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Superfluid flow and critical current through single barrier and periodic potentials

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Trieste ICTP, 5 May 2009

Superfluid flow and critical current through single barrier and periodic potentials

Gentaro Watanabe, Franco Dalfovo, Francesco Piazza,
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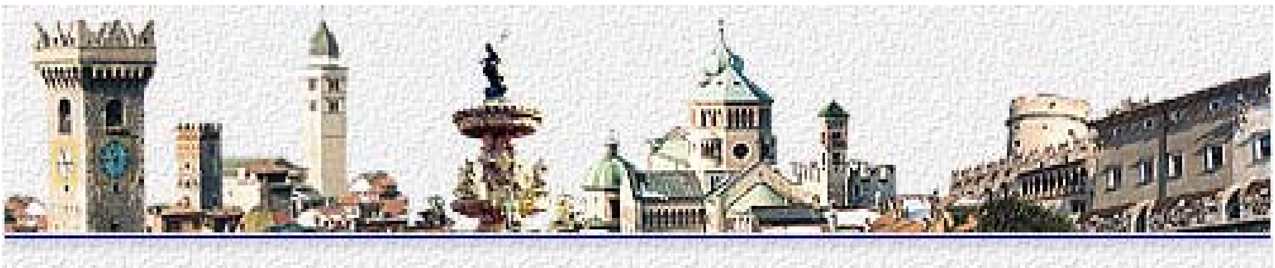
University of Trento



BEC

CNR-INFM

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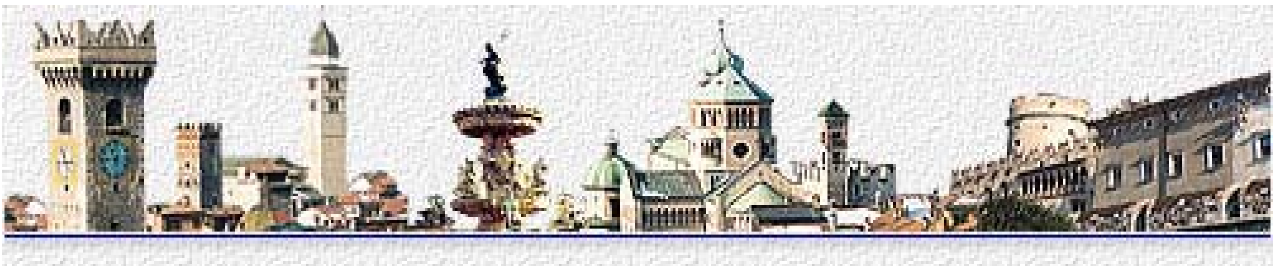


BEC

CNR-INFM

The concept of **critical current** plays a fundamental role in several **superfluid phenomena**.

Examples:

- **Landau critical velocity** for breaking of superfluidity and onset of viscous effects.
  Fixed by **excitation spectrum**:
  - phonons, rotons, vortices in BEC superfluids
  - single particle gap in BCS superfluids

- Critical current in **Josephson junctions.**
  Fixed by **quantum tunneling**

GENERAL QUESTION:

How is the **superfluid flow** affected by the presence of an **external perturbation** (barrier or periodic potential ?)

New experimental possibilities available in ultracold atomic gases (new **trapping conditions**, tuning of **scattering length**):

- Motion of **macroscopic impurities** (laser beam) has revealed the onset of heating effect (MIT 2000)

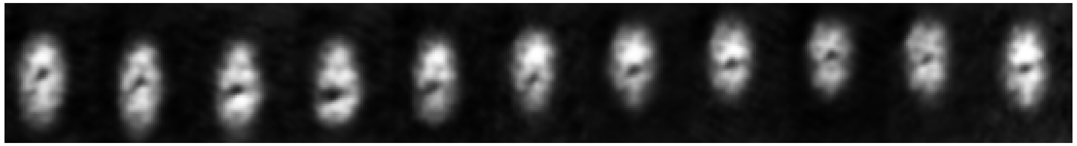
- Double well potentials are well suited to explore **Josephson** oscillations (Heidelberg 2004)

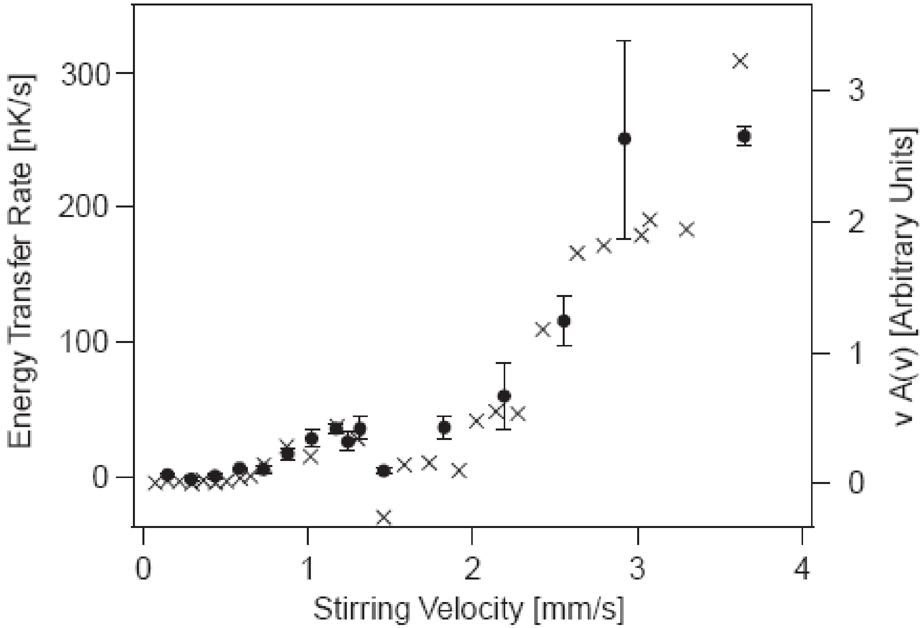
- **Moving periodic potentials** allow for the investigation of Landau critical velocity as well as for dynamic instability effects (Florence 2004, MIT 2007)

(MIT 2000)

Observation of Superfluid Flow in a Bose-Einstein Condensed Gas

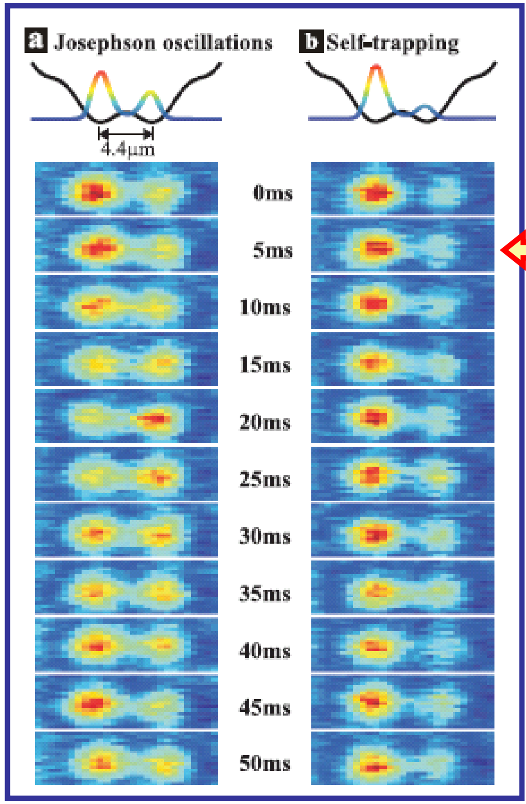
R. Onofrio, C. Raman, J. M. Vogels, J. Abo-Shaeer, A. P. Chikkatur, and W. Ketterle
Department of Physics and Research Laboratory of Electronics,
Massachusetts Institute of Technology, Cambridge, MA 02139



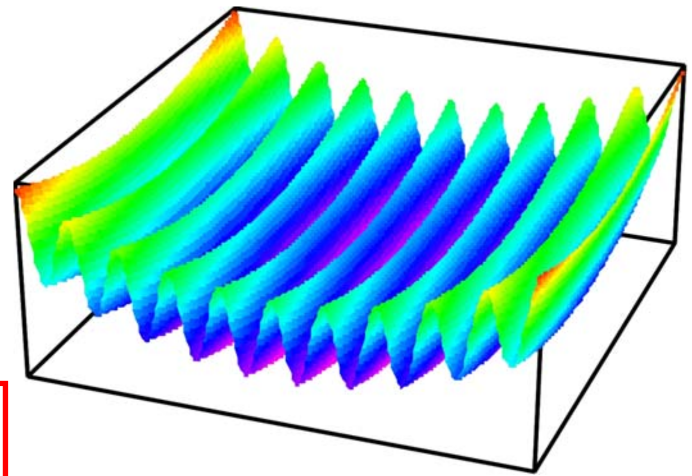


Critical velocity several factors smaller than central sound velocity

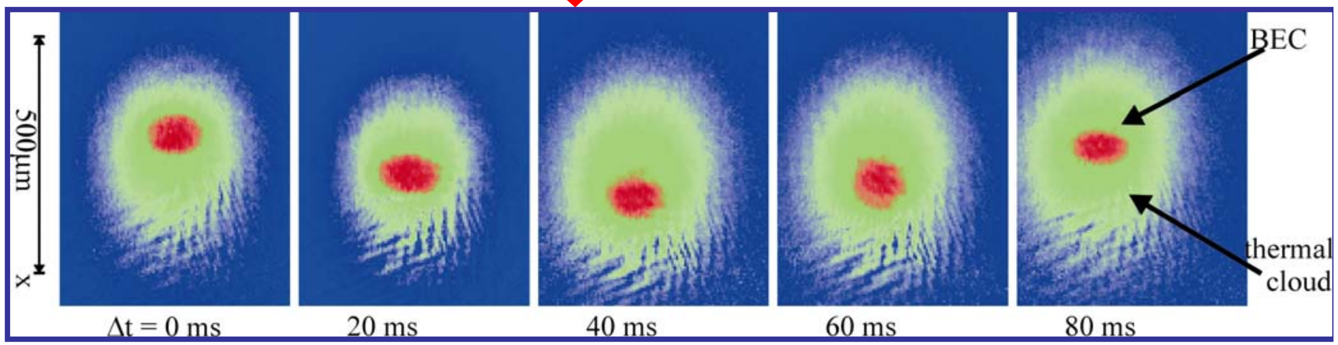
JOSEPHSON (PLASMA) OSCILLATION



Double well
(Heidelberg 2004)

