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Swimming in the Fermi sea

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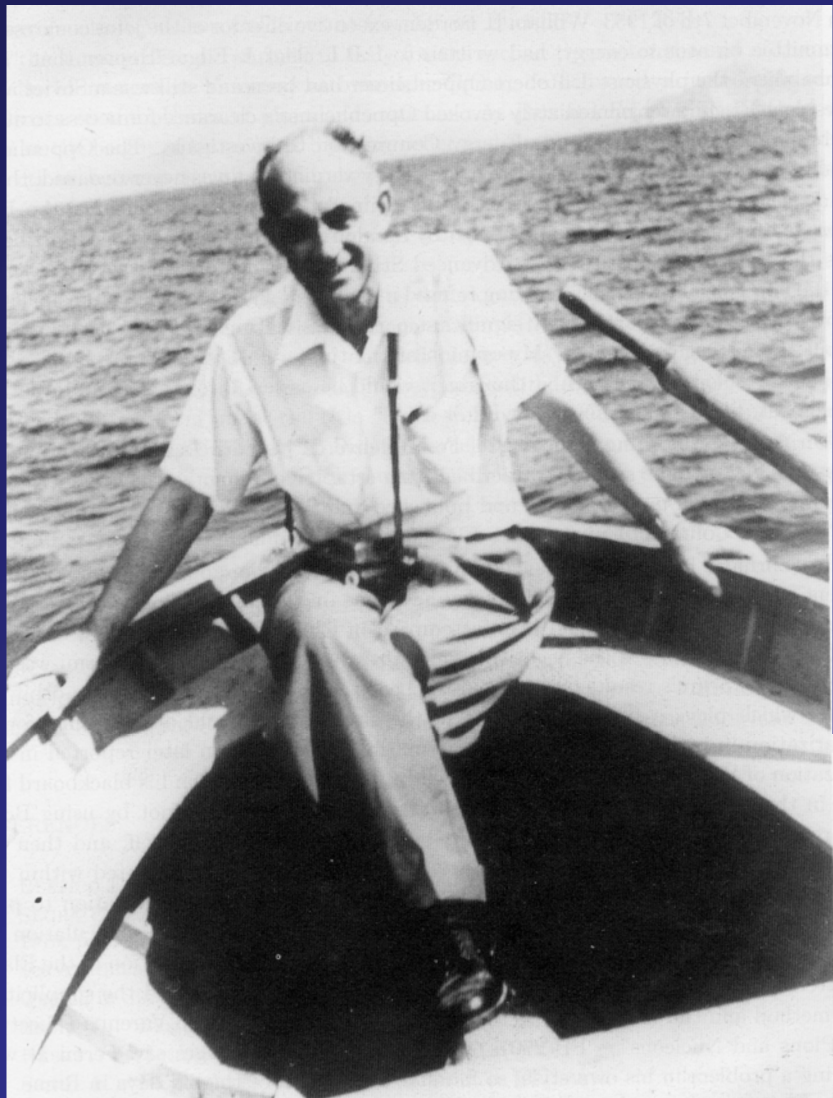
Swimming in the Fermi Sea

F. Chevy, R. Combescot, C. Mora (Paris), C. Lobo and A. Recati (Trento)
Laboratoire Kastler Brossel, Ecole normale supérieure, Paris



BCS theory and beyond

BCS theory describes weakly attractive spin ½ fermions. Meanfield theory, pairing at the surface of Fermi Sea (Cooper pairs).

Extension of the BCS model

- Strongly attractive fermions (BEC-BCS crossover)
- Pairing between different species (Atom mixture, *eg* LiK)
- Spin>1/2 (Color superfluidity)
- Imbalanced spin populations (MIT-Rice)

Ex:
Superconductors in magnetic fields
Quark matter
Ultra cold atoms

Fermionic superfluid with population imbalance

Chandrasekar and Clogston: robustness of the paired state : $\mu_\uparrow > \mu_\downarrow$

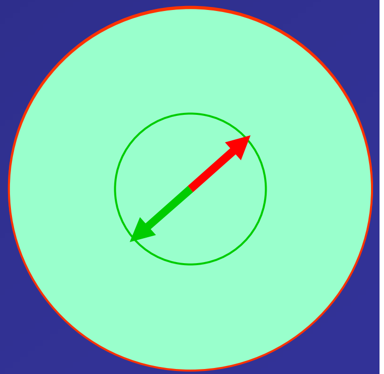
Paired state stable for $\mu_\uparrow - \mu_\downarrow < \Delta$

And beyond?

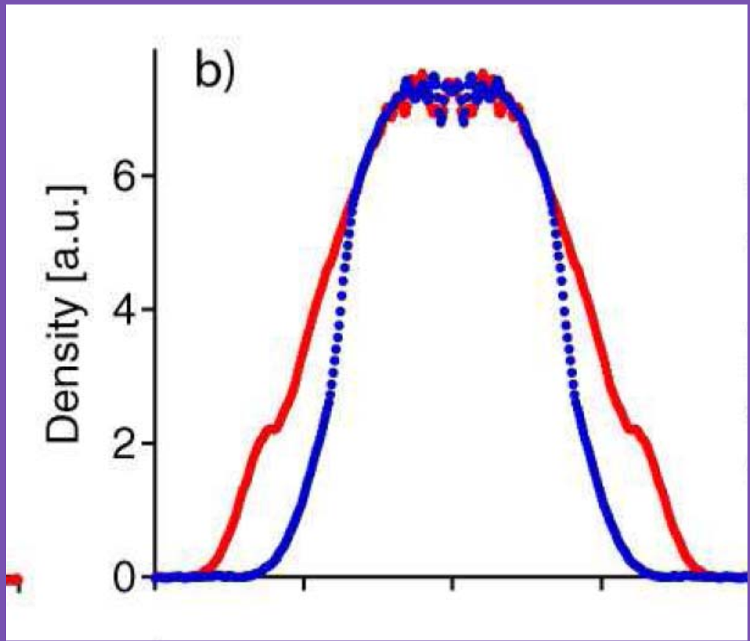
Polarized phase : One spin species (Carlson, PRL **95**, 060401 (2005))

