Workshop on Topics in Quantum Turbulence

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Measurement Methods of Vibrating Mechanical Objects Used for Generation and Detection of Quantum Turbulence

P. Skyba
Slovak Academy of Sciences
Inst. of Experimental Physics
Kosice
Slovak Republic
Notes on measurement methods of vibrating objects in low temperature physics

Peter Skyba

Talk outline:

Introduction

Voltage measurement (vibrating wires technique)

Current measurement (tuning forks, torsional oscillators, grids technique)

Conclusion

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Vibrating objects

Traditional resonators used: vibrating wires, spheres, grids, oscillating discs,..
- resonators are immersed in liquid and bring into forced oscillations,
- properties of the liquids inferred from the resonance characteristics

Equation of motion:

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = f_0 \exp(i \omega t)$$

Physical quantity being measured - voltage or current.

Vibrating wire

Tuning fork

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S.N. Fisher et al., PRL 63 2566 (1989)

Dave Clubb et al., JLTP 136 Nos.1/2 (2004), 1
Rob Blaauwgeers et al., JLTP 146 Nos. 5/6 (2007)
plus many others

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Mechanical resonators - basic properties

Equation of motion:
\[
\ddot{x} + \gamma \dot{x} + \omega_0^2 x = f_0 \exp(i \omega t)
\]

Velocity of the object:
\[
v(\omega) = v(\omega_0) \frac{\gamma_2 \omega^2 + i \omega (\omega_0^2 - \omega^2 - \gamma_1 \omega)}{(\omega_0^2 - \omega^2 - \gamma_1 \omega)^2 + \gamma_2^2 \omega^2}
\]

A Vibrating Wire Resonator

\[I(t) = \alpha \frac{dx(t)}{dt}\]

tuning fork

\[\alpha - \text{is the constant}\]

vibrating wire

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Voltage measurements of vibrating wires - theory

Phase sensitive detection technique (lock in) in frequency range up to few kHz.

Typical experimental arrangement

Analysis of the circuitry – superposition theorem

For the total voltage $V_{out}$ being measured one can get (simplified form):

$$V_{out} = V_i \left(1 - \frac{i \omega L_w}{R'}\right) + V_g \left(\frac{R_w}{R} - \frac{i \omega L_w}{R}\right)$$

$$V_i \approx Blv(\omega)$$

$$R' = \frac{R.R_i}{R + R_i}$$

Transformer !
Voltage measurements of vibrating wires - experiment

Total measured voltage:

\[ V_{out} = V_i \left( 1 - \frac{i \omega L_w}{R} \right) + V_g \left( \frac{R_w}{R} - \frac{i \omega L_w}{R} \right) \]

If \( B = 0 \) T, then \( V_i = 0 \) Volts, and \( V_{out} \) is:

\[ V_{out} = \frac{R_w}{R} V_g - \frac{i \omega L_w}{R} V_g \]

If \( R_w = 0 \), then in phase component is zero

In phase component

Quadrature component

\( R_w \) and \( L_w \) of the wire can be measured. Slope is \( L_w/R \)

Comment on \( L_w \)!
Vibrating wire with transformer – theory

As $T \rightarrow 0K$, in superfluid 3He-B dominant scattering process is Andreev reflection. Therefore, the wire (and any other mechanical resonator) is driven at low velocities.

$$V_i \approx Blv$$

This induced voltage becomes comparable with noise voltage of preamplifier as it operates at room temperature. Low temperature transformers are used to improve the signal to noise ratio.

**Typical experimental arrangement:**

For transformer the “T” model has been used.
Vibrating wire with transformer – theory

Typical experimental arrangement:

Let assume that $R_i >> R_s$, $R_i >> \text{abs}(i\omega M)$ and

Superposition theorem:

$$V_{out} = V_{bgrd} + k(\omega)V_i$$

$$V_{bgrd} = N \frac{V_g}{RR_p} \left[ R_w R_p + \omega^2 L_w (L_p + M) - i\omega L_w R_p \right]$$

$$V_{sig} = NV_i \left[ 1 - \frac{i\omega^3 R(L_p + M)^2 L_w}{(RR_p)^2 + \omega^2 R^2 (L_p + M)^2} \right]$$

Higher transformer gain leads to a nontrivial amplitude-frequency response.
Vibrating wire with transformer – experiment

Measurement performed on wire of diameter 4.5 μm having \( f_0 = 700 \) Hz. Transformer with gain \( N=30 \) and \( B=0 \) Tesla.

\[
V_{bgrd} = N \frac{V_g}{RR_p} \left[ R_w R_p + \omega^2 L_w (L_p + M) - i \omega L_w R_p \right]
\]

Allows determine \( R_w \) and slope determines \( L_w(L_p+M)/RR_p \) Allows to determine \( L_w \)

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Current measurements – tuning forks

Typical measurement setup

\[ I_F(t) = \alpha \frac{dx(t)}{dt} \]