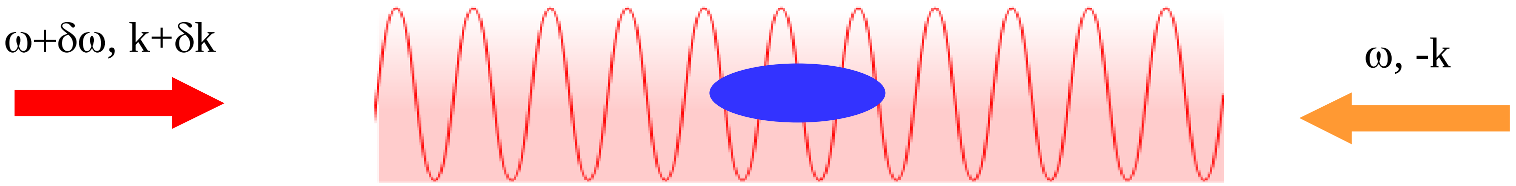


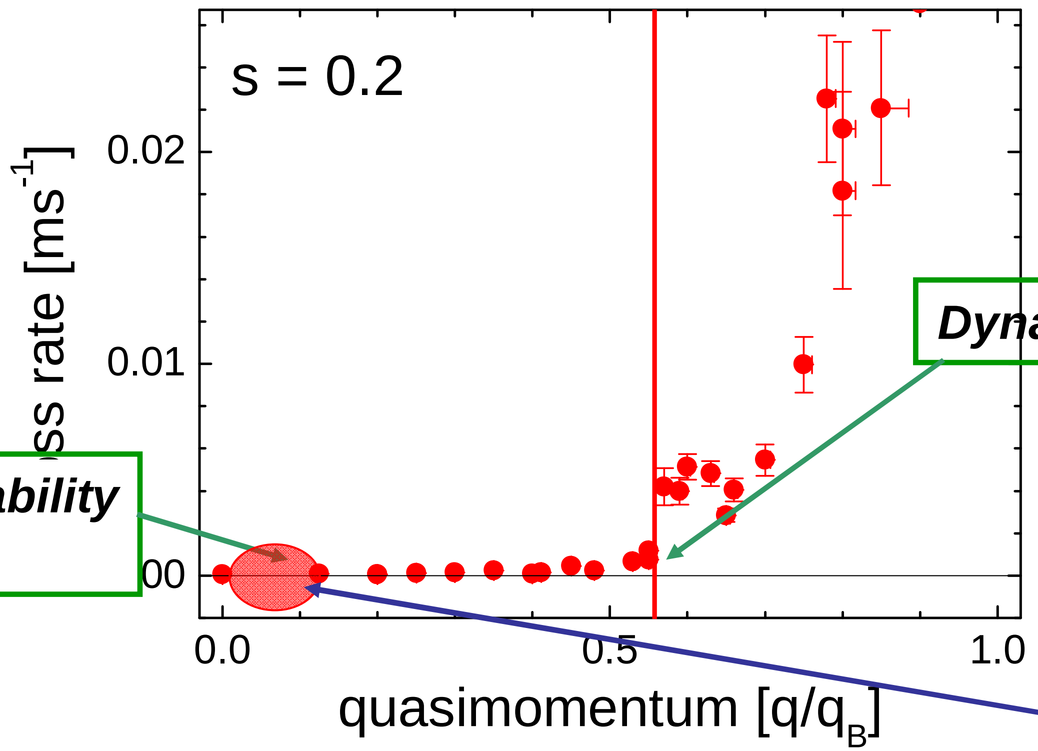
Periodic potential
(Firenze 2001)



Only condensate coherently tunnels through the barrier

Energetic ad dynamic instability in the presence of moving periodic potential





(Fallani et al., 2004)

Dynamic instability

energetic instability at finite T

energetic critical velocity several factors smaller than central sound velocity

Energetic instability of a Fermi gas in a moving periodic potential

Critical Velocity for Superfluid Flow across the BEC-BCS Crossover

D. E. Miller, J. K. Chin, C. A. Stan,* Y. Liu, W. Setiawan, C. Sanner, and W. Ketterle†

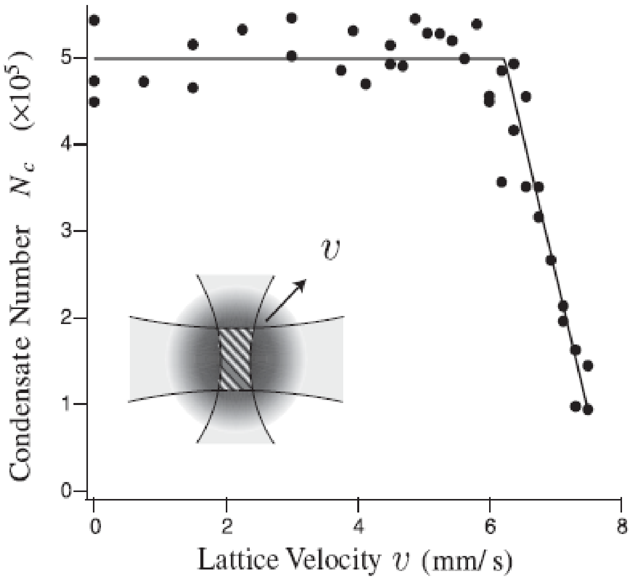


FIG. 1. Onset of dissipation for superfluid fermions in a moving optical lattice. (Inset) Schematic of the experiment in which two intersecting laser beams produced a moving optical lattice at the center of an optically trapped cloud (trapping beams not shown). Number of fermion pairs which remained in the condensate N_c after being subjected to a $V_0 = 0.2E_F$ deep optical lattice for 500 ms, moving with velocity v_L, at a magnetic field of 822 G ($1/k_F a = 0.15$). An abrupt onset of dissipation occurred above a critical velocity v_c, which we identify from a fit to Eq. (1).

- With respect to Florence experiment, lattice produced in **central** region (**local** measurement of critical velocity)
- **Onset of dissipation** more evident than in previous MIT exps with BEC
- Observed **Landau velocity** closer to **sound velocity**

Plan of the talk

Landau's criterion of superfluidity: summary and application to **2007 MIT exp**

Hydrodynamics and LDA: we use a **hydrodynamic** scheme in the local density approximation (**LDA**) to obtain an **analytic expression** for the critical current as a function of the barrier height or the lattice intensity, which applies to **both Bose and Fermi** superfluids.

Many-body theories: we compare the results of LDA with those of **Gross-Pitaevskii** and **Bogoliubov-de Gennes** equations.

We compare the **LDA** with the opposite **quantum** regimes and discuss the conditions required to observe **Josephson phenomena.**

Comparison with experiments with moving optical lattices

Landau's critical velocity

$$v_{cr} = \min_p \frac{\varepsilon(p)}{p}$$

← **Dispersion law of elementary excitations**

- **Landau's criterion for superfluidity** (**metastability**): fluid moving with velocity **smaller** than critical velocity cannot decay (**persistent current**). Assumption: driving potential does not affect dispersion of elementary excitations.

- In ideal Bose gas and ideal Fermi gas one has $v_{cr} = 0$

- In interacting gases one predicts two asymptotic behaviors:

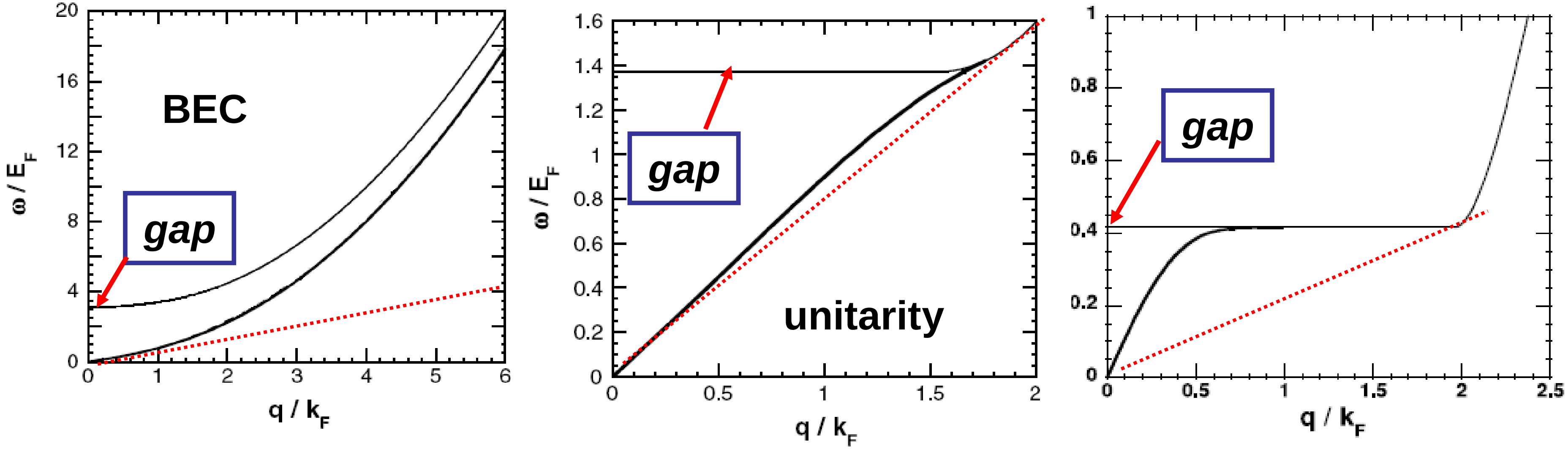
BEC (Bogoliubov dispersion)
(small and positive a)

BCS (role of the gap)
(small and negative a)

