The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2030-10

Conference on Research Frontiers in Ultra-Cold Atoms

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Number squeezing and entanglement in a Bose-Einstein condensate

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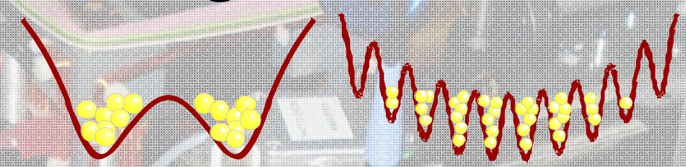
From 'simple' to 'fragmented' condensates

Number squeezing and entanglement

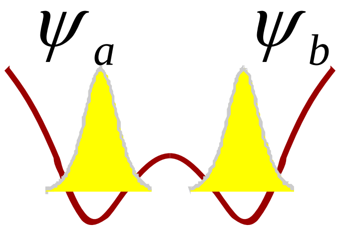
Slicing a BEC



Kirchhoff Institut für Physik
University Heidelberg

two mode approximation

meanfield

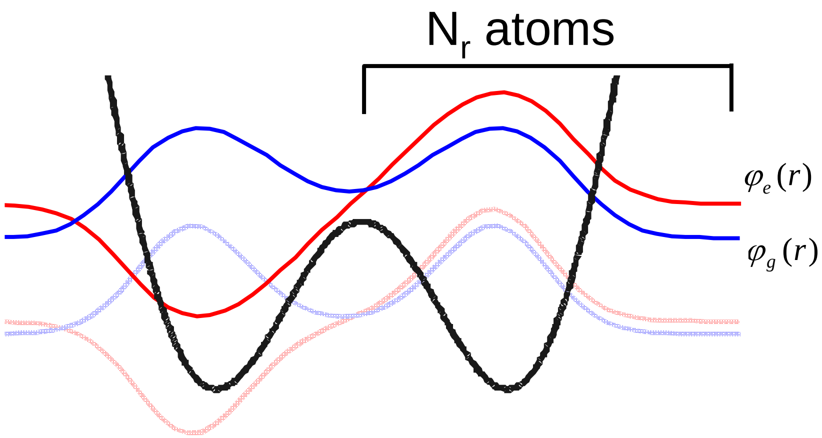


Gross-Pitaevskii equation

$$\imath\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\Delta + V_{\mathrm{ext}}(\mathbf{r},t) + g\left|\Psi(\mathbf{r},t)\right|^2\right]\Psi(\mathbf{r},t)$$

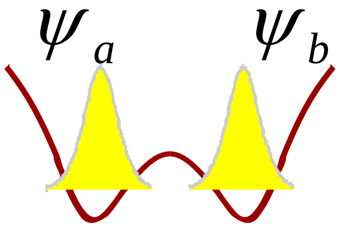
Two-mode ansatz

$$\Psi(\mathbf{r},t) = \psi_1(t)\varphi_1(\mathbf{r}) + \psi_2(t)\varphi_2(\mathbf{r})$$



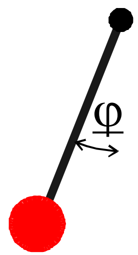
meanfield

two mode approximation



$\hat{=}$

pendulum analog



momentum shortened length

$$\Delta n = \frac{N_l - N_r}{2}$$
$$\varphi = \varphi_r - \varphi_l$$

$$H = E_c \Delta n^2 - E_j \sqrt{1 - \frac{4\Delta n^2}{N^2}} \cos\varphi$$

Charging energy: E_c

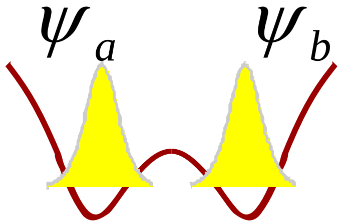
$$E_c \approx 4g \int |\Phi_l|^4 dr$$

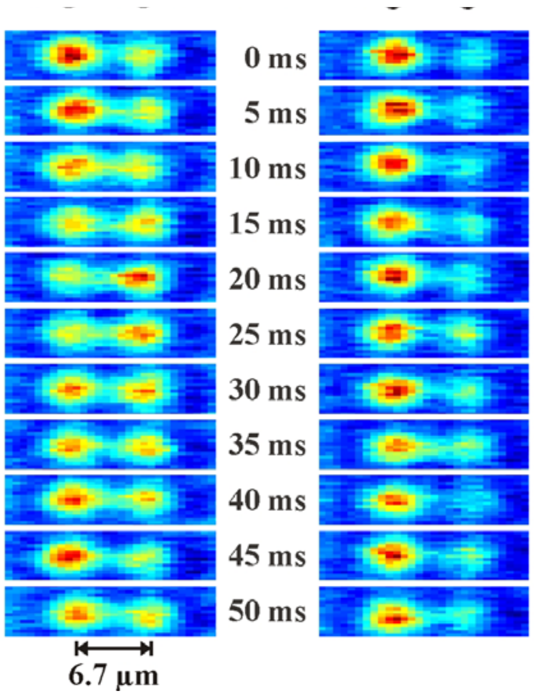
Josephson energy: E_j

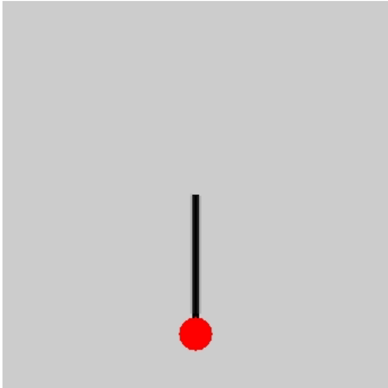
$$E_J \approx \frac{N}{2}(\mu_e - \mu_g) = NK$$

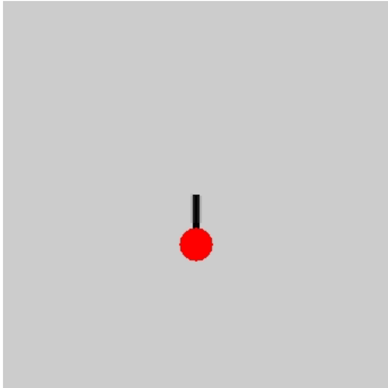
Josephson dynamics

meanfield



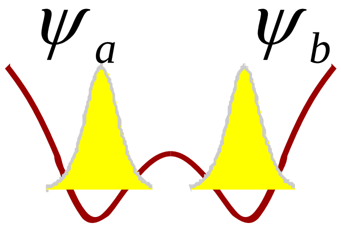






Phys. Rev. Lett. 95, 10402 (2005)

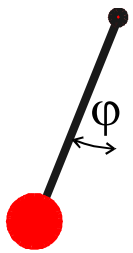
meanfield & beyond



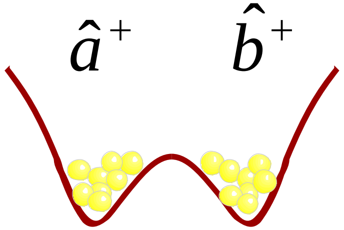
$$\Delta n = \frac{N_l - N_r}{2}$$
$$\varphi = \varphi_r - \varphi_l$$

$$H \cong E_c \Delta n^2 - E_j \cos\varphi$$

pendulum analog



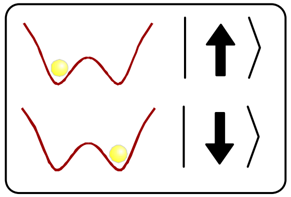
two mode Bose Hubbard



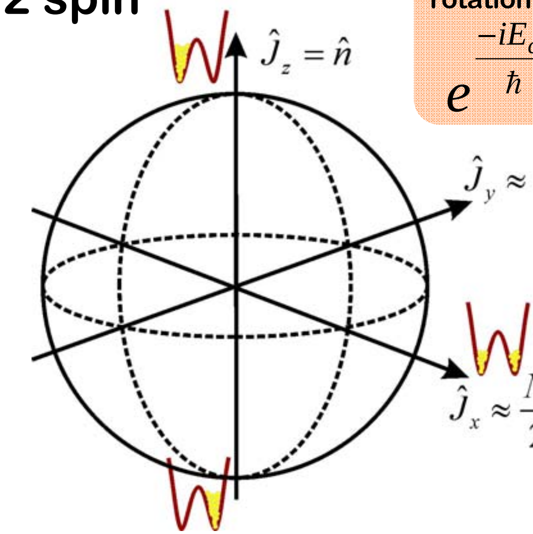
$$\hat{H} = \frac{E_c}{4}(\hat{a}^+\hat{a} - \hat{b}^+\hat{b})^2 - K(\hat{a}^+\hat{b} + \hat{b}^+\hat{a})$$

$$\hat{H} = E_c \hat{J}_z^{\,2} - 2K\,\hat{J}_x$$

(Fig. 1) (Fig. 2)



J=N 1/2 spin



rotation z axis

$$e^{\frac{-iE_c t}{\hbar}\hat{J}_z \hat{J}_z}$$

(Fig. 1)

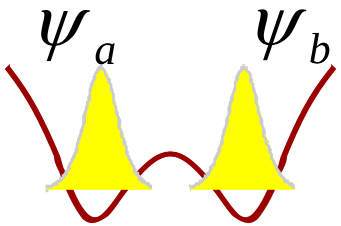
rotation x axis

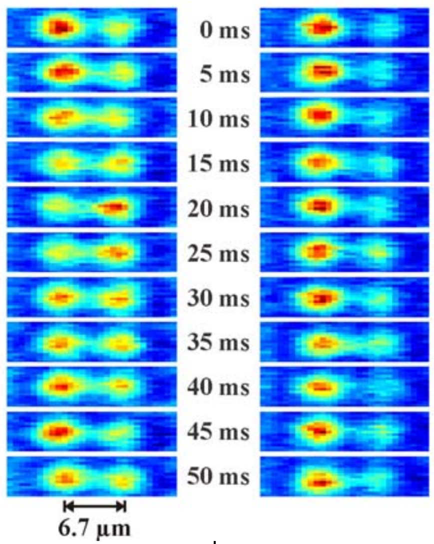
$$e^{\frac{i2Kt}{\hbar}\hat{J}_x}$$

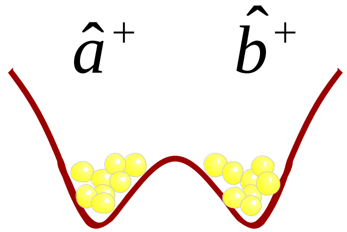
(Fig. 2)

