The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2030-3

Conference on Research Frontiers in Ultra-Cold Atoms

4 - 8 May 2009

Coherence and condensation in 2D atomic gases

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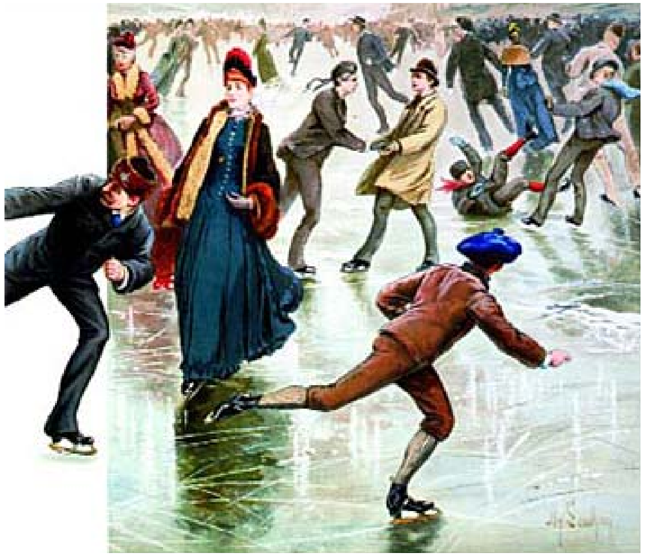
Coherence and condensation in a 2D Bose gas



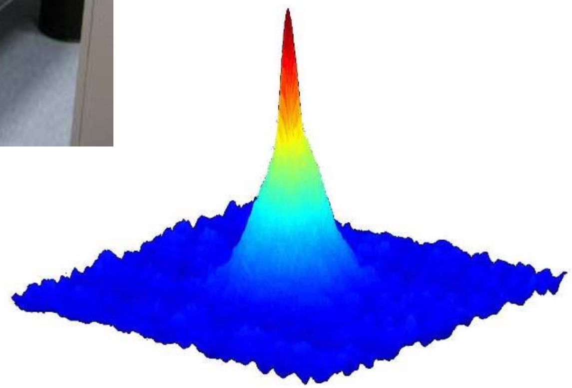
Zoran Hadzibabic
University of Cambridge

ICTP, May 2009

New experiments in Cambridge…

March 2008

February 2009



Looking for a
new postdoc
to join us in summer 2009!

Outline of the talk

2D basics (infinite uniform system)

Berezinskii-Kosterlitz-Thouless (BKT) physics
Quasi-2D Bose gases

Experimental data

ENS, Z.H. w/ J. Dalibard, P. Krüger, M. Cheneau, S.P. Rath, B. Battelier, S. Stock
+ NIST, P. Cladé, C. Ryu, A. Ramanathan, K. Helmerson, W. Phillips

Critical point
Coherence properties

Thoughts & Words (finite trapped system)

BEC vs. BKT
BEC vs. "BEC"
Quasicondensate vs. Quasicondensate
…

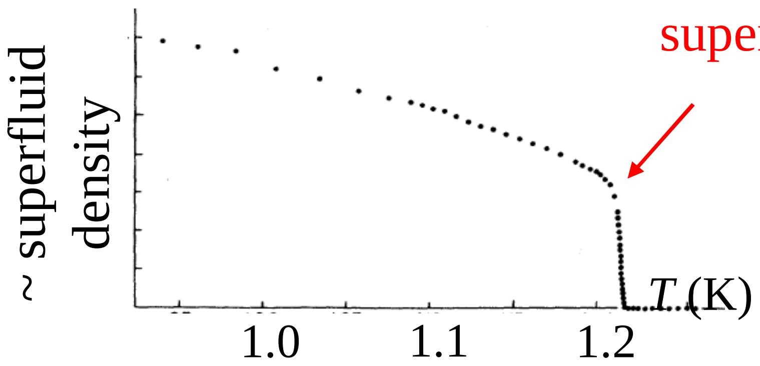
BEC, coherence, and superfluidity in 2D
in an infinite uniform system

Peierls (1935), Bogoliubov… Mermin-Wagner-Hohenberg (1966-67):
No long-range order at any $T \neq 0$
- destroyed by long wavelength phase fluctuations (phonons)

No BEC

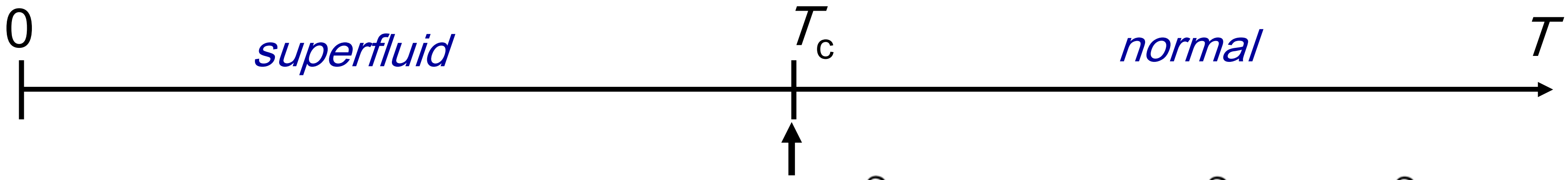
But still a superfluid transition at finite T_C
Described by the (Berezinskii-)Kosterlitz-Thouless (BKT) theory

superfluidity in liquid He films
Bishop and Reppy 1978



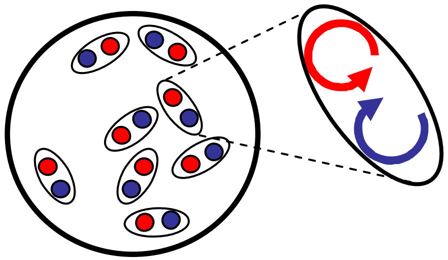
"universal jump in superfluid density" at $T = T_c$

Berezinskii - Kosterlitz - Thouless (BKT) 1971-73



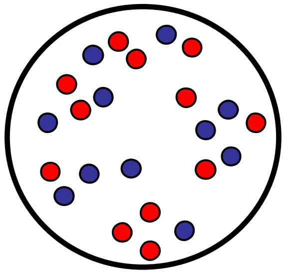
$$n_{\text{superfluid}}\lambda^2 = 4 \qquad \lambda^2 = 2\pi\hbar^2/(mkT)$$

