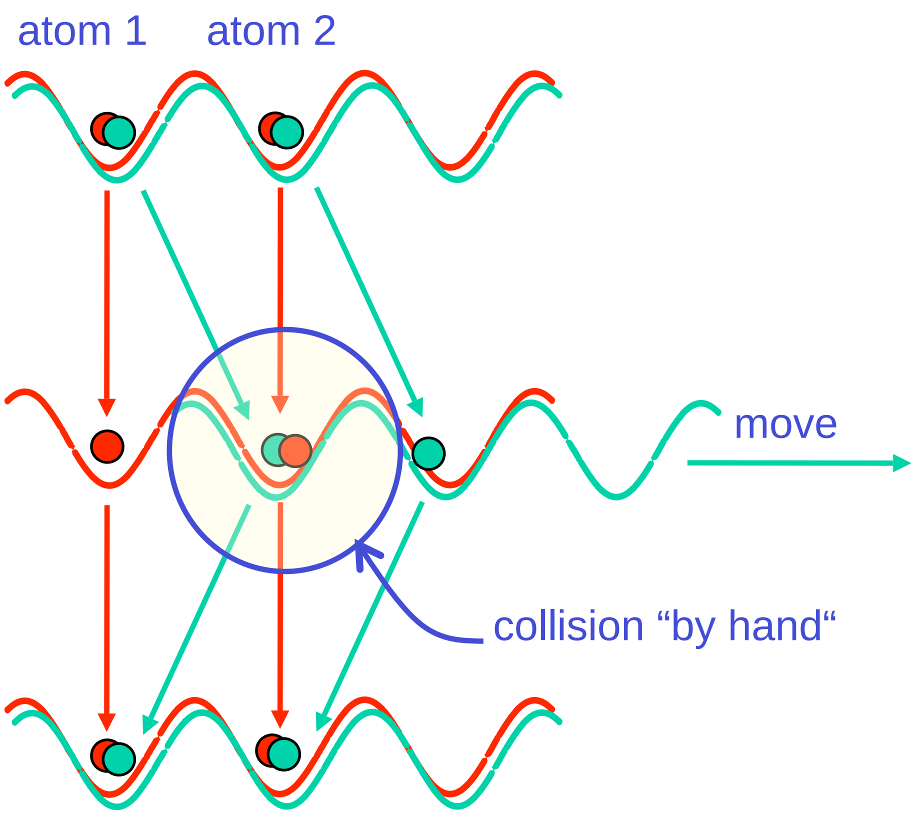
nearest neighbor, next to nearest neighbor ....

exp.: Bloch et al.

3. Digital Quantum Simulators

- interactions by moving the lattice: “colliding atoms by hand”





- Ising interaction as a building block for quantum simulators

$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

nearest neighbor, next to nearest neighbor ....

exp.: Bloch et al.

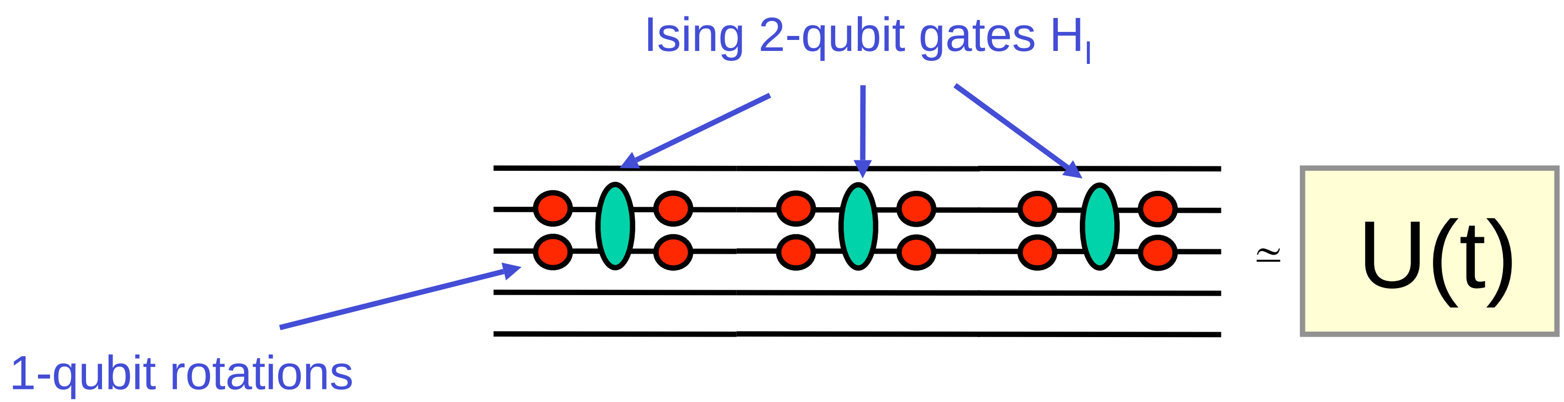
- Digital Simulator Idea: build a Hamiltonian stroboscobically from one and two-qubit gates

- Example: given Ising

$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

- ... we simulate the Heisenberg Hamiltonian

$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \left(\sigma_x^{(a)} \otimes \sigma_x^{(b)} + \sigma_y^{(a)} \otimes \sigma_y^{(b)} + \sigma_z^{(a)} \otimes \sigma_z^{(b)} \right)$$

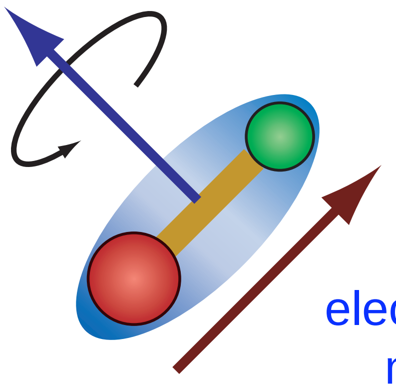


error correction?

A few slides on ...

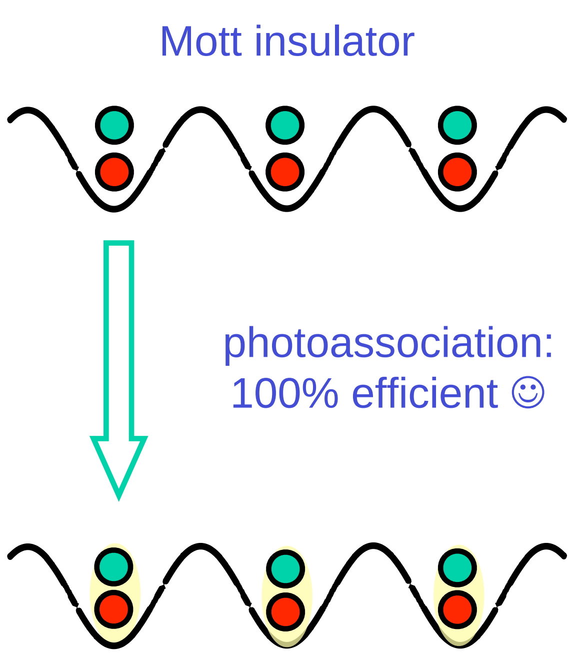
Polar molecules

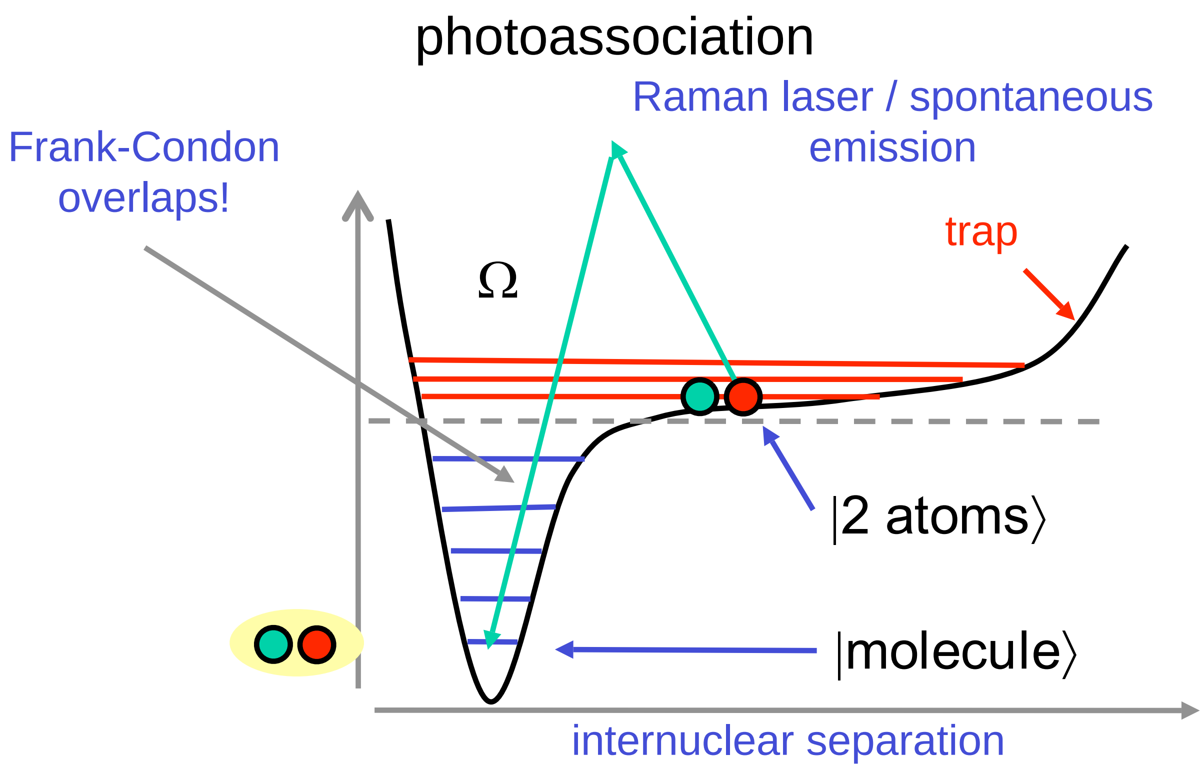
molecules in electronic and vibrational ground state



Polar Molecules in electronic and vibrational ground state

- Techniques are being developed for
  - cooling and trapping
  - preparation via …
    - ➡ photoassociation (e.g. from two-species BEC) — exp: all cold atom labs
    - buffer gas cooling — exp: Demille, Doyle, Mejer, Rempe, Ye …



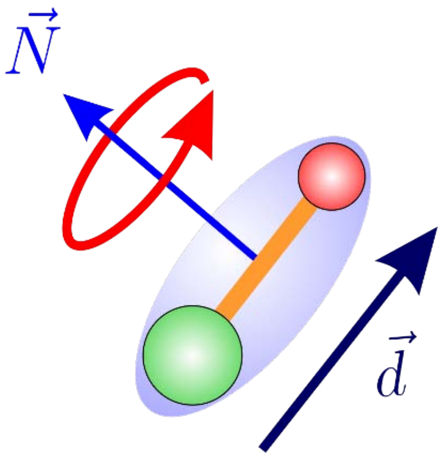


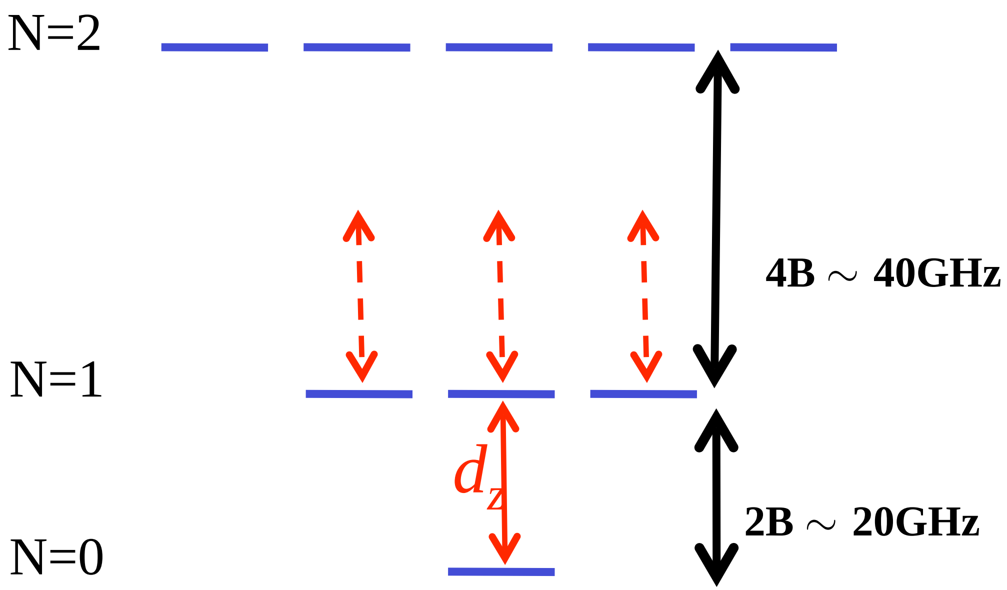
See, e.g., Special Issue on Ultracold Polar Molecules, *Eur. Phys. J. D* **31,** 149–444 (2004).

Single polar molecule I: Rotational spectroscopy

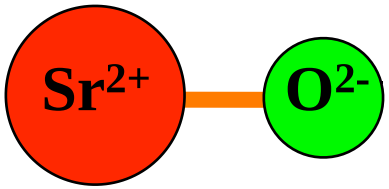
1) Rigid Rotor:

$$H = B\,\mathbf{N}^2$$





$X^1\Sigma_g^+$ closed shell molecules (SrO, CsRb, …)



Single polar molecule I: Rotational spectroscopy

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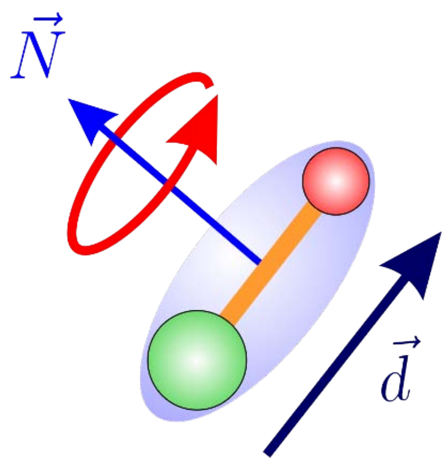

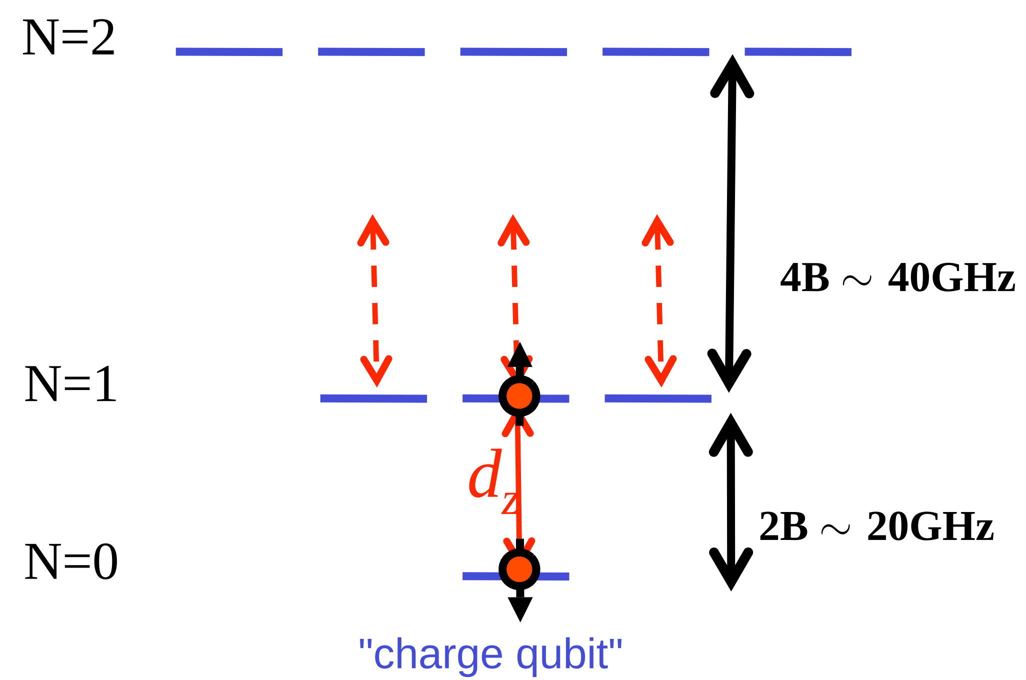

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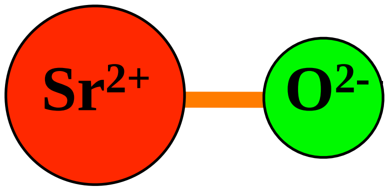
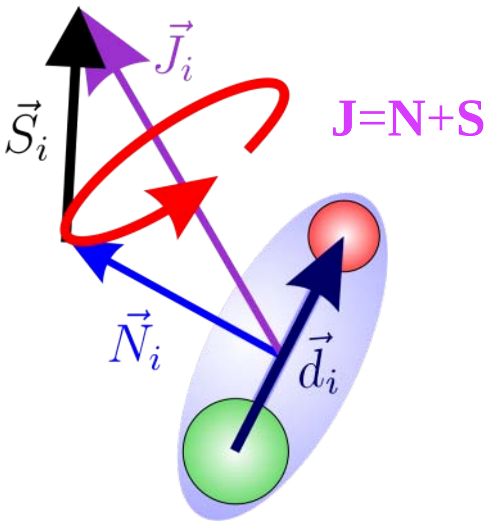


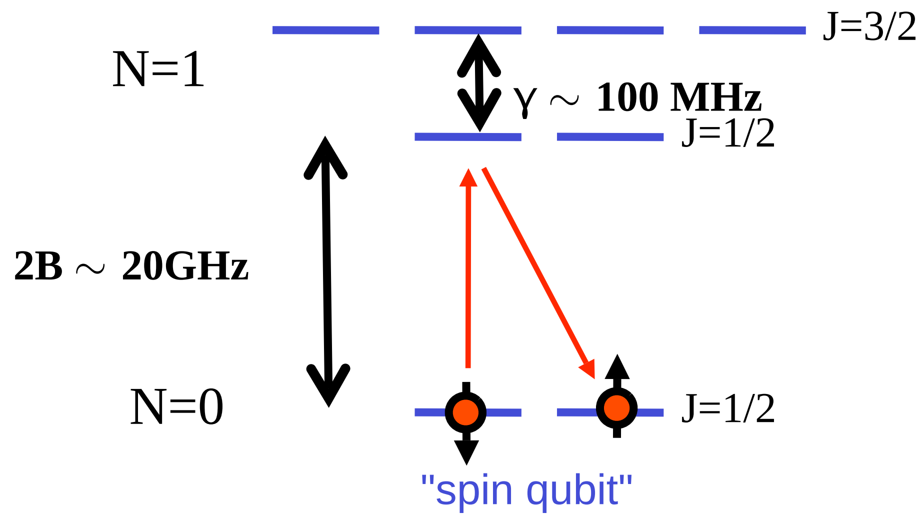
- anharmonic spectrum $E_N = B\,N(N+1)$
- electric dipole transitions $d \sim 3\text{-}10$ Debye
  - microwave transition frequencies
- no spontaneous emission $\Gamma < 0.1$ mHz
  - excited states are "useable"
- encode qubit

Single polar molecule II: Rotational spectroscopy

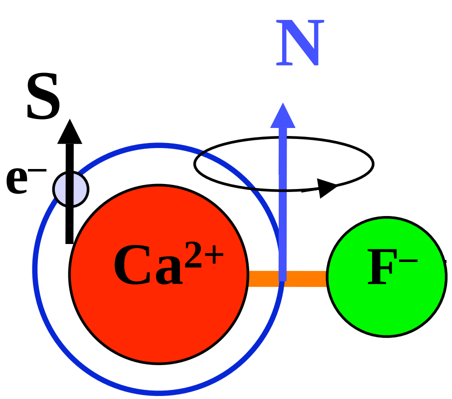
2) Spin Rotation Coupling



$$H = B\,\mathbf{N}^2 + \gamma\,\mathbf{N}\cdot\mathbf{S}$$



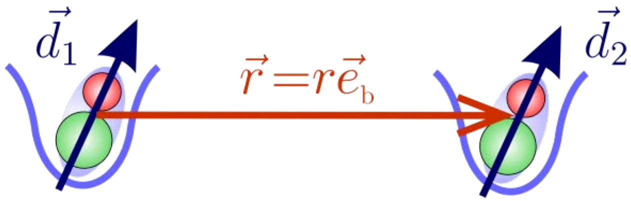
$X^2\Sigma_g^+$ molecules with an unpaired electron spin (CaF,CaCl,...)



- for e⁻ providing spin degree of freedom
  - encode qubit in rot. ground states
- strong spin-rotational mixing in N>0
  - Raman transitions

Two polar molecules: dipole – dipole interaction

- dipole moment gives rise to interaction of two molecules

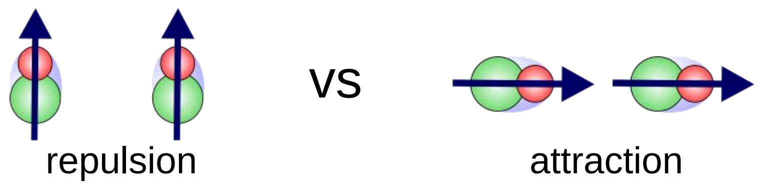


$$V_{\mathrm{dd}} = \frac{\vec{d}_1 \cdot \vec{d}_2 - 3\left(\vec{d}_1 \cdot \vec{e}_b\right)\left(\vec{e}_b \cdot \vec{d}_2\right)}{r^3}$$

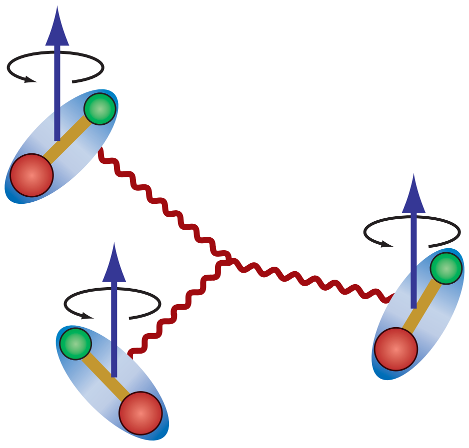
features of dipole-dipole interaction

✓ long range ~$1/r^3$

✓ angular dependence



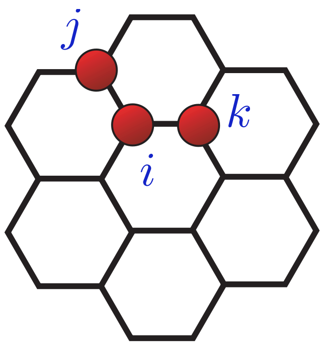
✓ strong! (temperature requirements)



Three-body interactions & extended Hubbard models

$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{1}{2} \sum_{i \neq j} U_{ij} n_i n_j + \frac{1}{6} \sum_{i \neq j \neq k} W_{ijk} n_i n_j n_k$$

hopping — *tunable* two-body interactions — *large* three-body interactions



compare: string net
Fidkowski et al., cond-mat/0610583

H.P. Büchler, A. Micheli, PZ, Nature Physics, July 2007 (article)

Dynamics with n-body interactions

We start in the continuum and add the optical lattice later

- Hamiltonians of condensed matter physics are effective Hamiltonians, obtained by integrating out the high energy excitations

$$H = \sum_i \left(\frac{\mathbf{p}_i^2}{2m} + V_{\mathrm{T}}(\mathbf{r}_i) \right) + V_{\mathrm{eff}}\left(\{\mathbf{r}_i\}\right)$$

effective interaction

$$V_{\mathrm{eff}}\left(\{\mathbf{r}_i\}\right) = \frac{1}{2}\sum_{i\neq j} V\left(\mathbf{r}_i - \mathbf{r}_j\right) + \frac{1}{6}\sum_{i\neq j\neq k} W\left(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k\right) + \ldots$$

usually small corrections

two particle interaction

three particle interaction

example: He

- Hamiltonians with three-body interactions
  - ground states with exotic phases & excitations (topological, spin liquids etc.)
  - difficult to find examples in nature (Fractional Quantum Hall Effect, ... AMO?)

