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International Centre for Theoretical Physics**



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**A fermionic quantum gas in an optical lattice**

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# A fermionic quantum gas in an optical lattice

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Funding: ETH, EU (OLAQUI, Scala), QSIT, SNF

[www.quantumoptics.ethz.ch](http://www.quantumoptics.ethz.ch)



**Superconductivity**



**BCS-Theory**

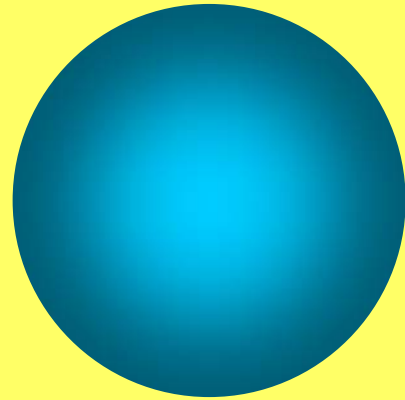
**Model**



**Experiment**

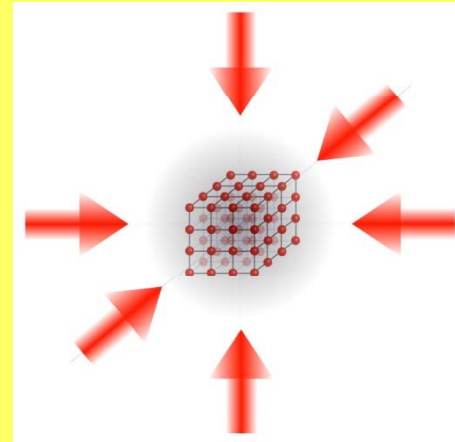
**Open Questions:**       $H \rightarrow \Psi?$

**Novel Models:**       $H = \dots$



Quantum Gases

+



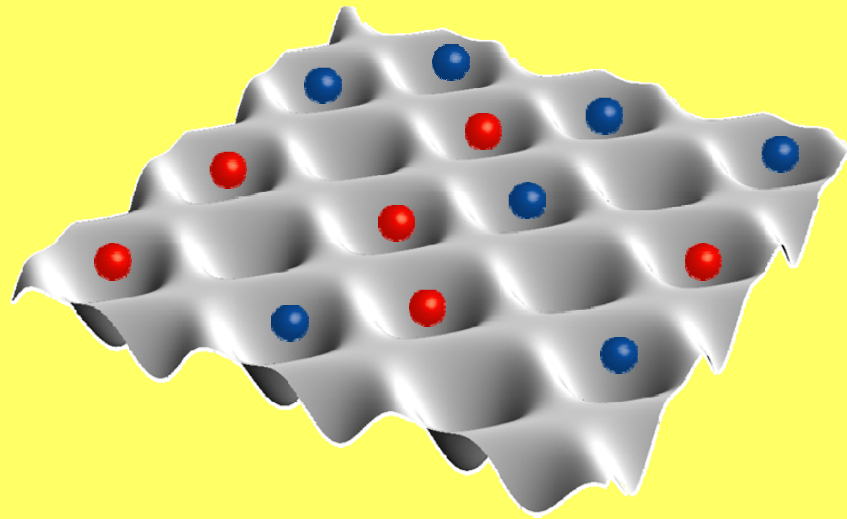
Optical Lattices

3D OL: Munich, NIST, ETH, Mainz, Texas, Innsbruck, Florence, MIT,  
Hamburg, Pisa, Illinois, Kyoto, Chicago, ...

# Fermi-Hubbard

- Static potential
- Single band
- Local interactions

# Fermi-Hubbard





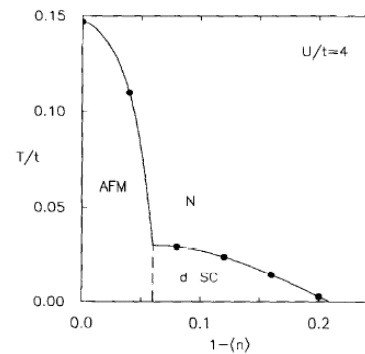
# Fermi-Hubbard

$$H = -J \sum_{\{i,j\},\sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

# Fermi-Hubbard ( $U>0$ )

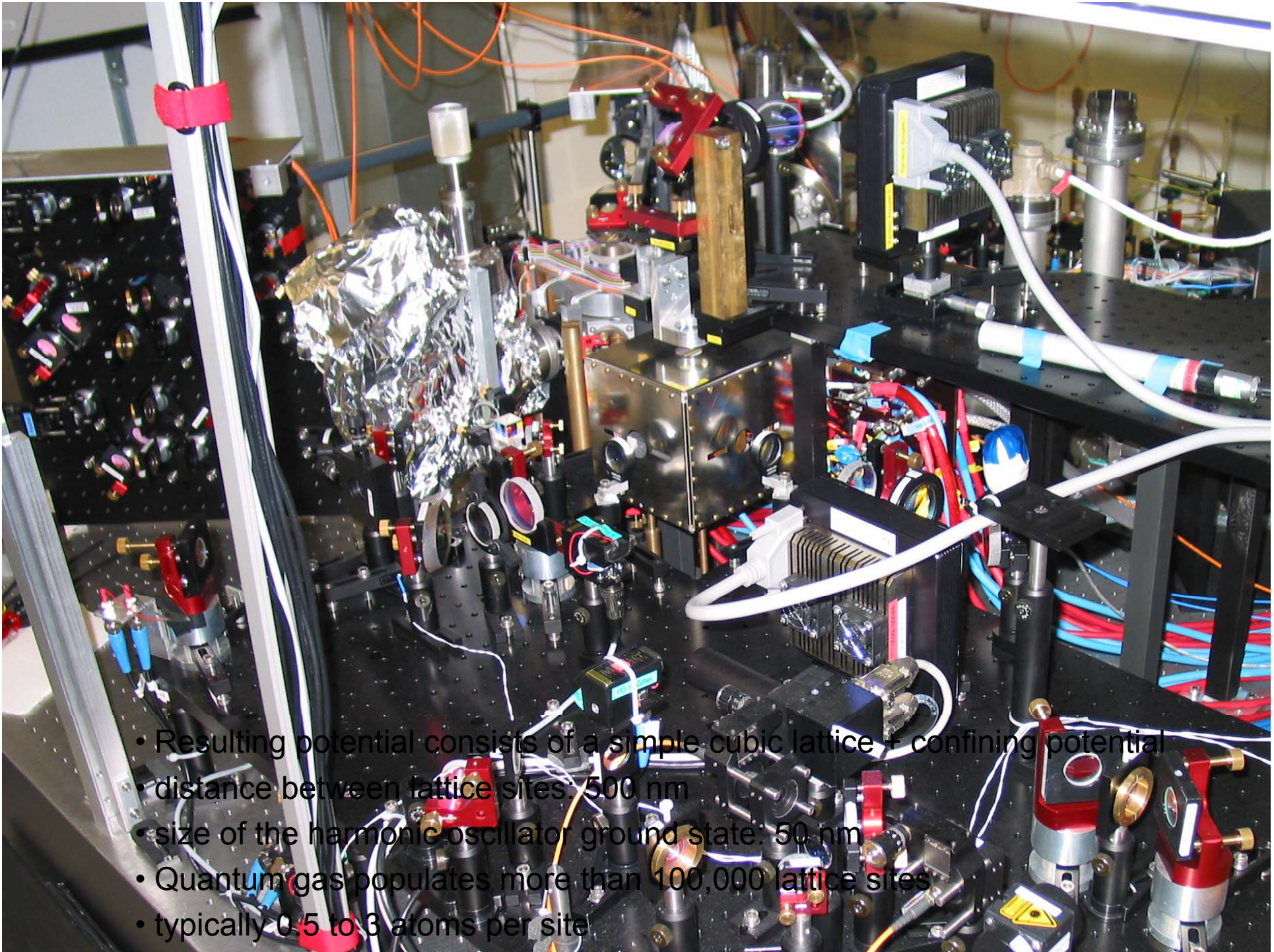
$$H = -J \sum_{\{i,j\},\sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

d-wave superconductor



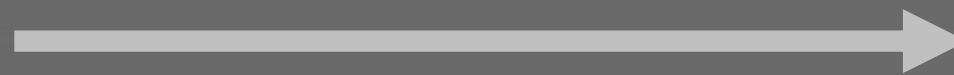
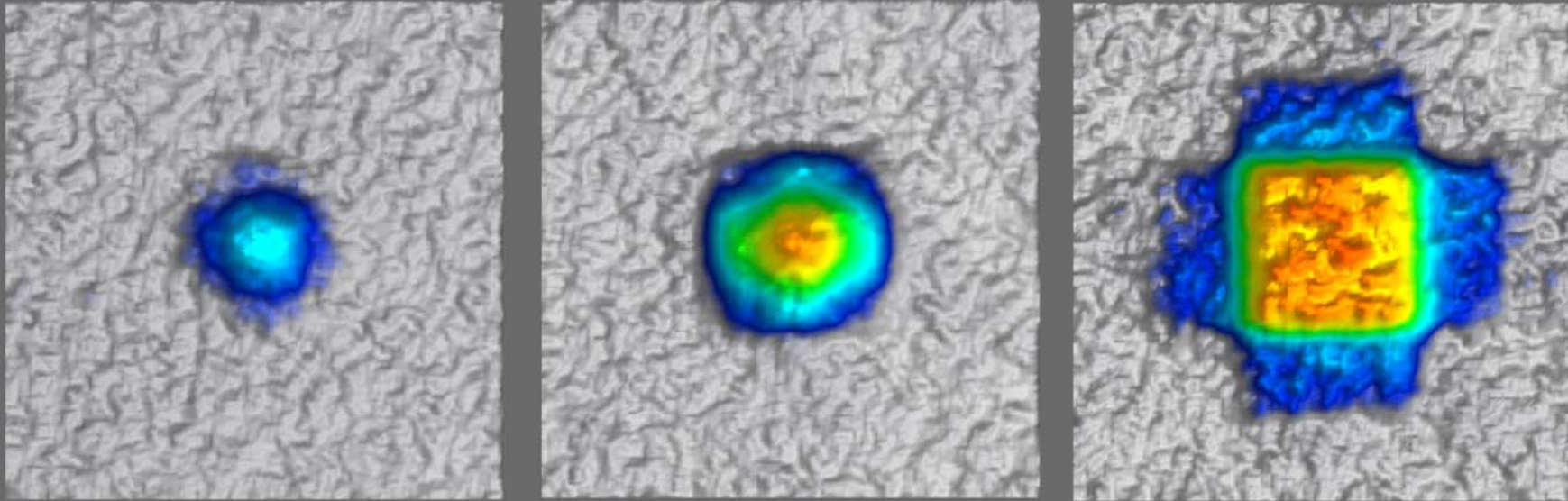
Scalapino, *Phys. Rep.* 250, 329 (1995)

Hofstetter et al., *Phys. Rev. Lett.* 89, 220407 (2002)



- Resulting potential consists of a simple cubic lattice + confining potential
- distance between lattice sites: 500 nm
- size of the harmonic oscillator ground state: 50 nm
- Quantum gas populates more than 100,000 lattice sites
- typically 0.5 to 3 atoms per site

# Fermi Surfaces



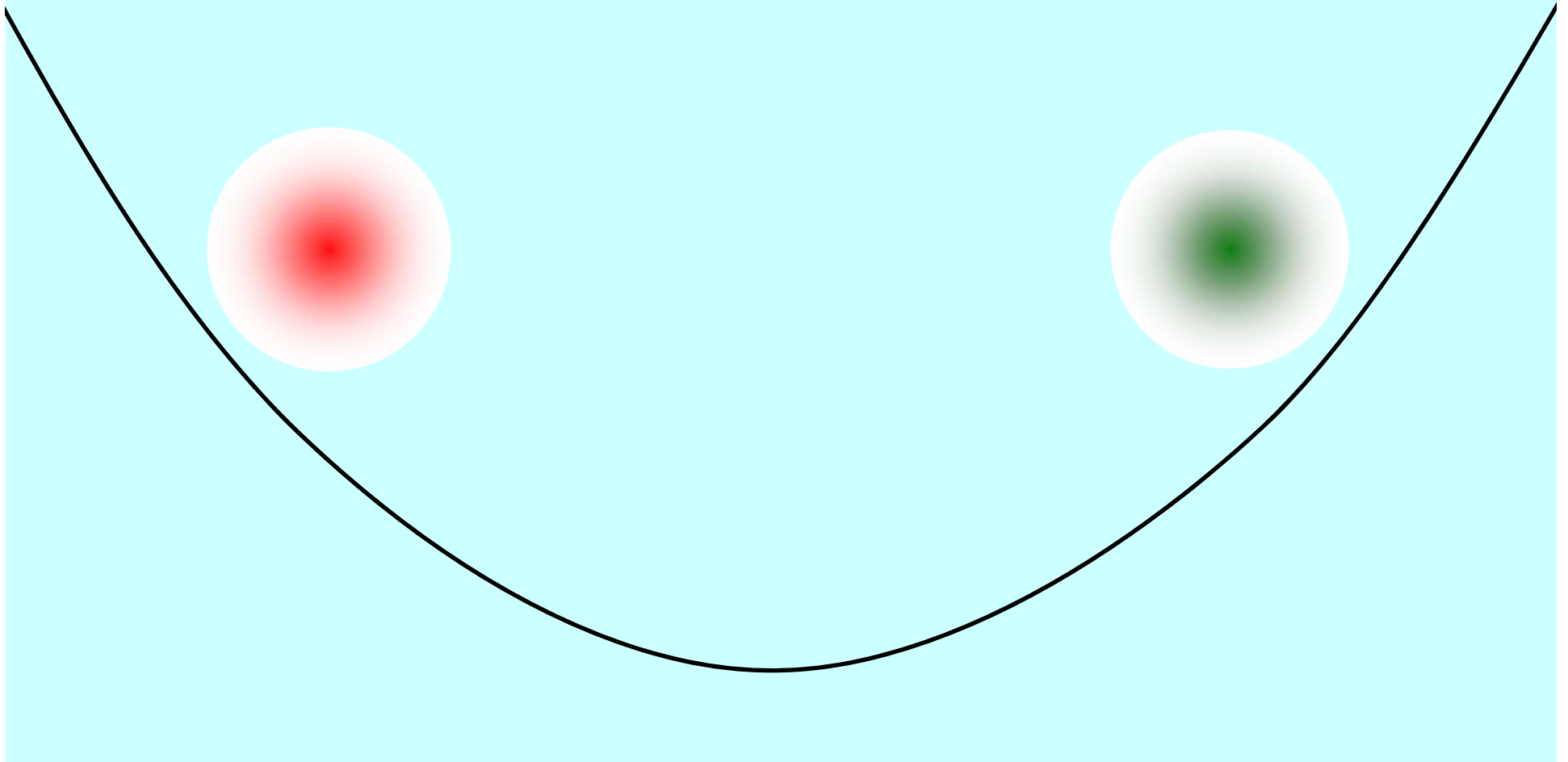
Increasing number of atoms in the lattice

M. Köhl, H. Moritz, T. Stöferle, K. Günter and T. Esslinger, PRL 94, 080403 (2005).

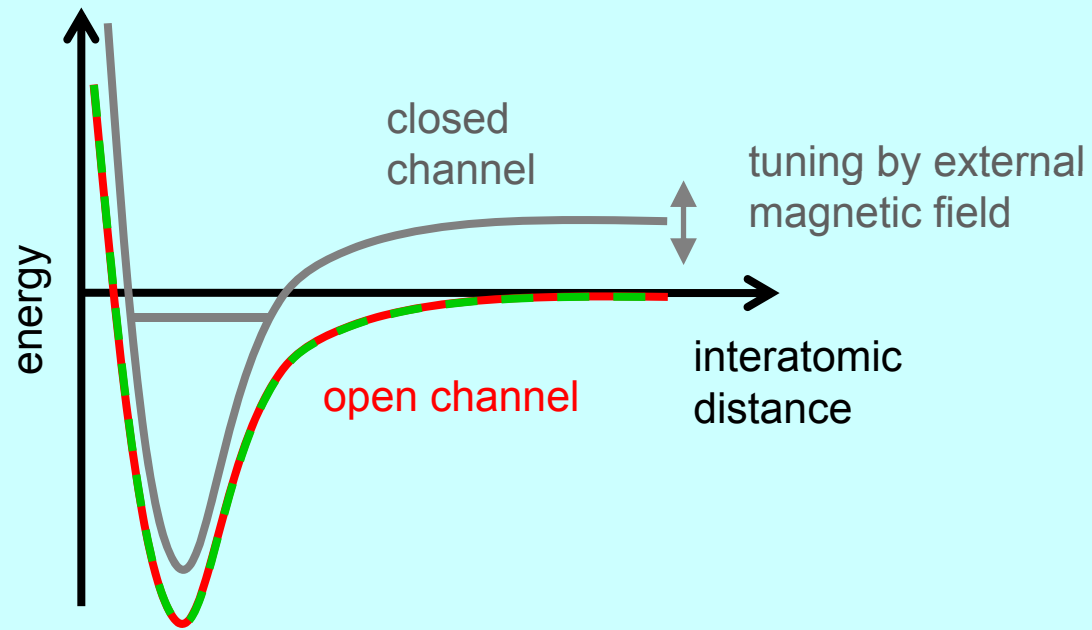
See also: Chin, J. K. et al., Nature, 443, 961-964 (2006).