(in addition to phonons…)



Bound vortex-antivortex pairs

Unbinding of vortex pairs



Proliferation of free vortices

$$g_1(x, y) = \langle \psi^\dagger(x, y)\, \psi(0) \rangle$$

Algebraic decay

$$g_1(r) \sim r^{-\alpha}$$

no LRO
either way

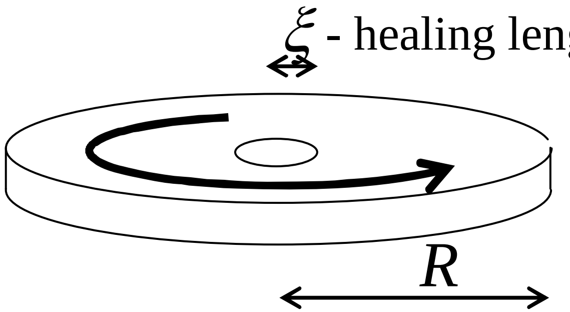
Exponential decay

$$g_1(r) \sim \exp(-r/\ell_0)$$

Quantitative predictions

1. Universal jump in superfluid density $n_s\lambda^2 = 4$ at the transition

Kosterlitz & Nelson 1977



Free energy for a free vortex: $$\frac{E-TS}{k_BT} \sim \frac{1}{2}\left(n_s\lambda^2 - 4\right)\ln\left(R/\xi\right)$$

---

2. Algebraic decay $g_1(r) \propto r^{-\alpha}$ with $\alpha = 1/n_s\lambda^2 \le 1/4$

---

3. *Total* critical density $D = n_{\text{total}}\lambda^2$ depends on interactions

Analytics by Fisher & Hohenberg + Monte-Carlo by Prokof'ev, Svistunov et al.:

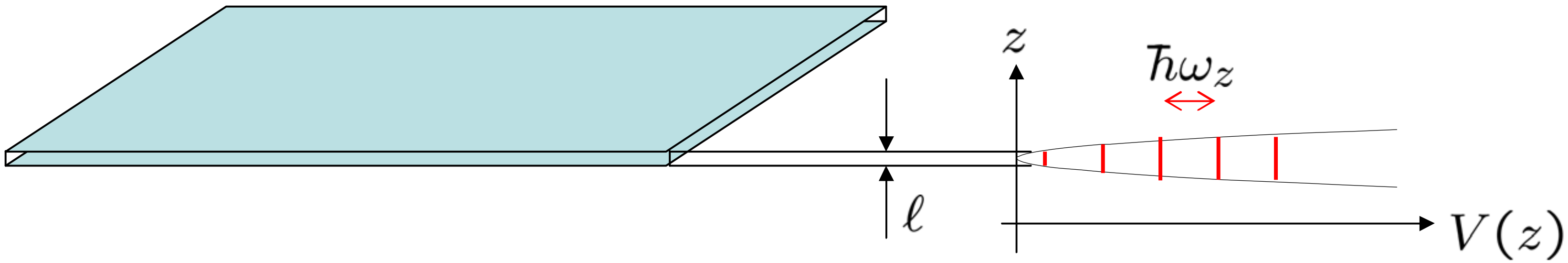
$$n_{\text{total}}\lambda^2 = \ln\left(\frac{C}{\bar{g}}\right)$$

$$\bar{g} = \frac{mg}{\hbar^2}$$

dimensionless interaction strength

$$C = 380 \pm 3$$

Quasi-2D atomic gases



Thermodynamically 2D if: $\hbar\omega_z > k_B T, \mu$ $\quad \ell \approx \sqrt{\hbar/m\omega_z} < \lambda, \xi$

If ℓ is larger than the 3D scattering length *a*, collisions still 3D

Interaction energy (to a good approximation): $\frac{g}{2}\int n^2(x,y)\,dx\,dy$

with $g = \frac{\hbar^2}{m}\tilde{g}$ and $\tilde{g} = \sqrt{8\pi}\frac{a}{\ell}$

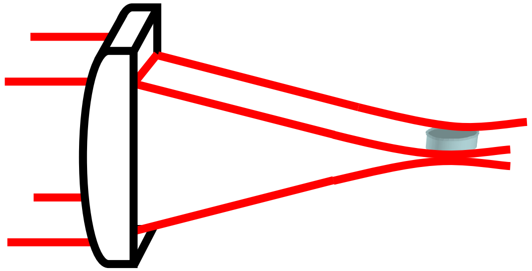
- Liquid helium films: $\tilde{g} \sim 1$
- ENS experiment (Rb): $\tilde{g} \sim 0.1$
- NIST experiment (Na): $\tilde{g} \sim 0.01$

For a more accurate treatment: Petrov-Shlyapnikov

Experimental realizations

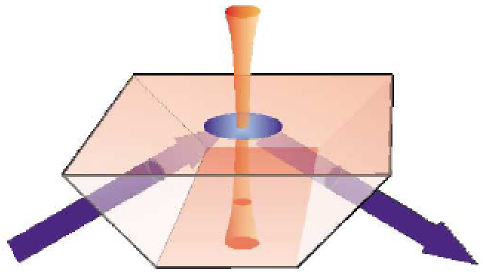
single plane

Focused beam



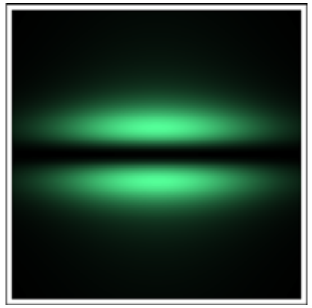
MIT, NIST

Evanescent wave



Innsbruck

Holographic mask



Oxford, Paris

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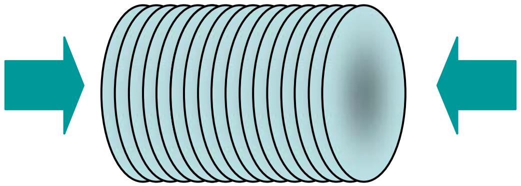
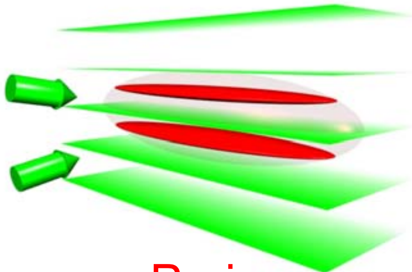
two or many planes

