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Magnetic order and transport in Bose gases with spin

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Magnetic Order and Transport in Bose Gases with Spin

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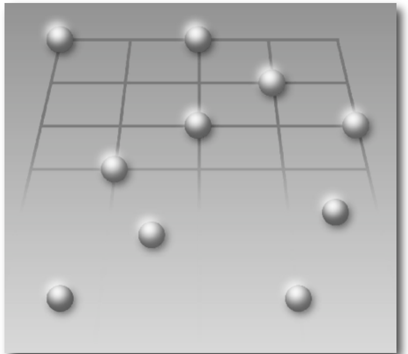

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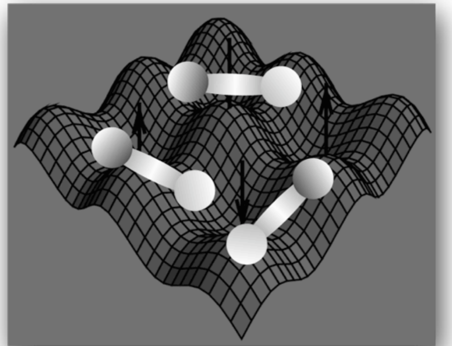

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German-Israeli
Project Cooperation

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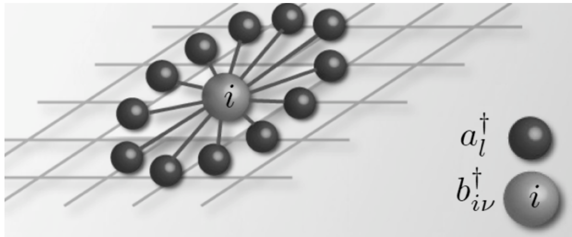
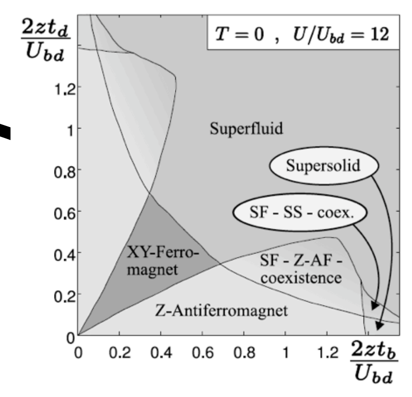
Outline

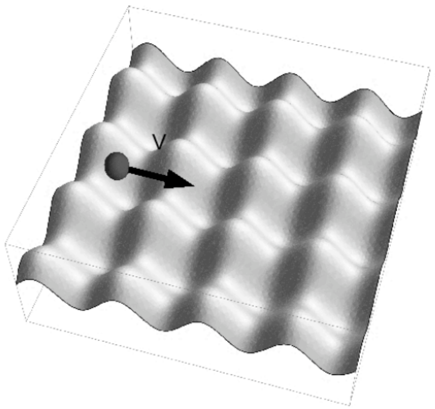


Hubener, Snoek and **WH**,
arXiv:0902.2212



Bosonic Dynamical
Mean-Field Theory

magnetic order
of Bosons



Anomalous Hall Effect in Spinor BEC

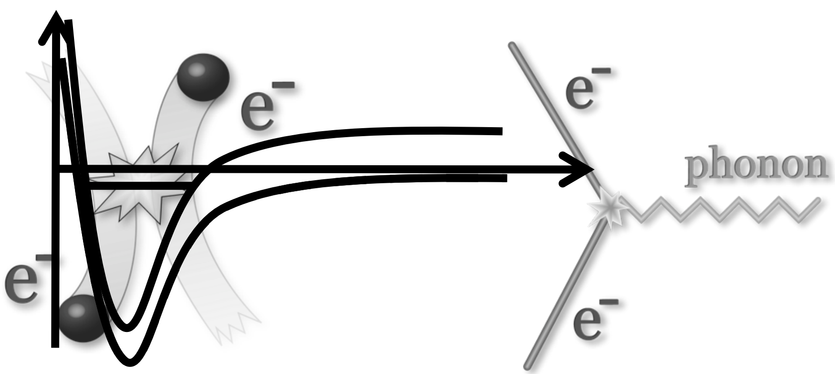
M. Taillefumier, E. Dahl, A. Brataas, and **WH**, arXiv:0901.1969

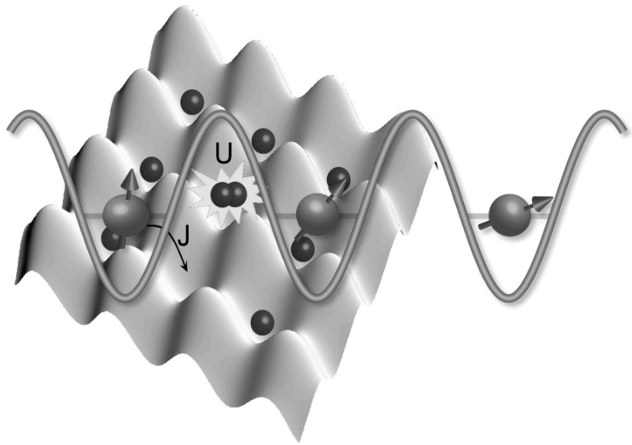
Strong correlations

Feshbach resonance

e[illegible]ed interactions

optical lattice





enhance interactions

reduce tunneling

strongly correlated system

Dynamical Mean-Field Theory

- interacting lattice model

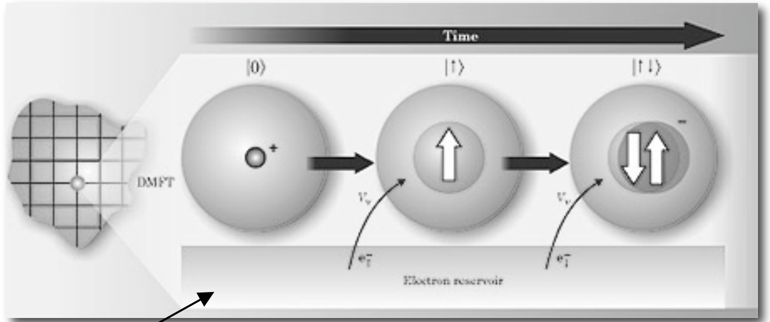
⇩

self-consistent local problem

$$G_{\text{lattice}} = G_{\text{imp}}\left(\epsilon_i, U, \Gamma(\omega)\right)$$

effective impurity

fermionic "bath"



Kotliar, Vollhardt,
Physics Today **57**, 53 ('04)

- DMFT uses local DOS as mean field

$$\Gamma(\omega) \sim \rho_{loc}(\omega)$$

nonperturbative approach

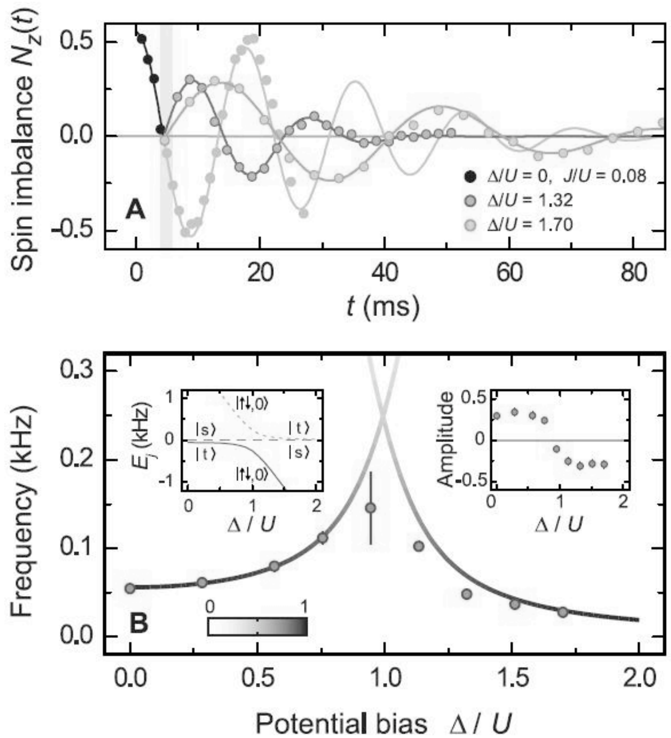
Strongly correlated Bosons

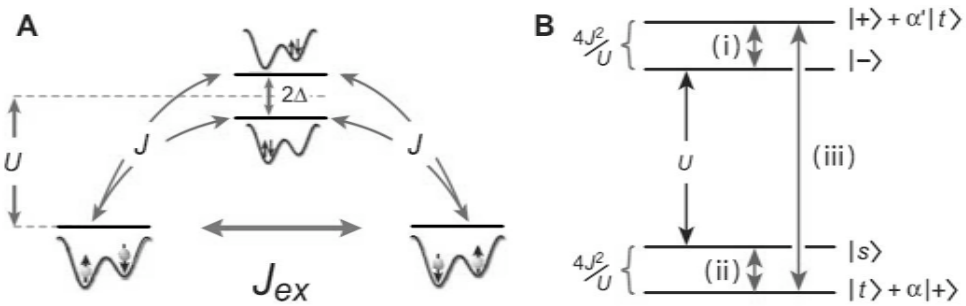
Recent experiments

Superexchange interactions in optical lattices

- double well, coupled via second order tunnelling process

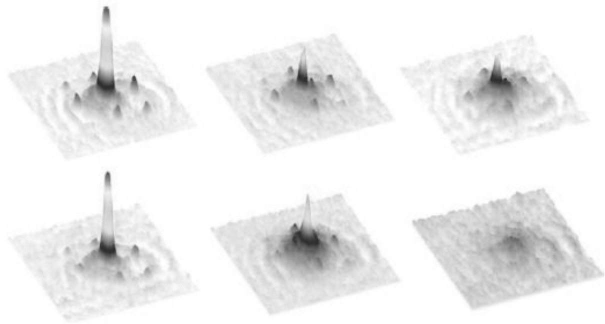
S. Trotzky *et al.*, Science **319**, 295 (2008)