FFLO Phase (Fulde Ferrell Larkin Ovshinikov) : pairing in $\mathbf{k}_\uparrow - \mathbf{k}_\downarrow \neq 0$
(C. Mora et R. Combescot, PRB **71**, 214504 (2005))

Sarma phase (internal gap) : pairing in $\mathbf{k}_\uparrow - \mathbf{k}_\downarrow = 0$
opening of a gap in the Fermi sea of majority species. (Liu, PRL **90**, 047002 (2003))



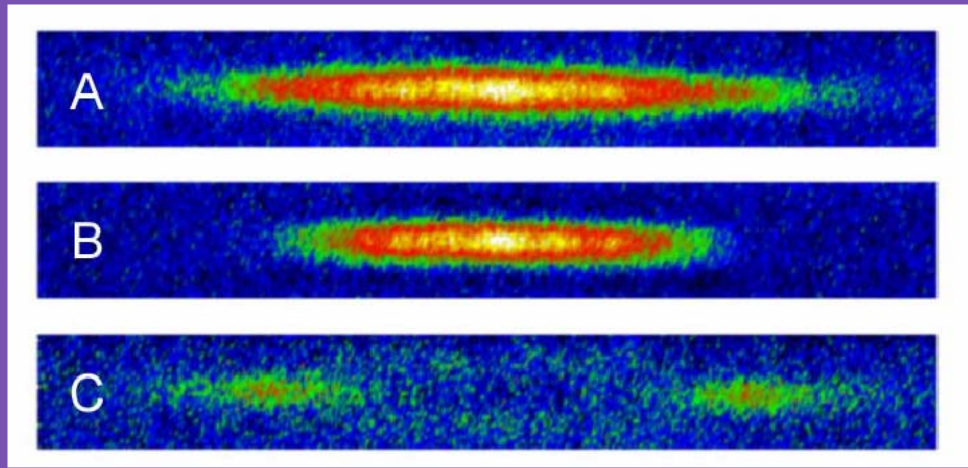
Experimental results at Feshbach resonance



MIT: 3 phases
- Fully paired superfluid core
-Intermediate mixture
-Fully polarized rim

M.W. Zwierlein, *et al.*, Science, **311** (2006) 492.

Rice: 2 phases
Fully paired superfluid core
Fully polarized rim



G.B. Partridge, W. Li , R.I. Kamar, Y.-A. Liao, R.G. Hulet, Science, **311** (2006) 503.

Unitary thermodynamics

(Or why you should'nt be afraid of working at a=◎
T.-L. Ho, Phys. Rev. Lett. **92**, 090402(2004))

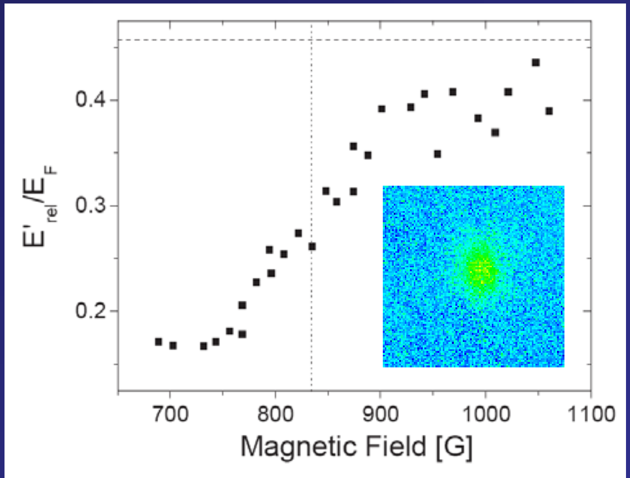
Balanced Fermi gas

Dimensional analysis: $\mu = E_F f(1/k_F a)$

At unitarity, $a = \infty \Rightarrow \mu = E_F f(0) = \xi E_F$

Measurement of ☒ by time of flight

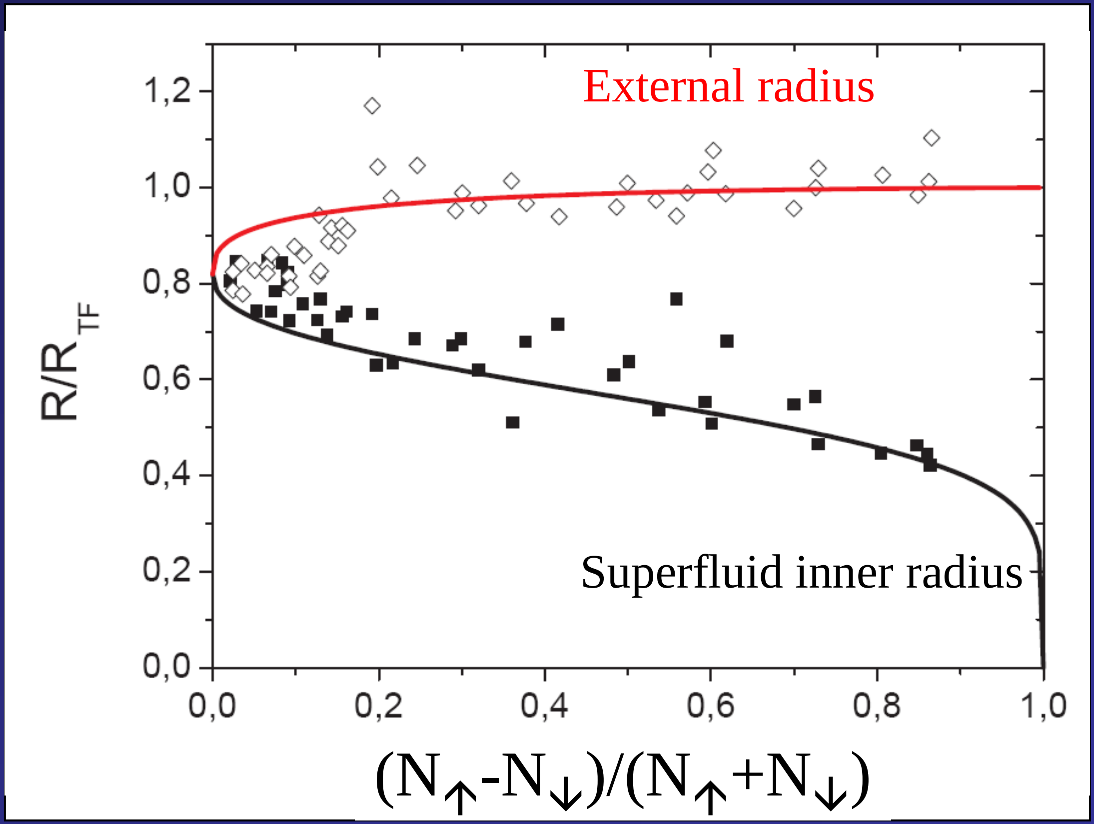
Release energy $E_R = E_{Kin} + E_{Int} = \sqrt{\xi} E_R^0$



