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IAEA International Atomic Energy Agency

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Conference on Research Frontiers in Ultra-Cold Atoms

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Relaxation of doublons in repulsive Hubbard model

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Dynamics of repulsively bound pairs in fermionic Hubbard model

David Pekker, Harvard University
Rajdeep Sensarma, Harvard University
Ehud Altman, Weizmann Institute
Eugene Demler , Harvard University

Collaboration with ETH (Zurich) quantum optics group
N. Strohmaier, D. Greif, L. Tarruell,
H. Moritz, T. Esslinger

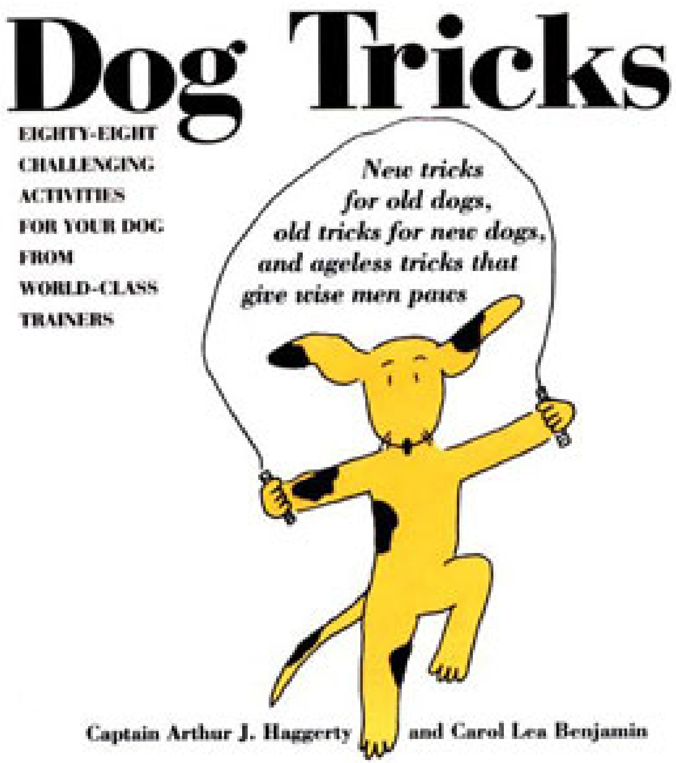
$$ NSF, AFOSR, MURI, DARPA,

Condensed Matter models for many-body systems of ultracold atoms

Old models, new physics

Using cold atoms to simulate condensed matter models.
Old Tricks for New Dogs

Using cold atoms to ask new questions about known models
New Tricks for Old Dogs

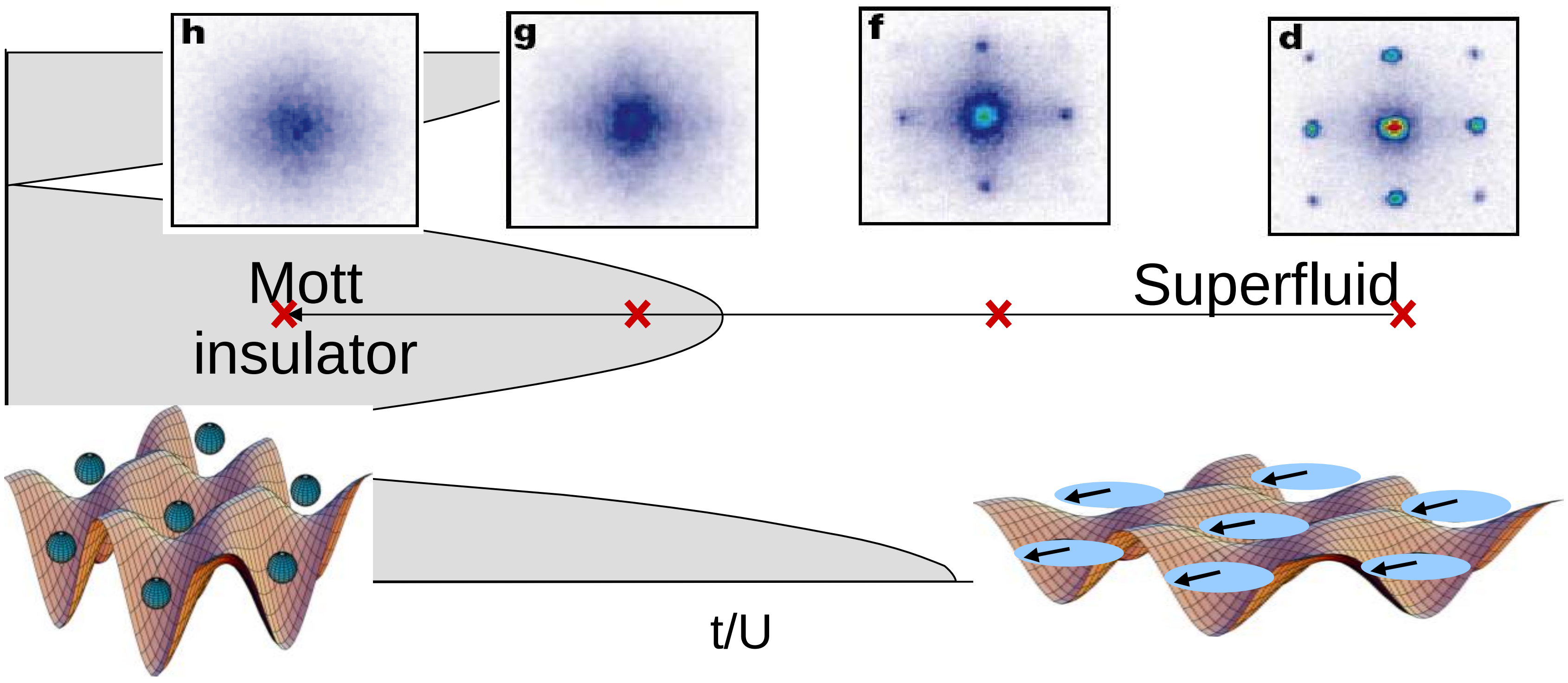


Bose Hubbard model

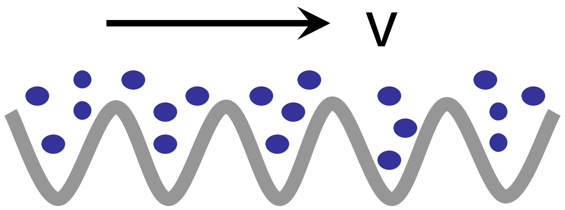
$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$

Superfluid to insulator transition in an optical lattice

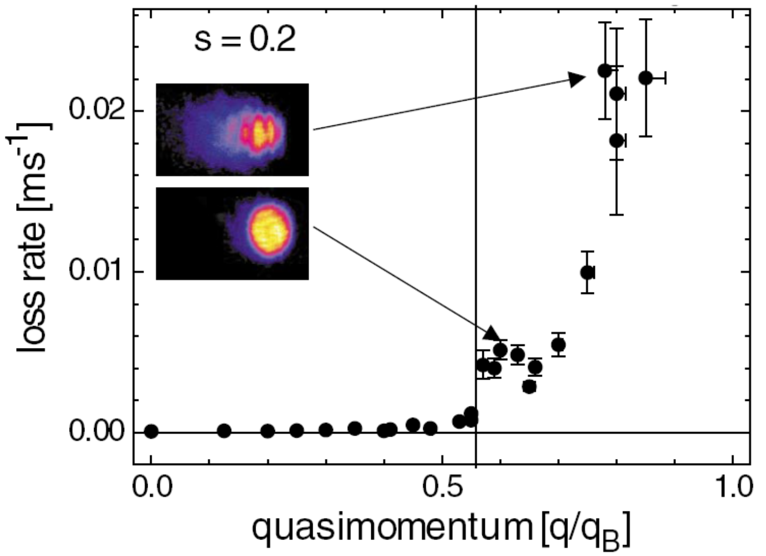
M. Greiner et al., Nature 415 (2002)