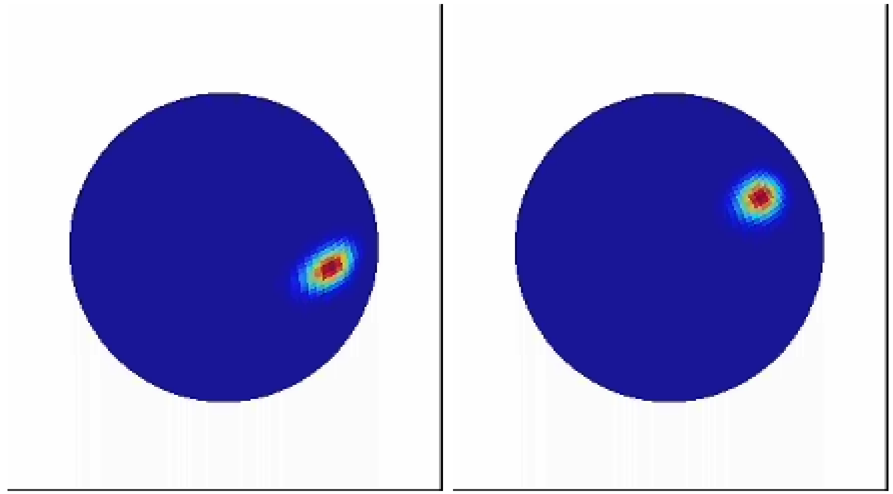
Josephson dynamics

meanfield/Bose Hubbard



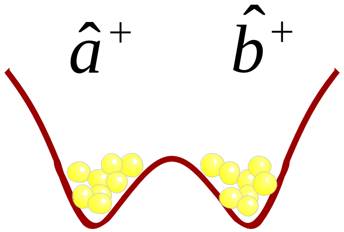






number squeezing

ground state properties

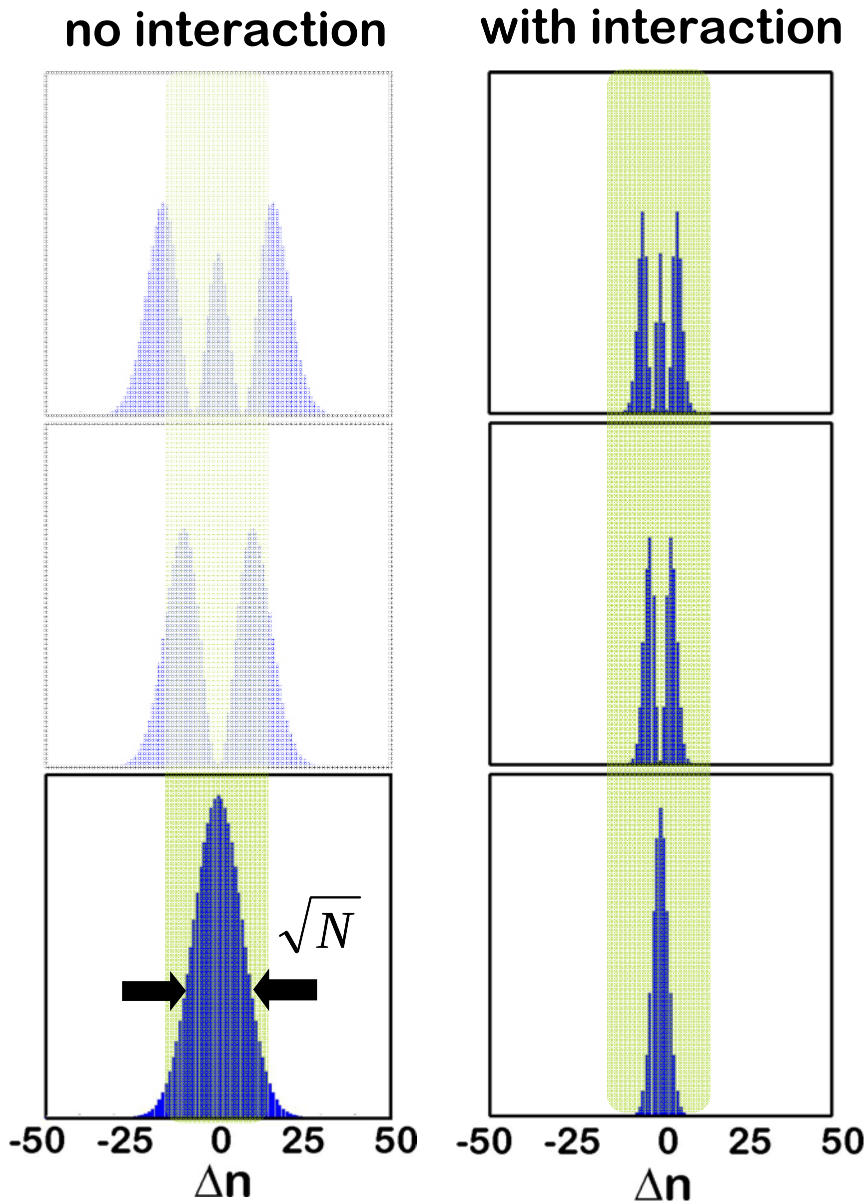


$$\hat{H} = \frac{E_c}{4}(\hat{a}^{+}\hat{a} - \hat{b}^{+}\hat{b})^2 - K(\hat{a}^{+}\hat{b} + \hat{b}^{+}\hat{a})$$

interaction

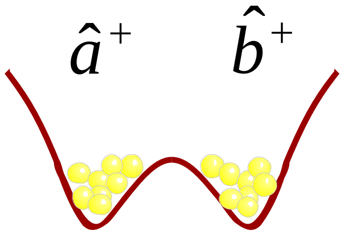
tunneling

$$|\Psi\rangle \cong (\hat{a}^{+} + \hat{b}^{+})^N |vac\rangle$$



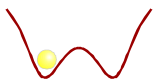
spin squeezing

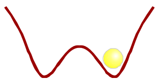
introduction



$$\hat{H} = \frac{E_c}{4}(\hat{a}^+\hat{a} - \hat{b}^+\hat{b})^2 - K(\hat{a}^+\hat{b} + \hat{b}^+\hat{a})$$

interaction — tunneling

 $|\uparrow\rangle$

 $|\downarrow\rangle$

N Spin 1/2

$$|\Psi\rangle = |\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\ldots\rangle$$

$$\hat{H} = E_c \hat{J}_z^{\,2} - 2K\,\hat{J}_x$$

$$\hat{a}^+ = \sqrt{n_a}\,e^{i\varphi_a}$$

$$\hat{J}_x = \frac{1}{2}\left(\hat{a}^+\hat{b} + \hat{b}^+\hat{a}\right)$$

$$\hat{J}_y = \frac{1}{2i}\left(\hat{a}^+\hat{b} - \hat{b}^+\hat{a}\right)$$

$$\hat{J}_z = \frac{1}{2}\left(\hat{a}^+\hat{a} - \hat{b}^+\hat{b}\right)$$

Bose Hubbard

$$(\Delta J_z)^2(\Delta J_y)^2 \geq \frac{1}{4}\left|\langle J_x\rangle\right|^2$$

$$(\Delta n)^2(\Delta\varphi)^2 \geq \frac{1}{4}$$

$$\hat{J}_x \cong \sqrt{n_a n_b}\cos\varphi$$

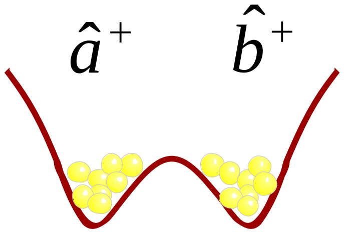
$$\hat{J}_y \cong \sqrt{n_a n_b}\sin\varphi$$

$$\hat{J}_z \cong \frac{n_a - n_b}{2}$$

'mean field limit'

'optics-like' squeezing

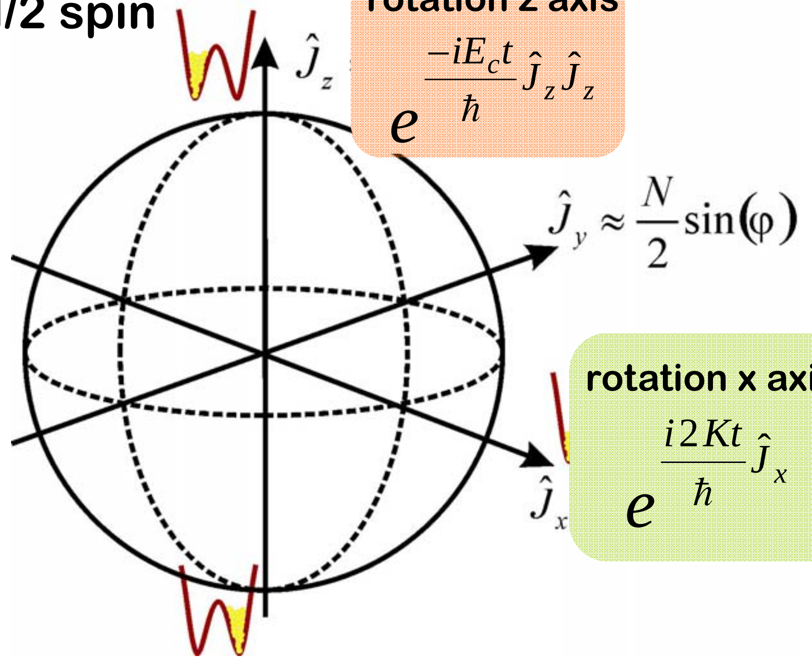
implementation



$$\hat{H} = E_c \hat{J}_z^{\,2} - 2K \hat{J}_x$$

(Fig. 2) (Fig. 1)