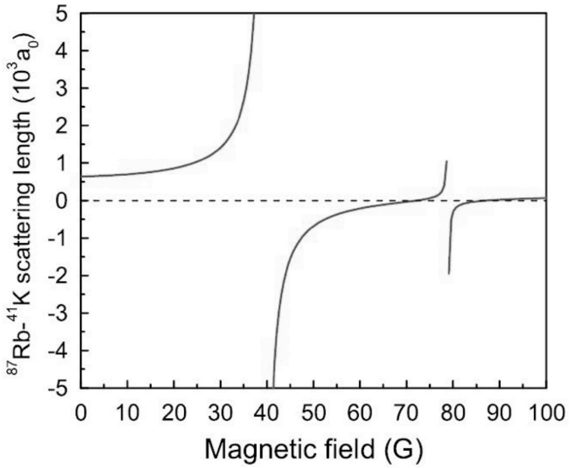


Double species Bose-Einstein Condensate

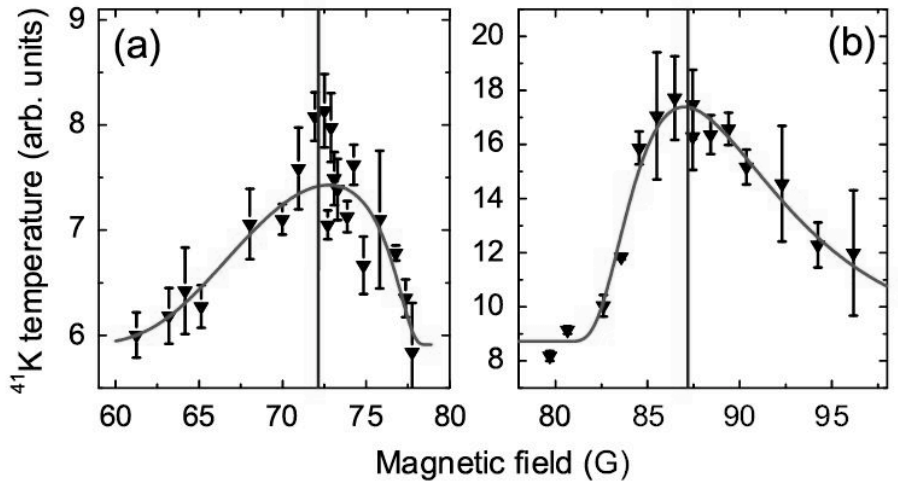
- superfluid ^{87}Rb-^{41}K-mixture
- K reduces visibility of Rb



- tunable Rb-K-scattering length via Feshbach-Resonances



- precise control of around zero



J. Catani *et al.*, Phys. Rev. A **77**, 011603 (2008)

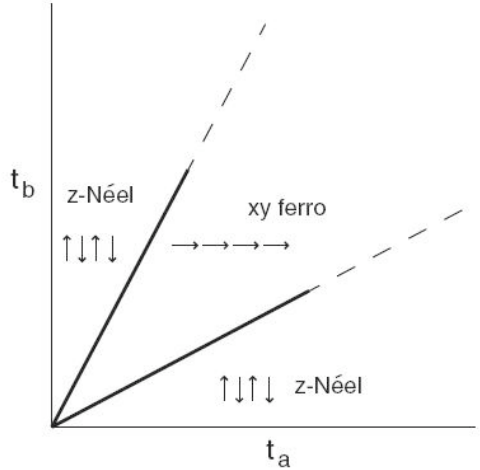
G. Thalhammer *et al.*, Phys. Rev. Lett. **100**, 210402 (2008)

Theoretical works

- Mapping of the two-species Bose Hubbard Model to effective spin Hamiltonian for MI

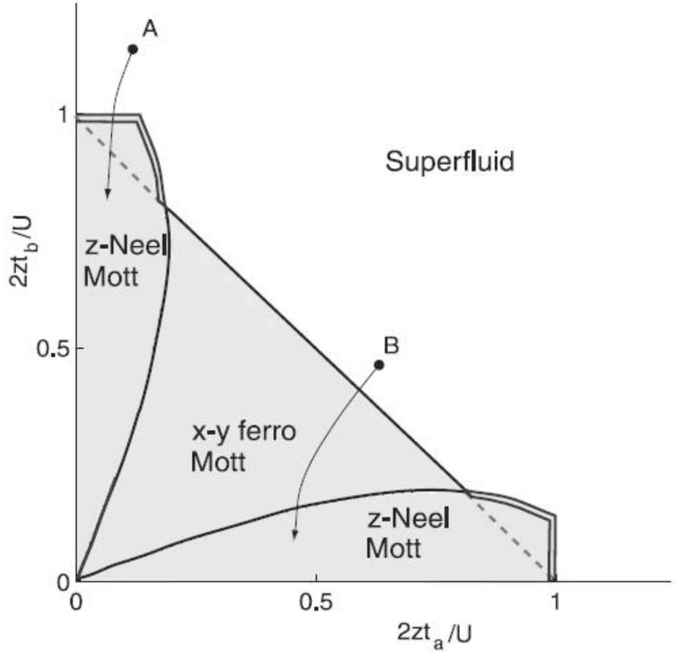
$$H_{\text{eff}} = J_z \sum_{\langle ij \rangle} S_i^z S_j^z - J_\perp \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - h \sum_i S_i^z.$$

Duan, Demler, Lukin, Phys. Rev. Lett. **91**, 090402 (2003)



- quantum fluctuation correction to mean-field:
  - spin order
  - hysteretic behavior
  - no supersolid

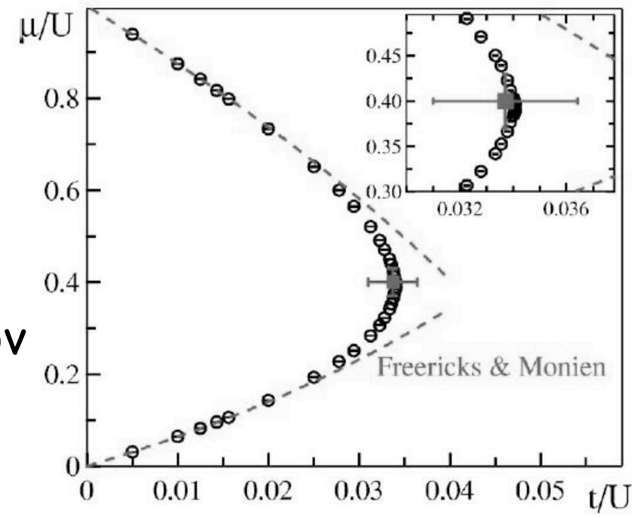
Altman, Hofstetter, Demler, Lukin New Journal of Physics **5**, 113 (2003)



Quantum Monte Carlo simulations

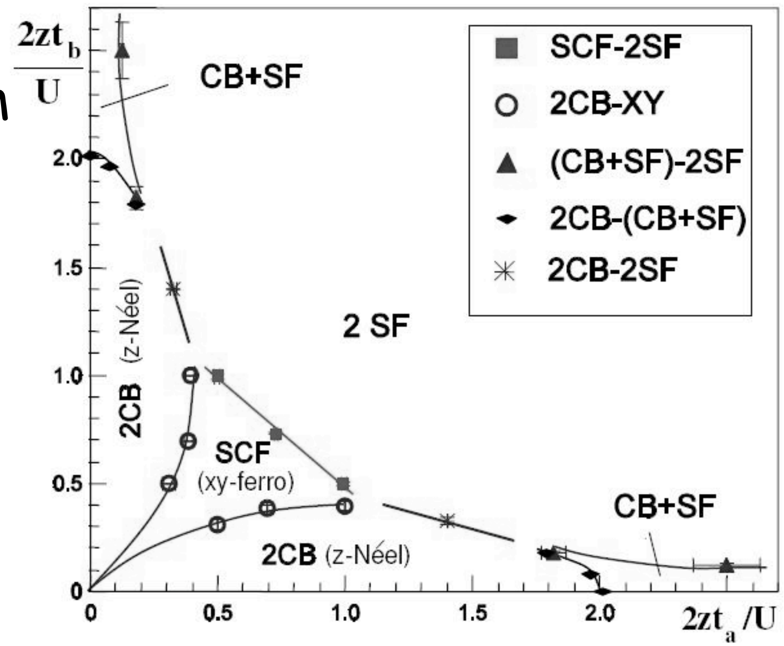
- Single-component MI-SF phase diagram

Capogrosso-Sansone, Prokof'ev, Svistunov
Phys. Rev. B **75**, 134302 (2007)



- two-component phase diagram on 2-D square lattice
→ supersolid phase

S.G. Söyler *et al.*,
e-print arXiv:0811.0397



Bosonic DMFT

Hubener, Snoek and WH, preprint arXiv:0902.2212

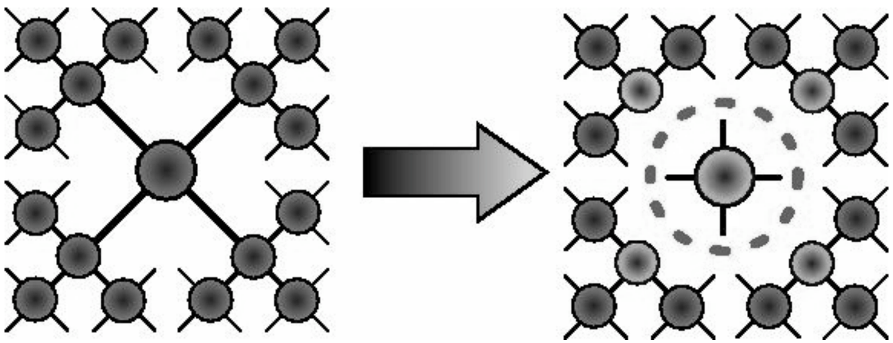
Investigation of the multi-species single band Bose Hubbard model:

$$\hat{H} = -\sum_{\langle ij\rangle,\nu}\left(t_\nu \hat{b}^\dagger_{i\nu}\hat{b}_{j\nu} + \mathrm{h.c.}\right) + \frac{1}{2}\sum_{i,\mu\nu} U_{\nu\mu}\hat{n}_{i\nu}\left(\hat{n}_{i\mu} - \delta_{\nu\mu}\right) - \sum_{i,\nu}\mu_\nu \hat{n}_{i\nu}$$