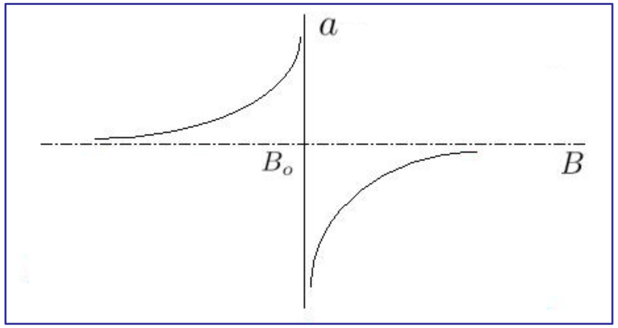
$$v_{cr} = c \propto \sqrt{a}$$ **(sound velocity)**

$$v_{cr} = \Delta / p_F \propto \exp(\pi / 2k_F a)$$

Dispersion law along BCS-BEC crossover of an interacting Fermi gas



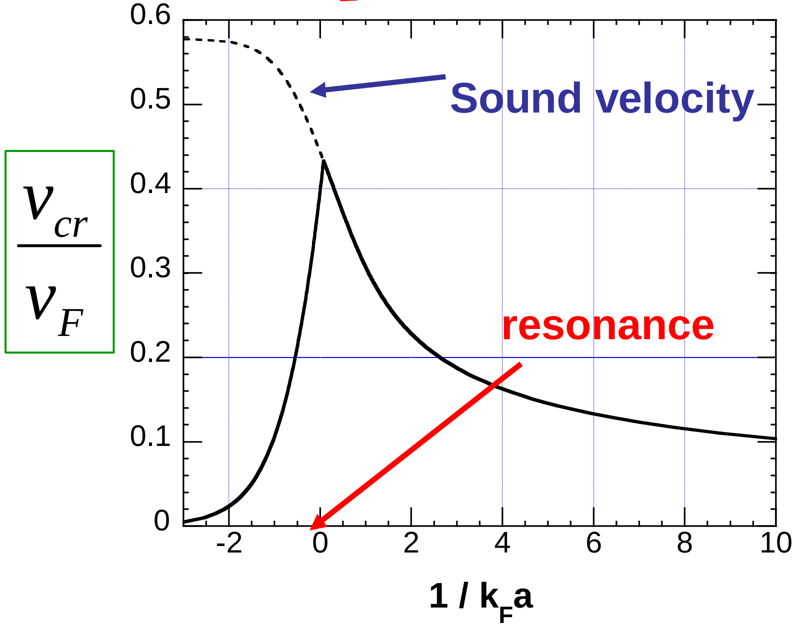
(R. Combescot, M. Kagan and S. Stringari 2006)



Fano-Feshbach resonance

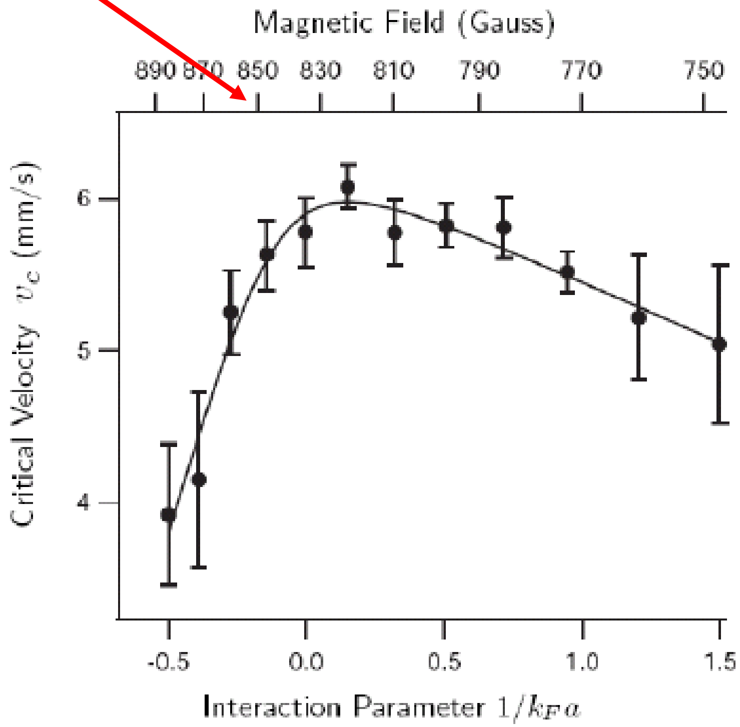
Results for Landau's critical velocity

theory



(Combescot et al, 2006)

experiment



(Miller et al, 2007)

Landau's critical velocity is highest near unitarity !!

Do we really understand 2007 MIT experiment?

The experiment reveals **fast decrease of critical velocity** as a function of the height of the barrier (laser intensity)

Critical Velocity for Superfluid Flow across the BEC-BCS Crossover

D. E. Miller, J. K. Chin, C. A. Stan,* Y. Liu, W. Setiawan, C. Sanner, and W. Ketterle†

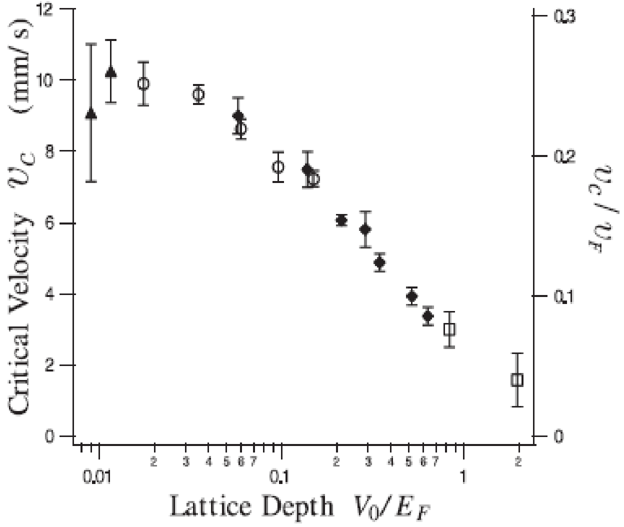
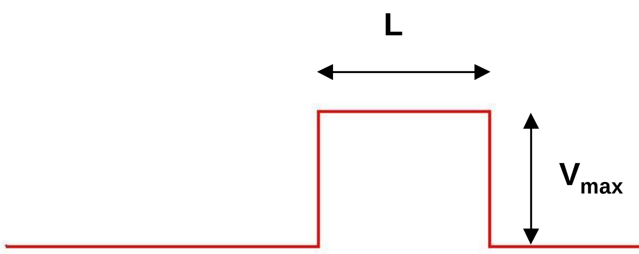
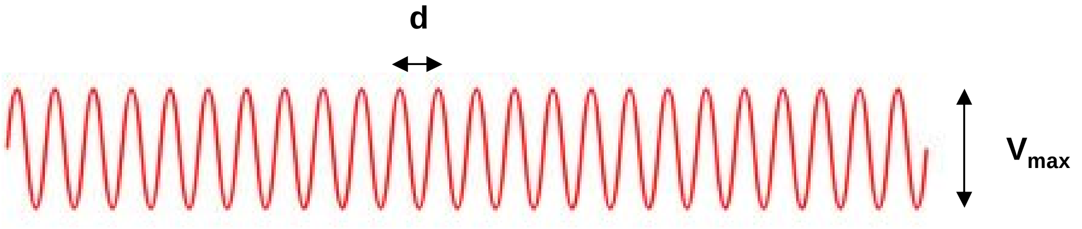


FIG. 4. Critical velocities at different lattice depths. The results show v_c to be a decreasing function of lattice depth V_0. In the limit of low V_0, v_c converges to a maximum value of 0.25 v_F. Data were taken near resonance, at 822 G ($1/k_F a = 0.15$) for hold times $t = 250$, 500, 1000, 2000 ms (squares, diamonds, circles, triangles).

Our goal:

establishing an appropriate framework in which critical current can be calculated in different situations (bosons vs. fermions and single barrier vs. optical lattice)





The simplest approach:
Hydrodynamics in Local Density Approximation (LDA)

Assumption: the system behaves locally as a uniform gas of density n, with energy density $e(n)$ and local chemical potential, $\mu(n)$.

The density profile of the gas at rest in the presence of an external potential is given by the Thomas-Fermi relation

$$\mu_0 = \mu(n(x)) + V_{\text{ext}}(x)$$

If the gas is flowing with a constant current density $j=n(x)v(x)$, the Bernoulli equation for the stationary velocity field $v(x)$ is

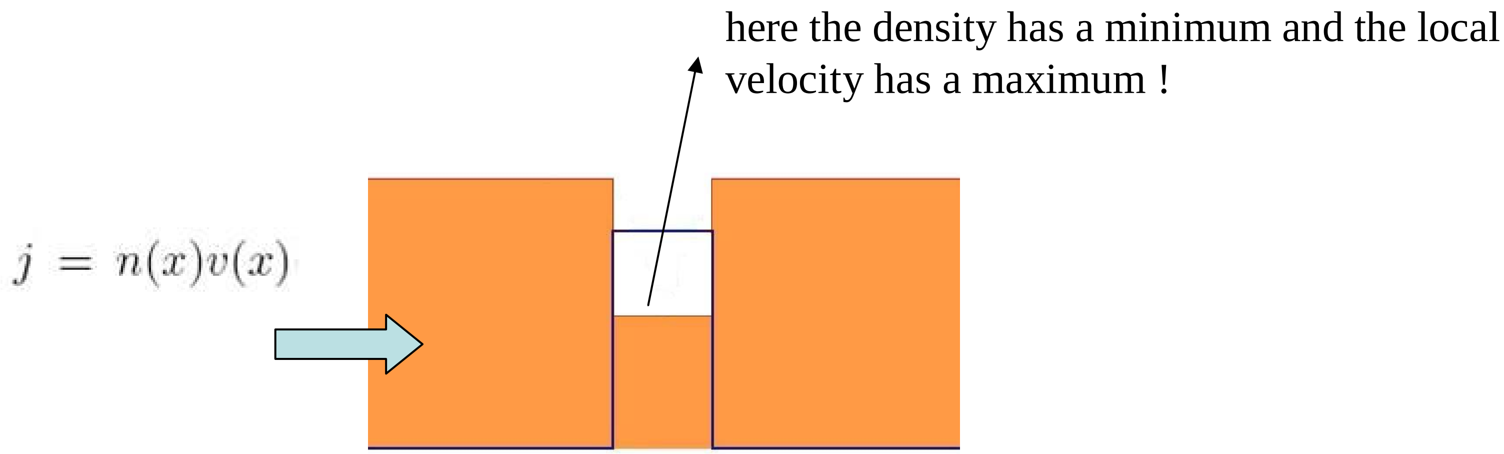
$$\mu_j = \frac{m}{2}\left[\frac{j}{n(x)}\right]^2 + \mu(n) + V_{\text{ext}}(x)$$

This equation fixes the density profile, $n(x)$, for any given current j.