Superposition theorem:

\[ V_0 = V_{\text{bgrd}}(V_g) + Z(\omega)I_F \]

\[ v(\omega) = v(\omega_0) \frac{\gamma_2^2 \omega^2 + i\omega(\omega_0^2 - \omega^2 - \gamma_1 \omega)}{\left(\omega_0^2 - \omega^2 - \gamma_1 \omega\right)^2 + \gamma_2^2 \omega^2} \]
Current measurements – tuning forks

Bgrd voltage:

\[
V_{\text{bgrd}} = -\frac{Z_2}{R_1 + Z_2} \frac{1}{1 + (\omega C_0 R_i)^2} \left( \omega^2 R_F R_i C C_0 - \frac{R_1}{Z_2} + i \omega R_F C \right) V_g
\]

\[
Z_2 = \frac{R_2}{1 + i \omega R_2 C_i}
\]

\[
k = 1 - \frac{R_1 C_0}{Z_2 C}
\]

Signal voltage:

\[
V_{\text{sig}} = -\frac{1 - i \omega C_0 R_i}{1 + (\omega R_i C_0)^2} R_F I_F(\omega)
\]
Current measurements – tuning forks - experiment

Let discuss first influence of the background voltage

\[ V_{bgrd} = -\frac{Z_2}{R_1 + Z_2} \frac{1}{1 + (\omega C_0 R_i)^2} \left[ \frac{\omega^2 R_F R_i C C_0}{Z_2} - \frac{R_1}{Z_2} + i k \omega R_F C \right] V_g \]

Proper I/V converter \( R_i \rightarrow 0\Omega \) and \( R_1 \rightarrow 0\Omega \), then

\[ V_{bgrd} = -i \omega C R_F V_g \]

Measurements in vacuum @ \( T \sim 1 \text{K} \)

Various values of \( R_F \)

C ~ 1.96 pF
Current measurements – tuning forks - experiment

Effect of non zero input resistance \((R_i \neq 0 \Omega)\) of an I/V converter on measurements

\[
V_{\text{sig}} = -\frac{1 - i \omega C_0 R_i}{1 + (\omega R_i C_0)^2} R_F I_F(\omega)
\]

Amplitude and phase change due to non-zero \(R_i\)

Nearly all commercial available lock-in amplifiers having I/V converters have \(R_i \neq 0\Omega\)

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Current measurements – grids and torsional oscillators

Typical measurement setup

\[ I_i(t) = V_{DC} \frac{dC_i}{dt} = -V_{DC} \frac{C_i}{d_i} v_i \]

Superposition theorem (I_1 is not contributed to V_0):

\[ v_i(\omega) = v_i(\omega_0) \frac{\gamma_2^2 \omega^2 + i \omega (\omega_0^2 - \omega^2 - \gamma_1 \omega)}{(\omega_0^2 - \omega^2 - \gamma_1 \omega)^2 + \gamma_2^2 \omega^2} \]

\[ V_0 = V_{bgrd}(V_g) + Z(\omega)I_2 \]
Current measurements – grids and torsional oscillators

Grid measurement setup:

\[ V_0 = V_{bgrd} (V_g) + Z(\omega)I_2 \]

\[ V_{bgrd} = -\frac{Z_2}{R_1 + Z_2} \frac{1}{1 + (\omega C_0 R_i)^2} \left[ \omega^2 R_F R_i C_P C_0 - \frac{R_1}{Z_2} + i k \omega R_F C_P \right] V_g \]

\[ V_{sig} = -\frac{1 - i \omega C_0 R_i}{1 + (\omega R_i C_0)^2} R_F I_2(\omega) \]

\[ I_2(t) = V_{DC} \frac{dC_2}{dt} \]
Current measurements – grids - experimental

Measurements performed using a Lancaster grid at $f_0 \sim 1\text{kHz}$ and $Q \sim 10^5$

Proper I/V converter having $R_i \rightarrow 0\Omega$, then $V_{bgrd}$ can be expressed as:

$$V_{bgrd} = -\frac{Z_2}{R_1 + Z_2} \left[ -\frac{R_1}{Z_2} + i\omega R_f C_P \right] V_g$$

Allows ratio $R_1/(R_1+Z_2)$ to be determined experimentally $\sim 0.0015$

Allows $C_P$ to be measured $C_P \ 7.7 \ \text{pF}$

Problem with ground!
Conclusion

Voltage measurements on vibrating wires:
- easy to measure and determine the background when B=0T;
- direct wire measurements – an agreement between the model and experiment;
- transformers improve signal to noise ratio, but they change the frequency dependence of background (can be measured) and reduce input impedance of the preamplifier (R_i/N^2);
- higher N results in nontrivial (resonance) response of the background voltage;
- Instead of using R to determine current – use proper V/I converter.

Current measurement tuning forks, grids and torsional oscillators:
- impossibility to measure pure background like for vw’s;
- non-zero input impedance of the I/V converter affects precision of measurements;
- ground problems – resistive attenuators dividers followed by a voltage follower providing impedance insulation and proper voltage source;
Acknowledgement

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Thank’s for your attention.