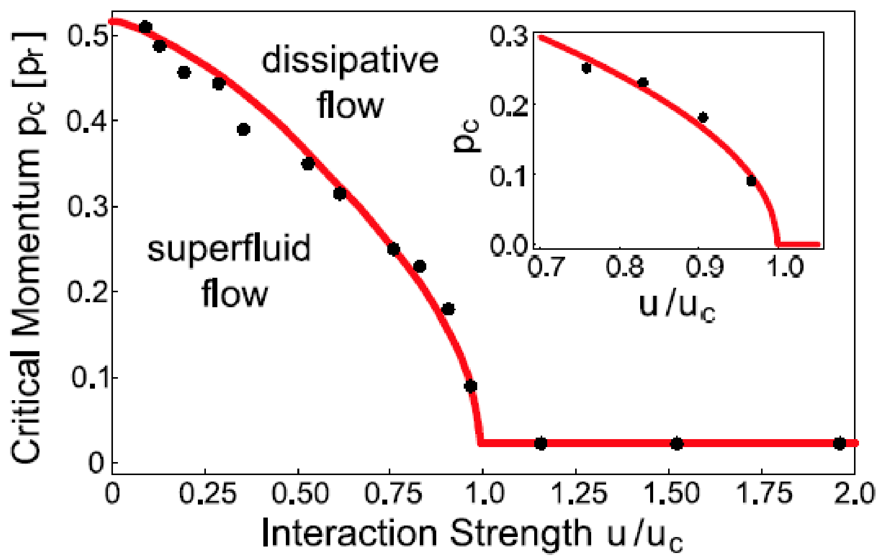
Instability of a moving condensate in an optical lattice



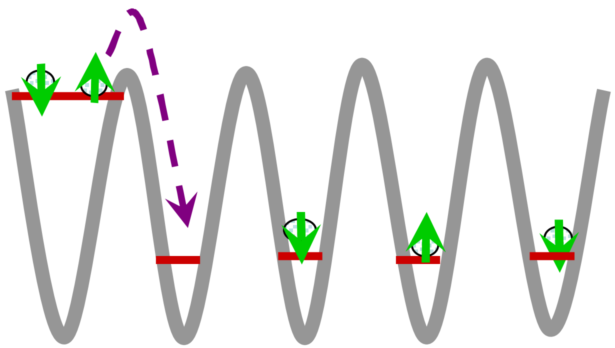
Dynamical instability for weak interactions, Fallani et al., PRL 04



Dynamical instability for strong interactions, Mun et al., PRL 07



Fermions in optical lattice.
Decay of repulsively bound pairs



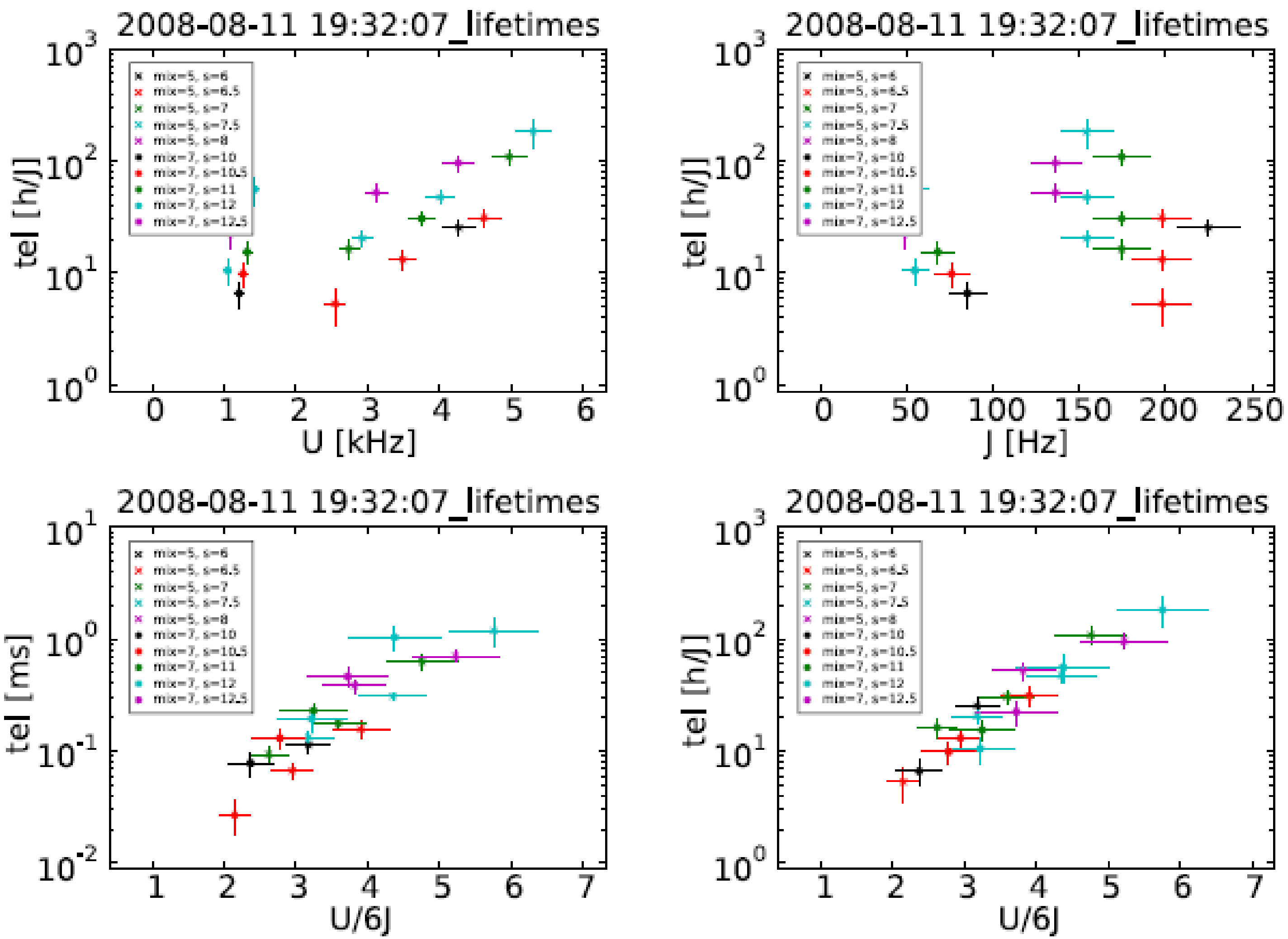
Experiment: ETH Zurich, Strohmaier et al.,

Outline of this talk

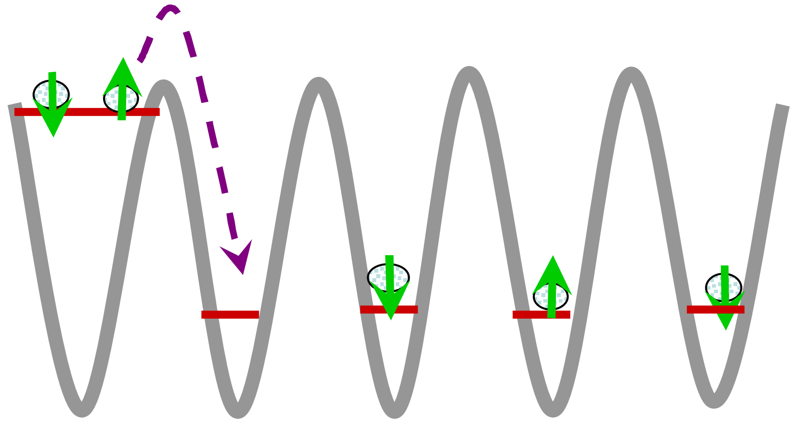
- Introduction
- Doublon decay in Mott state
- Doublon decay in compressible states
- General perspective

Fermions in optical lattice.
Decay of repulsively bound pairs

Experiments: N. Strohmaier et. al.



