Workshop on Topics in Quantum Turbulence

16 - 20 March 2009

Vortex State in hcp Solid He: Vortex Fluid to Supersolid Transition, and Vortex Dynamics of the Vortex Fluid State

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Vortex state in \textit{hcp} solid He:

Vortex fluid to Supersolid transition, and vortex dynamics of the vortex fluid state

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Supersolid state of matter: history and present rapid developments

History:
BEC: 1923
BEC & Superfluidity: 1938, F. London; Kapitsa,
Landau Theory of Superfluidity and BEC
BEC and Solid State: Penrose & 1956
Fermi liquid: 1957, BCS theory: 1957
Quantized flow: Vinen & Hall

Discussions about Supersolid state:
Reatto & Chester, Chester, Andreev & Lifshitz,
Long history of solid $^4$He study, mechanical properties, dislocation dynamics,
sound, …. Etc.

Acoustics measurements and anomaly near 200mK: Goodkind (~2000)
Renaissance of solid He study;
Torsional oscillation(TO) study of He in porous glass; Kim and Chan(2003~)
Superfluidity and Dimensionality

BEC occurs only in $D \geq 3$ for Ideal Gases.

→ Superfluidity in 2D: Kosterlitz-Thouless mechanism of paired quantized vortices play the role of keeping macroscopic Coherence in 2D. Confirmed both by experiments & Theory, but…

→ Superfluidity in 1D systems: Not known, but Shevchenko’s 3D network of 1D system Discussion; Still under Discussion
Vortex Fluid State in New Superconductors

Study of Superfluidity has been changed after the discovery of Cuprate High $T_c$ Superconductors:
The words “Vortex State” and/or “Vortex matter” appeared: Fisher, Fisher, Huse (PRB1991)

Essence
CuO$_2$ plane: 2D conducting planes coupled into 3D system. These 2D subsystems supply thermally excited vortices and/or field induced vortex state.

Later modern Superconductors: Organic SC, Layered SC, MgB$_2$, Fe compounds SC…, They, so far studied, all have 2D Subsystems as CuO$_2$ plane in Cuprates.

And also vortex state, namely vortex liquid and various vortex states are common in all these systems.

Quantized vortex dynamics may not have been discussed so far for such systems.
There is no excuse!!

P.W. Anderson pointed out that so far reported “Supersolid” data are actually that of vortex fluid and real Tc for Supersolid should be found at lower T. Nature Phys. Vol.3 160 (2007).

And furthermore a possible transition from VF state to real Supersolid state.

How do we study superfluidity and vortex dynamics?
Highly Sensitive and Stable Torsional Oscillator (TO)
to Study VF as well as Supersolid Properties of Solid $^4$He

**Frequency Shift $\Delta f$** → Moment of Inertia Change → Supersolid Density $\rho_{ss}(T)$, and Nonlinear Rotational Susceptibility (NLRS)
And
**Amplitude** → **Q Value** of TO → detailed energy dissipation → Vortex Dynamics

Kubota, Vortex state in hcp solid He
Experiments under DC rotation $\rightarrow$ Vortex line penetration into bulk superfluids; so far in $^4$He, $^3$He, Atomic BEC’s
Unique Activity of Kubota Group, ISSP U-Tokyo: Study of Essence of Superfluidity

Superfluidity in 3D Connected Monolayer He Films:

1]. Can one make a 3D Superfluid out of 2D Films, where there is no BE Condensate?
2]. What is the inter-relation between BEC and Superfluidity?

⇒ 3D Superfluid is Possible out of 2D Films !!!
Study of artificial 3D superfluids: (Previous activity)
He “monolayer” films on 3D connected pore surface

• Obata and Kubota, PHYSICAL REVIEW B 66, 140506(R) (2002)
Energy dissipation goes up when excitation is increased for all known SF
ISSP High Speed Rotating DR

Vortex line penetration induces extra $\Omega$-linear dissipation $dQ^{-1}$ increase!!