Standard 1D optical lattice



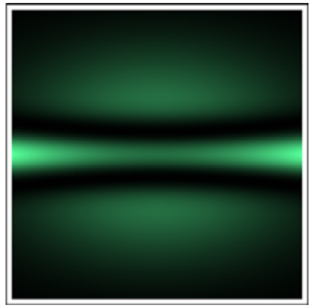
Yale, Florence, Zurich, Heidelberg,…

Large period optical lattice



Paris

Holographic mask



Paris

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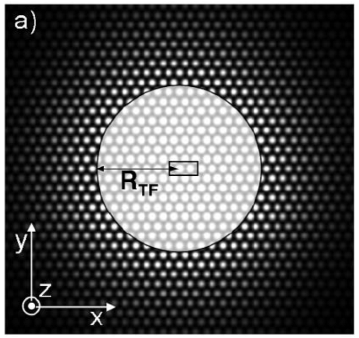
lattice model



~ 2D periodic array of Josephson junctions

Boulder

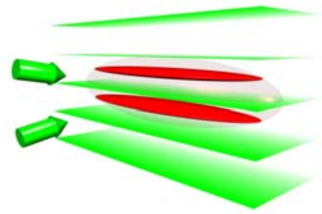
(nicely agrees with BKT, but not confusing…)

Harmonic trap and finite size effects?

Trust me for now…

Phase transition in a trapped 2D atomic gas

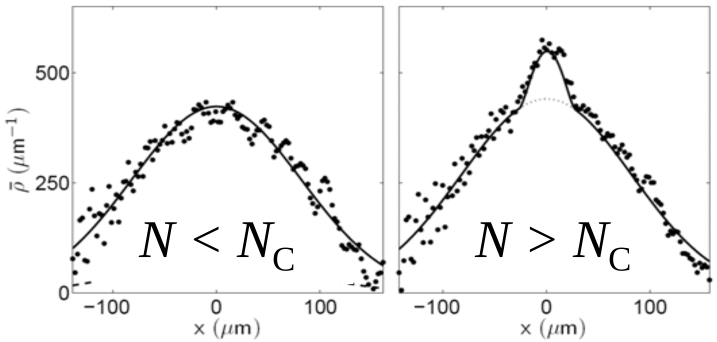
Constant T, vary the atom number N



two independent planes (no tunneling)

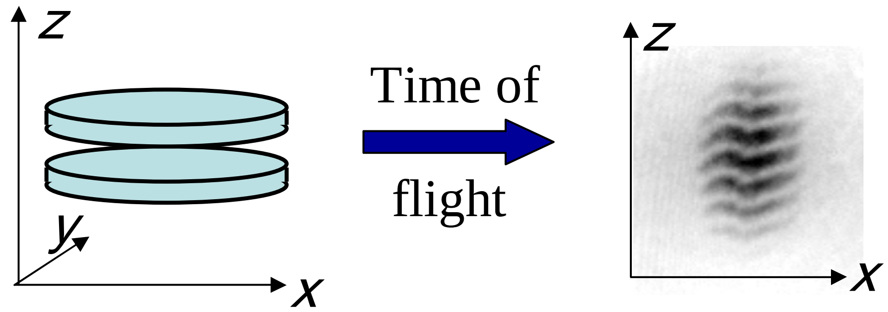
For $N > N_C$:

1. Bimodal density distribution



(in TOF)

2. Interference between two planes



Onsets of bimodality and interference coincide

Interfering part is the central feature

(reminiscent of ordinary 3D BEC, but let's not jump to conclusions…)

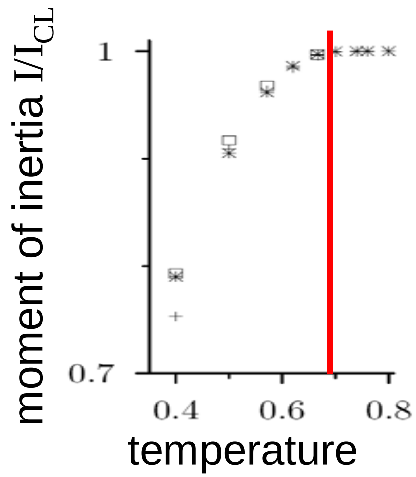
Krüger et al, PRL 99, 040402 (2007)

Nature of the critical point?

Local Density Approximation (LDA) hypothesis:
critical point = BKT transition in the trap center
confirmed by Quantum Monte-Carlo

Holzmann-Krauth, PRL 100, 190402 (2008)

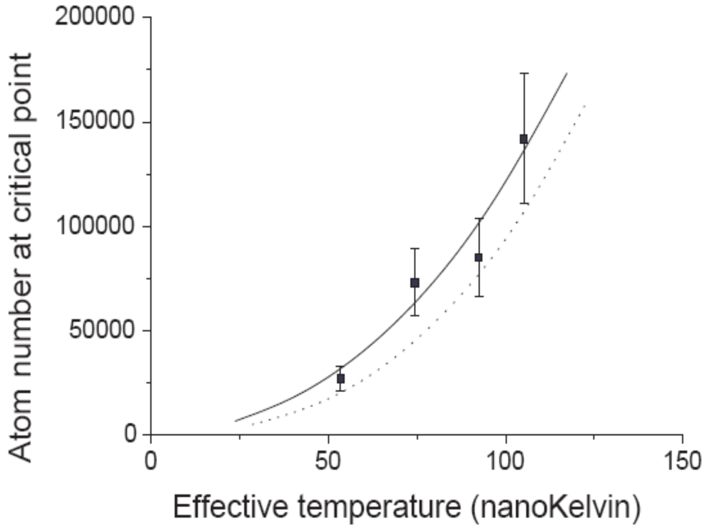
see also: Bisset et al., PRA 79, 033626 (2009)