Relaxation of repulsively bound pairs in the Fermionic Hubbard model



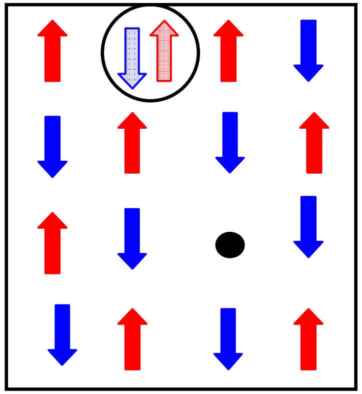
U >> t

For a repulsive bound pair to decay,
energy U needs to be absorbed
by other degrees of freedom in the system

Relaxation timescale is determined by
many-body dynamics of strongly correlated
system of interacting fermions

Doublon relaxation in the Mott state

Relaxation of doublon- hole pairs in the Mott state



Energy U needs to be absorbed by spin excitations

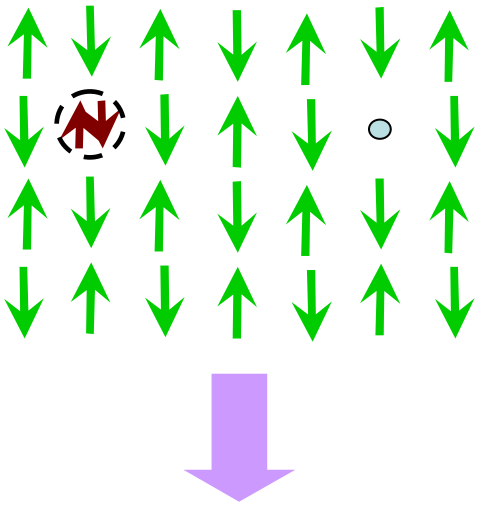
❖Energy carried by spin excitations $\sim J = 4t^2/U$

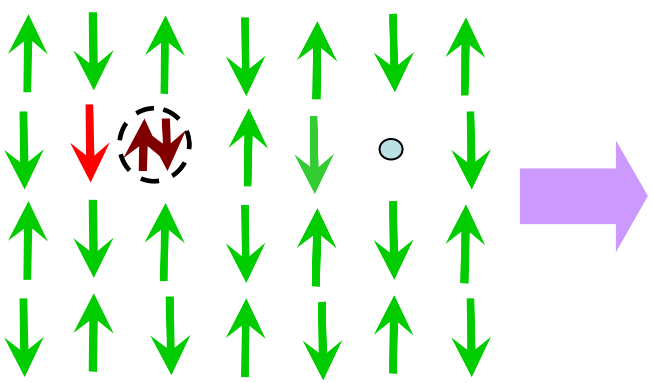
❖ Relaxation requires creation of $\sim U^2/t^2$ spin excitations

Need to create many spin excitations to absorb initial energy of doublon

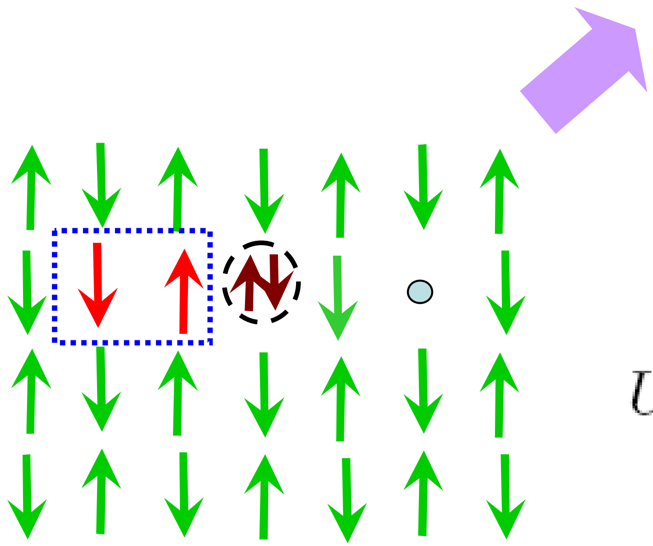
Relaxation of doublon-hole pairs in the Mott state

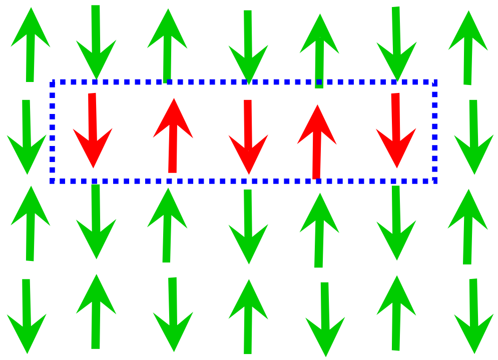
Doublon propogation creates a string of flipped spins





Total energy of flipped spins should match U



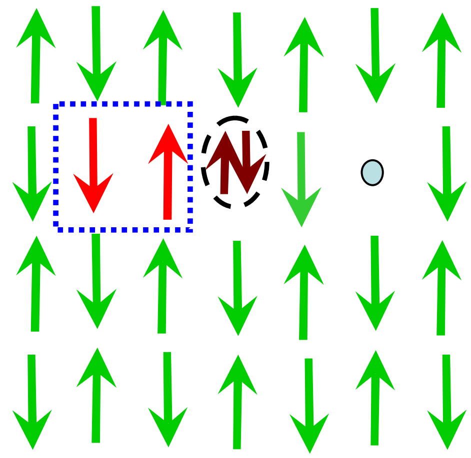


Number of flipped spins

$$U \approx (z-2)\frac{J_{\mathrm{ex}}}{2} N$$

$$J_{\mathrm{ex}} = 4t^2/U$$

Relaxation of doublon-hole pairs in the Mott state



High order perturbation theory in $V = t$

$$V_{\text{eff}} \sim V \prod_{n=1}^{N} \frac{V}{(E_n - E_0)}$$

$$E_n - E_0 \approx (z-2)\,\frac{J_{\text{ex}}}{2}\,n$$

$$V_{\text{eff}} \sim \prod_{n=1}^{N} \left[\frac{t}{(z-2)\,n\,J_{\text{ex}}} \right] \sim \frac{t}{N!} \left(\frac{t}{(z-2)J_{\text{ex}}} \right)^{N}$$

N itself is a function of *U/t* : $N \sim \dfrac{U}{(z-2)J_{\text{ex}}} \sim \dfrac{U^2}{4(z-2)t^2}$

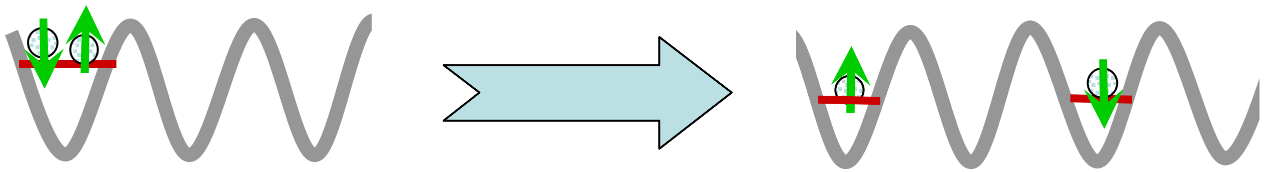
$$V_{\text{eff}} \sim t \left(\frac{t}{U} \right)^{N} \sim t \left(\frac{t}{U} \right)^{\frac{U^2}{4(z-2)t^2}}$$

Relaxation rate $W \sim e^{-\text{const} \times \frac{U^2}{t^2}}$

Slow superexponential relaxation

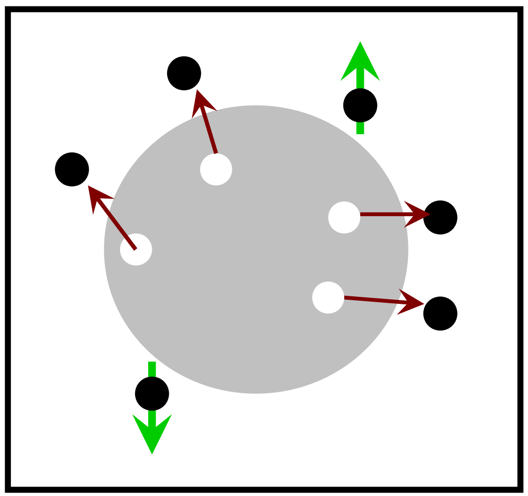
Doublon relaxation in a compressible state

