



**The Abdus Salam  
International Centre for Theoretical Physics**



**2030-28**

**Conference on Research Frontiers in Ultra-Cold Atoms**

*4 - 8 May 2009*

**Generation of a synthetic vector potential in ultracold neutral Rubidium**

SPIELMAN Ian  
*National Institute of Standards and Technology  
Laser Cooling and Trapping Group  
100 Bureau Drive  
Gaithersburg MD 20899-8424  
U.S.A.*

# Generation of a synthetic vector potential and an $E$ field

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I. B. Spielman

Team

Y.-J. Lin, R. L. Compton, A. R. Perry, and K. Jimenez-Garcia

Senior coworkers

J. V. Porto, and W. D. Phillips

**NIST**

National Institute of Standards and Technology  
Technology Administration, U.S. Department of Commerce



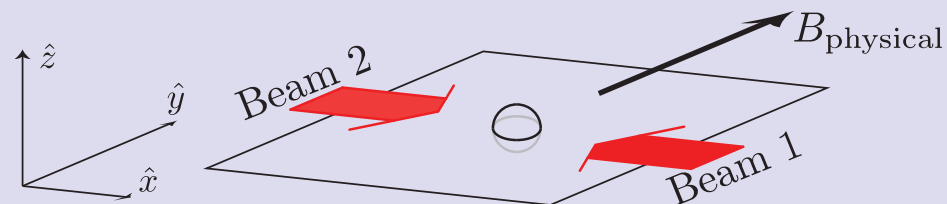
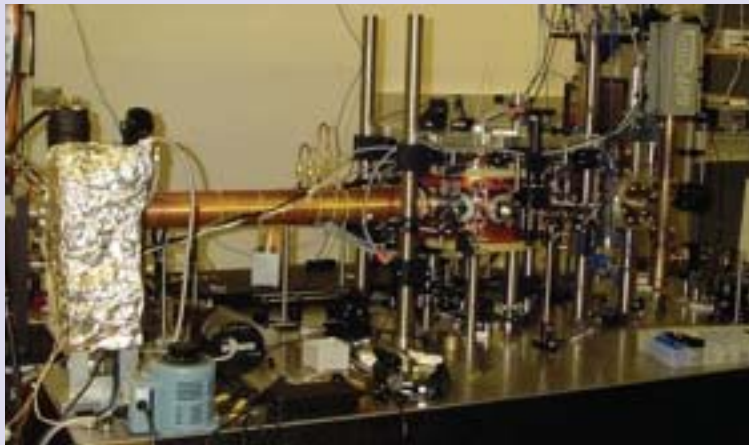
Funded by the DARPA OLE program, ONR, and  
the NSF through the PFC at JQI.

May, 2009

# Outline for today

## Raman dressed states

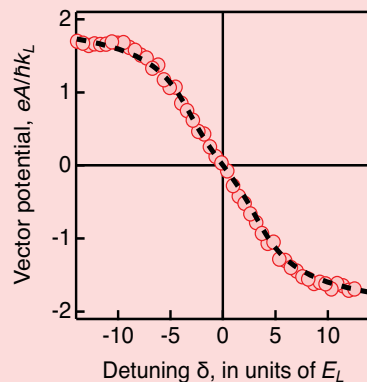
Brief description of theory and implementation



## A synthetic vector potential

Experimentally verify a vector potential appears

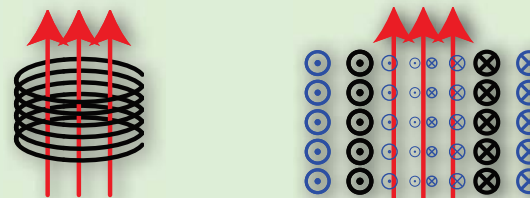
$$\hat{H} = \frac{1}{2m} \left[ (\hat{p}_y + q\hat{A}_y)^2 + (\hat{p}_x + q\hat{A}_x)^2 \right] + V(\hat{x})$$



## An electric field appears

Temporal variation of  $A$  gives rise to an electric field.

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$$



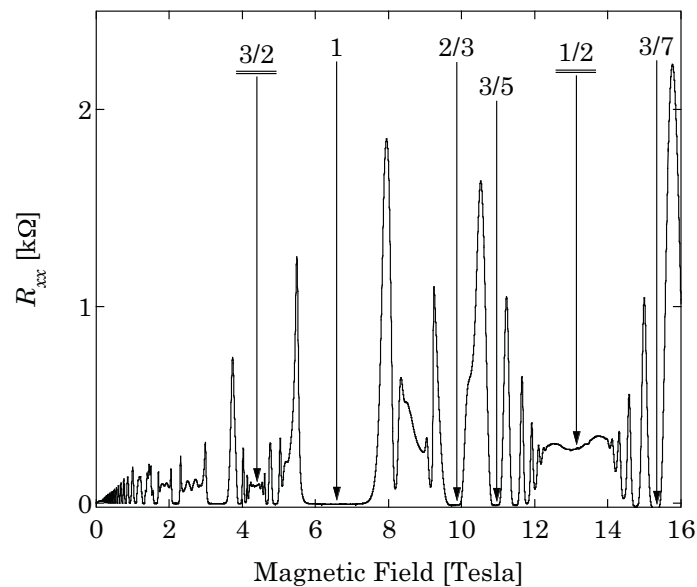
# Motivation

## Fundamental physics

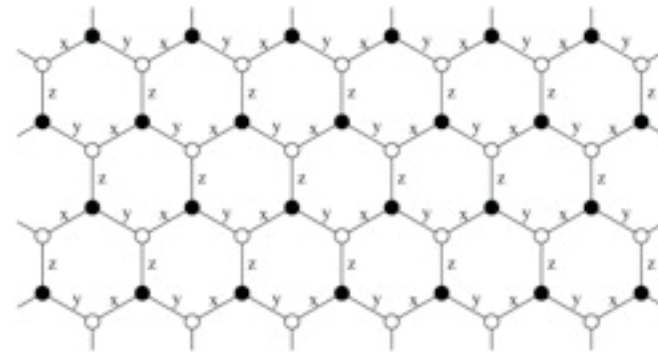
Under what general conditions can physical systems support excitations with quantum numbers and statistical angles which are not simple multiples of the constituent particles?

E.g., quantum Hall systems, quantum magnets,  $p$ -wave superconductivity, ...  
(all can potentially be studied in cold atom systems)

## FQHE Systems



## Spin 1/2 system: Kitaev lattice



## Refs.

- [1] R. B. Laughlin. PRL **50** p1395 (1983).
- [2] A. Y. Kitaev, Ann. Phys. **321**, 2 (2006).

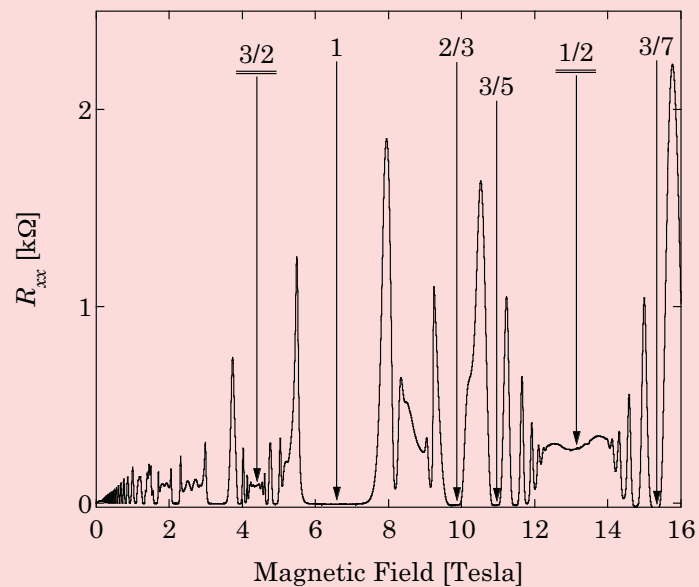
# Motivation

## Fundamental physics

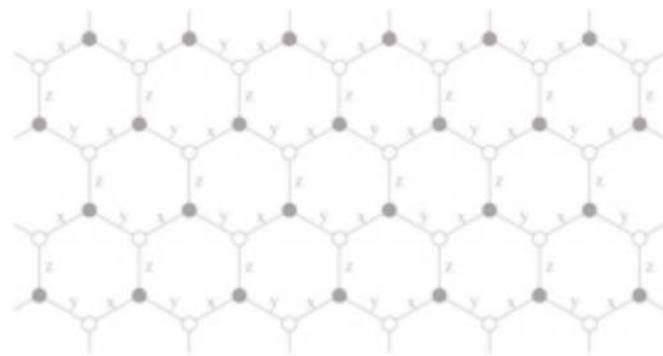
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### FQHE Systems



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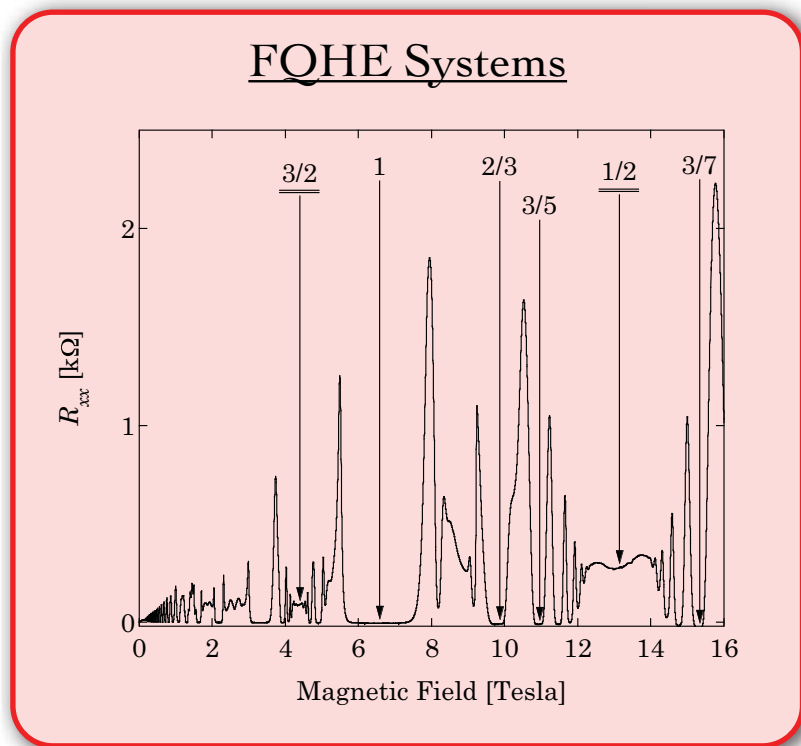


### Refs.

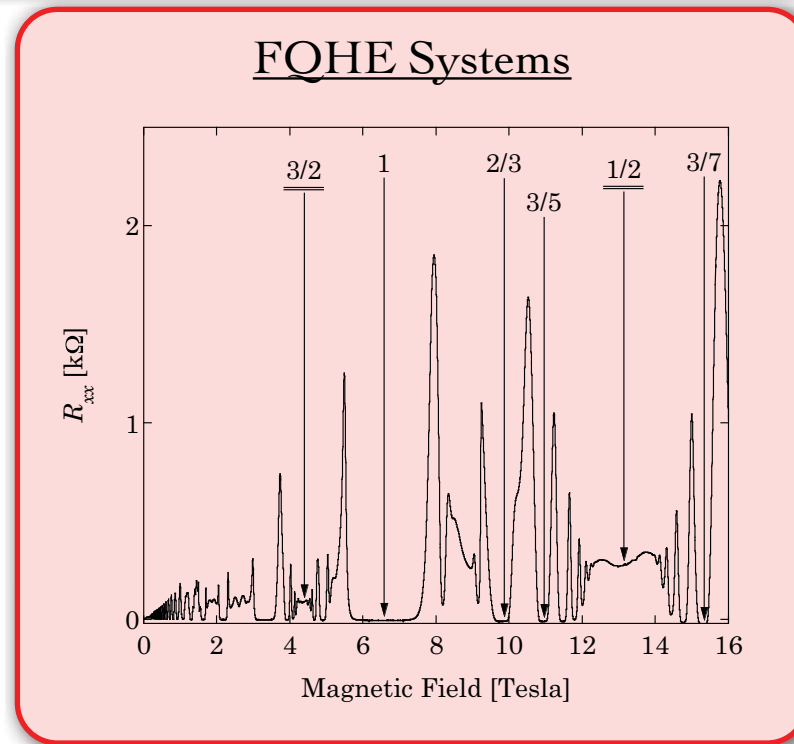
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# Motivation: magnetic fields

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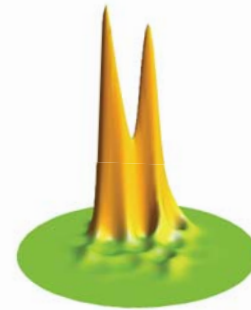
# Motivation: magnetic fields



## Bosons at high field

At filling factor  $1/2$  the Laughlin state is the *exact* ground state.

Binary contact interactions *are* sufficient to generate some non-abelian states (Moore-Read), but not all (Read-Rezayi).

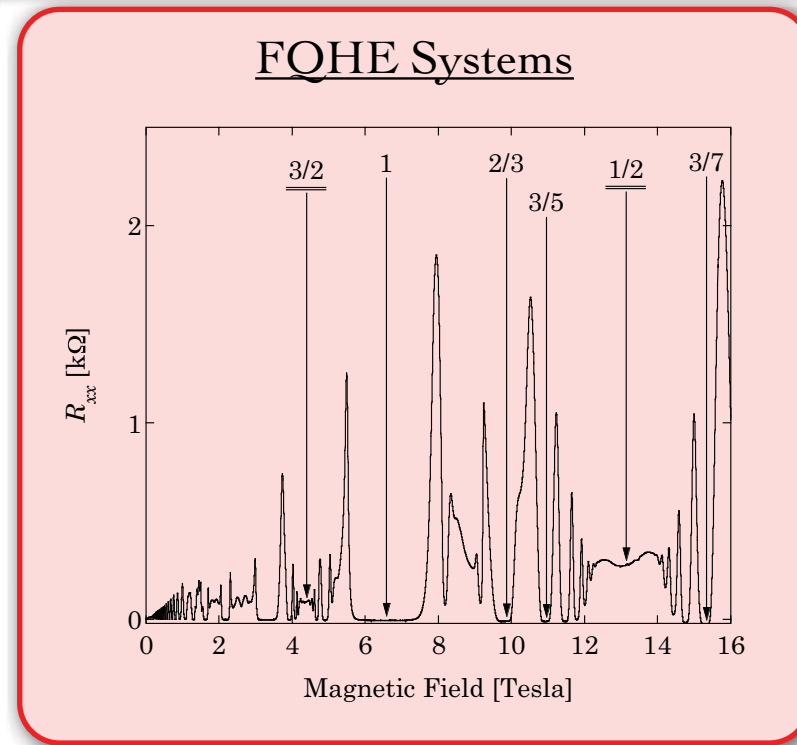


Single atom probability distribution in Laughlin  $1/2$  state, fixing coordinates of other atoms.