Rom, T. et al., Nature, 444, 733-736 (2006).

# Repulsive Interactions



# Tuning interactions: Feshbach Resonance



# Tuning interactions: Feshbach Resonance

Weak interactions:

$^{40}\text{K}$ :  $|F=9/2, m_F=-9/2\rangle$

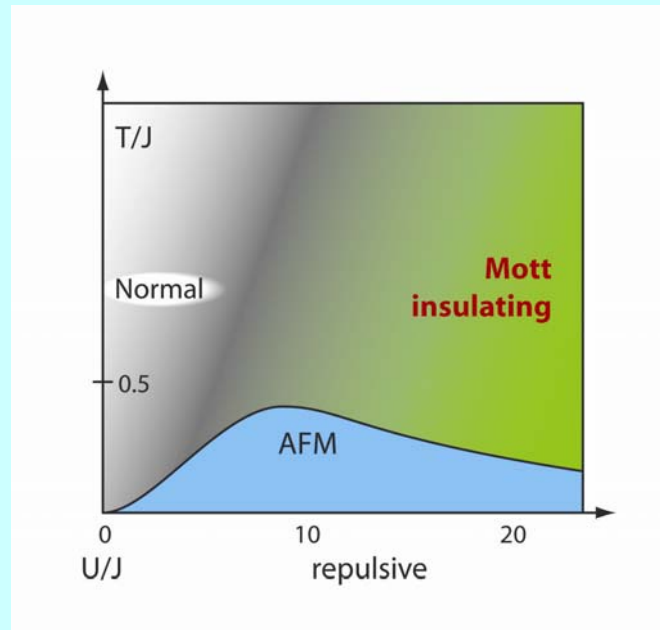
$|F=9/2, m_F=-7/2\rangle$

Strong repulsive interactions:  $^{40}\text{K}$ :  $|F=9/2, m_F=-9/2\rangle$

$|F=9/2, m_F=-5/2\rangle$

See: C. Regal, D. Jin, PRL 90, 230404

# Conductor to Mott-Insulator crossover

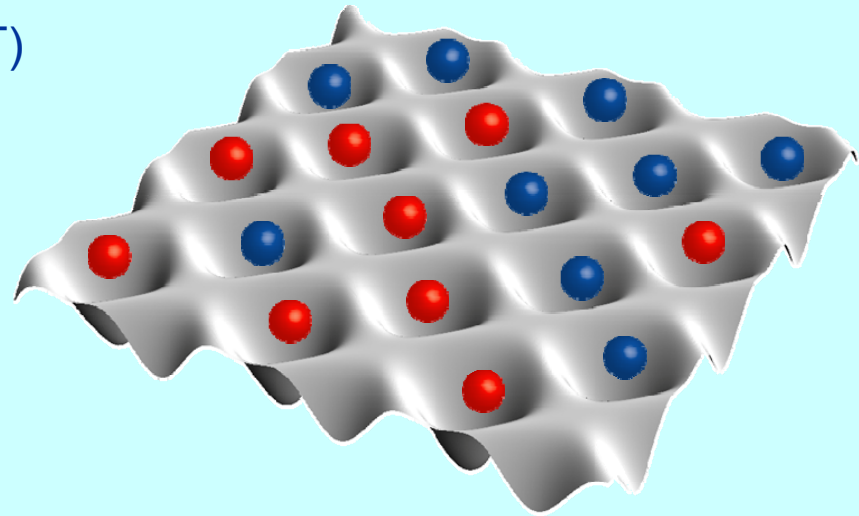


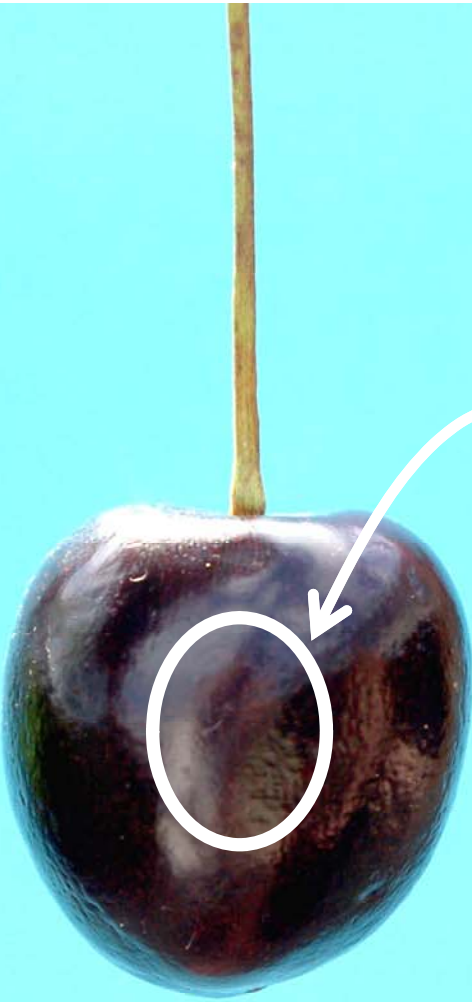


# Mott insulator

Interaction-induced localization ( $U \gg T$ )