Doublon decay in a compressible state

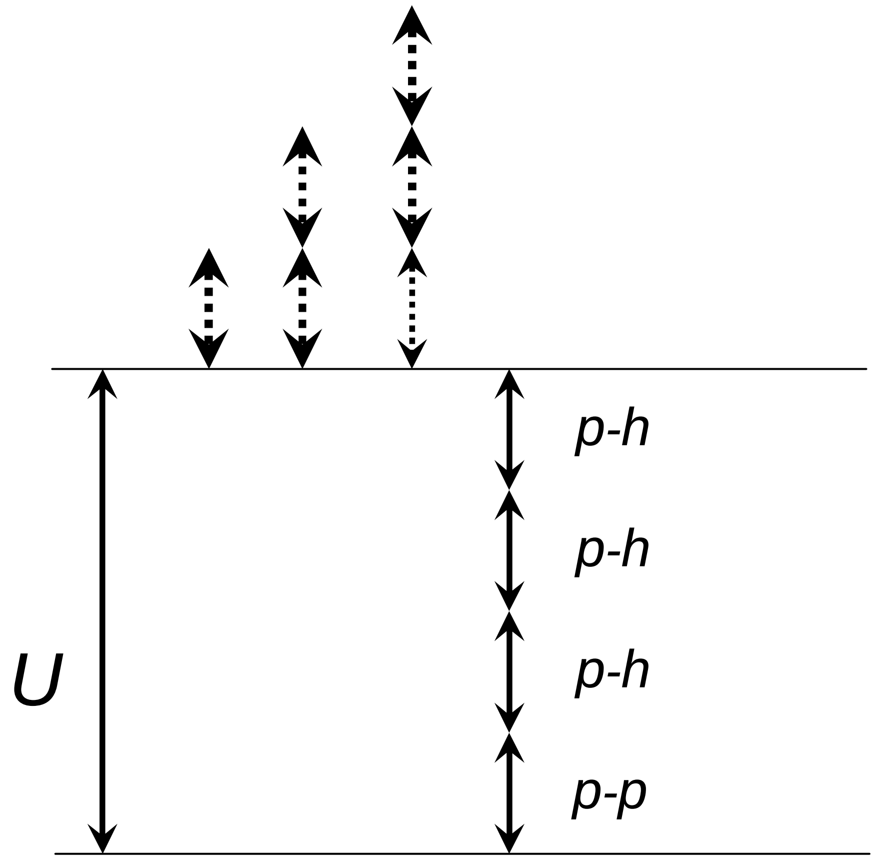


Excess energy U is converted to kinetic energy of single atoms

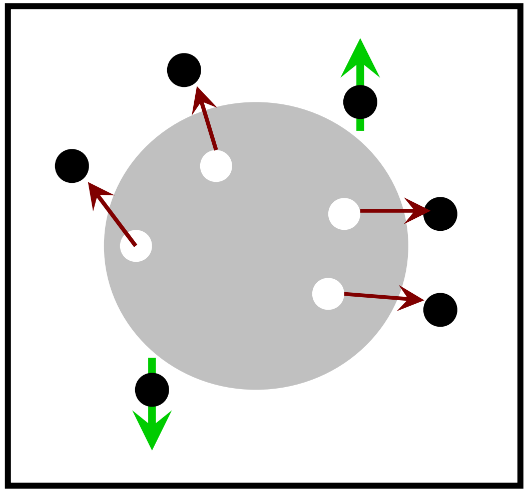
Compressible state: Fermi liquid description



Doublon can decay into a pair of quasiparticles with many particle-hole pairs

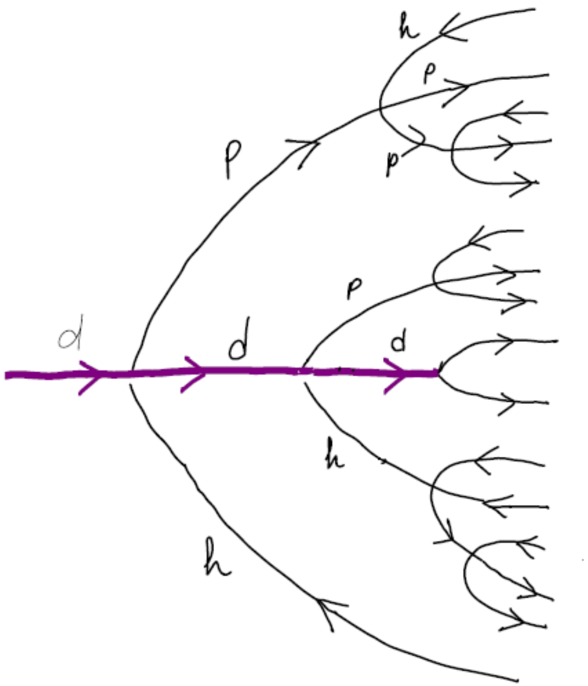


Doublon decay in a compressible state



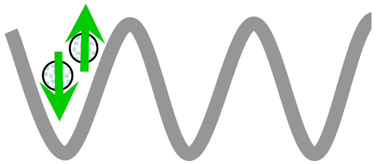
Perturbation theory to order n=U/6t
Decay probability

$$P \sim \left(\frac{t}{U}\right)^{\mathrm{const}\cdot\frac{U}{6t}} \sim e^{-\mathrm{const}\cdot\frac{U}{6t}\cdot\log(\frac{U}{t})}$$



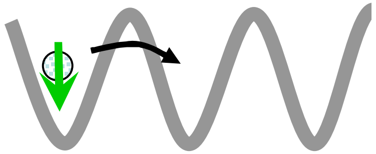
To calculate the rate: consider processes which maximize the number of particle-hole excitations

Doublon decay in a compressible state



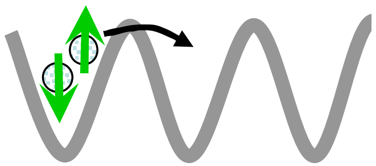
$$\mathcal{H}_{\mathrm{U}} = U \sum_i d_i^\dagger d_i$$

Doublon



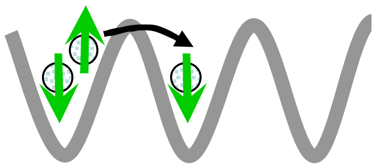
$$\mathcal{H}_{\mathrm{f}} = -t \sum_{i\sigma} c_{i\sigma}^\dagger c_{j\sigma}$$

Single fermion hopping



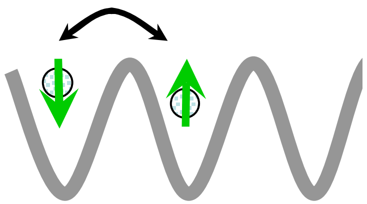
$$\mathcal{H}_{\mathrm{int1}} = -t \sum_{\langle ij \rangle \sigma} d_i \, c_{i\sigma}^\dagger \, c_{j-\sigma}^\dagger$$

Doublon decay



$$\mathcal{H}_{\mathrm{int2}} = -t \sum_{\langle ij \rangle \sigma} d_i^\dagger \, c_{j\sigma}^\dagger \, d_j \, c_{i\sigma}$$

Doublon-fermion scattering



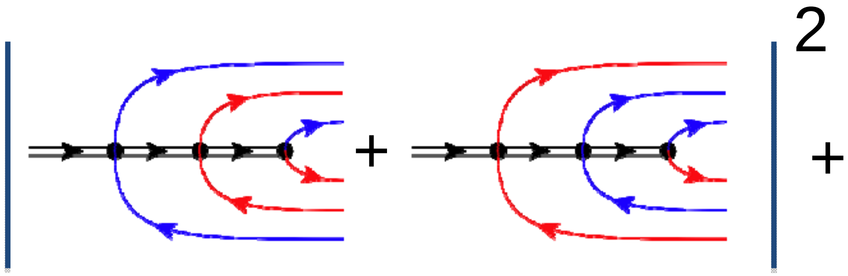
$$\mathcal{H}_{\mathrm{int3}} = -t \sum_{i\sigma} c_{i\sigma}^\dagger c_{j\sigma} \, n_{j-\sigma}$$