- effective impurity action by cavity method

$$Z_{\mathrm{imp}} = \frac{Z}{Z^{(0)}} = \int \prod_\nu \mathcal{D}b^*_{0,\nu}\,\mathcal{D}b_{0,\nu}\,e^{-S_{\mathrm{imp}}},$$

Byczuk and Vollhardt, PRB 77, 235106 (2008)



- For bosons anomalous cavity system expectation values do not vanish.

$$\left\langle b_{i\nu}(\tau)\right\rangle_0 = \phi_{i\nu} \geq 0$$

- Bethe lattice: product expectation values of different neighbors decouple.

$$\left\langle b_{i\nu}(\tau_1)\, b_{j\mu}(\tau_2)\right\rangle_0 = \left\langle b_{i\nu}(\tau_1)\right\rangle_0 \left\langle b_{j\mu}(\tau_2)\right\rangle_0 = \phi_{i\nu}\, \phi_{j\mu} \qquad \text{for} \quad i \neq j$$

- Identification of the expectation values of the cavity system with expectation values on the impurity

$$\left\langle b_{i,\nu}(\tau)\right\rangle_0 = \left\langle b_{0,\nu}(\tau)\right\rangle_{\rm imp}$$

$$\left\langle b_{i,\nu}(\tau_1)\, b^*_{i,\mu}(\tau_2)\right\rangle_0 = \left\langle b_{0,\nu}(\tau_1)\, b^*_{0,\mu}(\tau_2)\right\rangle_{\rm imp}$$

- Rescaling all hopping parameters $t_\nu = t^*_\nu / z$ and keeping terms up to subleading order 1/z → effective impurity action:

$$S_{\rm imp} = \int_0^\beta d\tau \sum_{\nu\mu} \frac{U_{\nu\mu}}{2}\, n_\nu(\tau) \Big(n_\mu(\tau) - \delta_{\nu\mu} \Big) + \int_0^\beta d\tau \sum_{\nu} z t_\nu \Big(b^*_\nu(\tau)\phi_\nu + b_\nu(\tau)\phi_\nu \Big)$$
$$+ \frac{1}{\beta} \sum_{n\geq 0} \sum_{\nu\mu} \mathbf{b}^*_{n,\nu} \bigg((-i\omega_n \hat{\sigma}_z - \mu\, \mathbb{1}_2)\delta_{\nu\mu} + z t_\nu t_\mu \mathbf{G}_{\nu\mu}(i\omega_n) \bigg) \mathbf{b}_{n,\mu}$$

$$S_{\text{imp}} = \int_0^\beta d\tau \sum_{\nu\mu} \frac{U_{\nu\mu}}{2} n_\nu(\tau) \Big(n_\mu(\tau) - \delta_{\nu\mu}\Big) + \int_0^\beta d\tau \sum_\nu z t_\nu \Big(b_\nu^*(\tau)\phi_\nu + b_\nu(\tau)\phi_\nu\Big)$$
$$+ \frac{1}{\beta} \sum_{n\geq 0} \sum_{\nu\mu} \mathbf{b}_{n,\nu}^* \Big((-i\omega_n \hat{\sigma}_z - \mu\, \mathbb{1}_2)\delta_{\nu\mu} + z t_\nu t_\mu \mathbf{G}_{\nu\mu}(i\omega_n) \Big) \mathbf{b}_{n,\mu}$$

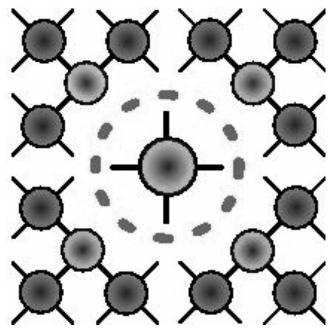
- Second term includes Greens function matrix with normal and anomalous terms:

$$\mathbf{G}_{\nu\mu} = \begin{pmatrix} G^1_{\nu\mu} + \phi_\nu\phi_\mu & G^2_{\nu\mu} + \phi_\nu\phi_\mu \\ G^2_{\nu\mu} + \phi_\nu\phi_\mu & G^1_{\nu\mu} + \phi_\nu\phi_\mu \end{pmatrix}$$

$$G^1_{\nu\mu}(\tau) = -\langle b_\nu(\tau)\, b^*_\mu \rangle$$
$$G^2_{\nu\mu}(\tau) = -\langle b_\nu(\tau)\, b_\mu \rangle$$

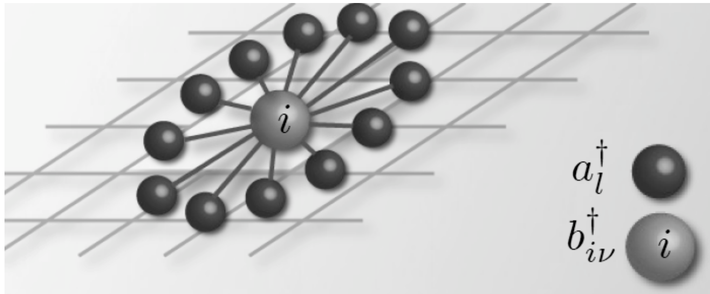
- These are 1/z-correction to the mean-field

- Additional 1/z-correction for mean-field itself: Sites in the cavity system have one neighbor less. We take this in a perturbative way into account:



$$\phi_\nu = \big\langle b_{i,\nu}(\tau) \big\rangle_0 \approx \big\langle b_{0,\nu}(\tau) \big\rangle^{\text{imp}}_{(z-1)}$$

$$
\begin{aligned}
\hat{H}_A =& \frac{1}{2}\sum_{\mu\nu} U_{\nu\mu}\hat{n}_\nu(\hat{n}_\mu - \delta_{\nu\mu}) - \sum_\nu \hat{n}_\nu \mu_\nu \\
& - \sum_\nu z t_\nu (\phi_\nu^* \hat{b}_\nu + \phi_\nu \hat{b}_\nu^\dagger) + \sum_l \epsilon_l \hat{a}_l^\dagger \hat{a}_l \\
& + \sum_{l,\nu} \Big(V_{\nu,l}(\hat{a}_l^\dagger \hat{b}_\nu + \hat{b}_\nu^\dagger \hat{a}_l) + W_{\nu,l}(\hat{a}_l \hat{b}_\nu + \hat{b}_\nu^\dagger \hat{a}_l^\dagger) \Big)
\end{aligned}
$$



effective
Anderson-impurity
Hamiltonian

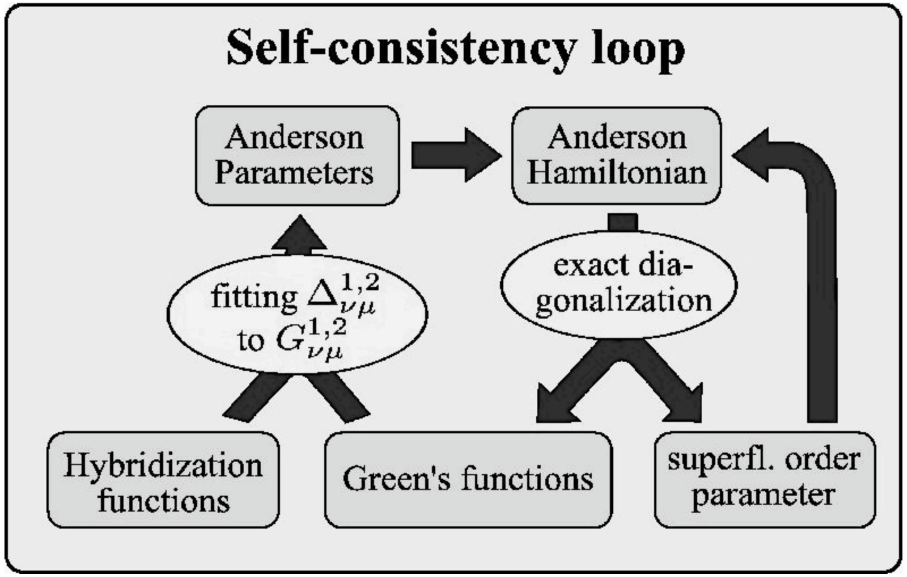
same effective action with the following identification:

$$
z t_\nu t_\mu G^1_{\nu\mu} \hat{=} \Delta^1_{\nu\mu} \quad \text{with} \quad \Delta^1_{\nu\mu} = -\sum_l \frac{V_{\nu,l}V_{\mu,l}}{\epsilon_l - i\omega_n} + \frac{W_{\nu,l}W_{\mu,l}}{\epsilon_l + i\omega_n}
$$

$$
z t_\nu t_\mu G^2_{\nu\mu} \hat{=} \Delta^2_{\nu\mu} \quad \text{with} \quad \Delta^2_{\nu\mu} = -\sum_l \frac{V_{\nu,l}W_{\mu,l}}{\epsilon_l - i\omega_n} + \frac{V_{\nu,l}W_{\mu,l}}{\epsilon_l + i\omega_n}
$$