ENS experiment, ☒~0.41(15), compatible with other experimental measurements, and theoretical calculations (Monte-Carlo ☒~0.42, BCS ☒~0.59).

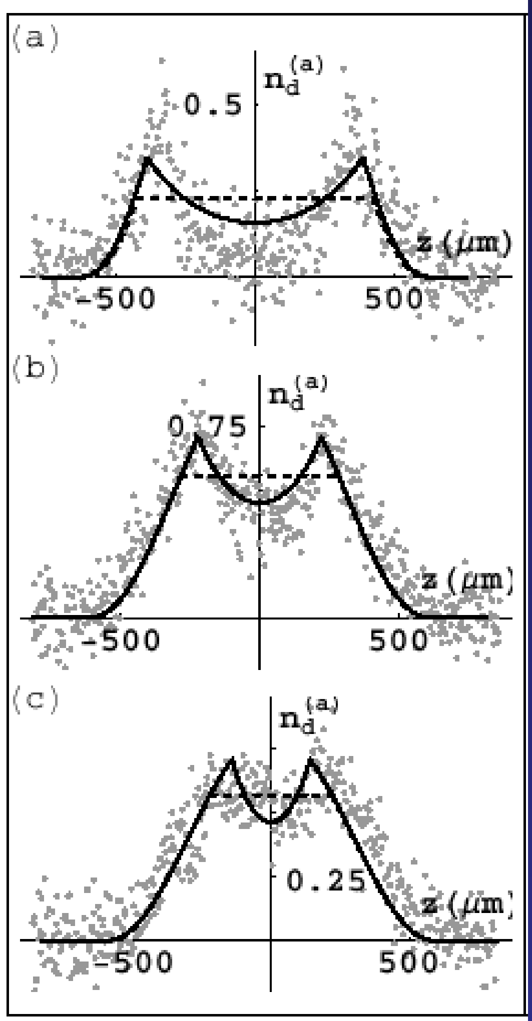
Rice : 2 phase model
(F. Chevy, PRL 96, 130401 (2006))

Rice experiment: fully described by a 2 phase model, without any adjustable parameter.



Surface tension

(T. N. De Silva and E. Mueller, PRL., 97 070402 (2006))



Local density approximation: - - - -

$$F_s = \gamma S_{\text{interface}}$$ (1st order transition)

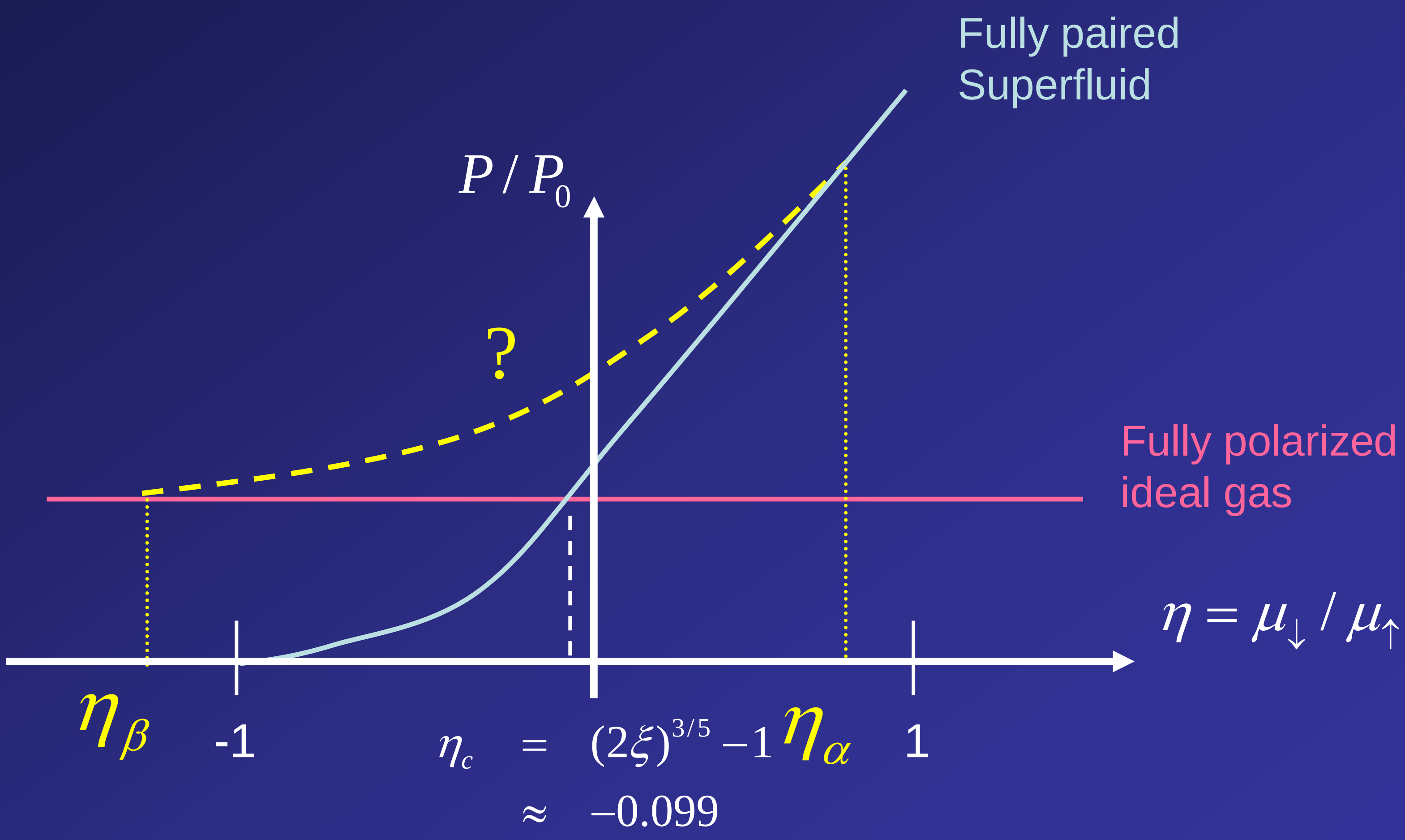
$$\gamma = \varepsilon \frac{\hbar^2 n^{4/3}}{2m}$$