Fermion-fermion scattering due to projected hopping

Fermi's golden rule

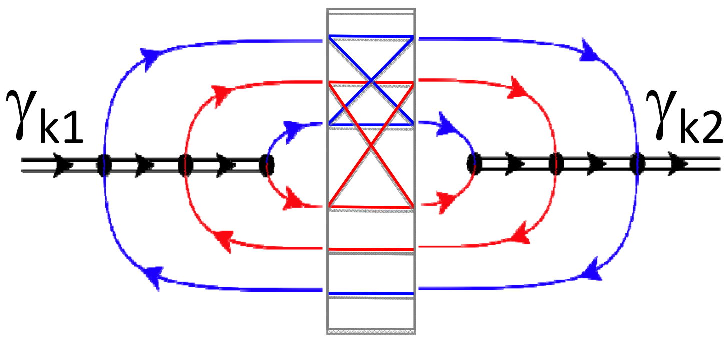
Neglect fermion-fermion scattering

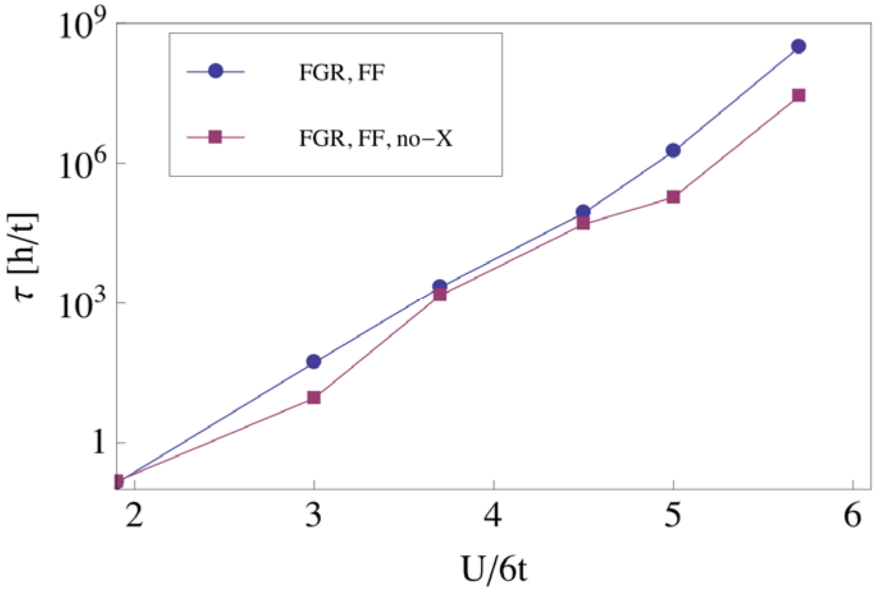
$\Gamma =$



+ other spin combinations

Particle-hole emission is incoherent: Crossed diagrams unimportant



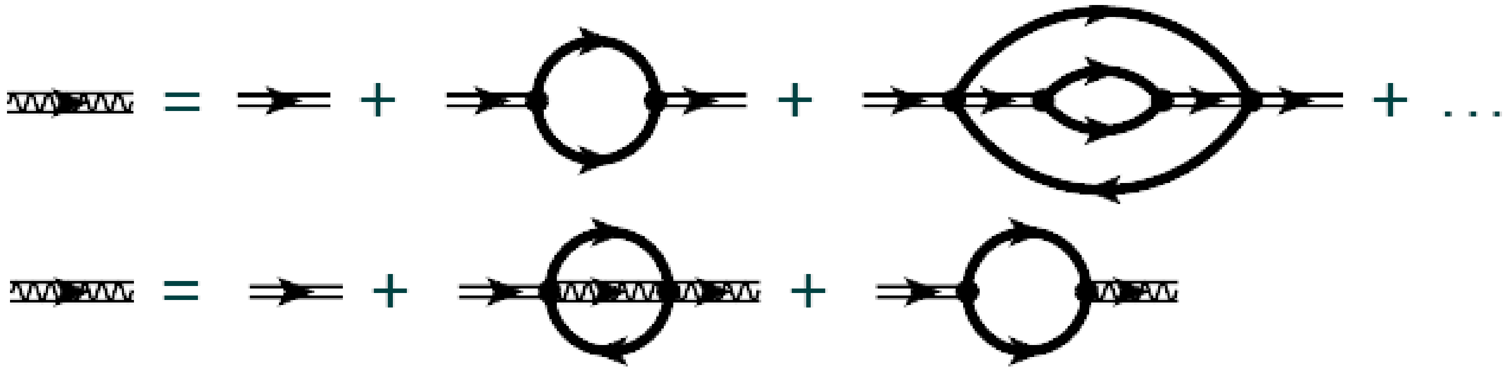


$\gamma_k = \cos k_x + \cos k_y + \cos k_z$

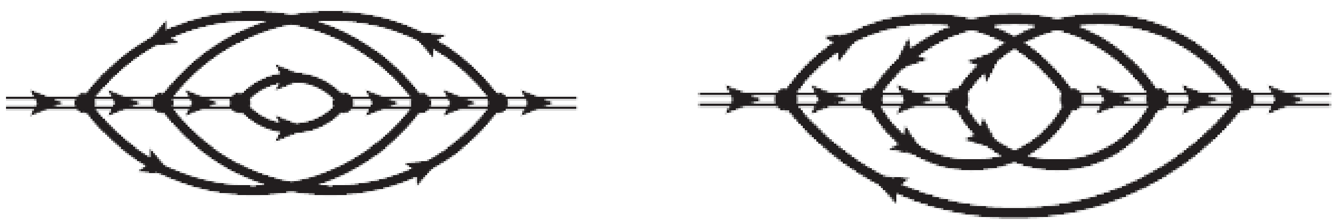
Self-consistent diagrammatics

Calculate doublon lifetime from Im Σ

Neglect fermion-fermion scattering



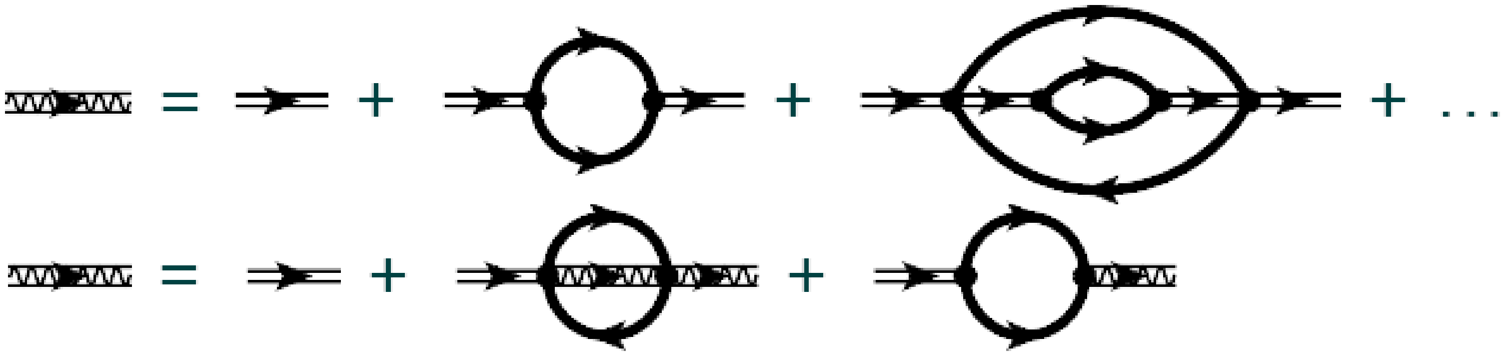
Emission of particle-hole pairs is incoherent:
Crossed diagrams are not important