Corresponding critical atom number N_C?
BKT + mean field density profile,
no adjustable parameters
(temperature calibrated from the simulations)

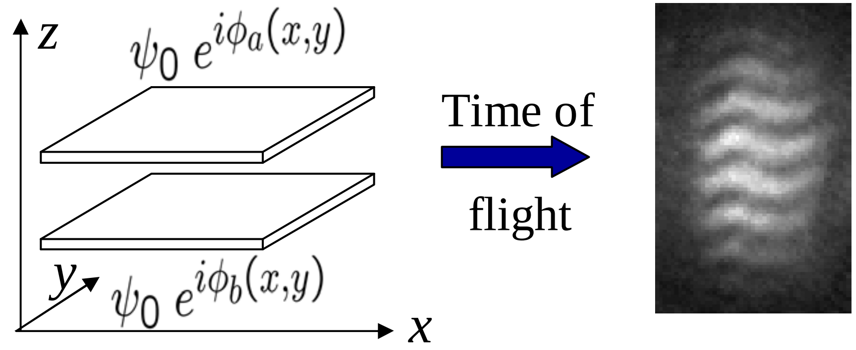
Hadzibabic et al., New J. Phys. 10, 045006 (2008)

also: Holzmann, Chevalier, Krauth, EPL 82, 30001 (2008)
Bisset, Baillie, Blakie, PRA 79, 013602 (2009)



At NIST peak density measured directly in a single plane, also agrees w/ BKT
Cladé et al., PRL 102, 170401 (2009)

Interference experiments



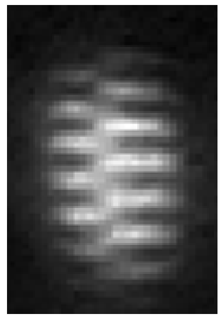
Fast expansion along *z*

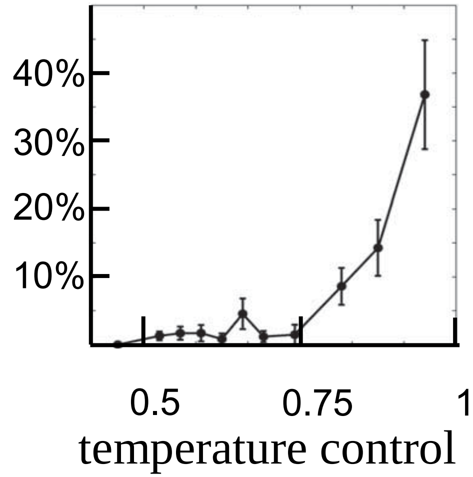
Fringe phase depends on ϕ_a - ϕ_b

Phonons – smooth phase variations

Direct evidence for vortices
appear as sharp dislocations

Hadzibabic et al, Nature 441, 1118 (2006)



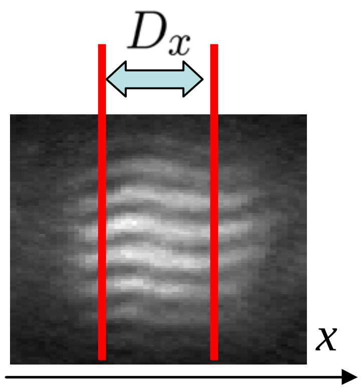


Complex contrast: $C(x) = \int dy\, \psi_a^*(x,y)\, \psi_b(x,y)$

… can extract various correlation functions…

At NIST Ramsey interference w/ a single 2D plane   Cladé et al., PRL 102, 170401 (2009)

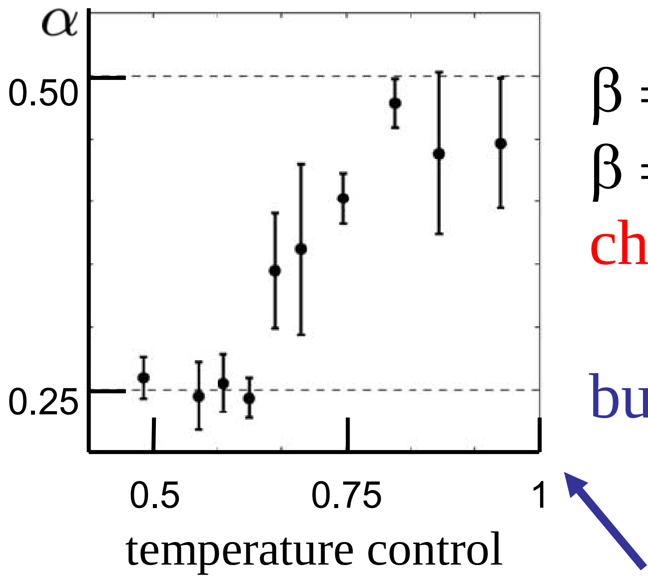
Average integrated contrast



Integrated contrast: $\langle C^2(D_x)\rangle \sim \frac{1}{D_x}\int_0^{D_x}(g_1(x,0))^2\,dx$

Polkovnikov, Altman, Demler,
PNAS 103, 6125 (2006)

$$\langle C^2(D_x)\rangle \sim 1/D_x^{\,2\beta}$$



$\beta = 1/2$ in the normal state

$\beta = \alpha = 1/n_s\lambda^2 \leq 1/4$ in the superfluid state

change in α coincides with vortex appearance

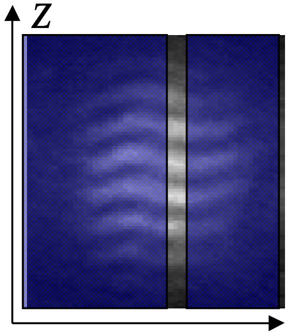
but quantitative agreement a bit serendipitous…

