Satellite Navigation Science and Technology for Africa

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Satellite Navigation Overview

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An Overview of Satellite Navigation

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Objectives

• To convey:
  – A broad understanding of the scientific and engineering principles of satellite navigation
  – The rudiments of GPS:
    • System
    • Signals and measurements
    • Performance
  – An outline of global navigation satellite systems under development: GLONASS, Galileo, Beidou

• Comprehensive discussions of these topics (and more) to follow later this week
Space age began with the launch of Sputnik I by the Soviet Union on 4 October 1957.

‘Beeps’ heard on short-wave radios tuned to 20 MHz or 40 MHz, Doppler shifted as the satellite moved in the sky.*

Within days, the idea of using radio signals from space for positioning on the earth was born.

*http://history.nasa.gov/sputnik
Doppler frequency shift = - range rate/ wavelength
Doppler Positioning
A Conceptual Exercise

- Record time when Doppler shift went through zero
- **Along-track position**
  - From train’s schedule
  - Error sources: Watch off, train off schedule
- **Cross-track position**
  - From Doppler profile
  - Ambiguity: Which side of the track?

Doppler frequency shift = - range rate/ wavelength
A Global Satellite Navigation System based on Doppler Positioning

• **Satellite transmits**
  – Frequency-stable signal
  – Time, orbital parameters, clock parameters

• **Receiver measures Doppler frequencies and records transmitted data for an entire pass**
  – Determine coordinates of the point on the ground track corresponding to the point of closest approach
  – Determine offset from the ground track

• **Error Sources**
  – Satellite clock frequency stability over 10-20 min
  – User velocity

Adapted from *Marine Electronic Navigation* by Appleyard et al.
Transit (1964-1996)

- 4-7 satellites in 1100-km, circular, polar orbits
- One satellite in view at a time
- A satellite pass lasted 10-20 min; up to 100-min wait between passes
- Satellite weight: 50 Kg (160 Kg)
- Signals at 150 MHz & 400 MHz
- Signal power: 1 watt
- 2-D Positioning accuracy (for a stationary users): 25 m
- ~ 10,000 receiver sets in 1980, cost: ~$25,000
Satellite Navigation Overview
Outline

• Principles of Satellite Navigation

• GPS Overview: System, Signals and measurements, Performance

• Applications and Performance Metrics

• Potential Partners/Rivals: GLONASS, Galileo, BeiDou/Compass, …
Triangulation

Method of determining the position of a fixed point from the angles to it from two fixed reference points a known distance apart.

from *Trigonometry Surveying and Navigation* by G.A. Wentworth
**Triangulation**

Method of determining the position of a fixed point from the angles to it from two fixed reference points a known distance apart.

**Trilateration**

Measure lengths of the sides of a triangle rather than angles.

A chain = 100 links = 66 feet long, 80 chains make a mile. A "rod" or "pole" is 1/4 of a chain, or 16-1/2 feet long. Thus "40 rods" is 10 chains, or 1/8 of a mile.

From *Trigonometry Surveying and Navigation* by G.A. Wentworth

Surveyor’s chain from [www.landsurveyinghistory.ab.ca](http://www.landsurveyinghistory.ab.ca)
2-D Trilateration
Trilateration

Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
Satellites: 24
Orbital planes: 6
Inclination: 55 deg
Altitude: 20,000 km
Period: 11 h, 58 min

• Actual number of satellites has exceeded 24 since 1995, and is currently 29
• U.S. Government intends to maintain at least 22 satellites in their nominal slots
Trilateration

Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)
Global and Regional Coordinate Frames

Global

- Geocentric Ellipsoid
- Geoid
- Reference Meridian
- Geodetic latitude defined relative to a geocentric ellipsoid

Regional

- Regional Ellipsoid
- Geoid
- Geodetic latitude defined relative to a regional ellipsoid

\( \phi \) Geodetic latitude defined relative to a geocentric ellipsoid

\( \tilde{\phi} \) Geodetic latitude defined relative to a regional ellipsoid
Trilateration

Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)
- Ultra-stable clocks aboard satellites to transmit synchronized signals

Frequency stability of $10^{13}$/day:

$$\frac{\Delta f}{f} = 10^{-13}$$

Timekeeping error:

$$\frac{\text{Timekeeping error}}{\text{Time interval}}$$

Timekeeping error: ~10 ns/day
Trilateration

**Satellite Navigation Enabling Technologies**

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)
- Ultra-stable clocks aboard satellites to transmit synchronized signals, but inexpensive clocks in receivers

![Trilateration Diagram]
Relativistic Effects: Circular 12-h Orbit

Atomic clock drift: \[ \frac{\Delta f}{f} \approx 10^{-13} \]

- Second-order Doppler shift (time dilation)
  \[ \frac{\Delta f}{f} = \frac{v^2}{2c^2} \approx 1 \times 10^{-10} \text{ (negative)} \]

- Gravitational frequency shift
  \[ \frac{\Delta f}{f} = \frac{\Delta \Phi}{c^2} \approx 5 \times 10^{-10} \text{ (positive)} \]

Combined effect accounted for by “factory offset” of satellite clock by
\[ \frac{\Delta f}{f} = -4.4645 \times 10^{-10} \]
Relativistic Effects: Elliptical Orbit

- **Eccentricity effect**
  - Periodic shift of clock rate with 12-h period
  - 1% eccentricity: Periodic error with amplitude 28 ns in signal transit time (~10 m)
  - Accounted for in GPS receivers

- **Smaller Effects, generally neglected but compensated for automatically in differential GPS**
  - Sagnac effect
  - Shapiro delay
  - Nonspherical gravity potential
  - Tidel effects from sun and moon
  - Lense-Thirring drag
Trilateration

Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)
- Ultra-stable clocks aboard satellites to transmit synchronized signals, but inexpensive clocks in receivers
- Integrated circuits: Compact, light, inexpensive receivers
Evolution of GPS Receivers
from 10 Kg to 100 g, 100 watts to 1 watt, $100k to $100

Early 1980s

2005
Satellite Navigation
Position Estimation by Trilateration

Error Sources
- Ephemeris
- Satellite Clock
- Propagation through Ionosphere
- Troposphere
- Multipath
- Receiver Noise

Measurements: Pseudoranges \( \{R_k\} \)

Given: Satellite Positions \( \{(x_k, y_k, z_k)\} \)

\[
R_k = \sqrt{(x_k - x)^2 + (y_k - y)^2 + (z_k - z)^2} - b, \\
k = 1, 2, \ldots, K
\]

Unknown: User Position \((x, y, z)\)
Receiver Clock Bias \(b\)
Satellite Navigation
Position Estimation by Trilateration

Your position coordinates are (in meters):

\[
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = \begin{bmatrix}
1510885.12 \\
-4463460.45 \\
4283906.78 \\
\end{bmatrix}
\]
Satellite Navigation Objectives

• To provide estimates of
  – Position [~10 m]
  – Velocity [~0.1 m/s]
  – Time [~0.1 ms]

• Instantaneously
• Continuously
• Globally
• Cheaply, etc.

• To any number of users

Misra 1999
Satellite Navigation Objectives

- To provide estimates of
  - Position \([\sim 10 \text{ m}]\)
  - Velocity \([\sim 0.1 \text{ m/s}]\)
  - Time \([\sim 0.1 \text{ µs}]\)

- Instantaneously, continuously, globally, cheaply, etc.

- To any number of users

Miura 1999

GPS Joint Program Office motto (Ca. 1975)
“The mission of this Program is: (1) Drop 5 bombs in the same hole, and (2) build a cheap set that navigates (< $10,000), and don’t you forget it!”
Satellite Navigation Overview
Outline

- Principles of Satellite Navigation
- GPS Overview: System, Signals and measurements, Performance
- Applications and Performance Metrics
- Potential Partners/Rivals: GLONASS, Galileo, BeiDou/Compass, …
GPS at a Glance

• Development began in early 1970s
  – First prototype satellite launched in 1978
  – Estimated number of receivers required: 27,000 (!)
  – Target cost of a receiver: $10,000 (!)