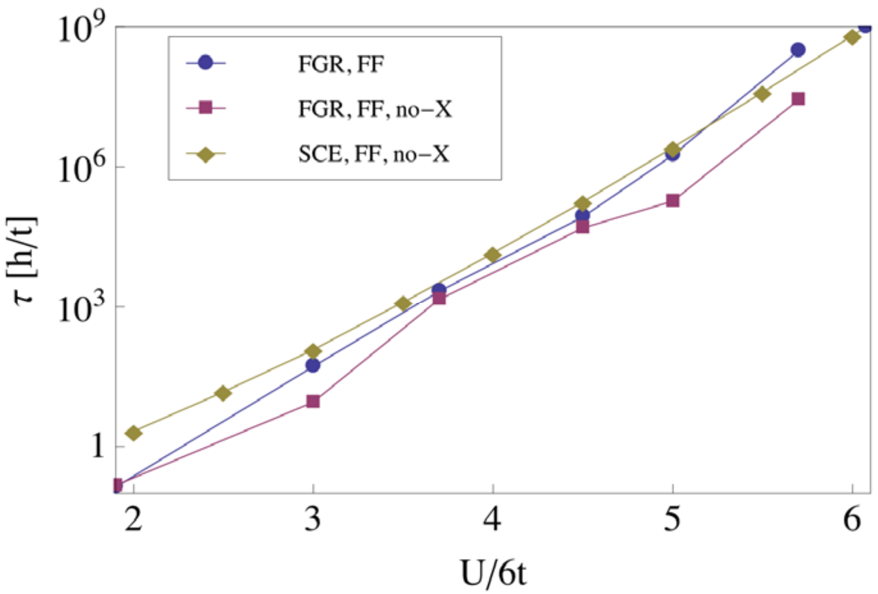


Suppressed by vertex functions

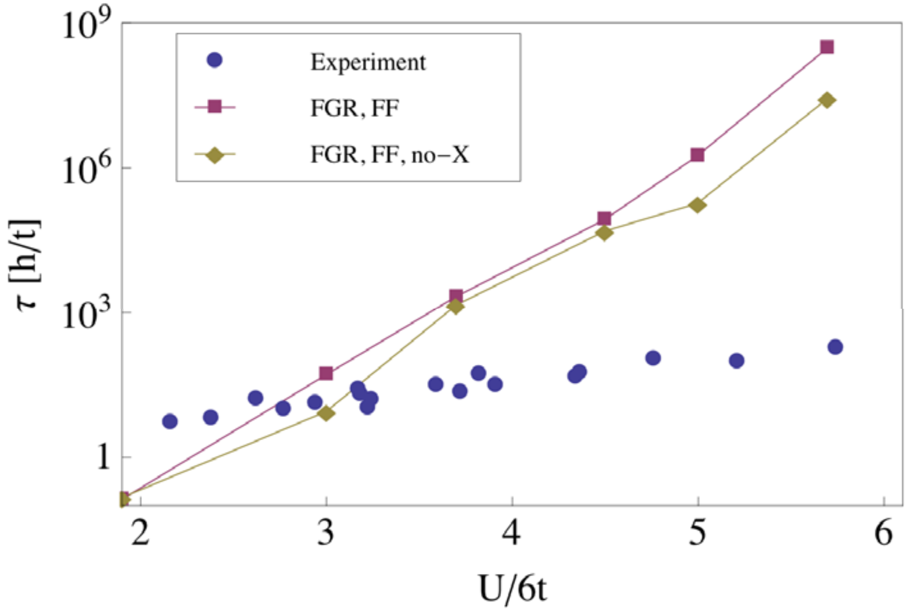
Self-consistent diagrammatics

Neglect fermion-fermion scattering





Comparison of Fermi's Golden rule and self-consistent diagrams

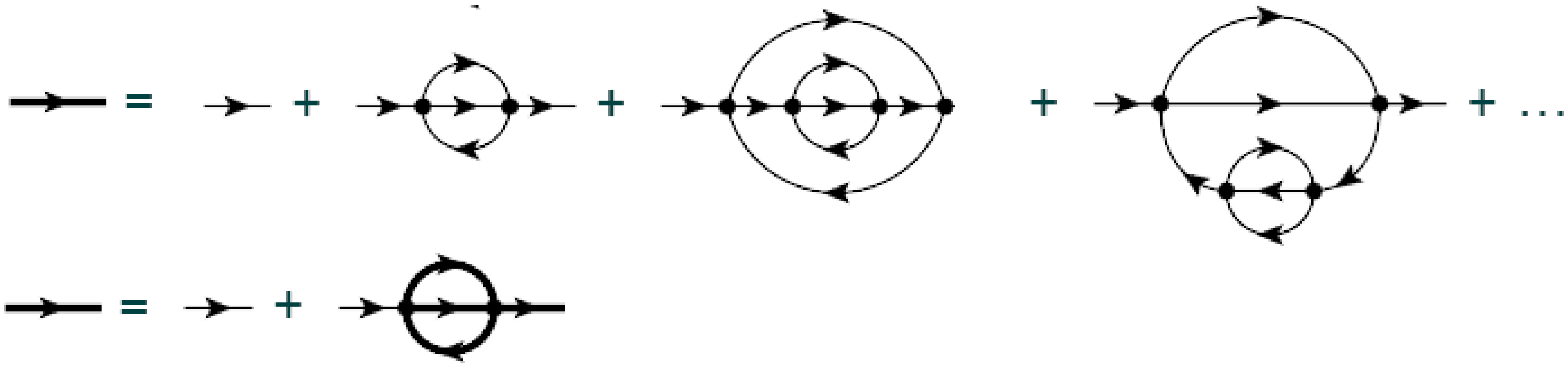


Need to include fermion-fermion scattering

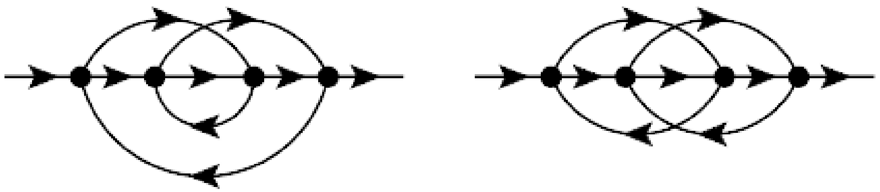
Self-consistent diagrammatics

Including fermion-fermion scattering

Treat emission of particle-hole pairs as incoherent
include only non-crossing diagrams



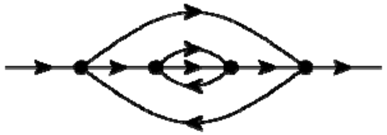
Analyzing particle-hole emission as coherent process requires adding decay amplitudes and then calculating net decay rate. Additional diagrams in self-energy need to be included



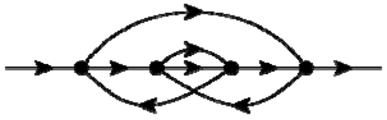
No vertex functions to justify neglecting crossed diagrams

Including fermion-fermion scattering

Correcting for missing diagrams



type present



type missing

Assume all amplitudes for particle-hole pair production are the same. Assume constructive interference between all decay amplitudes

For a given energy diagrams of a certain order dominate.

Lower order diagrams do not have enough p-h pairs to absorb energy

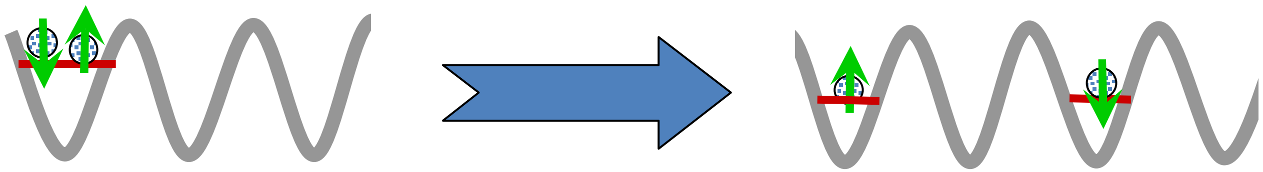
Higher order diagrams suppressed by additional powers of $(t/U)^2$

For each energy count number of missing crossed diagrams

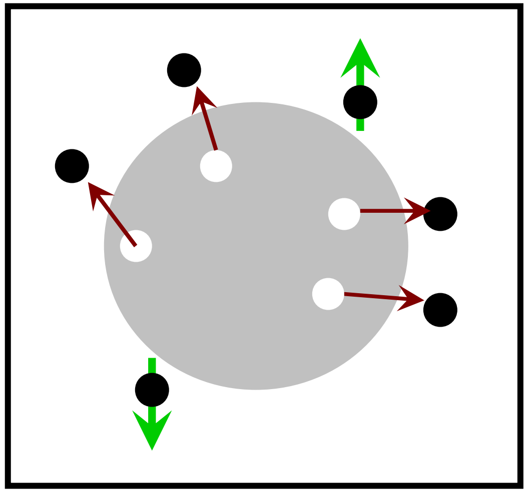
$$\Sigma_f''(\omega) \to \Sigma_f''(\omega) R[n_0(\omega)]$$