critical point in the cloud center

messy non-uniform effects

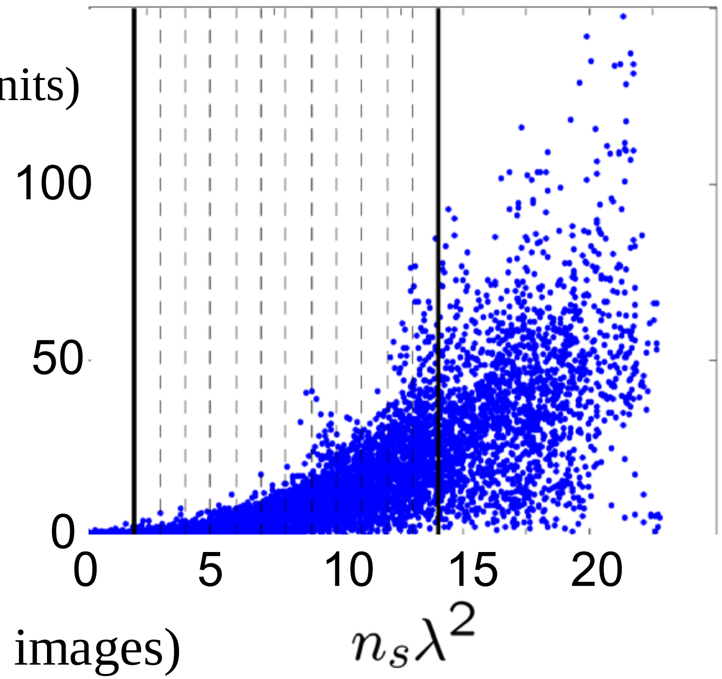
Full statistics of contrasts

Theory: Gritsev, Altman, Demler, Polkovnikov, Imambekov
1D experiment: Hofferberth et al., Nature Phys. 4, 489 (2008)



$$C(x) = \int dy\, \psi_a^*(x,y)\, \psi_b(x,y)$$

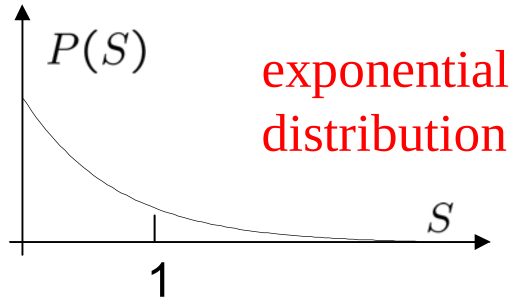
depends on coherence along the line of sight y,
each image ~30 columns w/ different $(C, n_s\lambda^2)$ pairs



(300 images)

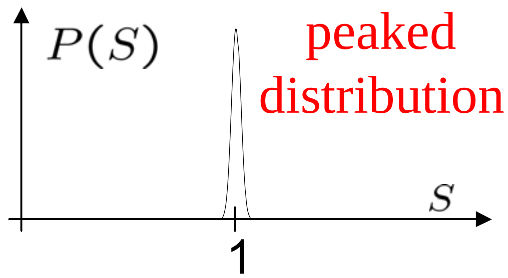
normalized contrast: $S = \frac{|C|^2}{\langle |C|^2 \rangle}$

many uncorrelated bits along y



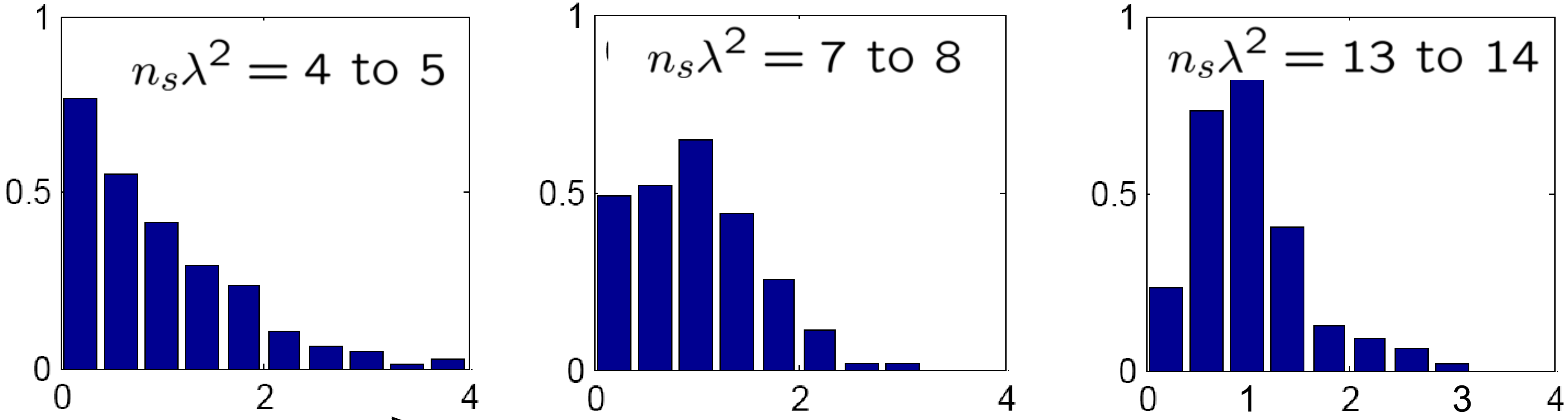
exponential distribution

full coherence (true BEC)