• Operational System
  – First operational satellite launched in 1989
  – System declared operational in 1995

• Expenditure
  – U.S. taxpayer investment (through 2007): $ 32b
  – Annual O&M costs: $ 1b

• Users: Millions
  – Most widely used military radio, albeit one way
  – Civil receivers manufactured annually: > 1 million

• Annual commerce in GPS products & services > $10 b
U.S. Policy on GPS

• Services
  – Standard Positioning Service (SPS) available to all
  – Precise Positioning Service (PPS) for “authorized” users

• Selective Availability (SA)
  – Purposeful degradation of the civil signal throughout 1990s, SPS horizontal positioning accuracy (95%): ~60 m
  – Foresworn for GPS III by Presidential Order (2007)

• Governance
GPS Segments

Space Segment

Ground control segment

User segment: civil and military
GPS Signals in mid-2005

- **L1**
  - 1575.42 MHz
  - P(Y) Code
    - Encrypted
  - C/A Code
    - Civil use
    - Degraded
  - Horizontal Error (95%) < 10* 100 m
  - Vertical Error (95%) < 15* 156

- **L2**
  - 1227.6 MHz
  - P(Y) Code
    - Encrypted

Specifications:

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Precise Positioning Service (PPS)</th>
<th>Standard Positioning Service (SPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Error (95%)</td>
<td>22 m</td>
<td>&lt; 10* 100 m</td>
</tr>
<tr>
<td>Vertical Error (95%)</td>
<td>27</td>
<td>&lt; 15* 156</td>
</tr>
</tbody>
</table>

* Since 2 May 2000 (empirical)
Satellite Signal
GPS C/A-Code

Satellite Signal: \[ [D(t) \oplus x(t)] \otimes \cos(2\pi f_L t) \]

\(\oplus\): Mod 2 Sum
\(\otimes\): Biphase Modulation
Spread Spectrum Signaling

Unmodulated Carrier

Two bits from data Stream \( D(t) \)

Two repeats of a code with 4 chips per data bit \( x(t) \)

Carrier modulated by code and data

\[
s_T(t) = \sqrt{2P_T} D(t) x(t) \cos(2\pi f_L t)
\]

Source: Prof. Per Enge, Stanford University
A Generic GPS Signal

Transmitted Signal

\[ s_T(t) = \sqrt{2P_T} \, D(t) \, x(t) \cos(2\pi f_L t) \]

Received Signal

\[ s_R(t) = \sqrt{2P_R} \, D(t - \tau) \, x(t - \tau) \cos\left(2\pi\left(f_L + f_D\right)t + \theta\right) \]

Estimate delay (\(\tau\)) and Doppler (\(f_D\))

Range = \(c \cdot \tau\); Range rate = \(\lambda \cdot f_D\)

\(D(t)\) : Nav data (\(\pm 1\)), \(x(t)\) : PRN code (\(\pm 1\)), \(f_L\) : Carrier frequency, \(f_D\) : Doppler frequency
Amplitude Spectrum of GPS Signals
1 November 2005

\[ f_{L2}: 1227.6 \text{ MHz} \]

\[ f_{L1}: 1575.42 \text{ MHz} \]

Source: Prof. Per Enge, Stanford University
GPS Signals are Extremely Weak-1

Freq: 1563-1587 MHz (L1)
Power: ~ 27 W (C/A-Code)

~ 20,000 km

\[ P_R = \frac{P_T G_T}{4\pi R^2} \cdot \frac{G_R \lambda^2}{4\pi} \]

- 160 dBW (10^{-16} W)
GPS Signals are Extremely Weak-2

Freq: 1563-1587 MHz (L1)
Power: ~27 W (C/A-Code)

Interference
Freq: 762-788 MHz
Power: 2000 kW
FCC: Out-of-band < -60 dB
Second harmonics: 1564-1576 MHz
Power: ~ 2 W

\[ P_R = \frac{P_I G_T}{4\pi R^2} \cdot \frac{G_R \lambda^2}{4\pi} \]

