Workshop on Topics in Quantum Turbulence

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Theory of Low Temperature Decay

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Theory of Low Temperature Decay

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The tangle can be chaotic (non-structured), or essentially polarized (Kolmogorov regime).

We will focus on the non-structured case.
BEC kinetics and superfluid turbulence:
Kibble-Zurek-type picture with a distinct non-trivial mechanism

Simulation of Gross-Pitaevskii equation. N. Berloff and B. Svistunov, 2002

Vortex line reconnections generate Kelvin waves.

Kelvin waves can, in principle, cascade down in the wavelength space, but...
Large and small parameters controlling kinetics of superfluid turbulence at T=0

Local induction: \[ \Lambda = \ln \left( \frac{R}{a_0} \right) \gg 1 \]

Pure Kelvin-wave cascade: \[ \frac{\text{Amplitude}}{\text{Wavelength}} \ll 1 \]
Becomes progressively smaller down the cascade

Kelvon-phonon interaction: \[ \frac{\kappa \lambda^{-1}}{c} \ll 1 \]
“Non-relativistic” parameter; guarantees weakness of coupling to phonons
Conservation laws controlling kinetics of superfluid turbulence at $T=0$

**Energy conservation**
To a good approximation, energy scales as the vortex line length.

**Momentum conservation**
For a vortex ring, momentum scales as the (algebraic) area.

**Angular momentum conservation**
= conservation of the number of kelvons:

Kelvons cannot scatter inelastically

**Integrability of the local-induction limit**
Suppression of kelvon scattering processes (!)
Absence of Feynman’s cascade

Feynman’s cascade is inconsistent with simultaneous conservation of energy and momentum.
The role of self-reconnections

A self-reconnection produces a small ring, the ring gets re-absorbed by the tangle. What’s left behind are Kelvin waves!
A smart model: Collapse supported cascade

canonical variables:
\( x \) – momentum
\( y \) – coordinate

\[ w(z) = x(z) + iy(z) \]

\[ i\Delta = \frac{\delta H[w]}{\delta w^*} \]

\[ H[w] = \int dz \sqrt{1 + |w'(z)|^2} \]

This model corresponds to the local induction approximation (and thus is integrable) as long as the function is smooth.

However, the time evolution naturally produces discontinuities, and these are qualitatively equivalent to self-crossings.
Pure Kelvin-wave cascade

Kozik and Svistunov, 2003

Small parameter \( = \frac{\text{Amplitude}}{\text{Wavelength}} \)

Becomes progressively smaller down the cascade.

1. Kelvon is a good elementary excitation

2. Number of kelvons is conserved (due to the rotational invariance)

**Kelvon Hamiltonian:**

\[ H_{kw} = \sum \varepsilon_k a_k^+ a_k + \ldots \]

**canonical variables:**

- \( x \) – momentum
- \( y \) – coordinate

\[ w(z) = x(z) + iy(z) \]

\[ a_k \propto \int w(z) e^{ikz} \, dz \]
Non-trivial two-kelvon scattering is absent because of 1D

Kinetics are driven by three-kelvon elastic scattering:

\[ n_k \propto k^{-17/5} \]

or for Kelvin wave amplitude \( b_k \propto \sqrt{n_k k} \):

\[ b_k \propto k^{-6/5} \]
Initial condition:
\[ n_k \propto k^{-3} \]

External forces:

*drain, no source*

Evolution picture:

a back-wave propagating from large-wavenumber region towards smaller wavenumbers transforms
\[ n_k \propto k^{-3} \] into \[ n_k \propto k^{-17/5} \].
Kelvin-wave cascade in the chaotic tangle

(Circumstantial) experimental evidence:
Davis, Hendry, and McClintock, 2000

\[ b_k \propto k^{-1} \sqrt{\ln k} \]
Svistunov, 1995 (theory)
Tsubota, Araki, Nemirovskii, 2000 (simulation; note however, authors' interpretation in terms of Feynman's cascade)

assisted by local self-crossings
Kozik and Svistunov, 2003 (kinetic theory, simulation)
Kozik and Svistunov, 2005

pure cascade (no reconnections)
Kivotides, Vassilicos, Samuels, and Barenghi, 2001 (simulation)
Vinen, Tsubota, and Mitani, 2003 (simulation)

energy flux

k_{ph} \ll a_0

wavenumber scales

\[ R_0^{-1} \quad k_* \quad k_{ph} \ll a_0 \quad \ln k \]
Emission of sound, and more…
What are phonons in the presence of vortices?