self-consistency equations for Anderson parameters

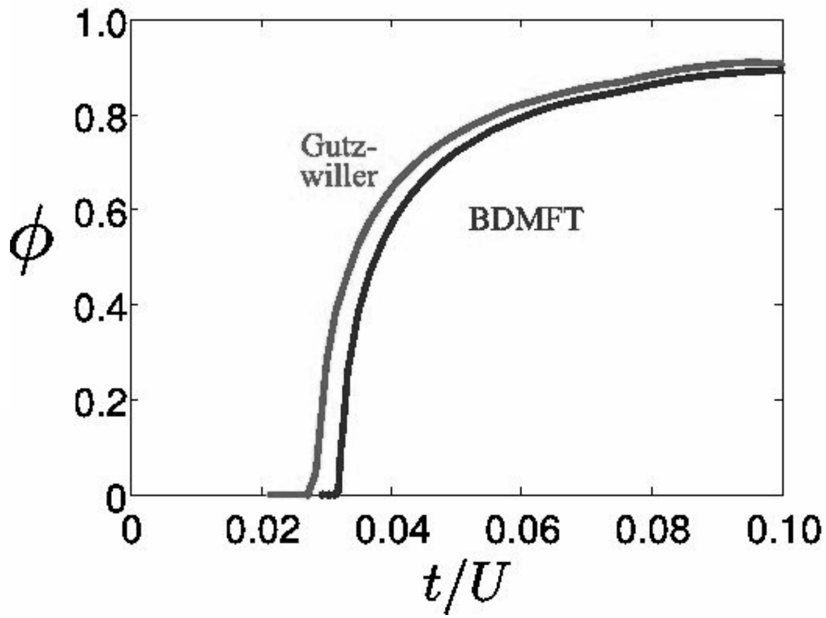
$$
\begin{aligned}
\phi_\nu &= \langle \hat{b}_\nu \rangle_{z-1} \\
z t_\nu t_\mu G^{1,2}_{\nu\mu}(i\omega_n) &= \Delta^{1,2}_{\nu\mu}(i\omega_n)
\end{aligned}
$$

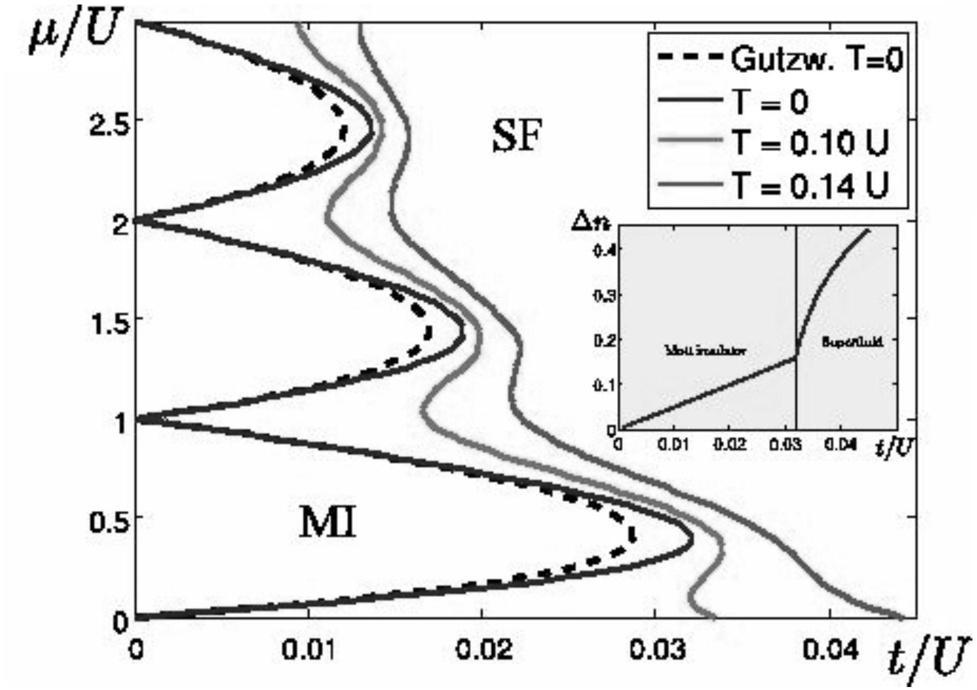


Results for a single species in three dimensions $(z = 6)$

Hubener, Snoek and WH, arXiv:0902.2212

- Superfluid order parameter reduced compared to mean-field due to quantum fluctuations

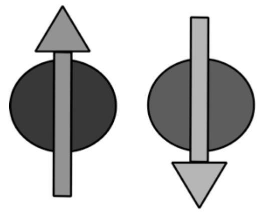




- Mott plateaus larger than in mean-field.

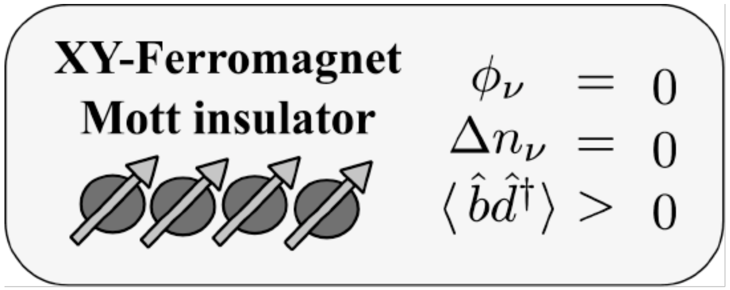
- finite number fluctuations in MI

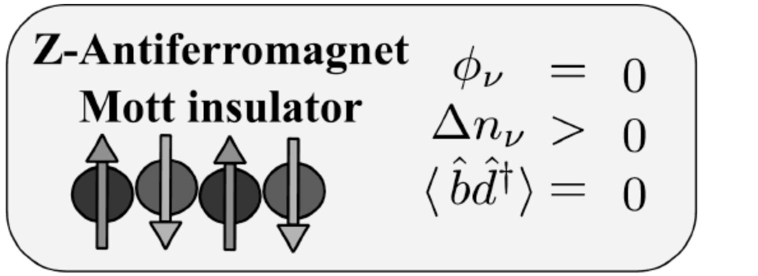
Results for two components at unit filling $(n = 1)$

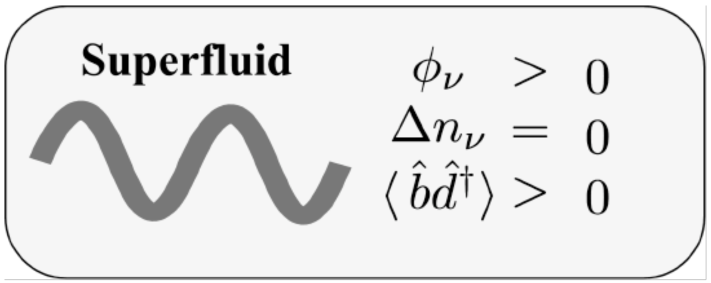


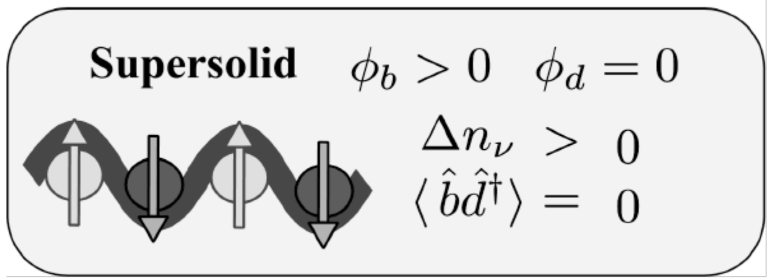
- two different species with pseudo-spin-½
- inter-species interaction less than intra-species interaction
- same intra-species interaction and chemical potential, but *different hopping*