Dynamics with n-body interactions

- Hamiltonians of condensed matter physics are effective Hamiltonians, obtained by integrating out the high energy excitations

$$H = \sum_i \left(\frac{\mathbf{p}_i^2}{2m} + V_{\mathrm{T}}(\mathbf{r}_i) \right) + V_{\mathrm{eff}}\left(\{\mathbf{r}_i\}\right)$$

effective interaction

$$V_{\mathrm{eff}}\left(\{\mathbf{r}_i\}\right) = \frac{1}{2} \sum_{i \neq j} V\left(\mathbf{r}_i - \mathbf{r}_j\right) + \frac{1}{6} \sum_{i \neq j \neq k} W\left(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k\right) + \ldots$$

turn off (?) — two particle interaction

three particle interaction — strong & repulsive (?)

- Cold gases of atoms and molecules
  - we know the high energy degrees of freedom & manipulate by external fields
  - Q.: switch off two-body, while generating strong repulsive three-body (?)

... with polar molecules dressed by external fields
(without introducing decoherence)

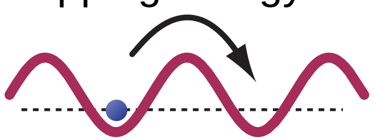
Hubbard models with three-body interactions

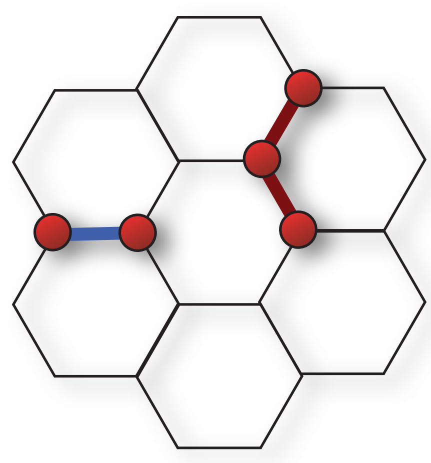
- Rem.: Typical Hubbard models with polar molecules involve strong dipole-dipole (two-body) offsite interactions
- Extended Hubbard models in 1D and 2D

+ small next-nearest neighbor interactions

$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{1}{2} \sum_{i \neq j} U_{ij} n_i n_j + \frac{1}{6} \sum_{i \neq j \neq k} W_{ijk} n_i n_j n_k .$$

hopping energy — two-body interaction — three-body interaction





- strong three-body interaction

$$W/J \sim 0...30$$

$$J \sim 0.1 E_r$$

- tunable two-body interaction

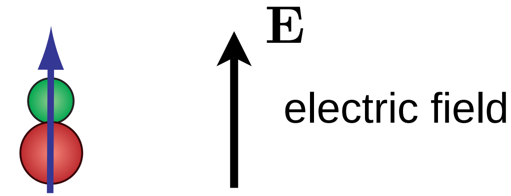
$$U/J \sim -300 \, ... \, 300$$

How to calculate effective n-body interactions ... basic idea

- Step 1: “dressed” single polar molecule

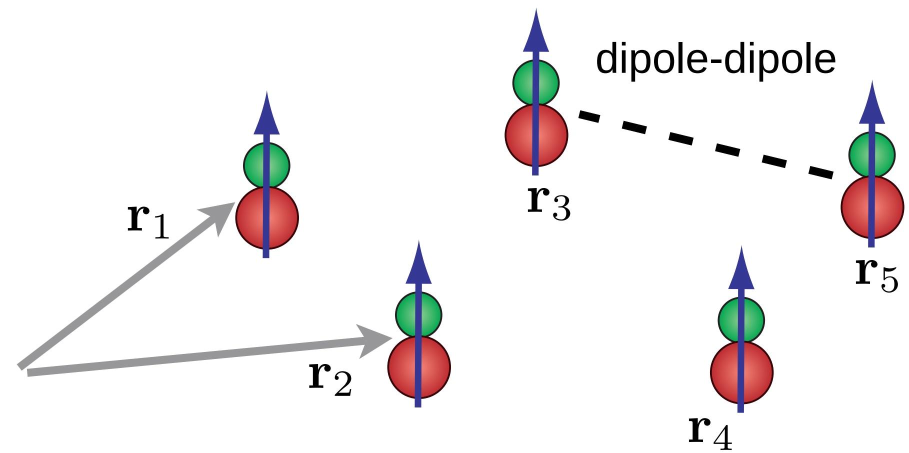
  We dress molecules prepared in the ground state by adiabatically switching on AC / DC electric fields.

polar molecules



- Step 2: interaction between molecules

  For *fixed* positions of the molecules we adiabatically switch on dipole-dipole interactions.



We identify the *interaction energy*

$$V_{\text{eff}}\left(\{\mathbf{r}_i\}\right) = \frac{1}{2}\sum_{i\neq j} V\left(\mathbf{r}_i - \mathbf{r}_j\right) + \frac{1}{6}\sum_{i\neq j\neq k} W\left(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k\right) + \dots$$

two particle interaction

three particle interaction

... with the *interaction potential* in the spirit of a Born-Oppenheimer approximation.

Our goal is now (i) to choose a molecular setup and (ii) calculate the BO potential.

Step 1: Single molecule as an effective spin-1/2

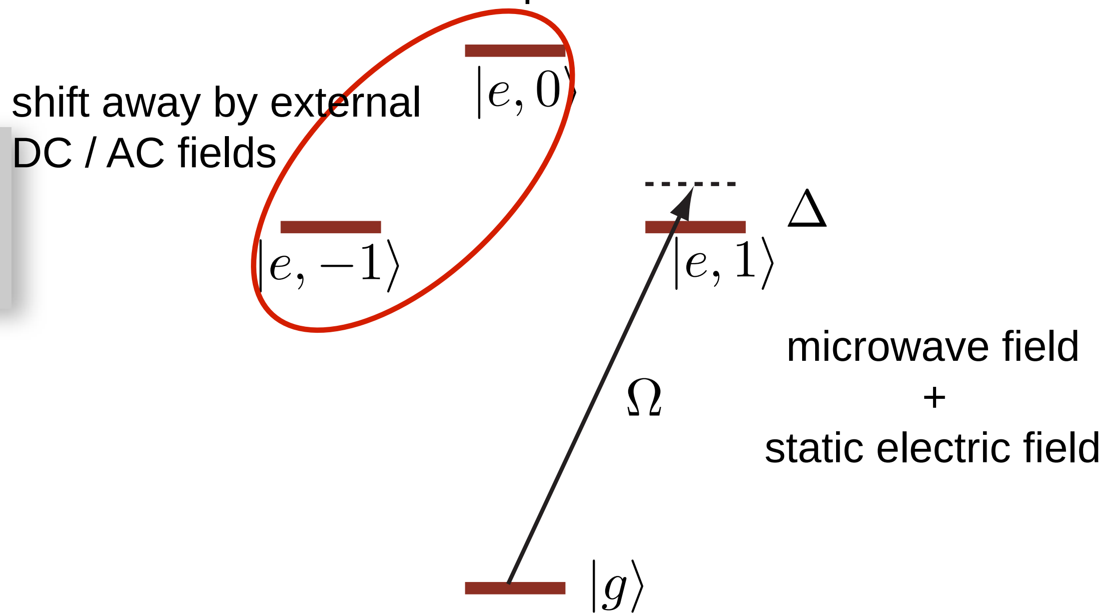
- Single molecule as a “spin-1/2 in an effective magnetic field”

Two-level System

- in rotating frame / RWA

$$H_0^{(i)} = \frac{1}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega & -\Delta \end{pmatrix} = \mathbf{h}\mathbf{S}_i$$

rotation spectrum:



- induced static dipole moments due to the static electric field

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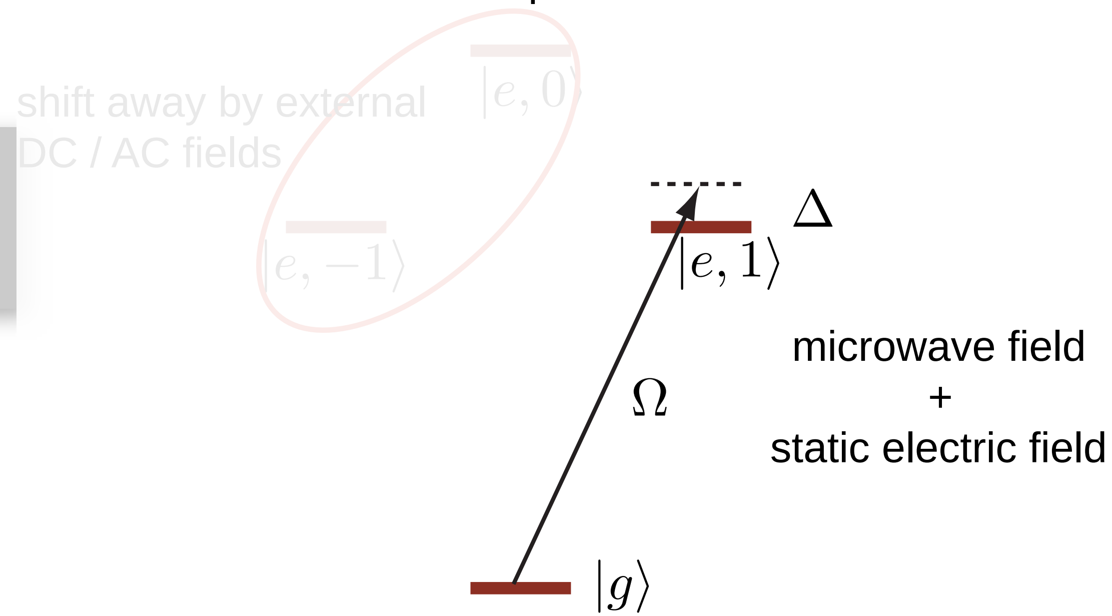
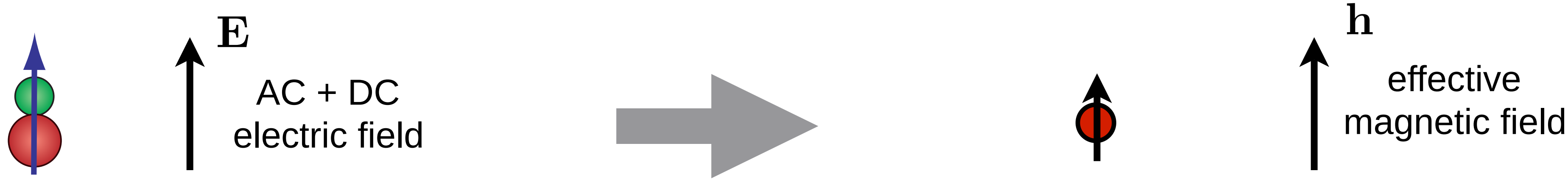


- induced static dipole moments due to the static electric field

Convenient mapping: (fixed) molecules to (fixed) spin-1/2

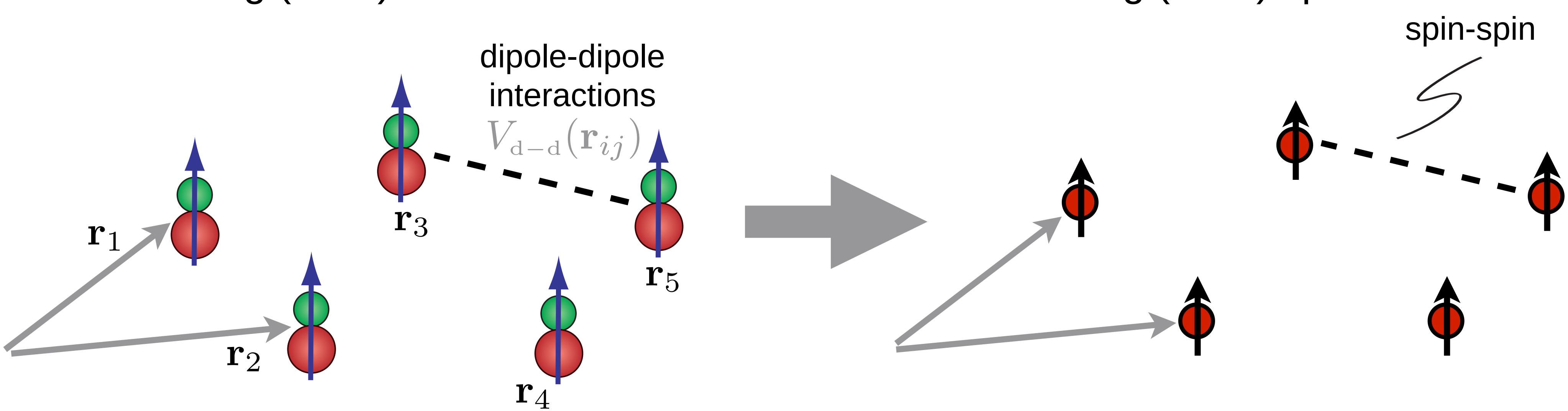
- Single molecule
- Spin-1/2 in magnetic field



$$H_{\text{rot}}^{(i)} = B\mathbf{N}_i^2 - \mathbf{d}_i\mathbf{E}(t)$$

$$H_0^{(i)} = \mathbf{h}\mathbf{S}_i$$

- Interacting (fixed) molecules
- Interacting (fixed) spins



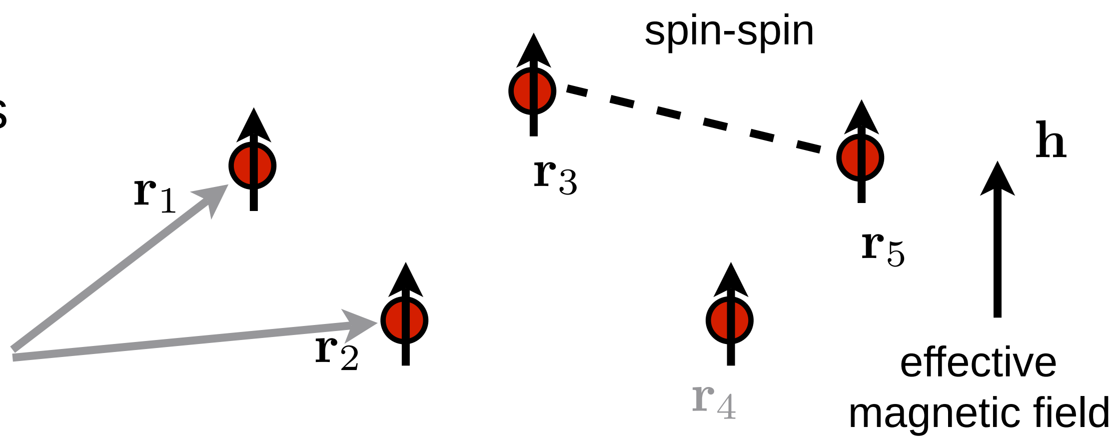
Our goal is to calculate the energy for fixed $\{\mathbf{r}_i\}$, i.e. the Born-Oppenheimer potential $V_{\text{eff}}(\{\mathbf{r}_i\})$. This is conveniently done in the spin-picture.

Step 2: Interactions

- Ensemble of (static) molecules as interacting spins in magnetic field

Dipole-dipole interaction

- in rotating frame / RWA



$$H = \sum_i \mathbf{h}\mathbf{S_i} + \sum_{i \neq j} D\left[(\cdots)(S_i^x S_j^x + S_i^y S_j^y) - (\cdots)S_i^z S_j^z + (\cdots)S_i^z\right]$$

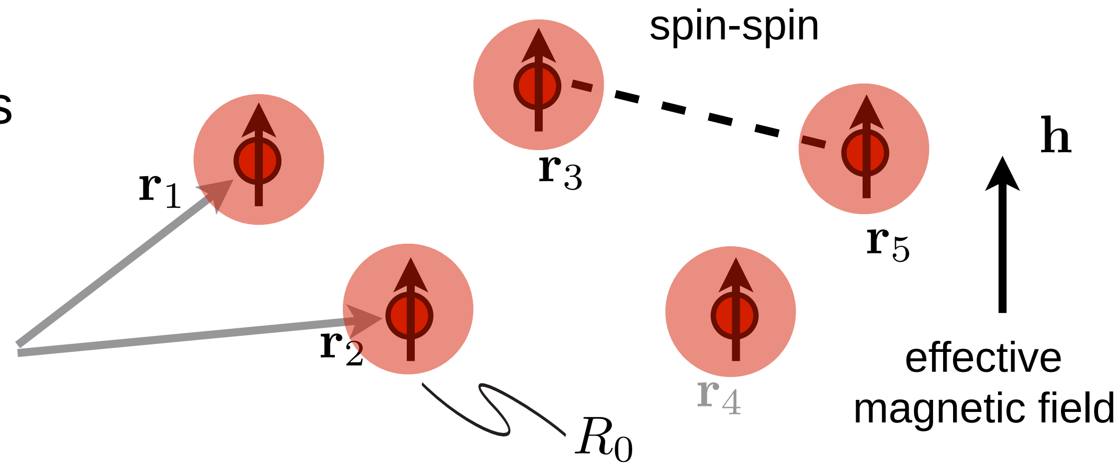
dipole-dipole interaction

$$\nu(\mathbf{r}) = \frac{1 - \cos\theta}{r^3}$$

XXZ- model in a magnetic field

Step 2: Interactions

- Ensemble of (static) molecules as interacting spins in magnetic field



Dipole-dipole interaction

- in rotating frame / RWA

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dipole-dipole interaction $\nu(\mathbf{r}) = \frac{1 - \cos\theta}{r^3}$

XXZ- model in a magnetic field

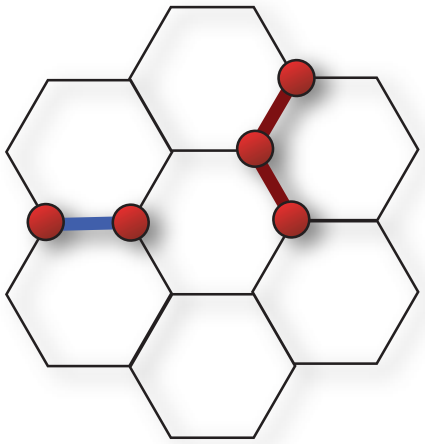
- Paramagnetic phase $\mathbf{h} >> D/a^3$ or $D/(a^3|\mathbf{h}|) = (R_0/a)^3 << 1$

weakly interacting regime:
interaction potential in perturbation theory

mean distance

- Provided $|\mathbf{r}_i - \mathbf{r}_j| > R_0$ we can calculate the interaction energy perturbatively

Extended Hubbard model



- Hamiltonian:

$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{1}{2} \sum_{i \neq j} U_{ij} n_i n_j + \frac{1}{6} \sum_{i \neq j \neq k} W_{ijk} n_i n_j n_k.$$

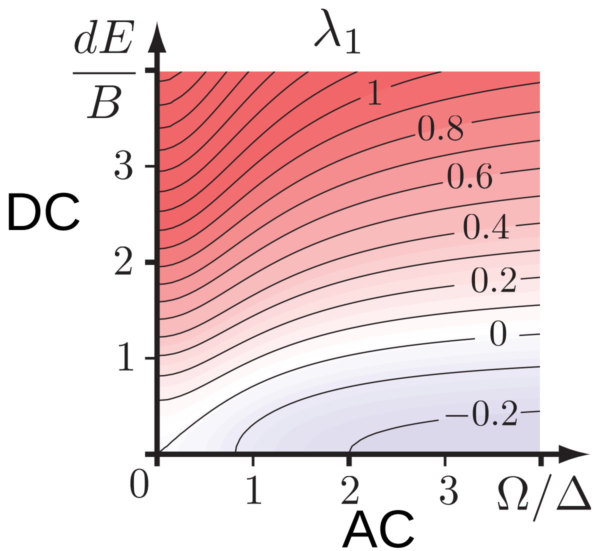
- two-body interaction

$$U_{ij} = U_0 \frac{a^3}{|\mathbf{R}_i - \mathbf{R}_j|^3} + U_1 \frac{a^6}{|\mathbf{R}_i - \mathbf{R}_j|^6}$$

$$U_0 = \lambda_1 D / a^3$$

tunable

repulsive



- three-body interaction

$$W_{ijk} = W_0 \left[\frac{a^6}{|\mathbf{R}_i - \mathbf{R}_j|^3 |\mathbf{R}_i - \mathbf{R}_k|^3} + \text{perm} \right].$$

repulsive

- hard core onsite condition ... $a_0 \ll R_0 \ll \lambda/2$

1D hard core Boson with three-body

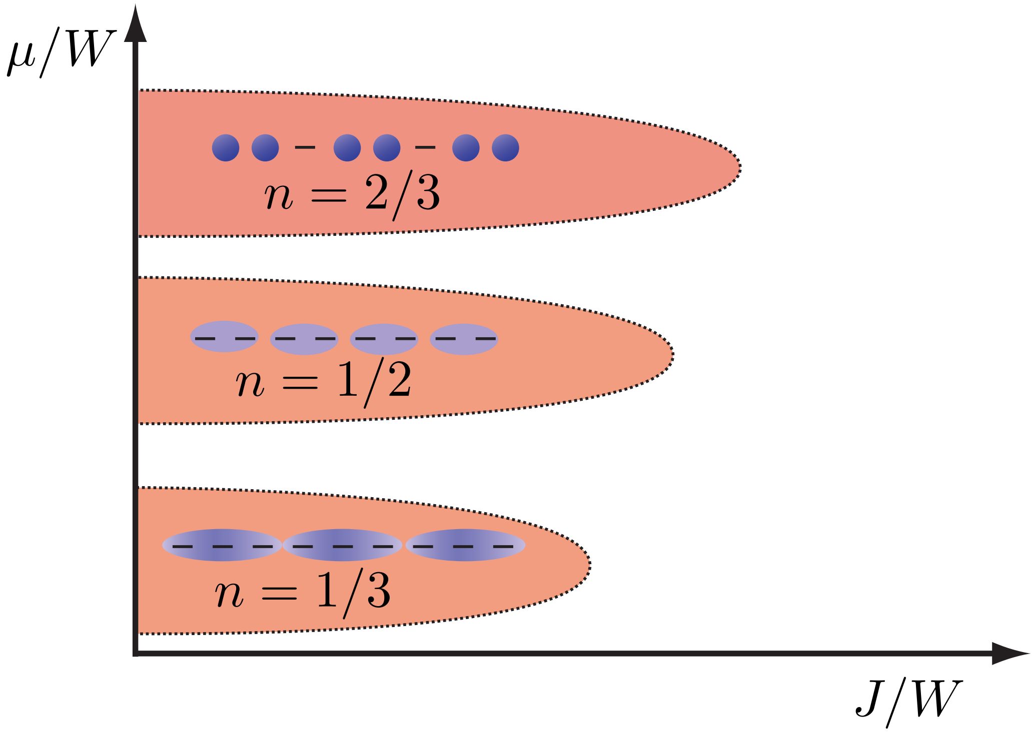
$$H = -J \sum_i \left[b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i \right] + W \sum_i n_{i-1} n_i n_{i+1}$$

Bosonization

- hard-core bosons
- instabilities for densities:

$$n = 2/3 \quad n = 1/2 \quad n = 1/3$$

- quantum Monte Carlo simulations (in progress)



Critical phase

- algebraic correlations
- compressible
- repulsive fermions

Solid phases

- excitation gap
- incompressible
- density-density correlations

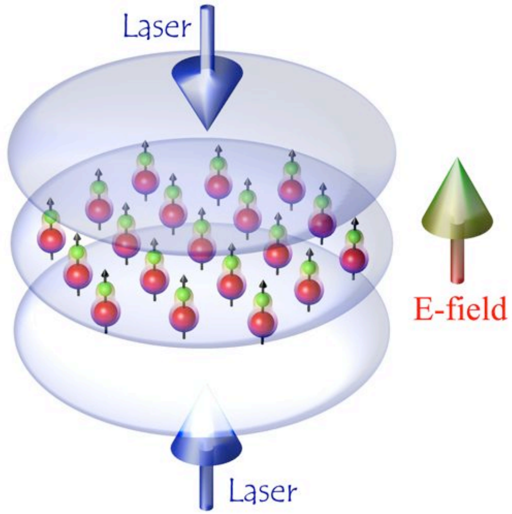
$$\langle \Delta n_i \Delta n_j \rangle$$

- hopping correlations (1D VBS)

$$\langle b_i^\dagger b_{i+1} b_j^\dagger b_{j+1} \rangle$$

Other AMO lattice models … besides optical lattices?