J=N/2 spin

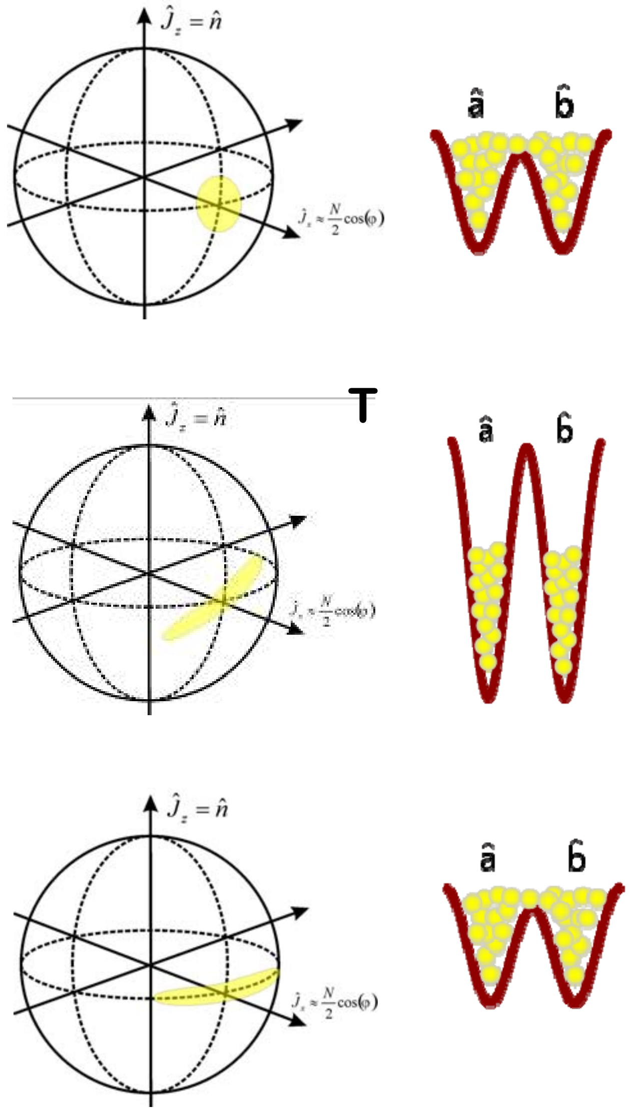


rotation z axis

$$e^{\frac{-iE_c t}{\hbar}\hat{J}_z\hat{J}_z}$$

rotation x axis

$$e^{\frac{i2Kt}{\hbar}\hat{J}_x}$$

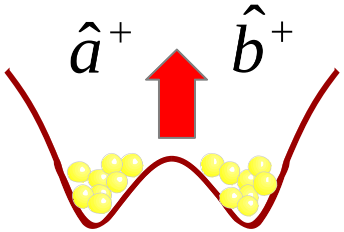


(Fig. 1)

(Fig. 2)