### Reference

N. R. Cooper *Advances in Physics* 57, 539 (2008)

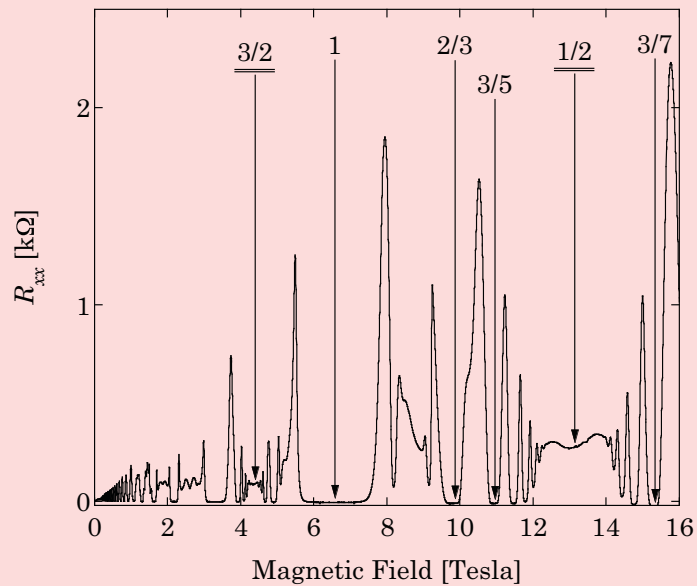
# How to “charge” neutral particles





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## FQHE Systems



## How to simulate magnetic fields

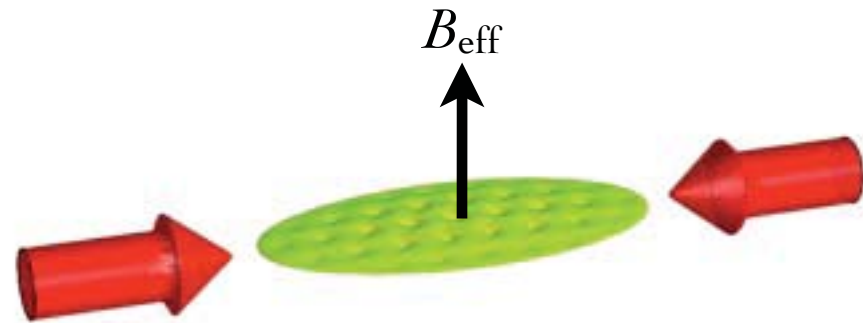
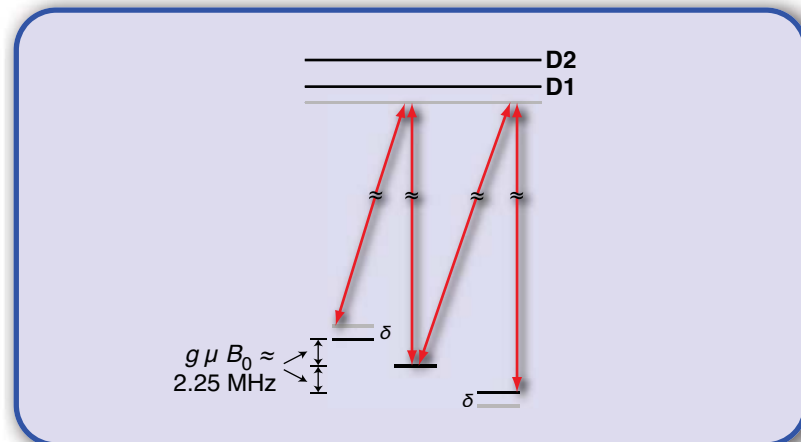
(1) Rotation: the Hamiltonian in the rotating frame has an effective field. To reach high fields fine tuning is required to compensate the centripetal term: small numbers.

(2) Stroboscopic proposal: precise modulation of lattices and background potentials.

**References:** V. Schweikhard et al PRL 92 p040404 (2004),  
A. Sørensen, et al PRL 94 p086803 (2005),

## Our approach

(3) Raman techniques.



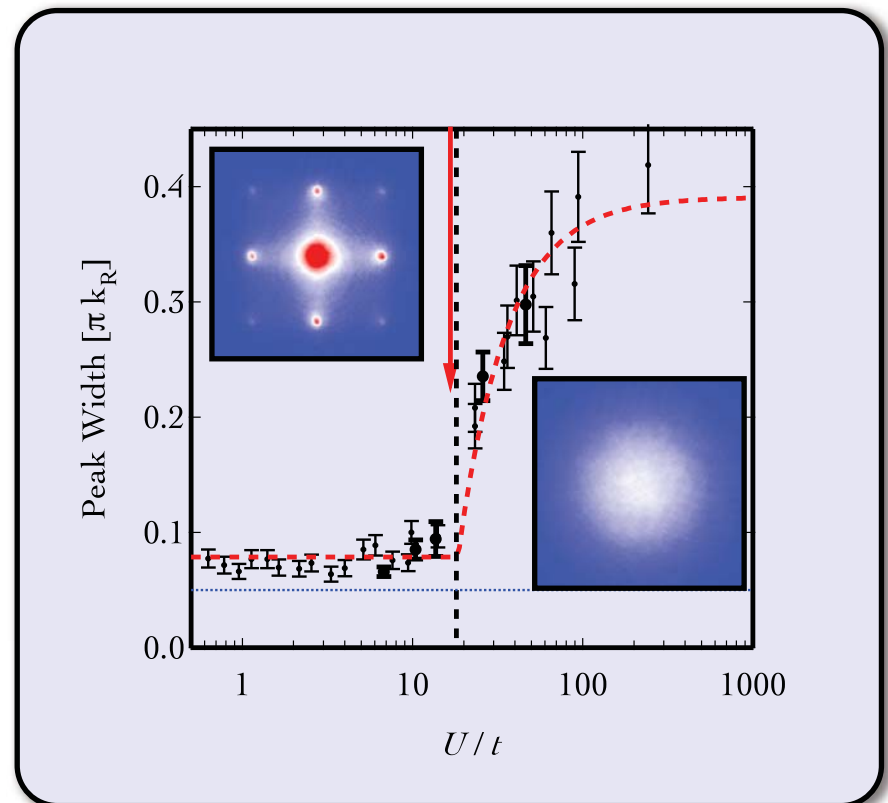
**Reference:** G. Juzeliūnas et al PRA 73 p025602 (2006)

# Cold atoms: a platform for many-body physics

We can control the hamiltonian for cold atoms in a number of ways.

$$\hat{H} = \sum_j \left[ \frac{\hat{p}_j^2}{2m} + V(x_j) \right] + \sum_{i < j} U(x_i - x_j)$$

*Potential:* optical and magnetic forces.  
**Lattice physics, 2D SF to MI transition**



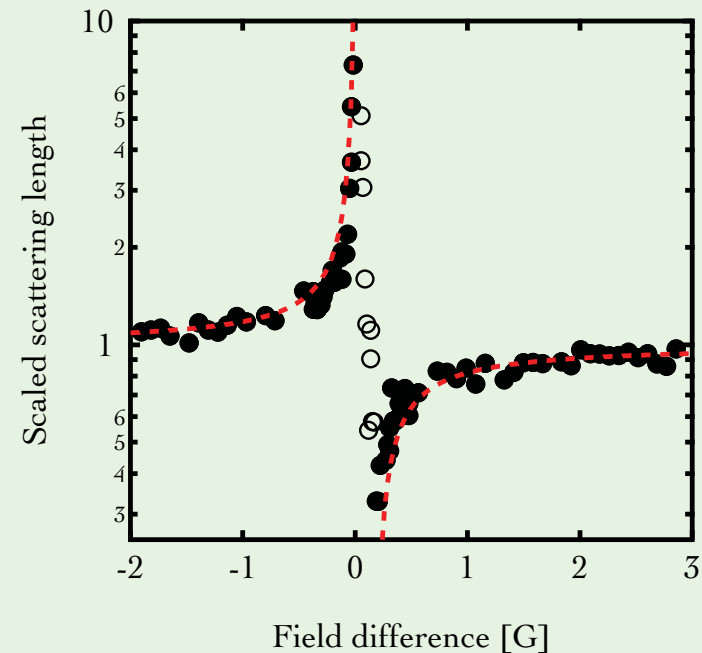
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*Interaction:* Choice of atom and collisional  
Feshbach resonances.  
Careful observation of  $^{87}\text{Rb}$  resonance



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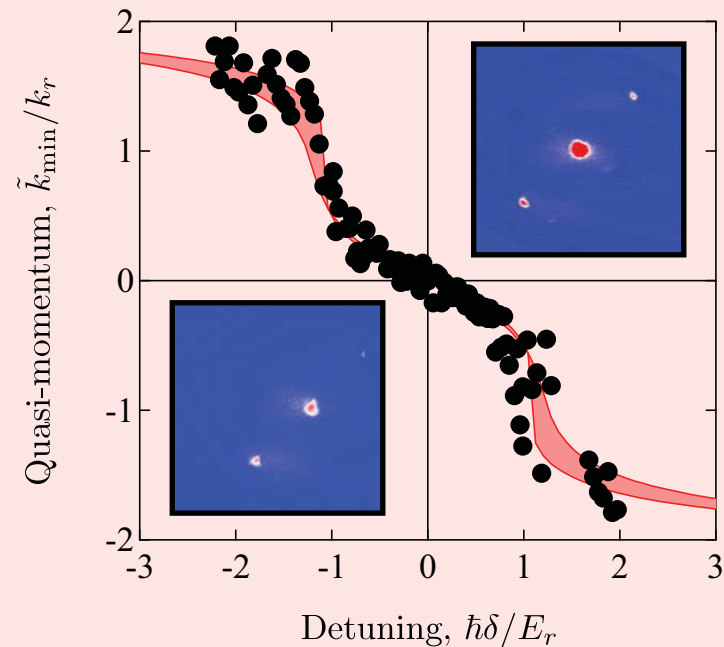
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*Potential:* optical and magnetic forces.  
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Feshbach resonances.  
Careful observation of  $^{87}\text{Rb}$  resonance

*Kinetic:* We have been working to control  
the kinetic energy term.  
**Synthetic vector potentials**



# Experimental control of cold atoms systems

We can control the hamiltonian for cold atoms in a number of ways.

$$\hat{H} = \sum_j \left[ \frac{\hat{p}_j^2}{2m} + V(x_j) \right] + \sum_{i < j} U(x_i - x_j)$$

We have been working on producing the same level of control with the *kinetic* energy term.

Here I will be interested in a synthetic field in the 2D plane.

Some common gauge choices are:

$$A = \left\{ -\frac{By}{2}, \frac{Bx}{2}, 0 \right\}$$

Symmetric gauge: natural for rotating systems

$$A = \{0, Bx, 0\}$$

Landau gauge: relevant here

Expect the usual relations for fields

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t} \quad B = \nabla \times \mathbf{A}$$

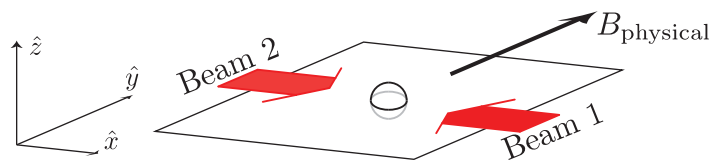
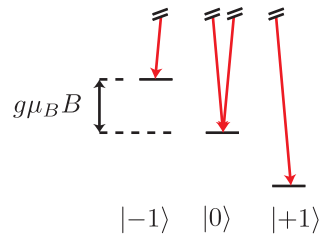
## References

J. C. Maxwell (1873)

# Atom light interaction

## Atom light interaction

Given the following geometry and levels



## Coupled Hamiltonian

We will want to label states, so I will start with the expected:

$$|k, \sigma\rangle$$

Absent the lasers the 1D Hamiltonian for motion along  $x$  is

$$H_0 = \sum_k \left( \frac{\hbar^2 k^2}{2m} - \frac{\delta}{2} \right) |k, 1\rangle\langle k, 1| + \left( \frac{\hbar^2 k^2}{2m} + \frac{\delta}{2} \right) |k, 2\rangle\langle k, 2|$$

The Raman beams couple states via

$$H_{\text{int}} = \sum_k \left( \frac{\Omega}{2} |k - 2k_r, 2\rangle\langle k, 1| + \frac{\Omega}{2} |k, 1\rangle\langle k - 2k_r, 2| \right)$$

(this is in the frame rotating at the frequency difference of the Raman beams, in with the RWA)

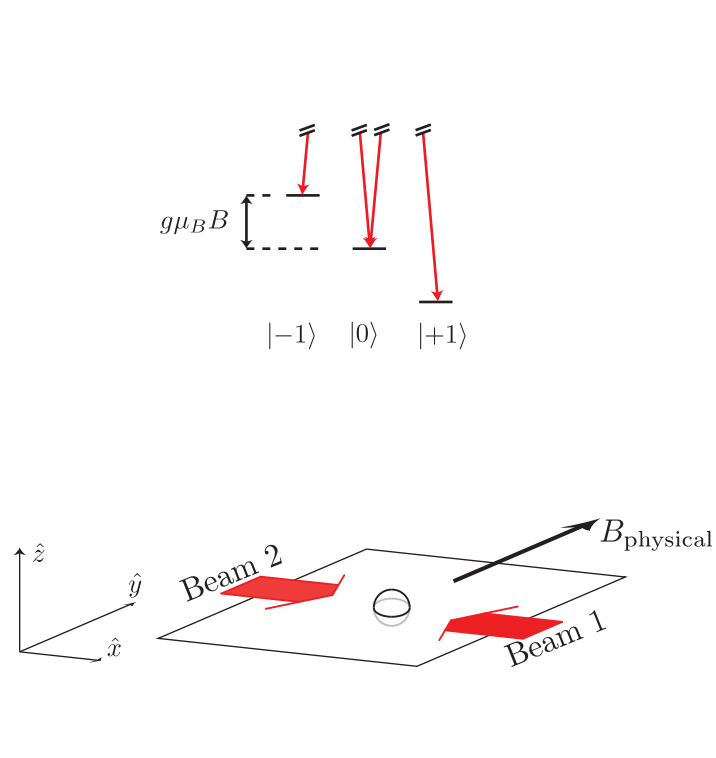
### References

- [1] Juzeliūnas, et al., PRA 025602 **73** (2006), + later pubs
- [2] S.-L. Zhu, et al., PRL 240401 **97** (2006)
- [3] IBS, submitted to PRA

# Atom light interaction: pictures

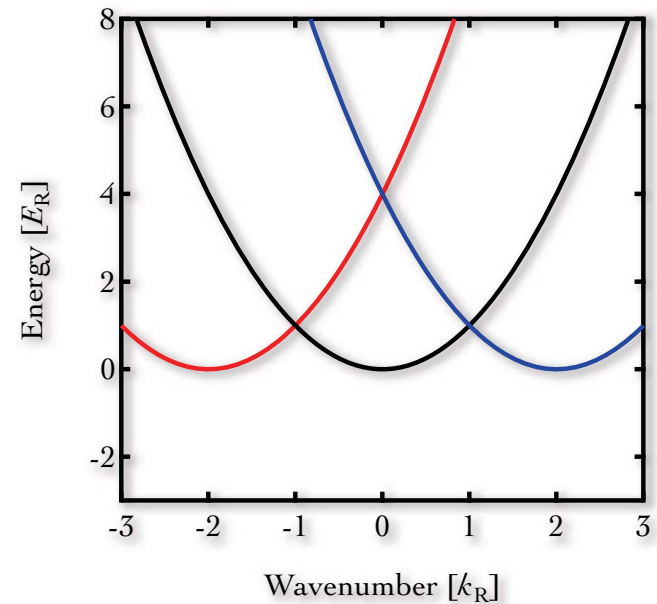
## Atom light interaction

Given the following geometry and levels



## Coupled States

States will be labeled by:  
(1) the “band index” and by  
(2) a quasi-momentum  $k$