Atoms (or Molecules) in Self-Assembled Dipolar Lattices

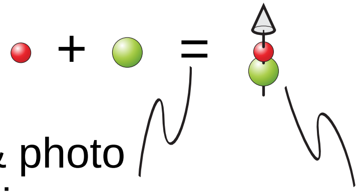


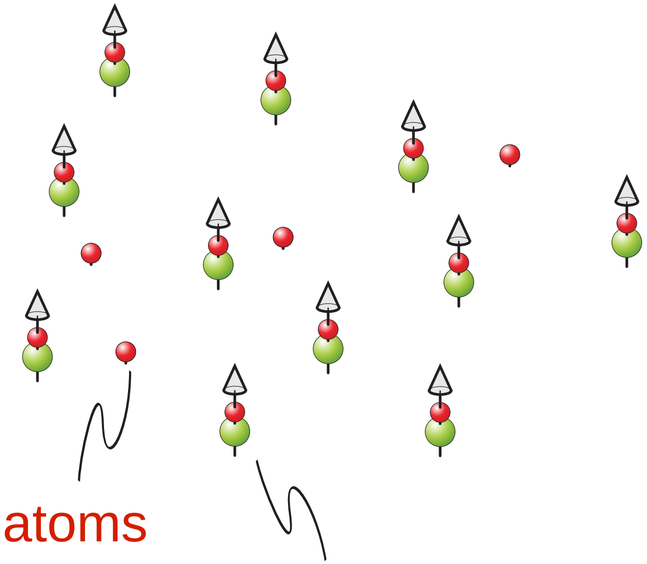
floating nanoscale self-assembled crystals
with polar molecules

G. Pupillo, A. Griessner, A. Micheli, M. Ortner, D.-W. Wang and P. Zoller, ArXiv Sep 2007

Atom + Polar Molecule Mixture

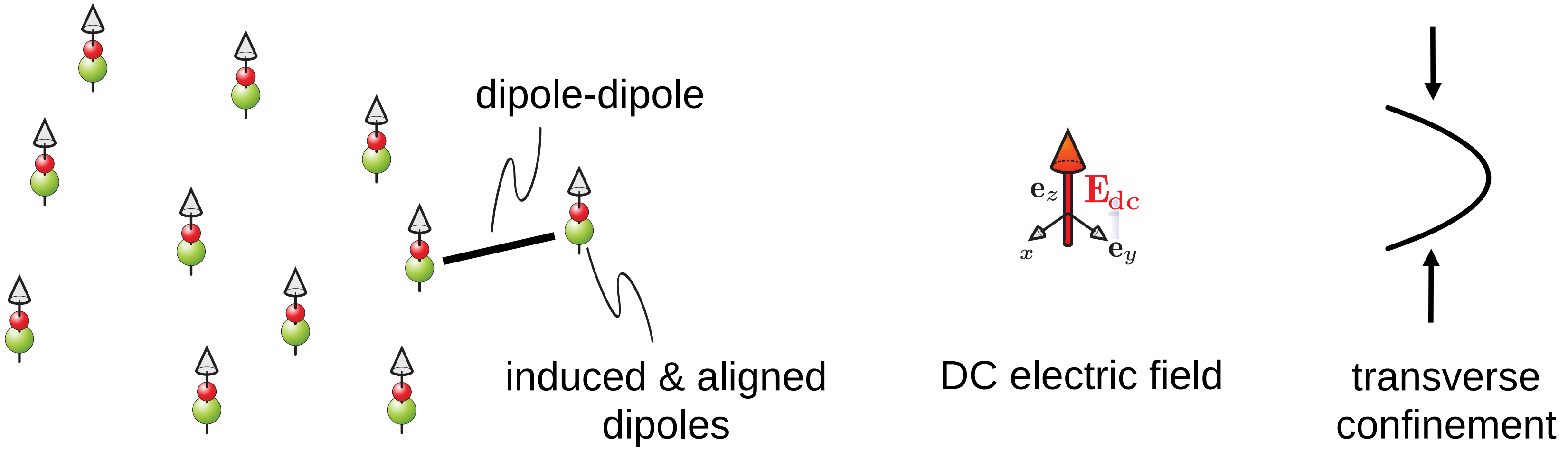
- Preparation of polar molecules:
  - e.g. two species BEC





Polar Molecules

- quantum gases with polar molecules

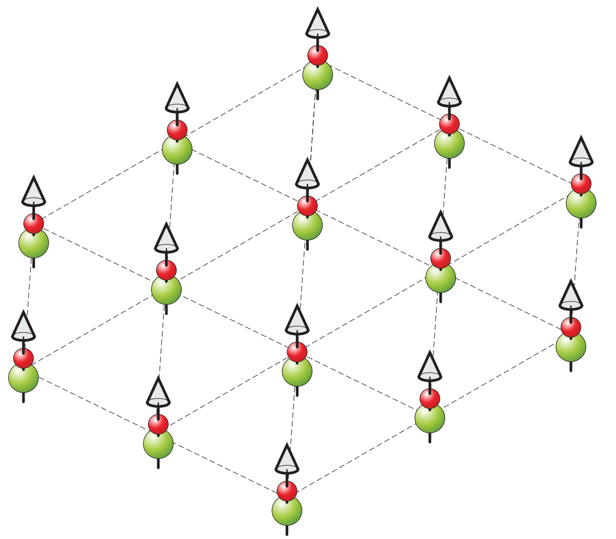


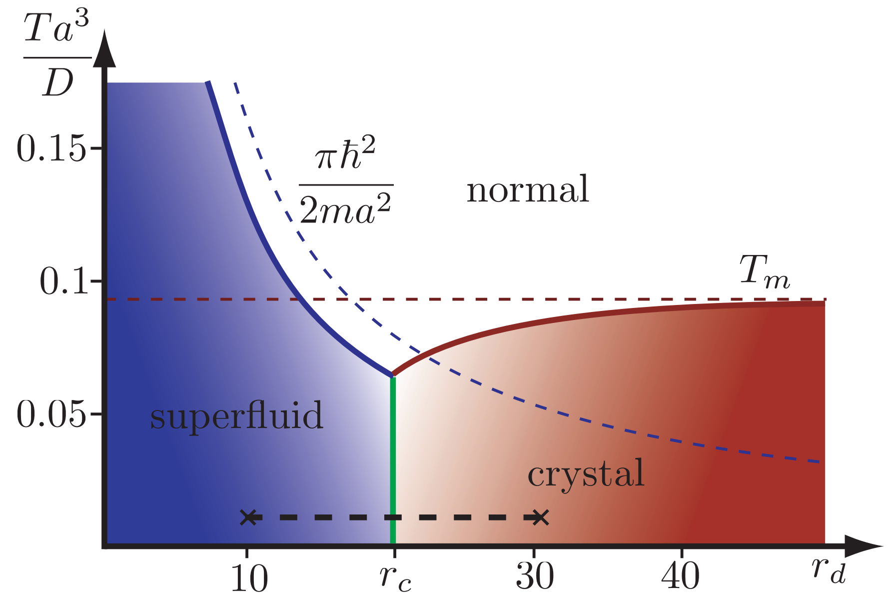
Theory: 1D/2D/3D gases, optical lattices, rotation, …
Experiment / other systems: magnetic dipoles, Rydberg, ...

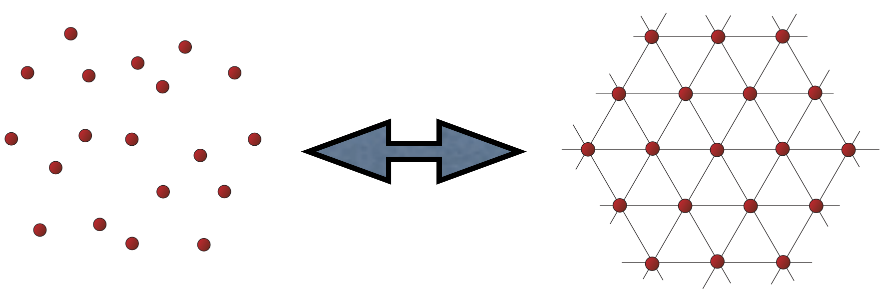
Polar Molecules

- dipolar quasi-crystal

HP. Büchler et al., PRL 2007
G.E. Astrakharchik et al., PRL 2007







dipolar superfluid
(low density)

dipolar crystal
(high density)

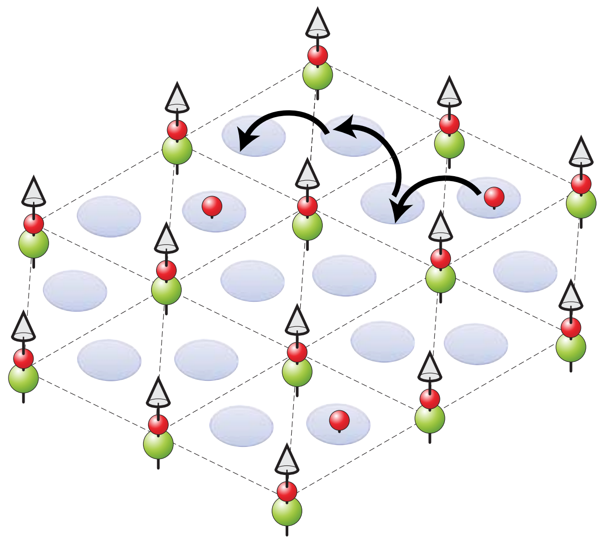
$$r_d = \frac{E_{\text{pot}}}{E_{\text{kin}}} \equiv \frac{d_C^2/a^3}{\hbar^2/m_C a^2} = \frac{d_C^2 m_C}{\hbar^2 a} >> 1$$

recoil energy
$E_{R,C} = \hbar^2\pi^2/2m_C a^2$

crystal forms at
high densities

Atom + Polar Molecule Mixture

- application of dipolar crystals



extra atom + dipolar crystal

floating nanoscale structures as atomic traps

- a self-assembled dipolar crystal provides a lattice for “extra atoms”

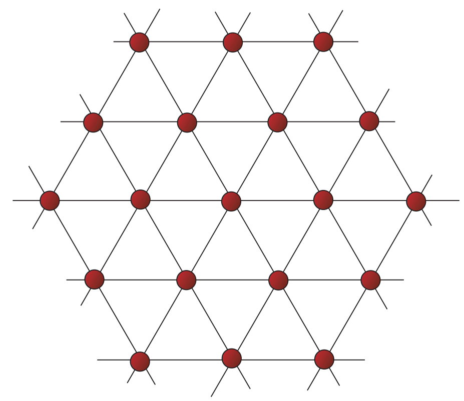
- features
  - crystal: *tunable* lattice spacings & *phonons*
  - extra particles: (single band) *Hubbard models*

Hubbard model: strong correlation

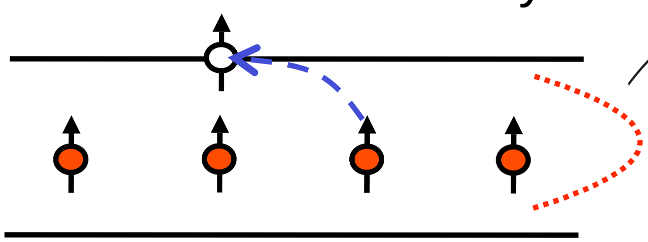
$$H = -J \sum_{<i,j>} c_i^\dagger c_j + \frac{1}{2} \sum_{i,j} V_{ij} c_i^\dagger c_j^\dagger c_j c_i + \sum_q \hbar\omega_q a_q^\dagger a_q + \sum_{q,j} M_q e^{i\mathbf{q}\cdot\mathbf{R}_j^0} c_j^\dagger c_j (a_q + a_{-q}^\dagger)$$

phonons as quantum dynamics of trap

Dipolar Crystal: Tuning the Lattice Spacing



transverse instability



$$a_{\mathrm{min}} < a < a_{\mathrm{max}}$$

quantum melting

$$r_d = \frac{E_{\mathrm{pot}}}{E_{\mathrm{kin}}} \equiv \frac{d_{\mathrm{c}}^2 m_{\mathrm{c}}}{\hbar^2 a} \gg 1$$

$$a_{\mathrm{min}} \sim 100nm$$

$$a_{\mathrm{max}} \sim 1\mu\mathrm{m}$$

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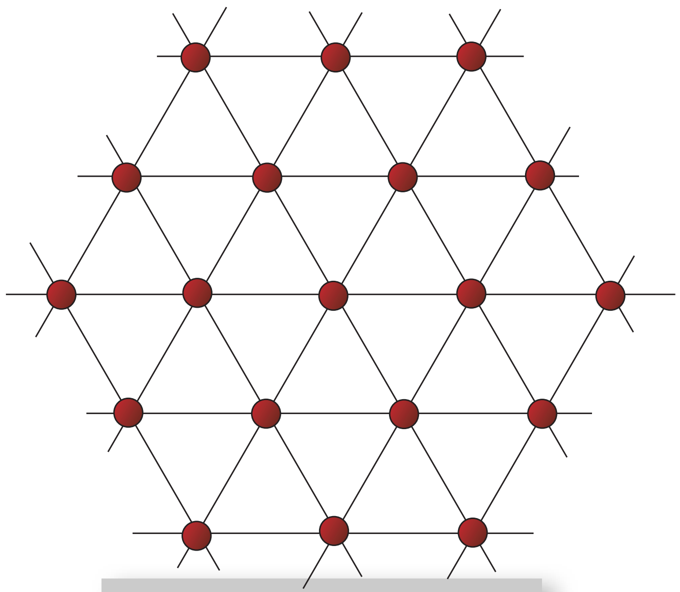

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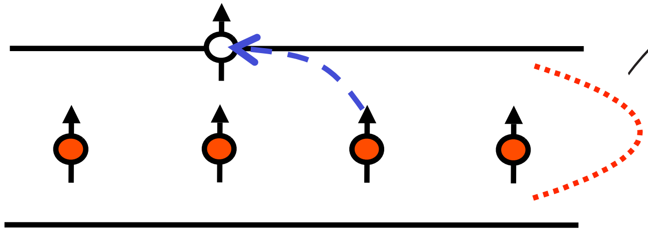

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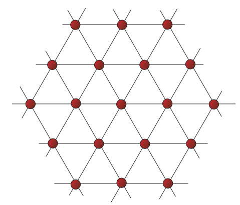

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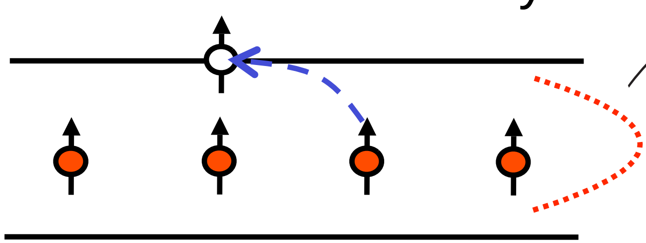

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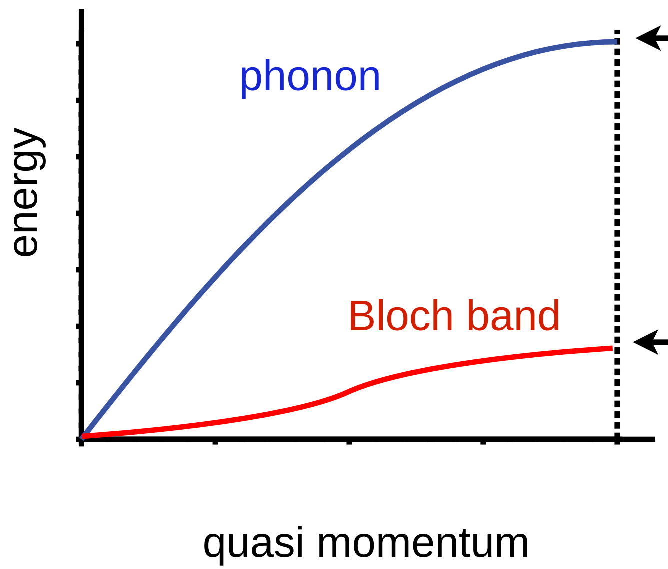
Phonons & Hubbard Dynamics

- phonon spectrum & Hubbard dynamics



$\hbar\omega_D \sim E_{\mathrm{R,c}}\sqrt{r_d}$

Debye frequency

$r_d \sim 100$

$J \lesssim E_{R,p}$

Bloch band width

$$J \ll \hbar\omega_D$$

Phonons & Hubbard Dynamics

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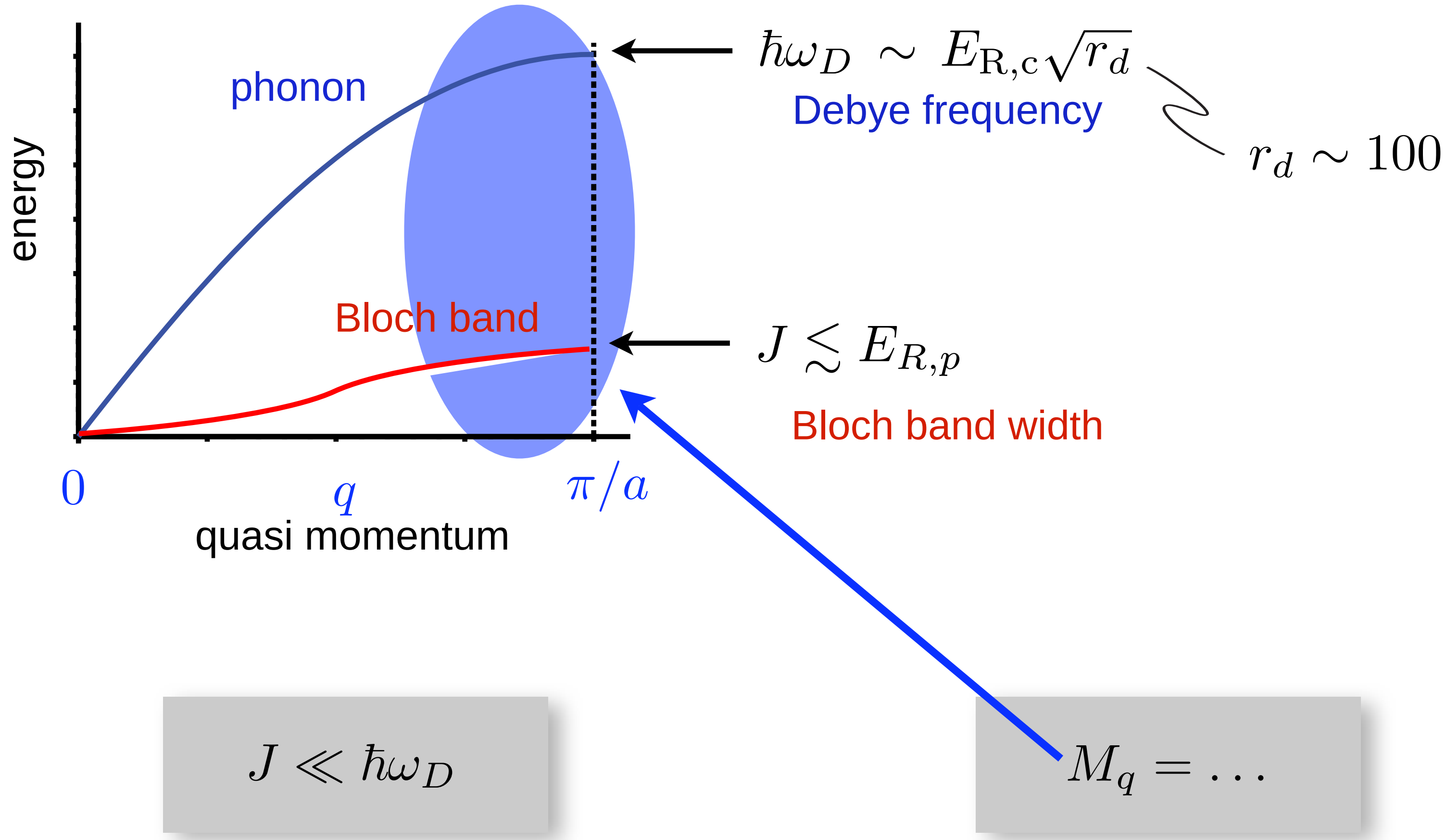


In a wide parameter regime ...

Slow atom dynamics vs. *fast* phonon dynamics (i.e. like optical phonons)

Polaron Dynamics: effective Hubbard models

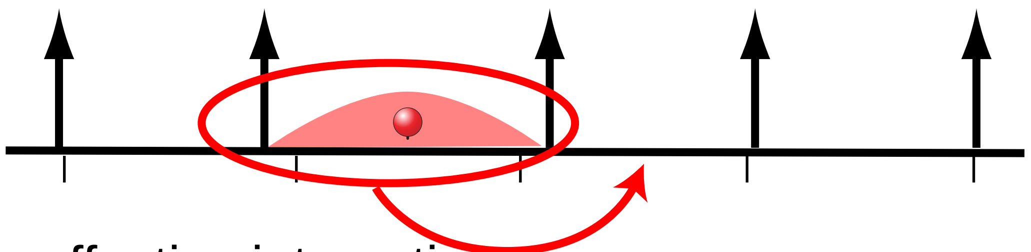
- effective Hubbard model for polarons (= atom dressed by phonons)

$$\tilde{H} = -\tilde{J} \sum_{<i,j>} c_i^\dagger c_j + \frac{1}{2} \sum_{i,j} \tilde{V}_{ij} c_i^\dagger c_j^\dagger c_j c_i$$

polaron hopping — polaron interactions

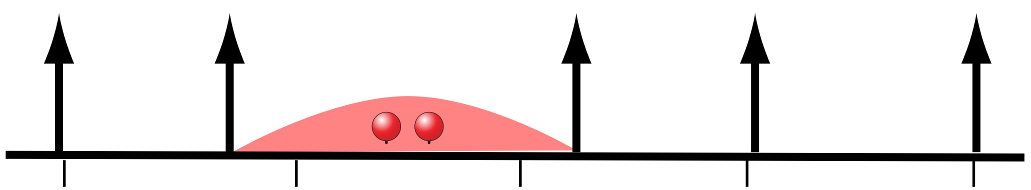
tunable over wide range

- hopping with effective mass

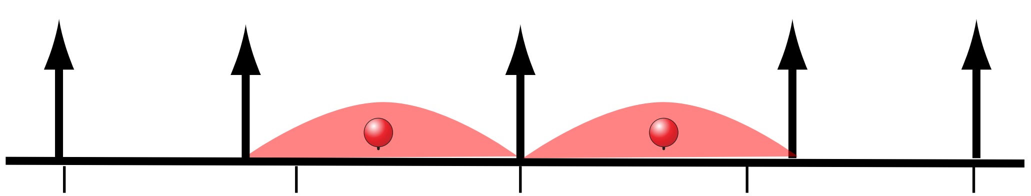


$$\tilde{J} = J e^{-S}$$

- effective interactions



$$\tilde{V}_{jj} = V_{jj} - 2 \sum_q \frac{M_q^2}{\hbar \omega_q}$$



$$\tilde{V}_{ij} = V_{ij} - 2 \sum_q \frac{M_q^2}{\hbar \omega_q} \cos(q(R_i^0 - R_j^0))$$

Quantum State Engineering *by Dissipation*

- quantum reservoir engineering for many body systems

Quantum State Engineering (in Many Body Systems)

- **Condensed matter physics: thermodynamic equilibrium**

  Hamiltonian $H\,|E_g\rangle = E\,|E_g\rangle$

  by cooling to zero temperature we obtain the ground state

$$\rho \sim e^{-H/k_B T} \xrightarrow{T\to 0} |E_g\rangle\,\langle E_g|$$

pure state of interest

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pure state of interest

- **driven dissipative system: non-equilibrium** (compare: laser)

  open quantum system: Master equation

$$\frac{d}{dt}\rho = -[H,\rho] + \mathcal{L}_d\rho$$

Hamiltonian dynamics

$$\mathcal{L}_d\rho = \sum \gamma_i \left(2c_i\rho c_i^\dagger - c_i^\dagger c_i\rho - \rho c_i^\dagger c_i\right)$$

dissipative dynamics

can we *design the reservoir couplings* so that in steady state ...

$$\Rightarrow \quad \rho \xrightarrow{t\to\infty} \rho_{ss} \equiv |\psi\rangle\,\langle\psi|$$

pure state of interest

Driven Dissipative Dynamics of Cold Atoms in Optical Lattices

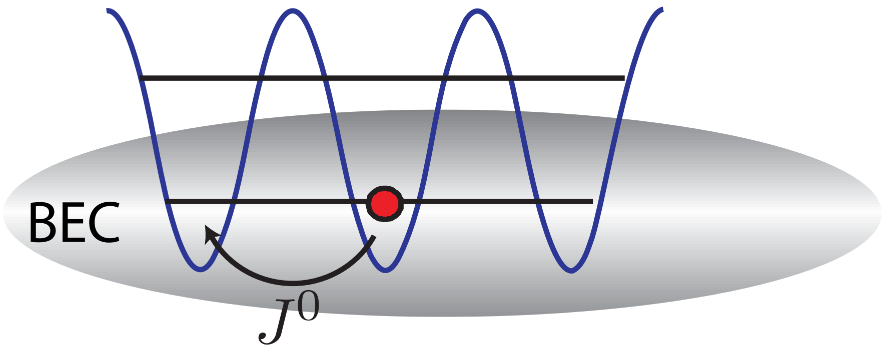
- quantum optics with cold atoms

A. Griessner et al. PRL 2006; NJP 2007
S. Diehl, A. Micheli et al., in preparation

Dissipative Hubbard dynamics

- BEC as a "phonon reservoir"
  - quantum reservoir engineering

1D model



- master equation:

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathscr{L}\hat{\rho}$$

- validity (as in quantum optics)
  - ✓ *inter*band transitions
  - ✓ RWA + Born + Markov

- *coherent* Hubbard dynamics

$$H = \ldots$$

✓ two band Hubbard model (1D)
✓ + Raman coupling

- *dissipative* dynamics

$$\mathscr{L}\rho = \sum_k \frac{\Gamma_k}{2}\left(2c_k\hat{\rho}c_k^\dagger - c_k^\dagger c_k\hat{\rho} - \hat{\rho}c_k^\dagger c_k\right)$$

Lindblad form

competing dynamics

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A. Griessner et al. PRL 2006; NJP 2007
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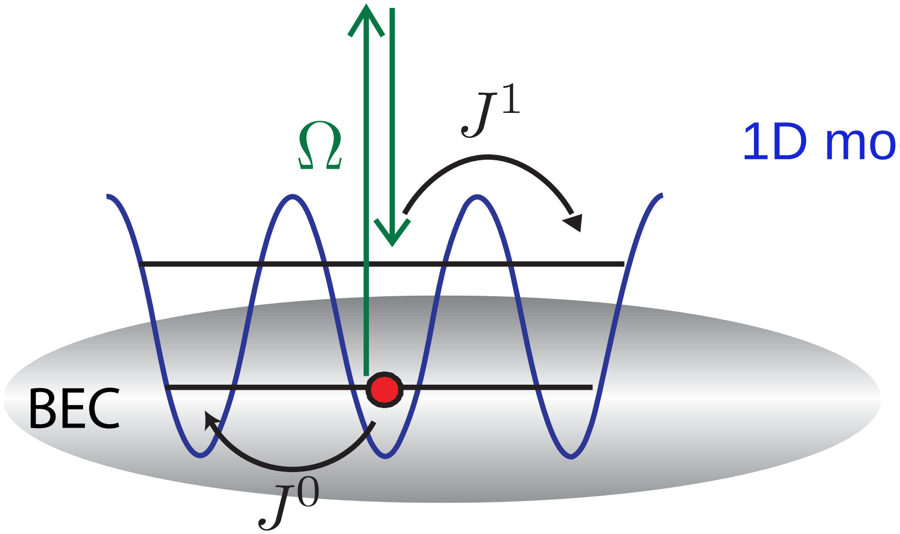

1D model

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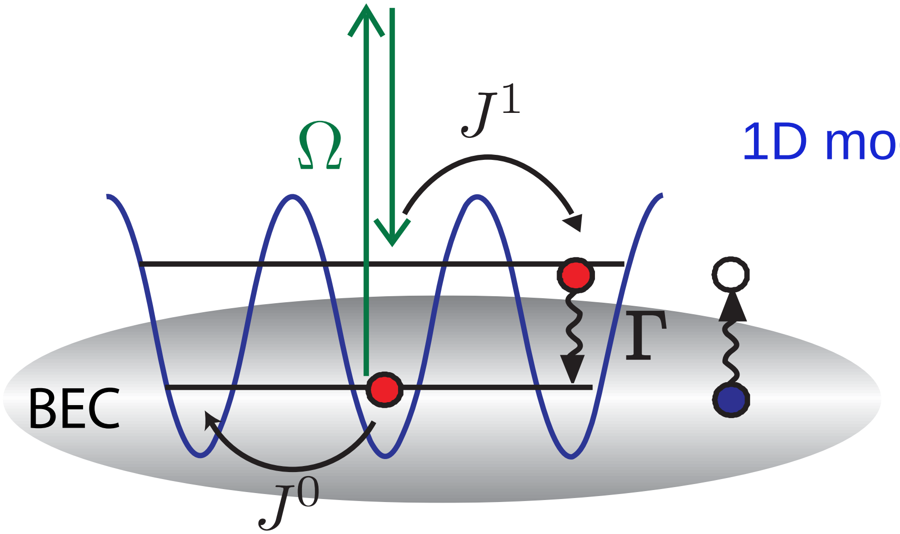

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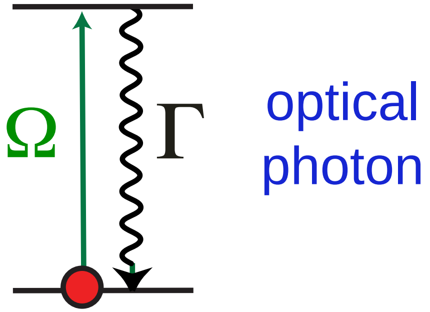
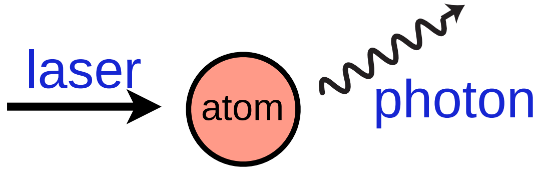
Lindblad form

competing dynamics

“think quantum optics”

- driven two-level atom + spontaneous emission





- reservoir: vacuum modes of the radiation field (T=0)

“think quantum optics”

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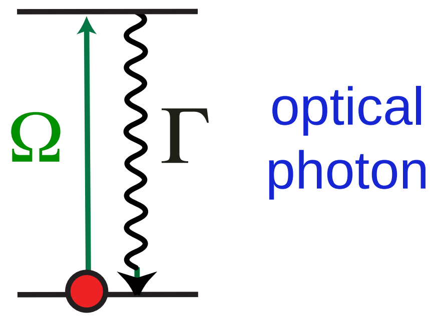

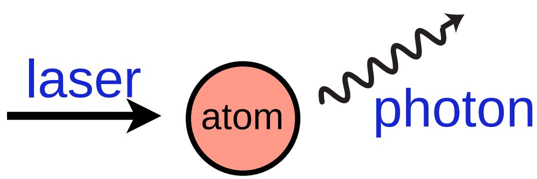

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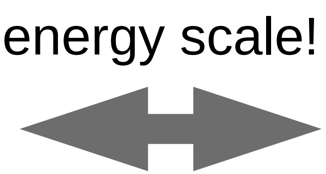
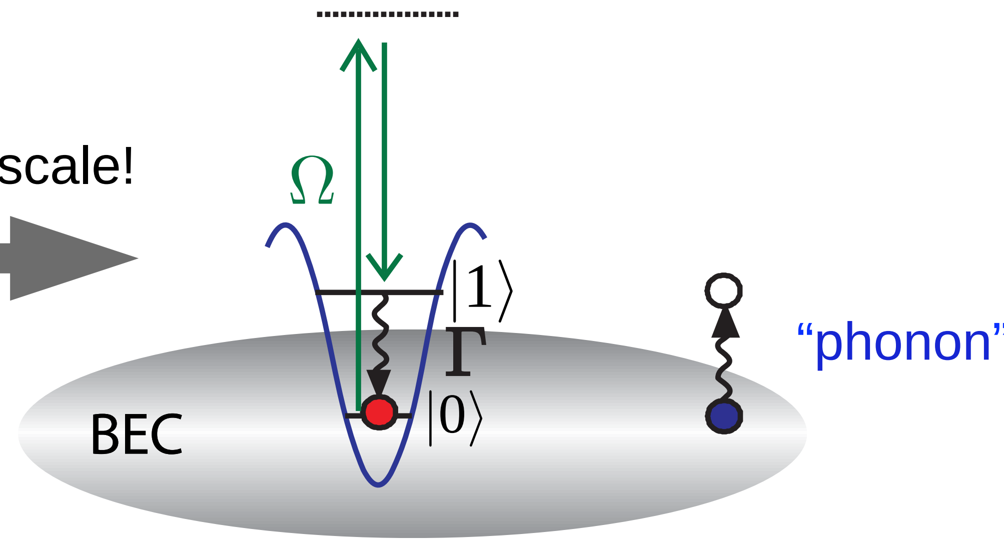


- trapped atom in a BEC reservoir

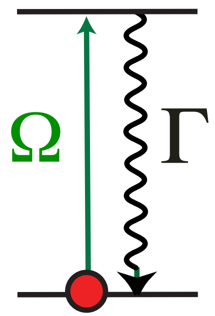


laser assisted atom + BEC collision

- reservoir: Bogoliubov excitations of the BEC (@ temperature T)

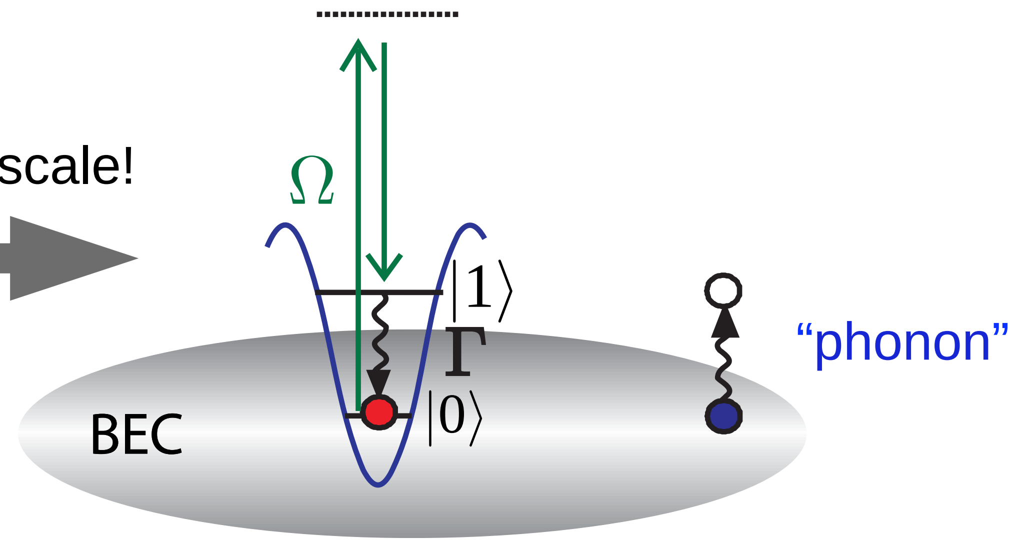
"think quantum optics"

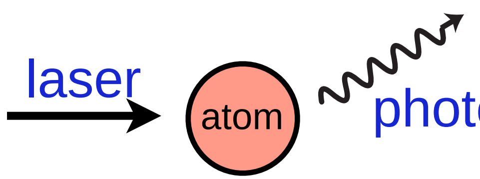
- driven two-level atom + spontaneous emission
- trapped atom in a BEC reservoir



optical photon

energy scale!





laser assisted atom + BEC collision

- reservoir: vacuum modes of the radiation field (T=0)
- reservoir: Bogoliubov excitations of the BEC (@ temperature T)

- optical pumping, laser cooling, ...
  - purification of electronic, and motional states

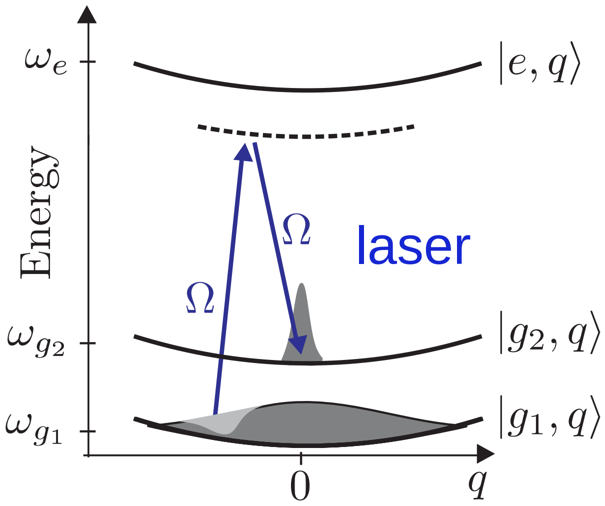
$$\rho_a \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}| \rightarrow |\psi_a\rangle\langle\psi_a| \otimes \rho'$$

?

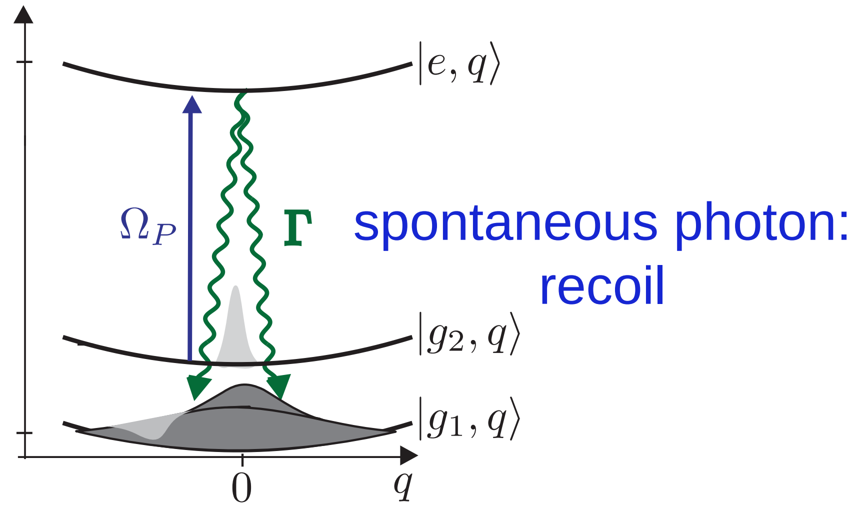
Subrecoil ("dark state") laser cooling

Raman subrecoil cooling (Kasevich and Chu) (see also: VSCPT Cohen et al.)

step 1: excitation & filtering



step 2: diffusion

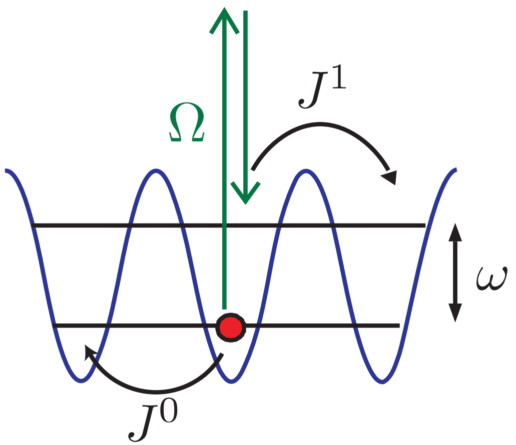


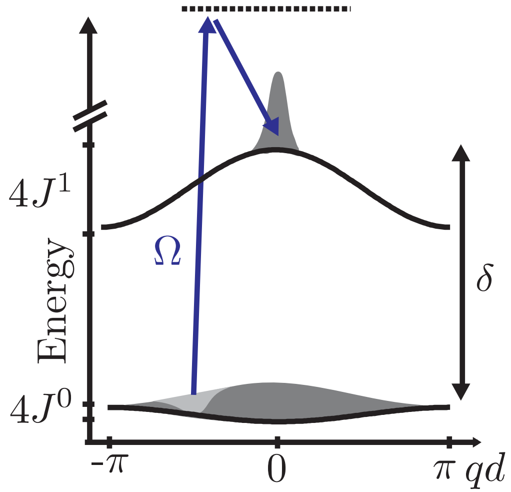
- "dark state" laser cooling: accumulate atoms near q≈0

A. Griessner et al. PRL 2006;
NJP 2007

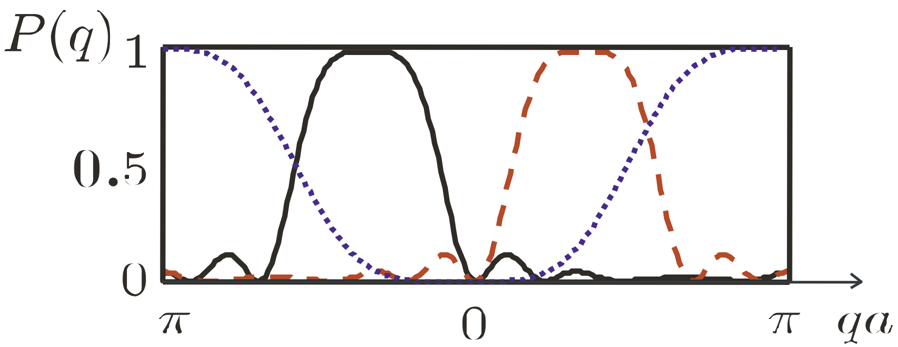
Raman cooling *within* a Bloch band

- step 1: (coherent) quasimomentum selective excitation





Laser: square pulse sequence



$$P(q) = \frac{\Omega^2}{(\delta^2_{q+\delta q} + \Omega^2)} \sin^2 \left(\sqrt{\delta^2_{q+\delta q} + \Omega^2} \tau / 2 \right)$$

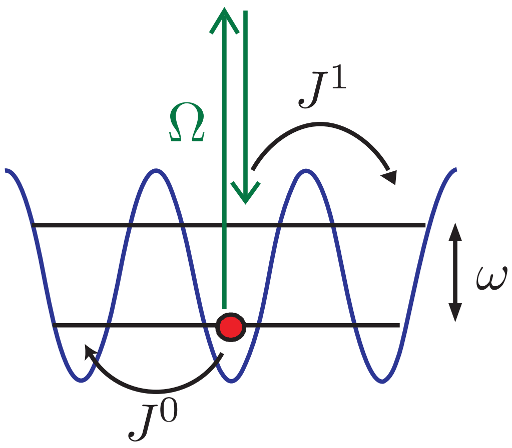
- requirements: $\Omega \ll 8|J^1|$
- Note: relevant energy scale given by $|J^1|$

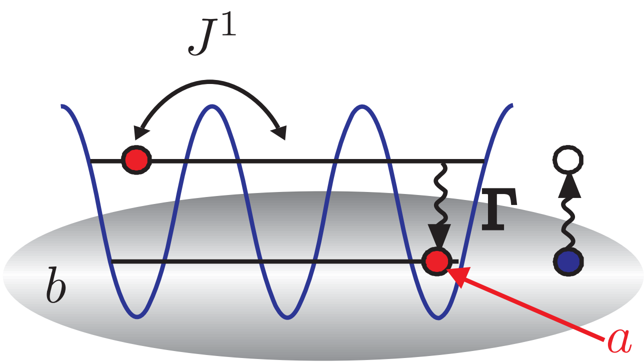
A. Griessner et al. PRL 2006;
NJP 2007

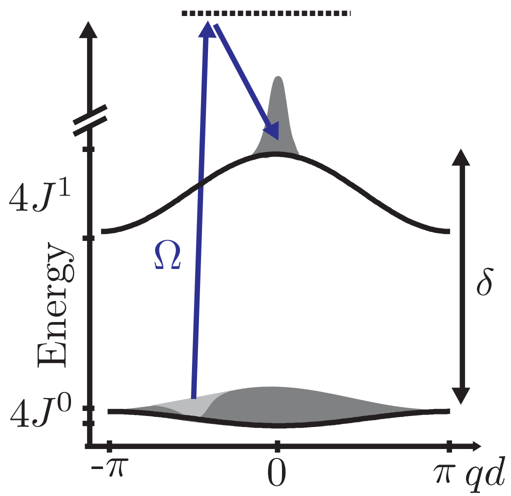
Raman cooling *within* a Bloch band

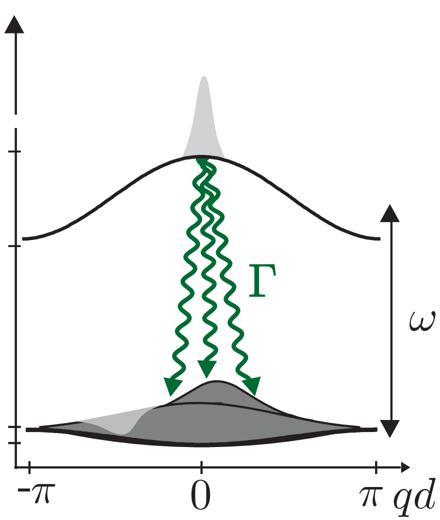
- step 1: (coherent) quasimomentum selective excitation

- step 2: (dissipative) decay to ground band





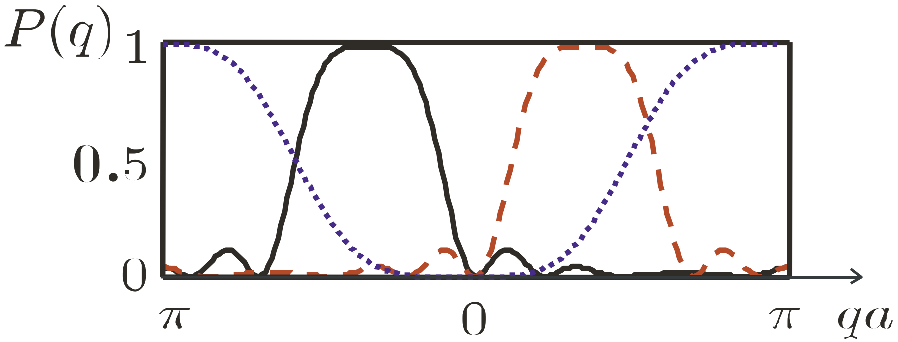




Results: single atoms

- Ground state q=0 momentum peak $4J^0 \ll k_B T \ll \omega$.
- Quantum trajectory simulation of the master equation

Laser: square pulse sequence



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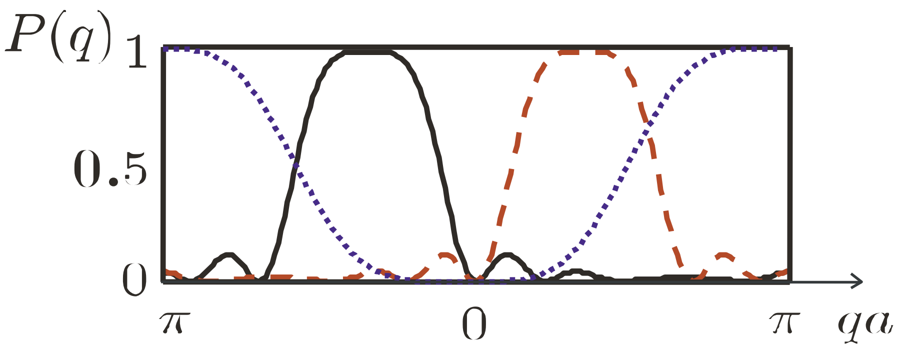

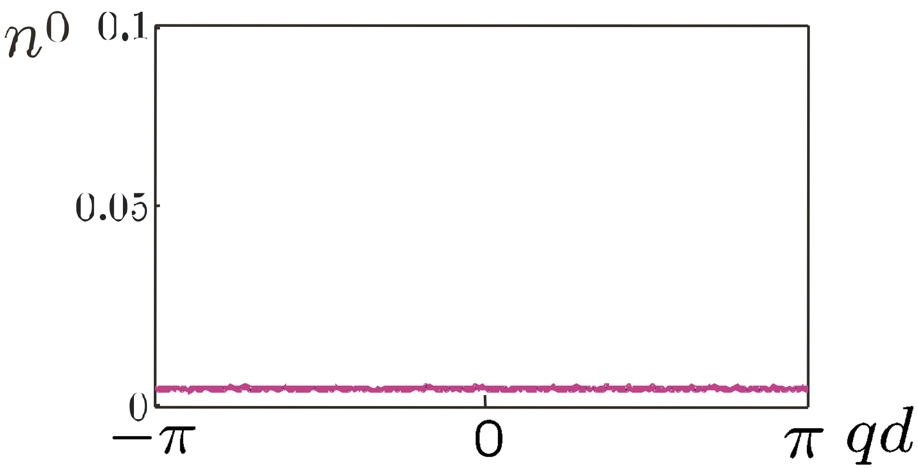

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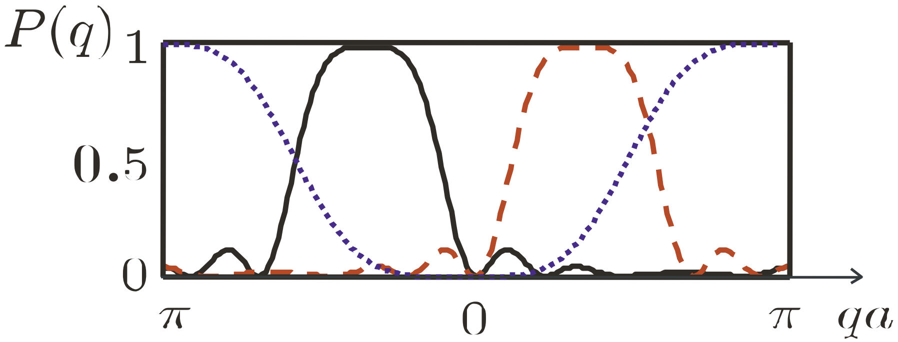