Fit of data: $\varepsilon \approx 10^{-3}$

With surface tension: ——

What about other phases.

Grand potential $\Omega=-PV$ ⇒ ground state has the highest pressure.

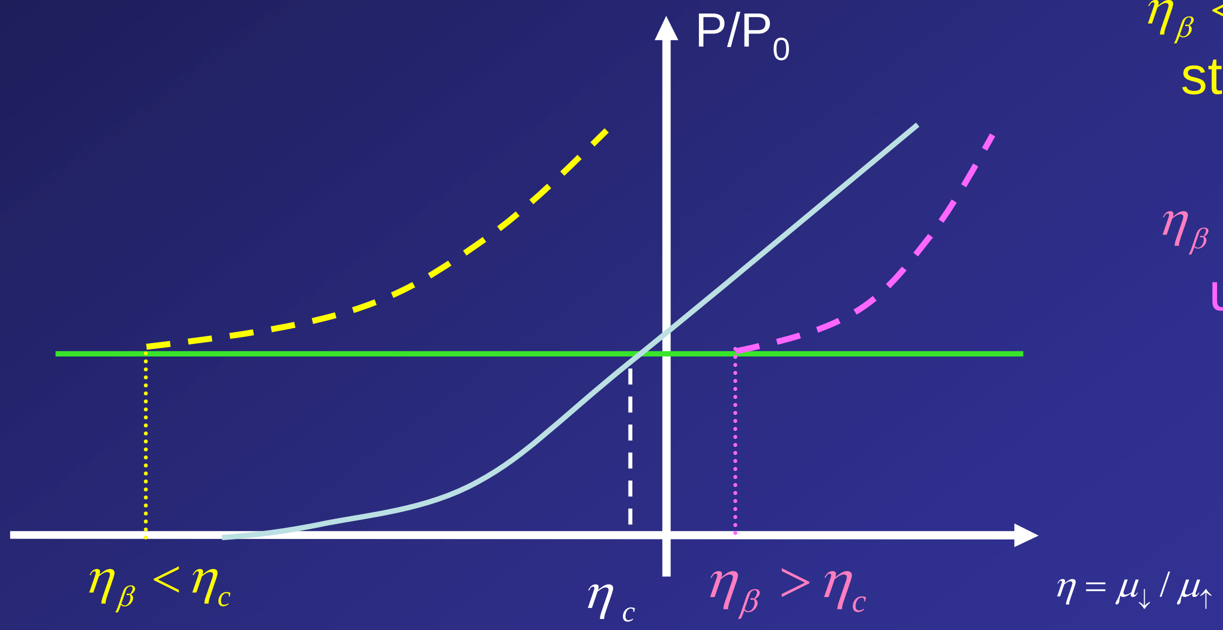


Theoretical evidence for an intermediate phase

General properties of a mixed branch?

Step 1: calculate the energy E of a single impurity atom immersed in a Fermi sea (E=❍$_{\downarrow}$($n_{\downarrow}$=0^+)).

Step 2: dP/d❍$_{\sigma}$=n⬧✇🗀



$\eta_\beta < \eta_c$: the new branch is stable

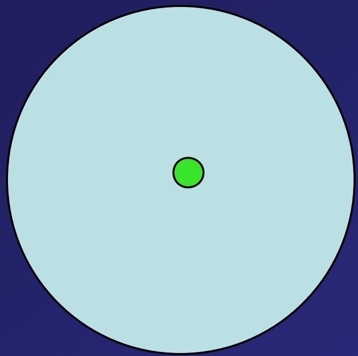
$\eta_\beta > \eta_c$: the new branch is unstable

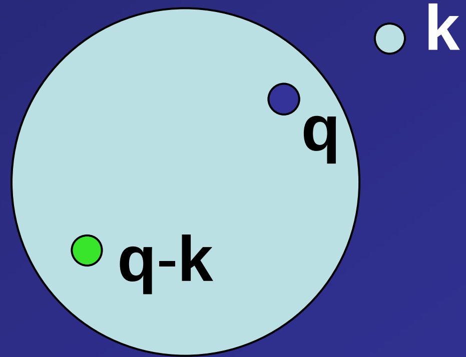
See A. Bulgac and M. McNeil Forbes, PRA **75**, 031605 (2007)

Variational upper bound for ♒$_{♌}$💻 the Fermi swimmer

One impurity: restrict the effect of interactions to the formation of a *single particle-hole pair*.