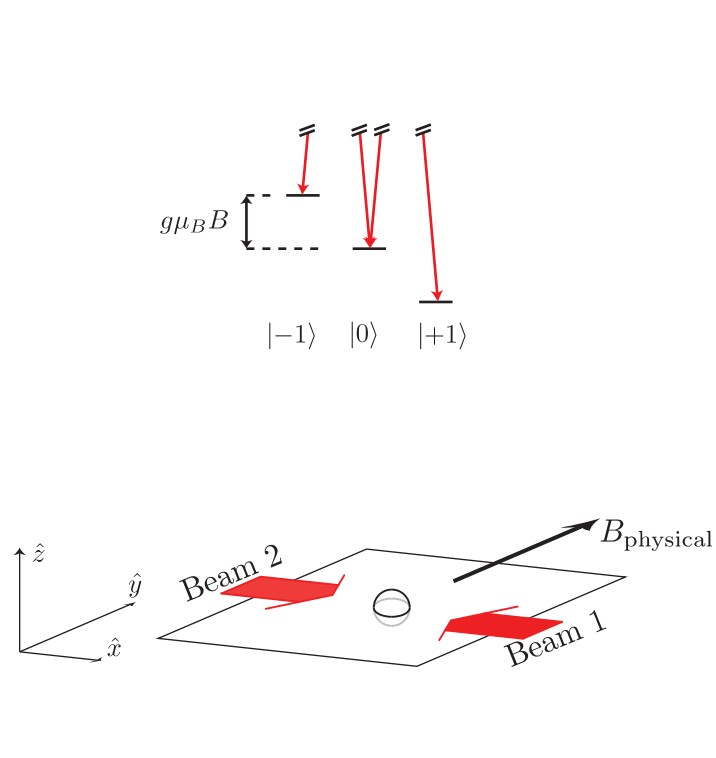
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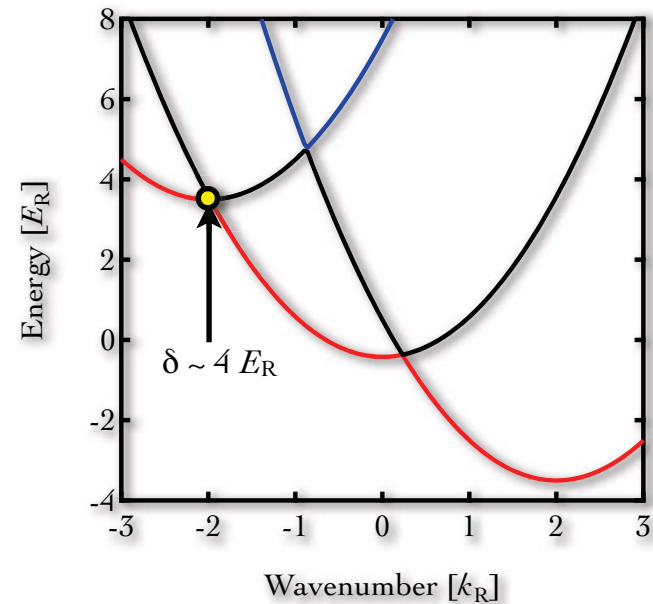
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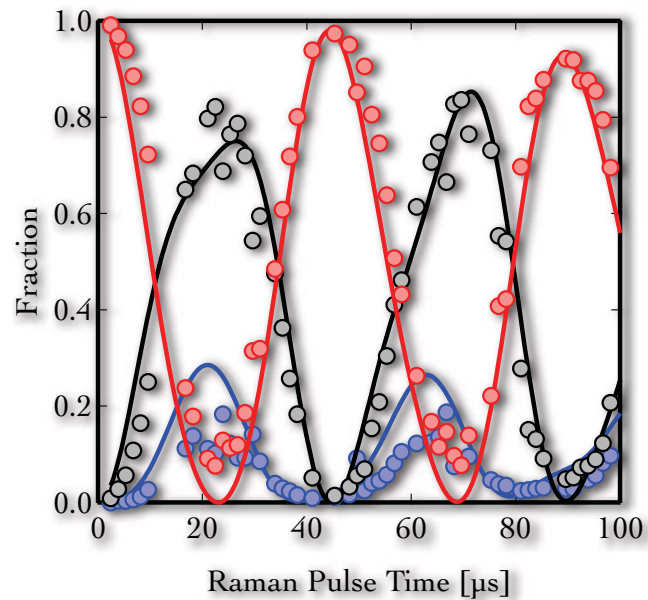
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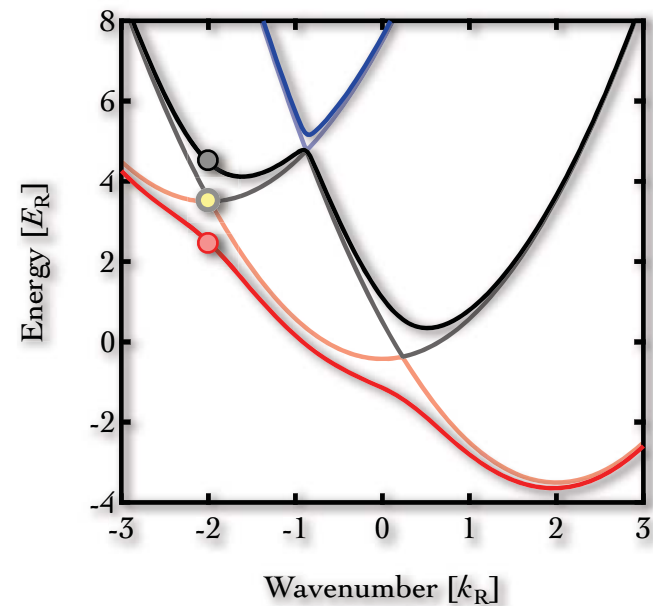
## Time evolution

In the sudden limit (Raman-Nath)  
Population oscillations yield coupling



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# Atom light interaction: vector potential

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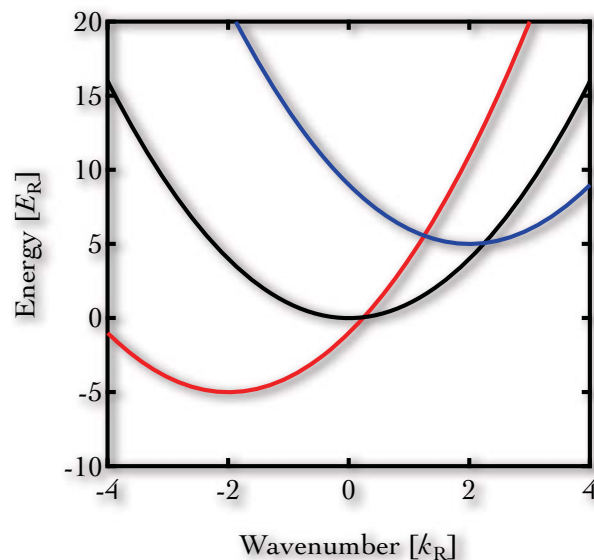
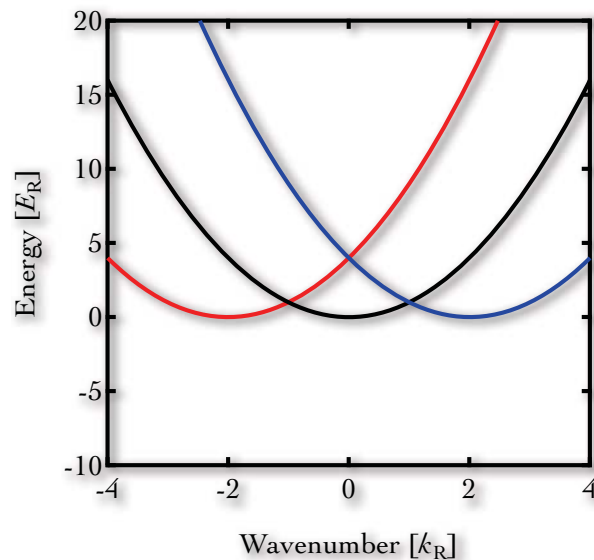
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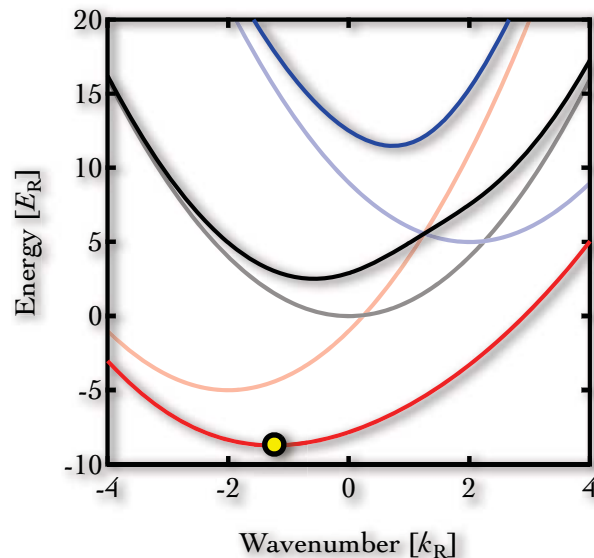
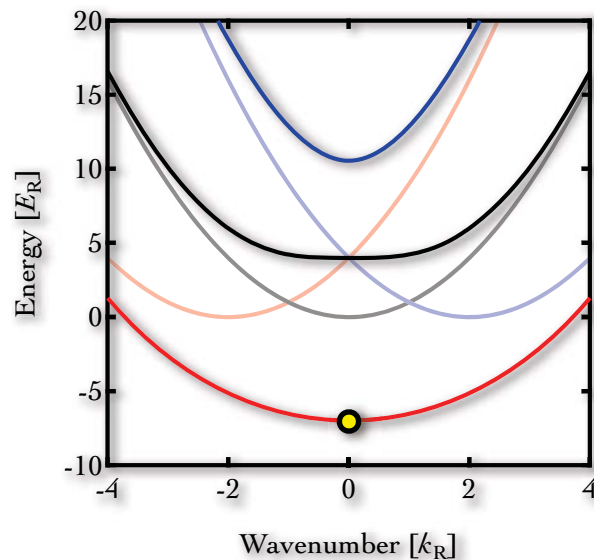
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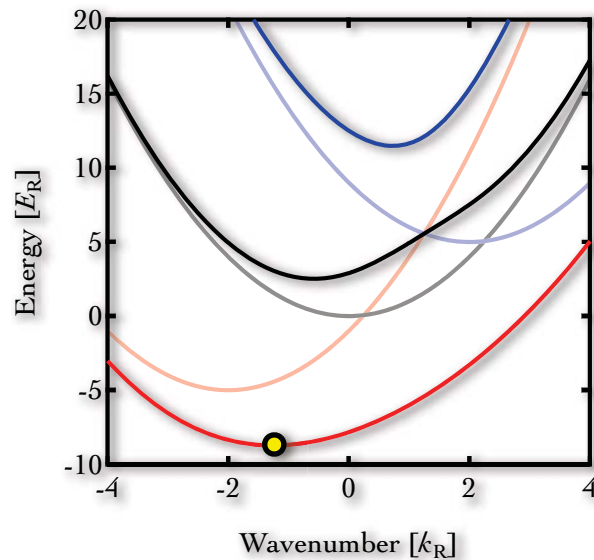
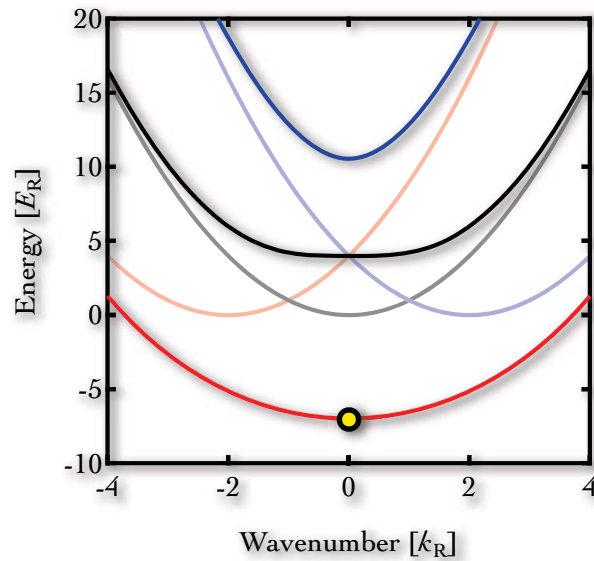
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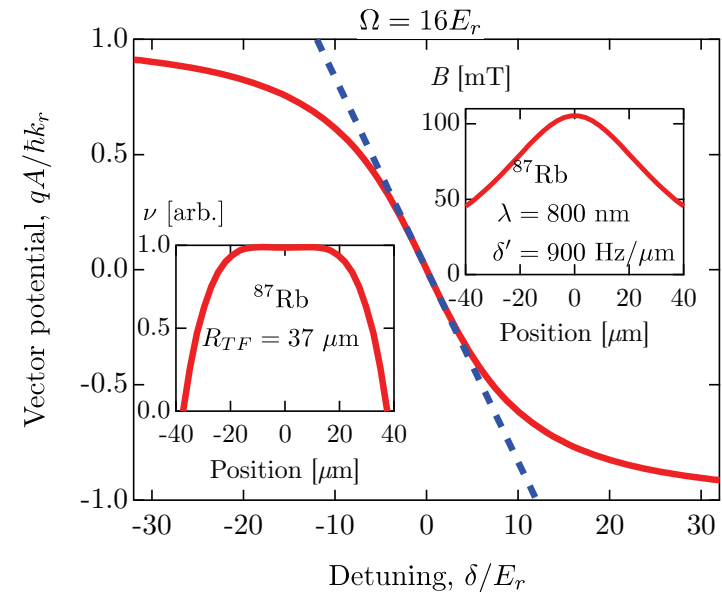
(this is in the frame rotating at the frequency difference of the Raman beams, in with the RWA)



# Atom light interaction: vector potential



## Effective vector potential



$$\frac{m^*}{m} \approx \frac{\Omega}{\Omega \pm 4}$$

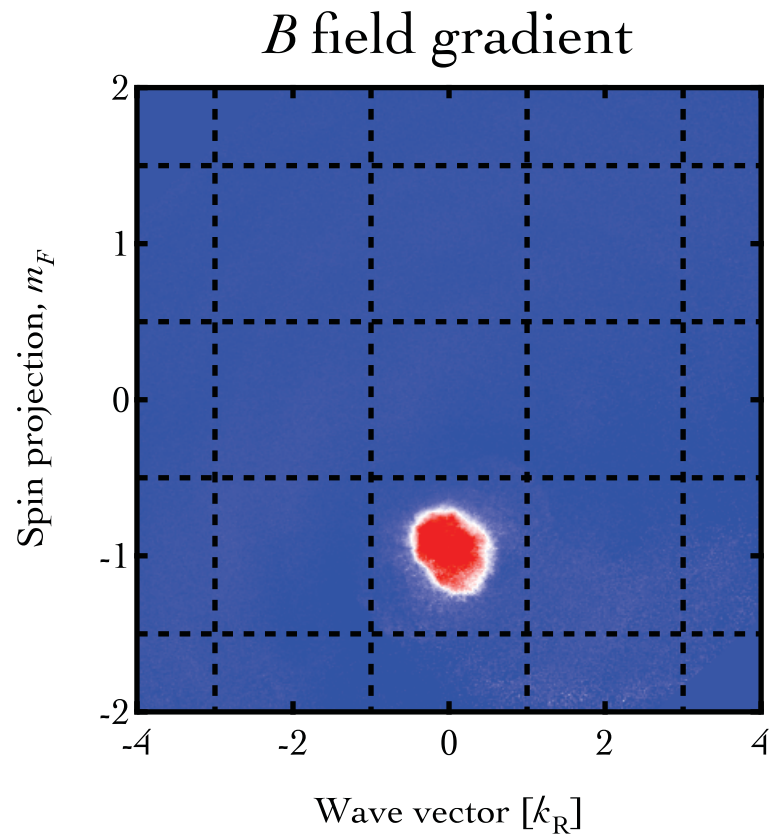
$$\frac{qA}{\hbar k_r} \approx \frac{\delta}{4 \pm \Omega}$$

$$\frac{\delta'}{E_r} \approx \frac{2 \pm \Omega}{2} + \frac{\delta^2(4 \pm \Omega)}{4(4 + \Omega)^2}$$

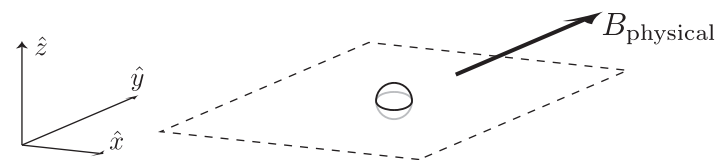
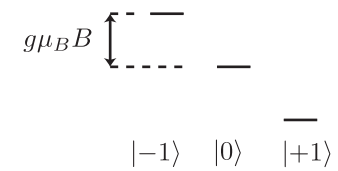
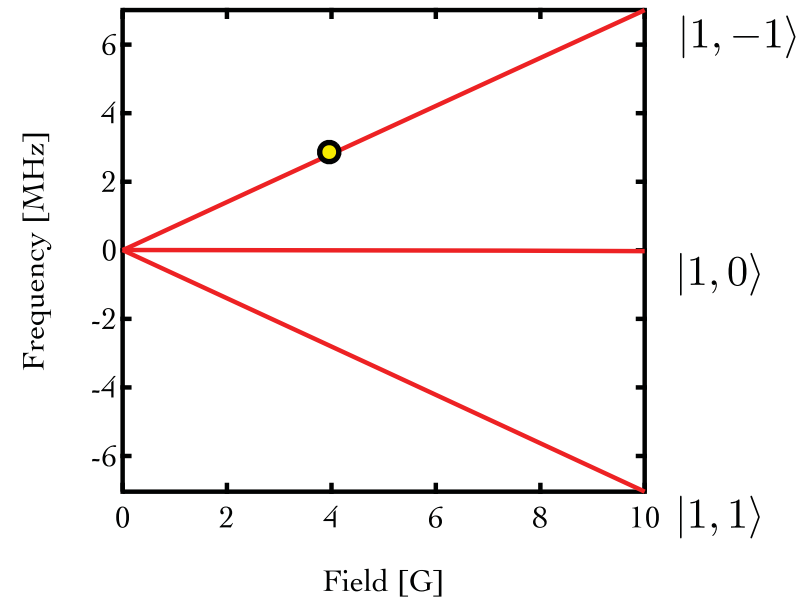
# Reality check

## Adiabatic manipulation of atoms

Initial state  $|F = 1, m_F = -1\rangle$



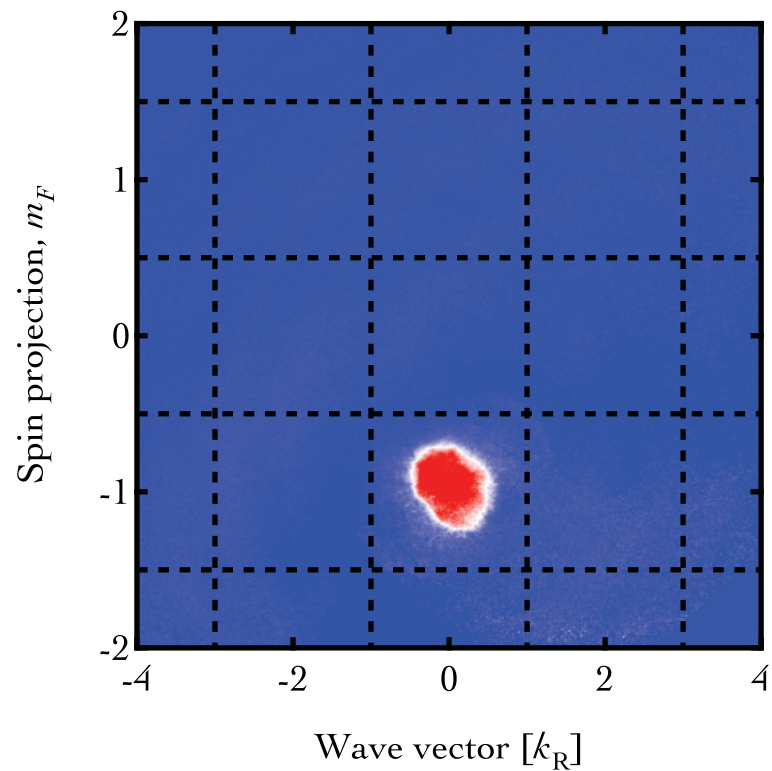
## Breit-Rabi result for $^{87}\text{Rb}$