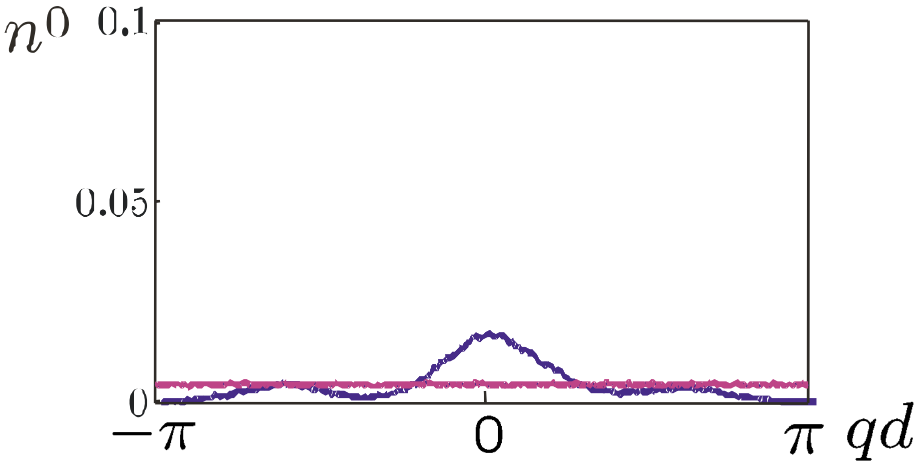

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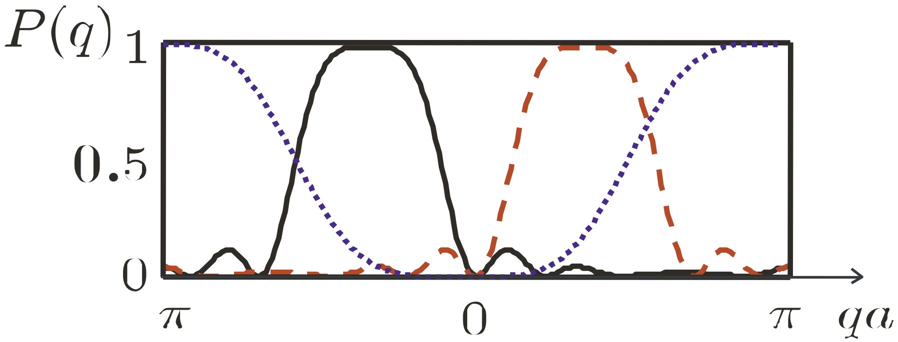

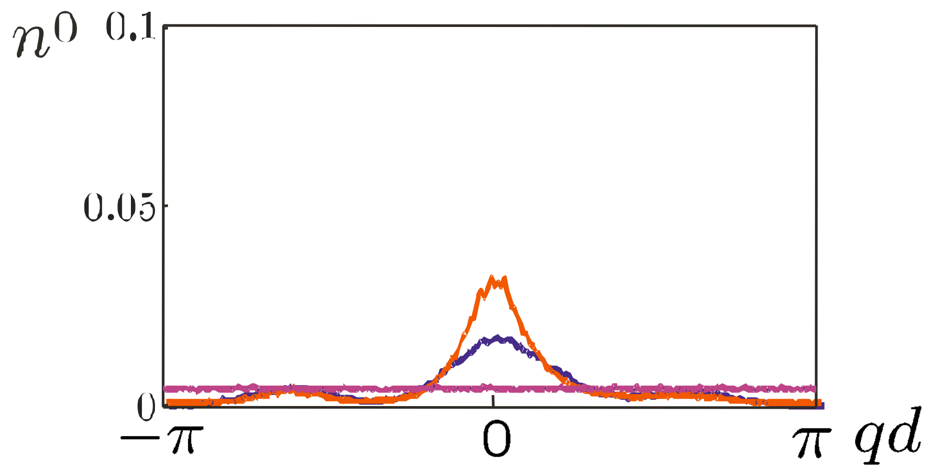

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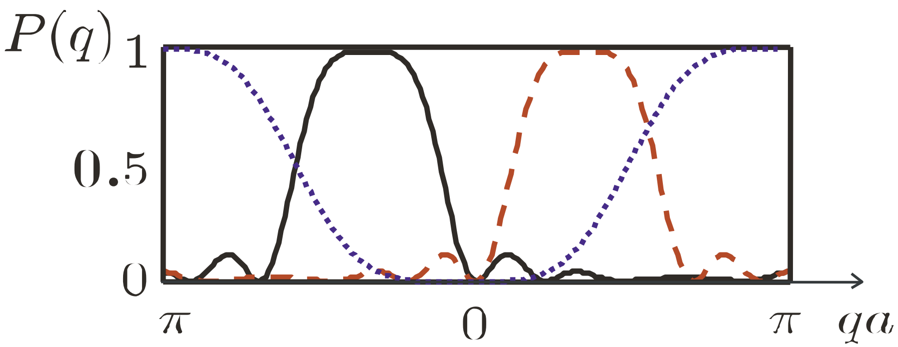

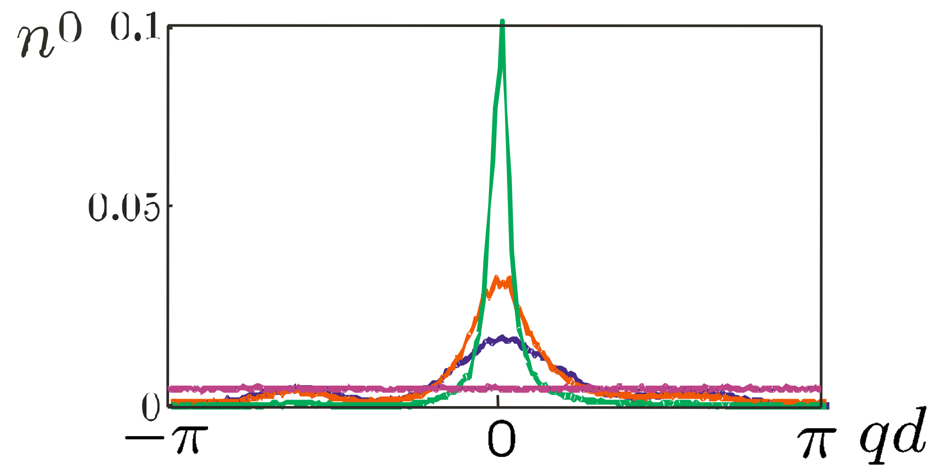

Results: single atoms

- Ground state q=0 momentum peak $4J^0 \ll k_B T \ll \omega_r$
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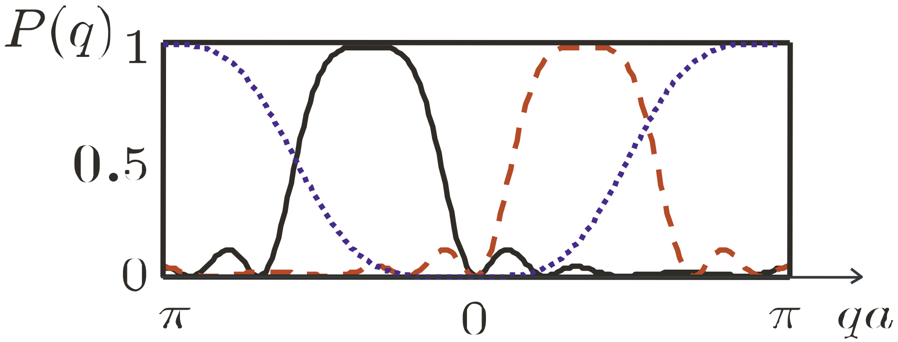
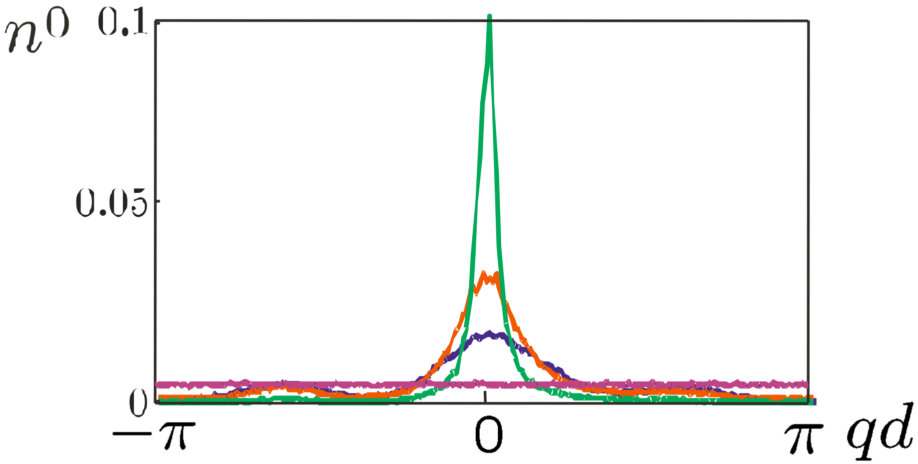
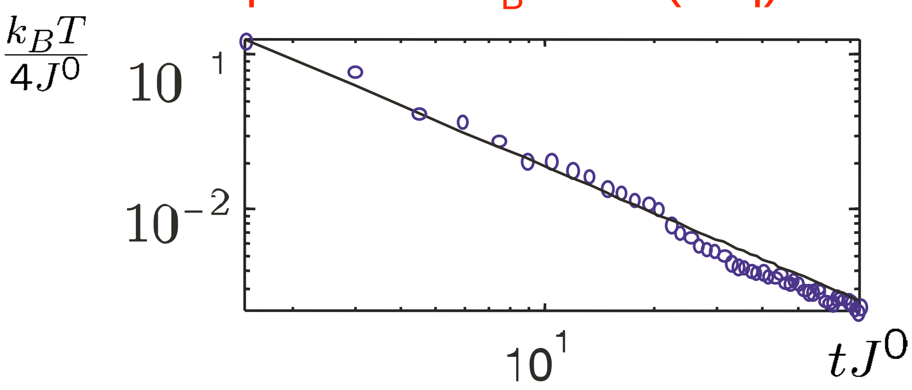
Laser: square pulse sequence

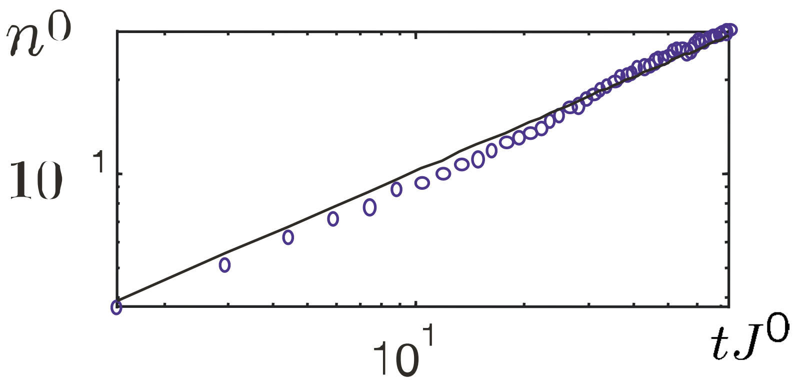




Temperature: $k_B T = 2J^0(\Delta q)^2$



Dark state occupation: $n^0(q=0)$



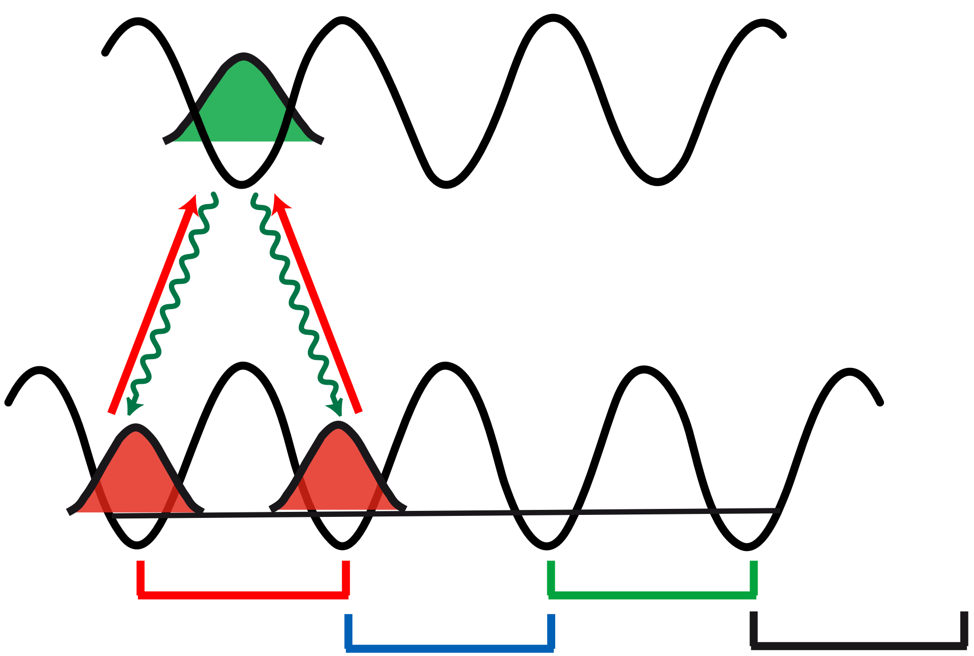
- Typical temperatures $k_B T/4J^0 \sim 2 \times 10^{-3}$ in $t_f J^0 \sim 50$
- Analysis in terms of Levy flights

S. Diehl, A. Micheli, H.P. Büchler, PZ,
in preparation

… ongoing work: extension to N atoms

- We know how to design reservoir / couplings so that we drive a dissipative system of cold atoms into a *pure BEC*

$$\rho \xrightarrow{t\to\infty} |\mathrm{BEC}\rangle\langle\mathrm{BEC}|$$



long range order
by local dissipation

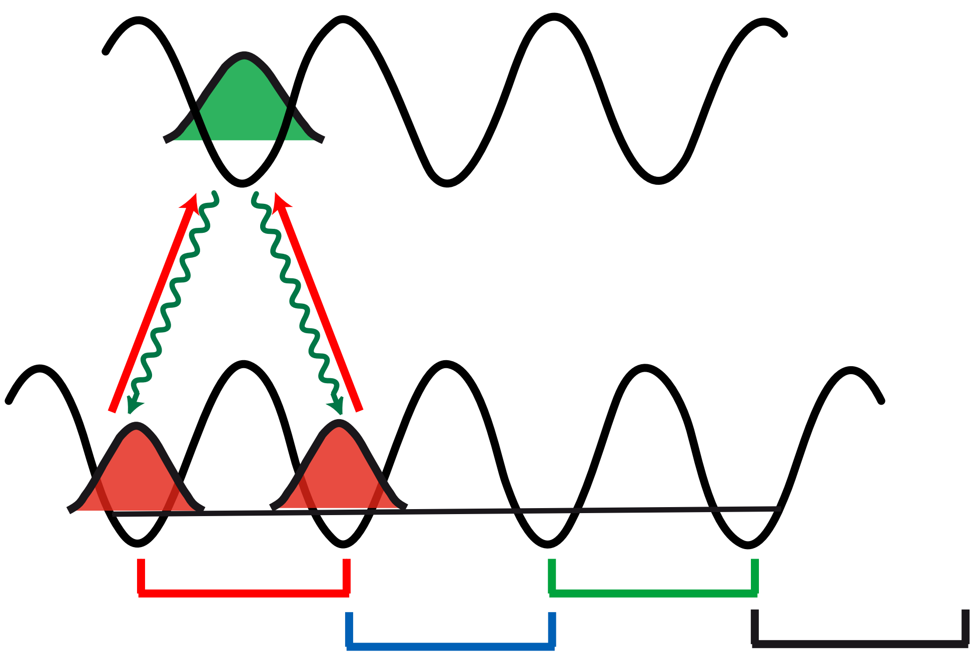
excite only the *antisymmetric* state, i.e.
lock phases between wells 1+2, 2+3, etc.

Master Equation

- Master equation for N atoms with a driven BEC as (unique) steady state with *local* dissipation?

local quantum jump operator $C_j = \frac{\sqrt{\kappa}}{2} (a_j + a_{j+1})^\dagger (a_j - a_{j+1})$

decay excitation



$$|\psi_{\mathrm{BEC}}\rangle = \frac{1}{\sqrt{N!}} \left(\sum_i a_i^\dagger \right)^N |\mathrm{vac}\rangle$$

… as solution of:

$$\forall j \quad C_j |\psi_{\mathrm{BEC}}\rangle = 0$$

- Effect of a “finite” Hubbard Hamiltonian: depletion

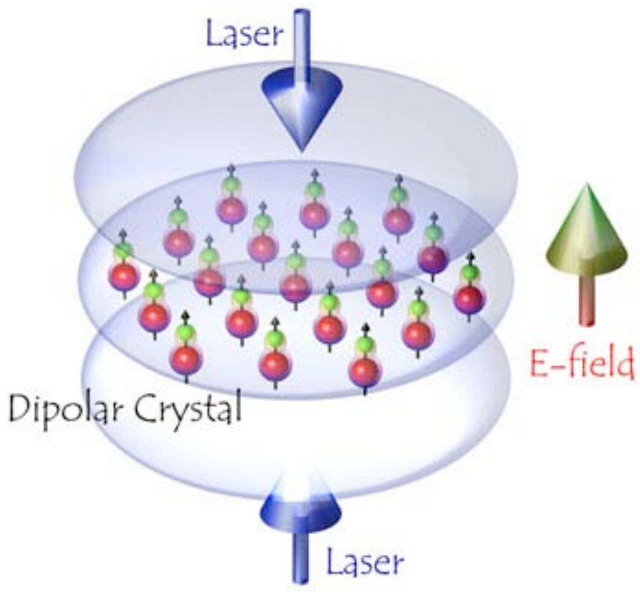
$$\rho \sim \exp(-\beta H)$$

effective finite “temperature”
~ dissipative driving

Conclusion and Outlook

Polar molecular crystal

- reduced three-body collisions
- strong coupling to cavity QED
- ideal quantum storage devices



Molecules with Spin

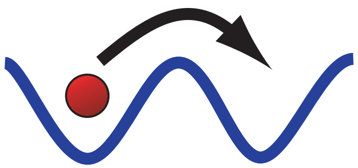
- spin toolbox
- ion trap like quantum computation

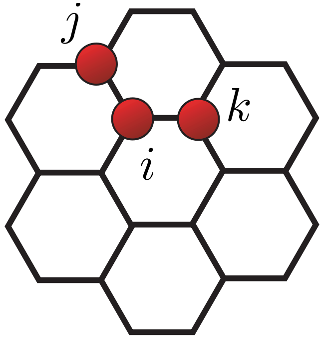
Lattice structure

- alternative to optical lattices
- tunable lattice parameters
- strong phonon coupling: polarons

$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + V \sum_{\langle\langle ijk \rangle\rangle} n_i n_j n_k$$

hopping energy — interaction energy



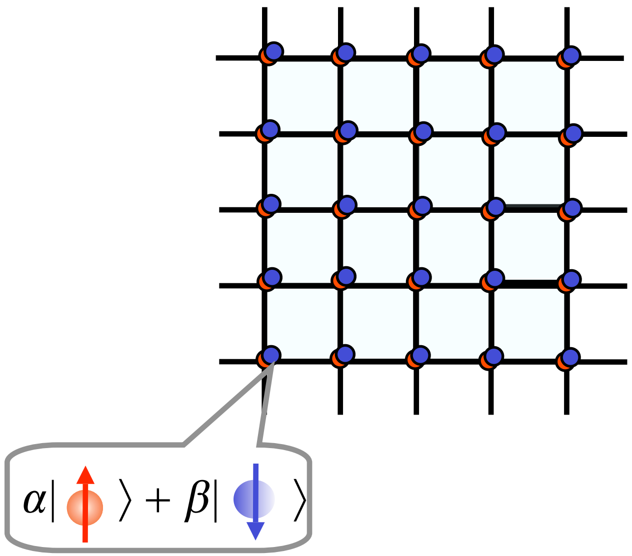


Three body interaction

- driving the interaction via microwave field

→ tunable three-body interaction

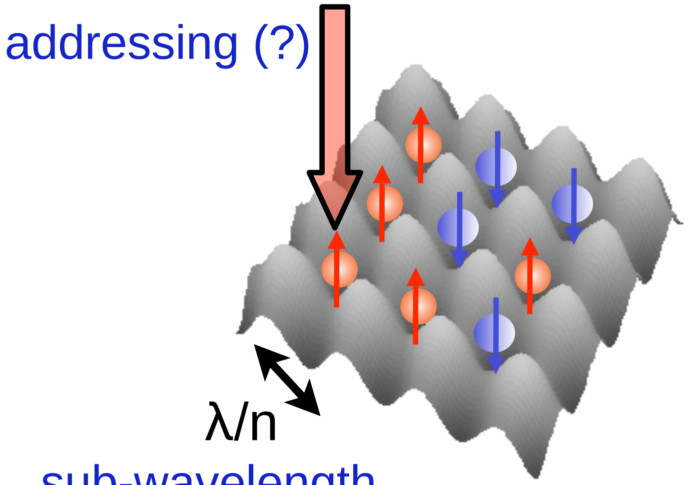
Quantum computing on a lattice



requirements:

- single site addressing
  - single qubit rotation
  - single qubit readout

Quantum optical implementation

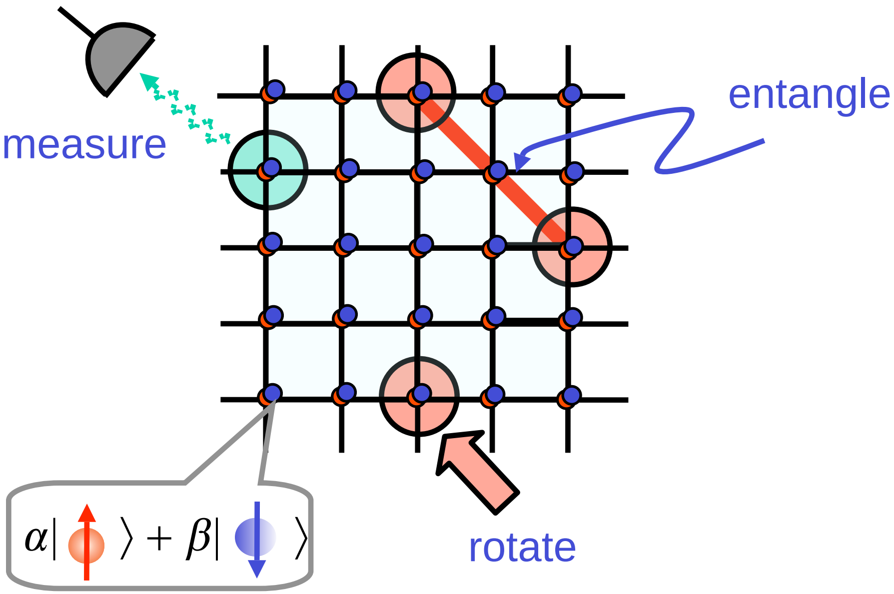


?

complaints / wishlist: ?