- insulator
- incompressible
- energy gap
- reduced number fluctuations



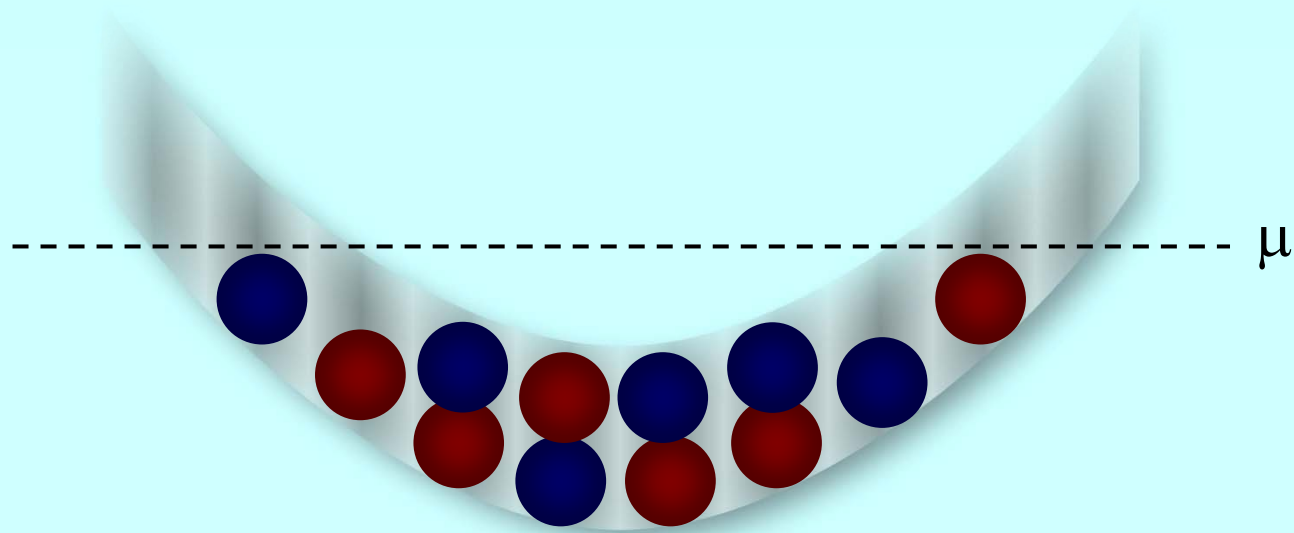


Incompressible core

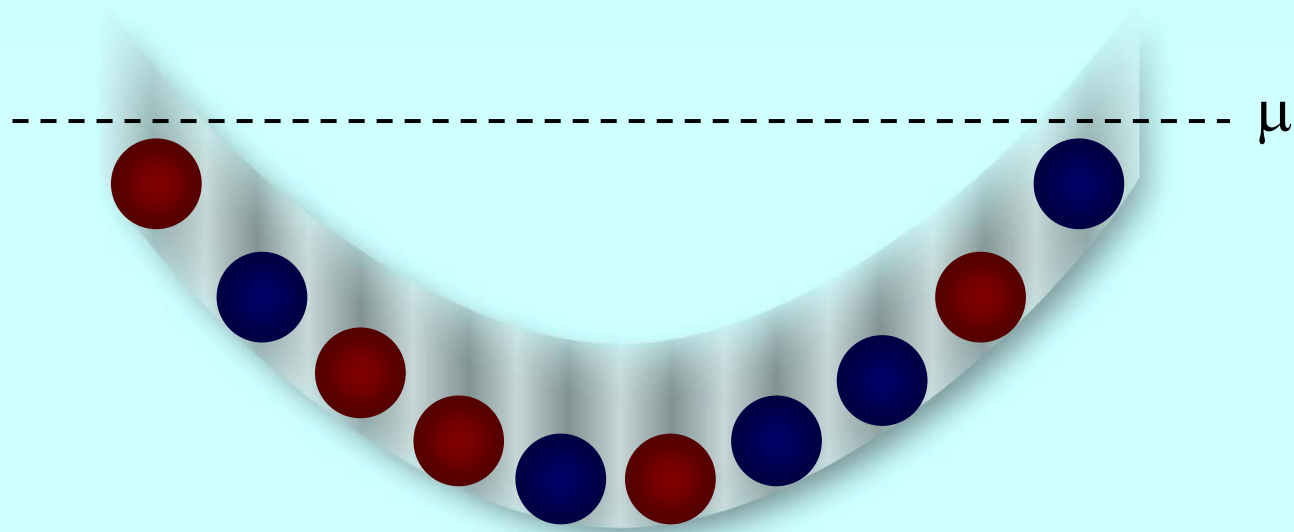


**Measure double occupancy**

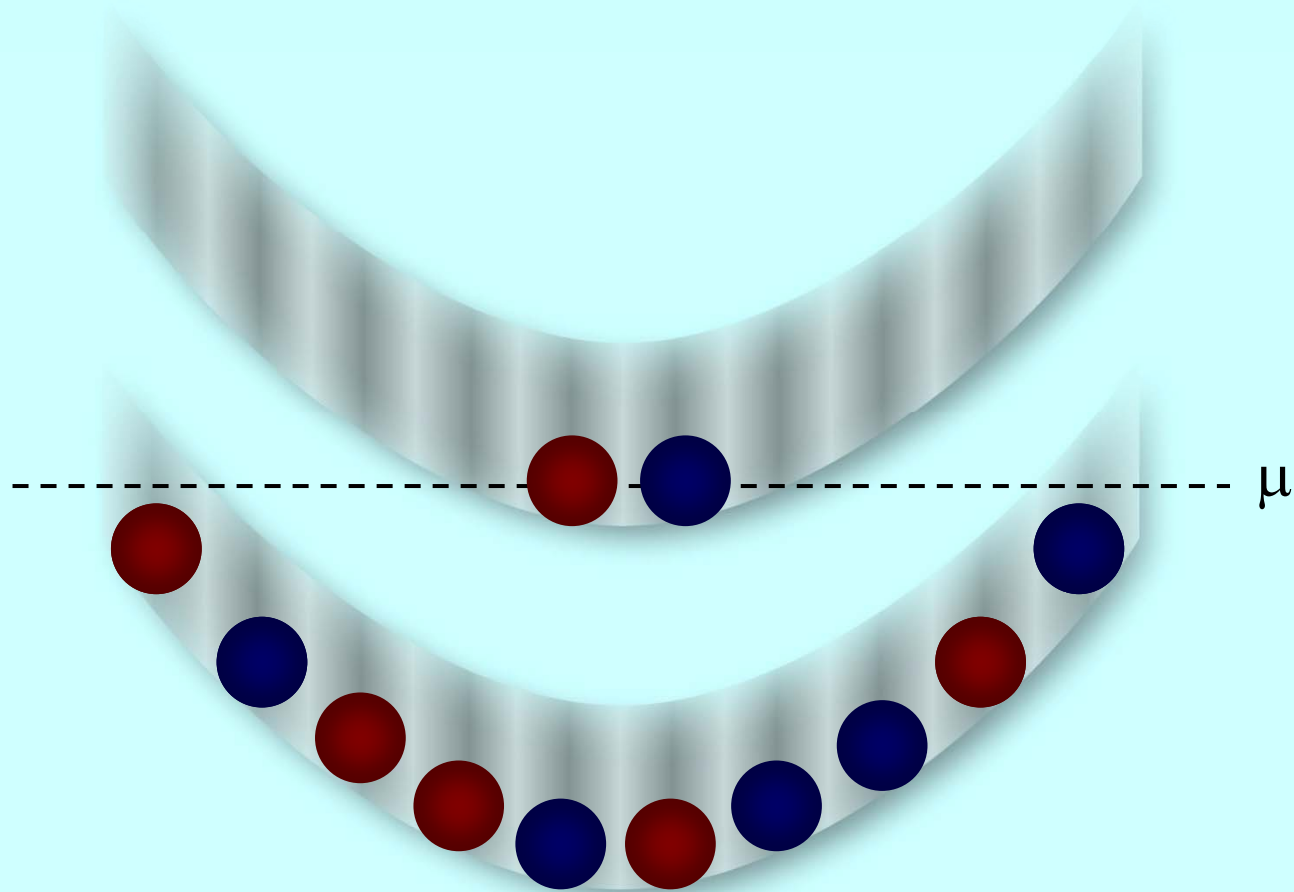
# Energy spectrum



# Energy spectrum

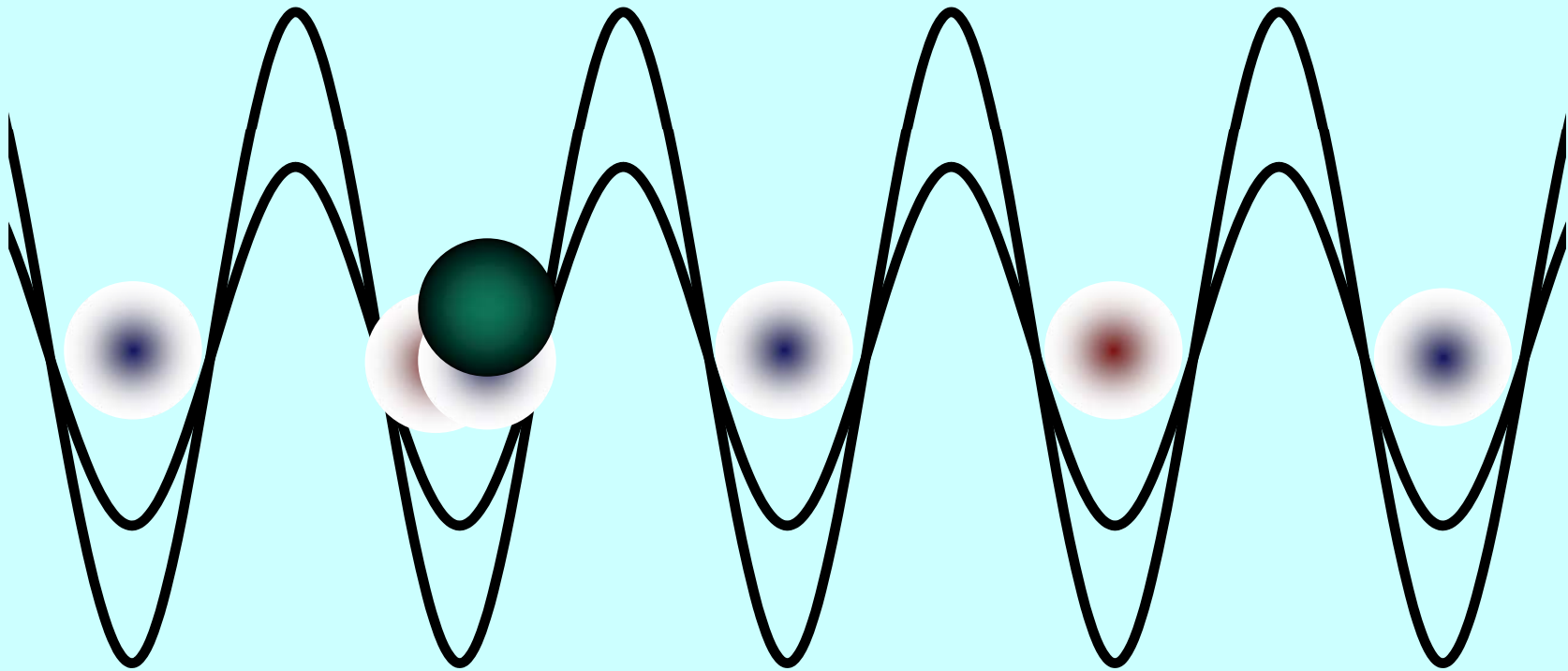


# Energy spectrum

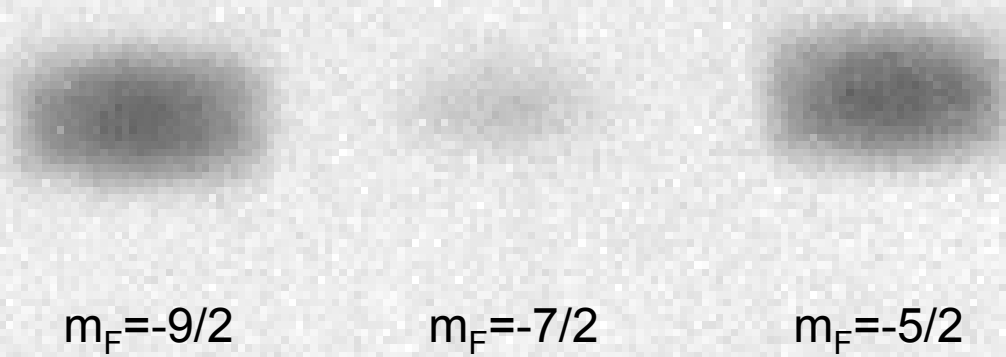


# Measuring Double Occupancy

1. suppress tunneling
2. Feshbach induced energy shift
3. Rf transition
4. release and Stern-Gerlach separation

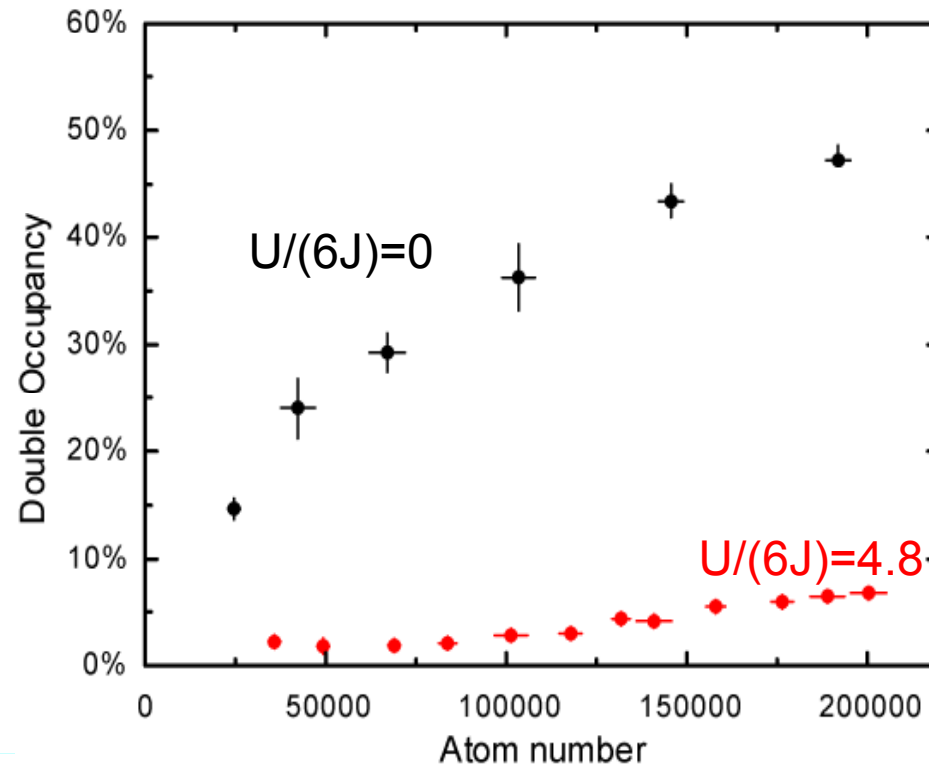


# Measuring Double Occupancy





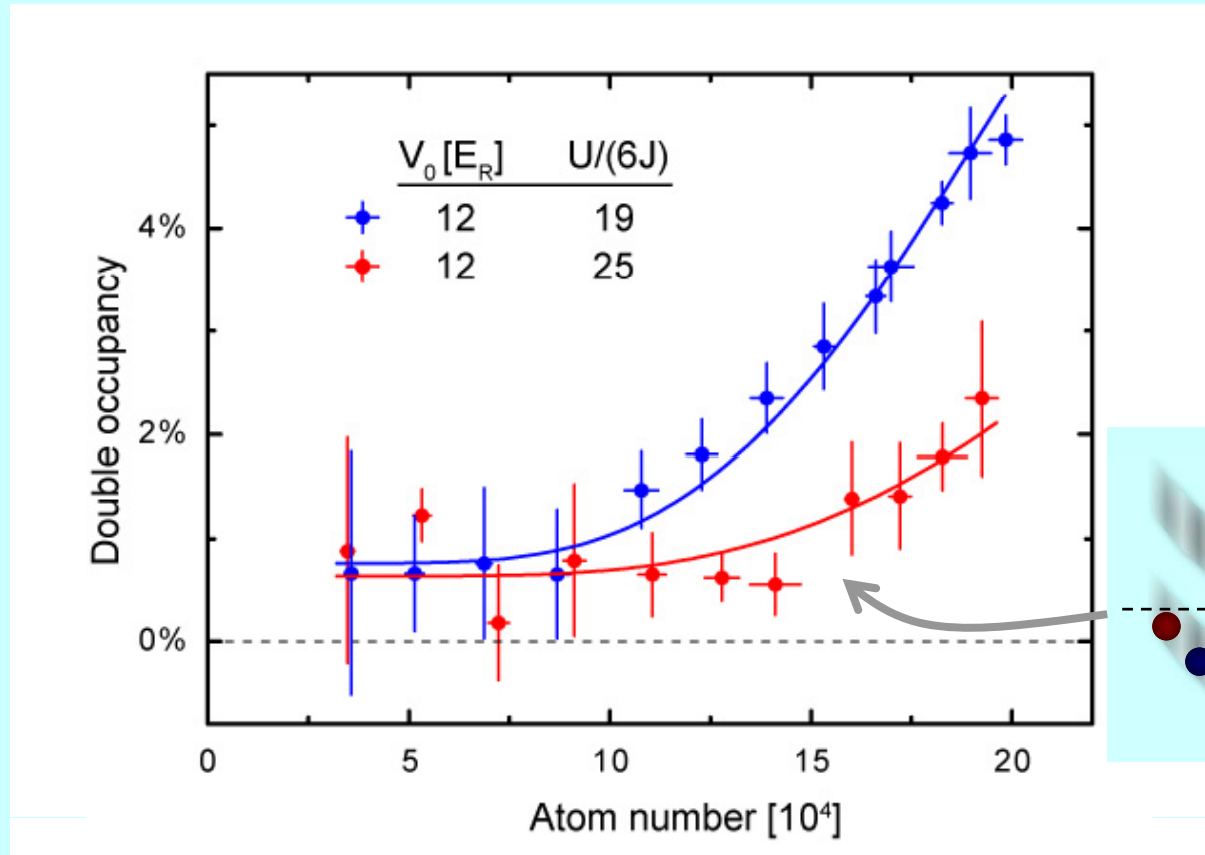
# Suppression of double occupancy



*R. Jördens, N. Strohmaier, K. Günter, H. Moritz, T. Esslinger, Nature 455, 204 (2008).*

See also: U. Schneider et al., Science 322, 1520 (2008).

# Occupation of upper Hubbard band



Fit:  $T=0.2 \pm 0.1 T_F$

*R. Jördens, N. Strohmaier, K. Günter, H. Moritz, T. Esslinger, Nature 455, 204 (2008).*

# Temperature

determine temperature in dipole trap:

$$T/T_F \sim 0.14, T_{\text{rev}}/T_F \sim 0.24 \Rightarrow \text{use } T/T_F \sim 0.195$$

constant entropy

→ 3 % vacancies in the center

$$T/U = 0.1$$

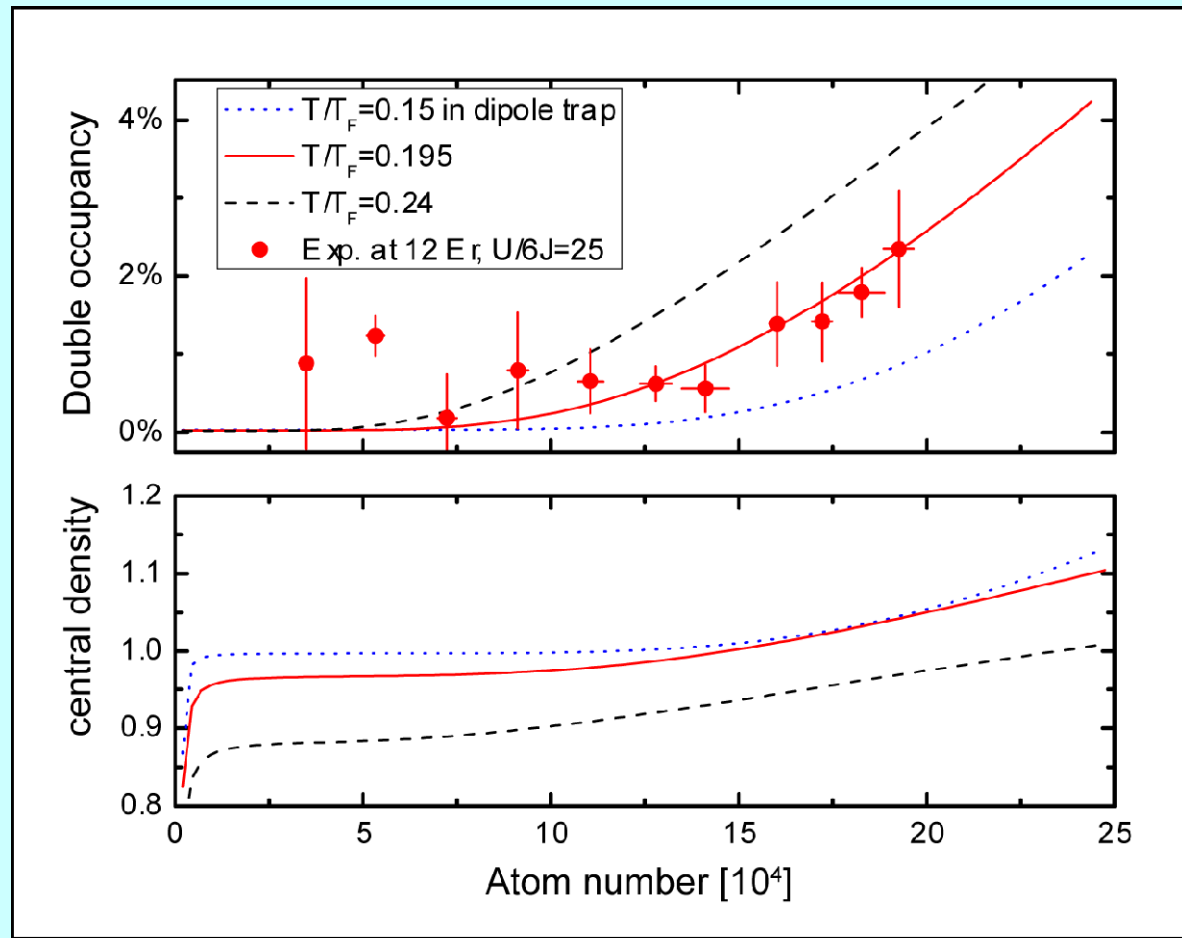
# Quantum Simulation

$$H = -J \sum_{\{i,j\},\sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + \sum_i \varepsilon_i \hat{n}_i$$

can be calculated for present experimental temperatures  
(DMFT and HTE)