Based on paper by Philip W. Ward, P.E.
Basic GPS Receiver Architecture-1

- Mismatch
- Correlators
- Replica signal generator
- Navigation filter
- Loop filter

Commands:
- Speed up
- Slow down

Pseudoranges, pseudorange rates

Thermal Noise

GPS Signal (C/A)

Power (dBW/MHz)

Frequency (GHz)

- Position (P)
- Velocity (V)
- Time (T)
Basic GPS Receiver Architecture-2

Processing Gain
\[ = \frac{BW_{RF}}{DataRate_{BPS}} \]

- Position (P)
- Velocity (V)
- Time (T)
Receiver Functions

- Condition input signal
  - Bandpass filter to suppress OOB interference
  - Down-convert
  - Digitize (A/D conversion)
- Separate signals from individual SVs
- Acquire and Track Signals
- Demodulate navigation data
- Calculate position, velocity, and time (PVT)
- Report results through user interface
GPS Receiver Functional Diagram

Courtesy: Prof. Per Enge, Stanford University
Code and Carrier Phase Measurements
A Conceptual Exercise
Code and Carrier Phase Measurements
Precision vs. Accuracy

- Carrier phase can be measured with a precision of millimeters, code phase with decimeters.
- Pseudoranges from each are affected by the same error sources, and the error in each can be several meters.
GPS Error Sources

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<th>Pseudorange Error Source</th>
<th>Size (typical)</th>
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<tr>
<td>Satellite clock/orbit error</td>
<td>1–2 m</td>
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<tr>
<td>Mis-modeled ionospheric delay</td>
<td>0–3</td>
</tr>
<tr>
<td>Mis-modeled tropospheric delay</td>
<td>1</td>
</tr>
<tr>
<td>Multipath</td>
<td>1–3</td>
</tr>
<tr>
<td>Receiver noise</td>
<td>&lt;1</td>
</tr>
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Pseudorange Error: 2–5 m
Horizontal position Error: 2–5 m
GPS-based Position Estimates (SPS)
Sampled over 24 hours (post-SA)*

S100 Receiver (L1 C/A-code only)
(1-minute samples) *

Error: 95% (empirical)
- Horizontal position: 10 m
- Vertical position: 15 m
- Time: 30 ns

N: # points in a cell

*Source: MIT Lincoln laboratory
GPS Augmentations

• Why augment?
  – For better accuracy: Mitigate measurement errors
  – For robustness: Mitigate effects of
    • RFI (intentional or not)
    • signal attenuation due to blockage (e.g., by foliage or building), or temporary loss of signal (e.g., going under a bridge or through a tunnel)

• How augment?
  – Transmit corrections for errors that are correlated spatially and temporally
    • Local Area Differential GPS
    • Space-Based Augmentation Systems (SBAS): WAAS, EGNOS, MSAS
  – Assist GPS receiver with complementary technologies (e.g., inertial), signals of opportunity (e.g., eLoran), or by offloading some functions (e.g., to a cell tower in E911)
Differential GPS (DGPS)
Mitigation of Error Sources

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<td>&lt;1 m</td>
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Pseudorange error: 2–5 m
Horizontal position error: 2–5 m

Differentially Corrected
(~10 km from reference receiver)

Pseudorange error: <1 m
Horizontal position error: 1–2 m
Local Area Differential GPS (DGPS)
Mitigation of Correlated Measurement Errors

Differential Corrections

DGPS Positioning Accuracy: ~1-3 m
National DGPS Coverage (2005)

In the Lower 48, single coverage: 87%, dual coverage: 55%
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• Potential Partners/Rivals: GLONASS, Galileo, BeiDou/Compass, …
Performance Metrics

- **Accuracy**
  - How good are the estimates? RMS error

- **Error Bounds**
  - Your error is no worse than x (with probability 0.999…9)

- **Integrity of Signals**
  - The signals on which your estimates are based are genuine. (Probability)

- **Availability of Service**
  - Consistent with your requirements. (Probability)