- single site addressing (?)
- slow couplings / small energies
  - latttice spacing ~ wavelength λ/2
  - hopping $J << E_R = \hbar^2 k_L^2/2m$
  - exchange ~ J^2/U

Quantum computing on a lattice



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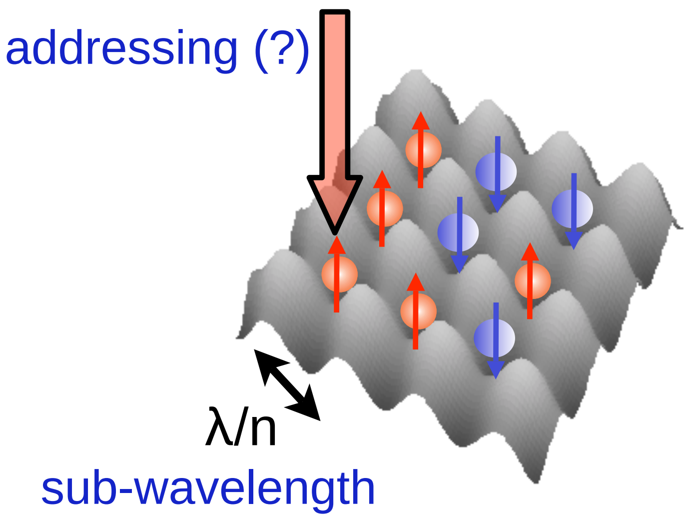

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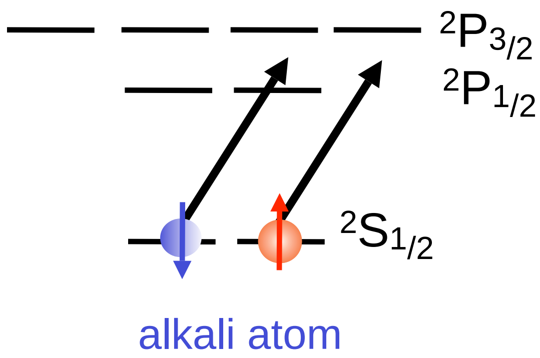
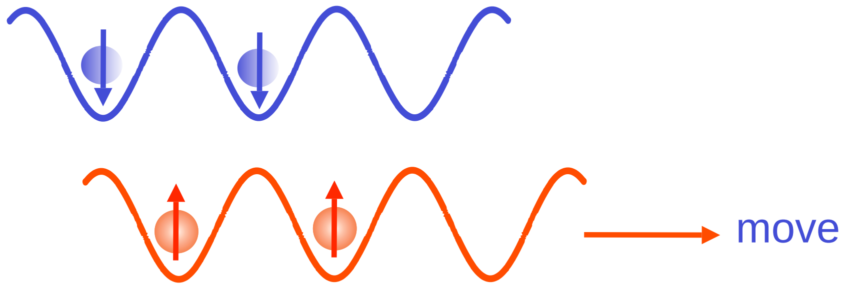
- spin / qubit dependent lattice





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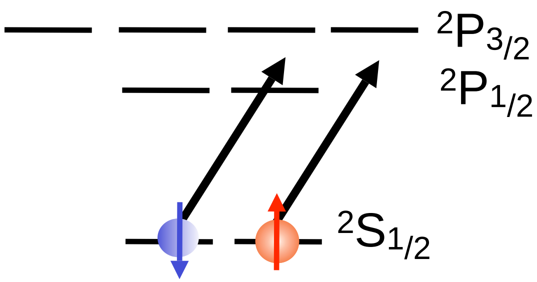
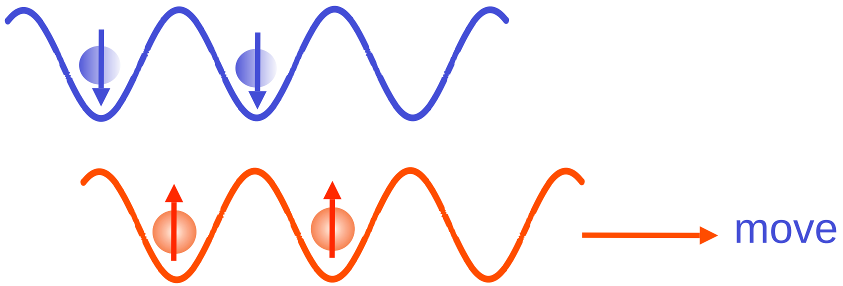
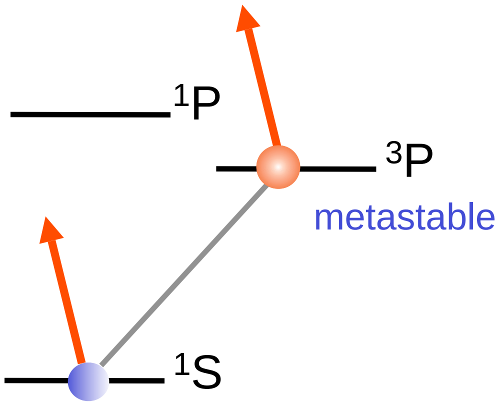


alkali atom

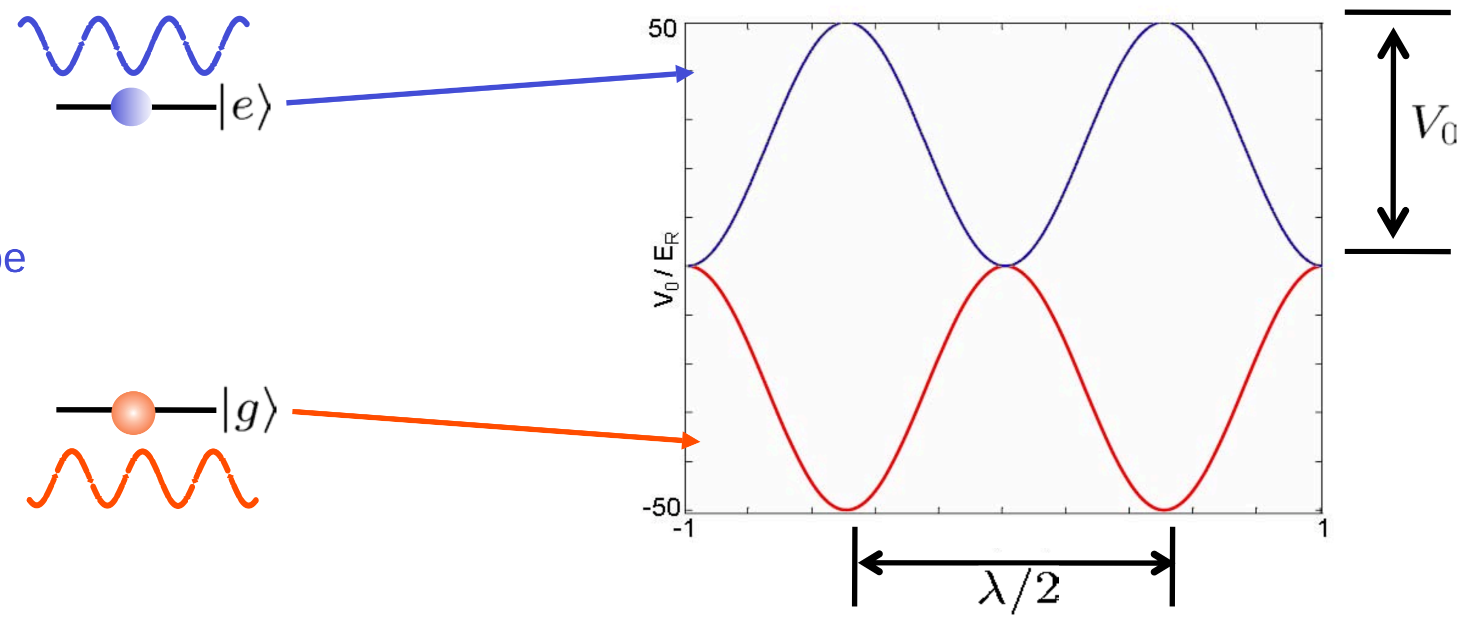




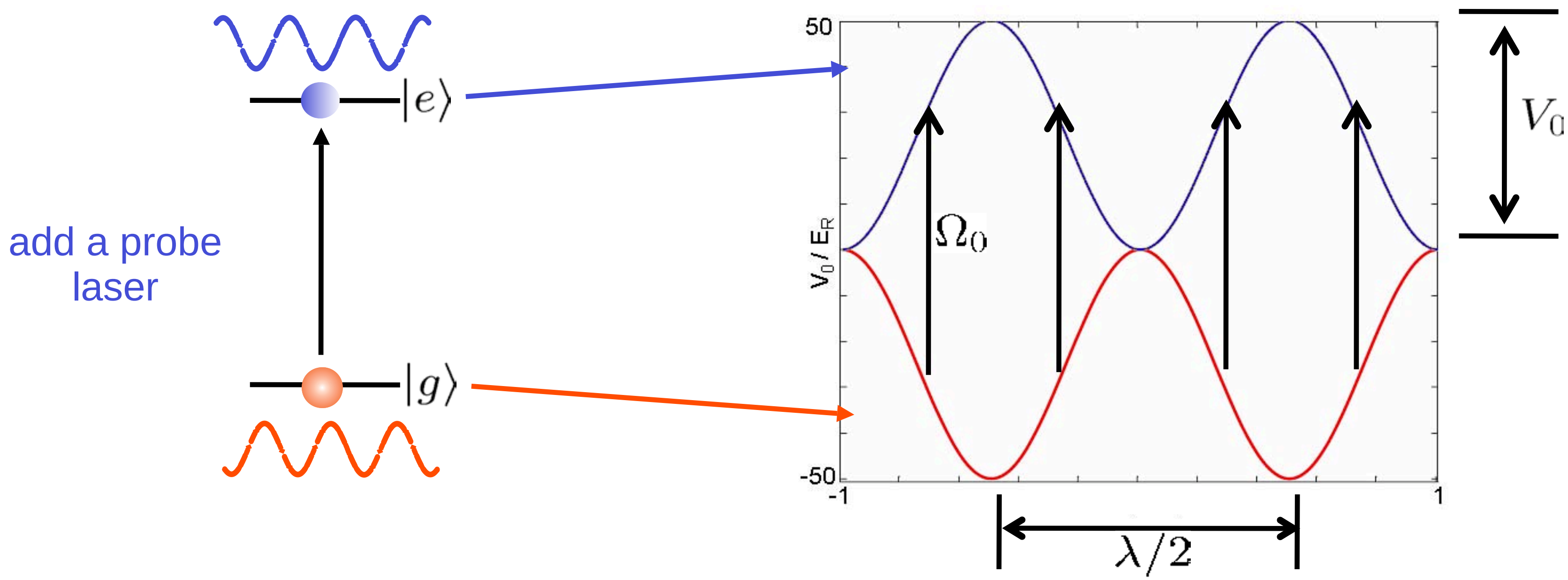
alkaline earth

$\lambda/2$ lattice with depth V_0

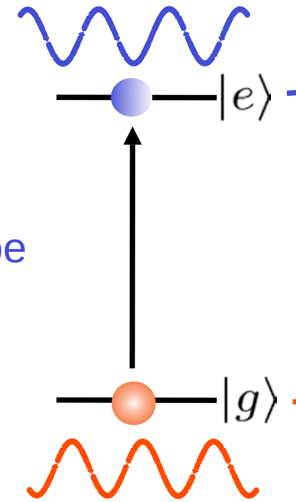
add a probe
laser



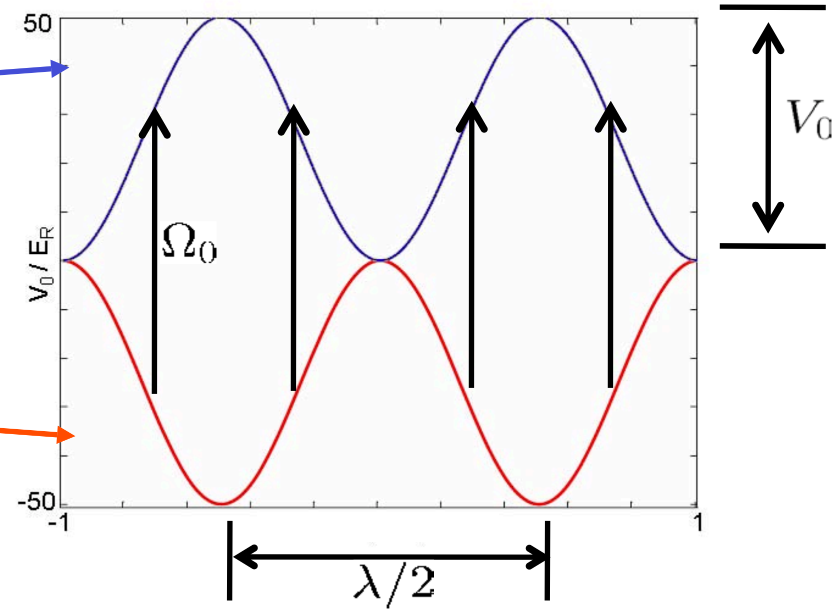
λ/2 lattice with depth V_0



$\lambda/2$ lattice with depth V_0

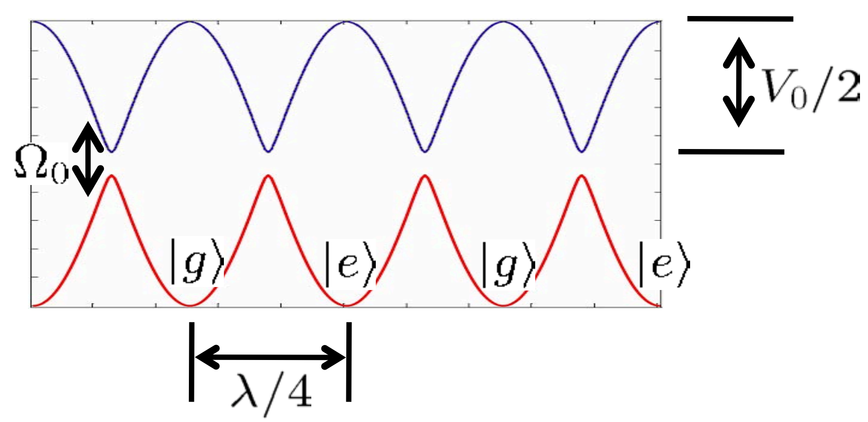


add a probe laser



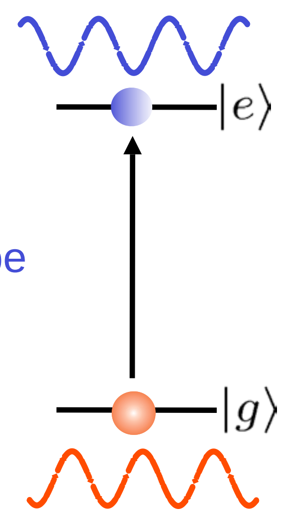
$\lambda/4$ lattice with depth $V_0/2$

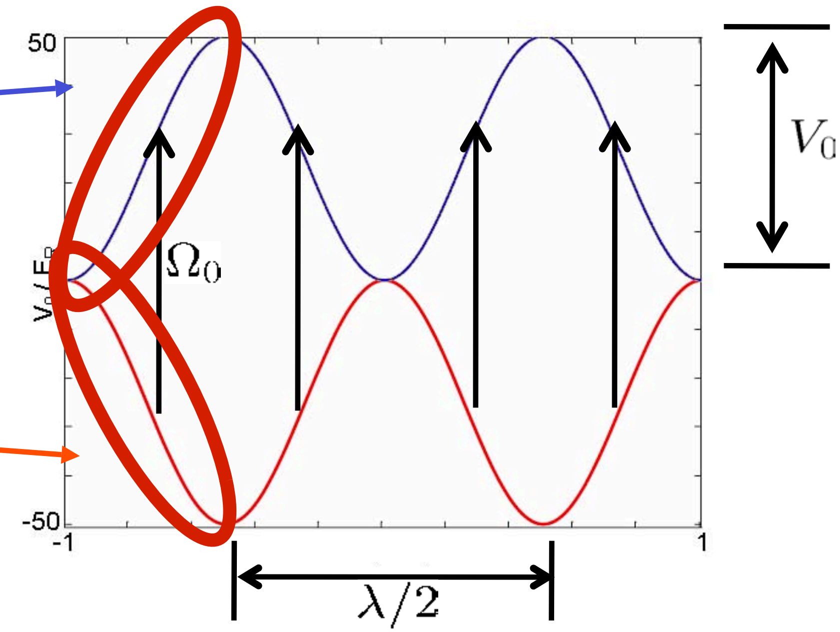
dressed adiabatic potential curves



$\lambda/2$ lattice with depth V_0

add a probe laser





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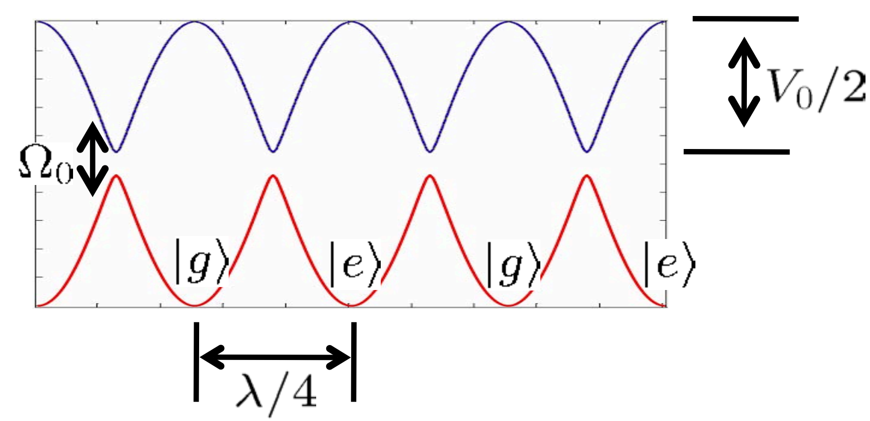
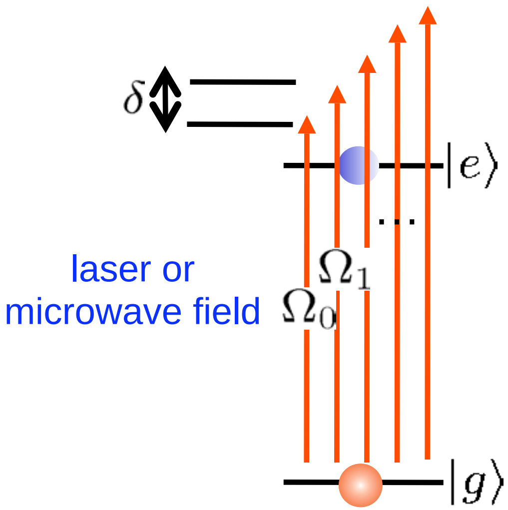
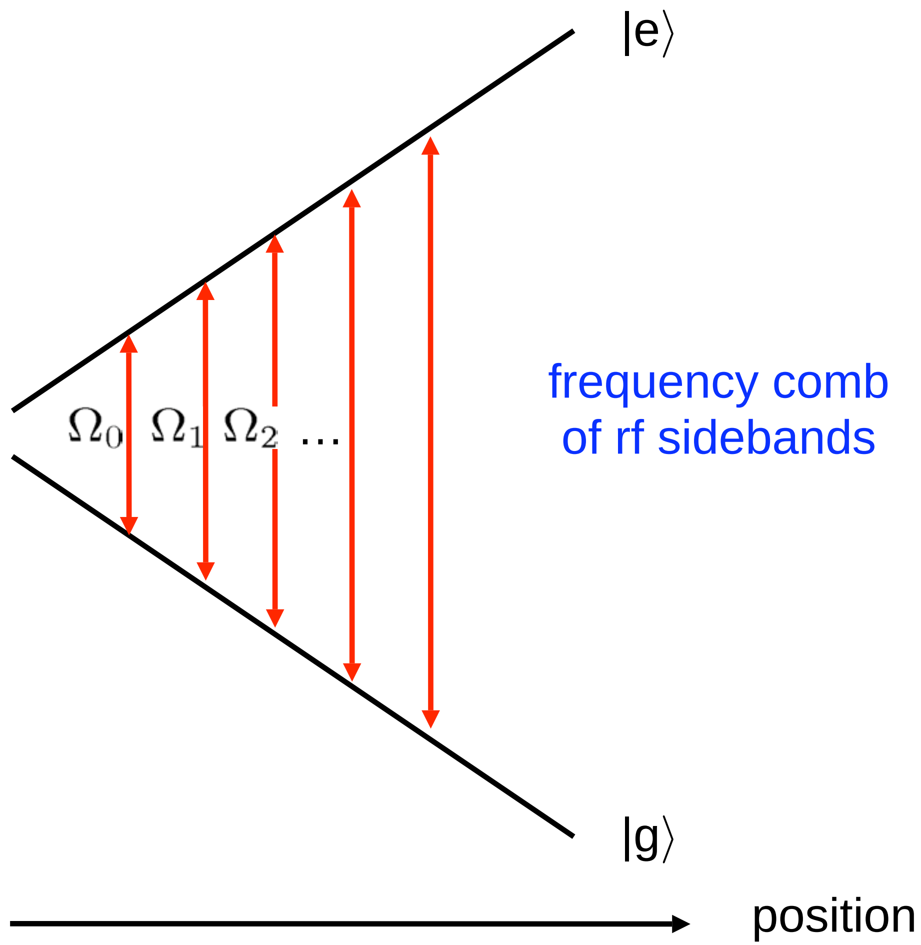


Custom Sub-wavelength lattice with a constant field gradient:

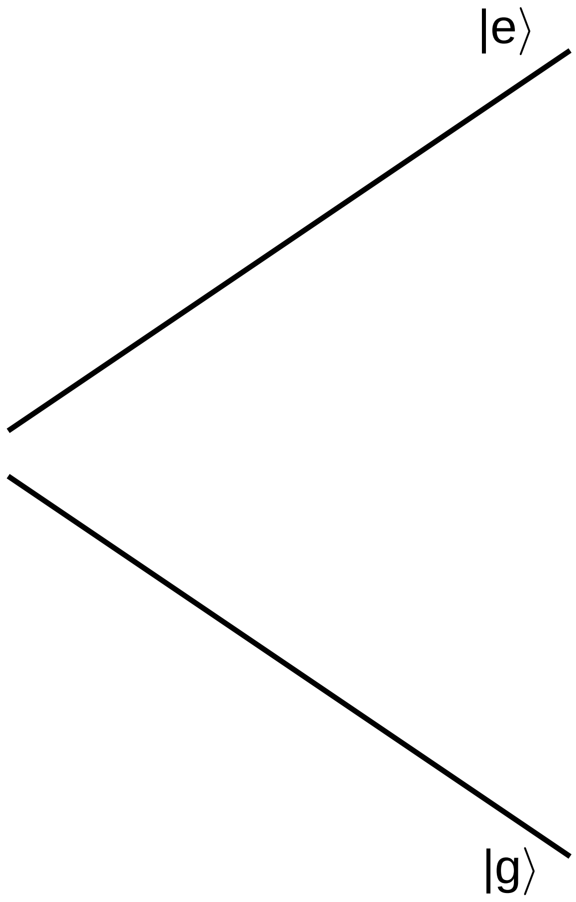
- Two level atom:

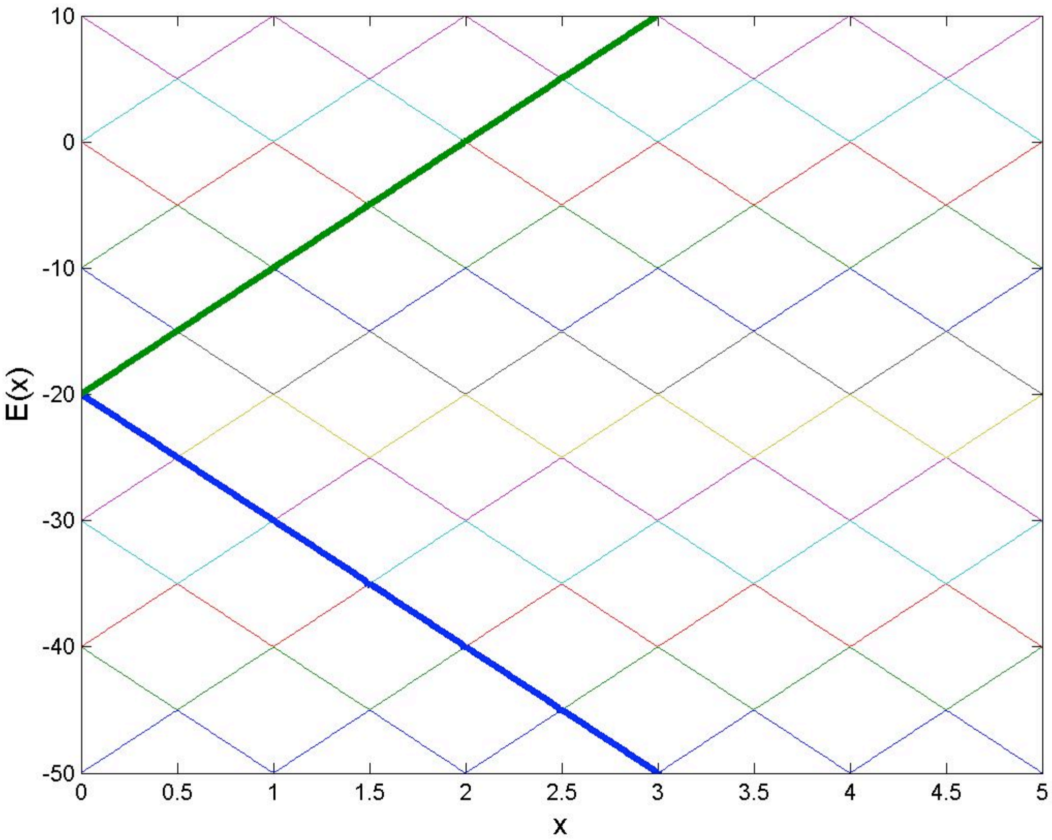


- Linear magnetic/electric field gradient



Solution in Floquet basis (Fourier series in frequency δ):