Solid $^4$He at 51 bars

Amplitude of oscillation is 7Å
A decrease in the resonant period, similar to that found in superfluid liquid helium, appears below 0.25K

$\tau_0 = 1.096,465$ns at 0 bar
$1.099,477$ns at 51 bars
(total mass loading=3012ns due to filling with helium)

The nonclassical rotational (NCRI) fraction is $\sim 1.3\%$

*Science 305, 1941 (2004);* Bulk solid
Our study of solid $^4$He Started with detailed TO excitation dependence. Oposit to usual Superfluids
Log($V_{ac}$) linear dependent suppression $\rightarrow$ Suppression by formation of vortex line!!


FIG. 2. $\delta$ (a) and $\Delta p/\Delta p_{load}$ (b) of 32 bar sample as a function of $V_{ac}$ at $T < 300$ mK. $\Delta p/\Delta p_{load}$ (c) is the data of the new sample at 49 bar. The solid lines in (c) show the linear dependence on log($V_{ac}$) for the Vac range; $\sim 30 < V_{ac} < 300$ µm/s and at higher $V_{ac}$ some other dependence appears. We observe practically the same log($V_{ac}$) linear dependence as in (c) also for a 32 bar sample (b) by fitting with linear lines. Extrapolated linear lines are found to converge at a point $\sim 600$ µm/s for the $V_{ac}$ range. This point of convergence also seems to coincide with the zero in Fig. 1(b).
High T limit behavior: the slope has $1/T^2$ Dependence!!

$1/T^2$ is not the T dependence of an order parameter!!
→ Nonlinear Rotational Susceptibility (NLRS) ! Cf. Anderson

FIG. 3 (color online). T dependence of the slope $d(\Delta p/\Delta p_{\text{load}})/d[\log(V_{ac})]$. Clear $1/T^2$ dependence is seen for both of the completely independent solid $^4$He samples at 32 and 49 bar pressure.
A detailed torsional oscillator (TO) study on a stable solid $^4$He sample at 49 bar with $T_o \sim 0.5$ K, is reported to $T$ below the dissipation peak at $T_p$. We find hysteretic behavior starting below $T_c \sim 75$ mK, in period shift, as well as in dissipation, with changes of AC excitation amplitude $V_{ac}$. The derived difference of non-linear rotational susceptibility $\Delta$NLRS($T$)$_{hys}$ across the hysteresis loop under a systematic condition is analyzed as a function of $V_{ac}$ and $T$. We propose that $\Delta$NLRS($T$)$_{hys} \propto$ non-classical rortalional inertia fraction, NCRIF is actually the supersolid density $\rho_{ss}$ of the 3D supersolid state below $T_c$. $\rho_{ss}$ changes linearly with $T$ down to $\sim 60$ mK and then increases much more steeply, and approaching a finite value towards $T=0$. We find an AC velocity of $\sim 40 \mu$m/s beyond which the hysteresis starts at $T<T_c$ and a ”critical AC velocity”, $\sim 1$ cm/s, above which $\rho_{ss}$ is completely destroyed. We discuss also the coherence length $\xi$ of the supersolid.

PACS numbers: 67.80.bd, 67.25.dk, 67.25.dt, 67.85.De.

arXiv:0903.1326
Vortex Fluid to Supersolid Transition in Solid $^4$He below $\sim 75$ mK*


FIG. 1: NLRS(T) at $V_{ac} \rightarrow 0$, is displayed as a function of $1/T^2$. The solid line through the data points is the Langevin function $f(x) = a\{\exp(bx)+\exp(-bx)\}/\{\exp(bx)-\exp(-bx)\}-1/(bx)$ with $a = 0.0878 \pm 0.0011$, and $b = 0.0148\pm 0.0004$. Inset shows the $V_{ac}$ dependence for data at each $T < 300$ mK and we can safely extrapolate to $V_{ac} \rightarrow 0$.

FIG. 2: The $V_{ac}$ dependence of NLRS of solid $^4$He sample at 49 bar pressure at constant representative $T$'s for clarity, obtained from the measurement of period change of $\Delta T$. Measurements are performed at various $V_{ac}$ as given in the figure.