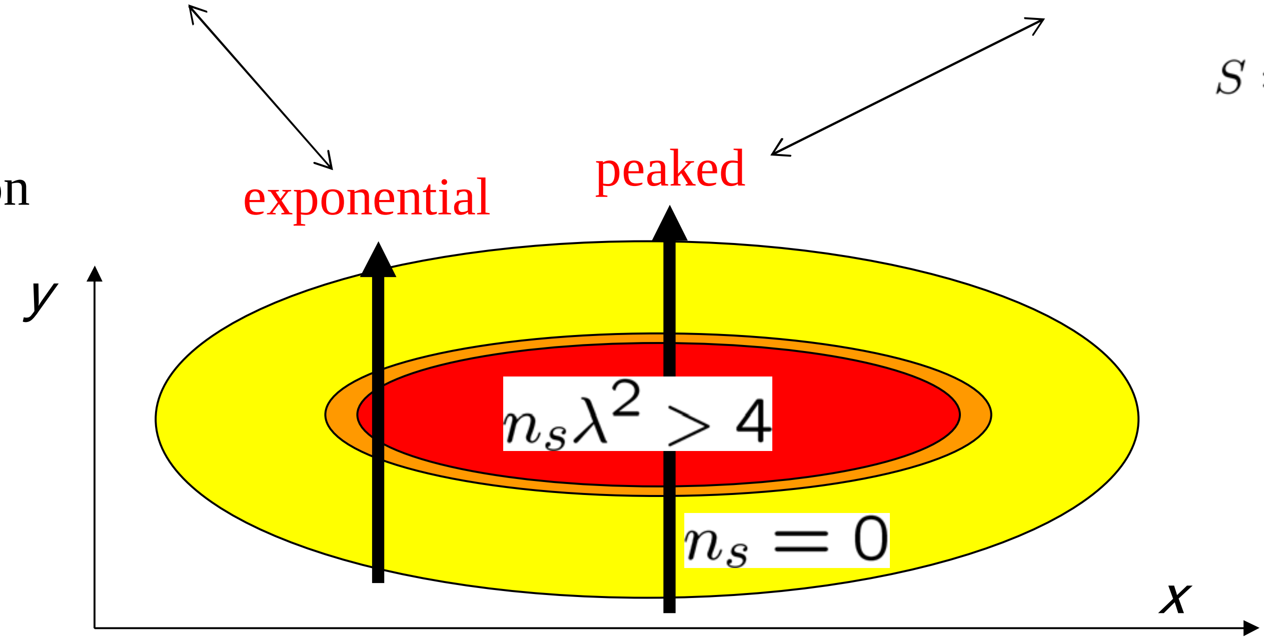
peaked distribution

From low to high coherence



(consistent
w/ β = 1/2
& observation
of vortices)

$$S = \frac{|C|^2}{\langle |C|^2 \rangle}$$



Outline of the talk

2D basics (infinite uniform system)

Berezinskii-Kosterlitz-Thouless (BKT) physics
Quasi-2D Bose gases

Experimental data

ENS, ZH w/ J. Dalibard, P. Krüger, M. Cheneau, S.P. Rath, B. Battelier, S. Stock
+ NIST, P. Cladé, C. Ryu, A. Ramanathan, K. Helmerson, W. Phillips

Critical point
Coherence properties

Thoughts & Words (finite trapped system)

BEC vs. BKT
BEC vs. "BEC"
Quasicondensate vs. Quasicondensate
…

BEC vs. BKT

harmonic trap, but neglect finite size effects

Thermodynamic limit: $\omega^2 \to 0 \quad N \to \infty \quad N\omega^2 = \mathrm{Ct}$

From 2D density of states, usual statistical argument (saturation of excited states) allows conventional BEC in an <u>ideal</u> Bose gas for:

$$N \; > \; N_c = 1.6 \left(\frac{kT}{\hbar\omega}\right)^2$$

Bagnato – Kleppner (1991)

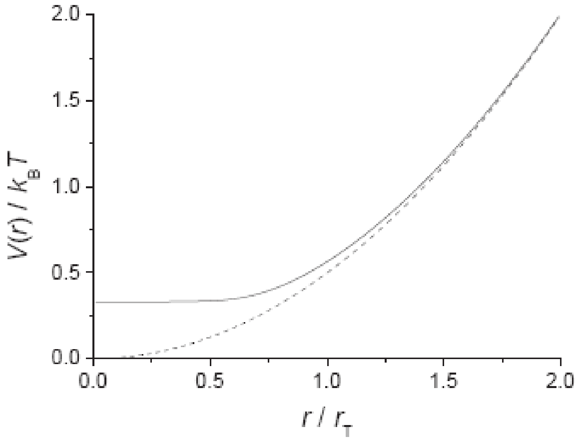
But BEC requires $n(0)\lambda^2 = \infty$, suppressed by any interactions

(in sharp contrast to 3D where $n(0)\lambda^3 = 2.6$ is finite, small effect of interactions)

Interactions flatten out the *effective* potential

excited states can accommodate any number of particles

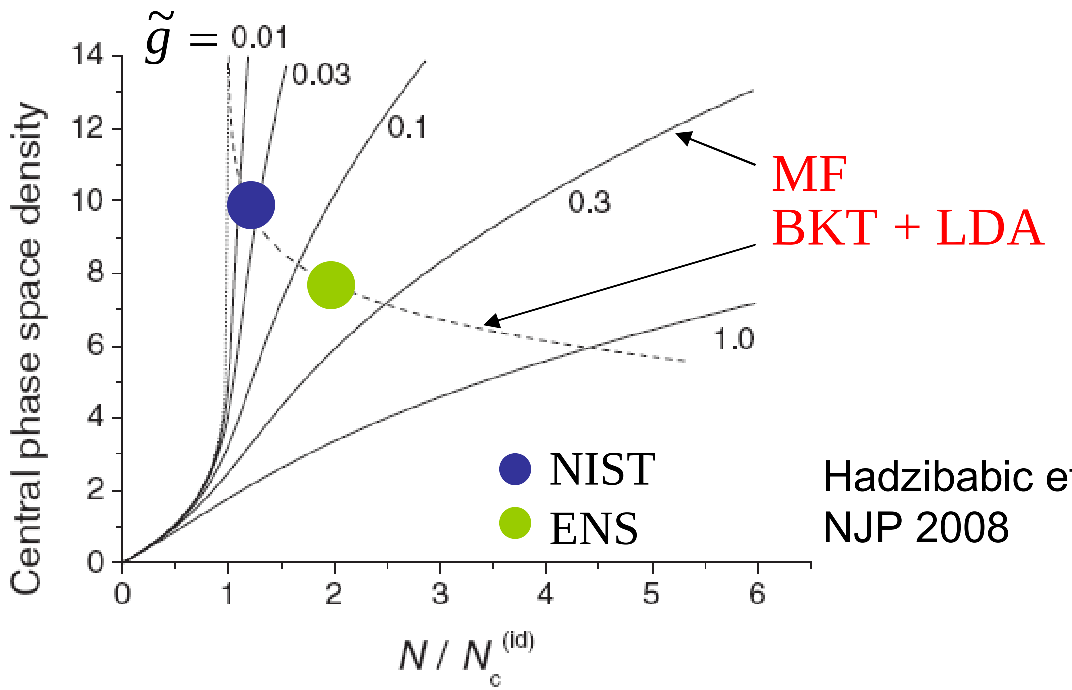
Holzmann, Baym, Blaizot, Laloe, PNAS 2007