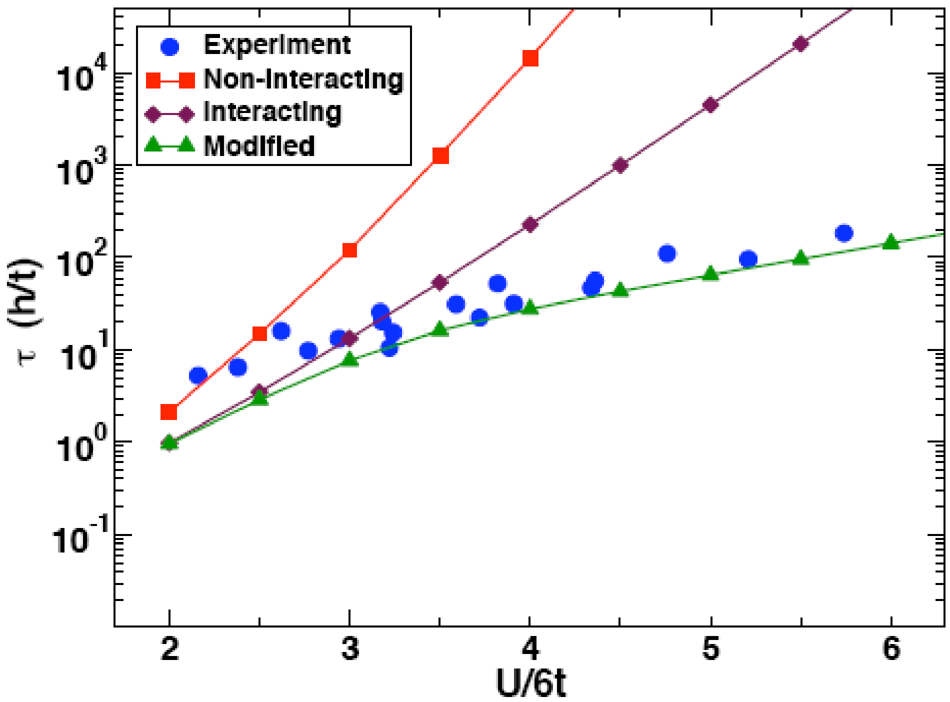
$R[n_0(\omega)]$ is renormalization of the number of diagrams

Doublon decay in a compressible state



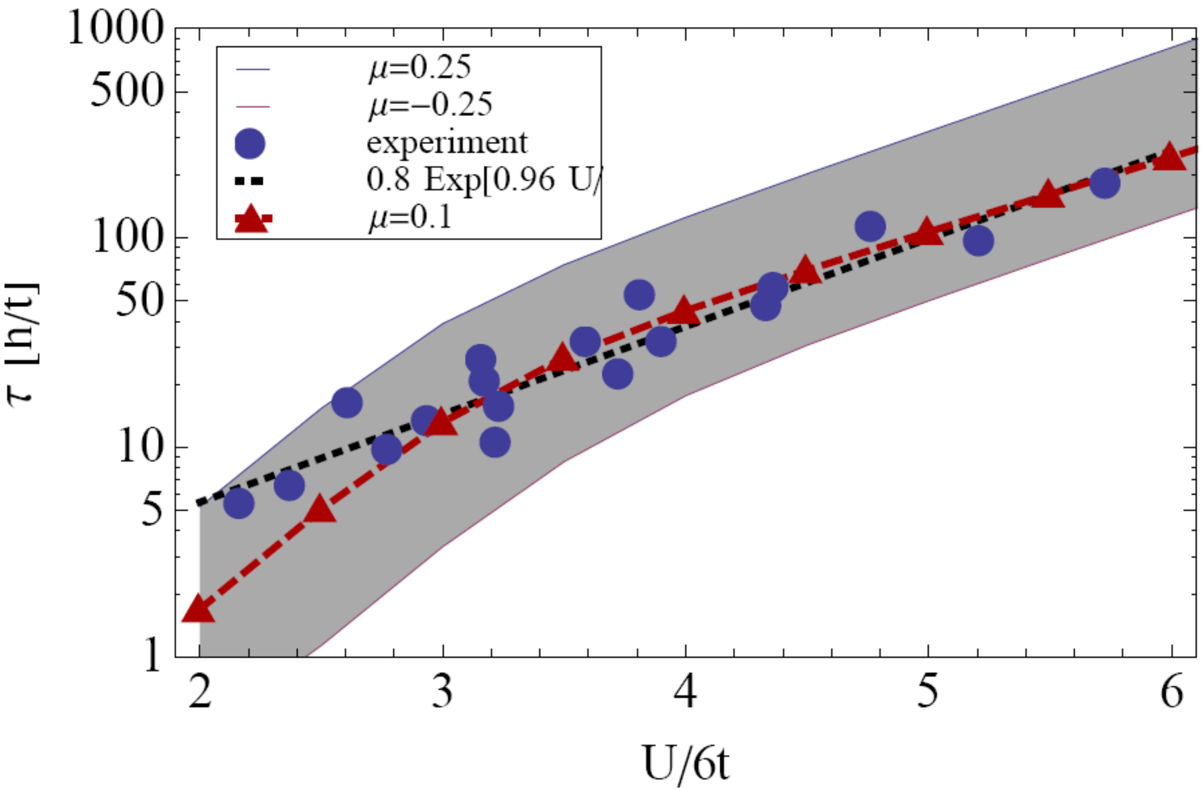
Doublon decay with generation of particle-hole pairs





Doublon decay in a compressible state

Close to half-filling decay rate
is not too sensitive to filling factor



Why understanding doublon decay rate is important

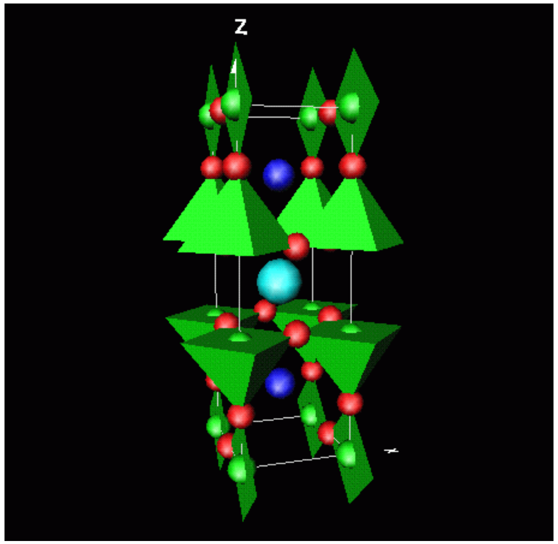
Prototype of decay processes with emission of many interacting particles. Example: jet production in the decay of massive particles in high energy physics: (e.g. top quarks)

Analogy to pump and probe experiments in condensed matter systems

Response functions of strongly correlated systems at high frequencies. Important for numerical analysis.

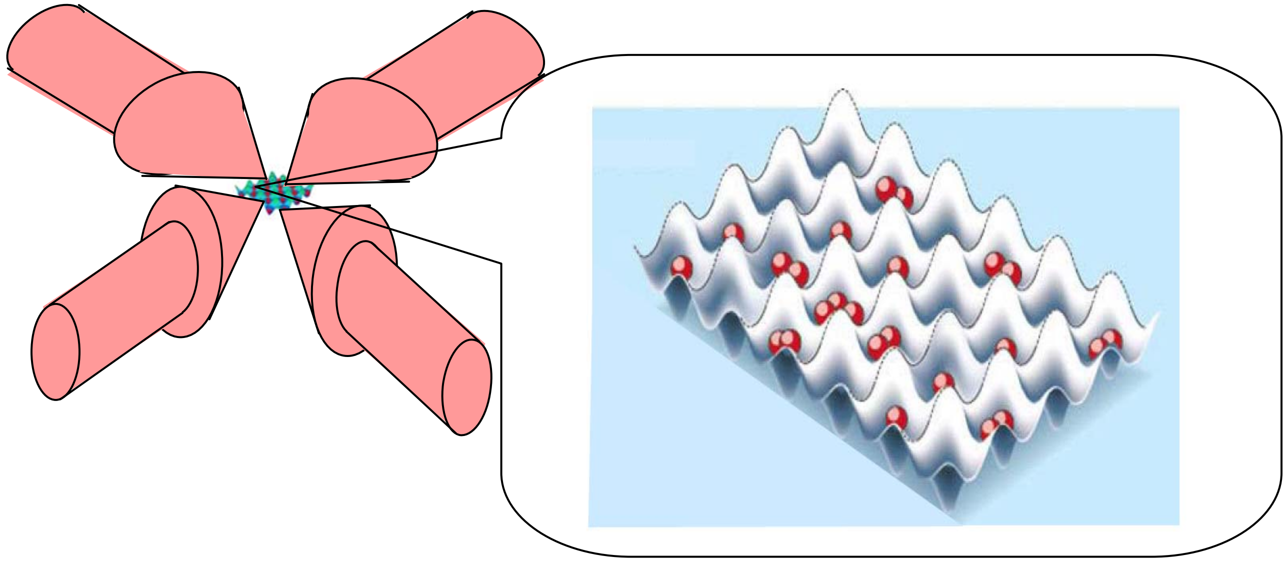
Important for adiabatic preparation of strongly correlated systems in optical lattices