$$\left|\Psi\right\rangle = \varphi_0 \left|0\right\rangle + \sum_{\mathbf{k},\mathbf{q}} \varphi_{\mathbf{k},\mathbf{q}} \left|\mathbf{k},\mathbf{q}\right\rangle$$

$\left|0\right\rangle =$ 

$\left|\mathbf{kq}\right\rangle =$ 

For a=◎, E=-0.606 $E_{F\uparrow}$ ⓿ η_β<-0.606<η_c~-0.1

Comparison with exact results :

♒$_{♌}$=-0.58(1) (*C. Lobo et al.* PRL. **97**, 200403 (2006));

♒$_{♌}$=-0.62 (Prokof'ev and Svistunov, Phys. Rev. B **77**, 020408 (2008))

Systematic expansion R. Combescot and S. Giraud, Phys. Rev. Lett. **101**, 050404 (2008)

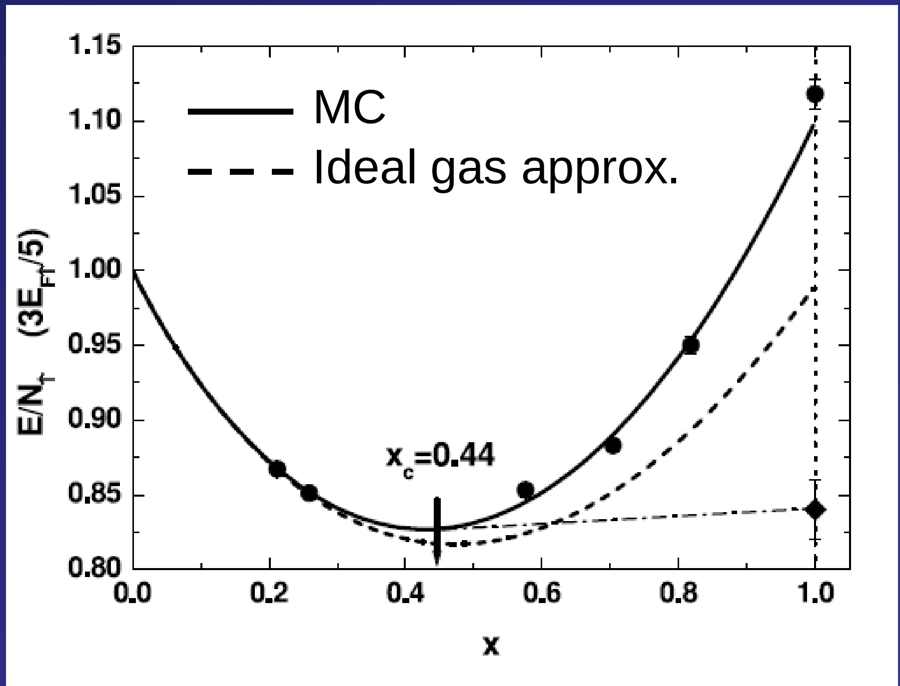
Structure of the intermediate phase

(Combescot et al.PRL 98, 180402 (2007))