adiabatic spin squeezing

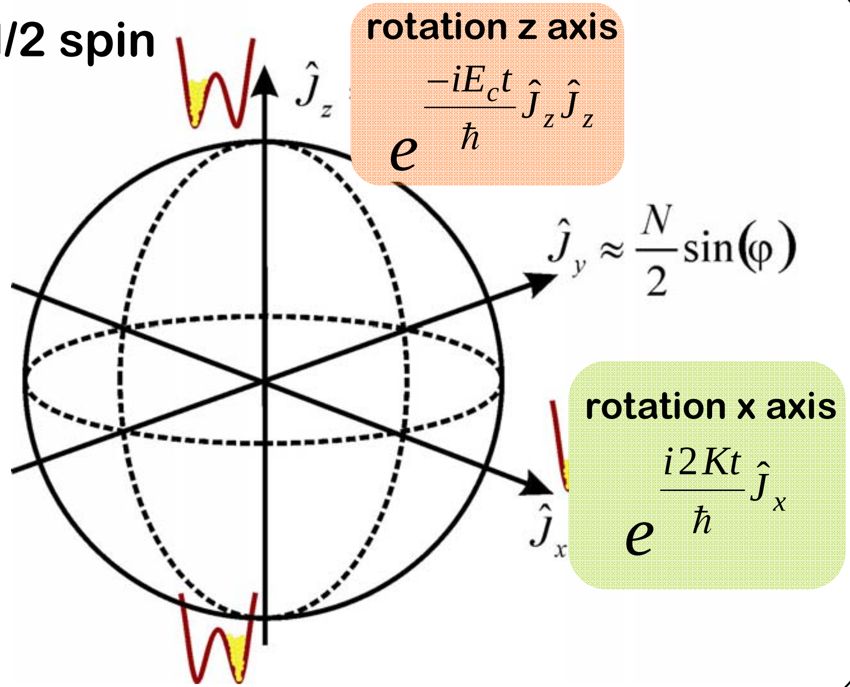
implementation

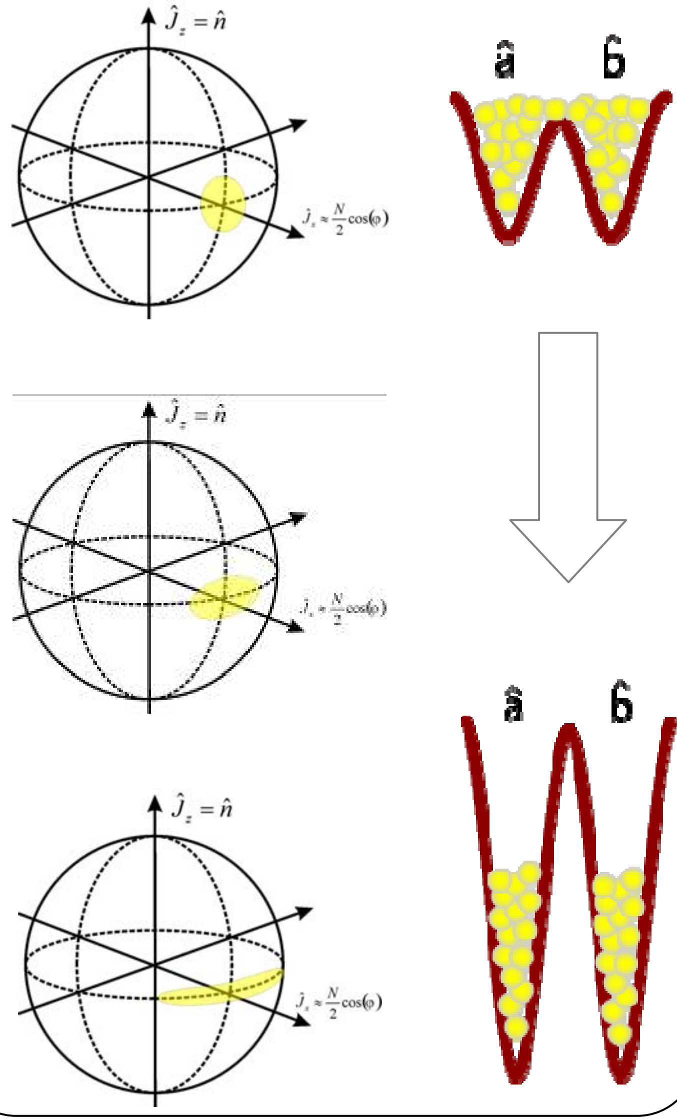


change

$$\hat{H} = E_c \hat{J}_z^{\,2} - 2K\hat{J}_x$$

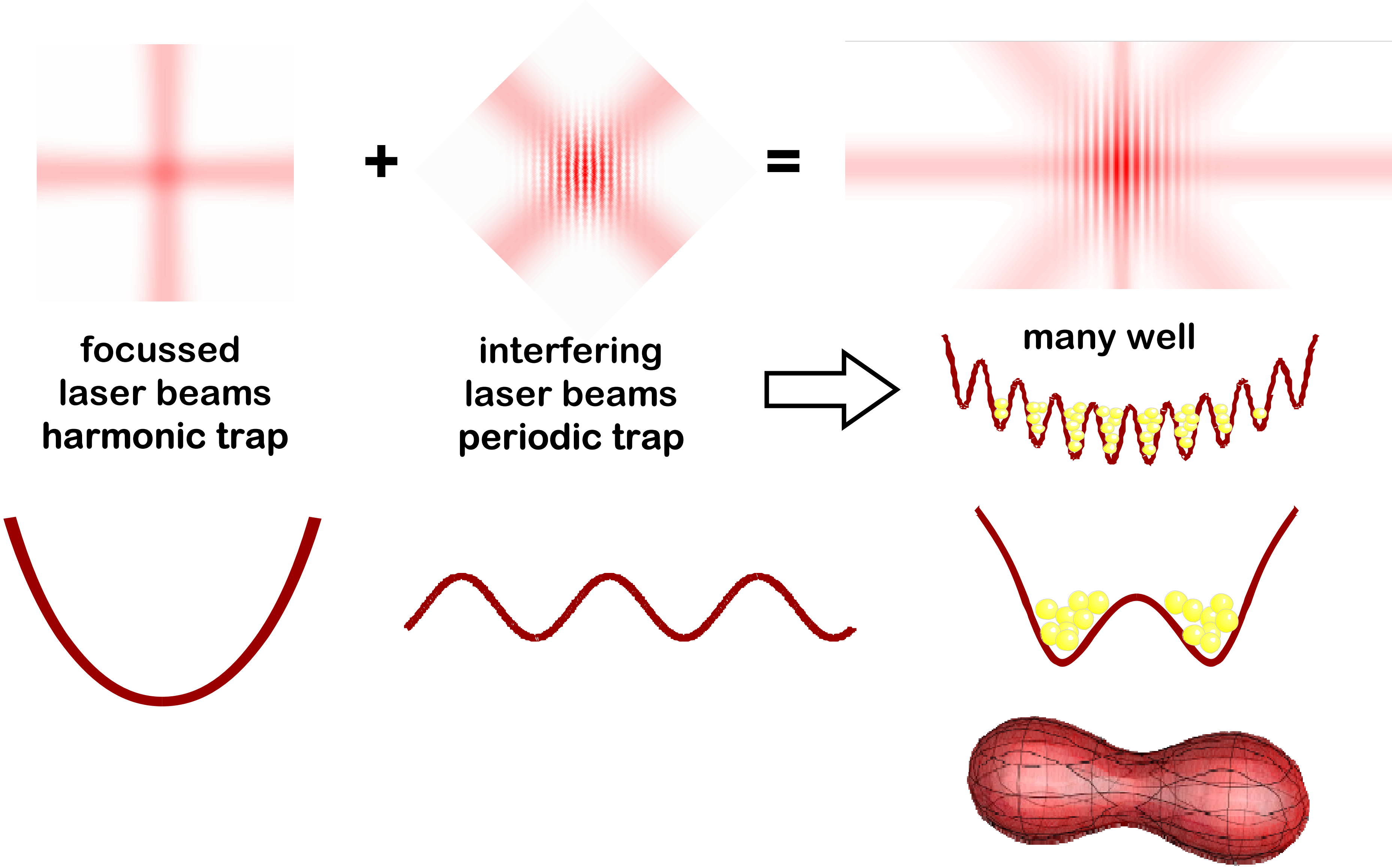
J=N/2 spin





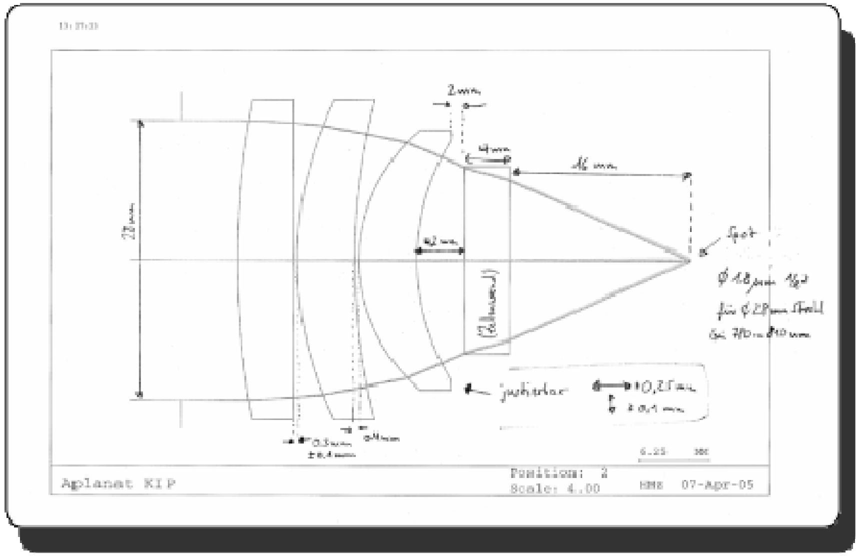
adding light fields

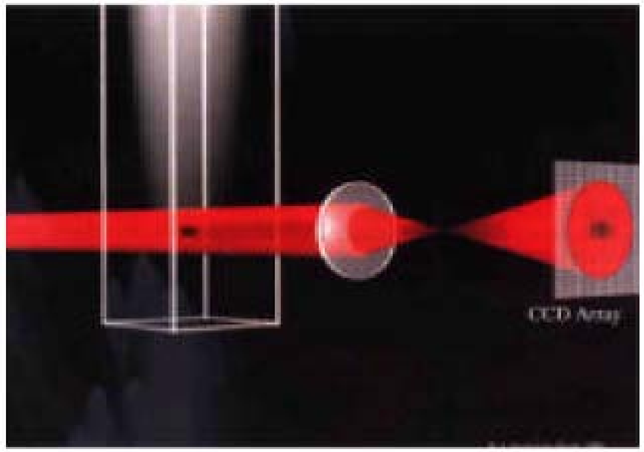
slicing BEC



imaging with high resolution

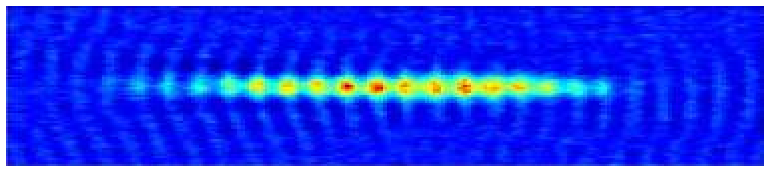
apparatus



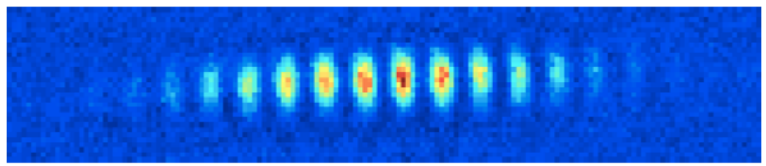


Atom detection using absorption imaging
high resolution (1 µm).

Before :

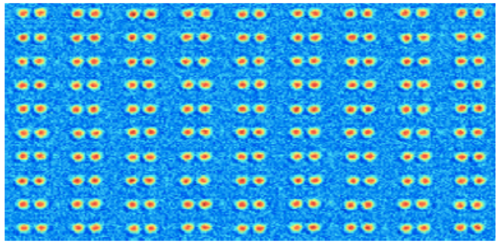


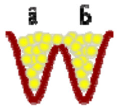
After :

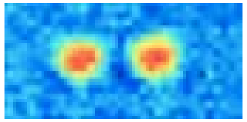


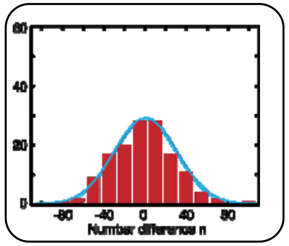
double well

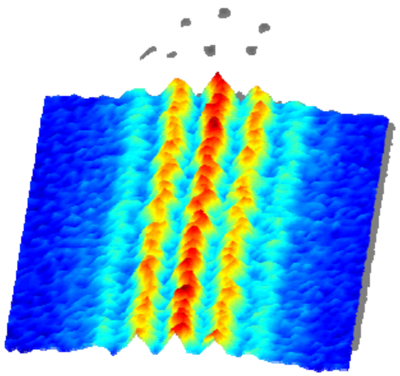
experimental results

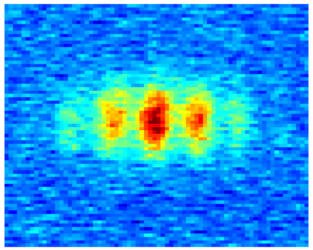






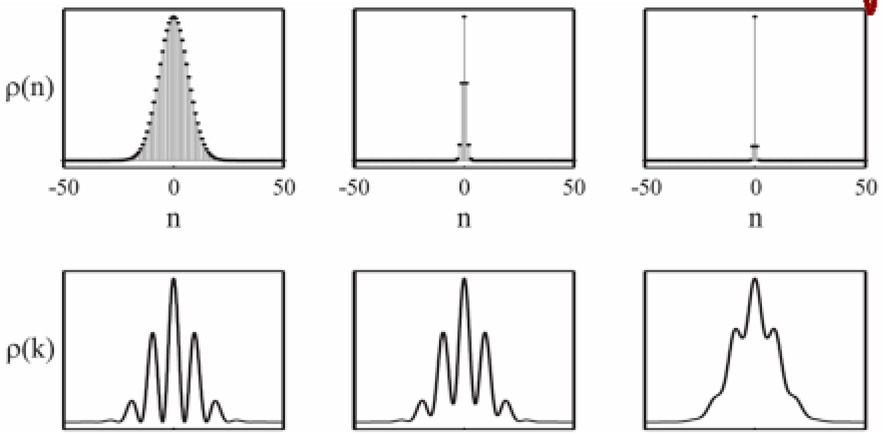


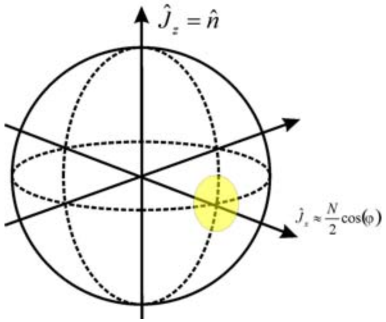


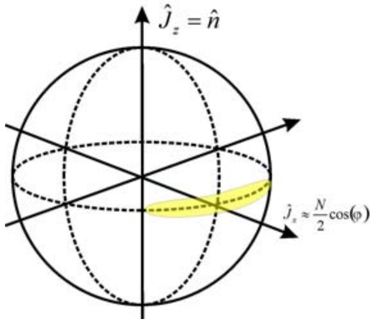


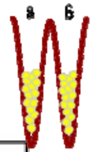
$$\frac{2\Delta n\,\Delta \sin(\varphi)}{<\cos\varphi>} \geq 10$$