BEC vs. BKT

harmonic trap, but neglect finite size effects

No BEC (so far), BKT occurs for: lower critical density (obviously)
higher critical number



Hadzibabic et al., NJP 2008

$$\frac{N_c^{(mf)}}{N_c^{(id)}} = 1 + \frac{3\tilde{g}}{\pi^3} D_c^2$$

$$D_C = \ln\left(380 / \tilde{g}\right)$$

Holzmann et al., PNAS 2007

1. <u>Only one</u> (BKT) phase transition!
2. Conventional BEC as a special "non-interacting limit" of BKT

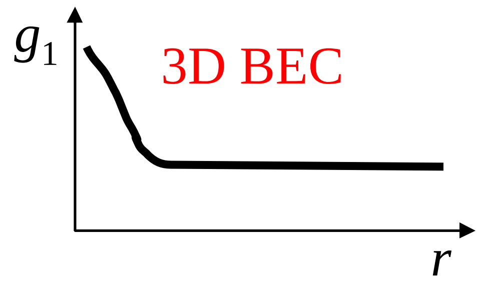
(could not have made this connection in a uniform system)

BEC vs. "BEC"

finite size effects (but don't worry about non-uniformity)

condensed fraction: $n_0 \sim \lim_{r \to \infty} g_1(r)$

if "infinity" = finite system size L, n_0 must be finite



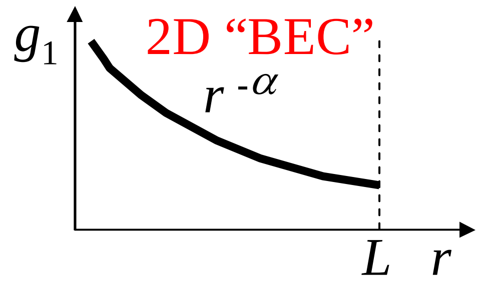
"BEC" in a 2D BKT superfluid:

L ~ 100 μm

healing length ξ ~ 0.1 μm

at the critical point α =1/4

$$n_0 \sim (\xi/L)^{\alpha} \sim 0.2$$



thermodynamic limit $\ln(L/\xi) >> 1$ experimentally <u>impossible</u>

"…the system would have to be bigger than Texas for Mermin-Wagner to apply …"

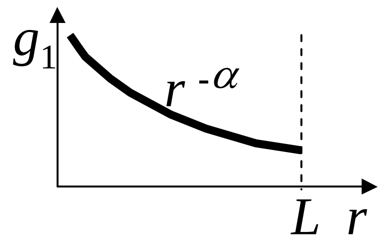
"*Magnetization as a signature of BKT,*" Bramwell – Holdsworth, 1994

1. Finite n_0 due to finite size, and <u>not</u> due to the harmonic trap
2. signature of the <u>BKT</u> phase transition
3. "BEC" ≠ BEC, but shows interference & sharp peak in TOF

Quasicondensate vs. Quasicondensate

(colloquially) two meanings of the word

1. QC = "not quite BEC" - no density fluctuations, but $g_1 \sim r^{-\alpha}$



appears at T_{BKT}, finite n_S, finite n_0 (for finite L)

"superfluid QC" = "BEC" $\quad n_{QC}^{(1)} = n_0, n_S \ldots ?$

"BEC" → ("true") BEC *crossover* at finite $T < T_{BKT}$

for finite L, all excitations (phonons, vortex pairs) gapped, exponentially suppressed below some finite T, so $\alpha \to 0$

Shlyapnikov et al, Simula et al.

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2. QC = "not just a thermal gas" - suppression of density fluctuations

$$\langle n^2 \rangle < 2\langle n \rangle^2 \qquad n_{QC}^{(2)} = \sqrt{2\langle n \rangle^2 - \langle n^2 \rangle} \neq n_{QC}^{(1)}$$

Kagan, Shlyapnikov, Svistunov, Prokof'ev et al.

prerequisite for BKT, n_S and n_0 might still be (essentially) 0

thermal gas → "non-superfluid QC" *crossover* at $T > T_{BKT}$

both in a finite and in an infinite system (in 2D!)

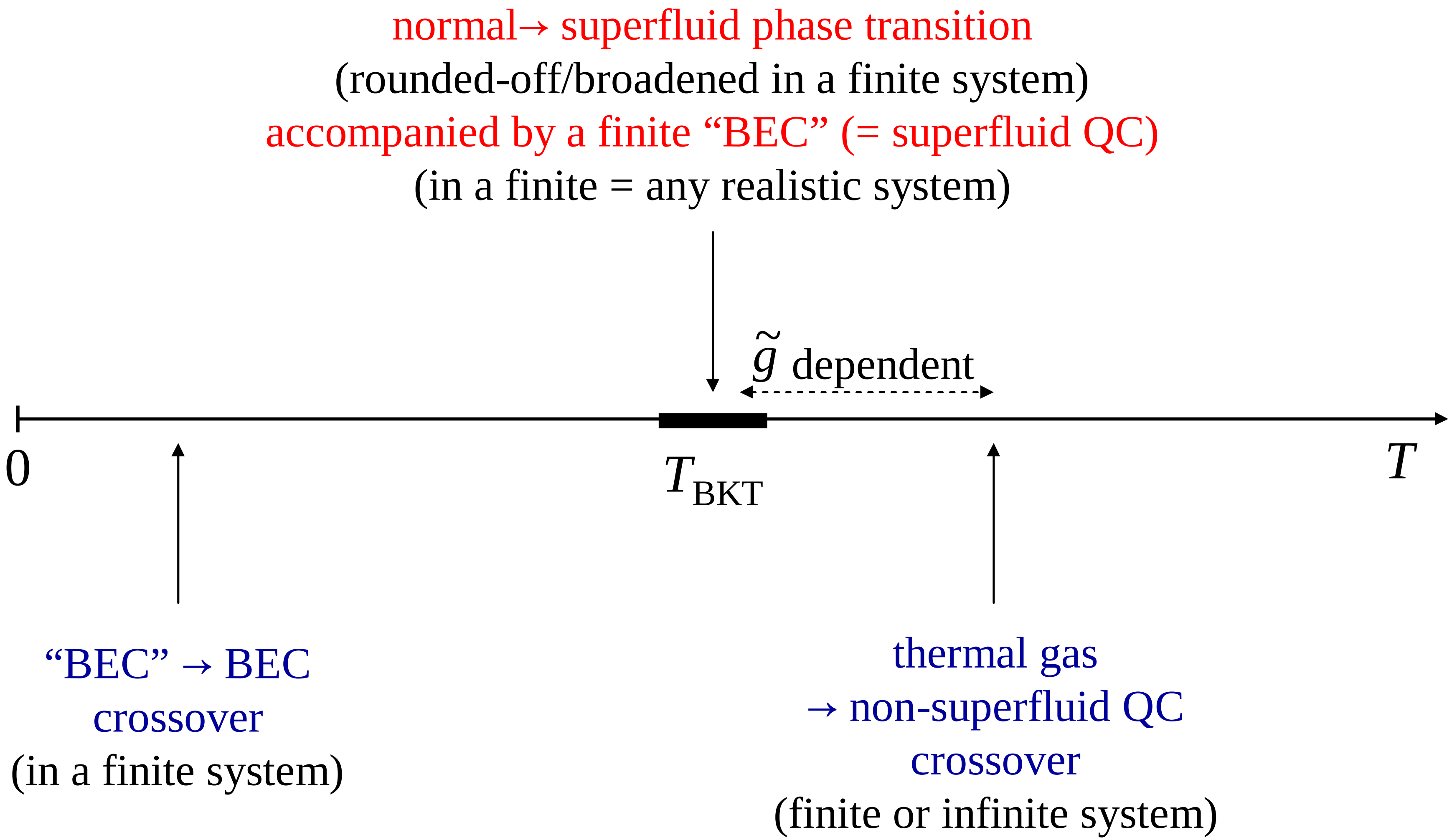
experimentally – suppression of 3-body recombination in 2D hydrogen