Ideal gas of fermionic impurities dressed by particle-hole pairs

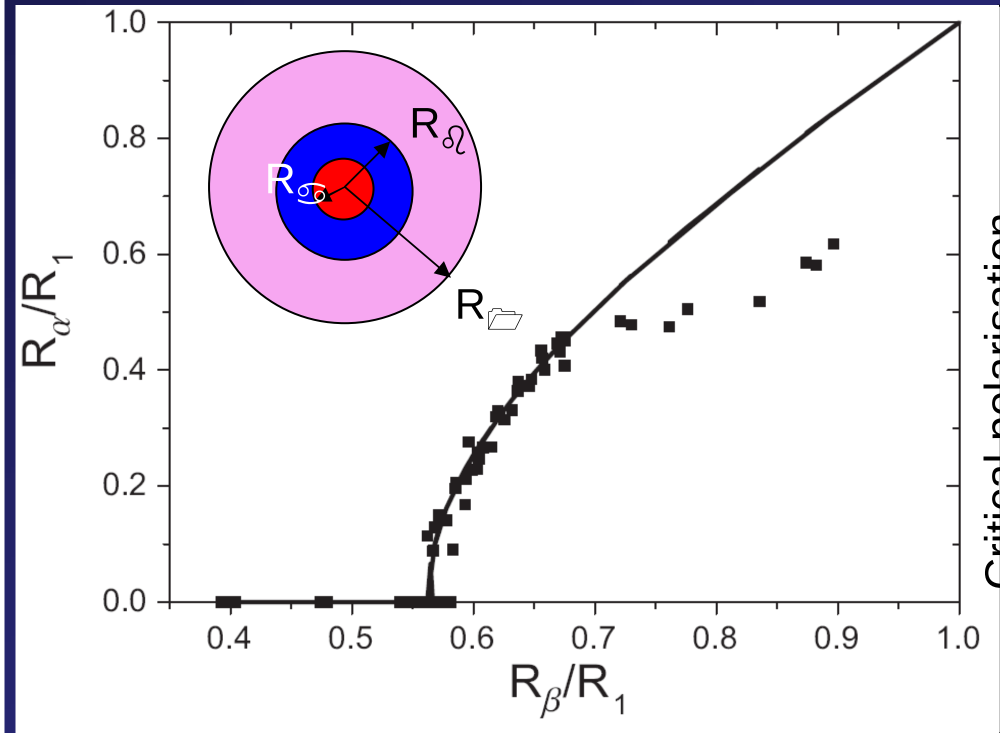
$$E(p) = \eta_\beta E_F + p^2 / 2m^*$$

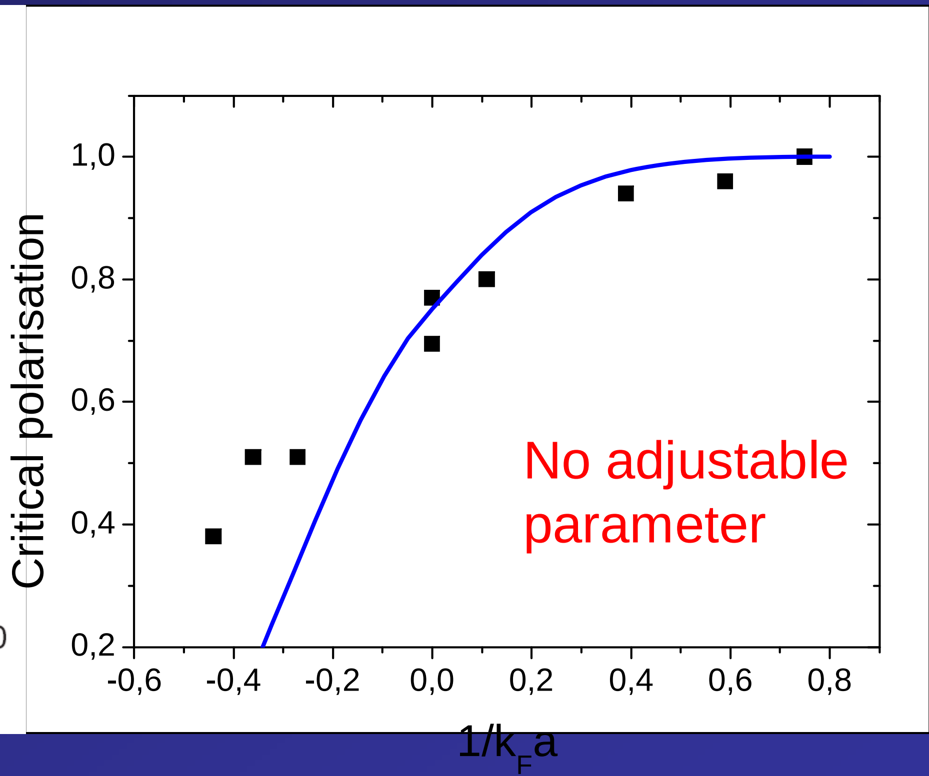
Variational calculation: $\eta_\beta = -0.6$ $m^* \sim 1.15m$



Comparison with experiment: Shell radii and critical polarisation

If polarisation $(N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$ is to large : superfluid core vanishes





See also Martin Zwierlein’s talk

BEC regime: Breakdown of the variational approach

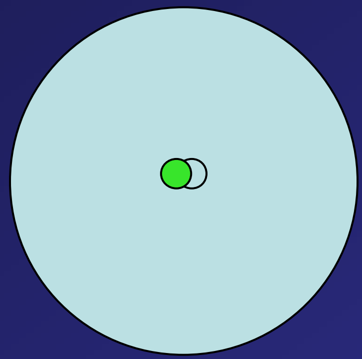
First signature: divergence and sign change of the effective mass for $1/k_F a \sim 1.7$.The variational ansatz is no longer a good approximation of the ground state ⇒ *New Ansatz required*

Prokof'ev and Svistunov: change of nature of the swimmer at $1/k_F a = 0.9$:
Sharp transition between a fermionic (polaron) and bosonic (molecule) swimmer.

Can we interpret this transition in the variational picture?

BEC variational Ansatz

Far in the BEC regime:

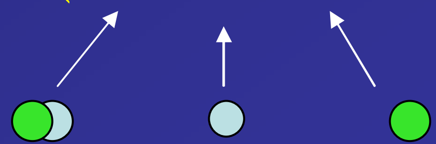


Deeply bound molecule + non-interacting Fermi sea: NO HOLE

Work in the two channel model

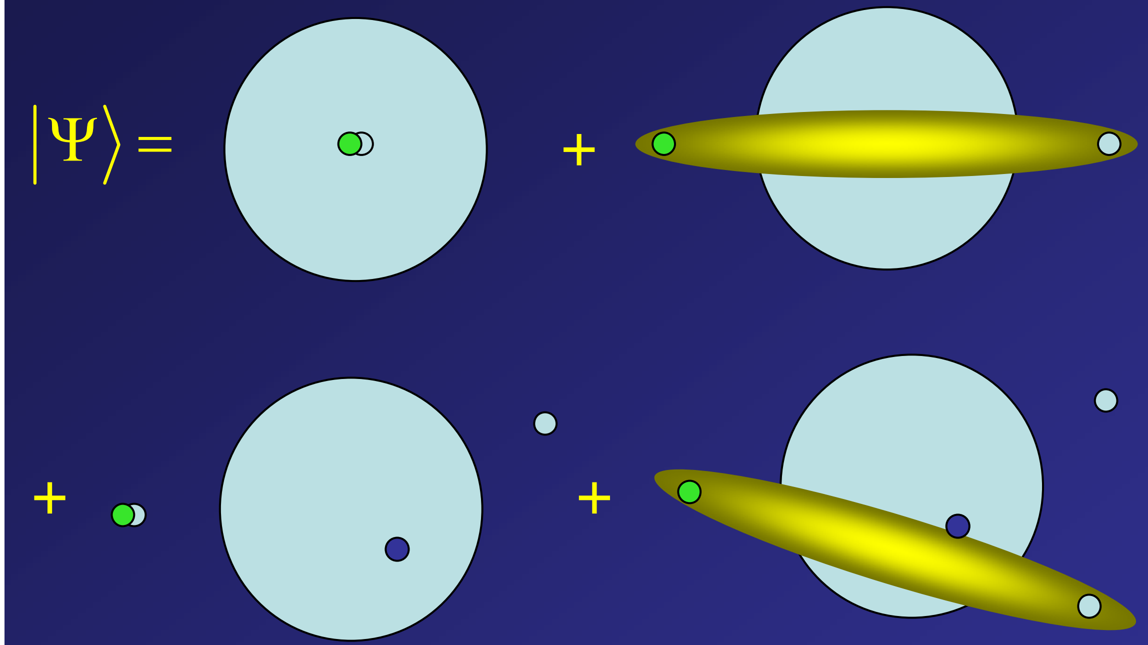
Advantage: introduces explicitely the bosonic molecular state