$$U_b = U_d \equiv U \qquad \mu_b = \mu_d \equiv \mu \qquad t_b \neq t_d$$





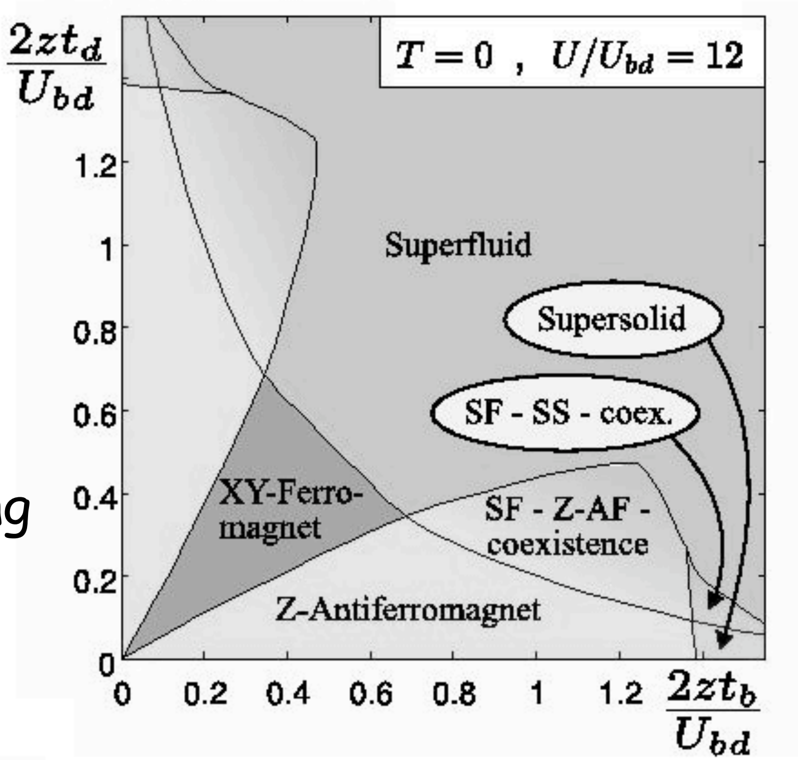


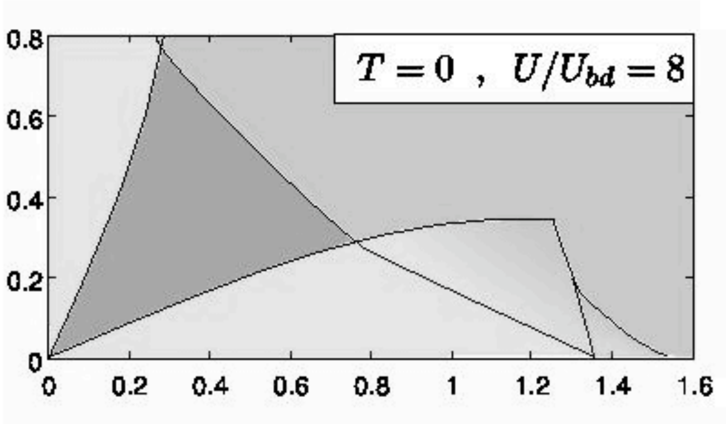


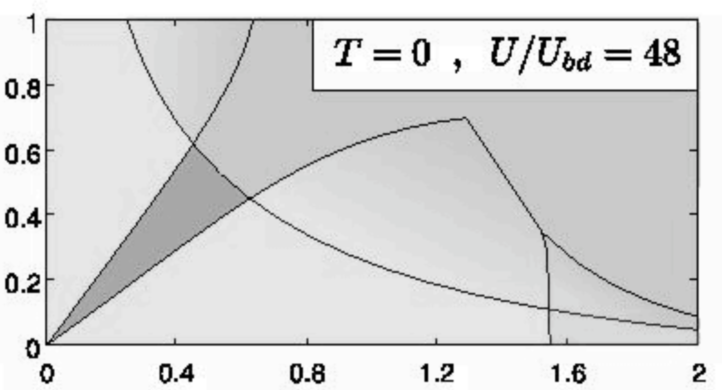
T=0 phase diagrams

Hubener, Snoek and WH,
arXiv:0902.2212

- $t_b \approx t_d \rightarrow$ ferromagnetic ordering
  $t_b > t_d \rightarrow$ antiferromagnetic ordering
- coexistence regions for superfluid and Z-antiferromagnet.
- high U/U_{bd} -ratios support AF-ordering
- supersolid for $t_b \gg t_d$



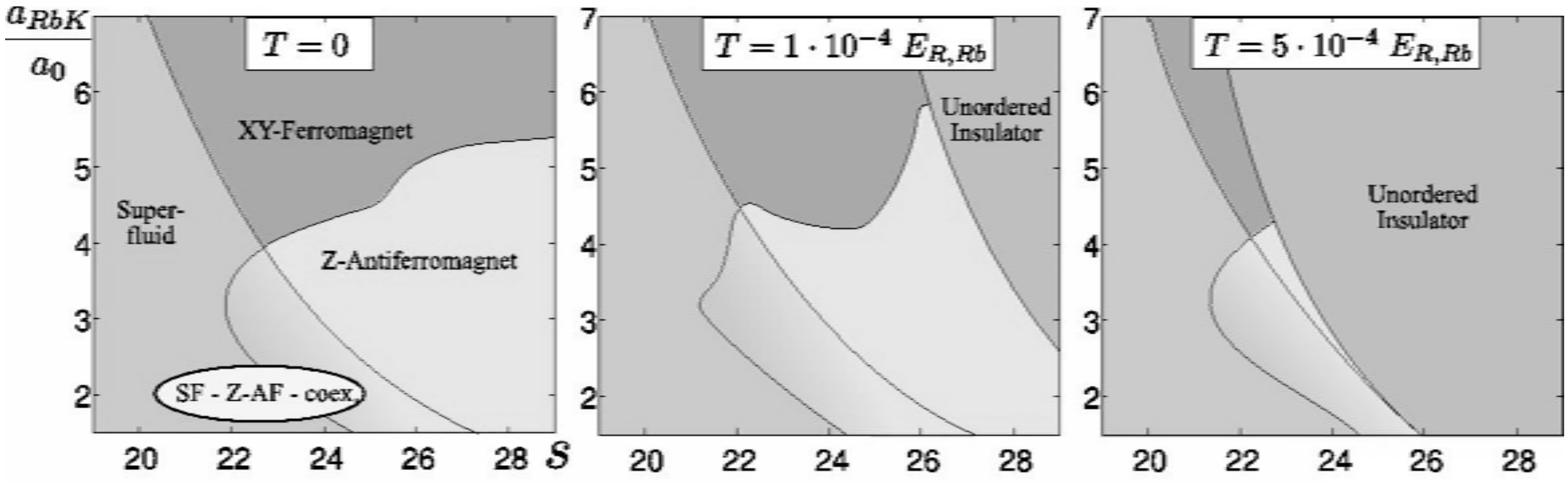


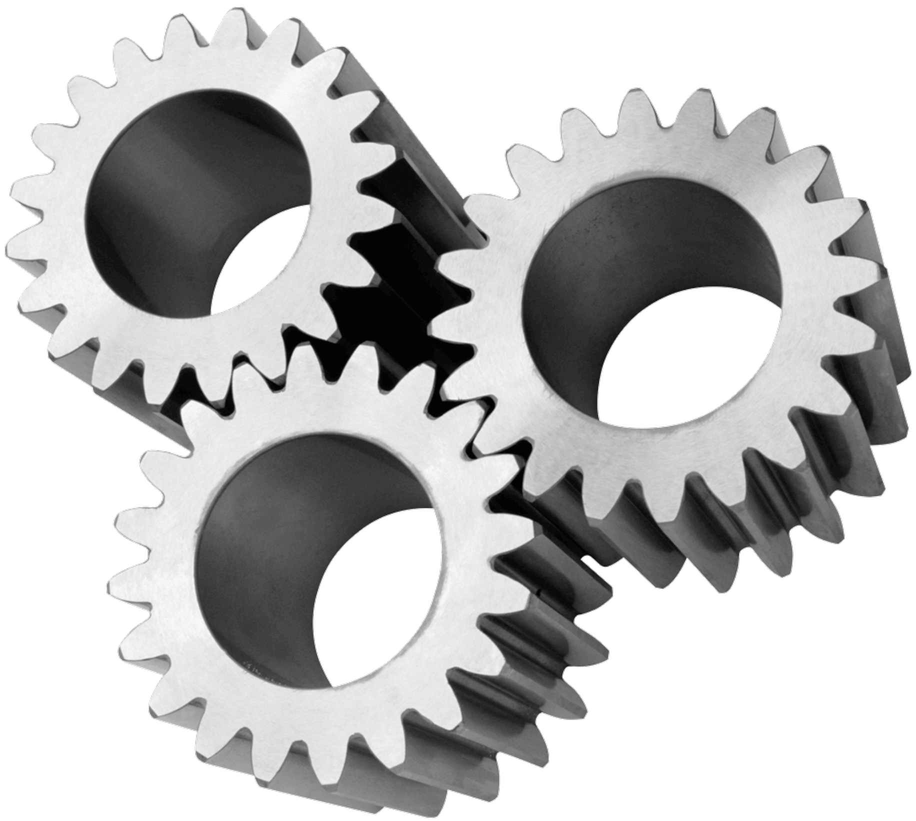


Finite temperature results for a ^{87}Rb-^{41}K-mixture

Hubener, Snoek and WH, arXiv:0902.2212

- hopping-ratio fixed but interspecies scattering length tunable
- all phases except supersolid appear.
- at finite temperatures: insulating phase without any ordering

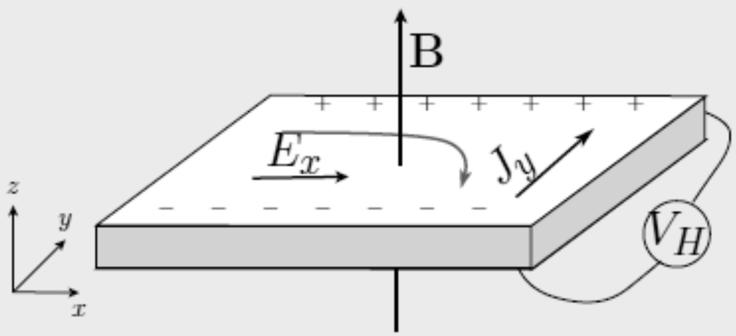




Anomalous Hall Effect in Spinor Condensates

M. Taillefumier, E. Dahl, A. Brataas, and **WH**, arXiv:0901.1969

Hall Effect in paramagnets



- Hall Effect : Lorentz Force $\mathbf{f} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \Rightarrow V_H \neq 0.$
- Definitions : $\mathbf{J} = \bar{\sigma}\mathbf{E}$, $\bar{\sigma}$ Conductivity tensor
- $\sigma_{xy} = J_y/E_x$ Off-diagonal conductivity
- $\rho_{xy} = [\bar{\sigma}^{-1}]_{xy} \approx \frac{\sigma_{xy}}{\sigma_{xx}^2}$ Hall resistivity

Anomalous Hall Effect in ferromagnets

Experimental Law (Pugh *et al*)

$$\rho_{xy} = \underbrace{R_0 B}_{\text{Classical Hall Effect}} + \underbrace{R_s M}_{\text{AHE}}$$

Origin

- Spontaneous magnetization (Acting like internal magnetic field on e^-).
- Strong spin-orbit coupling (lattice and impurities)

$$V_{SO} \propto (\nabla V \times p) \cdot \sigma$$

- Additional term in the velocity operator proportional to the spin

History of AHE

- 1880 : Discovery of the Hall effect (E. R. Hall)
- 1881 : Discovery of the Anomalous Hall effect (E. R. Hall)
- 1881-1950 : Numerous experimental studies of AHE (A. Kundt, A.W. Smith, A. Perrier, E.M. Pugh, J. Smit, ...)
- 1954 : First quantitative theory (Karplus and Luttinger) anomalous velocity current due to the spin-orbit coupling
- 1955 : skew scattering contribution (J. Smit)
- 1957-1958 : systematic theory of AHE with effect of scattering (Kohn, Luttinger)
- 1970 : Side Jump (Berger)

Intrinsic mechanism for AHE

Assumptions

- spin-orbit coupling and homogeneous magnetization field.
- OR inhomogeneous magnetization field (skyrmions or frustration)
- There is NO IMPURITIES.

