Workshop on Topics in Quantum Turbulence

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Numerical simulation of counterflow turbulence.

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Numerical simulation of counterflow turbulence

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1. Making a steady state of counterflow turbulence by fully nonlocal Biot-Savart calculation

2. Obtaining the velocity distribution of vortices compared with the Maryland’s observation

Thanks to M. Paoletti, D. Lathrop

Visualization of counterflow by Paoletti et al.
A vortex makes the superflow of the Biot-Savart law, and moves with this local flow. At a finite temperature, the mutual friction should be considered.

\[
\dot{s}_0 = \frac{\beta}{4\pi} \mathbf{s}' \times \mathbf{s}'' + \frac{\kappa}{4\pi} \int_L \frac{\left(\mathbf{s}_1 - \mathbf{r}\right) \times ds_1}{|\mathbf{s}_1 - \mathbf{r}|^3} + \mathbf{v}_{s,a}(\mathbf{s})
\]

\[
\dot{s} = \dot{s}_0 + \alpha \mathbf{s}' \times (\mathbf{v}_n - \dot{s}_0) - \alpha' \mathbf{s}' \times \left[\mathbf{s}' \times (\mathbf{v}_n - \dot{s}_0)\right]
\]

The approximation neglecting the nonlocal term is called the LIA (Localized Induction Approximation).

\[
\dot{s}_0 = \frac{\beta}{4\pi} \mathbf{s}' \times \mathbf{s}'' + \mathbf{v}_{s,a}(\mathbf{s})
\]
Schwarz simulated the counterflow turbulence by the vortex filament model and obtained the steady state.

However, this simulation was unsatisfactory.

1. All calculations were performed by the LIA.
Schwarz’s simulation(2) PRB38, 2398(1988)

However, this simulation was unsatisfactory.

1. All calculation was performed by the LIA.
2. He used an artificial mixing procedure in order to obtain the steady state.
After Schwarz, there has been no progress on the counterflow simulation.

In this work we made the steady state of counterflow turbulence by fully nonlocal simulation to understand the Maryland’s observation.

An important criterion of the steady state is to obtain

\[ L = \gamma^2 |v_n - v_s|^2 \quad \gamma = \frac{\pi B \rho \kappa}{\kappa \rho} \]
Simulation by the full Biot-Savart law

BOX 2mm×2mm×2mm
T=1.9(K)
Vn=3mm/s
Vs=2.71(mm/s)
Periodic boundary conditions for all three directions
Time-development of the line-length density $L$

![Graph showing the time-development of the line-length density $L$ for different velocities $V_n$. The graph includes lines for $V_n = 0.3$ (cm/s), $V_n = 0.2$ (cm/s), and $V_n = 0.15$ (cm/s). The graph indicates fluctuations in $L$ over time, with one line specifically denoted as $T=1.9K$.](image-url)
The results are consistent with the observations by Childers and Tough, PRB13, 1040(1976).
Observation of the velocity by the solid hydrogen particles


**Upward particles**

![Graph showing upward particles at different temperatures](image)

**Downward particles**

![Graph showing downward particles at different temperatures](image)

The broken line shows

\[ v_n = \frac{q}{\rho_{ST}} \quad v_s = -\frac{\rho_n}{\rho_s} v_n \]

The downward particles should be related with the velocity of vortices!
As the first trial, we obtained the velocity distribution on the vortices.

\[ v_s = -\frac{\rho_n}{\rho_s} v_n = -0.271 \text{cm/s} \]

The peak is just shifted from the mean superflow velocity.

\[ T = 1.9 \text{(K)} \]

BOX 2mm × 2mm × 2mm

\[ V_n = 3 \text{(mm/s)} \]
The simulation shows some qualitative behavior, but not quantitative agreement with the experiments.
Velocity distribution (2)

Direction of the downward particles

Experiments

Simulation

We obtained the similar distribution for other temperatures and velocities.
Summary

We performed the numerical simulation for counterflow turbulence by the full nonlocal Biot-Savart law.

1. We obtained statistically steady states.
2. We calculated the superfluid velocity distribution on the vortices and compared it with the Maryland’s observation. They were qualitatively consistent.

Future developments

By considering the force acting on the particles, we will develop the analysis.