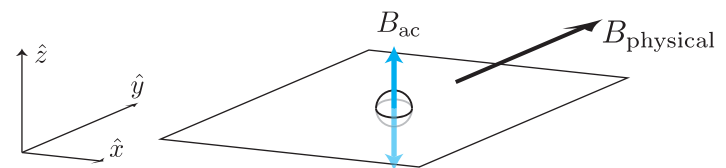
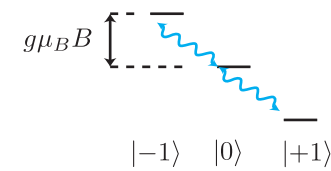
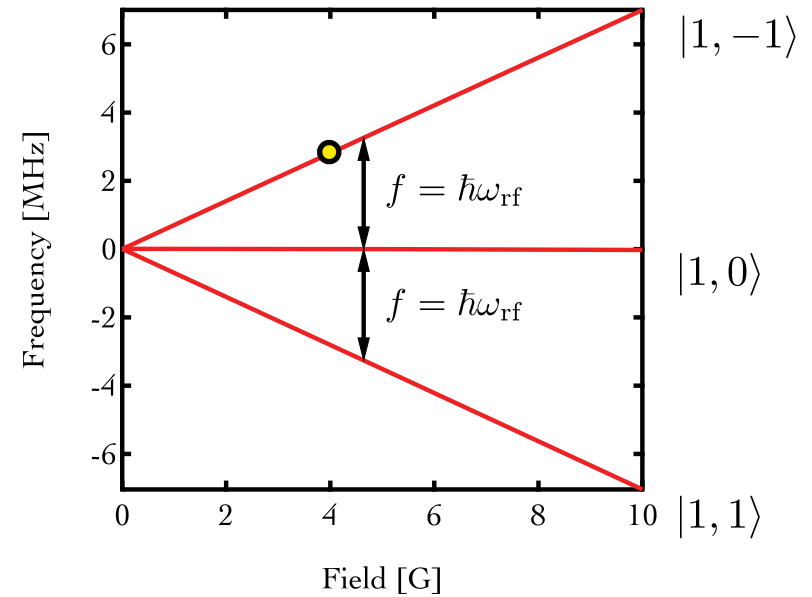
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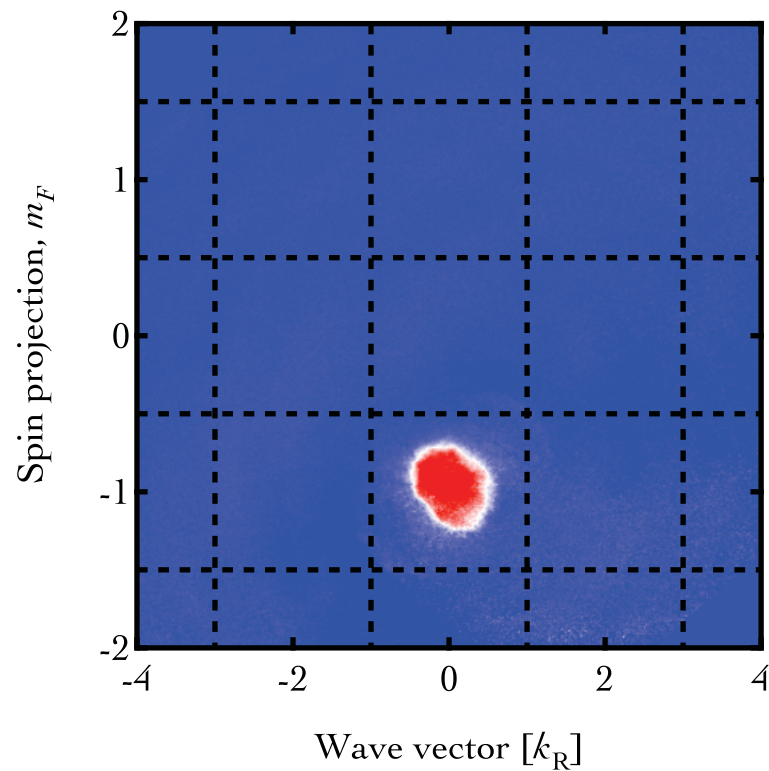


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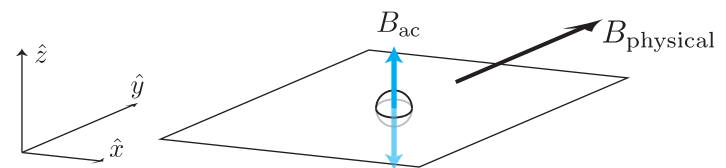
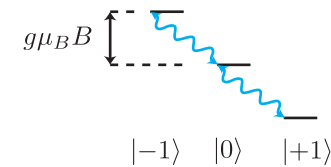
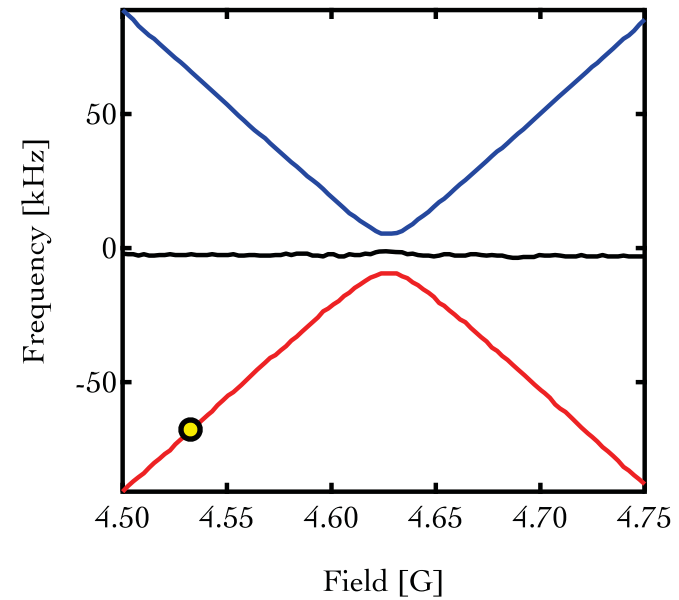
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RF dressed state (RF on, ramp  $B$  to resonance)



## RF Dressed

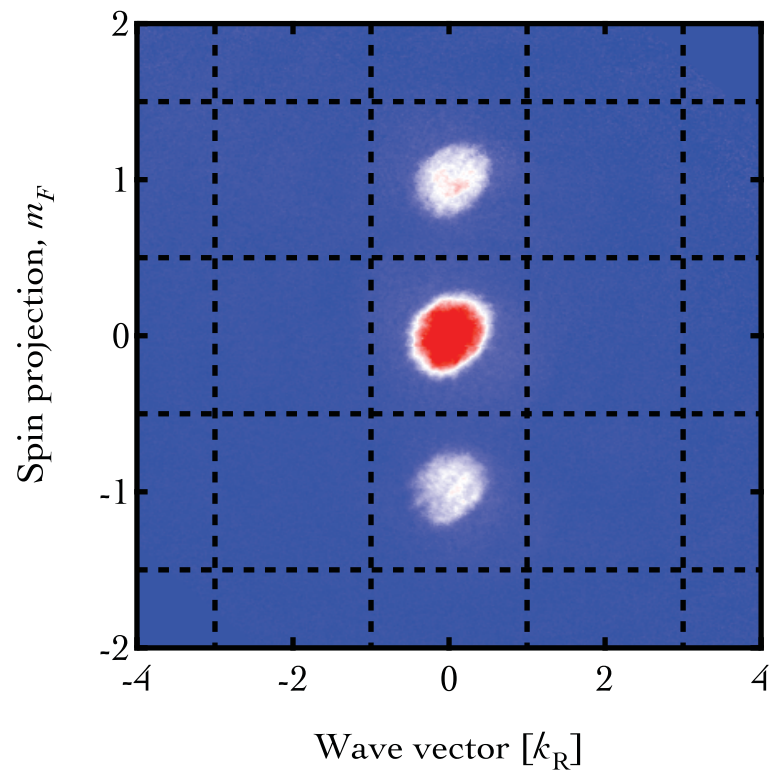


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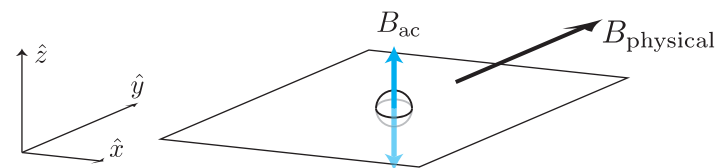
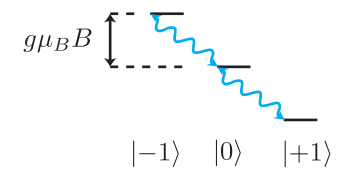
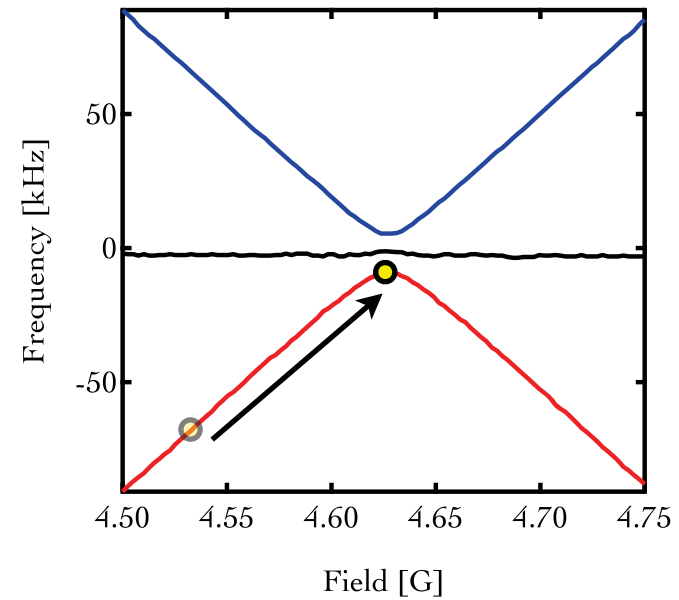
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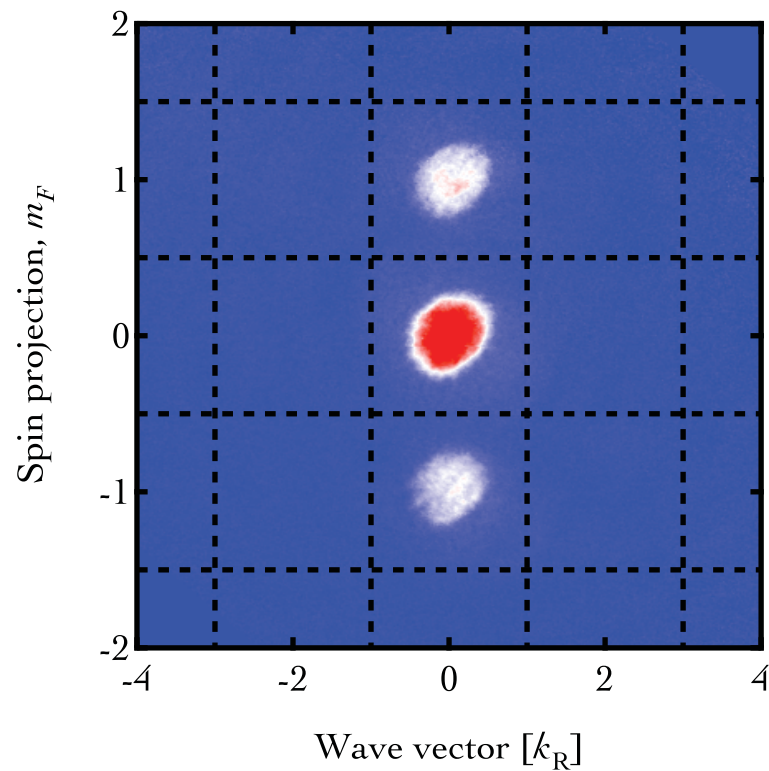


# Loading: momentum

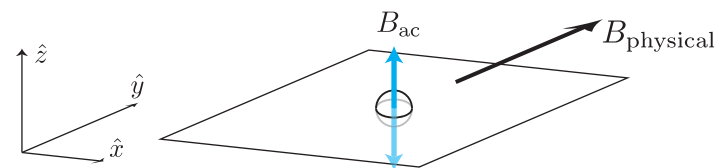
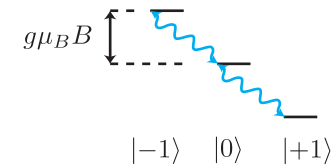
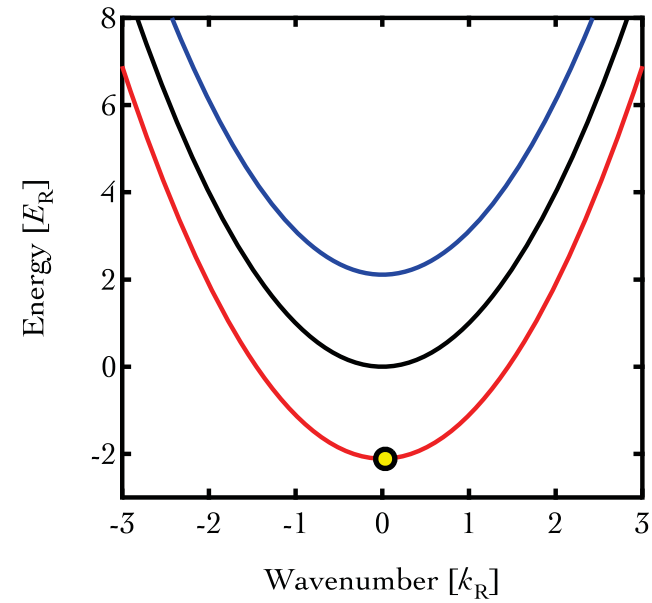
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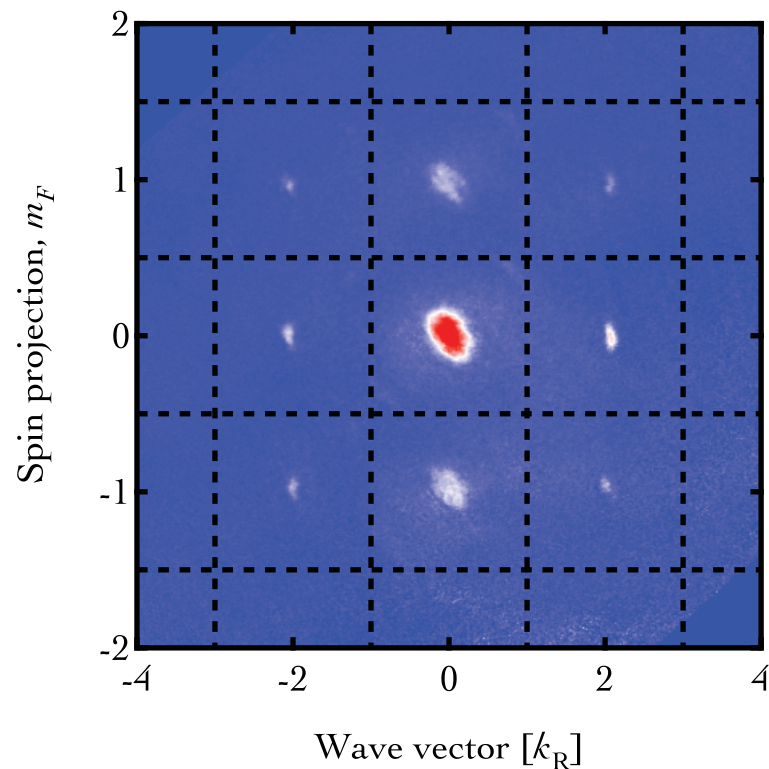
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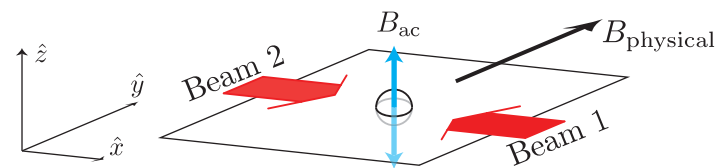
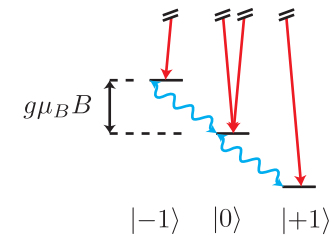
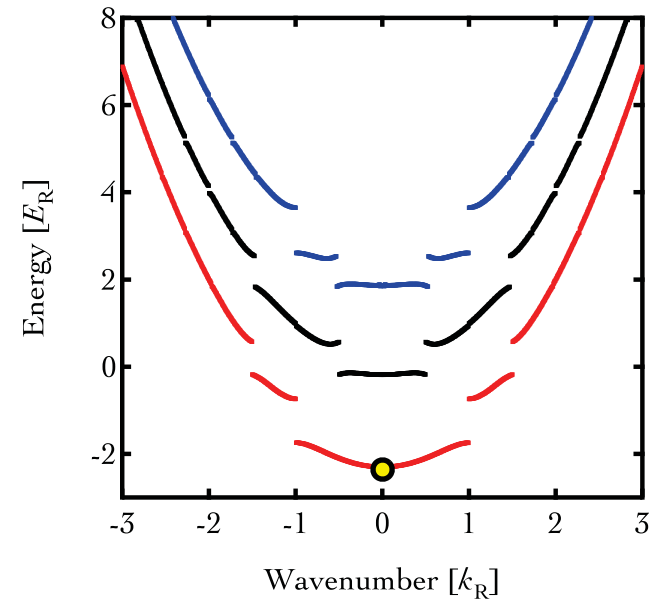
Initial state  $|F = 1, m_F = -1\rangle$