FIG. 3: "hysteretic" NCRIF as well as dissipation component vs log $V_{ac}$. From the linear extension of the log $V_{ac}$ dependence we obtain a critical velocity, 6–10 mm/s to suppress the "hysteretic" $\Delta NLRS_{hys} = NCRIF$ to zero.
Vortex Fluid to Supersolid Transition in Solid $^4$He below $\sim 75$ mK

FIG. 4: The temperature dependence of the "persistent" N C R I F $\Delta$N LRShys appeared as a result of the "hysteretic Process". It represents the persistent N C R I F produced by the process, which supports persistent circulation even after the process finished and it may represent macroscopic persistent current by a set of vortex rings. It would be an evidence of 3D macroscopic phase appearance from a certain $T_c$. It compares with "paramagnetic" behavior of N LRS(open symbols), extrapolated to Vac $= 0$, whose $1/T^2$ dependence at high $T$ as well as Langevin function dependence suggests "susceptibility" feature of this quantity.

FIG. 5: Critical behavior of $\xi$ obtained from the absolute evaluation of supersolid fraction $\rho_{ss}$ by extrapolating to Vac $=0$. We took $T_c = 56.7$ mK and obtain $\xi_0 \sim 25$ to 50 nm at $t = 1$ or $T = 0$K by simple extrapolations, horizontal and straightextension of linear relation. Inset shows $\xi$ for SS state andVF state(open symbols). The latter is smaller for a $T$.

Is it really a supersolid below $T_c$?

Vortex line penetration through the sample under DC rotation?

-Experimental Procedure of Measurement under DC Rotation-

1. At high Temperature ($\sim 0.5$ K) Change AC excitation to $V_{ac}=200$ $\mu$m/sec (Equilibrium)

2. DC Rotation Start ($\Omega=0\rightarrow 0.2$ rps)

3. Cooling down

4. Measurement under T sweep ($T=50$mK$\rightarrow 150$mK / 3h, 9h,..)

5. Repeat with different DC Rotation Speeds
Results under DC Rotation

- Energy Dissipation below $T \sim 80$ mK Changes under DC Rotation: Faster Rotation $\implies$ Larger Change !!

- No Change in NLRS (non-linear rotational susceptibility)
How does Energy Dissipation Change as DC Rotational Speed $\Omega$?

Linear Change!!

→ Vortex lines!!, Otherwise $\Omega^2$ Dependence Should Occur!

If Dissipation $\Delta Q^{-1}$ is Caused by Supersolid Vortices, Then It Should Be Proportional to $\rho_s(T)$!
Transition Temperature(s) $T_c(s)$ to Supersolid State and $\rho_s(T)$?

Above graph looks just the same as $\Delta$NCRIF, observation in the hysteretic behavior vs $T$!

Slope $\Delta Q^{-1}/\Omega$ vs $T$ (mK)

We find two characteristic temperatures, 

$\sim 57$ mK, $\sim 76$ mK
OK. Vortex line penetration is occurring at $T < T_c \sim 75$ mK! Then what about vortex fluid (VF) state?

What should characterize VF state?

We have started study on vortex dynamics by a relaxation model, where we started similar considerations as in Quantum turbulence.

N. Shimizu, S. Nemirovskii, and M. Kubota (2009)
Quantitative Analysis of Vortex Fluid State of solid $^4$He

Vortex Fluid Relaxation Model for Torsional Oscillation Responses of Solid $^4$He

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A set of detailed torsional oscillator(TO) response data of hcp $^4$He to below energy dissipation peak is presented and a phenomenological model for relaxation processes in the response of Vortex fluid (vortex tangle) on rotation and torsional oscillations have been studied. Dependence of the both Nonlinear Rotational Susceptibility(NLRS) and Dissipation on $T$, and rim velocity $V_{ac}$ was studied. We could reproduce not only the unique $V_{ac}$ dependence, but also obtain new information of the vortex tangle system by the model calculation using parameters obtained from measured data. The results obtained may serve as a good qualitative description for the according measurements in the vortex fluid state in solid $^4$He and we find an interesting peculiarity in the relaxation time at extrapolated temperature to $\sim$30 mK from the measurement down to 50 mK. We discuss further the consequence.
Angular momentum of superfluid fraction appears only due to presence of either aligned vortices (vortex array) or due to the polarized vortex tangle having nonzero total polarization \( \mathcal{P} = \mathcal{L}(s_i(\xi)) \) along the applied angular velocity \( \Omega \) (axis \( \tau \)). For pure thermodynamic reasons it is clear that in the steady rotation the arrangement of the polarized vortex tangle is such that the whole set of vortices rotates with angular velocity \( \Omega \). That is quite natural supposition otherwise superfluid component would rotate either with larger angular and leave behind the whole rotating frame, or with smaller \( \Omega \) and be behind. Both variants seem to be unrealistic, so in steady (direct) rotation superfluid component rotates with the applied angular velocity \( \Omega \) and, of course there is no deficit of moment of inertia at all. Thus, in the steady case there is strictly fixed relation between total polarization \( \mathcal{L}(s_i(\xi)) \) and applied angular velocity \( \Omega \) (\( \kappa \) is the quantum of circulation).