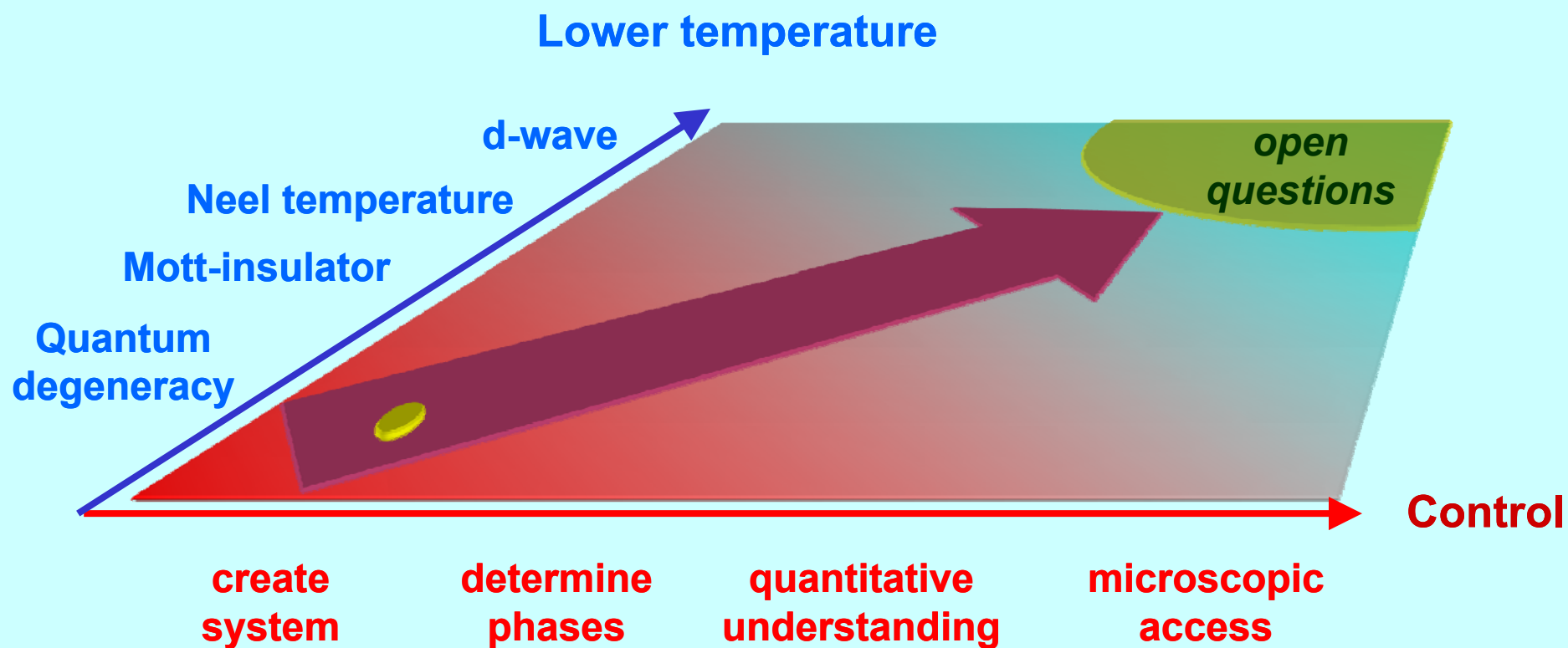
it should be possible to obtain very good agreement  
between theory and experiment

# Comparison with theory



Theory: 10th order series expansion as in  
V. Scarola, Pollet, Oitmaa, Troyer, cond-mat:0809.3239 (2008)

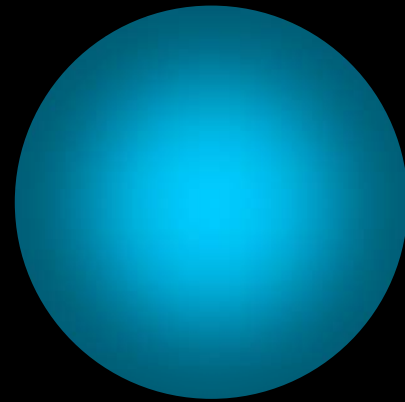
# Quantum Simulation



# Excitations

Spectrum

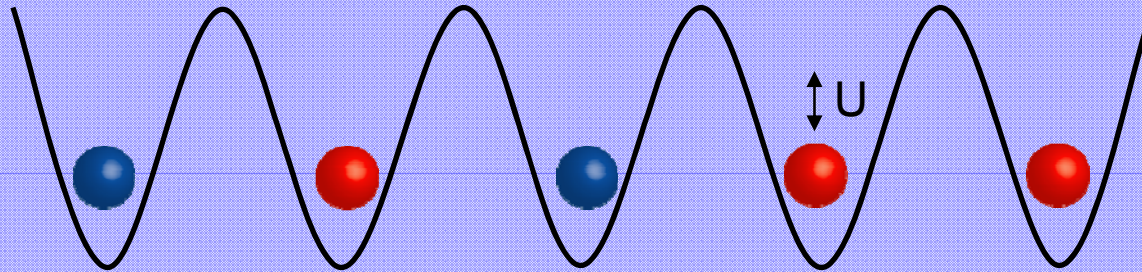
Relaxation



environment



# Gapped Mode

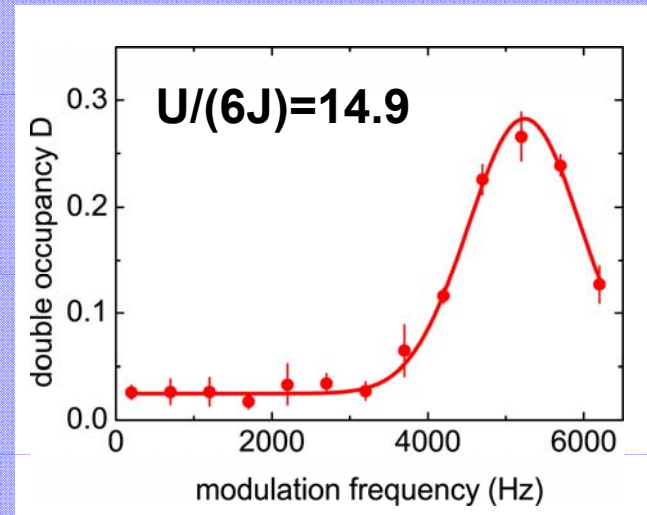
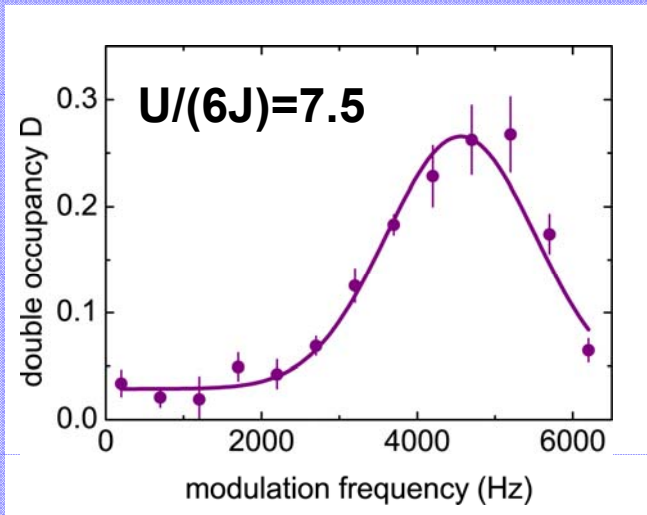
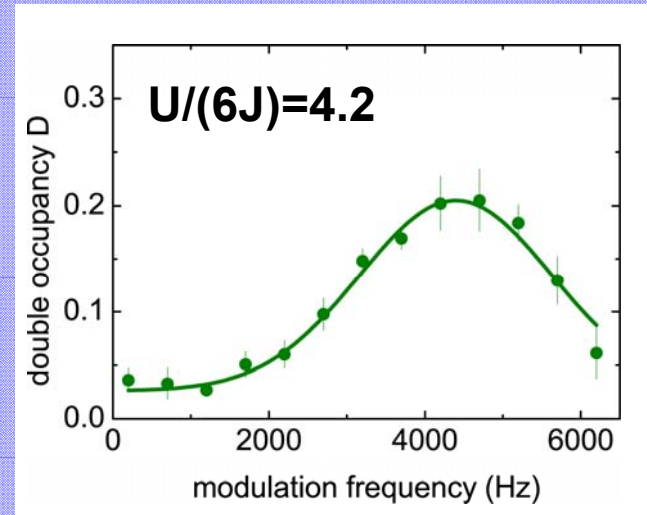
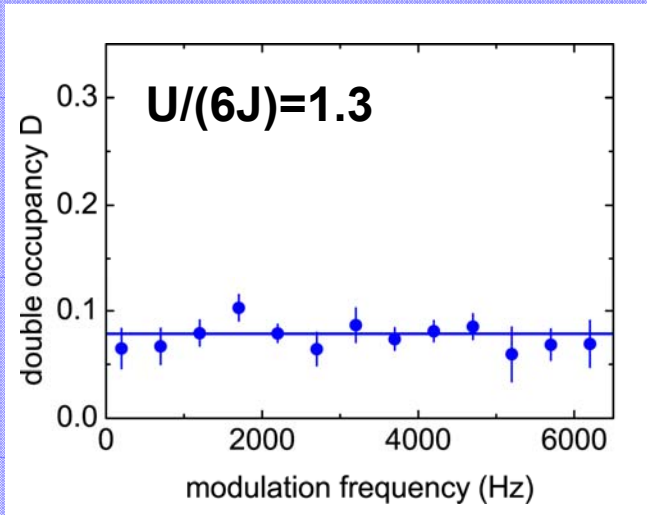


Modulation of the lattice amplitude with frequency  $U/h$ : Particle-hole excitation

*C. Kollath et.al, Phys.Rev.A., 74, 041604 (2006)*

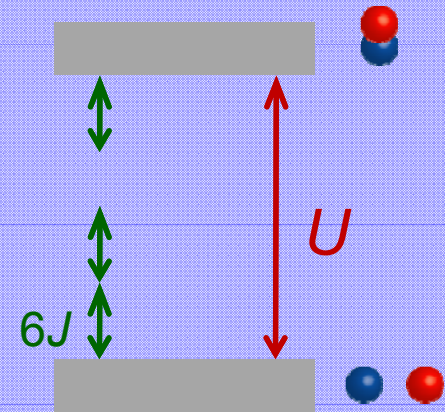
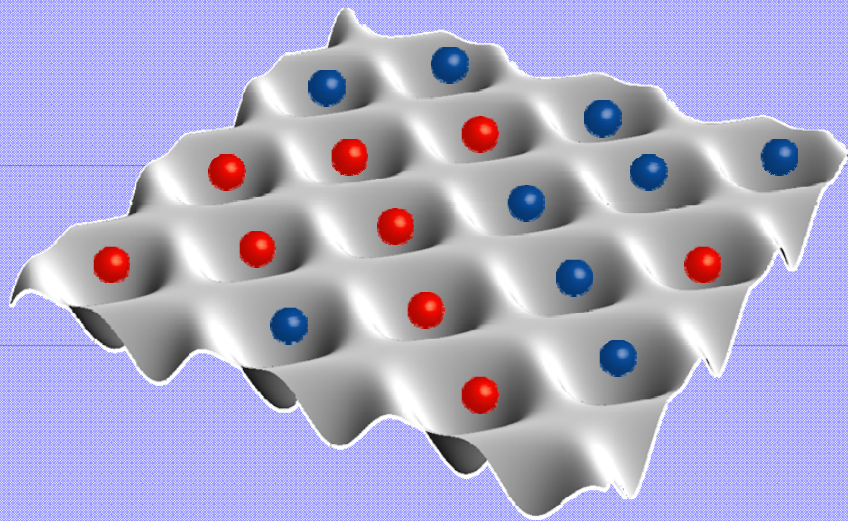
*T. Stöferle et.al., Phys.Rev.Lett., 92, 130403 (2004)*

# Gapped excitation mode



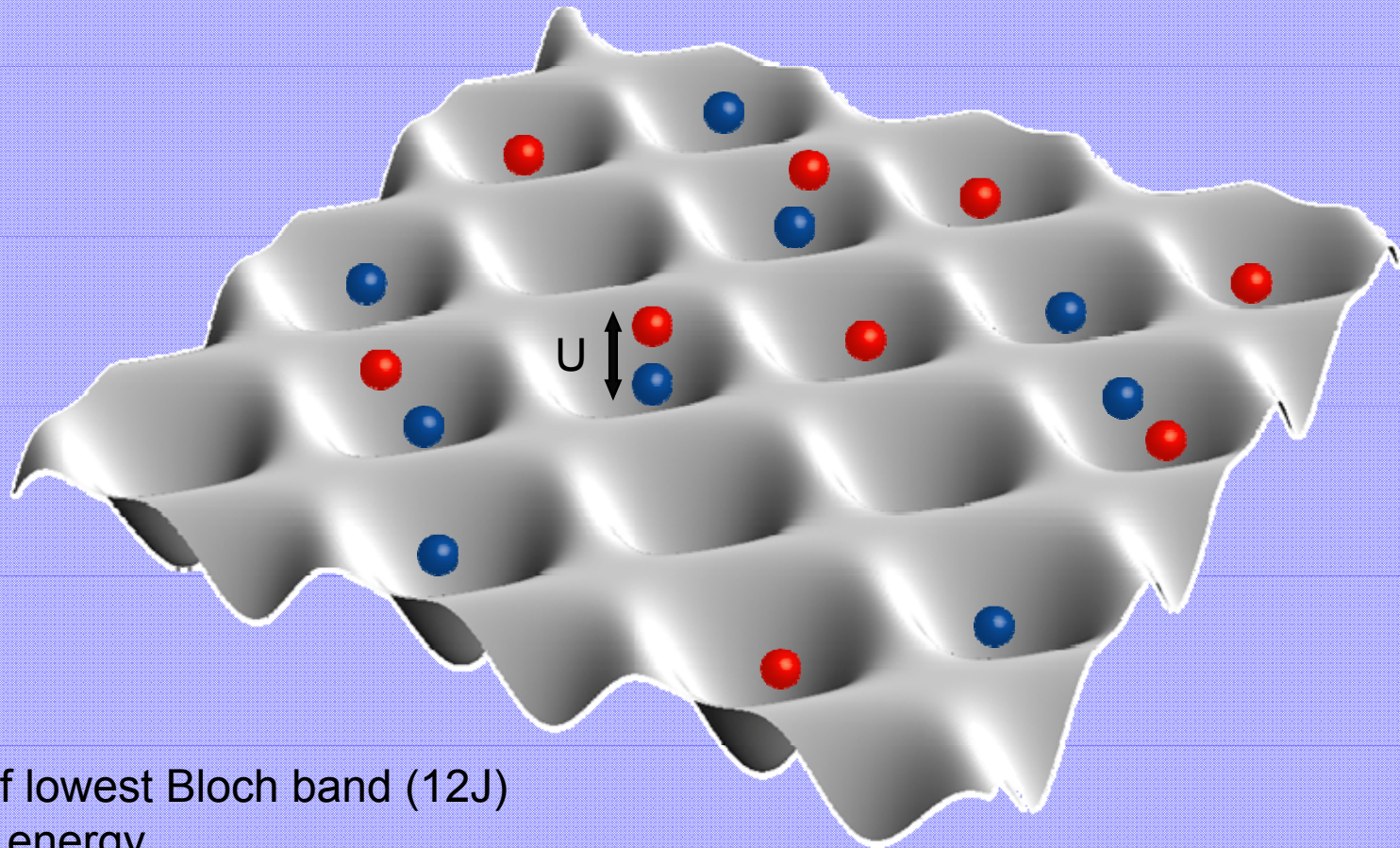
# Fermi-Hubbard

$$H = -t \sum_{\{i,j\},\sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$



(collaboration with Demler group, Harvard)