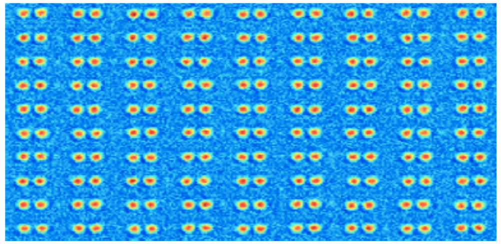
finite temperature

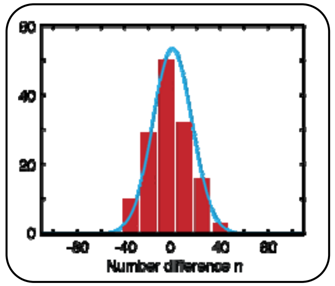


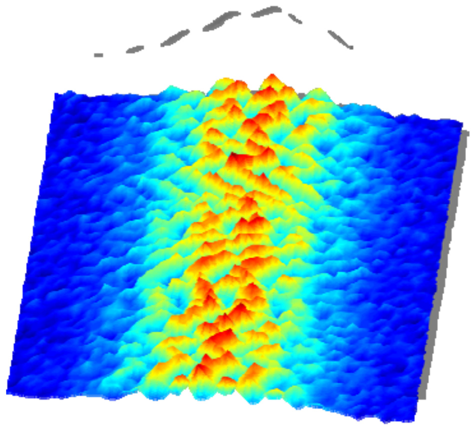




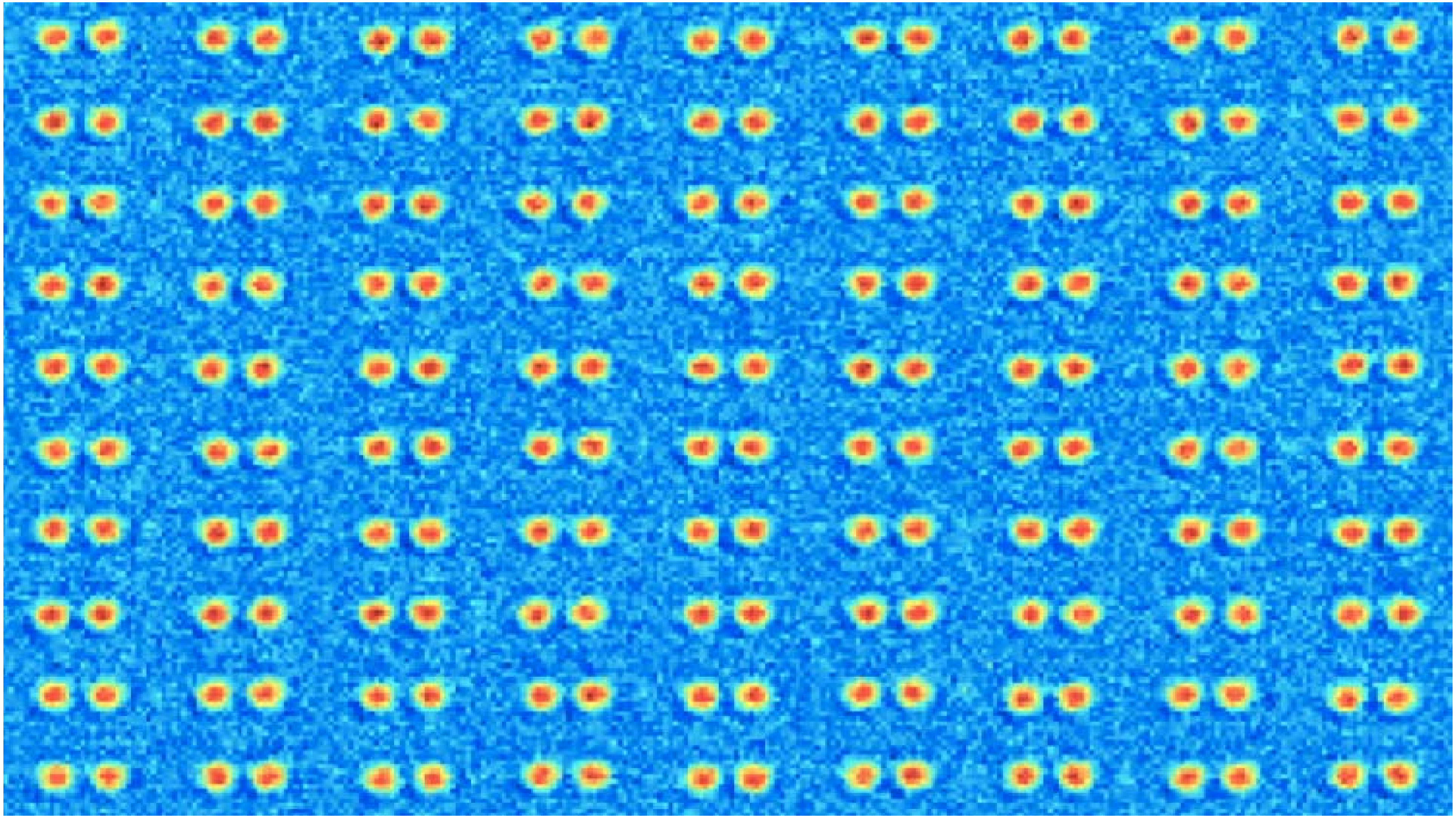


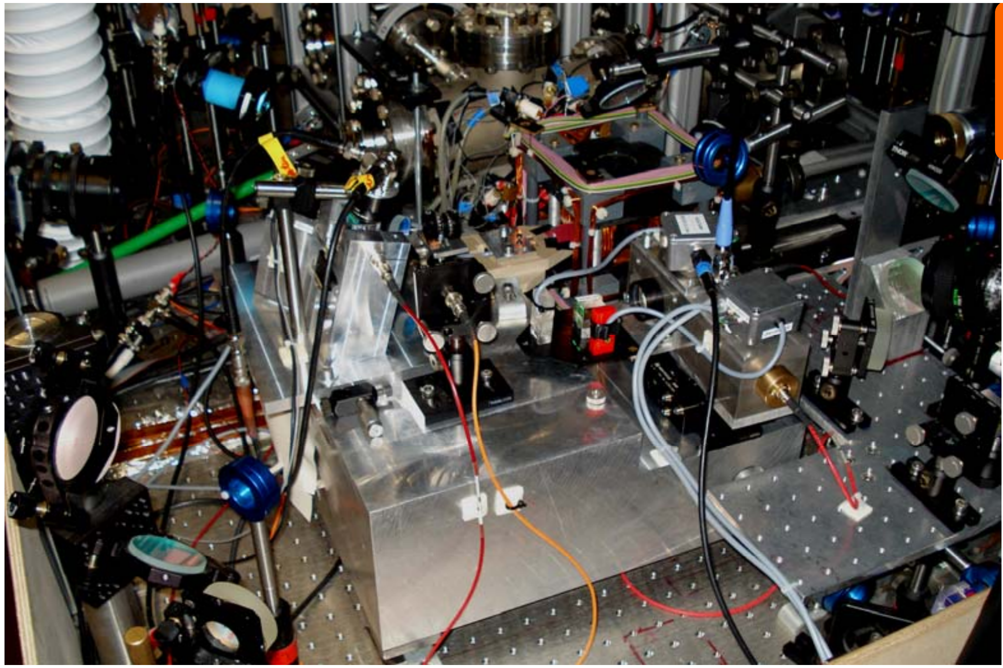






double well



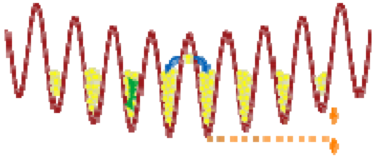


double well

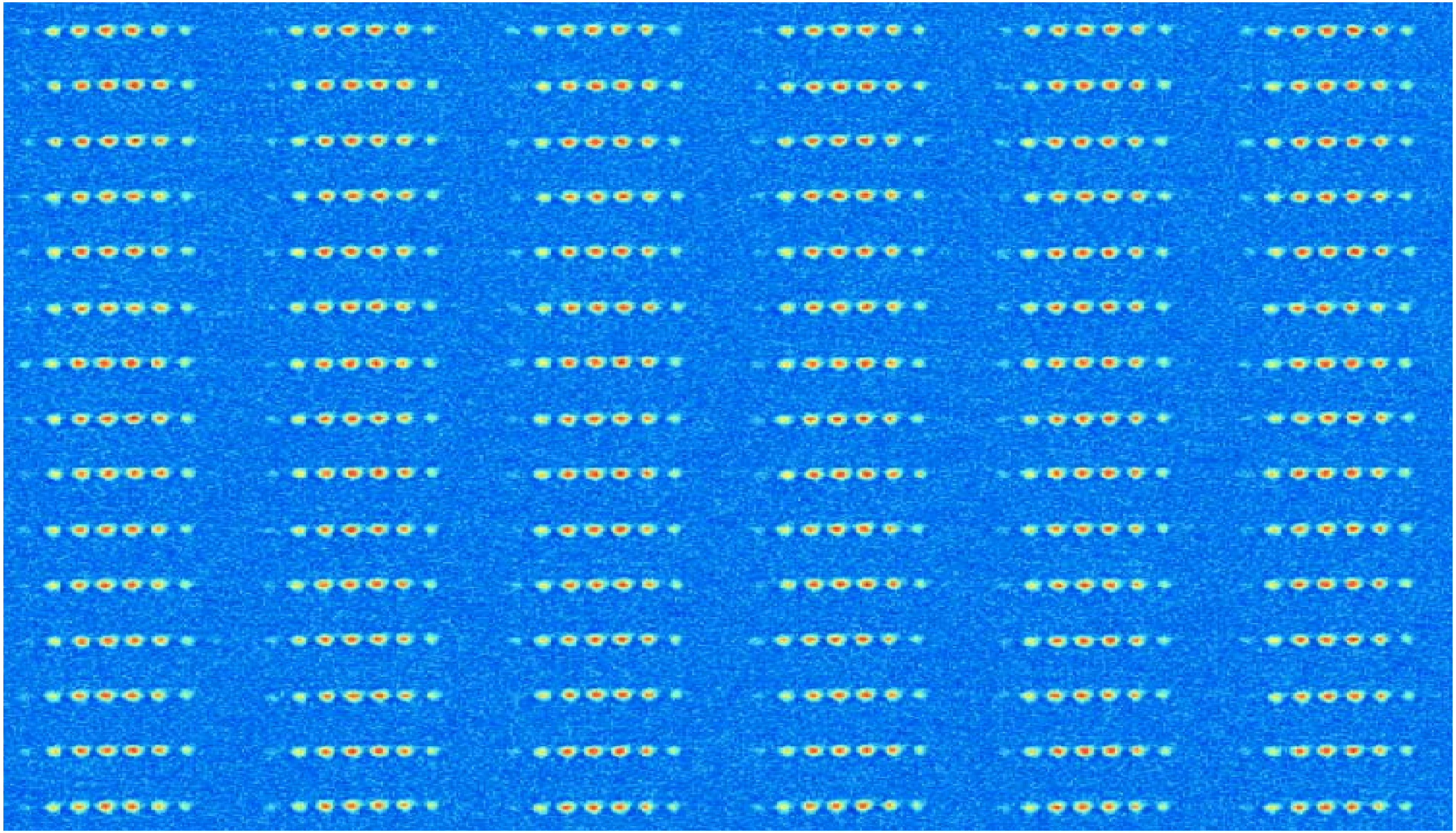
~40nm / 8 hours

experimental results

many well situation



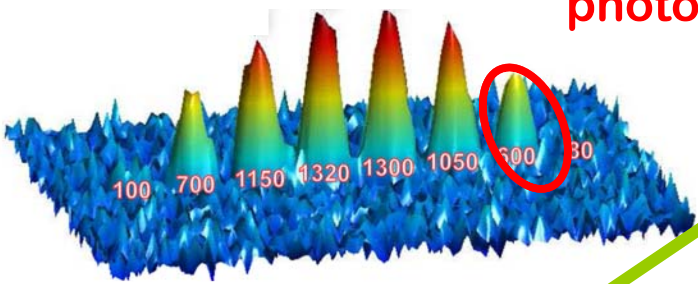
more stable situation – used for systematics