- **Continuity of Service**
  - For the next x seconds, consistent with your requirements. (Probability)
Positioning Accuracy Hierarchy

GPS and Its Augmentations

- SPS (1990 – 2000)
- SPS (2000 – )
- PPS
- WADGPS
- DGPS
- Relative Navigation
- Surveying & Geodesy

Position Error:
- 1 mm
- 1 cm
- 10 cm
- 1 m
- 10 m
- 100 m

Types of Measurements:
- Carrier Phase Measurements
- Code Phase Measurements
Real-Time Position Estimates from GPS (1997)*

*Source: MIT Lincoln Laboratory
GPS Applications
Assisted GPS (AGPS)

Assistance:
Time, Navigation data
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Evolution of GPS Signals

Block II / IIA / IIR
1989 - 2005

Block IIR-M
Starting in 2005

Block IIF
Starting in 2009

C/A
P(Y)
M
L2C
L5
1176.45 MHz
1227.6 MHz
1575.42 MHz
Frequency Plans*

*Adapted from T. Grelier et al., Inside GNSS, May/June 2007
• History
  – Developed by Soviet Union, first launch: 1982
  – Declined under Russia, but newly revived
  – Similar to GPS: Passive, one-way ranging
  – 10-12 working satellites over the past 5 of years, currently 16
  – No significant user base

• Constellation
  – 24 satellites in 3 orbital planes, 64.8º inclination
  – 19,100 km altitude, 11 ¼ hour period

• Signals
  – 3 allocated bands: G1 (1602 MHz), G2: (1245 MHz), G3 (?)
  – C/A-like code: 511 chips, 1 ms code period, 50 bps data
  – All SVs use same PRN with frequency division multiple access (FDMA) using 16 frequency channels, reused for antipodal SVs

• Plans: 18 SVs in 2008, full constellation in 2011 (?)
GPS+GLONASS Satellite Visibility

Graphs showing satellite visibility and dilution of precision (DOP) for GLONASS-21 and GPS+GLONASS (2x21) systems.
GPS & GLONASS Position Estimates*
1-Minute Samples, 15 June 1996

*Source: MIT Lincoln Laboratory
GPS & GLONASS Position Estimates*
1-Minute Samples, 15 June 1996

* Source: MIT Lincoln Laboratory
Galileo

- European-owned: planned public-private partnership didn’t work out
- “Seen” as a civil system, but military role may emerge
- 5 services
  - Free: Open Service
  - For a Fee:
    - Commercial Service
    - Safety-of-Life Service
    - Public Regulated Service
    - Search & Rescue Service
- 30 MEOs in 3 planes inclined at 56°
- First experimental satellite launched in 2005
- Appears to have recovered from recent setbacks; system operational around 2013

Objectives of Galileo:
- Increased overall performance
- Civil system in contrast to GPS
- Independent and interoperable with GPS
- Better robustness
- Certified quality of services
- Qualified for safety critical applications
- ...
BeiDou/Compass

- Chinese
- BeiDou: Regional System
  - Active system
  - 2 - 3 geostationary satellites orbited in 2000 – 2003
- Compass: GNSS
  - 1 MEO launched in 2007
Satellite navigation systems exploit basic properties of radio waves: Transit exploited the Doppler effect, GPS exploits the known speed of propagation.

GPS is based on the old idea of trilateration, but implemented with the technology of the second-half of the 20th century: space-based radio transmitters, ultra-stable clocks, and spread spectrum signals.

A GPS receiver measures pseudoranges to the satellites by measuring pseudo-transit times of radio signals. It takes 4 satellites (i.e., 4 pseudoranges) in order to estimate position \((x, y, z)\) and time \(t\).

With a clear view of the sky, it's easy to get positioning accuracy of several meters with a $100 GPS receiver, or relative positioning accuracy of millimeters with a pair of $1000 receivers.

GPS satellites are 20-watt transmitters 20,000 km away, so the signals reaching the earth are very weak and, therefore susceptible to interference.

The success and breadth of GPS applications is attributable largely to “the chip.” The VLSI revolution was well-timed for GPS.