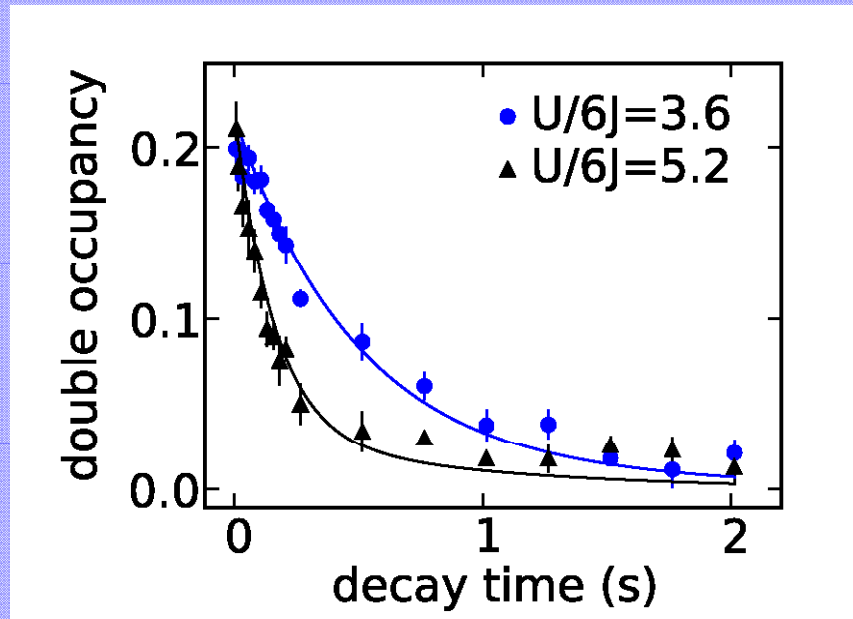
# Doublons



Bandwidth of lowest Bloch band (12J)  
limits kinetic energy  
> Repulsively bound pair

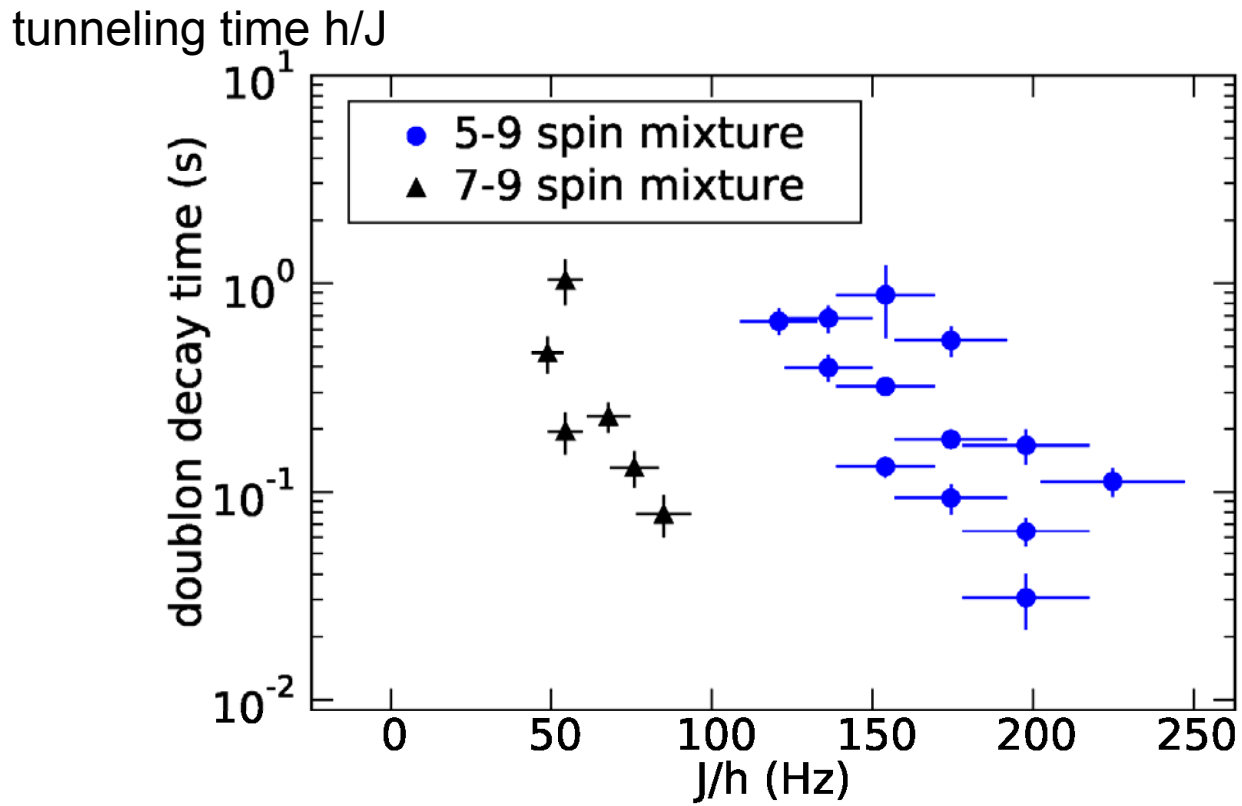
*Winkler et.al., Nature, 441 (2006)*

# Doublon decay

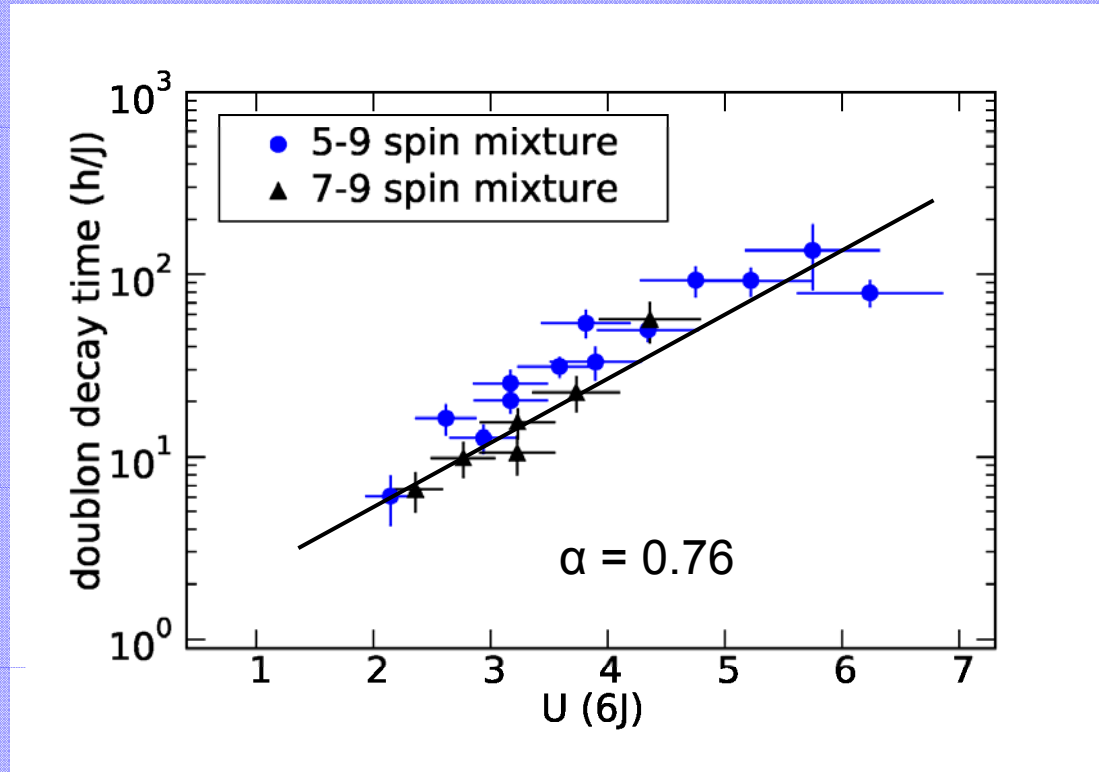


- decay of doublon into single particles
- lifetime depends on interaction and/or tunneling

# Parameter space



# Scaling



- Lifetime of doublon  $\tau/(h/J) \sim \exp(\alpha U/(6J))$
- multiple scattering
- Perturbation theory:  $\alpha$  of order one

Theory collaboration: Rajdeep Sensarma, David Pekker, Eugene Demler

# Novel Models?

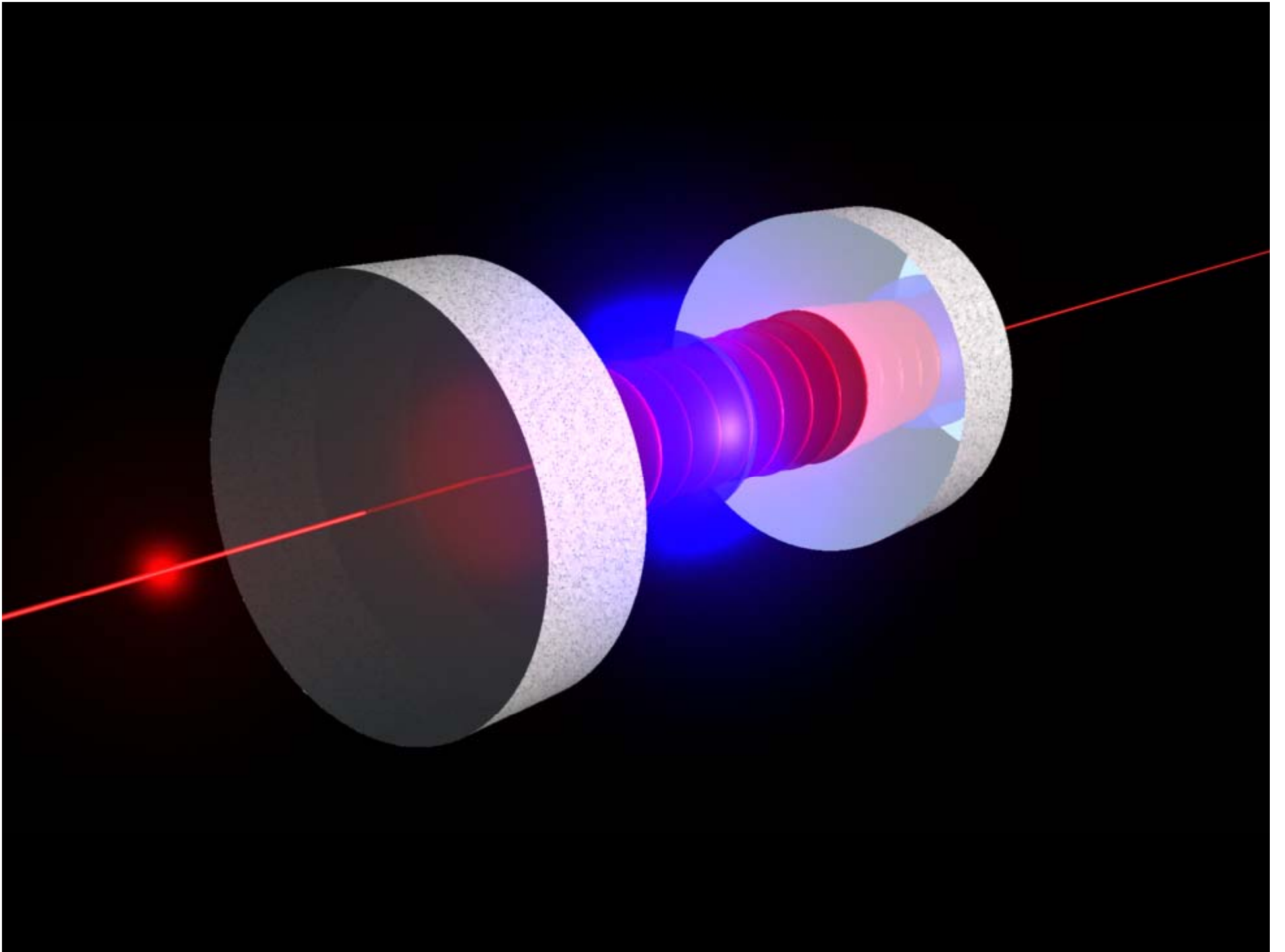
## long-range interactions

dipolar molecules

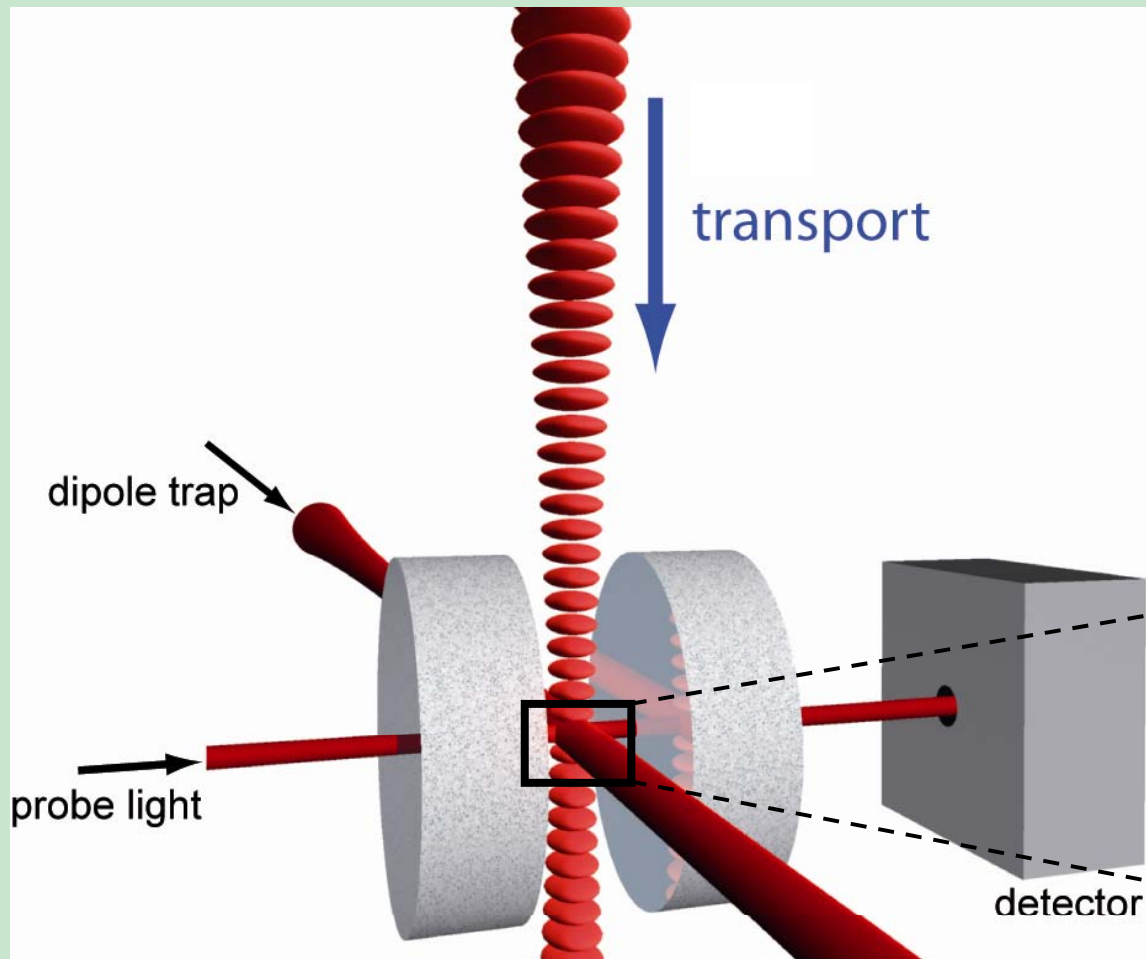
Rydberg atoms

cavity mediated interactions

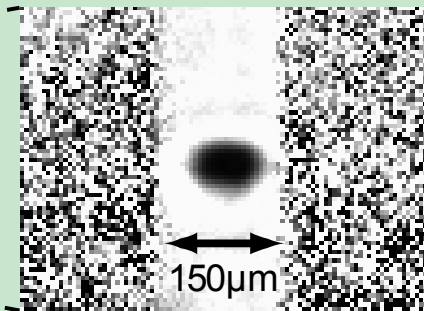




# Experimental setup



$$\begin{aligned}g_0 &= 2\pi \times 10.6 \text{ MHz} \\ \kappa &= 2\pi \times 1.3 \text{ MHz} \\ \gamma &= 2\pi \times 3.0 \text{ MHz} \\ \mathcal{F} &= 3 \times 10^5 \\ L &= 178 \mu\text{m}\end{aligned}$$