Results

$$\sigma_{xy} = \frac{ie^2}{h} \sum_n \int \frac{d^2\mathbf{k}}{4\pi^2} n_f(\varepsilon_{n,\mathbf{k}}) \underbrace{\mathrm{Im}\left[\left\langle \frac{\partial u_{n,k}}{\partial k_x} \middle| \frac{\partial u_{n,k}}{\partial k_y} \right\rangle\right]}_{\text{Berry curvature}} .$$

Karplus, Luttinger '54

- Depends only the topology of the eigenstates.
- All occupied states are needed. Haldane (2004) shows that this contribution can be described also as a contribution of states close to the Fermi level.

Topological Hall Effect

- AHE can arise due to non-trivial magnetic structure, e.g. in frustrated ferromagnets ($Nd_2Mo_2O_7$)

- most simple model:
  - free particles (e.g. electrons) $\mathbf{p}^2/2m$
  - Zeeman coupling $\mathbf{B}(\mathbf{r}) \cdot \sigma$

    where $B(\mathbf{r})$ simulates magnetic structure
  - neglect spin-orbit coupling

- Hamiltonian

$$\mathcal{H} = -\frac{\hbar^2}{2m}\nabla^2 + g\mu_B \mathbf{B}(\mathbf{r}) \cdot \sigma$$

P. Bruno et al., PRL 93, 096806 ('04)

Why study AHE in spinor BEC

Why studying AHE in BEC

- External environment is known
- several hyperfine states can be trapped
- It is possible to study the geometrical (Berry) phases contributions of the AHE only.

Advantages

- They are very clean systems (ideal to study the KL term)
- They are relatively simple to manipulate
- Large number of particles in the same state (stronger effect)
- They are already used to study transport phenomenas: (Bloch oscillations, Josephson effect, etc...)

related: A. Dudarev et al., PRL 92, 153005 ('04)

Challenges

Problems

- neutral gases so no classical Hall effect nor classical Lorentz force.
- no impurities nor spin-orbit coupling (no SJ and Skew scattering).
- Only possibility : induce Lorentz force through geometrical phases (Aharonov and Stern, PRL).

Solution to the problem

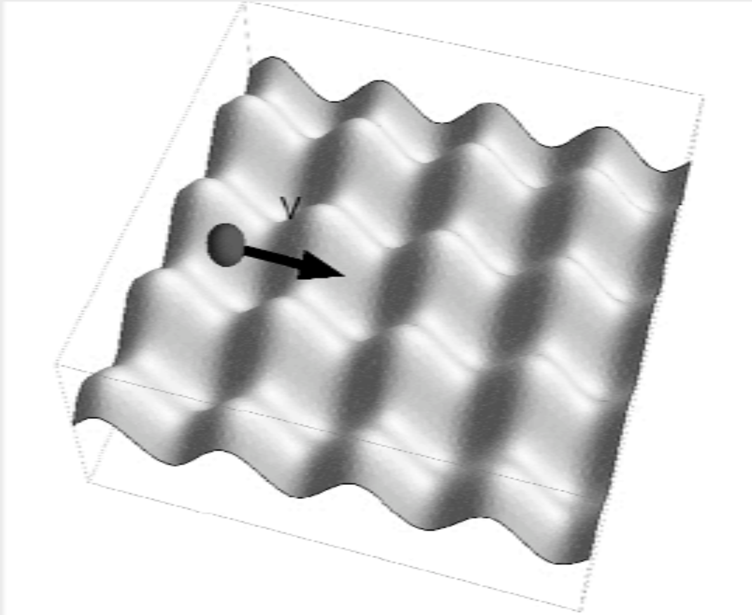
- use spin-dependent optical lattices or specially designed magnetic microtraps.
- gravity can provide the external electrical field.
- But at least two internal degrees of freedom should be coupled

Setup

M. Taillefumier, E. Dahl, A. Brataas, and **WH**, arXiv:0901.1969

System

- condensate of spin $F = 1$.
- microtraps with 2D array of magnetic cylinders.
- additional magnetic field can be applied.



Goal

see what are the effects of the initial velocity, add. mag. field., etc on the dynamic of the BEC

Hamiltonian

M. Taillefumier, E. Dahl, A. Brataas, and **WH**, arXiv:0901.1969

Characteristics

- $F = 1$ spinor condensate in xOy-plane (not specific to "spin one")
- "Magnetic field" $\mathbf{B}(\mathbf{r})$ with $\langle B_z(\mathbf{r})\rangle = 0$ per unit cell.
- No classical Hall effect.
- spin-dependent two-body interactions.

Hamiltonian

$$\begin{aligned}\mathcal{H} &= \int d^2\mathbf{r}\psi^\dagger_\alpha(\mathbf{r})\left(\left[-\frac{\hbar^2}{2M}\nabla^2 + V(\mathbf{r})\right]\delta_{\alpha\gamma} + (\mathbf{B}(\mathbf{r})\cdot\mathbf{F})_{\alpha\delta}\right.\\ &+ \left.\sum_{s=0}^{F}\frac{\sqrt{2\pi}\hbar^2 a_{2s}}{Ma_z}(\mathcal{P}^{2s})_{\alpha\beta\gamma\delta}\psi^\dagger_\beta(\mathbf{r})\psi_\gamma(\mathbf{r})\right)\psi_\delta(\mathbf{r}) \qquad (1)\end{aligned}$$

Local transformation

- „rotating frame": choose spin quantization axis along magnetic field $\mathbf{B}(\mathbf{r}) = B(\mathbf{r})\mathbf{n}(\mathbf{r})$ via local rotation $\mathcal{T}(\mathbf{r})$

- effective Hamiltonian

$$\mathcal{H}_{\mathcal{T}} = \int d^2\mathbf{r}\,\psi_\alpha^\dagger(\mathbf{r}) \left(\left[-\frac{\hbar^2}{2M} (\nabla - i\mathbf{A}_g(\mathbf{r}))^2 + V(\mathbf{r})\delta_{\alpha\gamma} \right] + |B(\mathbf{r})| F_z^{\alpha\delta} + \sum_{s=0}^{F} \frac{\sqrt{2\pi}\hbar^2 a_{2s}}{M a_z} (\mathcal{P}^{2s})_{\alpha\beta\gamma\delta} \psi_\beta^\dagger(\mathbf{r}) \psi_\gamma(\mathbf{r}) \right) \psi_\delta(\mathbf{r})$$

with spin-dependent gauge field

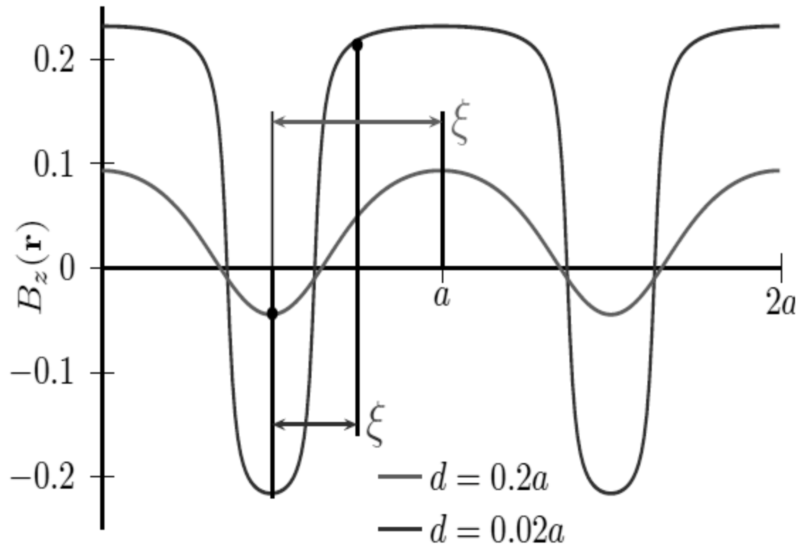
$$\mathbf{A}_g(\mathbf{r}) = -i\mathcal{T}^\dagger(\mathbf{r}) \nabla_{\mathbf{r}} \mathcal{T}(\mathbf{r}) = \mathbf{A}_g^i(\mathbf{r}) F^i$$

Adiabatic approximation

- spin **F** parallel to **B**(**r**)
- important parameters:

τ_0 precession time along $\mathbf{n}(\mathbf{r})$

BEC group velocity $\tau = \xi / v_g$



- adiabaticity criterion

$$\lambda = \frac{\tau_0}{\tau} = \frac{\hbar v_g}{\epsilon_0 \xi} \ll 1$$

ϵ_0 Zeeman splitting

Adiabatic approximation

M. Taillefumier, E. Dahl, A. Brataas, and **WH**, arXiv:0901.1969

Adiabaticity criteria verified: $\lambda \ll 1$, i.e. fully polarized condensate

- the condensate is in the spin state m (ferromagnetic or polar).

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \int d^3\mathbf{r}\psi^\dagger_m(\mathbf{r})\left[\frac{\hbar^2}{2M}\left(-i\partial_\mathbf{r} + m\mathbf{A}^z_g(\mathbf{r})\right)^2 + V(\mathbf{r})\right. \\ &+ \left. m|\mathbf{B}(\mathbf{r})|\right]\psi_\alpha(\mathbf{r}), \end{aligned}$$

- $\mathcal{H}_{\text{eff}}$ describes a spinless condensate particle of “charge” m moving in presence of a force **f** similar to a Lorentz force (Aharonov *et al* 1992).
- Equivalent field $\mathbf{b}_g(\mathbf{r}) = \nabla_\mathbf{r} \times \mathbf{A}_g(\mathbf{r})$
- $\mathbf{b}_g(\mathbf{r})$ is non zero because $\mathbf{B}(\mathbf{r})$ has spatial variations.