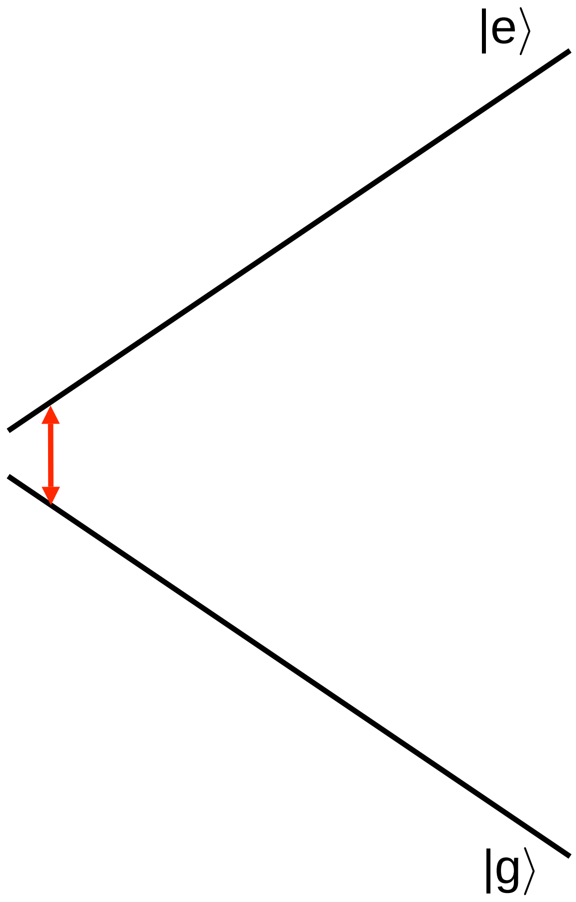


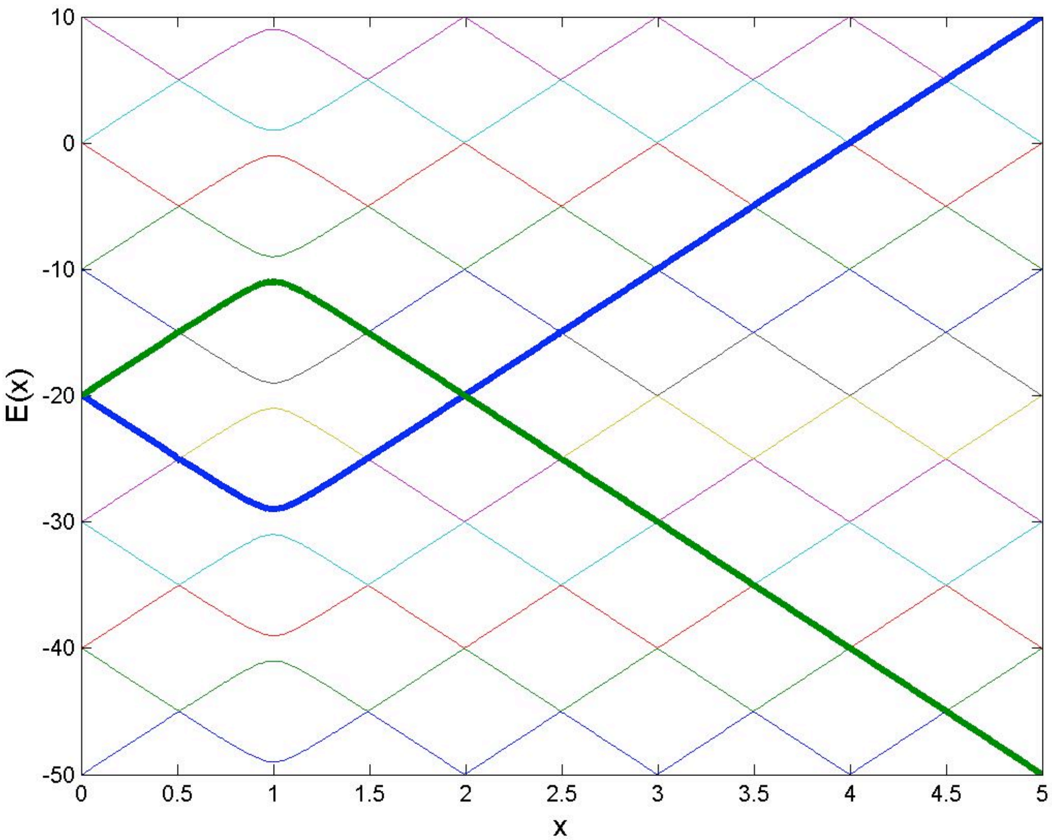


$E_e(x)=5x$, $E_g(x)=-5x$

$\Omega_m=2$, $\delta=10$

Solution in Floquet basis (Fourier series in frequency δ):





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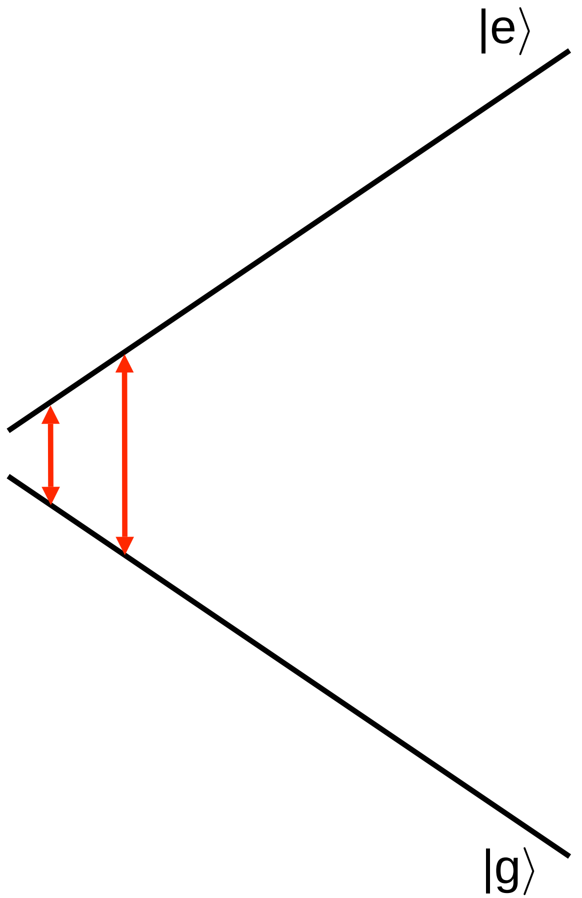

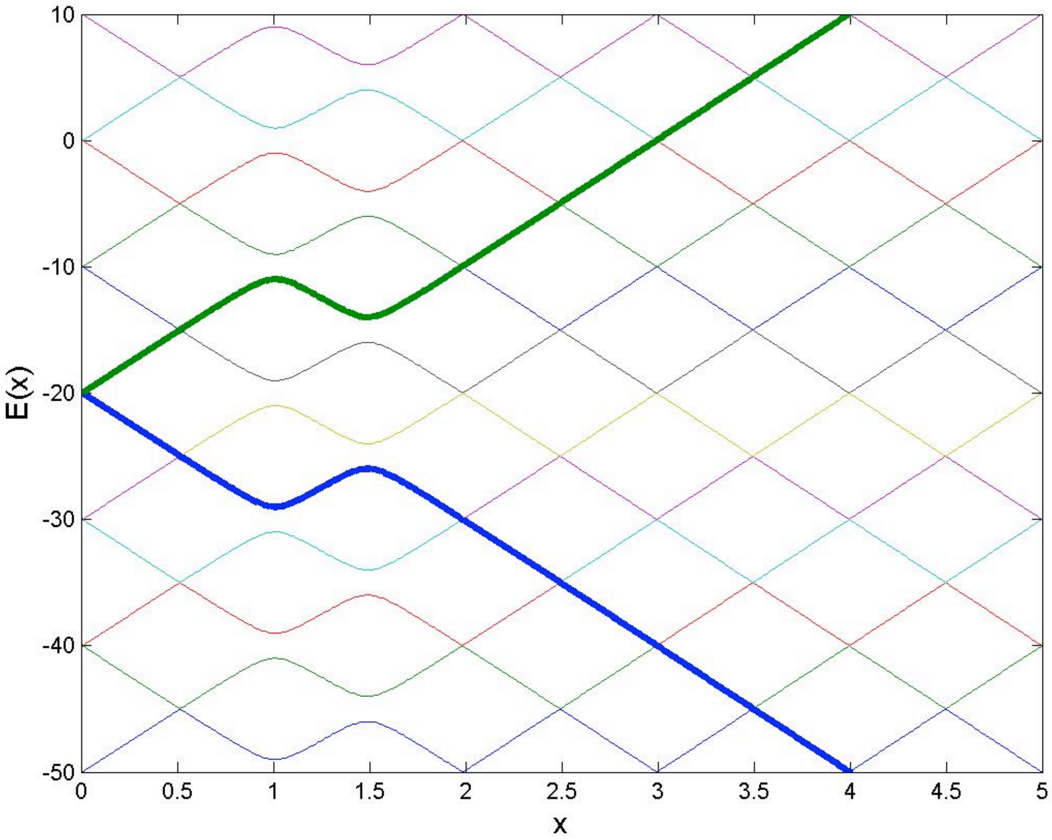

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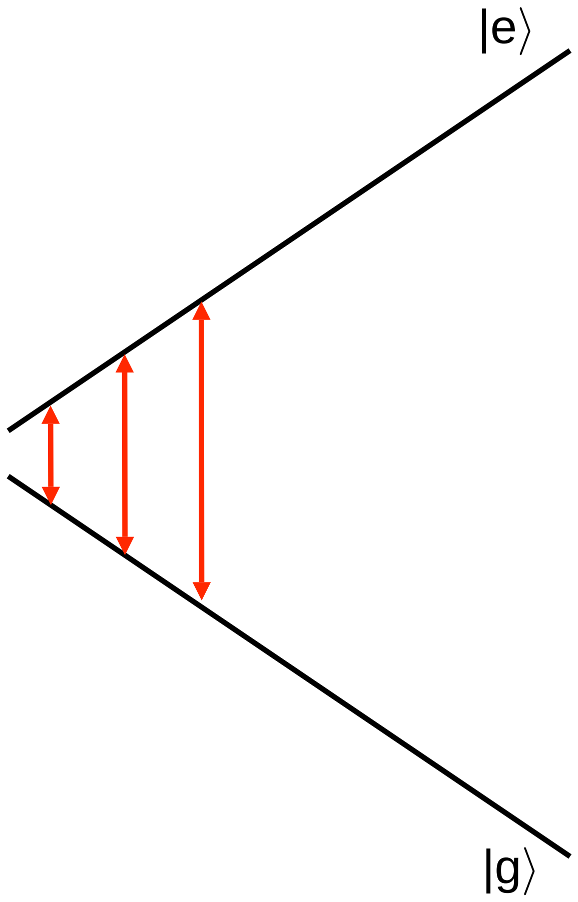

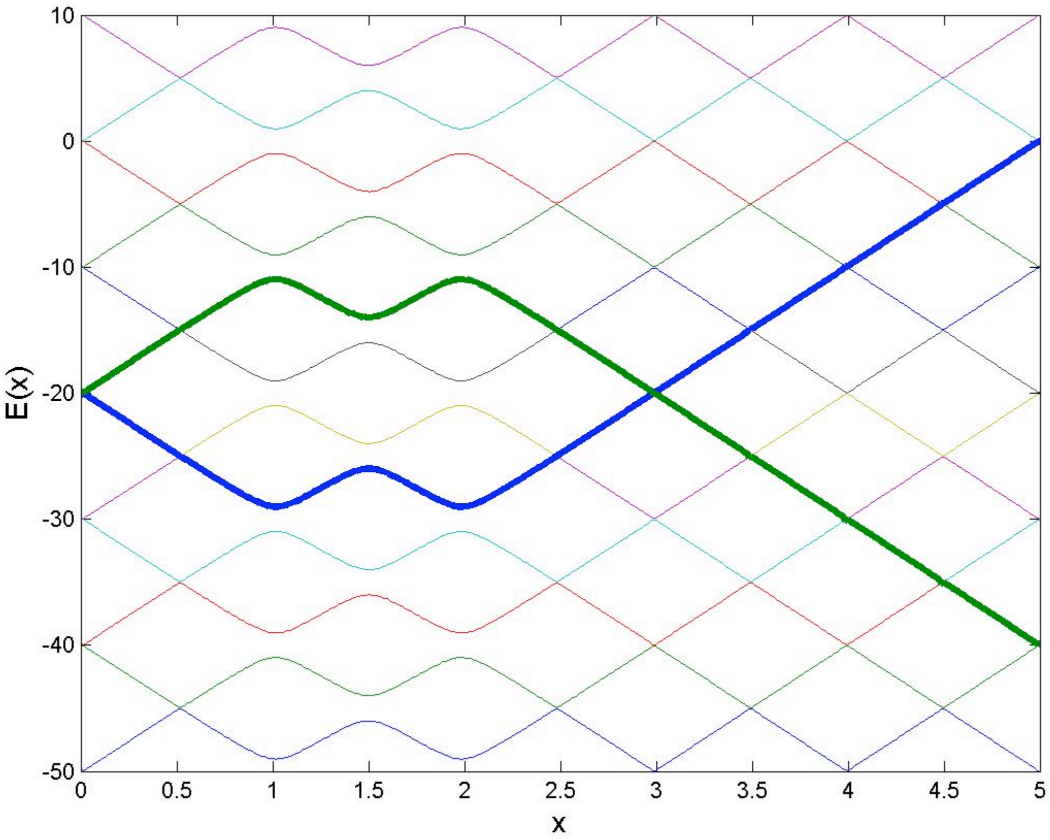

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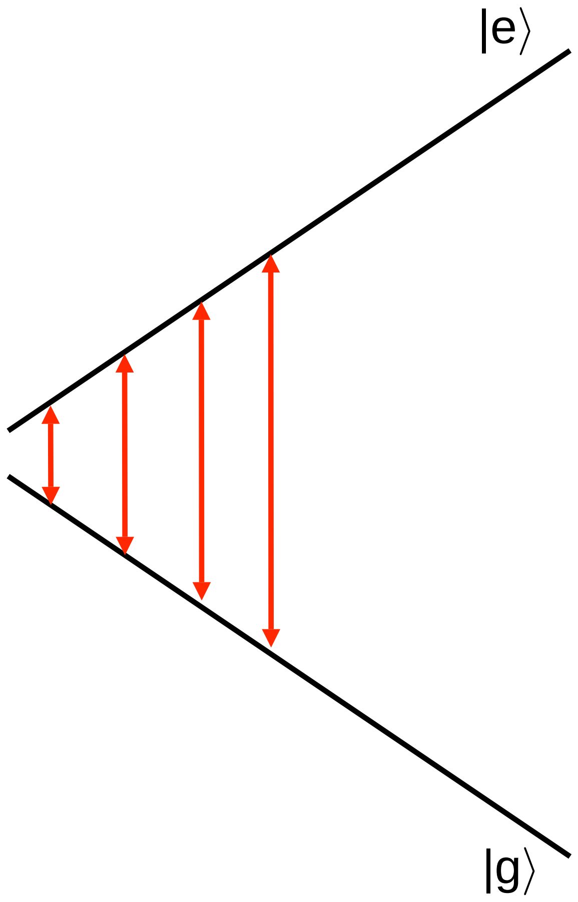

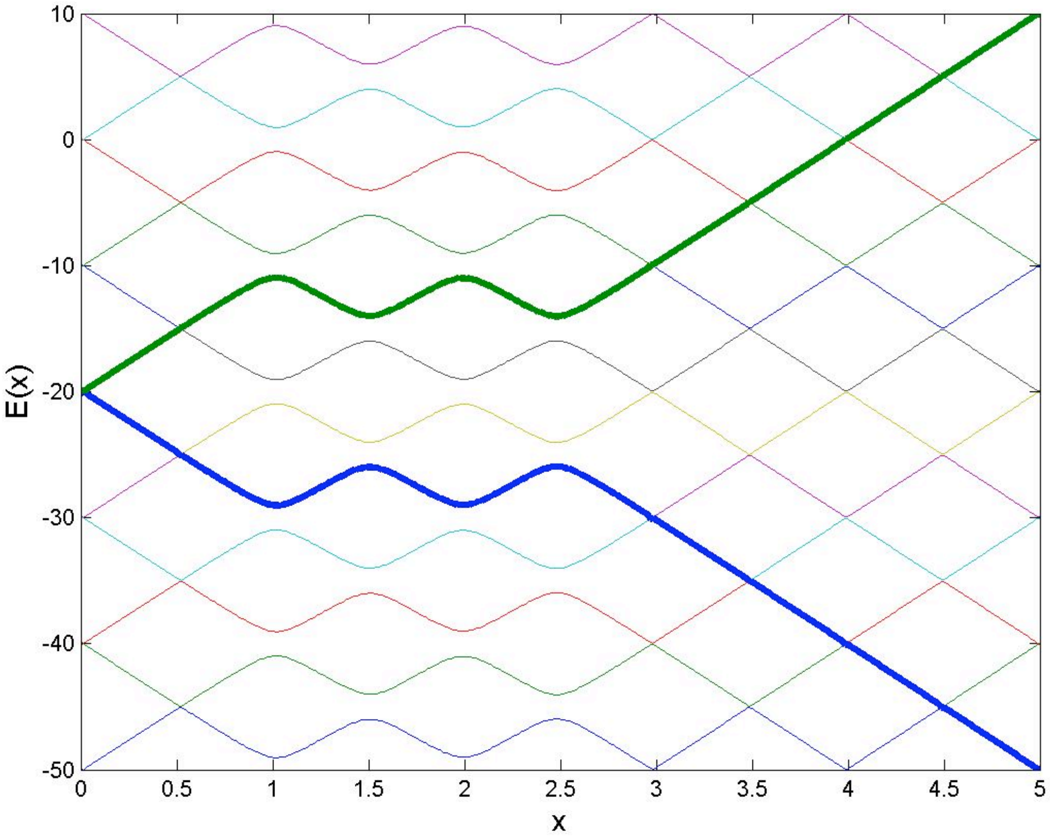

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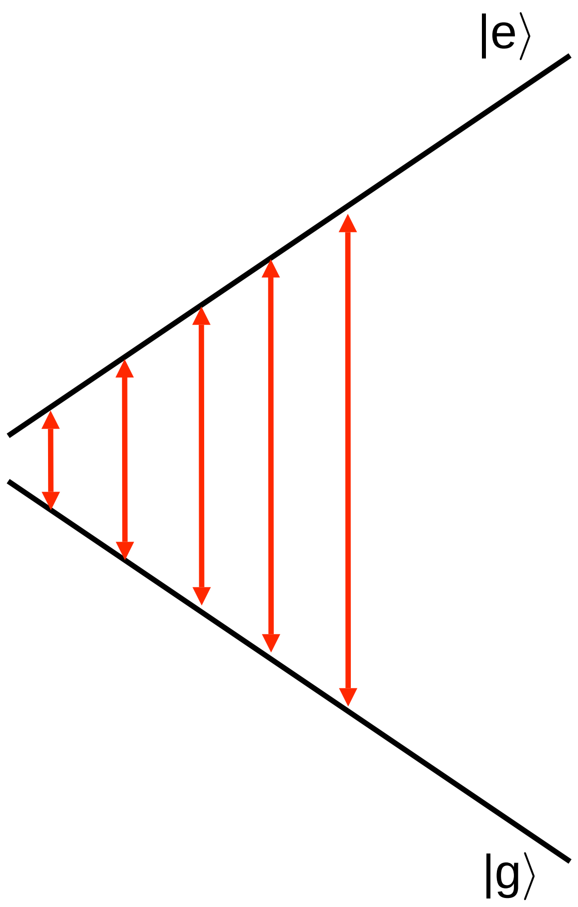

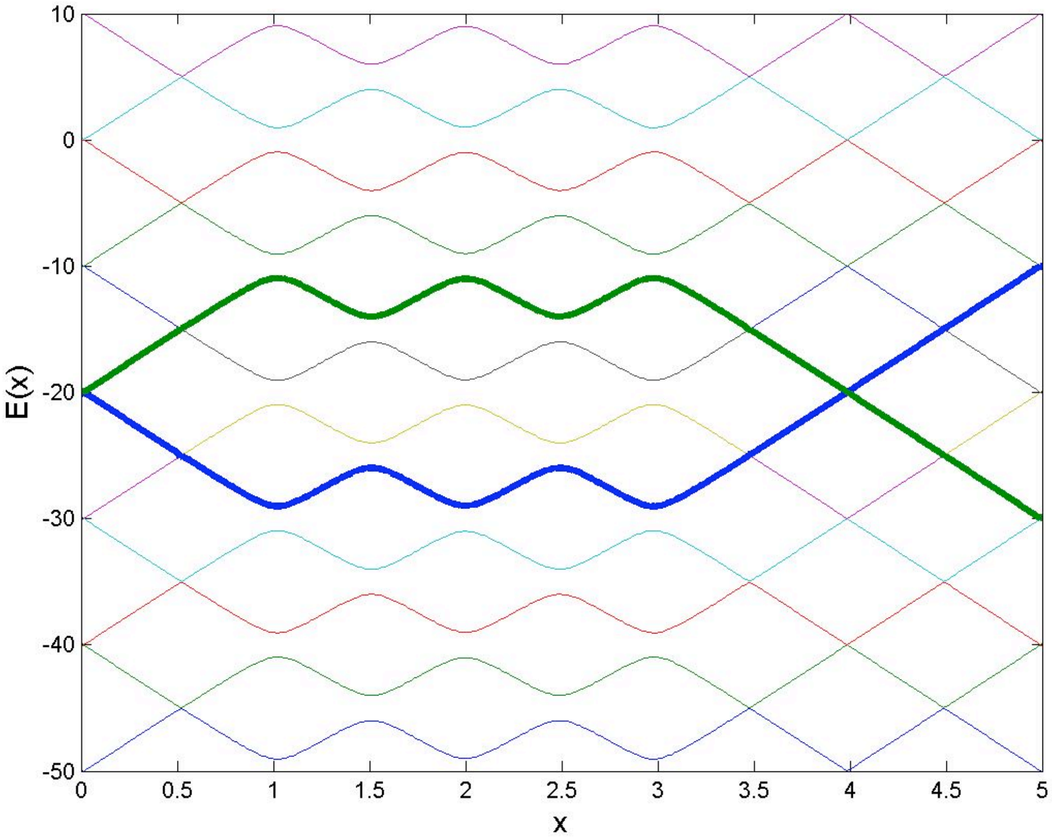

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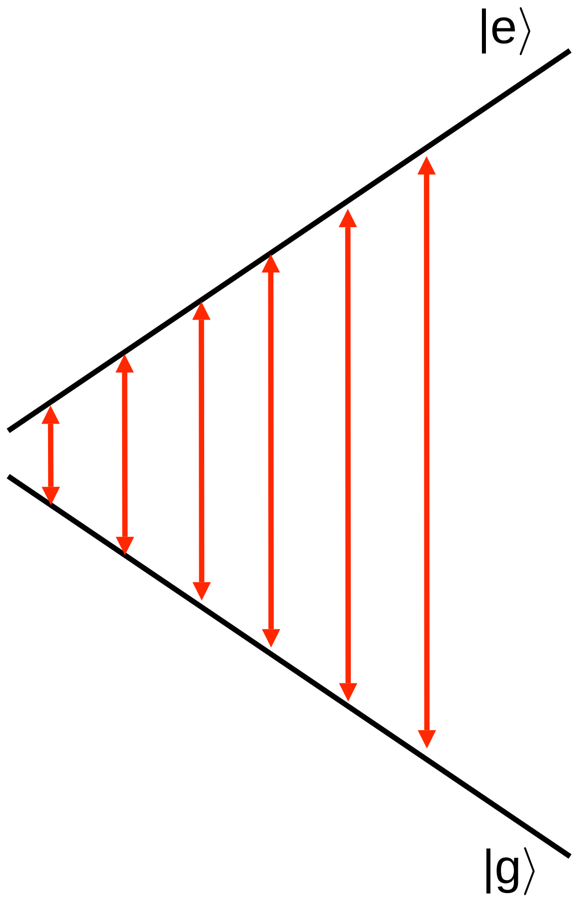