The simplest approach:
Hydrodynamics in Local Density Approximation (LDA)

The system becomes energetically unstable when the local velocity, $v(x)$, at some point x becomes equal to the local sound velocity, $c_s[n(x)]$.

For a given current j, this condition is first reached at the point of minimum density, where $v(x)$ is maximum and $c_s(x)$ is minimum.

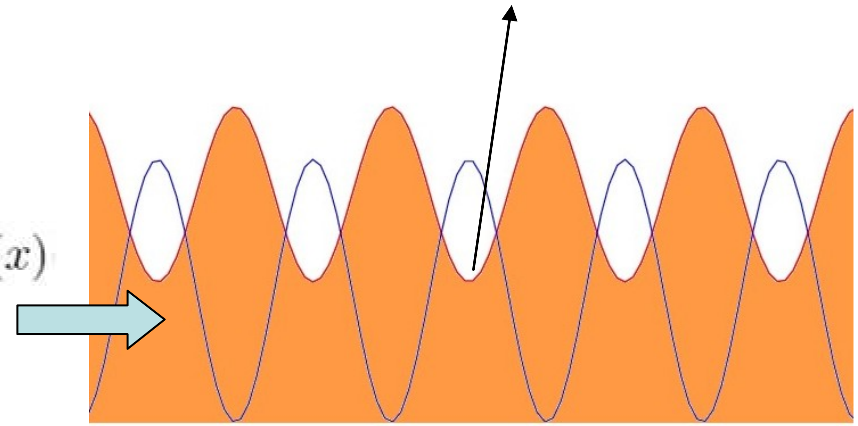


The simplest approach:
Hydrodynamics in Local Density Approximation (LDA)

The same happens in a periodic potential

here the density has a minimum and the local velocity has a maximum !

$$j = n(x)v(x)$$



The simplest approach:
Hydrodynamics in Local Density Approximation (LDA)

$$\mu_j = \frac{m}{2}\left[\frac{j}{n(x)}\right]^2 + \mu(n) + V_{\text{ext}}(x)$$

To calculate the critical velocity, one needs the equation of state $\mu(n)$ of the uniform gas!

We use a **polytropic equation of state**:

$$\mu(n) = \alpha n^\gamma$$

Bosons (BEC)

$\gamma = 1$

$\alpha = g = 4\pi\hbar^2 a_s/m$

Unitary Fermions

$\gamma = 2/3$

$\alpha = (1+\beta)(3\pi^2)^{2/3}\hbar^2/2m$

Local sound velocity: $$mc_s^2(n) = n\frac{\partial}{\partial n}\mu(n) = \gamma\mu(n)$$

The simplest approach:
Hydrodynamics in Local Density Approximation (LDA)

Inserting the critical condition

$$m\left(\frac{j_c}{n_c(0)}\right)^2 = \gamma\mu(n_c(0)) = \gamma\alpha n_c^\gamma(0)$$

into the Bernoulli equation

$$\mu_j = \frac{m}{2}\left[\frac{j}{n(x)}\right]^2 + \mu(n) + V_{\text{ext}}(x)$$

one gets an implicit relation for the critical current:

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}}\left[\frac{2\mu_{j_c}}{2+\gamma}\left(1-\frac{V_{\max}}{\mu_{j_c}}\right)\right]^{\frac{2+\gamma}{\gamma}}$$

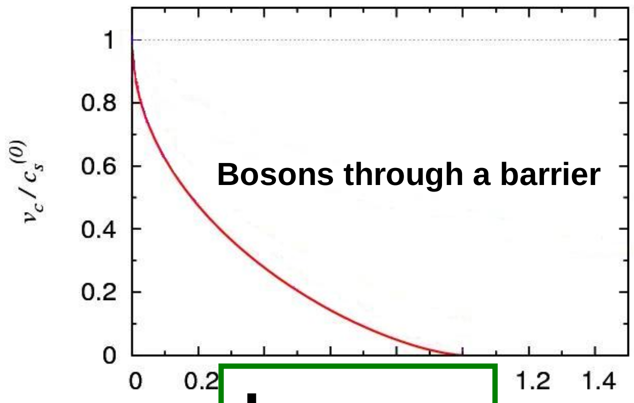
Universal !!
Bosons and Fermions in any 1D potential

Note: for bosons through a single barrier this has been discussed by V.Hakim, PRE 55, 285 (1997)

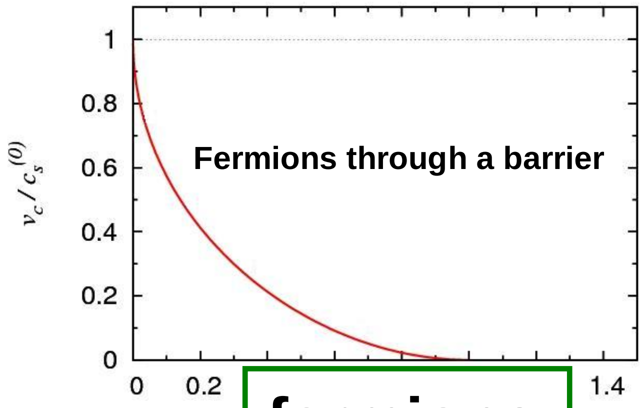
LDA

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}} \left[\frac{2\mu_{j_c}}{2+\gamma} \left(1 - \frac{V_{\max}}{\mu_{j_c}} \right) \right]^{\frac{2+\gamma}{\gamma}}$$

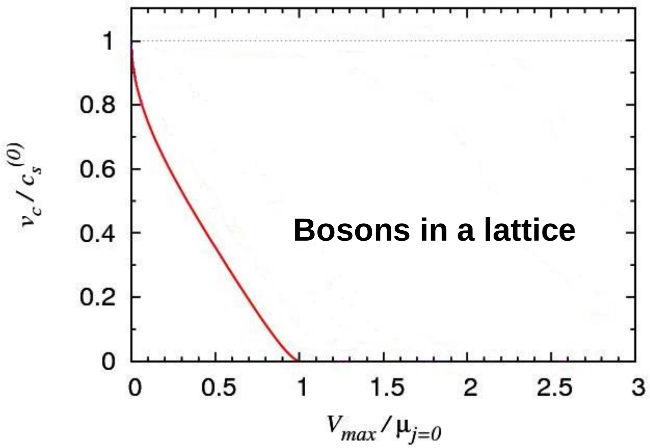
Results for the critical velocity depend on the actual equation of state

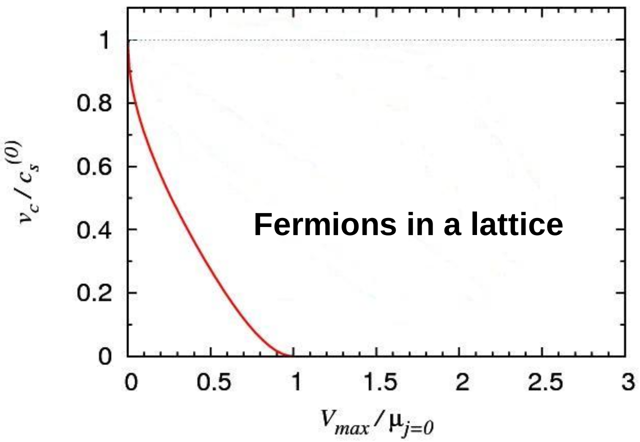


bosons



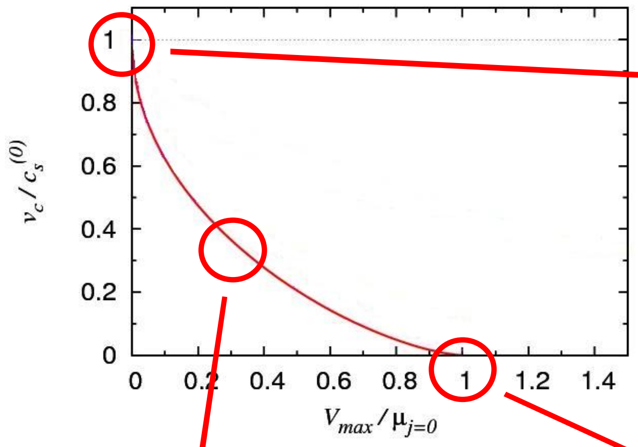
fermions

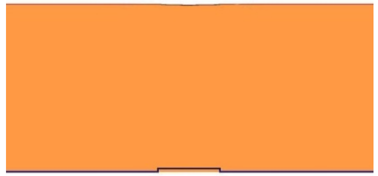




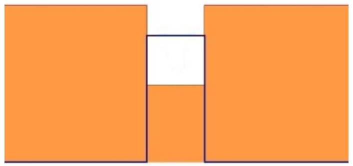
LDA

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}}\left[\frac{2\mu_{j_c}}{2+\gamma}\left(1-\frac{V_{\max}}{\mu_{j_c}}\right)\right]^{\frac{2+\gamma}{\gamma}}$$

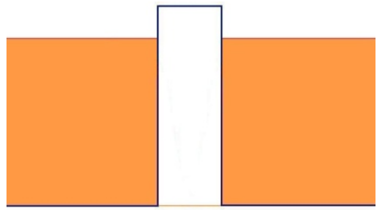




The limit V_{max} << μ corresponds to the usual Landau criterion for a uniform superfluid flow in the presence of a small external perturbation, i.e., a critical velocity equal to the sound velocity of the gas.



the critical velocity decreases because the density has a local depletion and the velocity has a corresponding local maximum



When V_{max} = μ the density vanishes and the critical velocity too.

LDA

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}}\left[\frac{2\mu_{j_c}}{2+\gamma}\left(1-\frac{V_{\max}}{\mu_{j_c}}\right)\right]^{\frac{2+\gamma}{\gamma}}$$

Question: when is LDA reliable?