BEC after resonant probing and free expansion

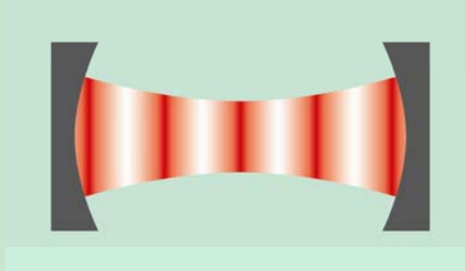
Related work: Stamper-Kurn (Berkeley), Reichel (ENS)

Single atoms, strong coupling: Kimble (Caltech), Rempe (MPQ)

Large volume cavities: Zimmermann (Tübingen), Hemmerich (Hamburg), Vuletic (MIT)

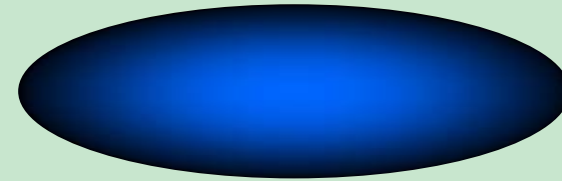
# Cavity QED meets BEC

Cavity QED



Single mode of light field

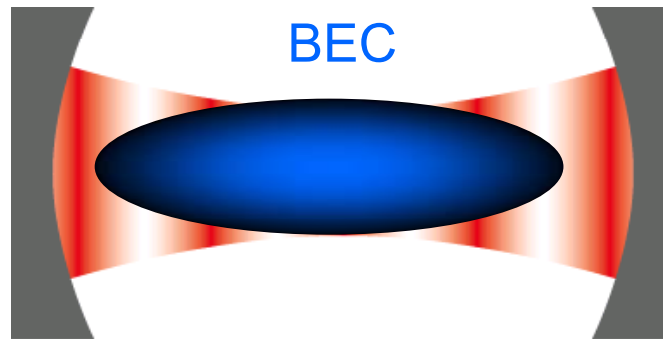
BEC



Single mode of matter field

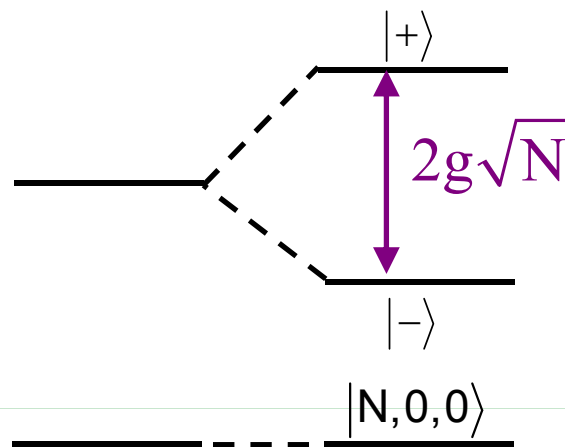
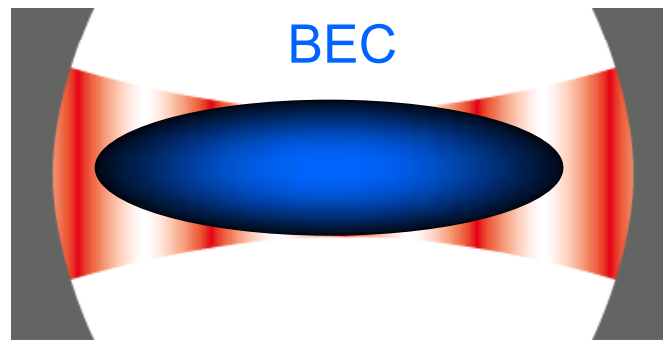
Theory, see e.g. Meystre, Ritsch, Lewenstein

# A BEC in a high-finesse cavity



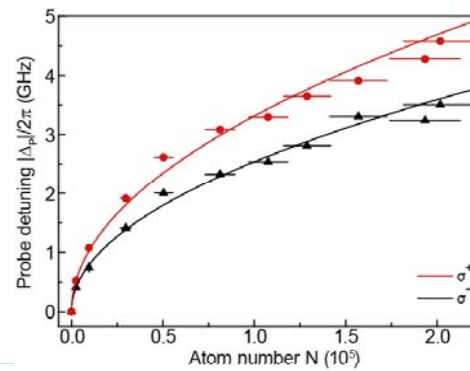
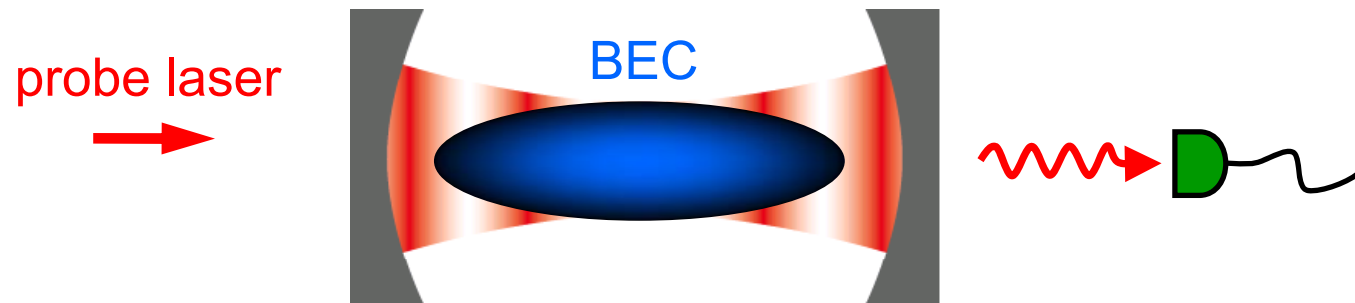
N atoms in delocalized wavefunction of BEC  
coupling to the cavity

# A BEC in a high-finesse cavity

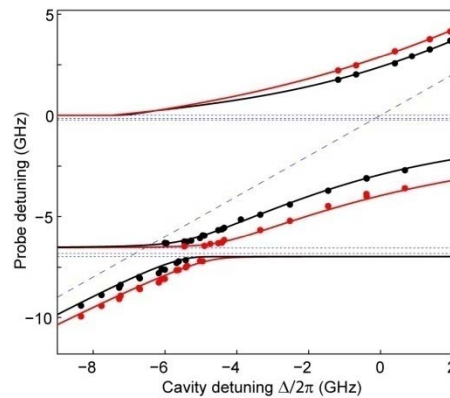
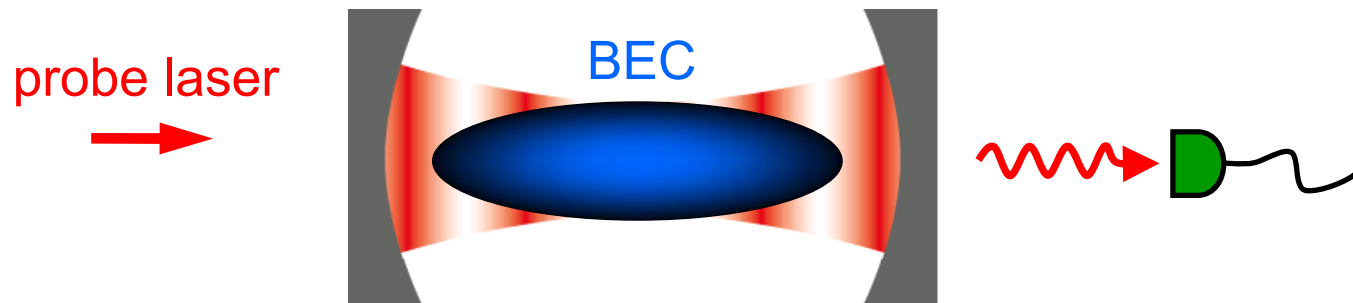


Tavis-Cummings Hamiltonian

# A BEC in a high-finesse cavity

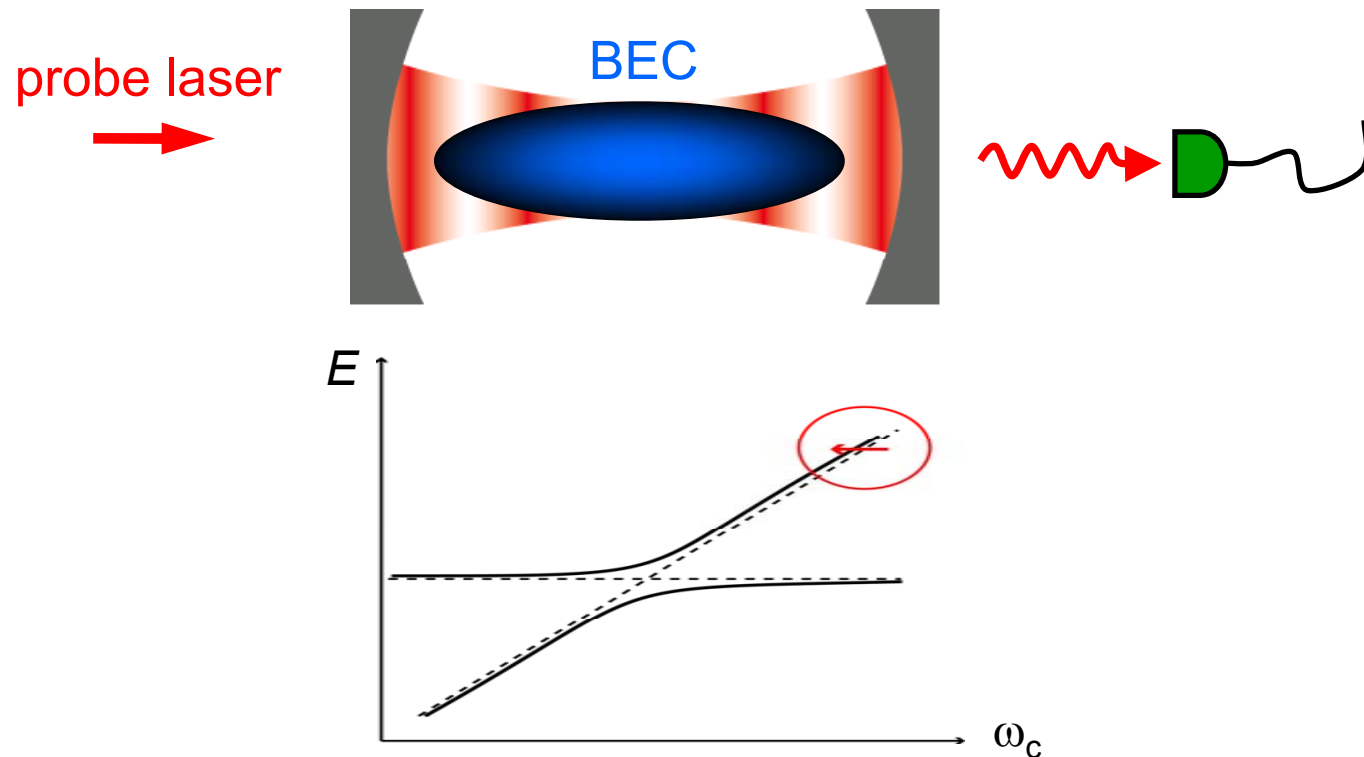


# A BEC in a high-finesse cavity



collective cooperativity:  $Ng^2/(2\kappa\gamma)=1.6 \times 10^6$

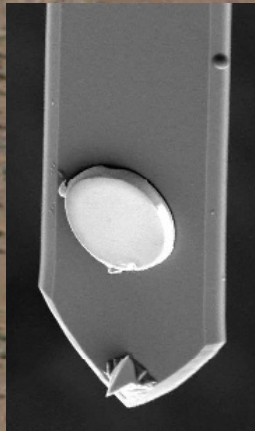
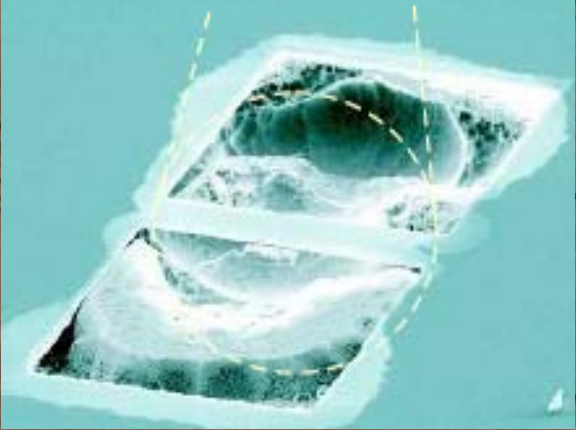
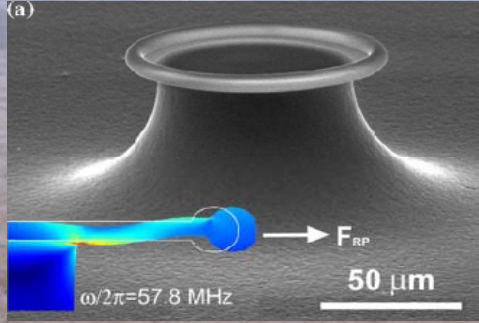
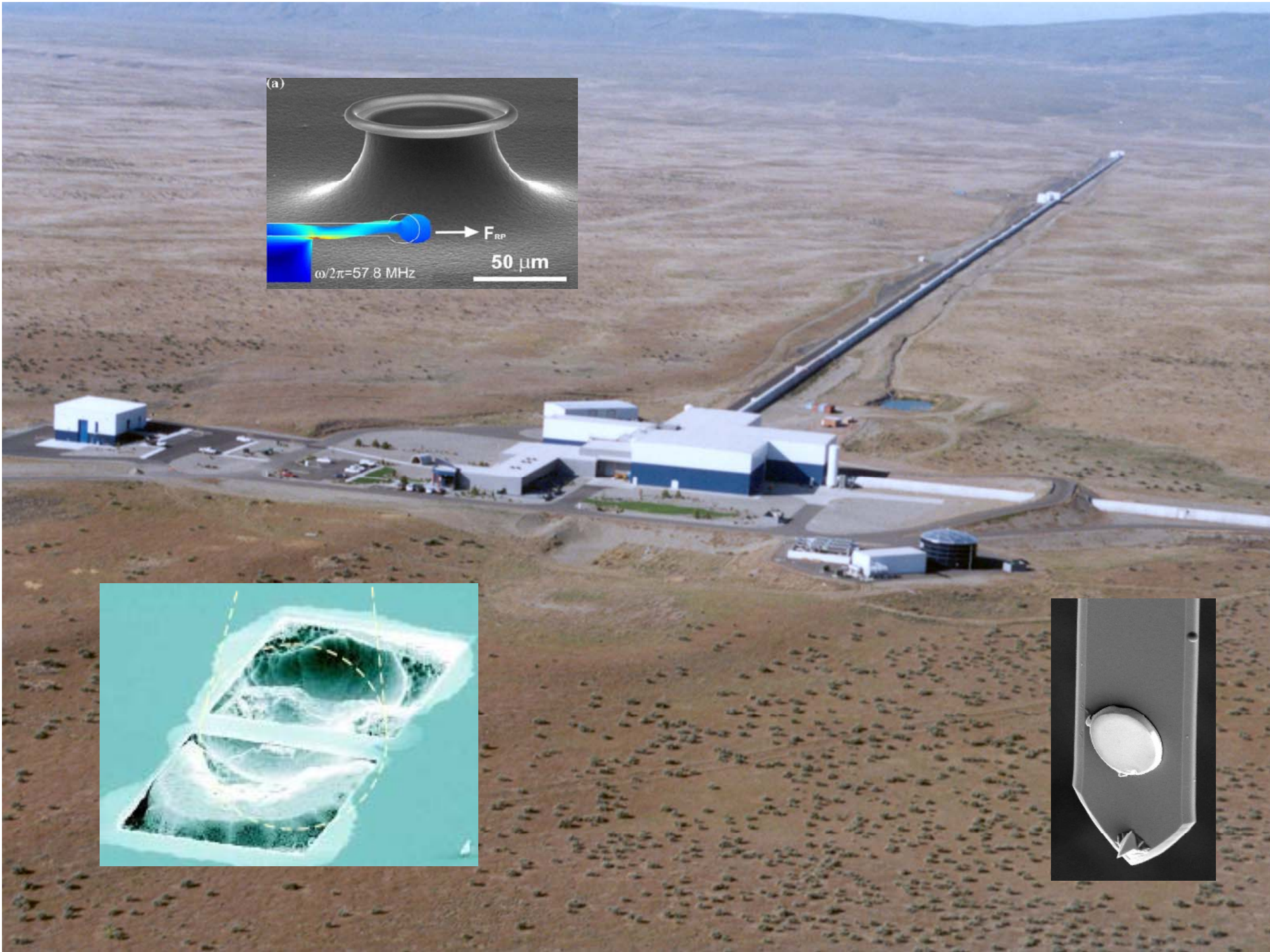
# Dispersive regime