RF dressed state (RF on, ramp  $B$  to resonance)

Raman + RF dressed state (Ramp Raman on)



## Raman + RF Dressed



# Loading: momentum

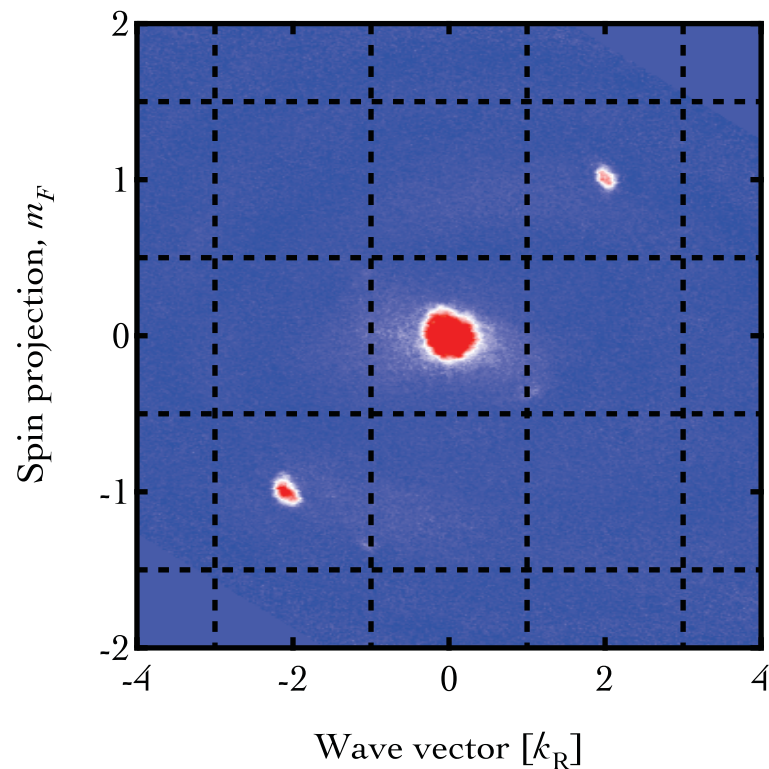
## Adiabatic manipulation of atoms

Initial state  $|F = 1, m_F = -1\rangle$

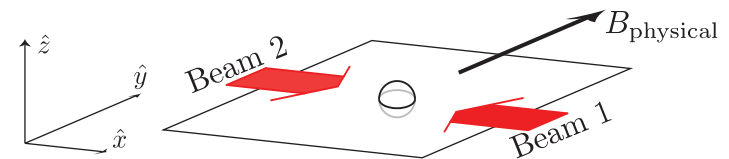
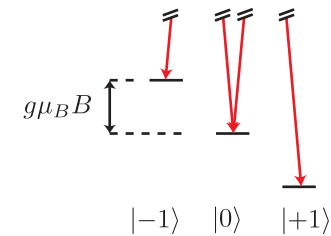
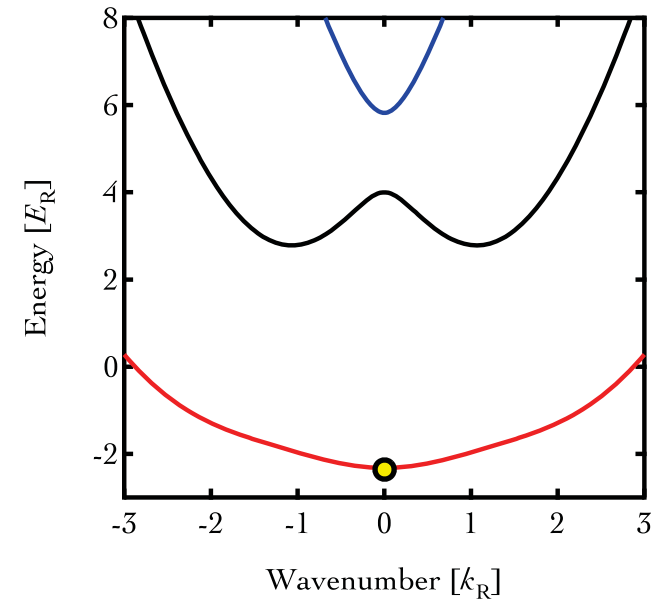
RF dressed state (RF on, ramp  $B$  to resonance)

Raman + RF dressed state (Ramp Raman on)

**Raman only dressed state (Ramp RF off)**



## Raman Dressed



# Displaced momentum distribution

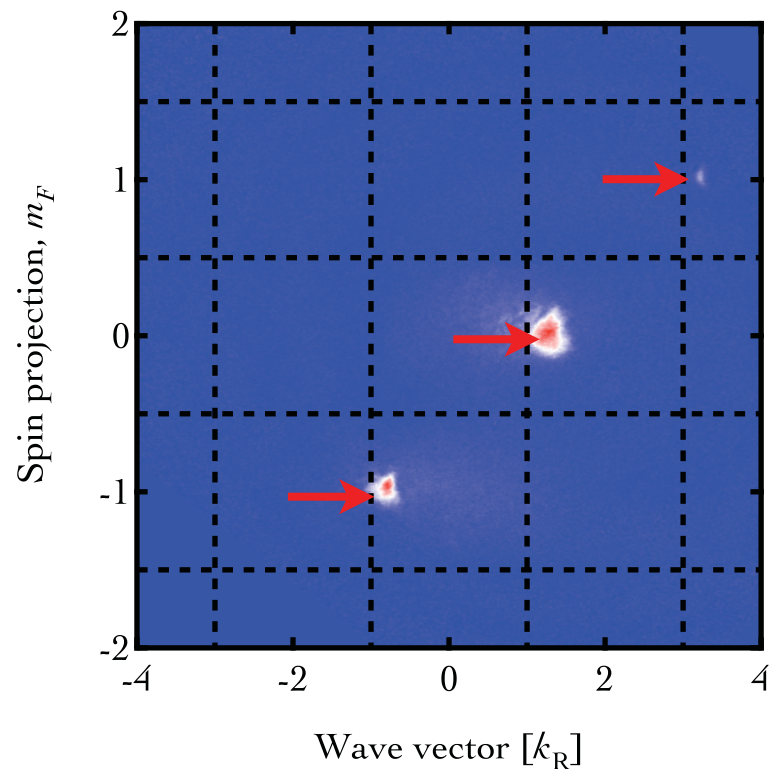
## Adiabatic manipulation of atoms

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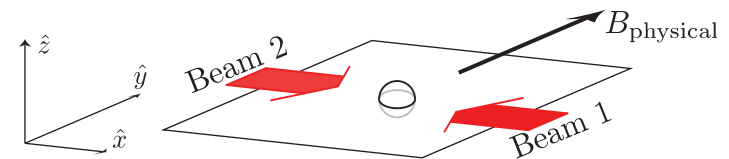
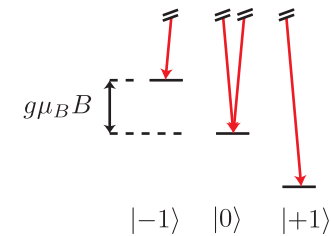
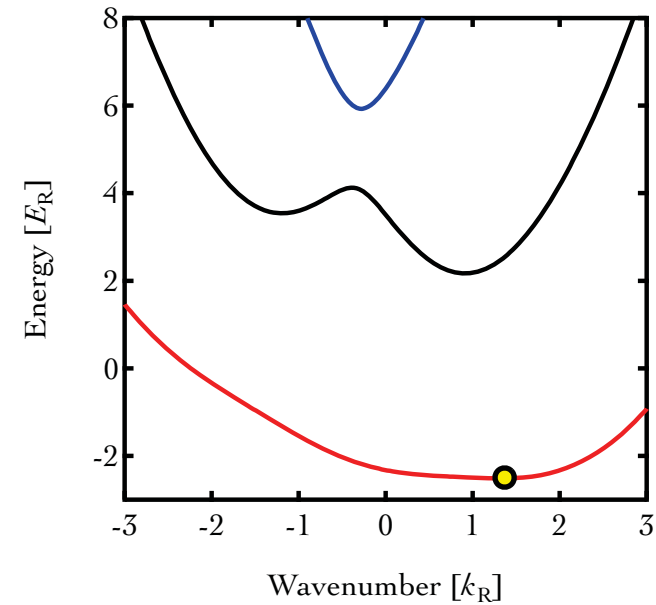
RF dressed state (RF on, ramp  $B$  to resonance)

Raman + RF dressed state (Ramp Raman on)

Raman only dressed state (Ramp RF off)



## Raman Dressed



# Displaced momentum distribution

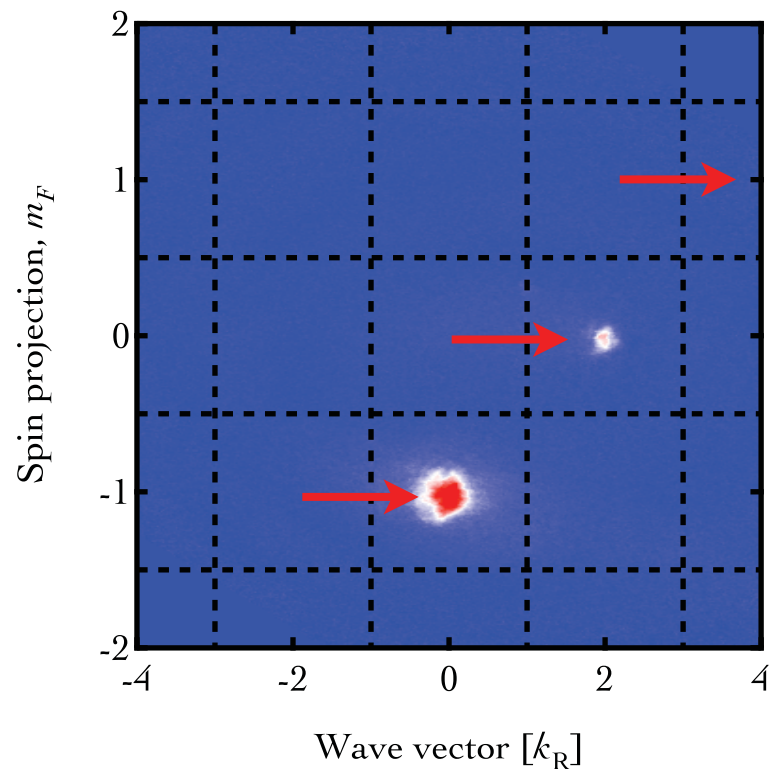
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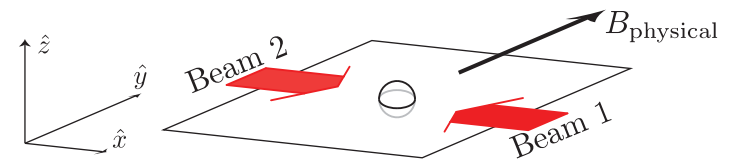
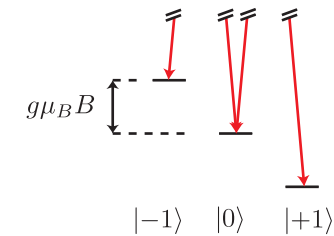
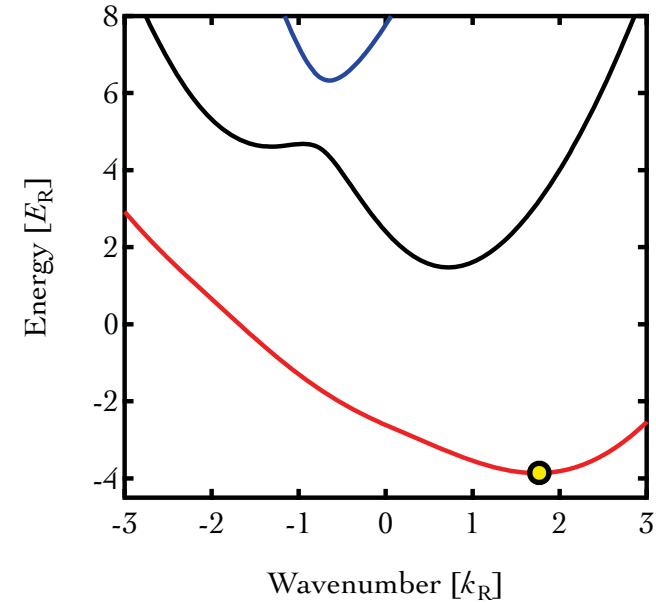
RF dressed state (RF on, ramp  $B$  to resonance)

Raman + RF dressed state (Ramp Raman on)

**Raman only dressed state (Ramp RF off)**



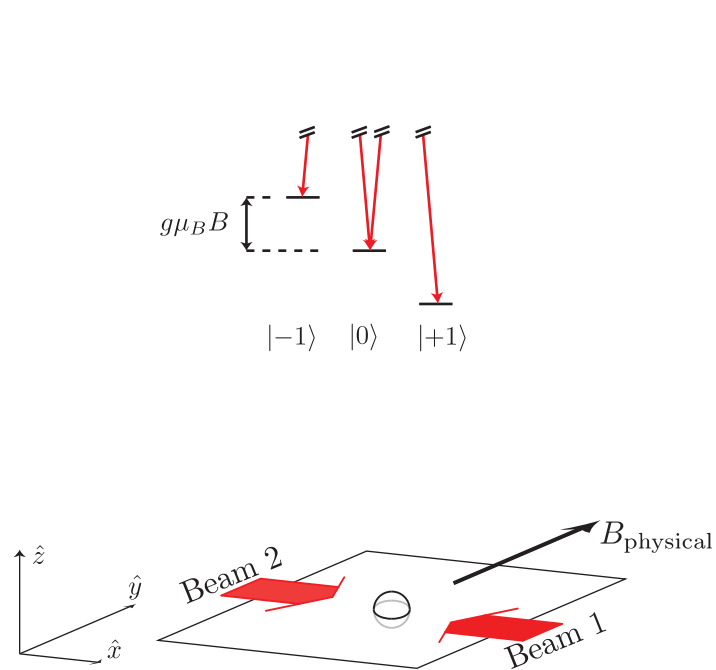
## Raman Dressed



# Reminder: dressed state vector potential

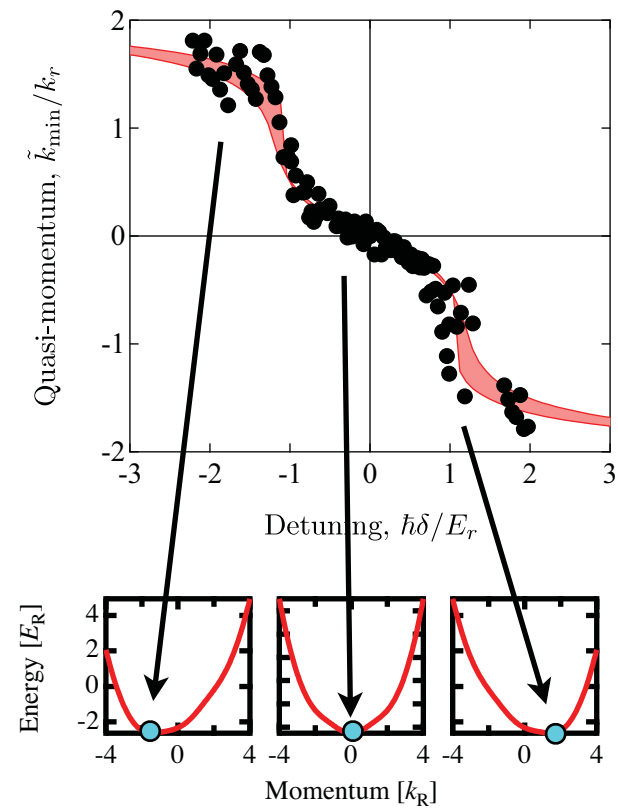
## Raman Coupling

Raman coupling between ground state manifold: “dressed” Energy-momentum curves.