Answer: the external potential must vary on a spatial scale much larger than the healing length of the superfluid.

$$L >> \xi$$

For a single square barrier, L is just its width.

For an optical lattice, L is of the order of the lattice spacing (we choose $L=d/2$).

For bosons with density n_0, the healing length is $\xi=\hbar/(2mgn_0)^{1/2}$.

For fermions at unitarity, one has $\xi \approx 1/k_F$, where $k_F = (3\pi^2 n_0)^{1/3}$

LDA

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}} \left[\frac{2\mu_{j_c}}{2+\gamma} \left(1 - \frac{V_{\max}}{\mu_{j_c}} \right) \right]^{\frac{2+\gamma}{\gamma}}$$

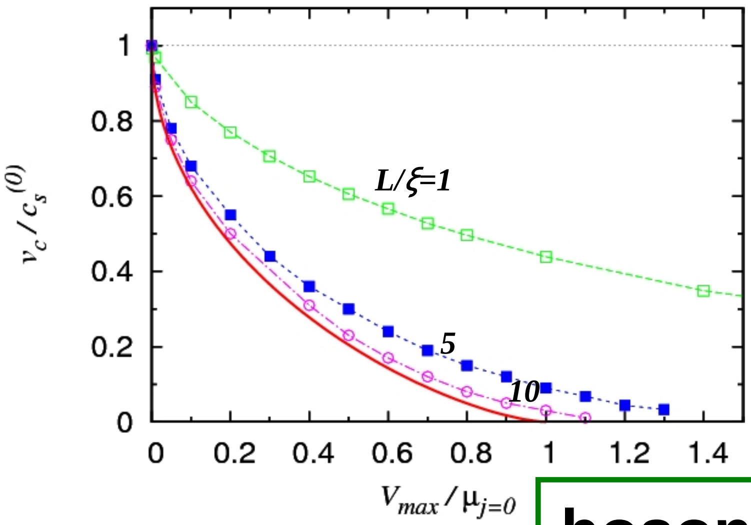
Quantum effects beyond LDA become important when

- ξ is of the same order or larger than ***L***; they cause a smoothing of both density and velocity distributions, as well as the emergence of solitonic excitations (and vortices in 3D).
- $V_{max} > \mu$; in this case LDA predicts a vanishing current, while quantum tunneling effects yield Josephson current.

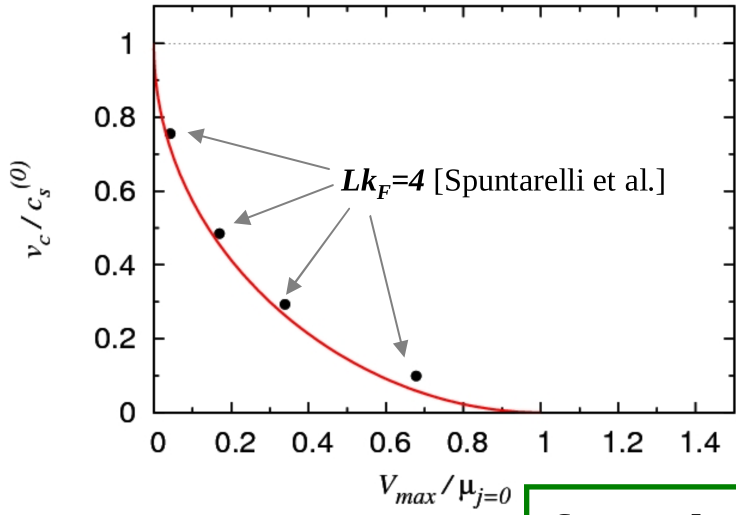
Quantitative estimates of deviations from LDA can be obtained by using **quantum many-body** theories, like **Gross-Pitaevskii** theory for dilute bosons and **Bogoliubov-de Gennes** equations for interacting fermions.