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} a^{+}_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} \sum_{\mathbf{k}} \left(\frac{\varepsilon_{\mathbf{k}}}{2} + E_b \right) b^{+}_{\mathbf{k}\sigma} b_{\mathbf{k}} + \sum_{k,k'} g_{\mathbf{k},\mathbf{k}'} \left(b^{+}_{\mathbf{k}+\mathbf{k}'} a_{\mathbf{k}\uparrow} a_{\mathbf{k}'\downarrow} + b_{\mathbf{k}+\mathbf{k}'} a^{+}_{\mathbf{k}\uparrow} a^{+}_{\mathbf{k}'\downarrow} \right)$$



BEC variational Ansatz in the two channel model

$$|\Psi\rangle =$$



$$E = -\frac{\hbar^2}{ma^2} + \cancel{\frac{8\pi\hbar^2 a}{m} n_\uparrow} + \ldots$$

$$\ldots - \cancel{\frac{8\pi\hbar^2 a}{m}} n_\uparrow + \frac{3\pi\hbar^2 a_{ad}}{m} n_\uparrow + \ldots$$

a_{ad}=1.18 a, free atom-dimer scattering length: Mean field correction of a point-like boson of mass 2m

Conclusion et future directions

What was demonstrated?

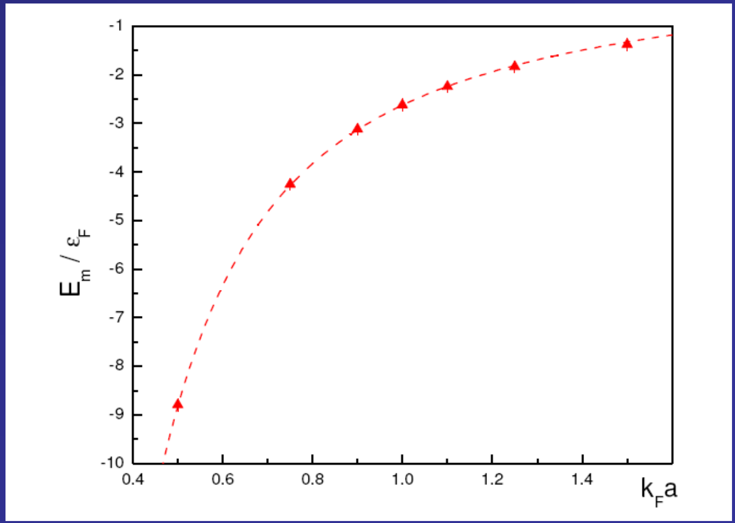
3 stable phases (at least) in the phase diagram of the polarized Fermi gas.

2 phases+Surface tension effects at Rice (Elongated trap? Fewer atoms? Metastability?)