Which quantities to measure

M. Taillefumier, E. Dahl, A. Brataas, and **WH**, arXiv:0901.1969

- neutral BEC ←→

  no voltage, no electric current

- measure velocity distribution

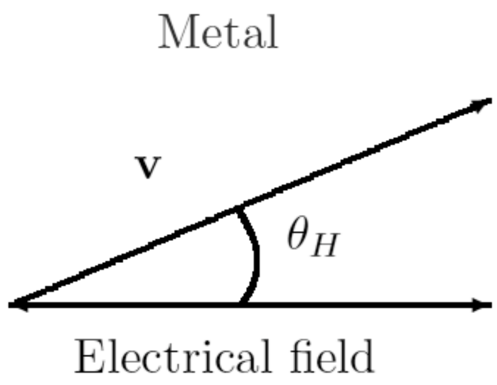
- „Hall" angle

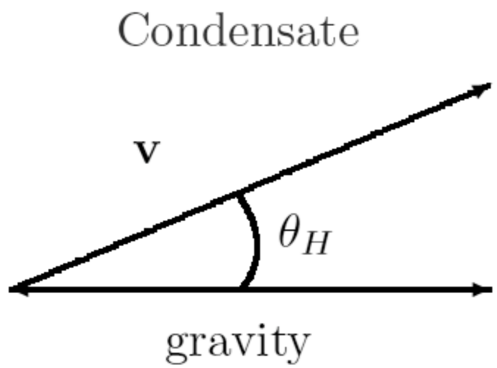
  $$\tan\theta_H = \langle v_y \rangle / \langle v_x \rangle$$

- **theory**:

  classical limit vs.

  full solution of Gross-Pitaevskii eq.





Classical limit

Assumptions

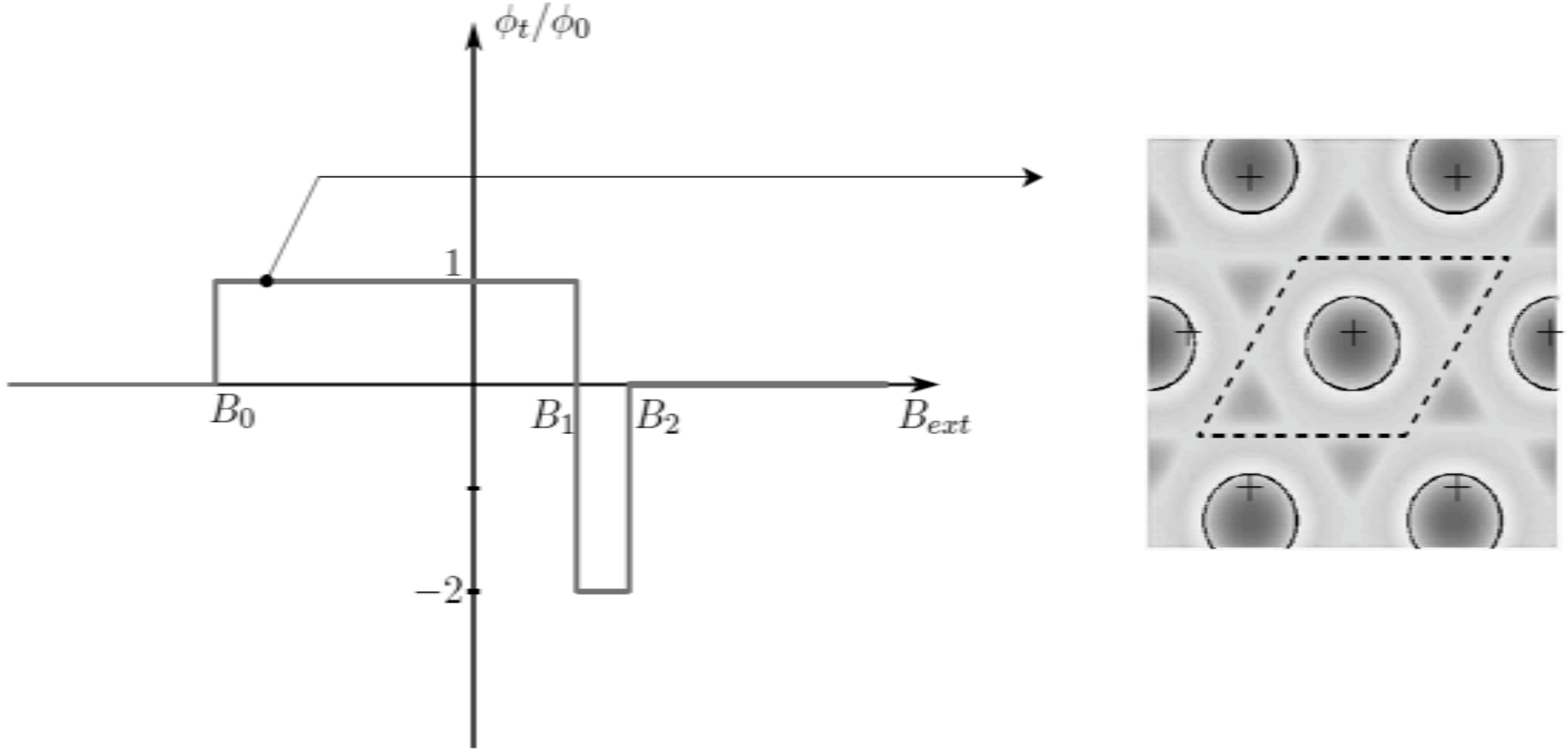
- condensate is fully polarized ($m = 1$ state) and treated as a particle
- we consider $< \mathbf{b}_g(\mathbf{r}) >= 2\pi n$ ($n \in \mathbb{Z}$) only. Can be modified with an additional B_{ext}
- spatial dependence of the periodic potential treated as scattering time τ in momentum space.
- It is the Drude model

Results

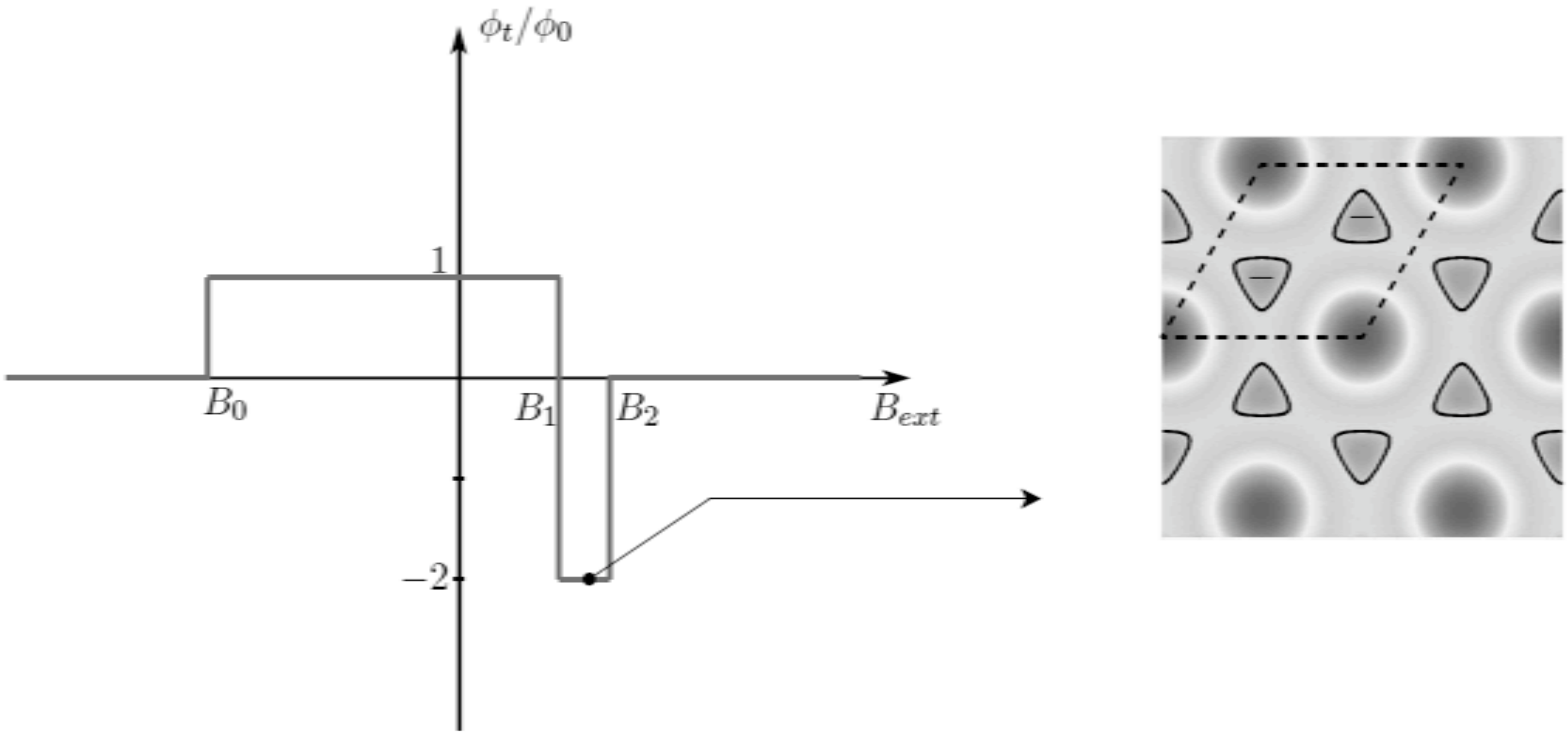
it is proportional to the flux ϕ_g of the gauge field $\mathbf{b}_g$