LDA (———) vs. quantum many-body

Bosons through a barrier



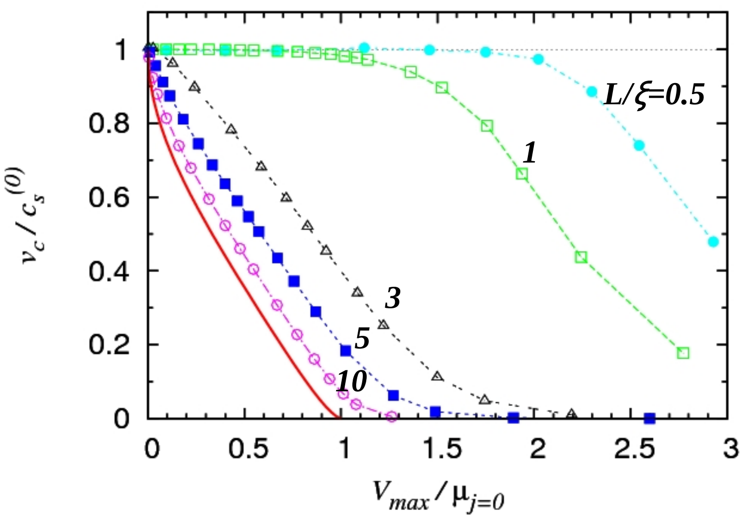
Fermions through a barrier



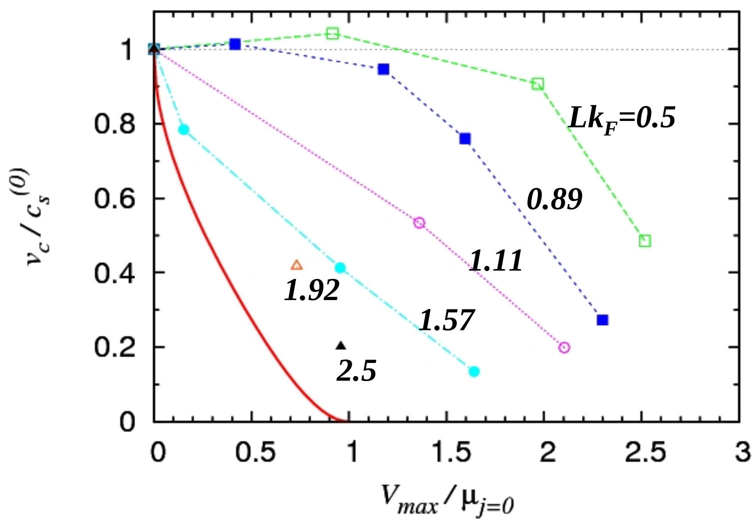
bosons

fermions

Bosons in a lattice



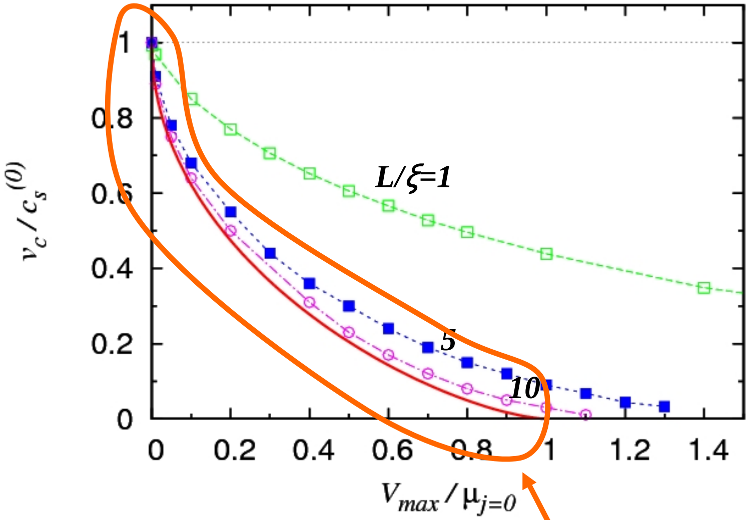
Fermions in a lattice

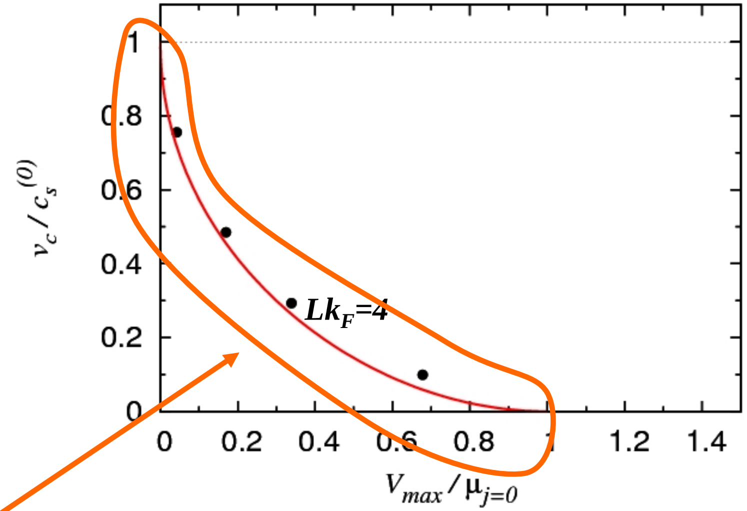


Bosons (left) and Fermions (right) through single barrier

bosons

fermions

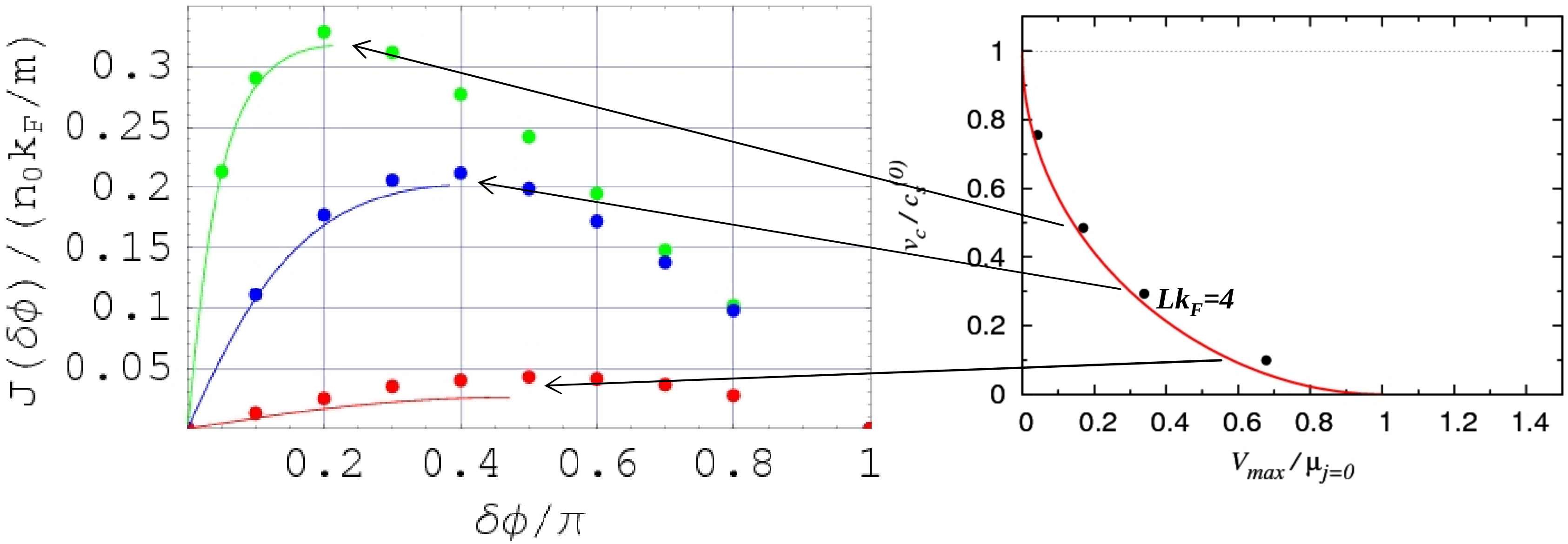




L >> ξ : Hydrodynamic flow in LDA

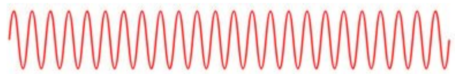
Fermions through single barrier

Current-phase relation.
Comparison LDA (lines) vs. BdG (points)
[BdG results from Spuntarelli et al. ,PRL 99, 040401 (2007)]



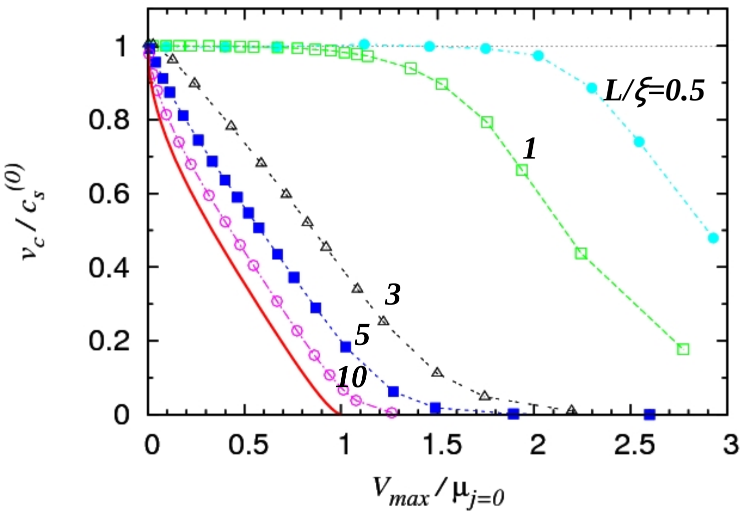
L >> ξ : Hydrodynamic flow in LDA

Bosons (left) and Fermions (right) in a periodic potential

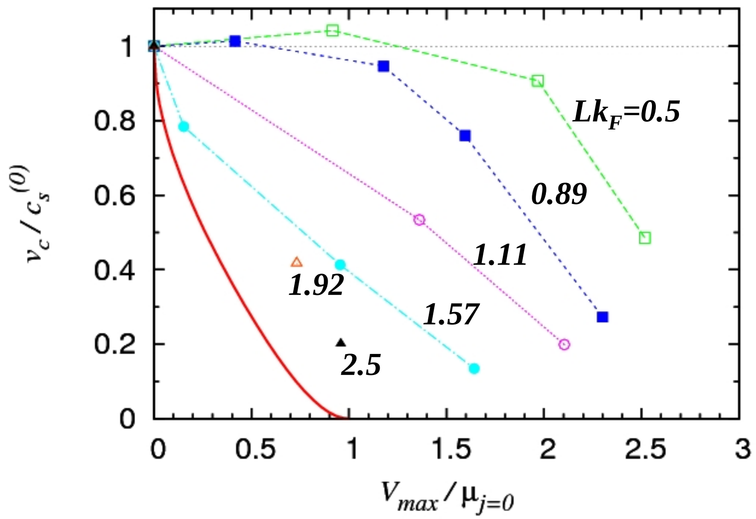


The periodic potential gives results similar to the case of single barrier

bosons



fermions



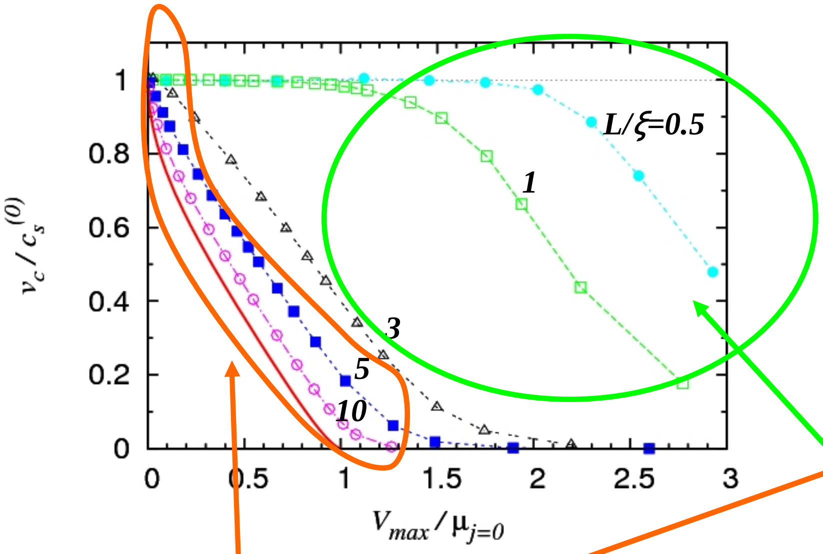
LDA (———) vs. quantum many-body

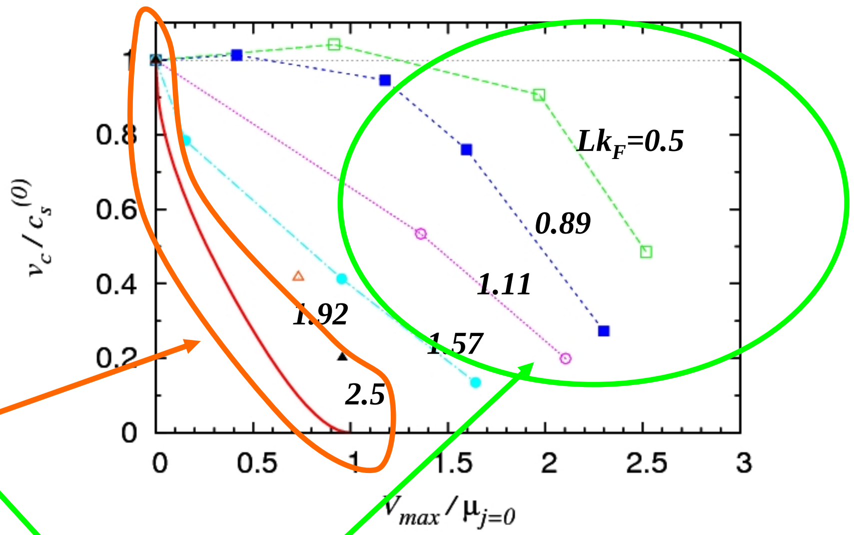
Bosons (left) and Fermions (right) in a periodic potential

L < ξ : Macroscopic flow with
quantum effects beyond LDA

bosons

fermions





L >> ξ : Hydrodynamic flow in LDA

Role of effective mass
in Josephson regime (V_{max}>> μ)

Periodic potential (bosons)

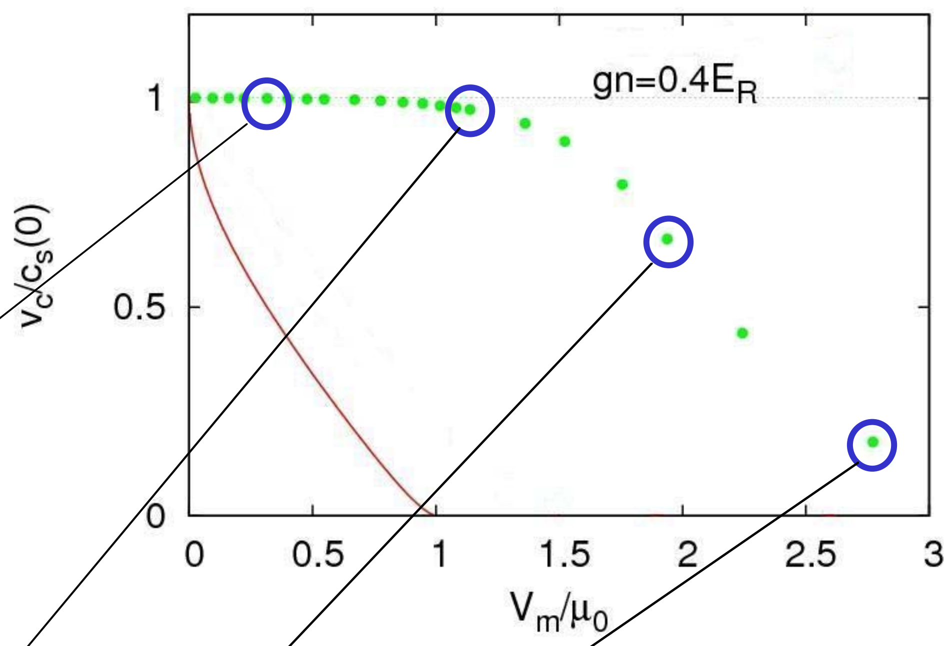
Bloch band structure.