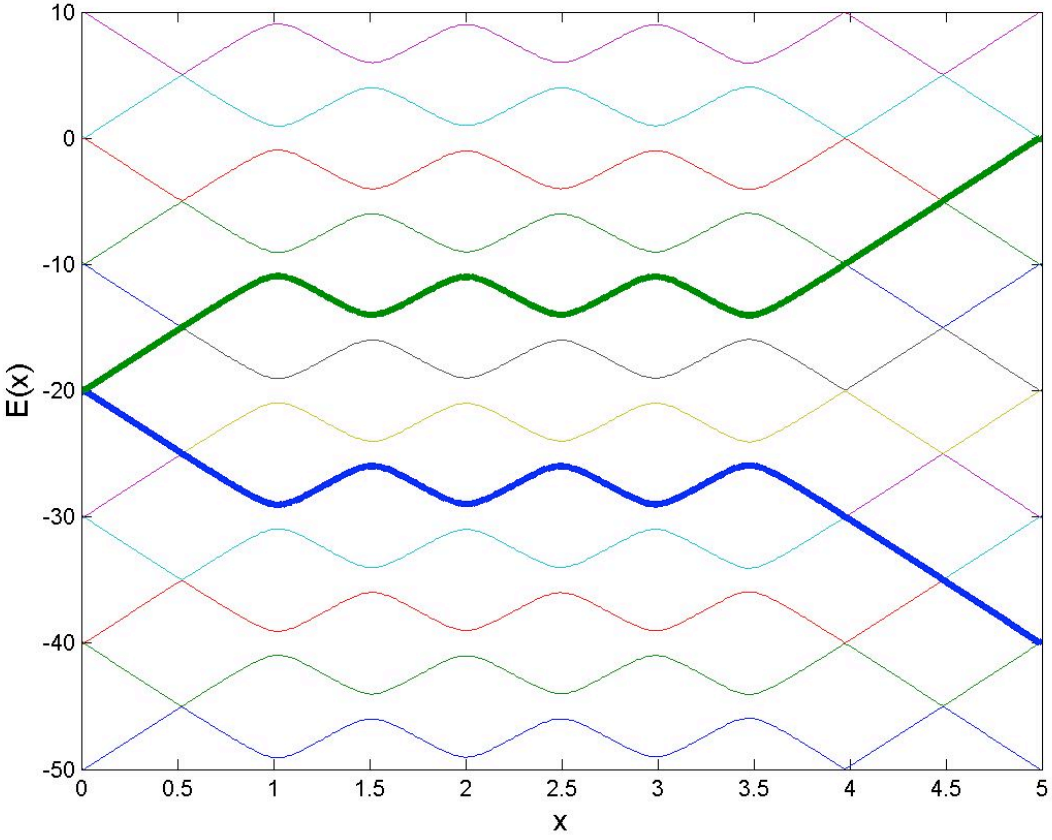

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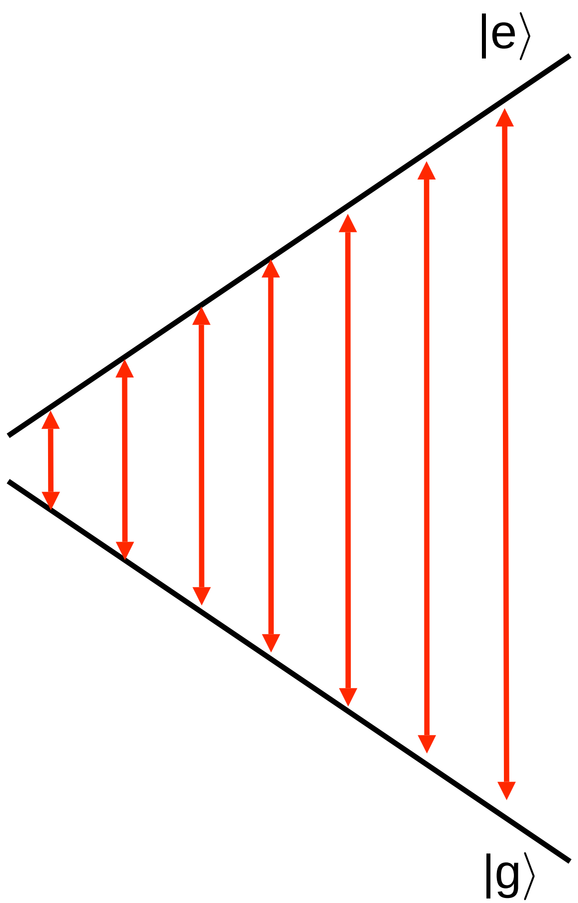

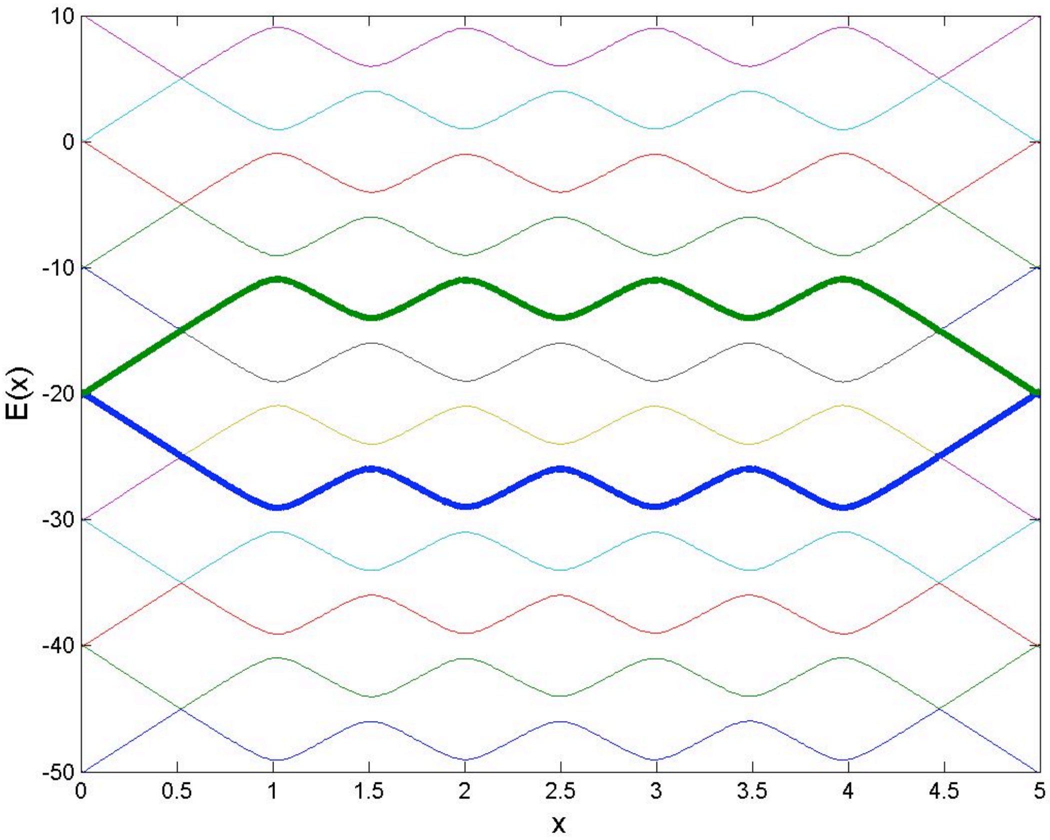

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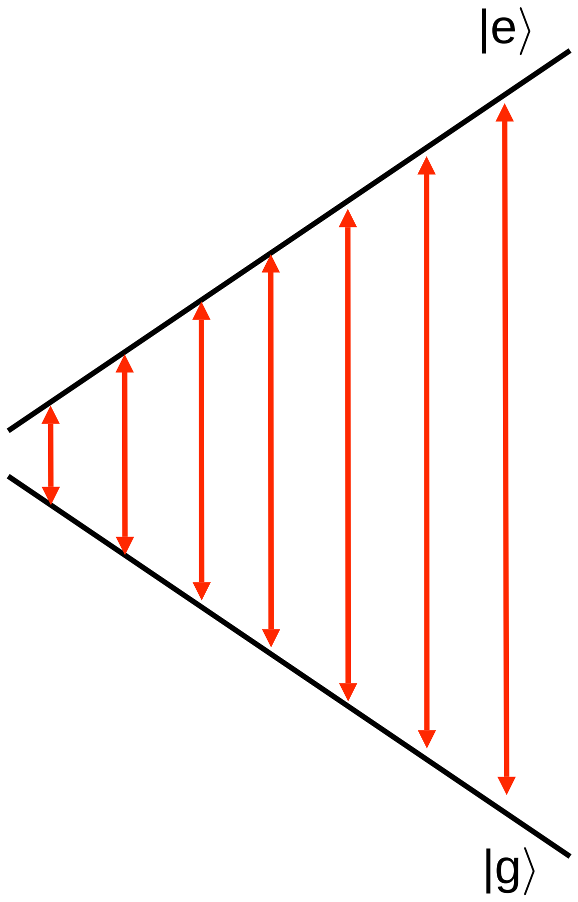

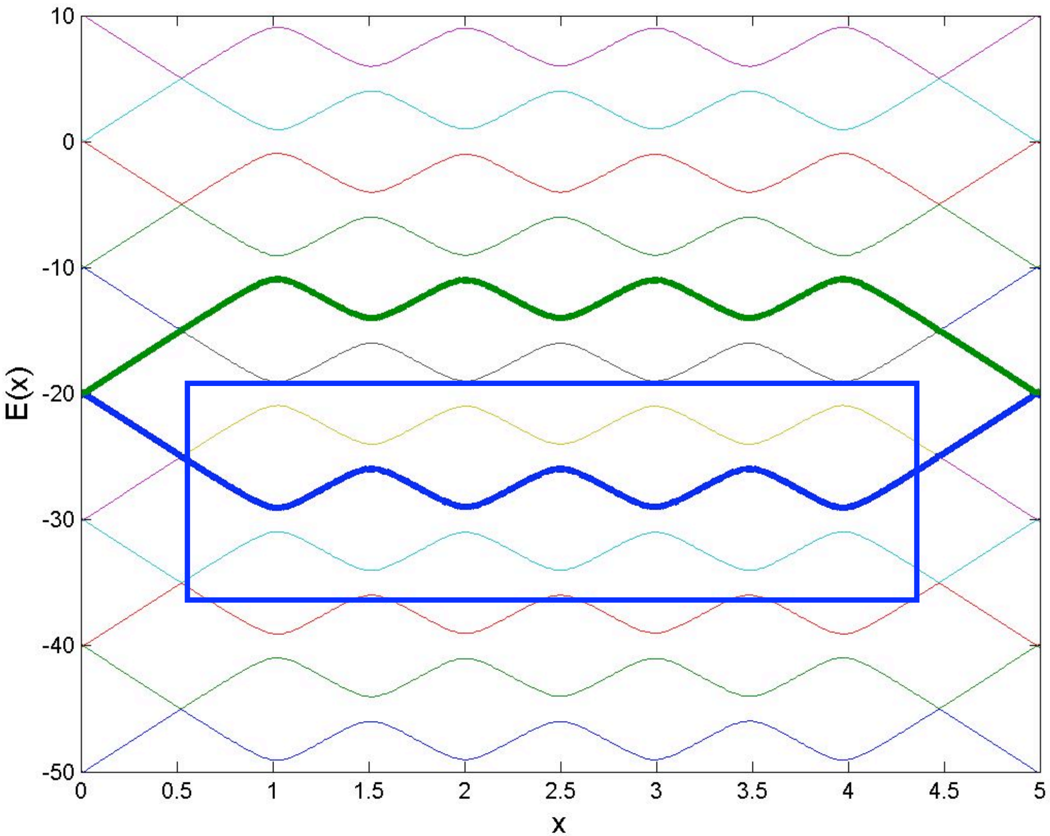

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flat bottom trap

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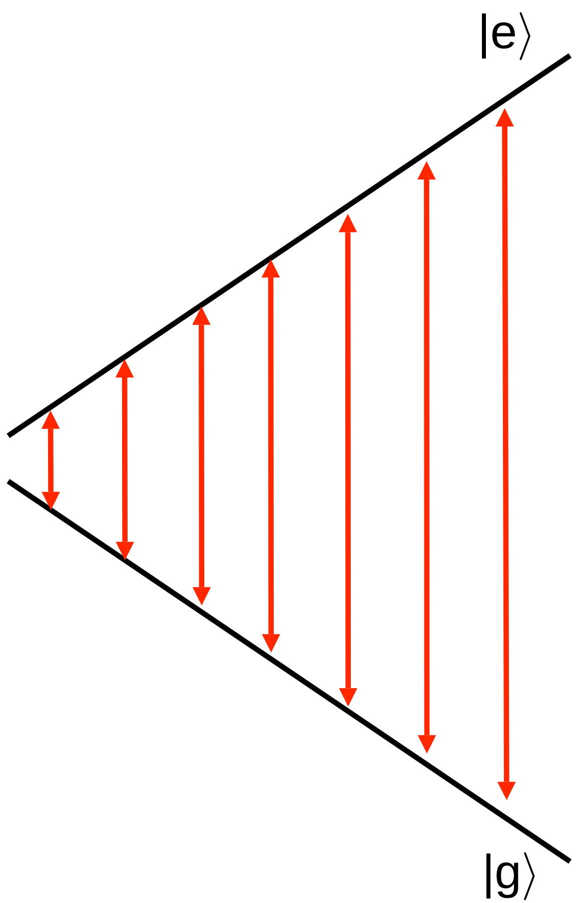

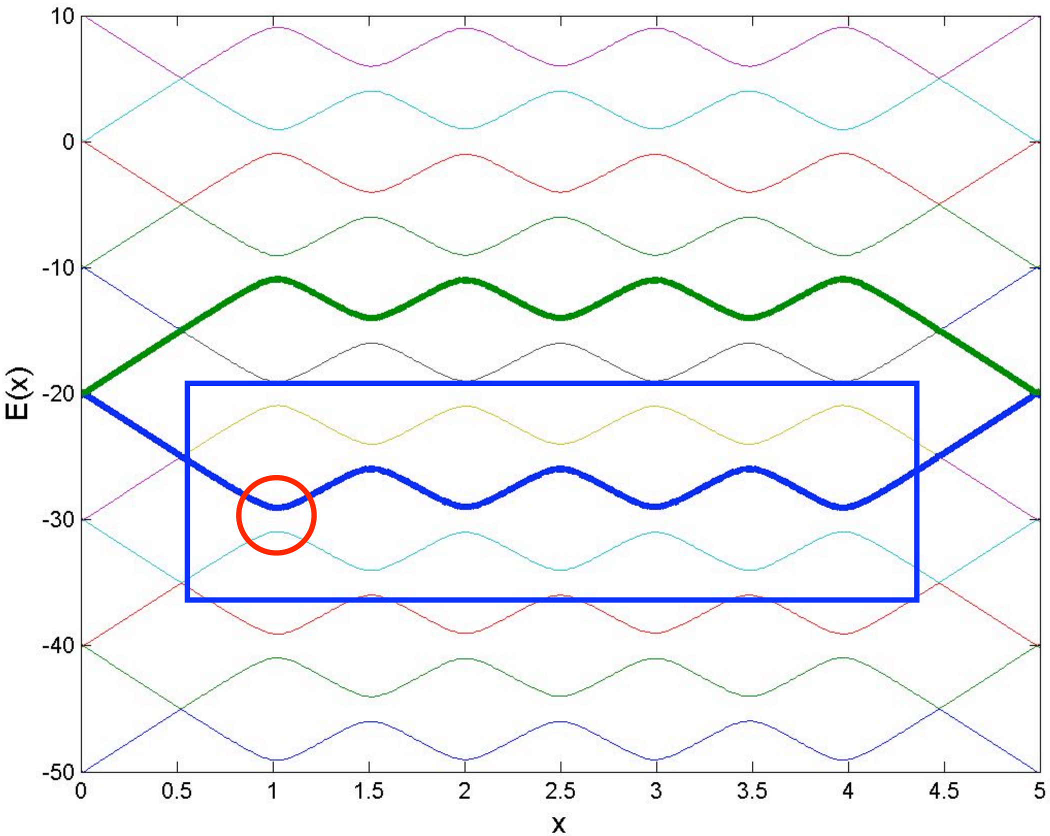

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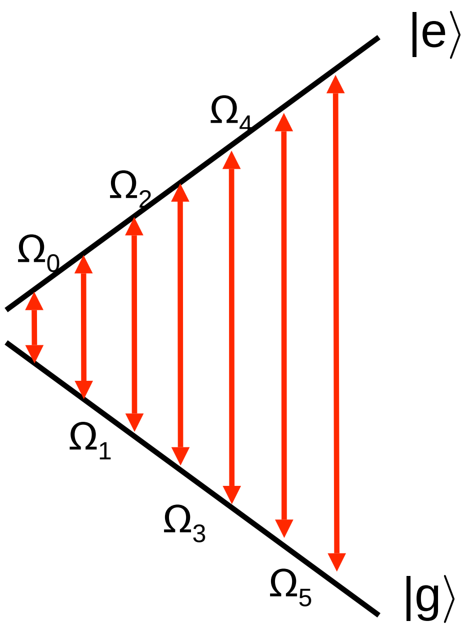
flat bottom trap

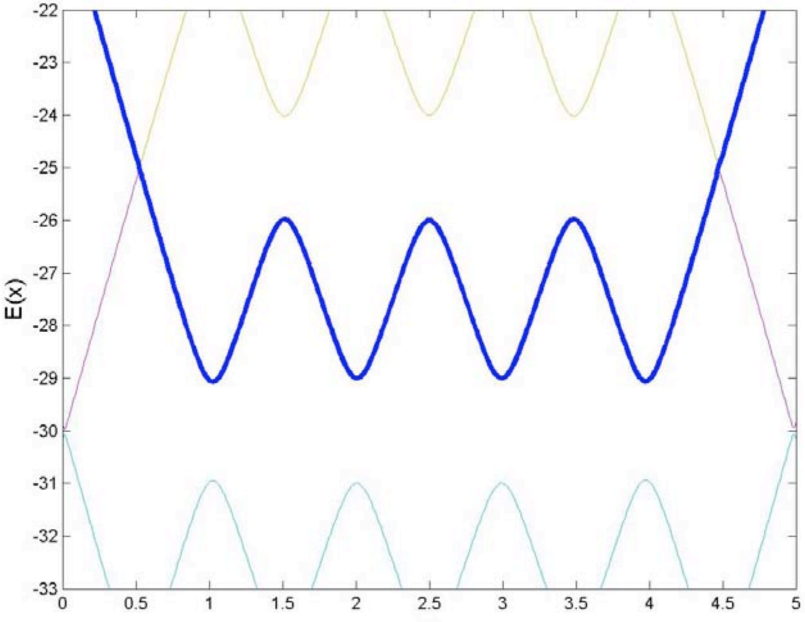
Control of Individual Wells:

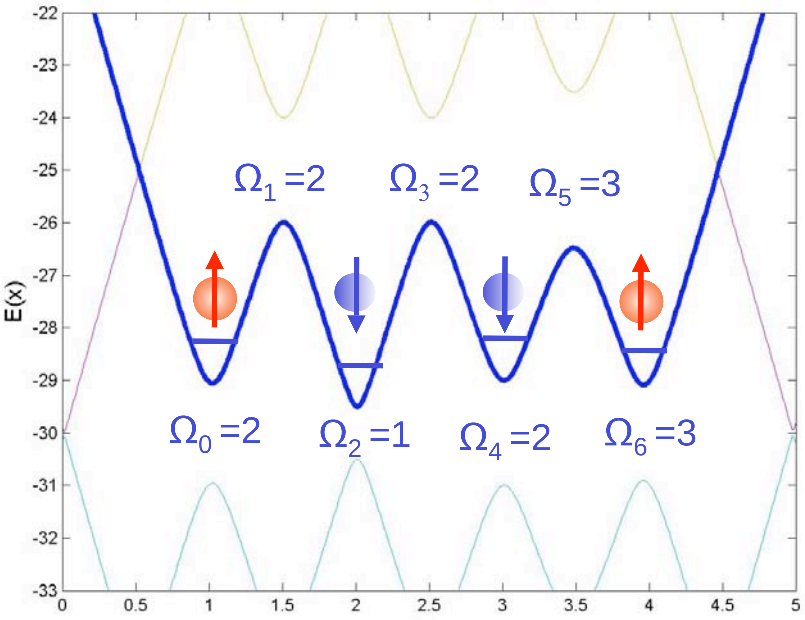


features:

- frequency / amplitude controls depth / height of barrier

$E_e(x)=5x$, $E_g(x)=-5x$, $\Omega_m=2$, $\delta=10$

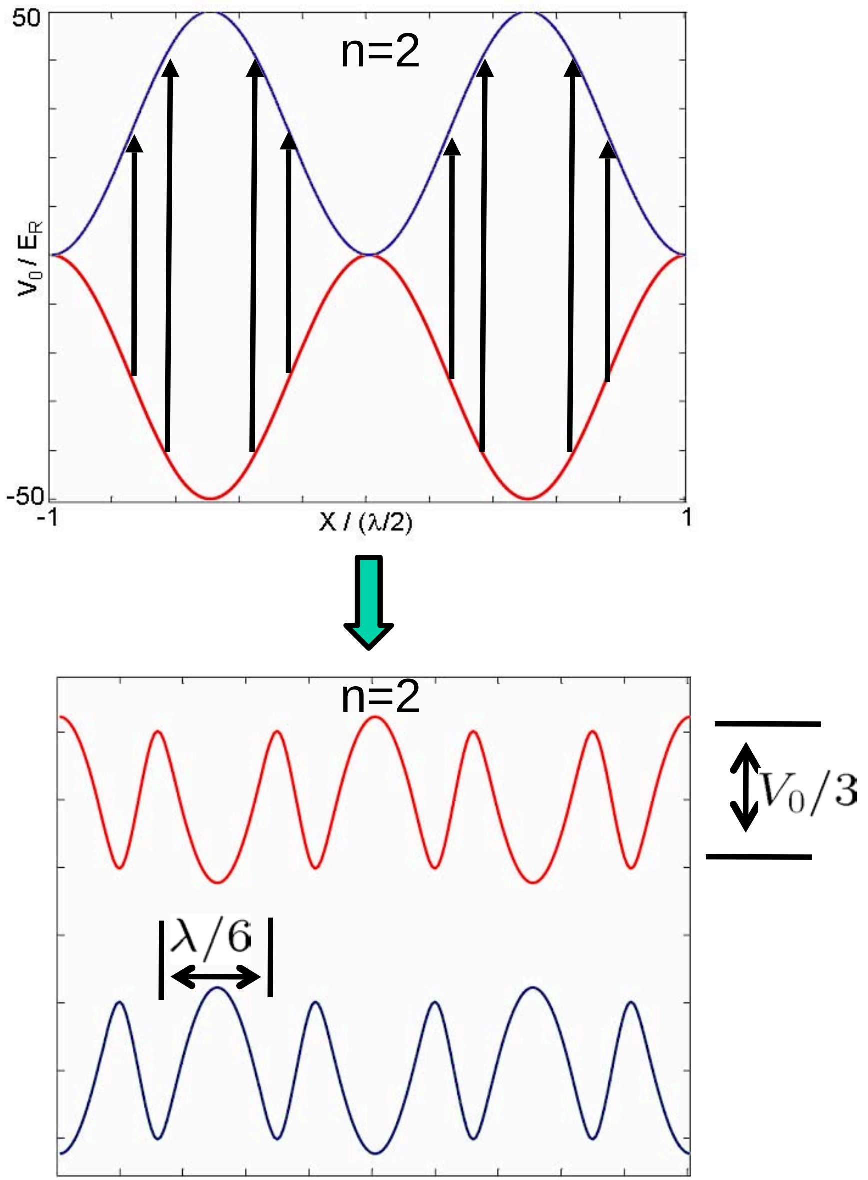




“Inverse PIANO“

… back to optical lattices with n=2,… probe lasers

- n lasers
  - n+1 wells
  - depth $V_0 \rightarrow V_0/(n+1)$



sub-wavelength
superlattice