$$\tan\theta_H \propto \langle \mathbf{b}_g(\mathbf{r}) \rangle \propto \phi_g$$

Properties of the gauge field



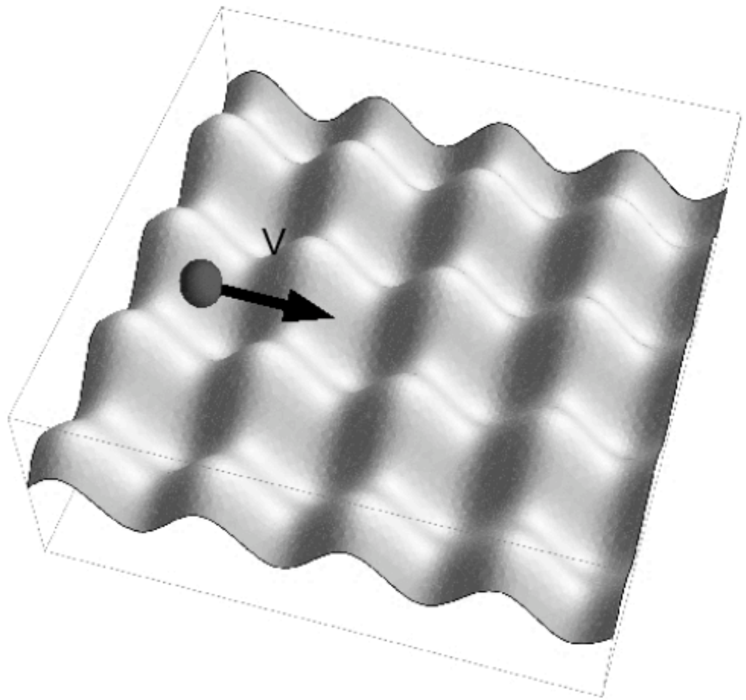
Properties of the gauge field



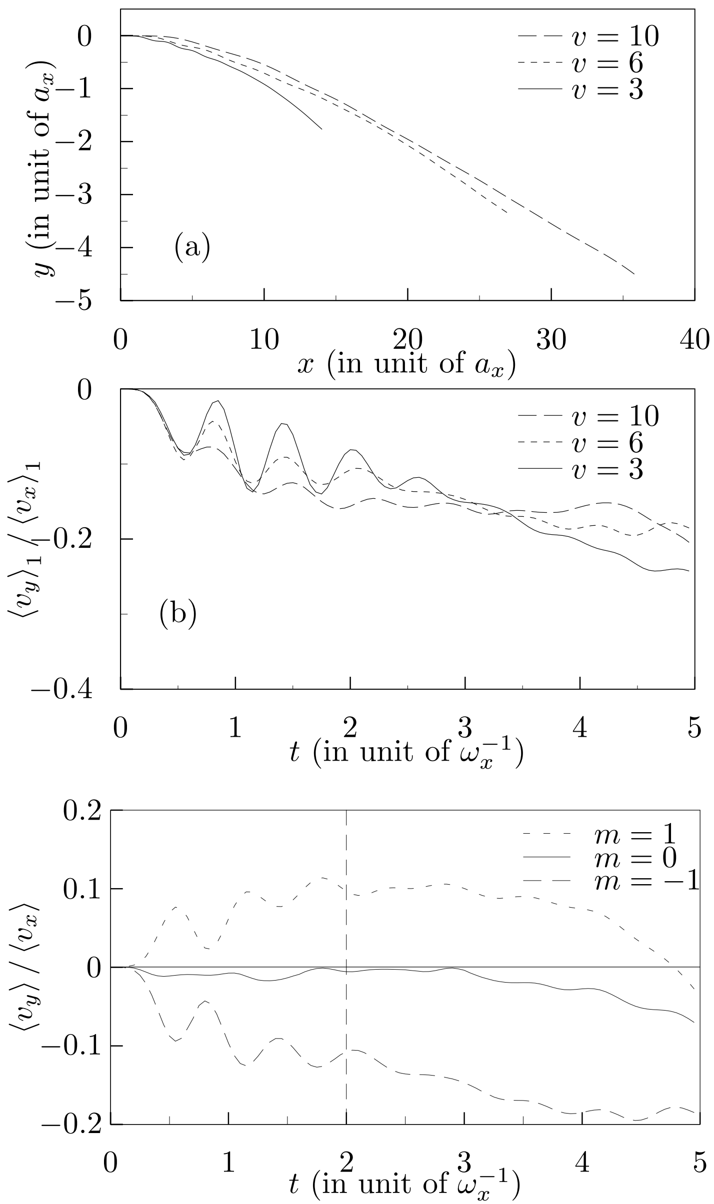
constant external magnetic field ➔
abrupt jumps + sign changes of gauge flux ϕ_t

Gross-Pitaevskii approach

- F=1 spinor condensate, ferromagnetic
- square magnetic lattice
- BEC smaller than unit cell
- initial conditions :
  BEC in harmonic trap (T=0)
  finite velocity v



Hall angle: Two dynamical regimes



(1) short time:

adiabatic approximation valid

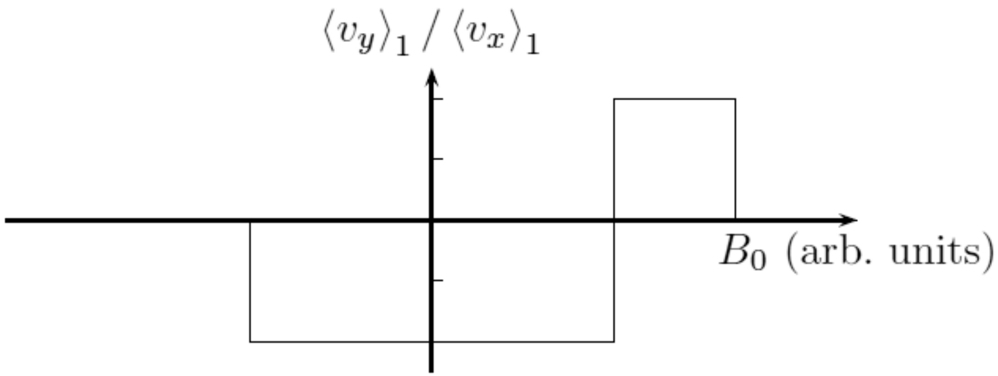
- oscillations of $\tan\theta_H$ due to sign changes in $b_g(\mathbf{r})$
- polar state not affected by magnetic lattice
- states m=1 and m=-1 have opposite Hall angles

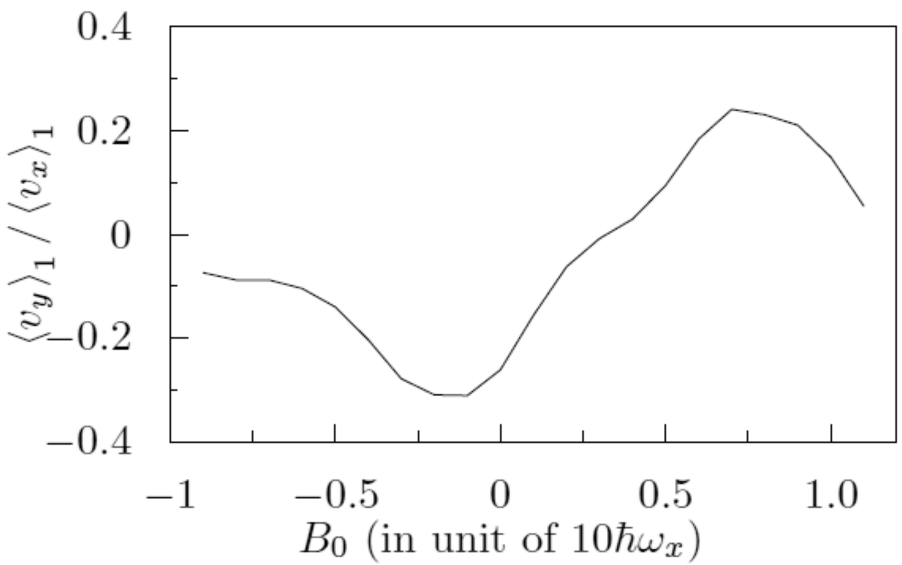
(2) long time:

classical description invalid

due to loss of polarization

Effect of constant external magnetic field





- classical limit:
  abrupt variations of Hall angle
- GP simulations:
  regular variation, but same qualitative behavior (sign)

detection of AHE:
small constant magnetic field
+ magnetic lattice
➔ measure velocity

Conclusion: AHE in Spinor BEC

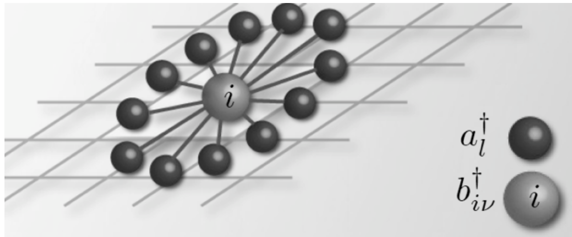
Main message of the presentation

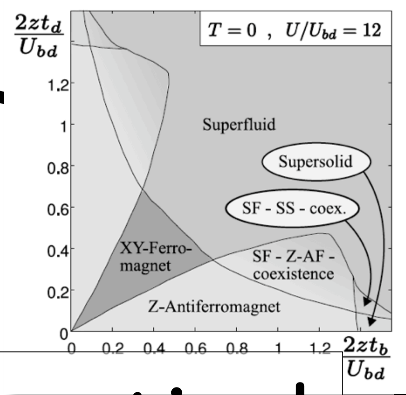
- AHE in Bose condensates induced by inhomogeneous magnetic distribution.
- Signature of the effect : sign change in the Hall angle when a constant magnetic field is applied.
- Experiment possible using magnetic microtraps.

Open questions

- Effect of the temperature (diffusion process)
- quantum corrections.
- repeat the same calculations in optical lattices

Summary

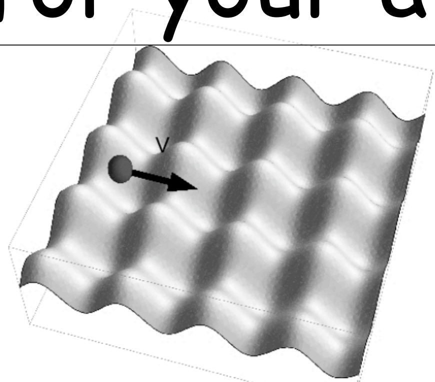




Bosonic DMFT

Thanks for your attention!

magnetic order of Bosons



Anomalous Hall Effect in Spinor BEC