systematics

squeezed atomic states

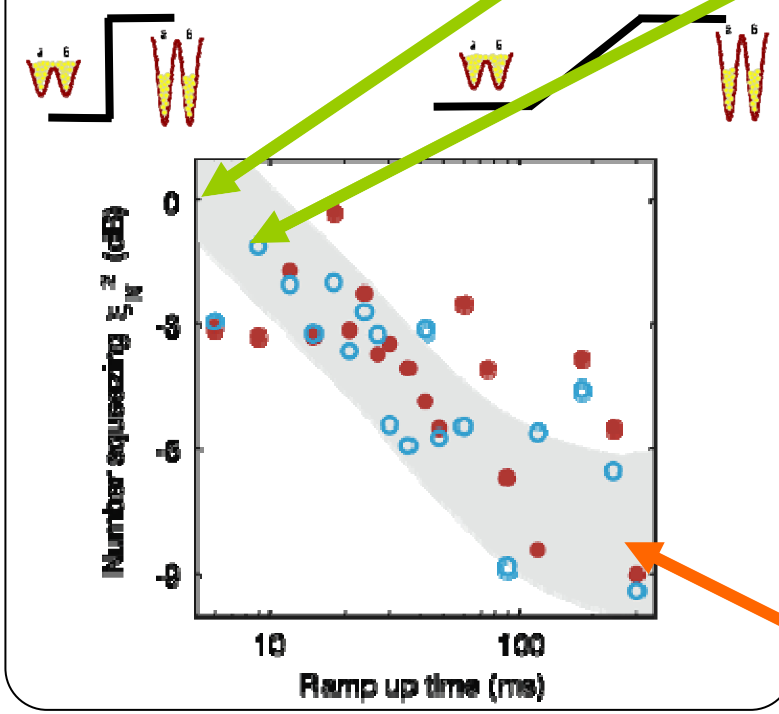
photon shot noise: ~10 atoms

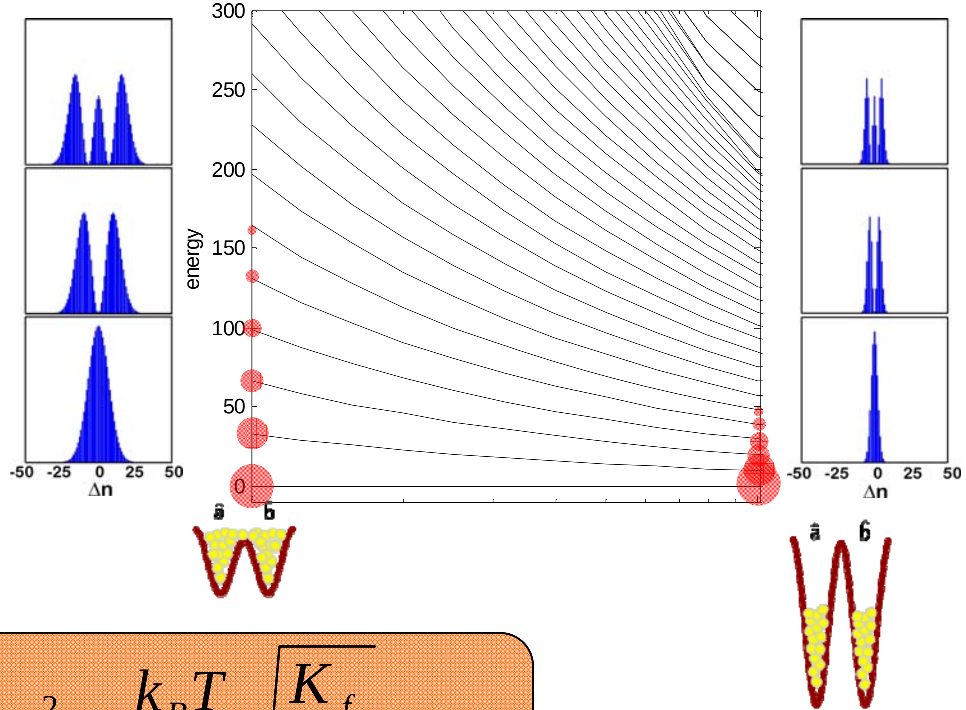


$$\xi_N^{\;2} = \frac{4\Delta n^2}{N}$$

$$\xi_N^{\;2} = \frac{4\Delta n^2}{N} \approx \frac{k_B T}{\mu}$$

adiabaticity





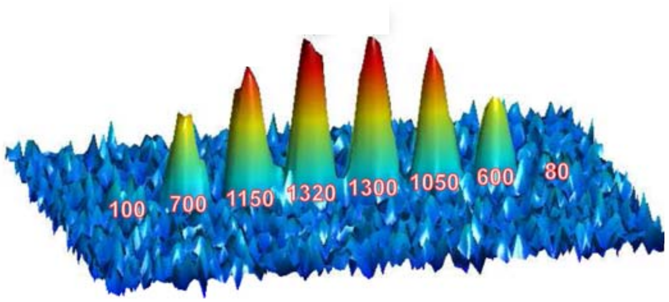
$$\xi_N^{\;2} = \frac{k_B T}{\mu}\sqrt{\frac{K_f}{K_i}} \approx 0.2$$

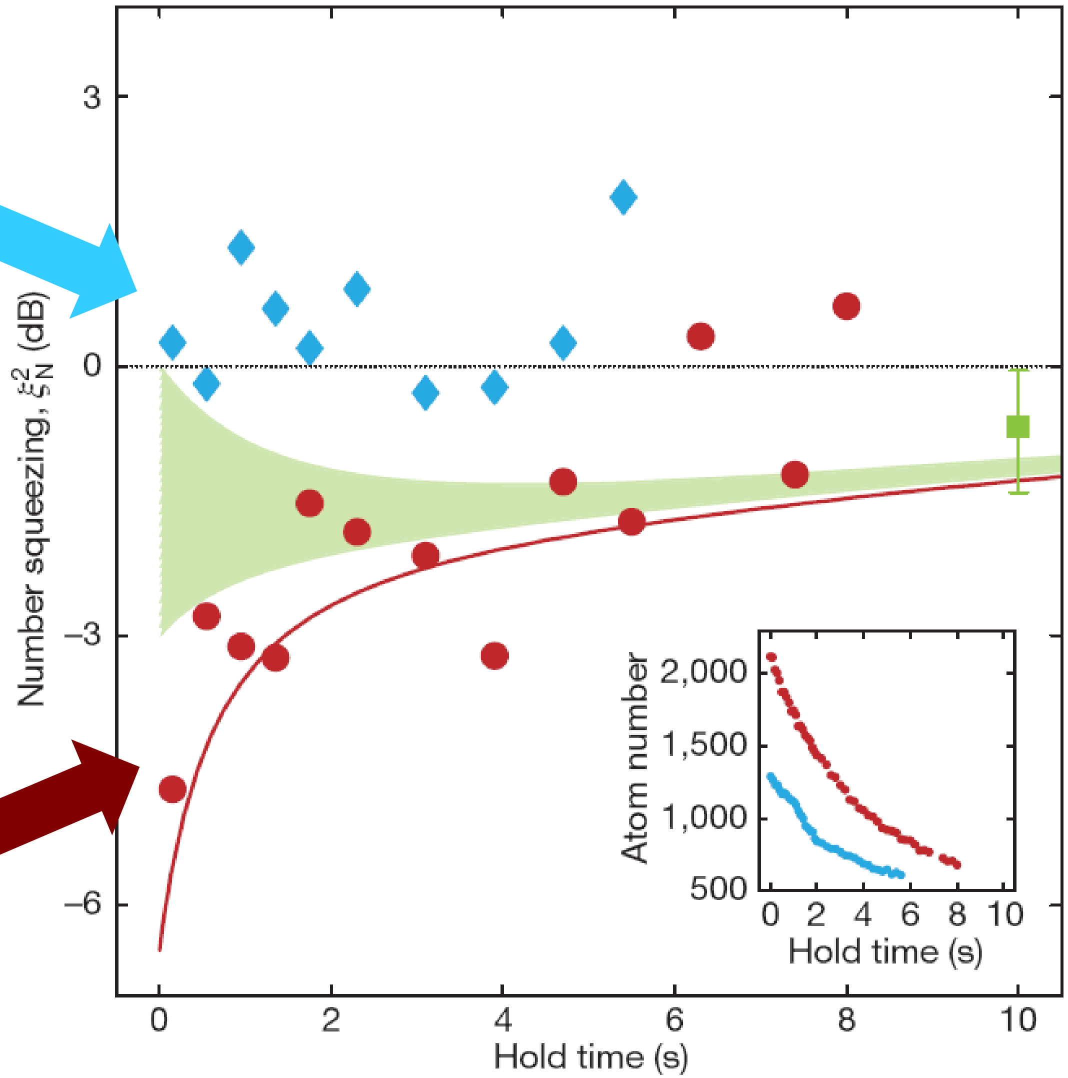
systematics

losses

independently condensed

number squeezed initial state

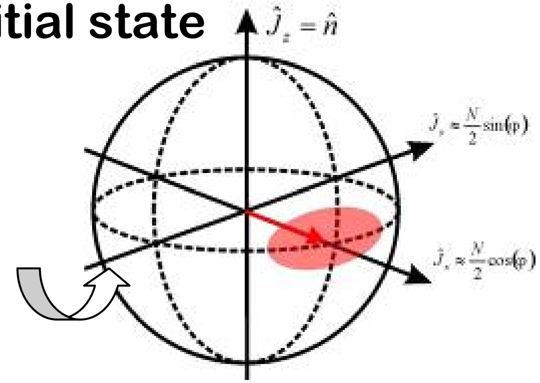




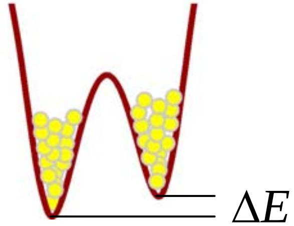
Ramsey- no interaction

application

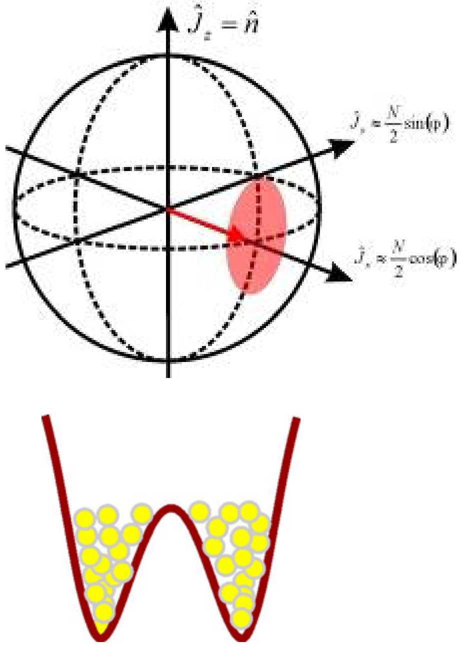
initial state

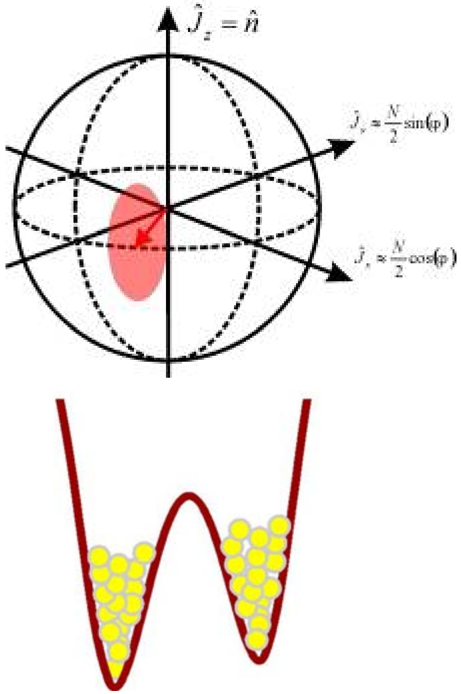


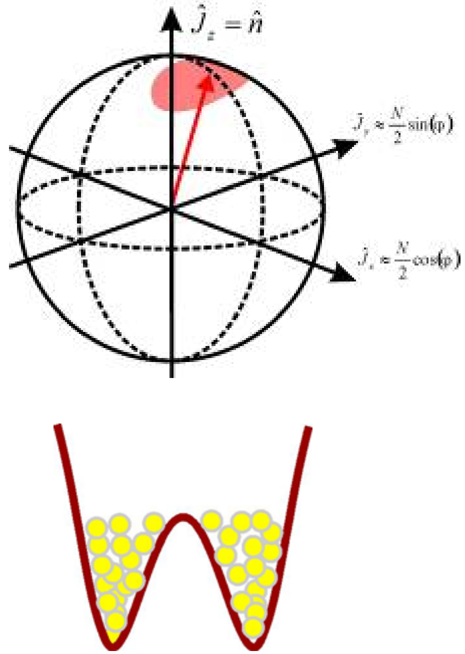
$$\hat{H} = -2K\ \hat{J}_x + \Delta E\ \hat{J}_z$$