## $E(k)$ minimum (large coupling)

Good agreement with theory



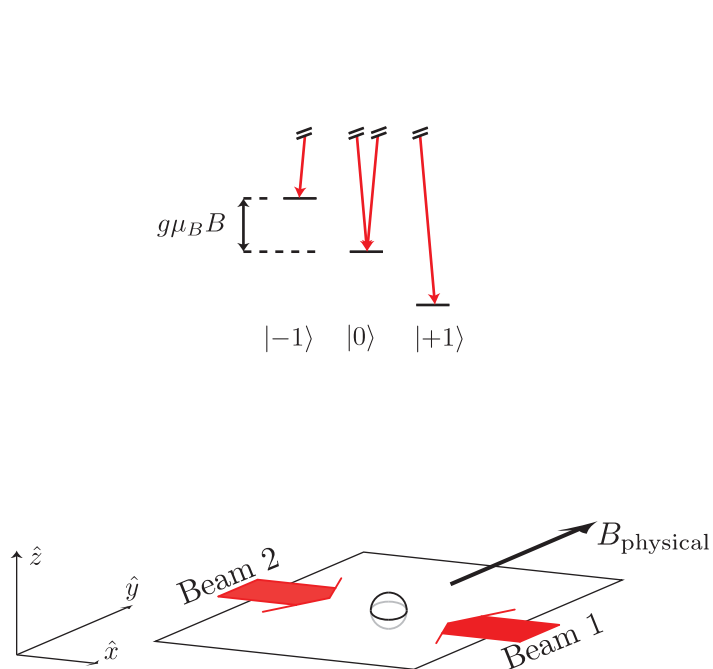
## References

- [1] Experiment: Y.-J. Lin et al, PRL **102** 130401 (2009)
- [2] Theory: IBS (Submitted to PRA)

# Neat digression: experiment

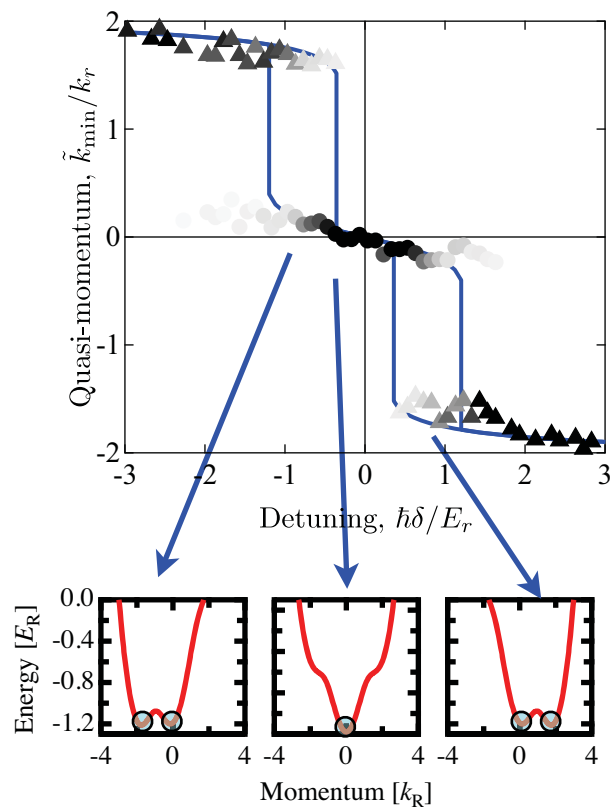
## Raman Coupling

Raman coupling between ground state manifold: “dressed” Energy-momentum curves.



## $E(k)$ minima (smaller coupling)

Still good agreement with theory



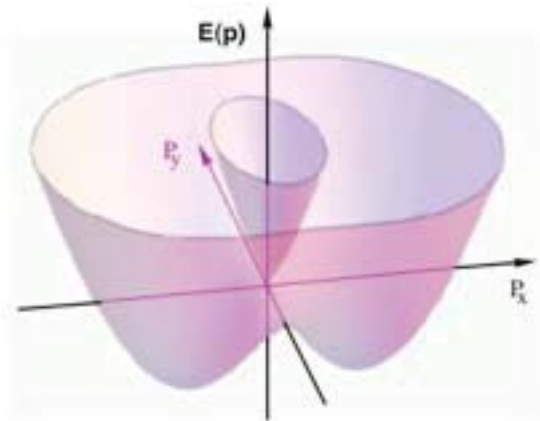
## References

- [1] Experiment: Y.-J. Lin et al, PRL **102** 130401 (2009)
- [2] Theory: IBS (Submitted to PRA)

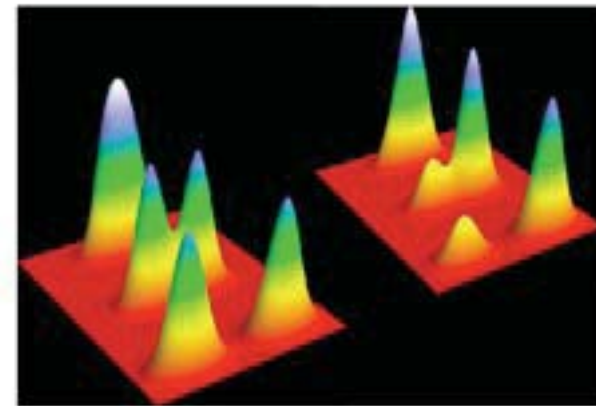
# Neat digression: theory

---

## Geometry



Two component BEC's  
Hey! This what we see.



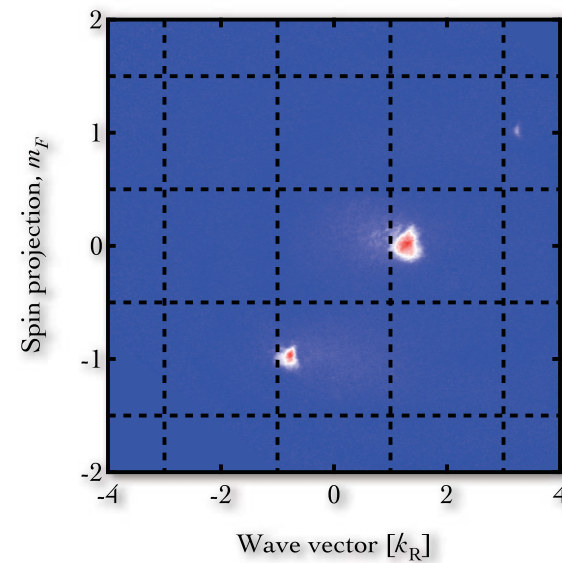
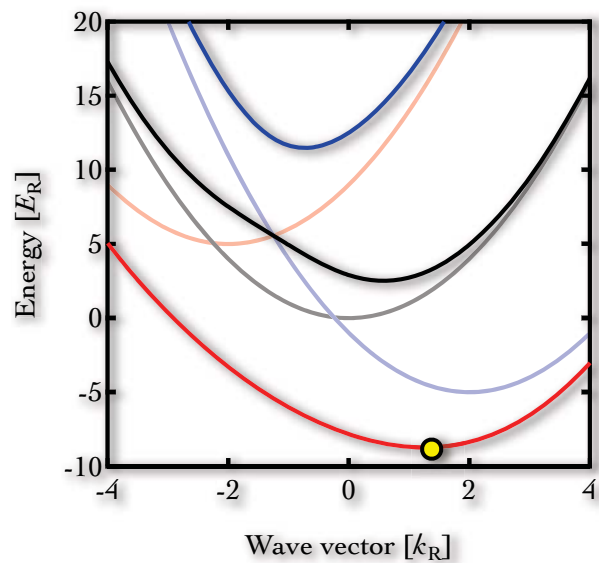
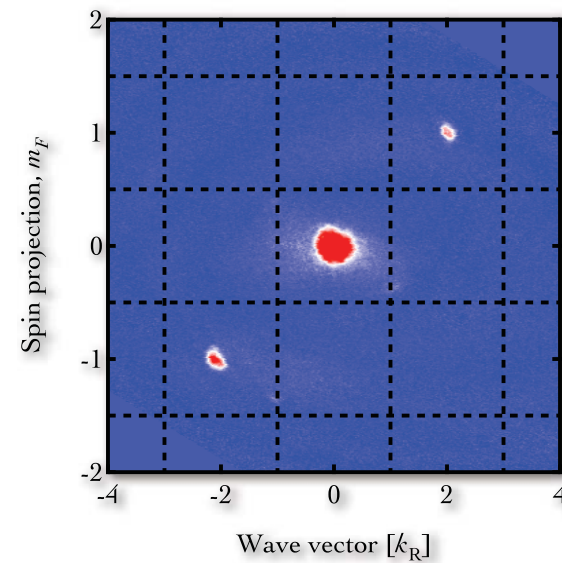
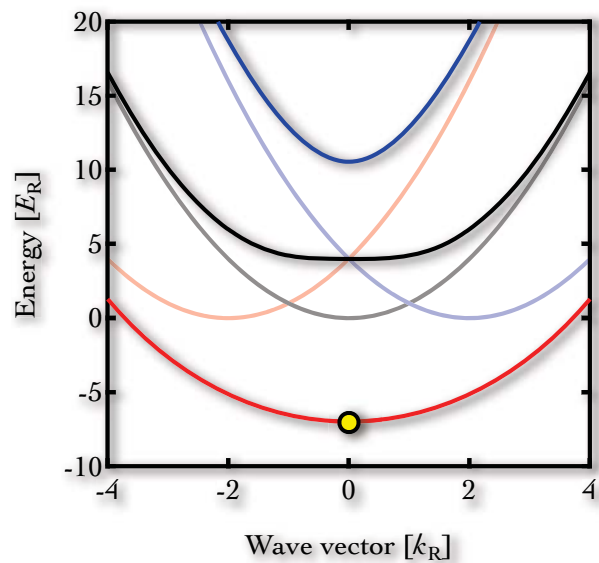
$x$  Momentum

## References

- [1] T. Stanescu and V. Galitski, Phys. Rev. A **78**, 023616 (2008)



# Atom light interaction: Summary

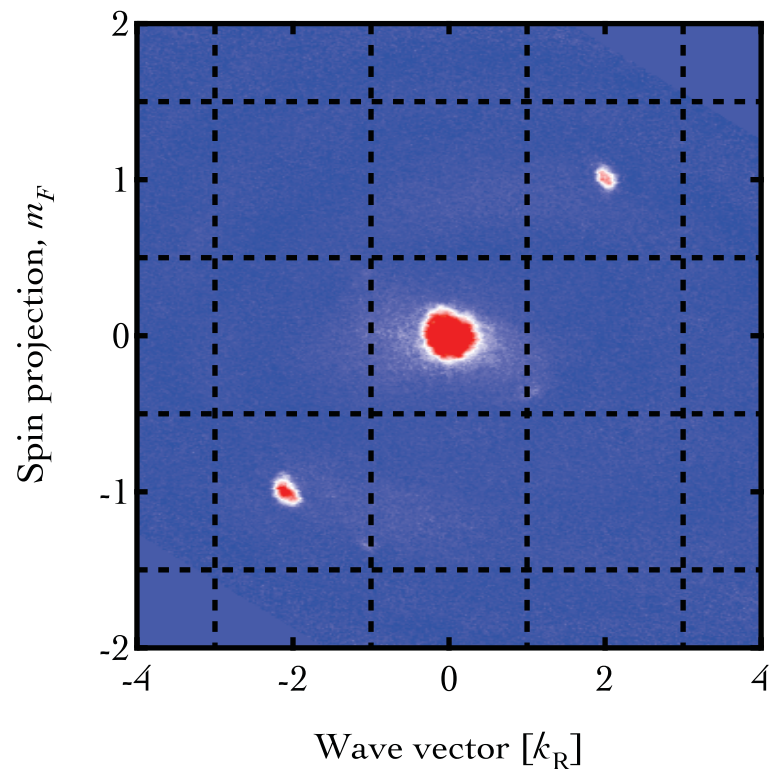


# Symmetric case

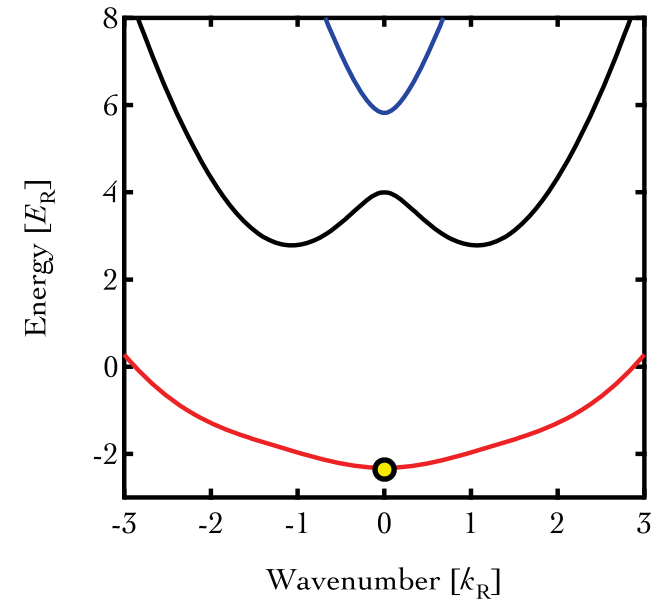
## Conserved

Abrupt turnoff conserves mechanical momentum

Mechanical momentum is averaged over all orders and is zero in equilibrium (of course).



## Raman Dressed

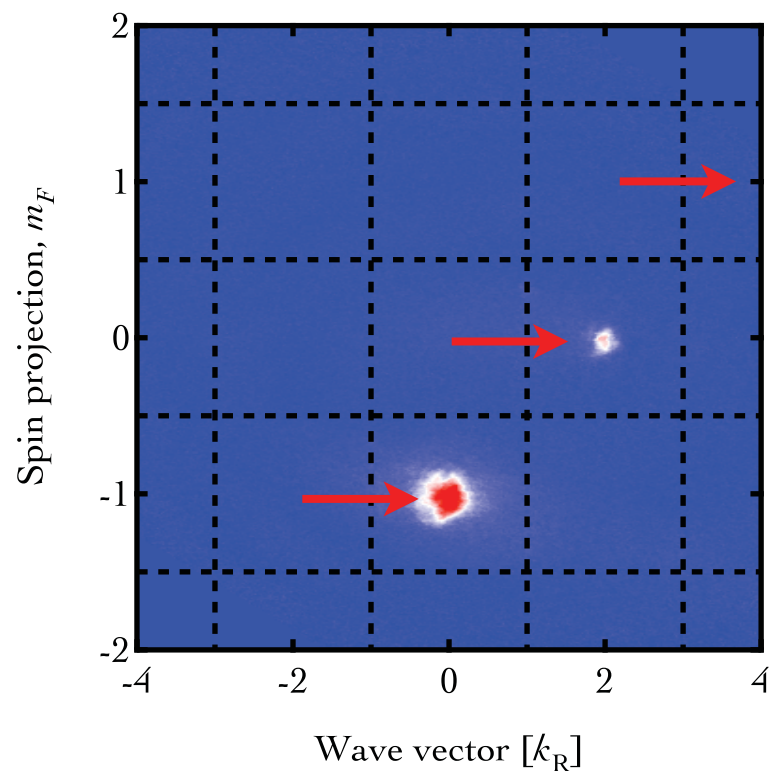


# Displaced momentum distribution

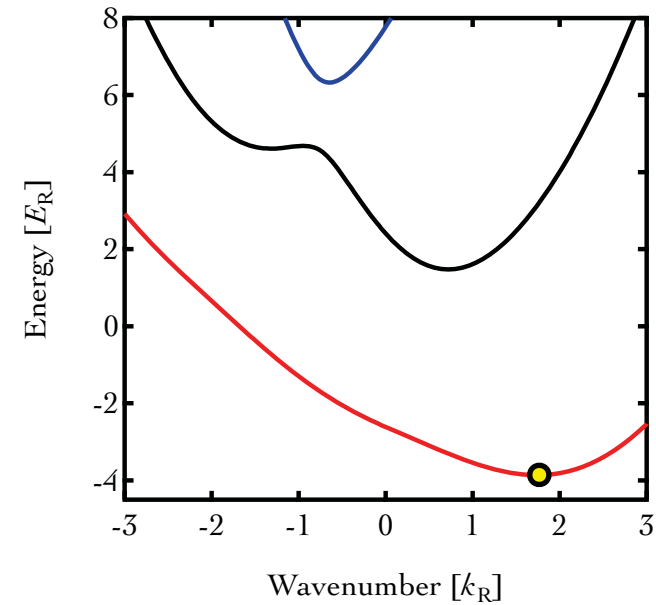
## Conserved

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## Raman Dressed



## Group velocity

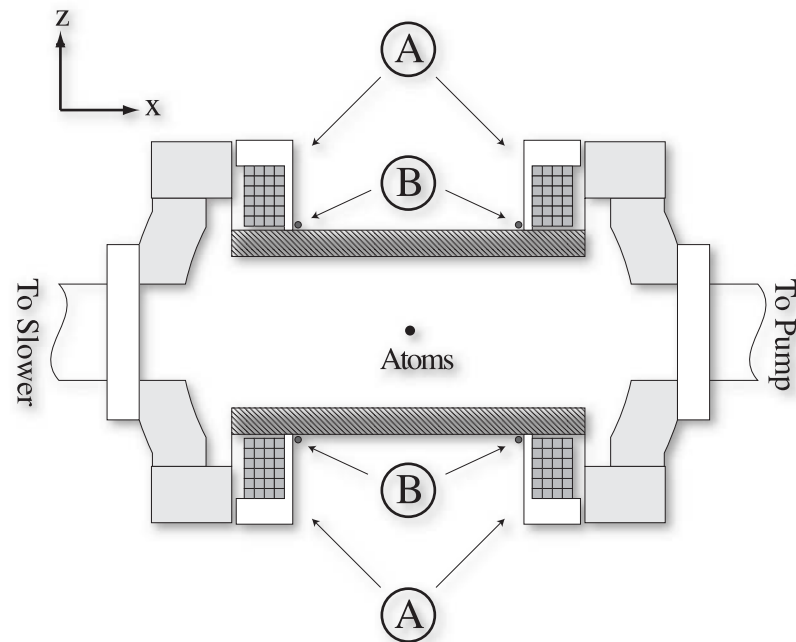
Since the 1st derivative is zero, a wave-packets group velocity is zero: no COM motion.