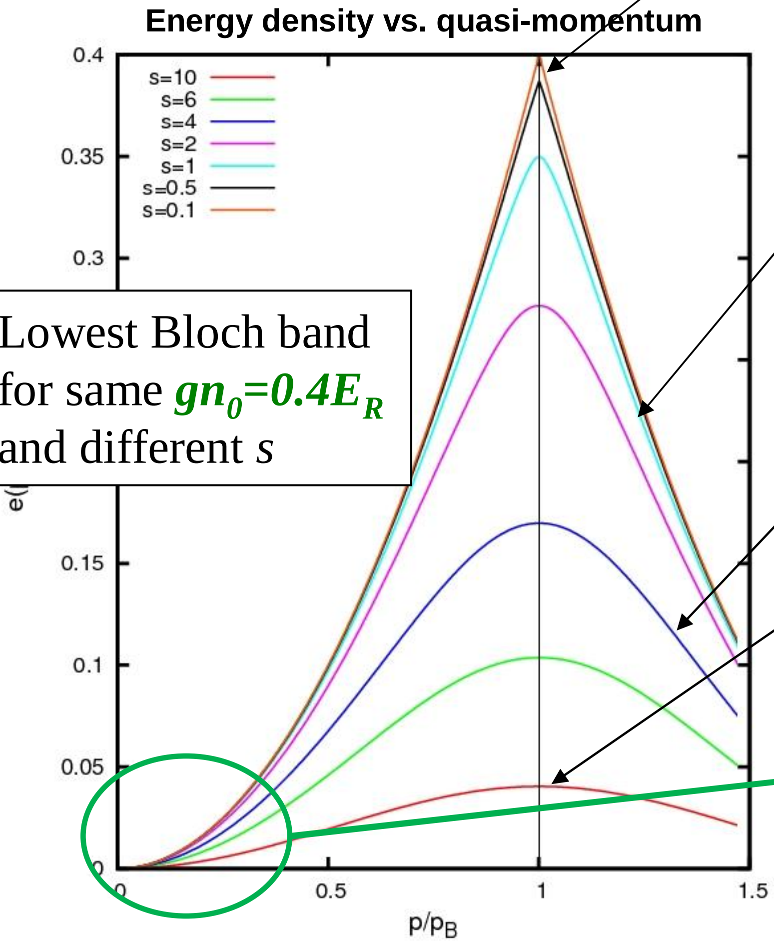
p = quasi-momentum

p_B= Bragg quasi-momentum

$E_R = p^2_B/2m$ = recoil energy

$V_{max} = sE_R$ = lattice strength





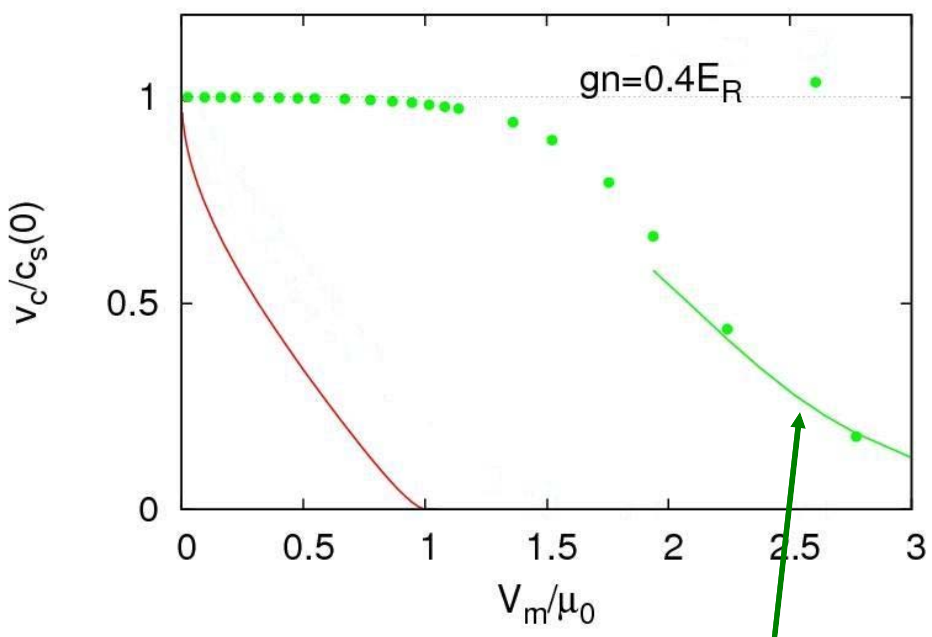
Lowest Bloch band for same gn_0=$0.4E_R$ and different s

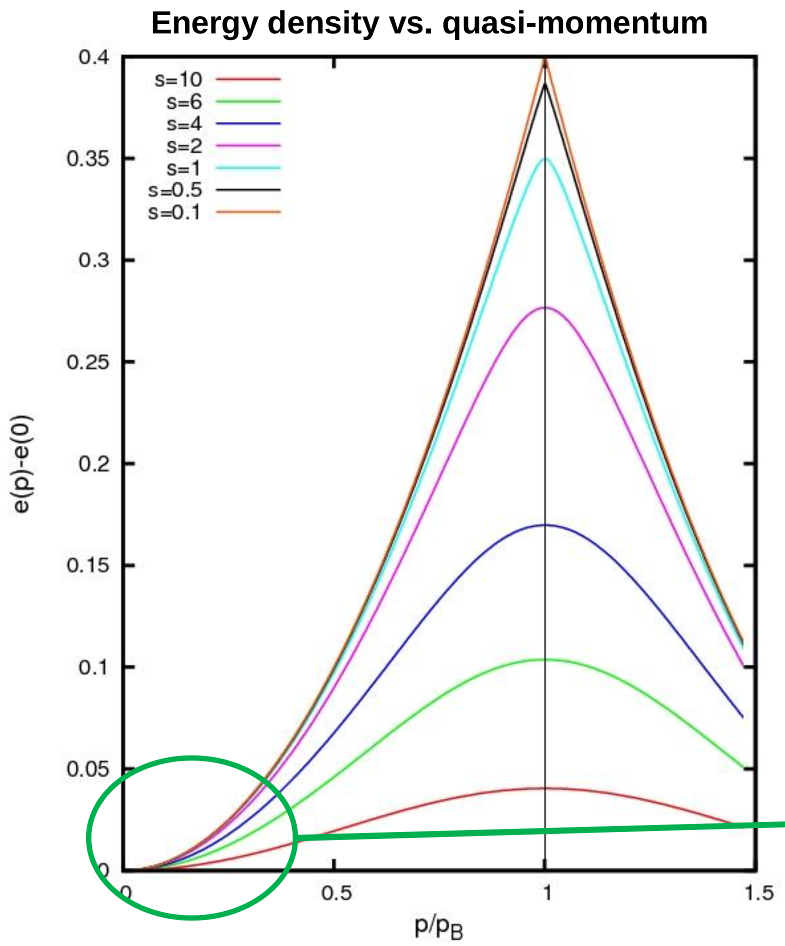
Curvature at $p=0$ gives effective mass:

$e = n_0\, p^2/2m^*$

Periodic potential (bosons)

Bloch band structure.





In tight-binding limit ($V_{max} >> \mu$):

Critical p: $p_c=0.5p_B$

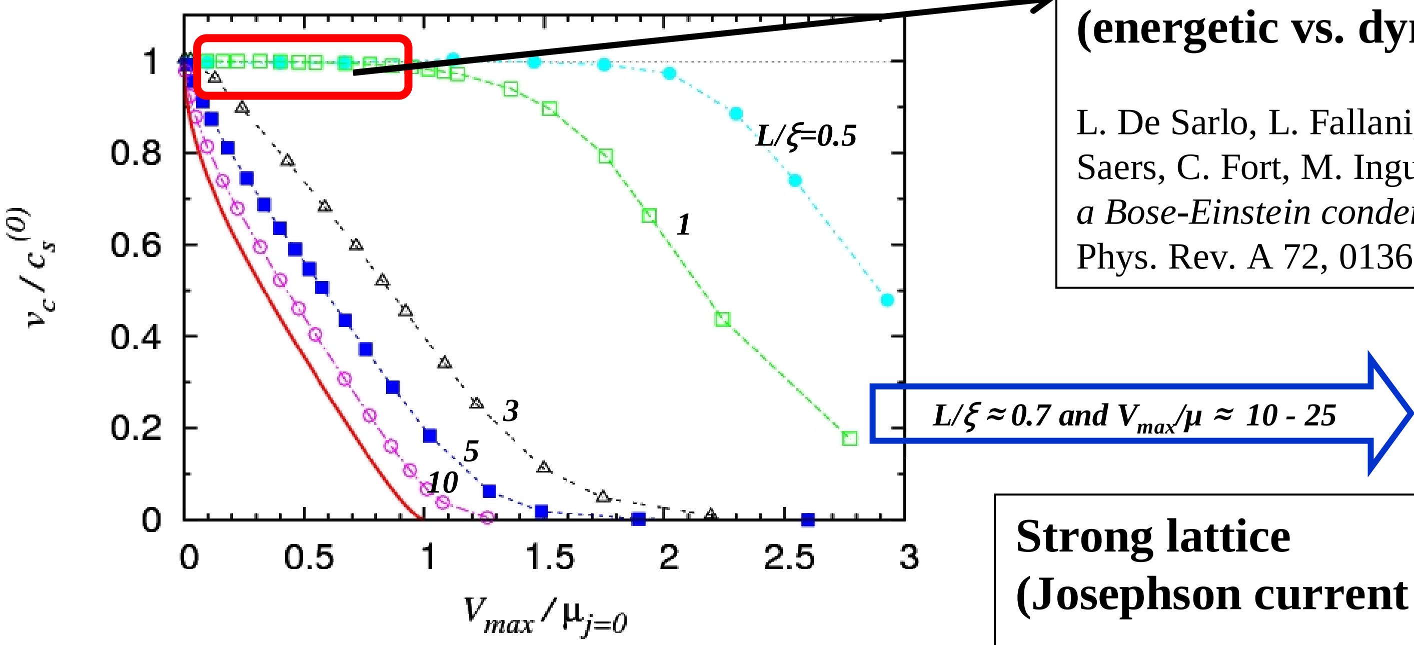
critical velocity is fixed by effective mass:
$v_c = (m/m^*)\, E_R/q_B$

m/m^* proportional to tunneling rate $e^{-2\sqrt{s}}$

Curvature at $p=0$ gives effective mass:
$e = n_0\, p^2/2m^*$

What about experiments?