π/2 pulse → **phase evolution** → **π/2 pulse**



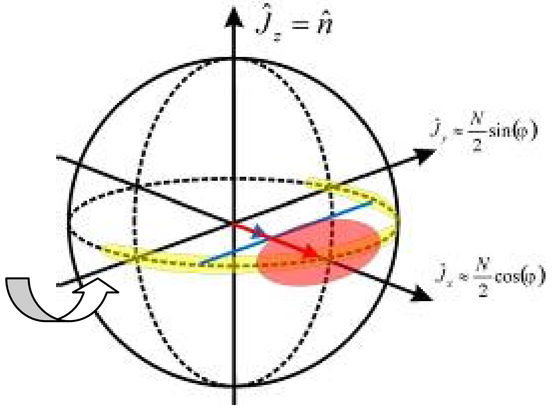




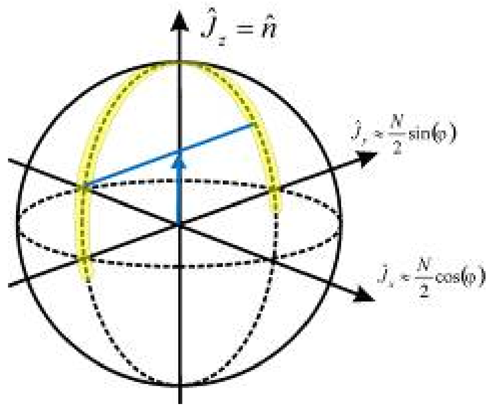
Ramsey - number squeezing

application

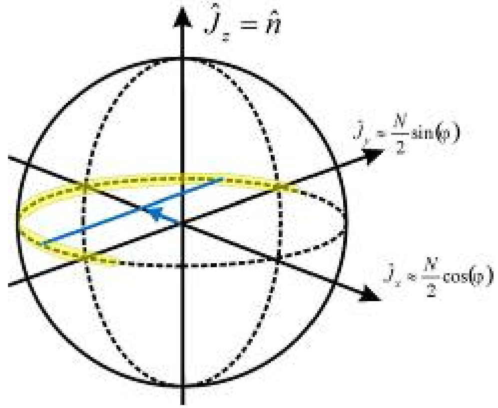
initial state

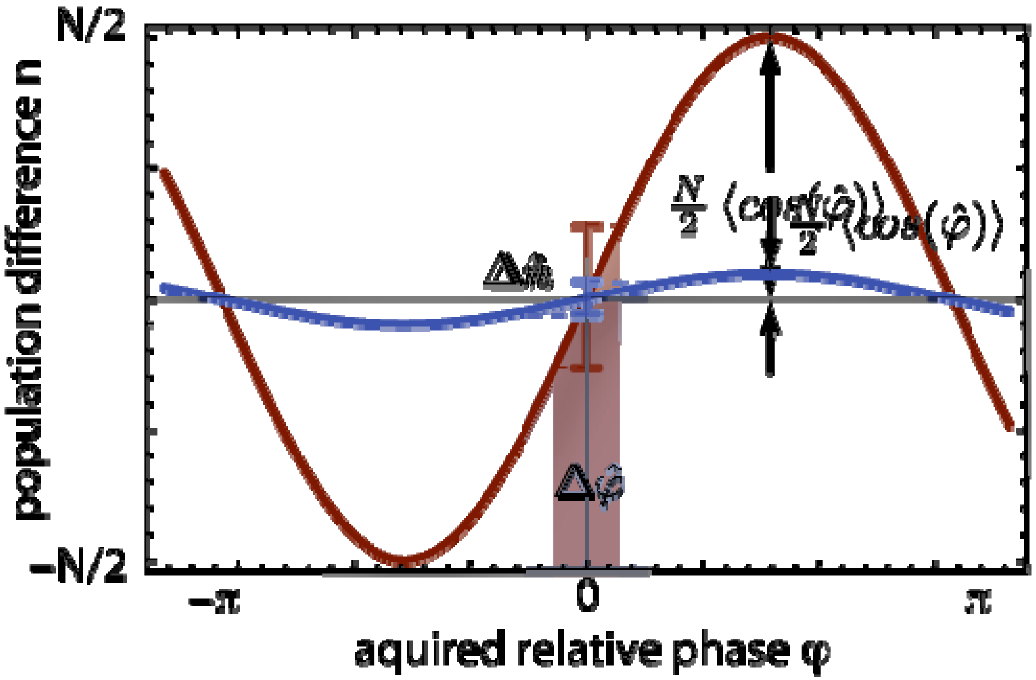


final state: $\phi=\pi/2$



final state: $\phi=\pi$

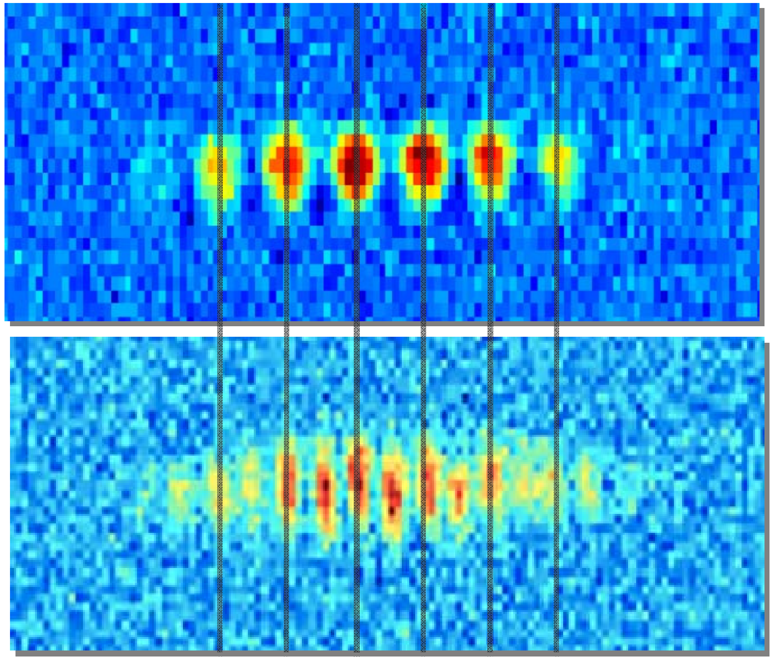




$$\xi_S{}^2 = \frac{4\Delta n^2}{N < \cos\varphi >^2}$$

D. Wineland, et al. Phys.Rev. A 50, 67 (1994) phase in many wells

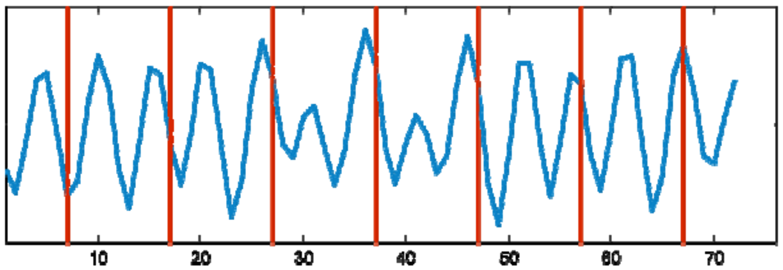
squeezed atomic states



In situ imaging

2.25 ms expansion

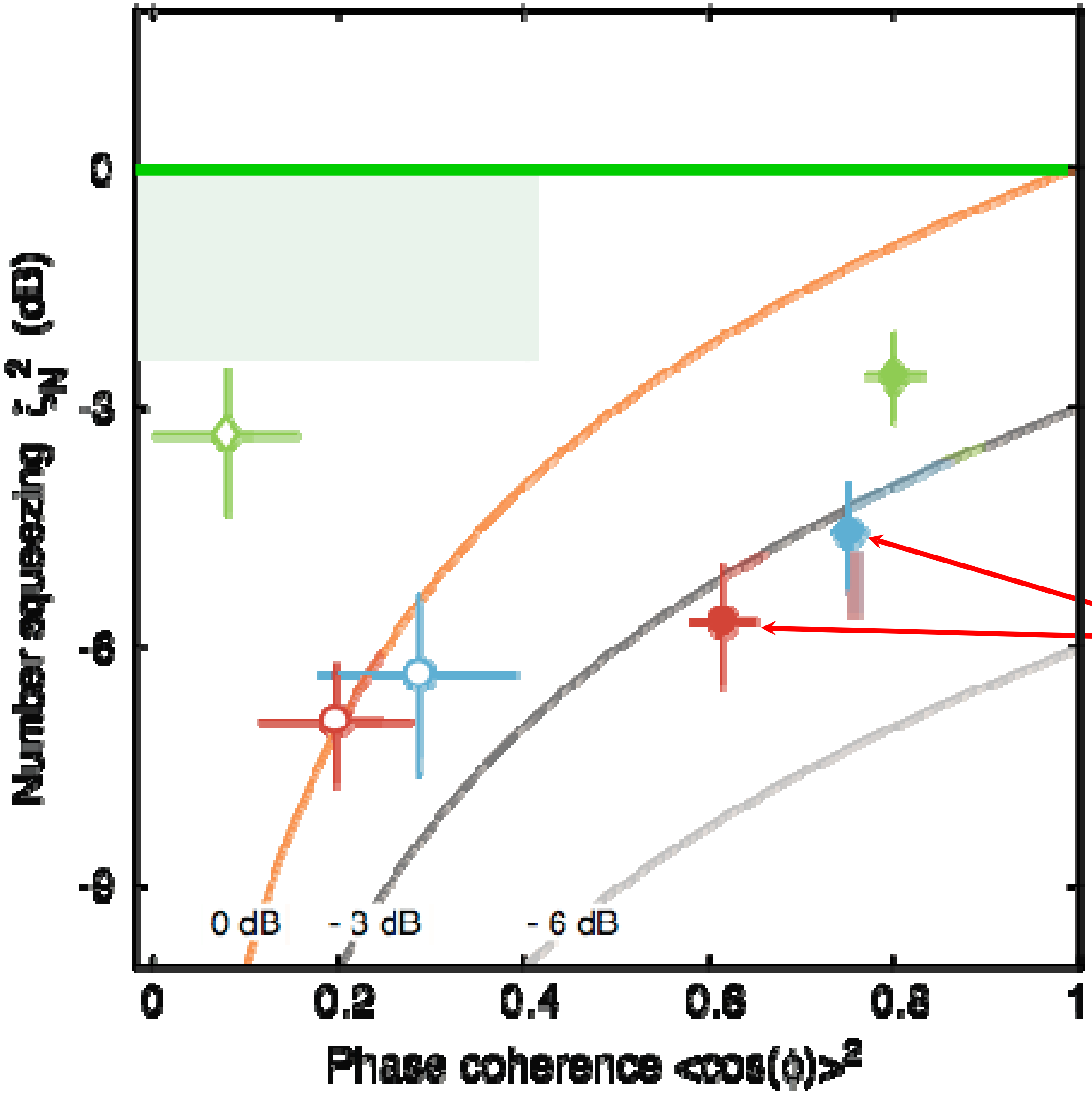
$$\xi_S^{\ 2} = \frac{4\Delta n^2}{N < \cos\varphi >^2}$$