The bare cavity resonance frequency is refractively shifted by BEC

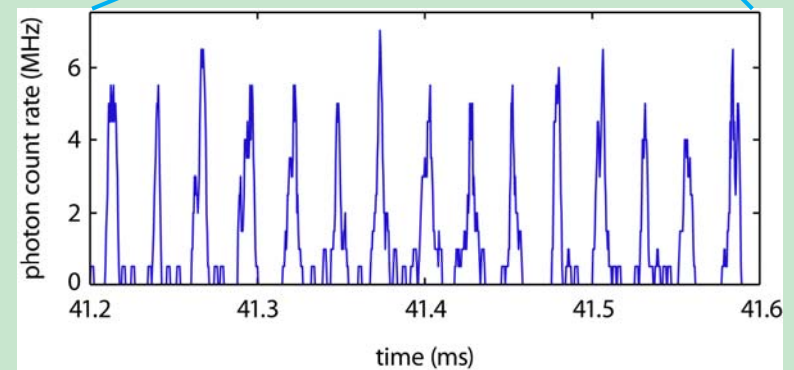
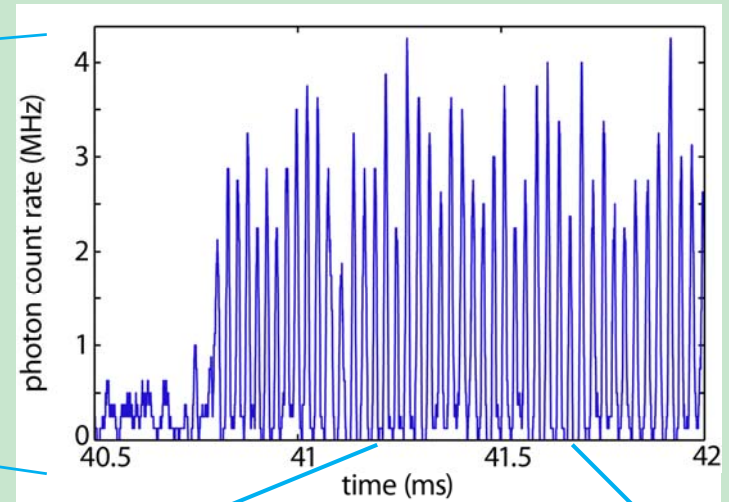
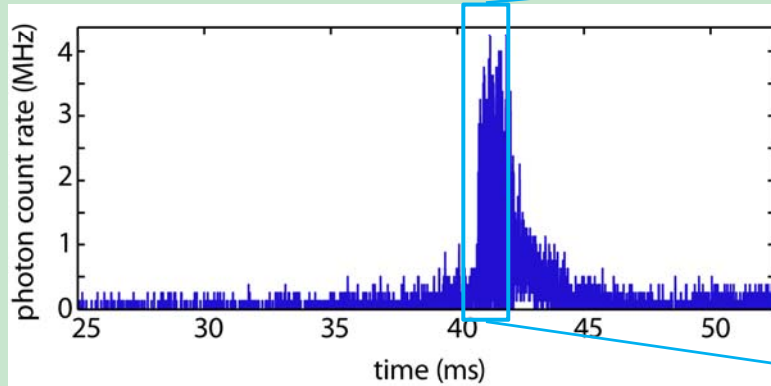
See also: D. Stamper-Kurn group: PRL 99, 213601 (2007), Nat. Phys. 4, 561 (2008)





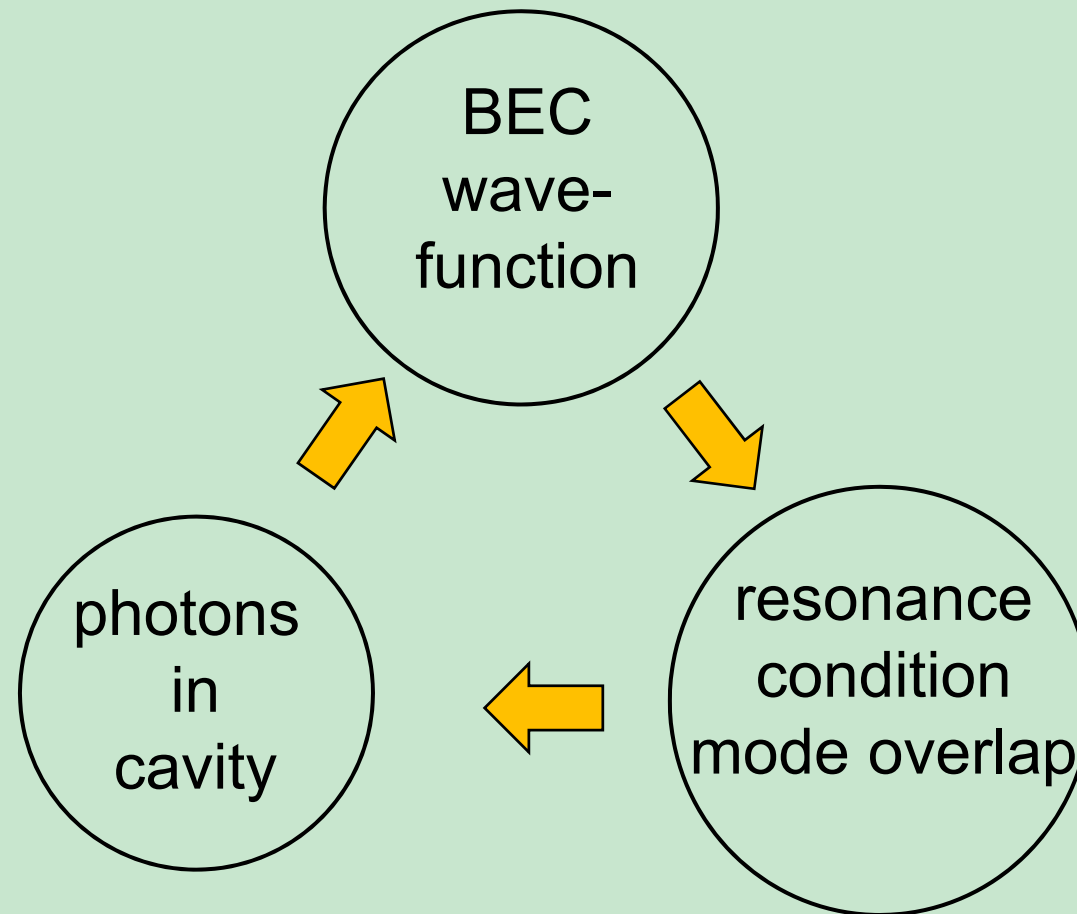
# BEC Cavity Dynamics

BEC



**35 kHz – 25 kHz**

# BEC Cavity Dynamics



# Equations of Motion

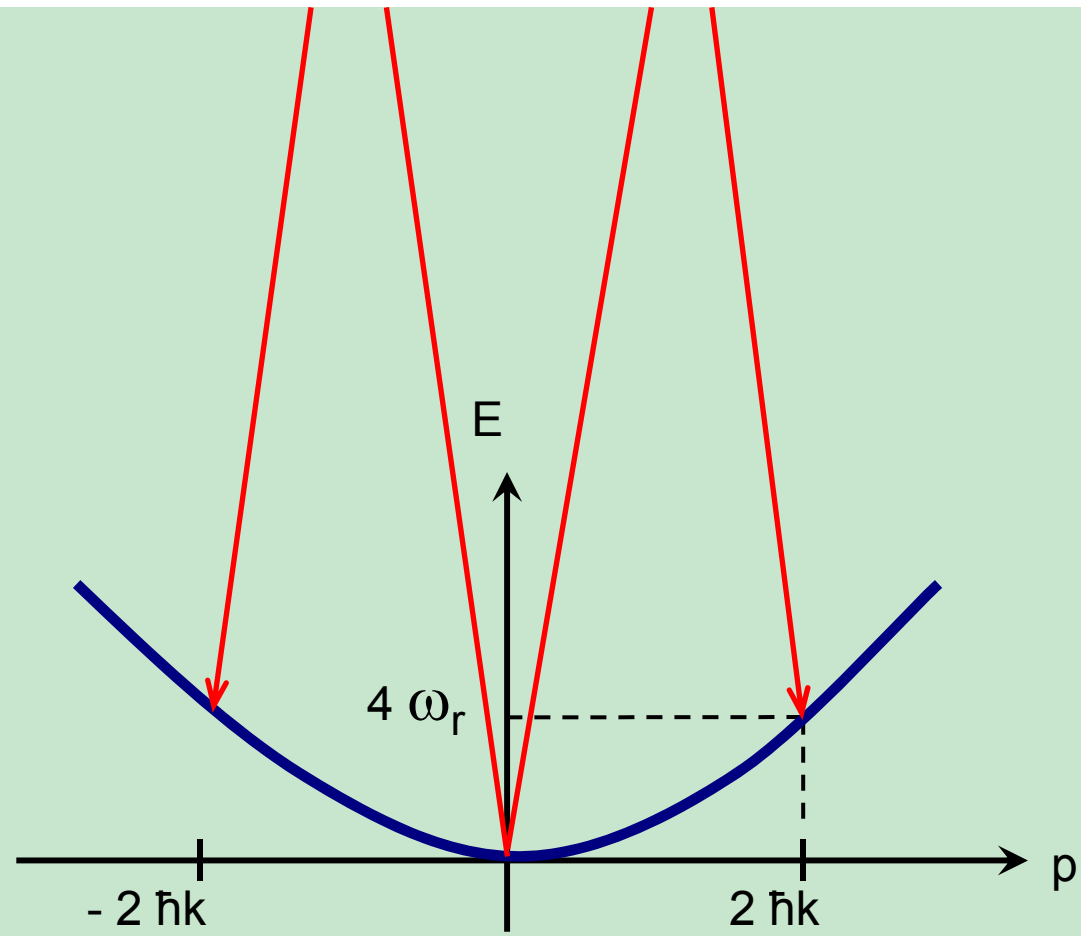
Mean field description

$$i\dot{\psi}(x, t) = \left( \frac{-\hbar}{2m} \frac{d^2}{dx^2} + |\alpha(t)|^2 U_0 \cos^2(kx) \right) \psi(x, t)$$
$$\alpha(t) = \frac{\eta}{\kappa - i(\Delta_c - NU_0 \langle \cos^2(kx) \rangle)}$$

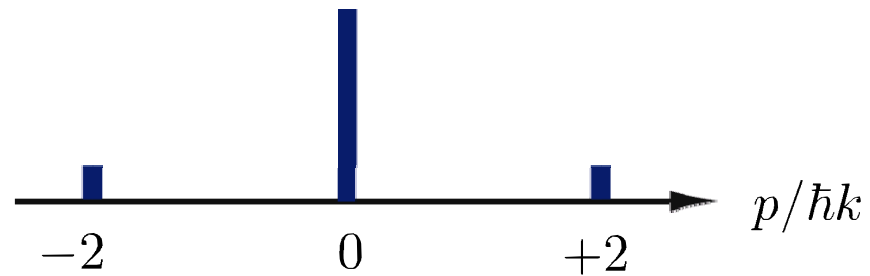
non-local and non-linear coupling – overlap integral