Safonov et al., PRL 1998

Reasonably complete (?) phase diagram

finite $\tilde{g}$, no need to mention the harmonic trap d.o.s.

(but ignores non-uniformity – assume quasi-local probes)



ENS vs. NIST

TOF for $N > N_C$

ENS (Rb)

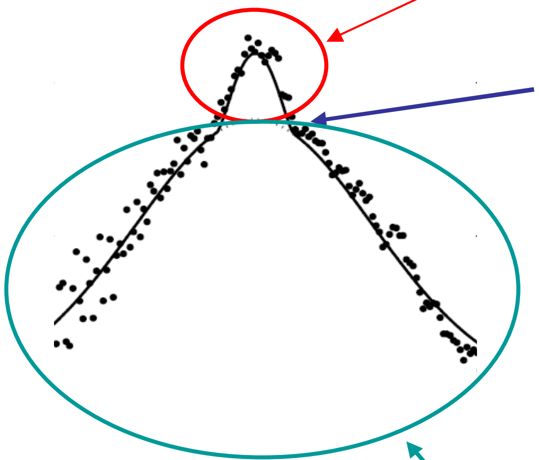
$\tilde{g} \sim 10^{-1}$

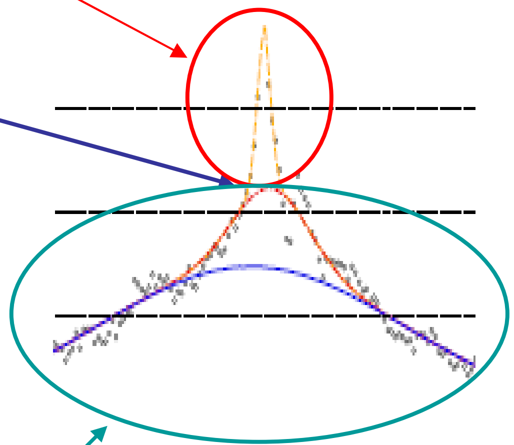
NIST (Na)

$\tilde{g} \sim 10^{-2}$

superfluid part

critical density agrees with BKT





~ Gaussian

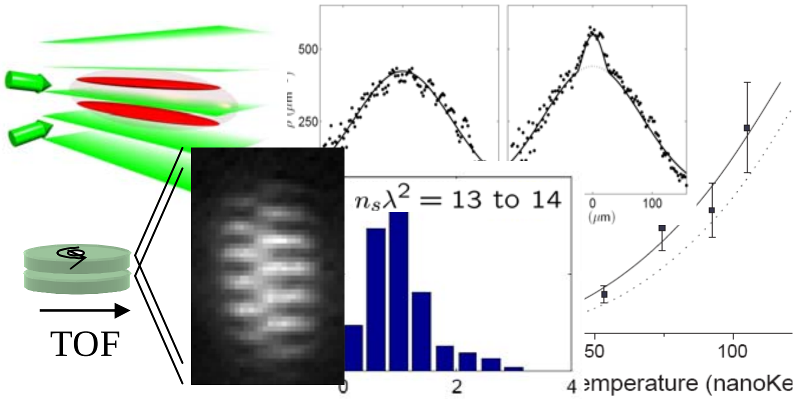
normal component different

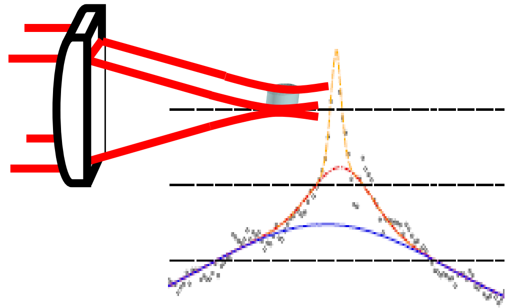
“bimodal”

distinguishable thermal gas and non-superfluid QC? supported by NIST interference exp.

NB: ideal 2D gas near BEC is not even close to a Gaussian

Summary





Critical point - agrees w/ BKT (+LDA) in two different experiments
Coherence measurements - support our pictures (but hard stuff)
(Crude) phase diagram - one phase transition, two crossovers, many words

Need: better understanding of the normal state
direct measurement of superfluidity
direct measurement of suppressed density fluctuations
experiments with different $\tilde{g}$
a postdoc

THE END