envelope removal, FFT

improvement of ifm

squeezed atomic states



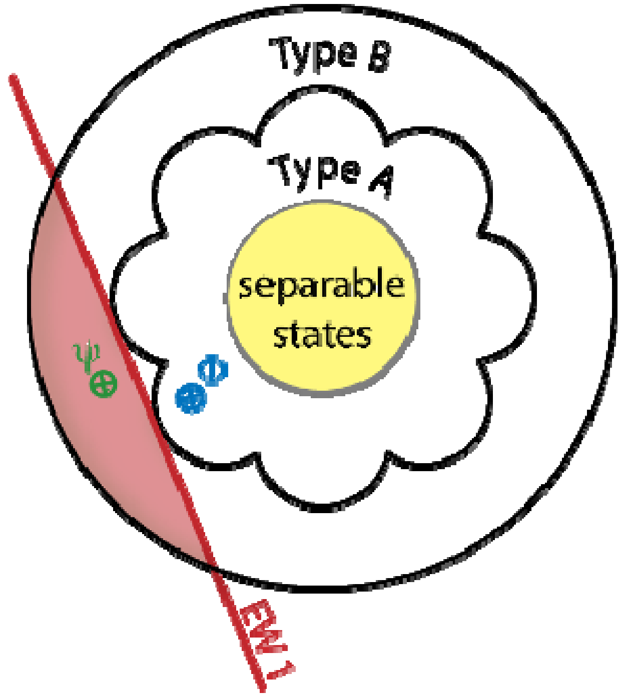
$$\xi_S^{\ 2} = \frac{4\Delta n^2}{N < \cos\varphi >^2} < 1$$

Over 1000 measurements

$$\xi_S^2 = -3.8^{+0.3}_{-0.4}\ \mathrm{dB}$$

entanglement criterion

squeezed atomic states



separable

$$\rho \stackrel{\downarrow}{=} \sum_k p_k \rho_1^{(k)} \otimes \rho_2^{(k)} \ldots \otimes \rho_N^{(k)}$$

$$\xi_S{}^2 = \frac{4\Delta n^2}{N < \cos\varphi >^2} \geq 1$$

Sorensen, et al. Nature 409, 63 (2001)

The same criterion as for precision enhancement

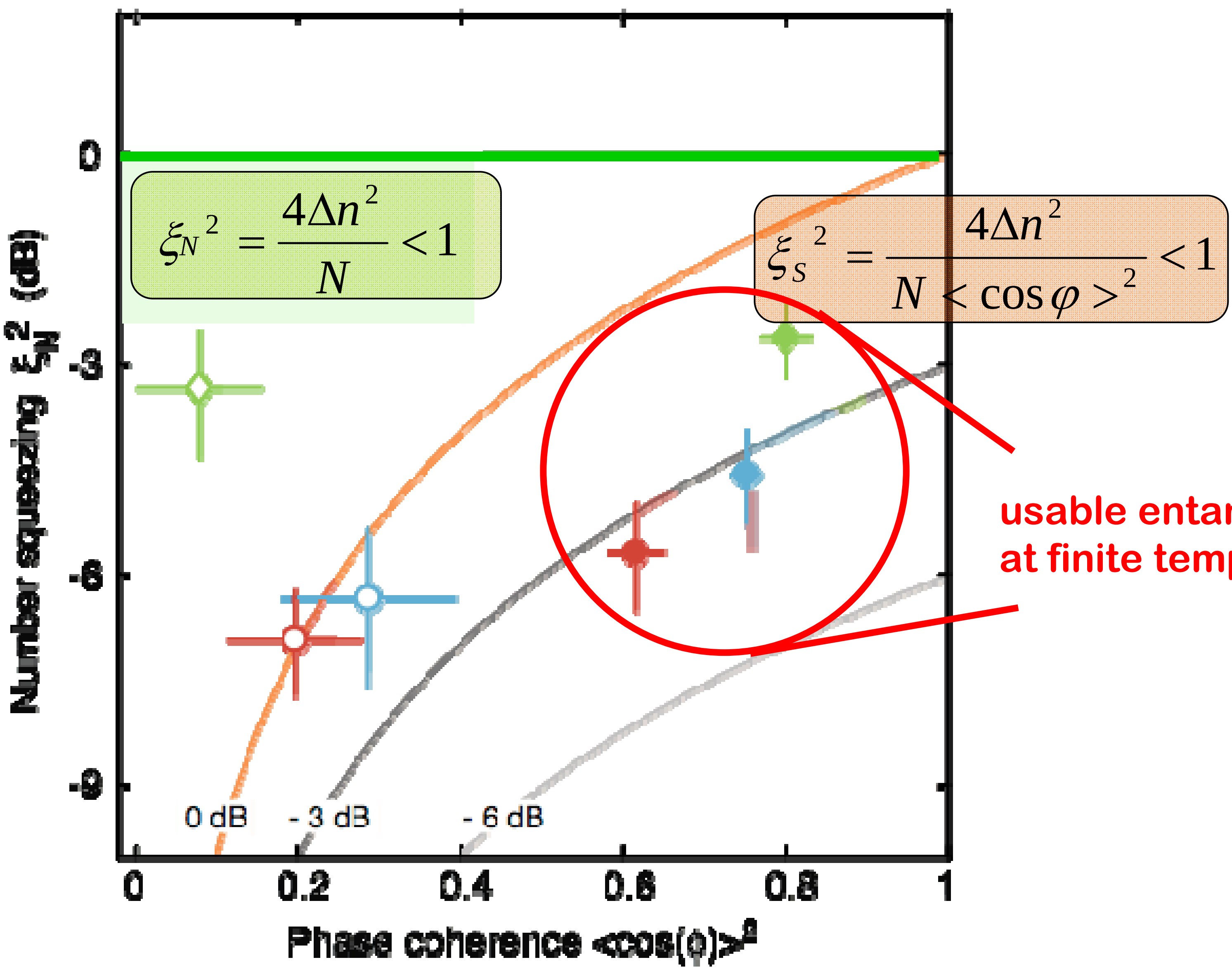
Criterion for indistinguishable particles

$$\rho_{2\mathrm{body}} = \sum_k p_k \rho_{1\mathrm{body}}^{(k)} \otimes \rho_{1\mathrm{body}}^{(k)}$$

$$\xi_N{}^2 = \frac{4\Delta n^2}{N} \geq 1$$

Wang et al. PRA 68, 012101 (2003) , Korbicz et al. PRL 95, 120502 (2005) entanglement

squeezed atomic states

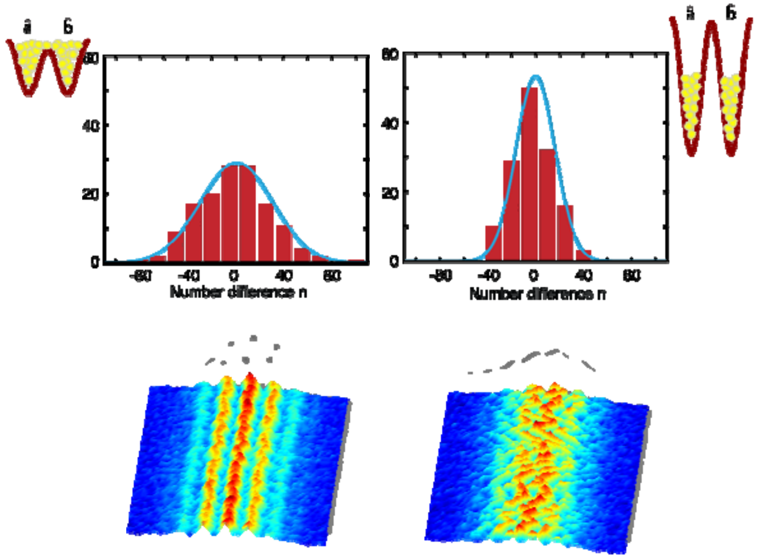


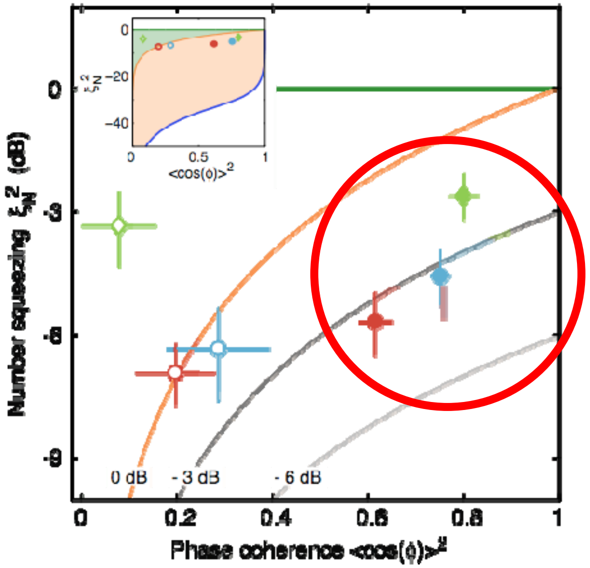
usable entanglement
at finite temperature

conclusion

Direct observation of coherent number squeezing

'a useful quantum resource'





Nature 455, 1216 (2008)