$$\langle \cos^2(kx) \rangle = \int |\psi(x, t)|^2 \cos^2(kx) dx$$

$$|\alpha^2| \approx 5$$



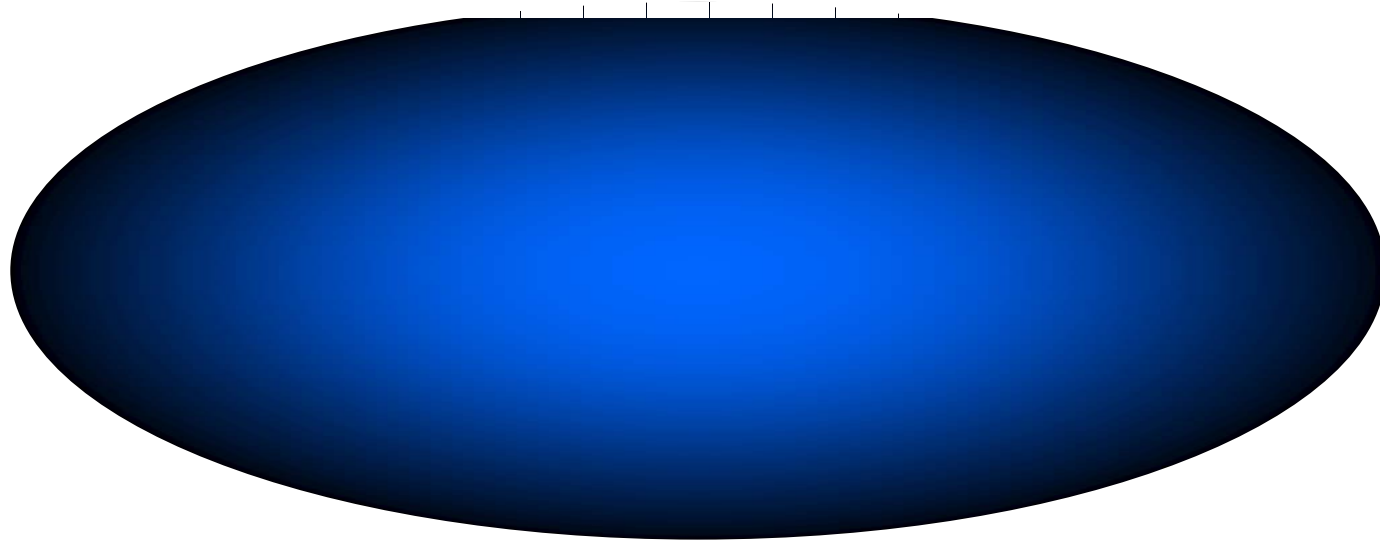
# BEC as a mechanical oscillator



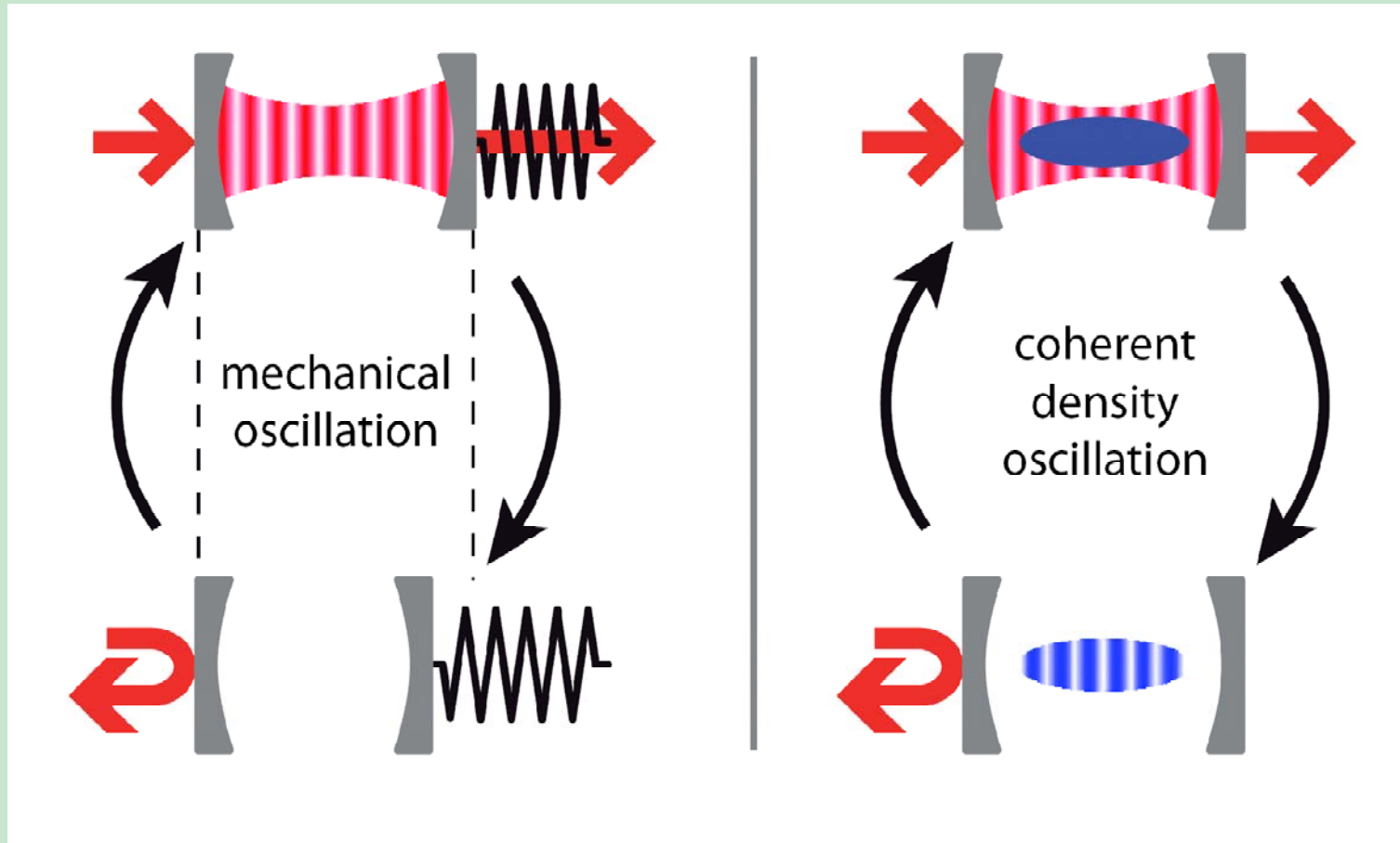
$$\Psi(x, t) = c_0(t) |p = 0\rangle + c_2(t) |p = \pm 2\hbar k\rangle$$

$$|\Psi(x, t)|^2 \approx N + c_2 \sqrt{N} \cos(2kx) \cos(4\omega_r t)$$

# BEC as a mechanical oscillator



# Cavity opto-mechanics with a BEC



F. Brennecke, S. Ritter, T. Donner, T. Esslinger, Science 322, 235 (2008)

See also: Stamper-Kurn (Berkeley), Nature Physics 2008



# BEC mechanically coupled to cavity mode

$$H = 4\hbar\omega_{rec} \hat{c}^\dagger \hat{c} - \Delta\hbar \hat{a}^\dagger \hat{a} + \hbar g (\hat{c} + \hat{c}^\dagger) \hat{a}^\dagger \hat{a} - i\hbar\eta (\hat{a} - \hat{a}^\dagger)$$

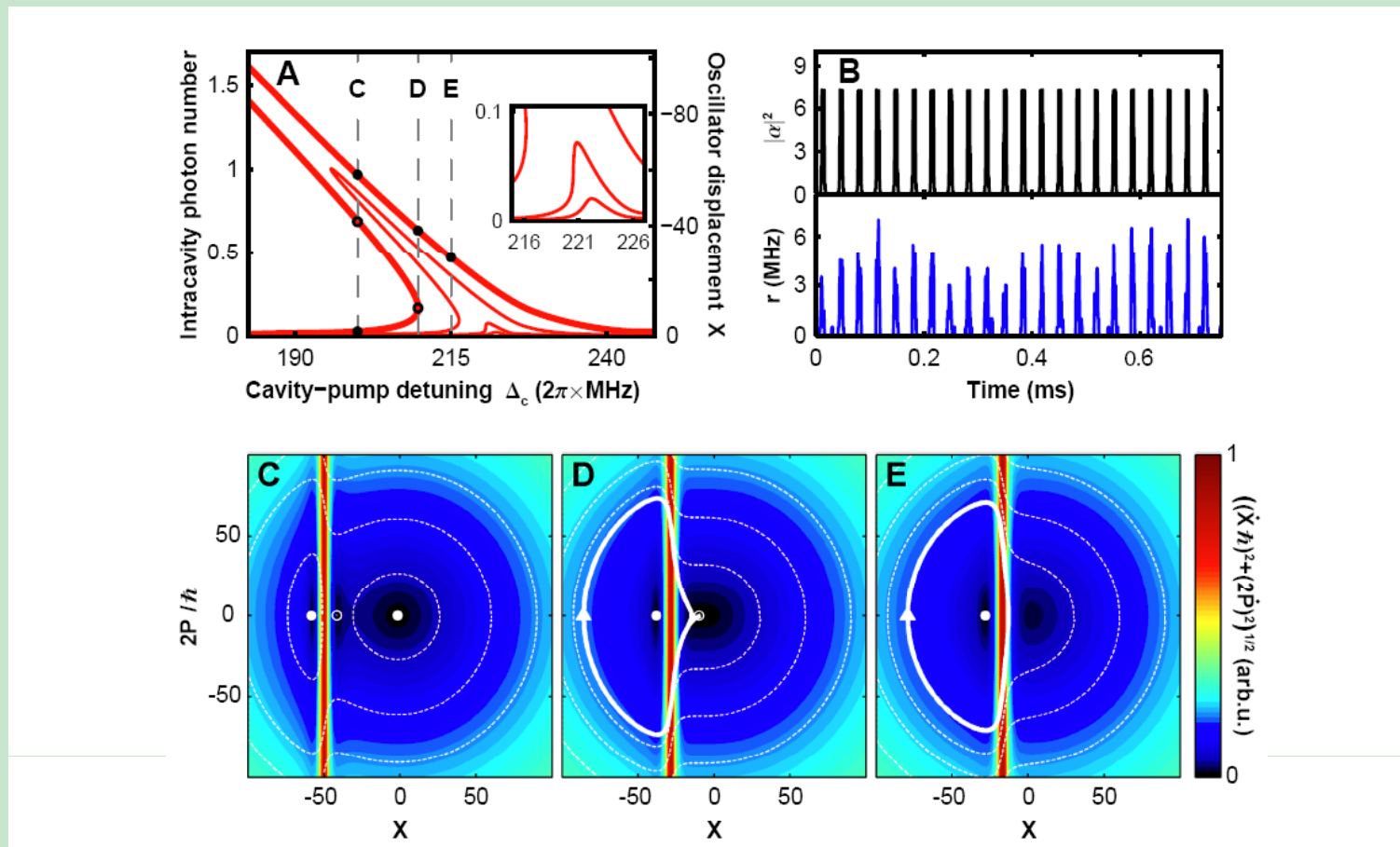
mechanical  
mode

optical  
mode

Strong coupling:  
 $g \approx 0.3 \kappa$

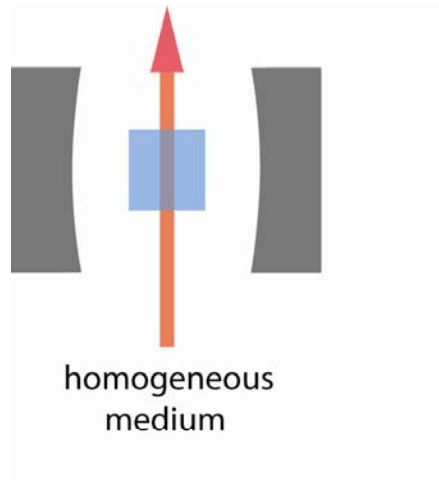
pump

# Dynamics of of BEC cavity system



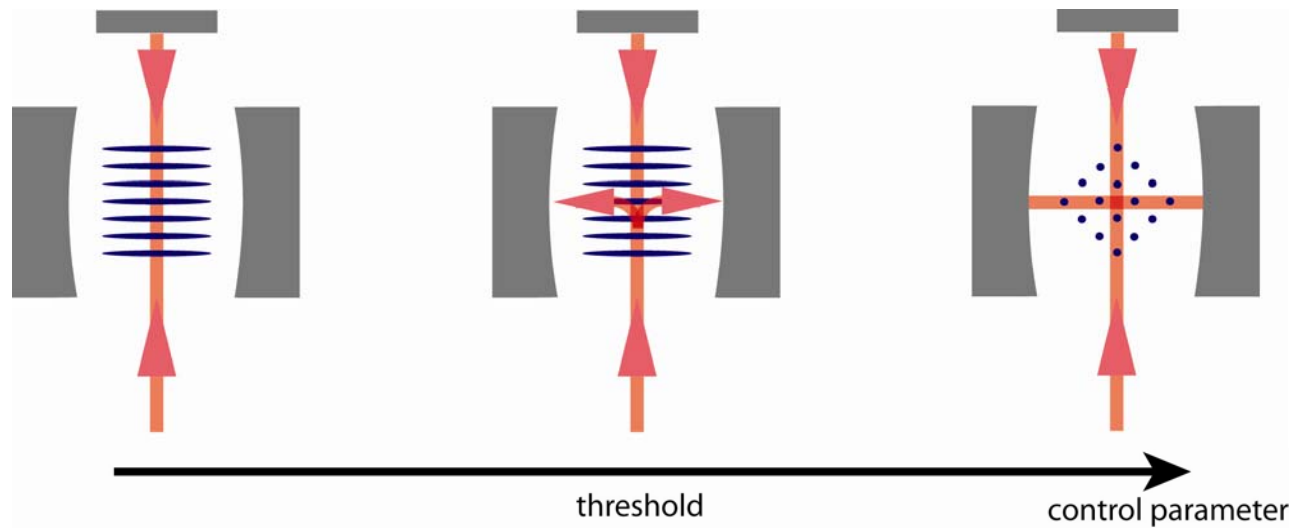
F. Brennecke, S. Ritter, T. Donner, T. Esslinger, Science 322, 235 (2008)

# Cavity Detection



See: H. Ritsch, P. Domokos

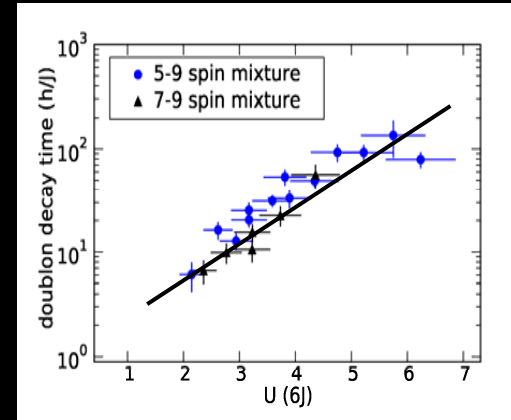
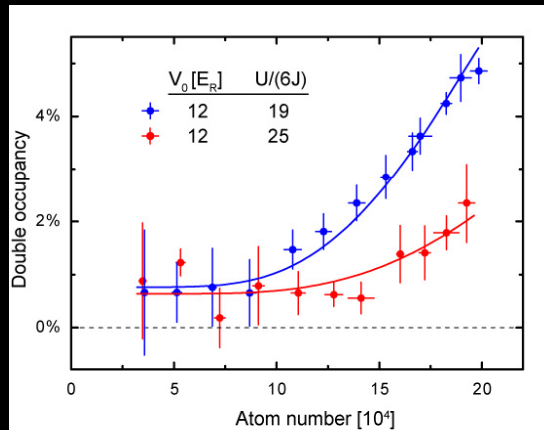
# Cavity Controlled Quantum Gas



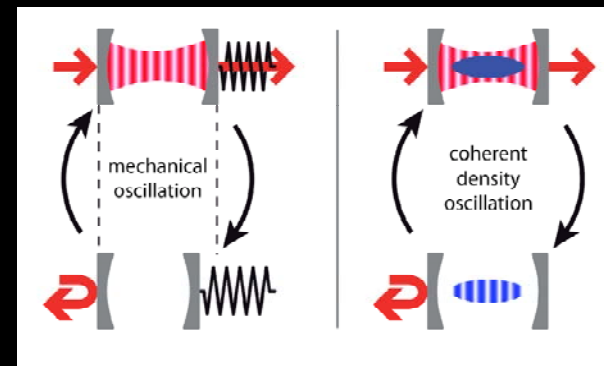
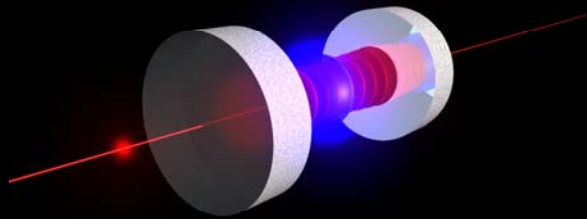
See: H. Ritsch, P. Domokos, M. Lewenstein, V. Vuletic

# Conclusions and Outlook

## Fermi-Hubbard



## Cavity opto-mechanics with a BEC



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Quantum Gases in  
Optical Lattices

Lithium:

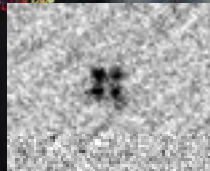
BEC and Cavity

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