Related issues

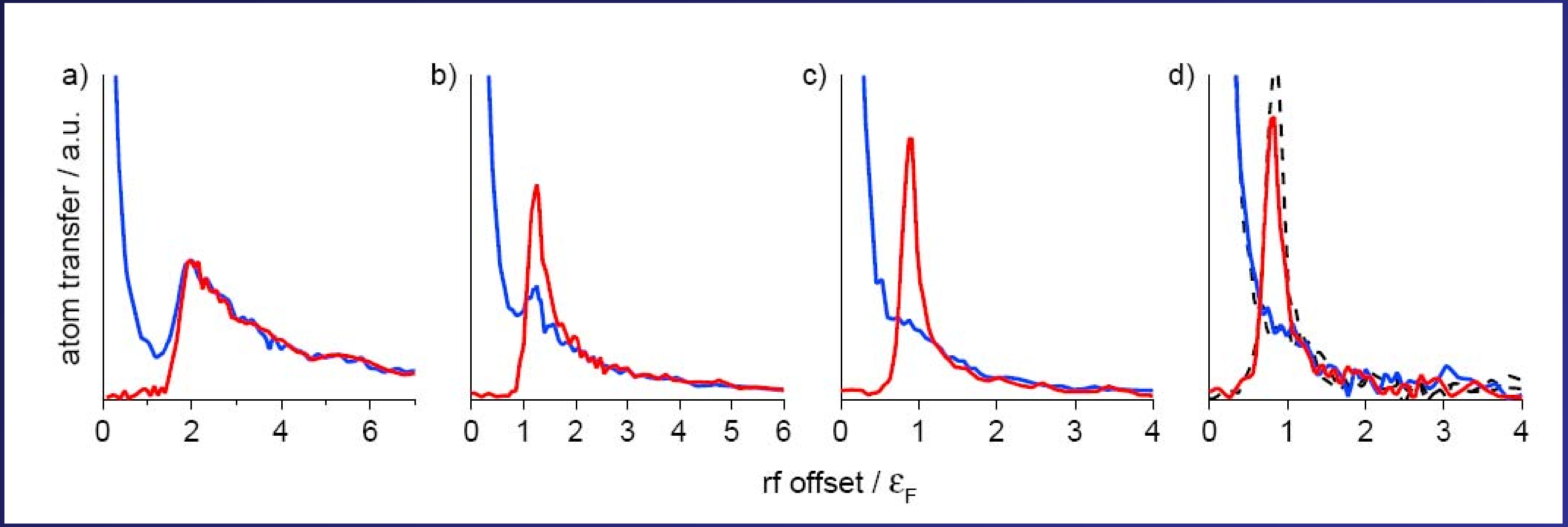
Reason for the Rice/MIT disagreement

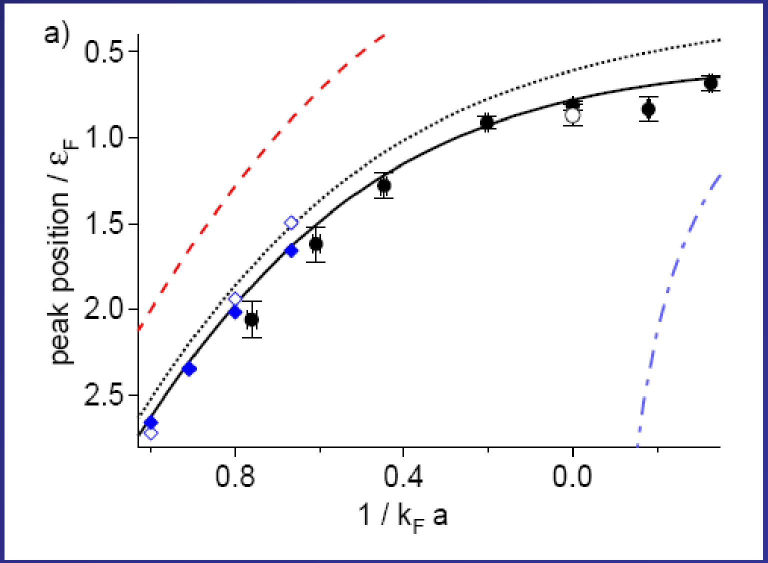
Real variational calculation of the molecular state



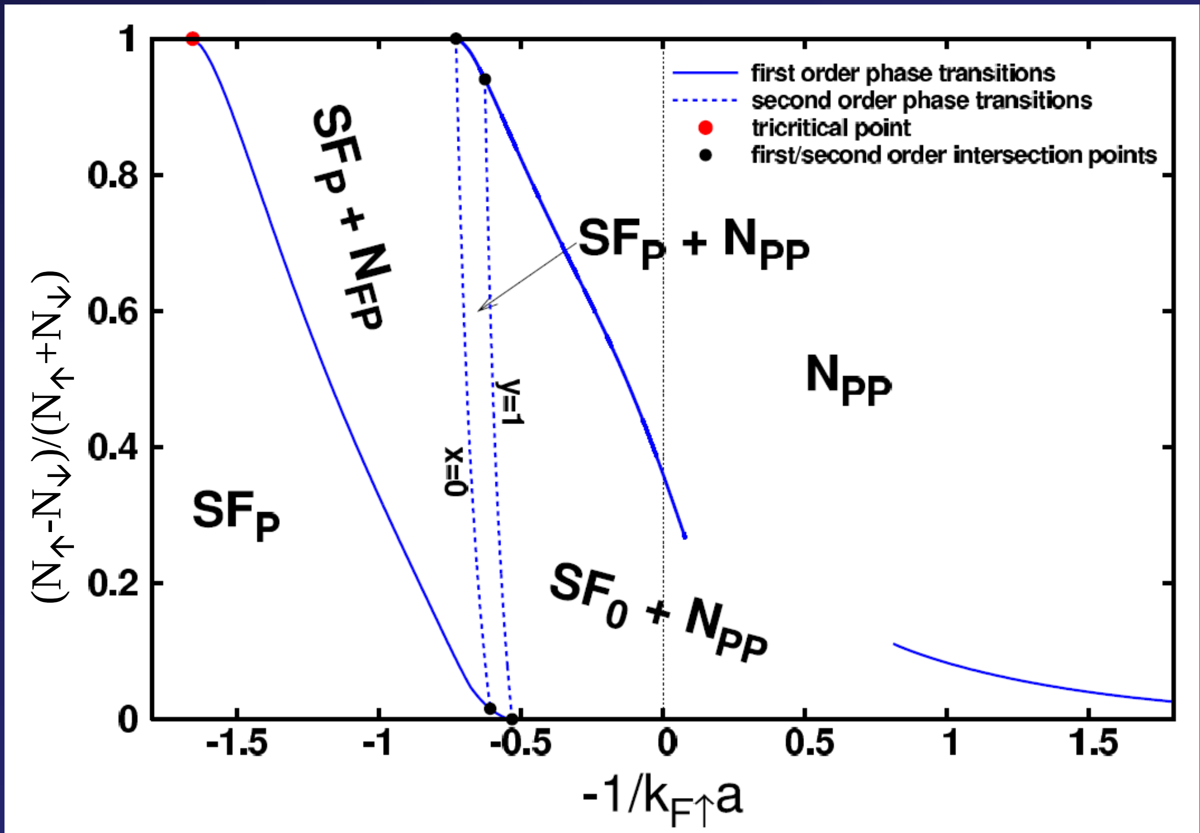
Comparison with experiments

(A. Schirotzek et al. arXiv:0902.3021)





Application to the zero temperature tricritical point



SF_0: fully paired superfluid
SF_P: polarized superfluid
N_{PP}: Polarized normal fluid
N_{FP}: Fully polarized normal fluid

- Tricritical point: Normal-superfluid transition goes from *first to second* order

(Pilati and Giorgini, Phys. Rev. Lett. **100**, 030401 (2008))

The Bose swimmer and the Tricritical point

Far in the BEC regime: partially polarized superfluid=Fermi sea+weakly interacting BEC of molecules.

$$E(N_B, N_F) = \frac{3}{5} N_F E_F + \frac{g_{bb} N_B^2}{2V} + \frac{g_{bf} N_B N_F}{V}$$

Homogeneous system thermodynamically stable for ✰^{2}E/✰N$_{♋}$✰N$_{♌}$>0.
For $N_B \rightarrow 0$, system stable for:

$$\frac{1}{k_F a} > \frac{m_F M_B}{2\pi\mu^2} \frac{a_{BF}^2}{a_{BB} a}$$

(❍ reduced mass)

$$\left.\begin{aligned} a_{BF} &= 1.2a \\ a_{BB} &= 0.6a \\ M &= 2m_F \end{aligned}\right\} \Rightarrow \left(\frac{1}{k_F a}\right)_c = 1.7$$

a_{BF} and a_{BB} known for any mass ratio (Petrov et al.): can be used for any atomic mixture – Li-K for instance (stable for $1/k_F a > 2.9$).