\[
\Omega = \kappa \mathcal{P} = \kappa \mathcal{L}(s_i(\xi)).
\]

Angular momentum of the superfluid part can be written as

\[
\mathbf{M}_{SF} = I_{SF} \dot{\Omega} = I_{SF} \kappa \mathcal{P}.
\]

Situation is drastically changed in nonstationary (transient or oscillating) situation. The total polarization \( \mathcal{P}(\tau) \) changes in time owing to that the both vortex line density \( \mathcal{L}(\tau) \) and the mean local polarization \( \langle s_i'(\xi) \rangle(\tau) \) change in time. Therefore the angular momentum of the superfluid part is

\[
\mathbf{M}_{SF}(\tau) - I_{SF} \kappa \mathcal{P}(\tau) = \frac{\partial \mathcal{V} R^2}{2} \mathcal{P}(\tau) - \frac{\mathcal{V} R^2}{2} \frac{\kappa \mathcal{P}(\tau)}{\Omega} \Omega - \mathcal{L}(\tau) \Omega.
\]
Exponential adjustment

Let us suppose that equilibrium polarization is being reached in usual exponential way (\( P_{eq} \) below is just the stationary value satisfying to \( \Omega = \kappa P \))

\[
P(t) = P_{eq}(1 - \exp(-t/\tau (V_{ac}))) \quad \text{and} \quad M(t) = M_{eq}(1 - \exp(-t/\tau (V_{ac})))
\]
Torsion oscillations with relaxation processes

\[ M = a\Omega(t) + b \int_0^\infty \Omega(t-t')\varphi\left(\frac{t'}{\tau}\right)\frac{dt'}{\tau}. \]

\[ \varphi\left(\frac{t'}{\tau}\right) \sim \exp\left(-\frac{t'}{\tau}\right), \quad \tau(V_{ac}) \sim \frac{1}{\alpha(T)V_{ac}/R + \beta(T)}. \]

\[ M_{\omega \to 0} = (a + b)\Omega, \quad M_{\omega \to \infty} = a\Omega \]

\[ M = I_N\Omega(t) + I_{SF} \int_0^\infty \Omega(t-t')\varphi\left(\frac{t'}{\tau}\right)\frac{dt'}{\tau} \]

**Final results**

\[ \frac{\Delta P}{P} = -\frac{1}{2} \frac{I_{SF}}{I_{full}} \frac{(\omega\tau)^2}{(\omega\tau)^2 + 1} \]

\[ Q^{-1} = \frac{2\text{Im}\omega}{\omega} = \frac{I_{SF}}{I_{full}} \frac{(\tau\omega)}{\tau^2\omega^2 + 1} \]

\[ \tau(V_{ac}) \sim \frac{1}{\alpha(T)V_{ac}/R + \beta(T)} \]
Obtained parameters of vortex dynamics
Our preliminary analysis based on vortex dynamics lead re-construction of TO responses. And we observe some extra features occurring at $T<\sim 75$ mK.

Phase Transition of the vortex system !?!!? Detailed study is still in progress.

Thank you for your attention.
We have been asking ourselves:
What is superfluid?

We are studying 3D connected $^4$He monolayer superfluid systems, as well as superfluid $^3$He in restricted geometry, especially under rotation using two rotating cryostats.