BOSONS: Experiments at LENS-Florence



Weak lattice
(energetic vs. dynamic instability)

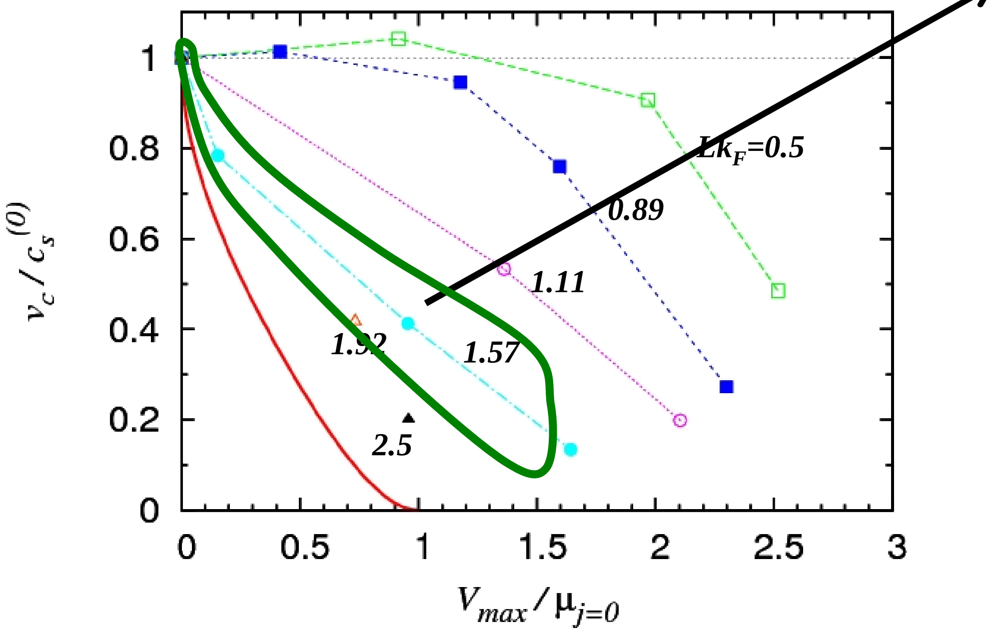
L. De Sarlo, L. Fallani, J. E. Lye, M. Modugno, R. Saers, C. Fort, M. Inguscio, *Unstable regimes for a Bose-Einstein condensate in an optical lattice* Phys. Rev. A 72, 013603 (2005)

Strong lattice
(Josephson current regime):

F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, M. Inguscio *Josephson Junction arrays with Bose-Einstein Condensates* Science 293, 843 (2001)

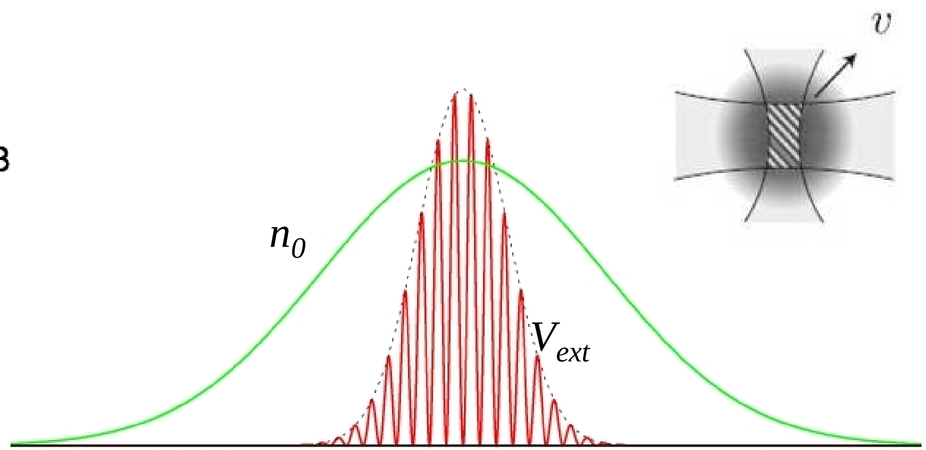
What about experiments?

FERMIONS: Experiments at MIT



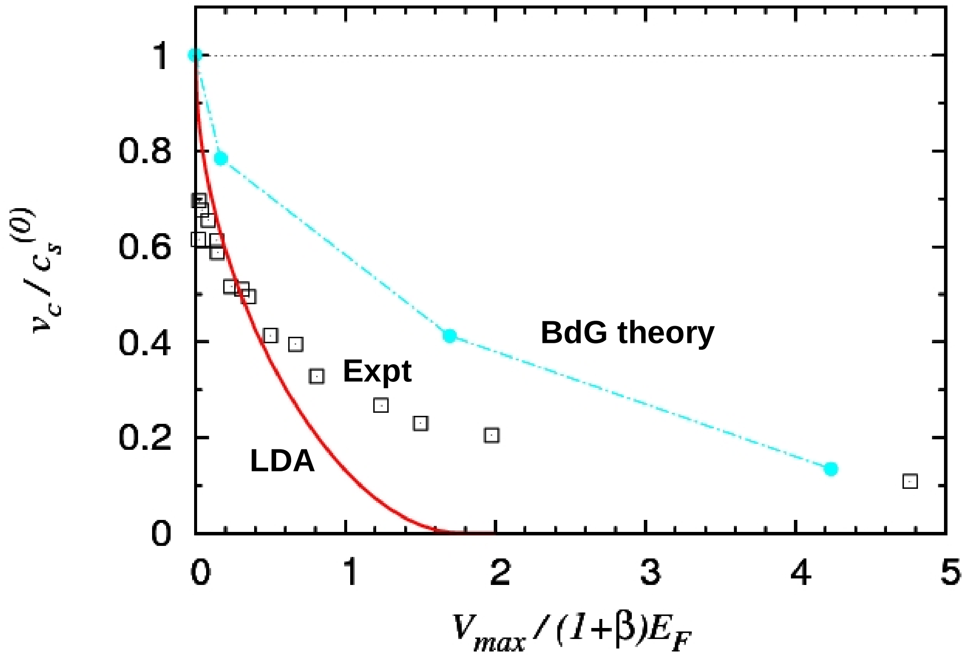
D. E. Miller, J. K. Chin, C. A. Stan, Y. Liu, W. Setiawan, C. Sanner, W. Ketterle
Critical velocity for superfluid flow across the BEC-BCS crossover
PRL 99, 070402 (2007)]

Problem: Which density n_0? Which V_{max}?



What about experiments?

FERMIONS: Experiments at MIT



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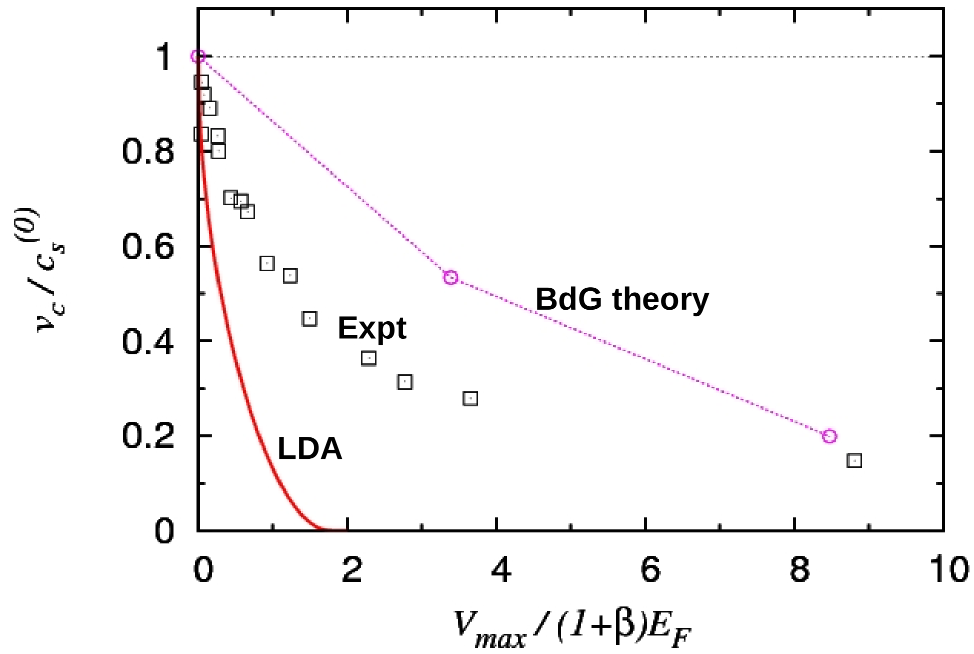
PRL 99, 070402 (2007)]

If E_F is determined by the total number of fermions in the trap:
$E_F/E_R \approx 1 \quad (Lk_F \approx 1.6)$

What about experiments?

FERMIONS: Experiments at MIT



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PRL 99, 070402 (2007)]

If E_F is by the density at e^{-2} beam waist:
$E_F/E_R \approx 0.5 \quad (Lk_F \approx 1.1)$

With both choices of Fermi energy significant discrepancies between theory and MIT data.

Conclusions and perspectives

❖ Remaining discrepancy with MIT 07 experiment remains to be explained:

- non-uniform nature of the gas
- 3D nature of geometry
- inadequacy of mean field Bogoliubov de Gennes theory

❖ Repeat MIT 07 experiment (localized laser lattice) with BEC's (more conclusive comparison with GP theory)

❖ Look for more suitable geometrical configurations. For example toroidal geometry with rotating barrier would provide new insight on criticality of superfluid phenomena (including role of quantum vorticity)

(arXiv:0903.2534)

Vortex-Induced Phase Slip Dissipation in a Toroidal Bose-Einstein Condensate Flowing Through a Barrier

F. Piazza,[1] L. A. Collins,[2] and A. Smerzi[1]