# Main point

## Idea

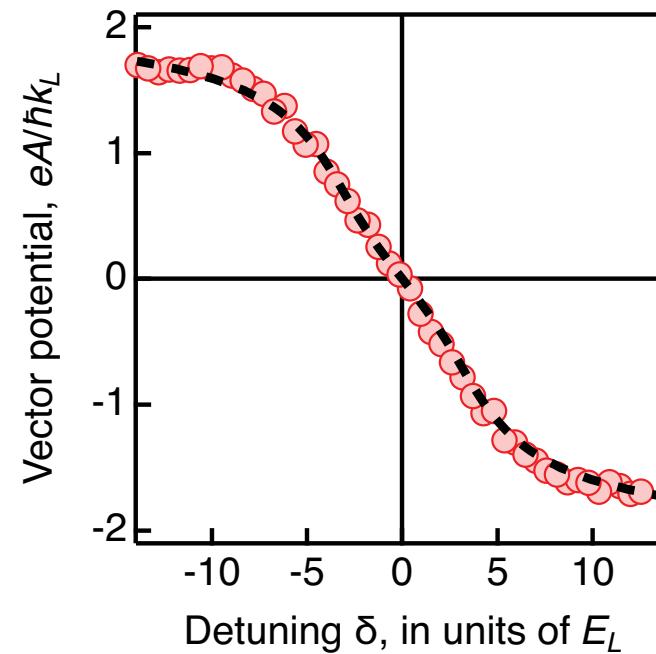
We can control the *synthetic* vector potential in time and space.

Bias and quadrupole  $B$  fields = offset and gradient in detuning.



## Transfer function

A given *local* detuning specifies the local synthetic vector potential



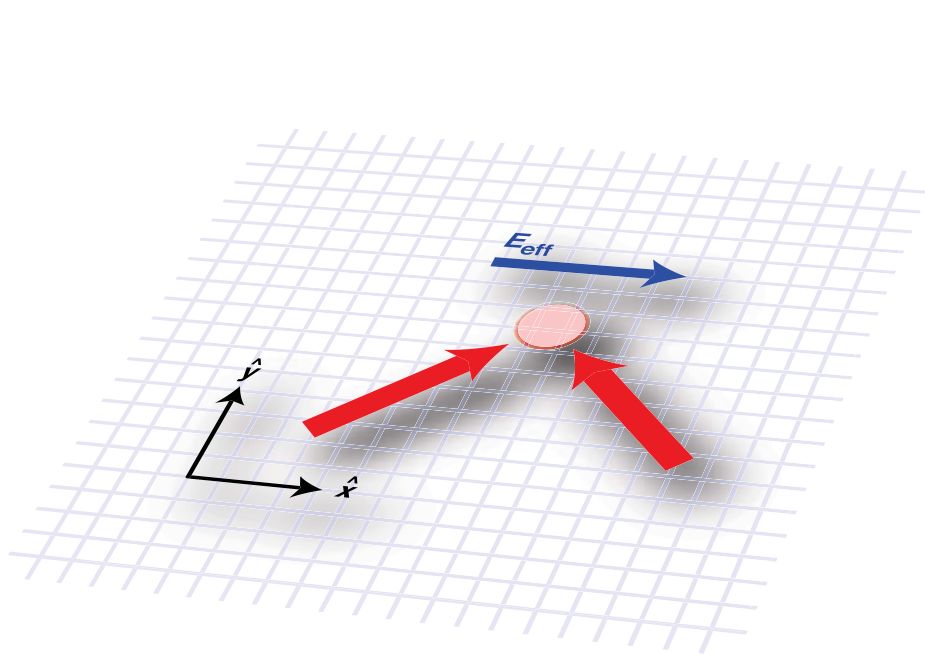
## References

[1] Y.-J. Lin et al, Submitted to PRA

# Electric fields

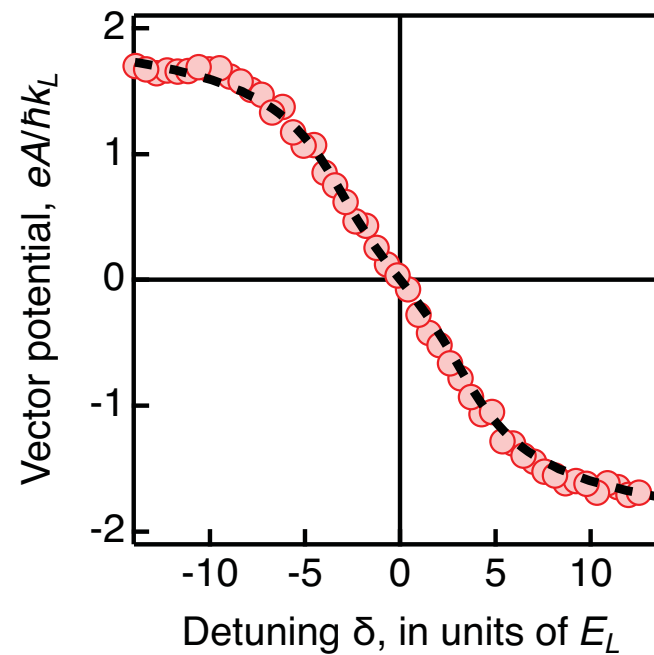
## Complete disclosure

Our beams now intersect at  $90^\circ$



## Transfer function

A given *local* detuning specifies the local synthetic vector potential



# Synthetic Electric Field

## A uniform vector potential: forces

Time dependence gives electric fields and forces

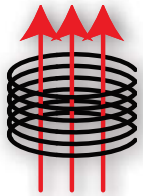
Make usual “quasi-static assumptions”

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$$

$$\Delta\mathbf{k} = \frac{e}{\hbar} \int \mathbf{E} dt = -\frac{e}{\hbar} \Delta\mathbf{A}$$

*Mechanical* not canonical momentum

Simple geometric example from grade-school



## Realization with dressed states

# Synthetic Electric Field

## A uniform vector potential: forces

Time dependence gives electric fields and forces

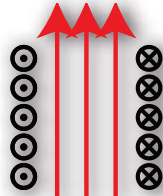
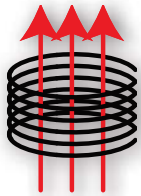
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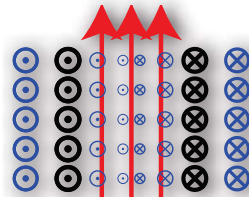
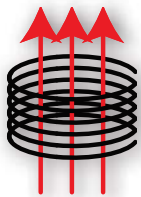
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## Realization with dressed states



# Synthetic Electric Field

## A uniform vector potential: forces

Time dependence gives electric fields and forces

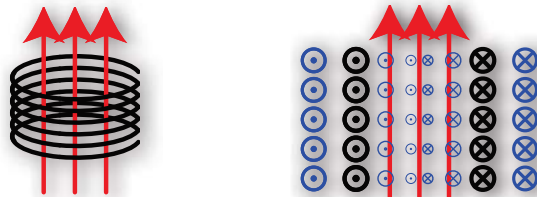
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Mechanical not canonical momentum

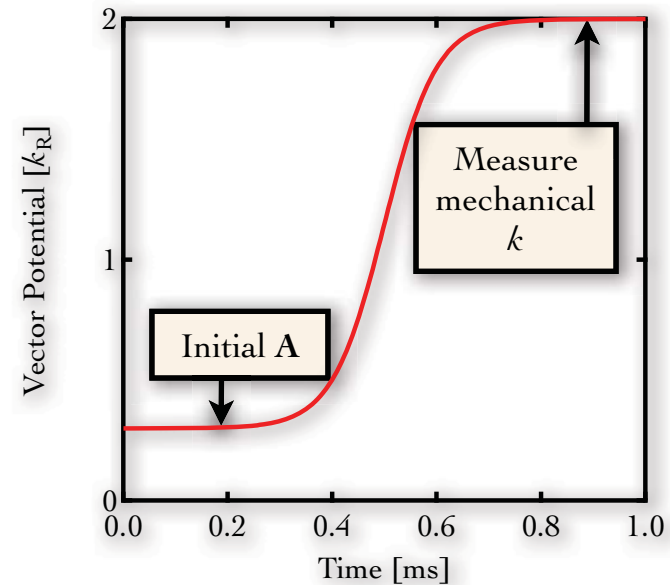
Simple geometric example from grade-school



## Realization with dressed states

*Experimental procedure*

1. Prepare initial state
2. Jump vector potential, always to  $k = 2k_R$
3. Measure mechanical momentum



# Synthetic Electric Field

## A uniform vector potential: forces

Time dependence gives electric fields and forces

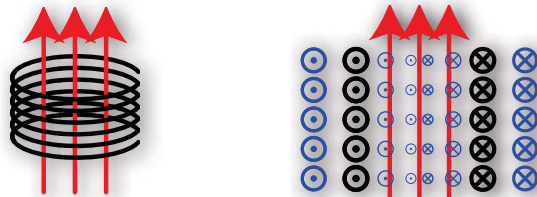
Make usual “quasi-static assumptions”

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$$

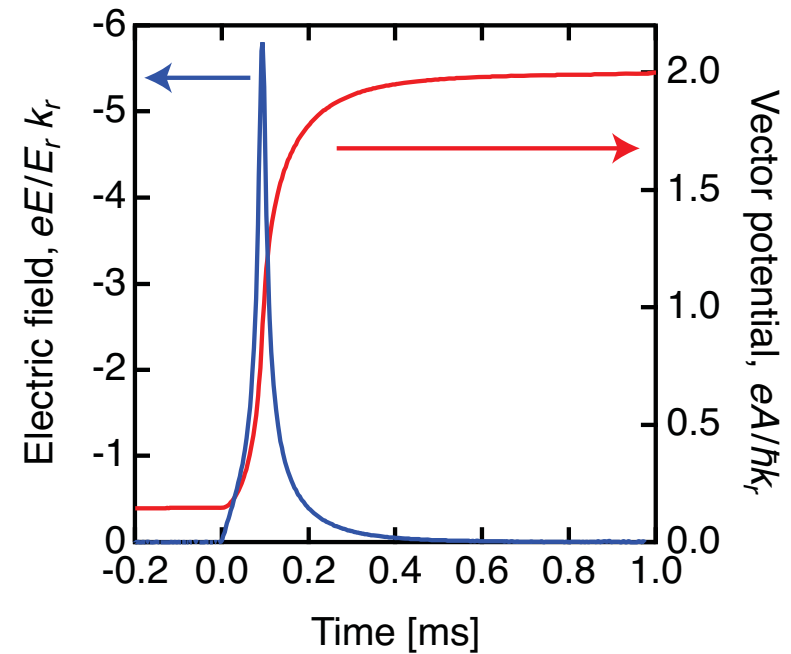
$$\Delta\mathbf{k} = \frac{e}{\hbar} \int \mathbf{E} dt = -\frac{e}{\hbar} \Delta\mathbf{A}$$

Mechanical not canonical momentum

Simple geometric example from grade-school



## Realization with dressed states



# Synthetic Electric Field

## A uniform vector potential: forces

Time dependence gives electric fields and forces

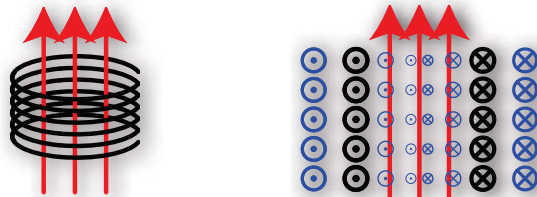
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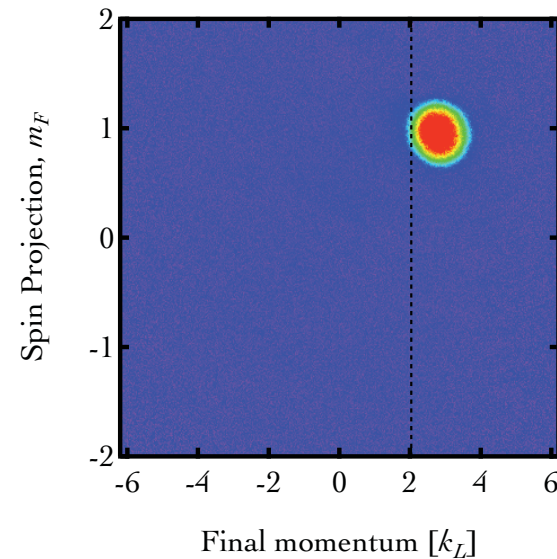
Mechanical not canonical momentum

Simple geometric examples from grade-school



## Realization with dressed states

Yes! Atoms acquire expected  $-2 k_R$  mechanical momentum kick.