Phase field: \( \Phi(\vec{r}) = \Phi_0(\vec{r}) + \varphi(\vec{r}) \)

\[ \oint_{\text{around vortex}} \nabla \Phi_0 \cdot d\vec{l} = \pm 2\pi \quad \text{singular} \]

\[ \oint \nabla \varphi \cdot d\vec{l} = 0 \quad \text{non-singular} \]

\[ \Delta \Phi_0(\vec{r}) = 0 \quad \text{(away from a vortex)} \]

But how about the causality?

To find the canonical variables, use the Lagrangian.
Vortex – Phonon Lagrangian

\[ L = \int d\mathbf{r} \left[ -n \dot{\Phi}_0 - \eta \dot{\varphi} - \eta \dot{\Phi}_0 \right] - H_{kw} - H_{ph} + H_{int} \]

vortex
\[ \oint \nabla \Phi_0 d\mathbf{r} = 2\pi \]
around vortex
\[ \nabla^2 \Phi_0 = 0 \]

phonons
\[ H_{kw} \approx \sum_k \varepsilon_k a_k^+ a_k \]
\[ H_{ph} \approx \sum_q \omega_q c_q^+ c_q \]
\[ \oint \varphi d\mathbf{r} = 0 \]
anywhere

\[ \{a_k, a_k^+\}, \{c_q, c_q^+\} \text{ are not canonical!} \]

variable transformation:
\[ \{a_k, a_k^+\}, \{c_q, c_q^+\} \quad \text{(eliminating } \eta \dot{\Phi}_0 \text{)} \rightarrow \{\tilde{a}_k, \tilde{a}_k^+\}, \{\tilde{c}_q, \tilde{c}_q^+\} \]
canonical
Vortex – phonon Hamiltonian

Energy = \sum \varepsilon_k a_k^+ a_k + \sum \omega_q c_q^+ c_q + H_{int}

Hamiltonian =

\sum \varepsilon_k \tilde{a}_k^+ \tilde{a}_k + \sum \omega_q \tilde{c}_q^+ \tilde{c}_q + m = -1 + m = -2 + m = -3 + \cdots + H_{int}

conserves
1. momentum along z
2. angular momentum along z

Fetter, '64
Sound radiation by superfluid turbulence

\[ \Pi_k = \text{Const.} \frac{\varepsilon_k^6 k}{c^5 \rho} \left( n_k n_{-k} \right), \quad \varepsilon_k = \frac{k}{4\pi} \ln(1/k a_0) k^2 \]

radiated power per unit length:

\[ n_k \propto k^{-3} / (\ln k)^\gamma \]

\[ n_k \propto k^{-17/5} \]

\[ k_{ph} \sim \frac{[a_0 / R_0]^{6/31}}{[\ln(R_0 / a_0)]^{24/31}} a_0^{-1} \]

\[ \lambda_{ph} \sim 1 \div 0.1 \mu m \]
Scattering

Elastic scattering

\[ \text{pinned vortex} \]

correct result: Pitaevskii, '58
Sonin, '76
(see also Iordanskii)

Inelastic scattering

\[ \text{Fetter, '64} \]

Demircan, Ao, Niu, '95

Kozik and Svistunov, 2005

Kozik and Svistunov, 2005
Conclusions

• Reasons for a physically rich Theory: Conserving quantities and small parameters

• Absence of Feynman’s cascade

• Self-reconnection supported range is inevitable.

• Theory of pure Kelvin wave cascade

• The Hamiltonian of vortex-phonon interaction: the answers for sound radiation by Kelvin wave cascade, and other vortex-phonon processes.