Importance of doublon relaxation for quantum simulations



$YBa_2Cu_3O_7$

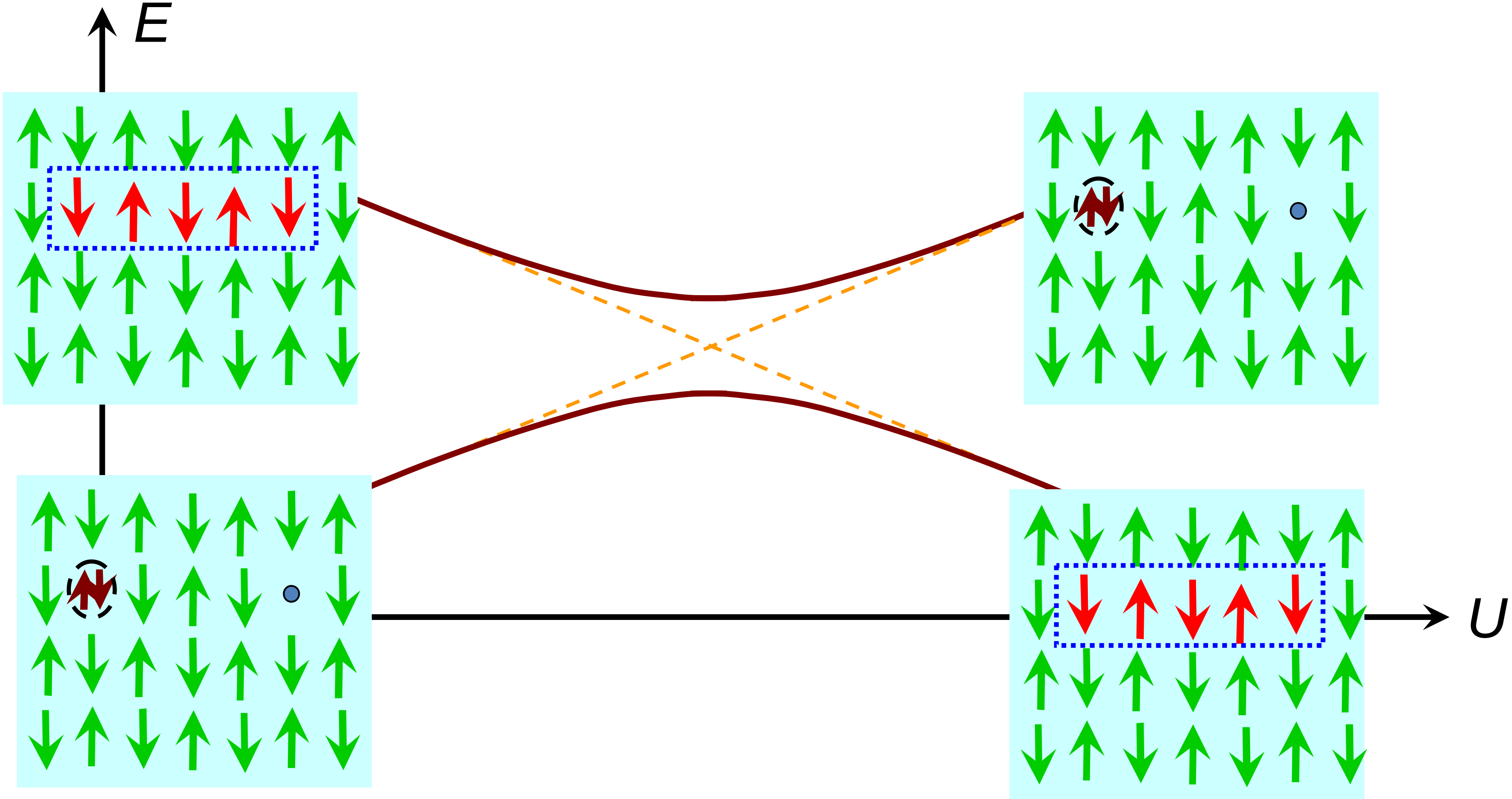
Antiferromagnetic and superconducting Tc of the order of 100 K



Atoms in optical lattice

Antiferromagnetism and pairing at sub-micro Kelvin temperatures

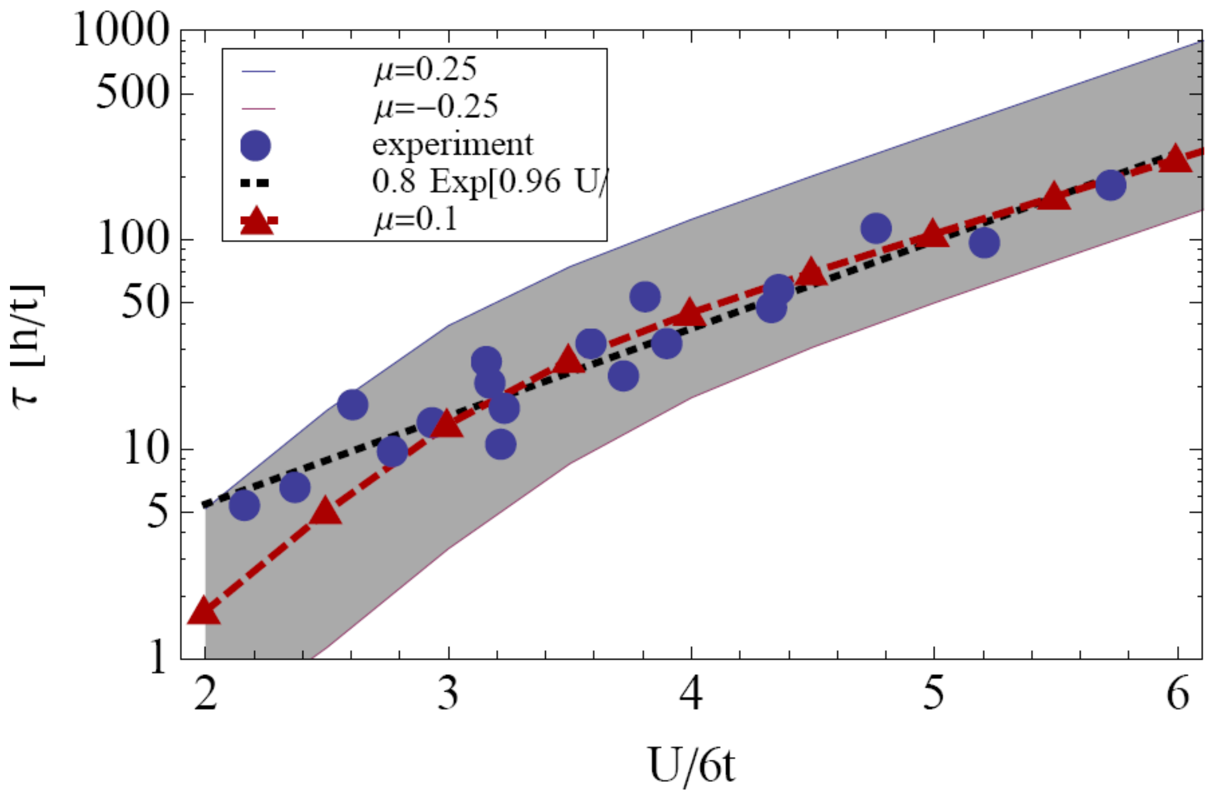
Adiabaticity at the level crossing



Relaxation rate provides constraints on the rate of change of interaction strength or the lattice height.

Summary

Fermions in optical lattice. Repulsively bound pairs decay via avalanches of particle-hole pairs



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Program in
Optical Lattices