Our synthetic vector potential behaves just like the real thing

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## A uniform vector potential: forces

Time dependence gives electric fields and forces

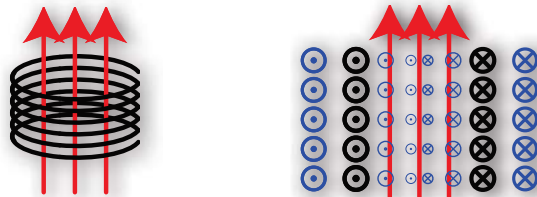
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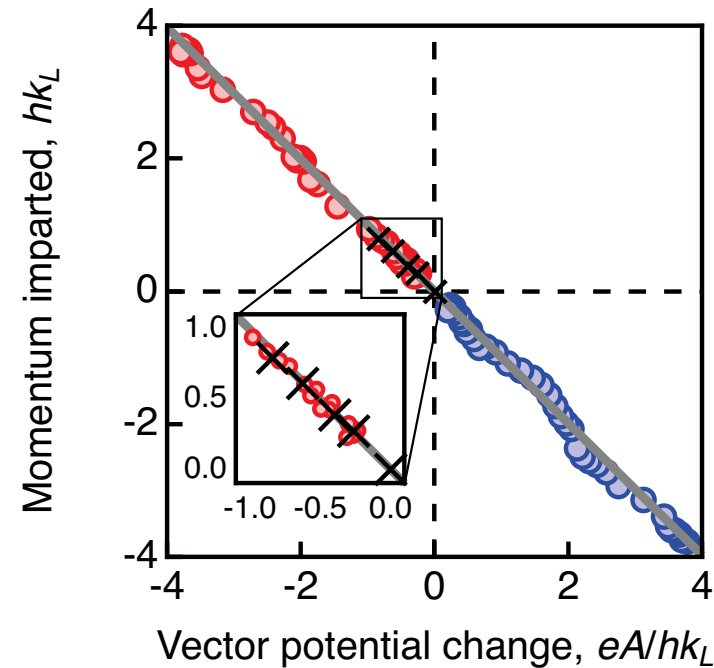
Mechanical not canonical momentum

Simple geometric examples from grade-school



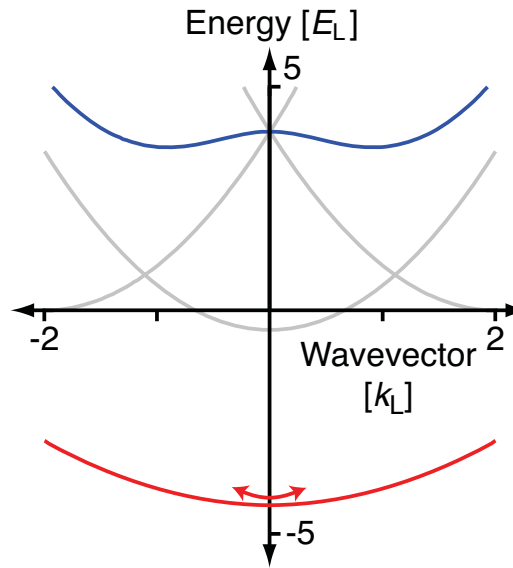
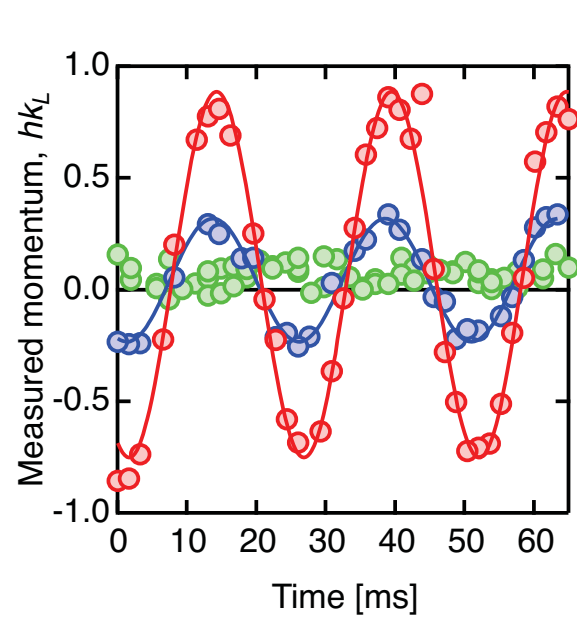
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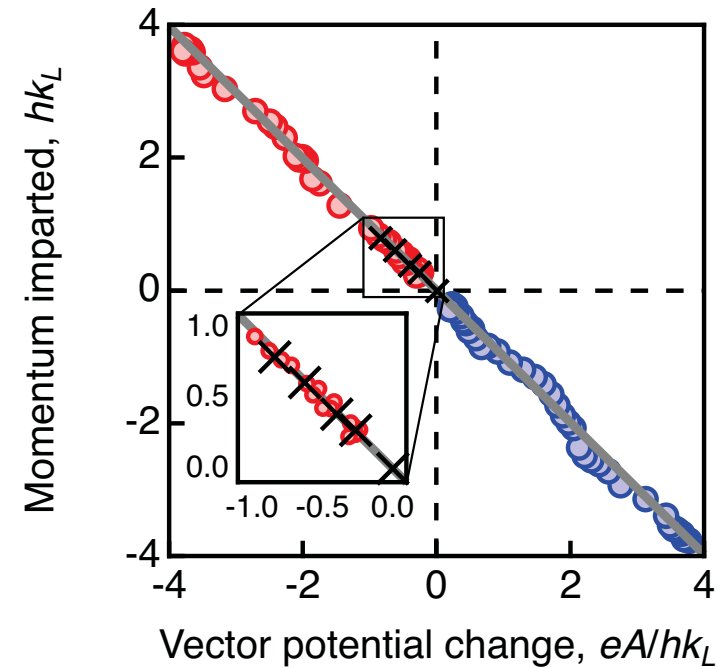
Our synthetic vector potential behaves just like the real thing

# Field in the dressed state

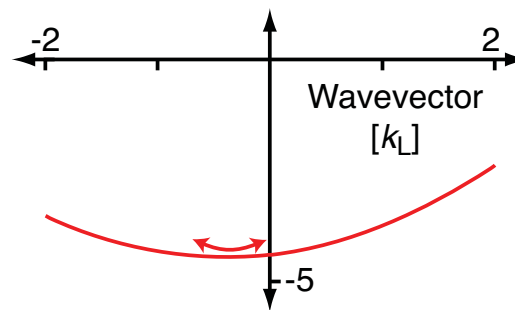
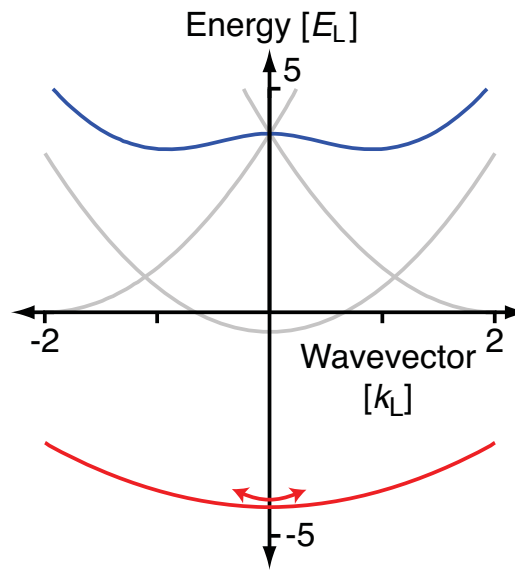
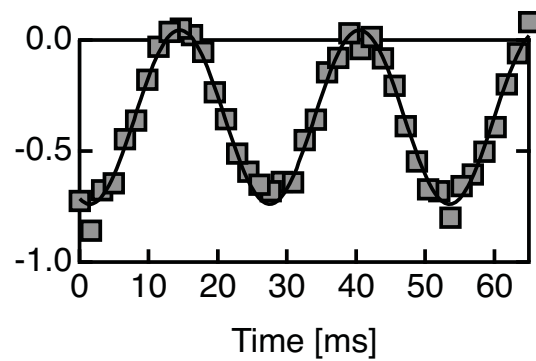
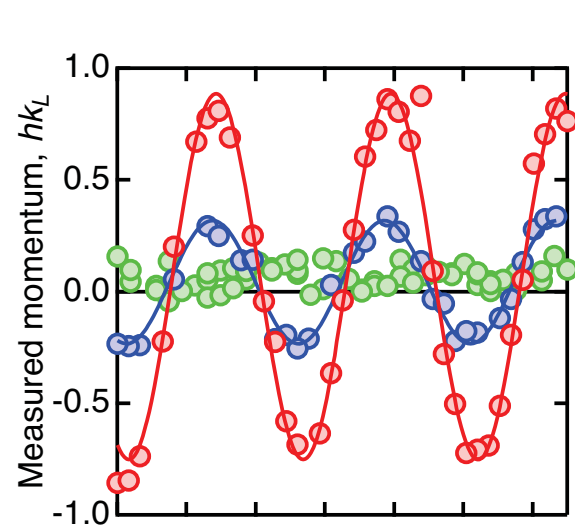


## Motion in dressed state

Even remaining in the dressed state the atoms feel the expected electric field "kick"



# Field in the dressed state: non-zero final $A$



## Motion about non-zero $k$

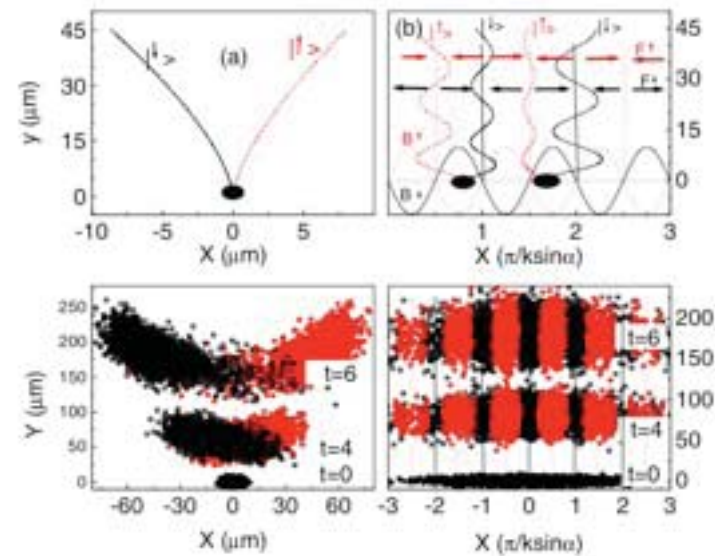
We really do measure effective canonical momentum in a specific gauge.

Remember canonical momentum is always an “observable” just not unique.

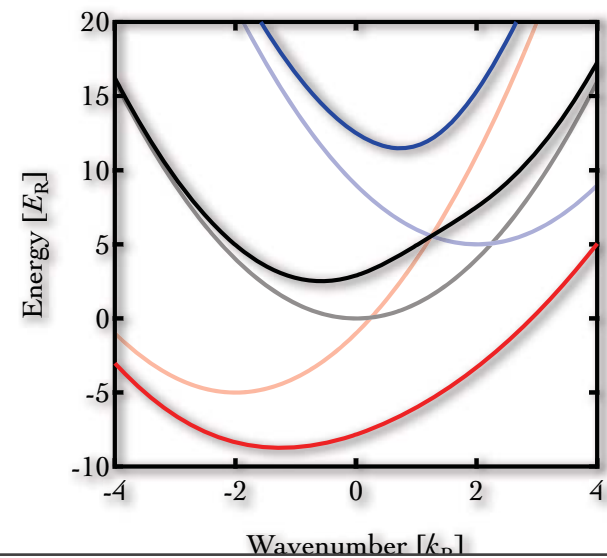
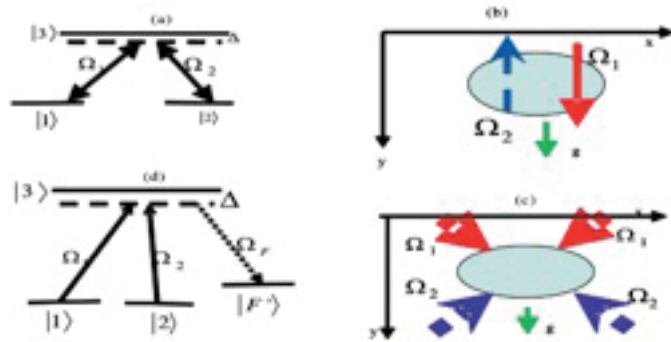
“Mechanical” momentum remains about zero.

# Other applications: spin-Hall physics

## Theory: "spin" dependant forces



## Geometry



## References

- [1] Shi-Liang Zhu et al., PRL 97 240401 (2006)

# New approaches for controlling cold atoms

We have been working on controlling terms in the *kinetic* energy.

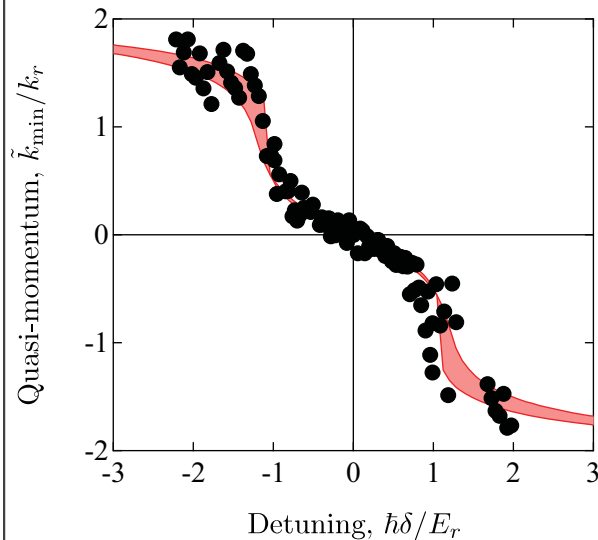
Not discussed today

Control of  $m^*$ :

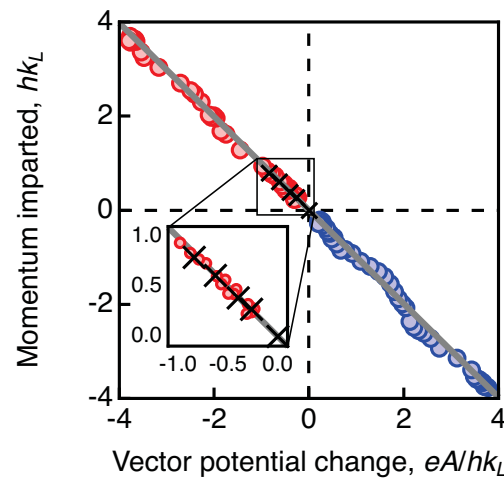
From 1 to  $\infty$ , and negative

$$\hat{H} = \sum_j \left[ \frac{\hat{p}_j^2}{2m} + V(x_j) \right] + \sum_{i < j} U(x_i - x_j)$$

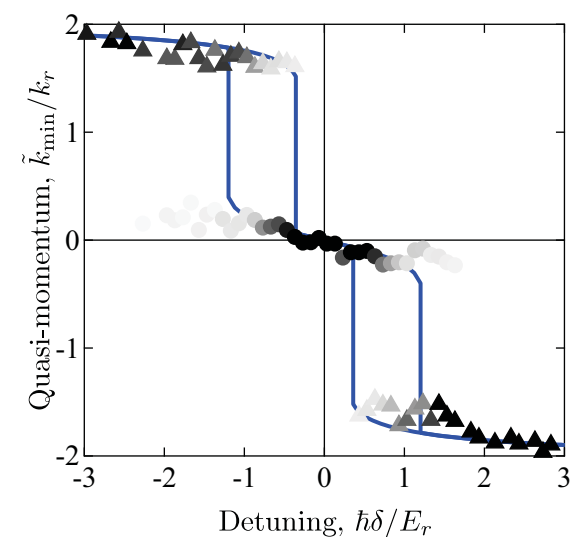
Synthetic vector potential



Synthetic electric field



Momentum double well



Our References

- [1] Experiment: Y.-J. Lin et al, PRL **102** 130401 (2009)
- [2] Electric field: in preparation
- [3] Theory: IBS (Submitted to PRA)



# Next experimental step: spatial gradients (in progress)

## Adiabatic manipulation of atoms



(1) Stabilize external magnetic fields and gradients  
(reduce unwanted heating and stabilize dressed state)

(2) Phase and intensity lock Raman coupling lasers  
(stabilize dressed state)

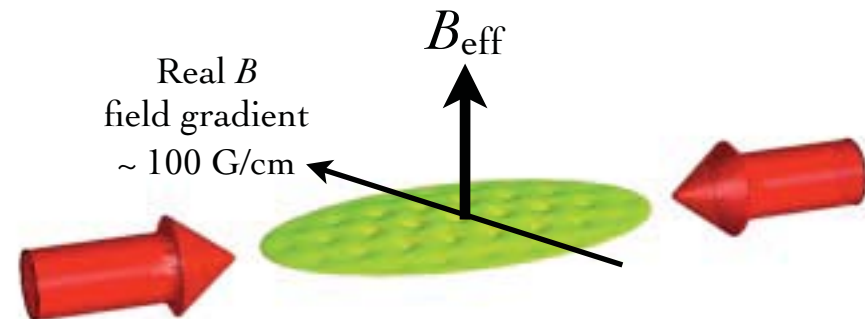
(3) Make dressed state actually dark  
(our system has both D1 and D2 excited states, but there exists a  
“real” dark state configuration detuned between D1 and D2)

## System

Start with a 2D BEC



Add Raman fields and a **spatial gradient** to create an effective magnetic field



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✓  
✓  
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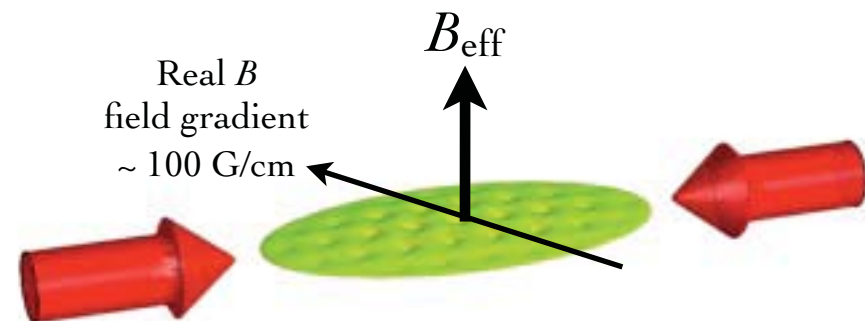
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Start with a 2D BEC



Add Raman fields and a **spatial gradient** to create an effective magnetic field



# In the lab!



Karina  
*"RF Slicer"*  
Jimenez-Garcia

IBS

Yu-Ju  
*"Fake fields"*  
Lin

Rob  
*"Feshbach"*  
Compton