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Collisional properties of heteronuclear mixtures with resonant interspecies interaction

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Collisional properties of heteronuclear mixtures with resonant interspecies interaction

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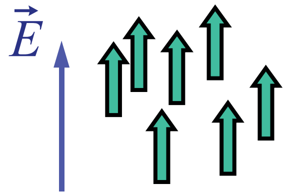
IFRAF

Heteronuclear mixtures

Dipolar gases



- Lots of interesting stuff
- ground state heteronuclear molecules, JILA, Freiburg

Few-body problem with different masses and statistics

- weakly bound dimers
- atom-dimer scattering
- trimers

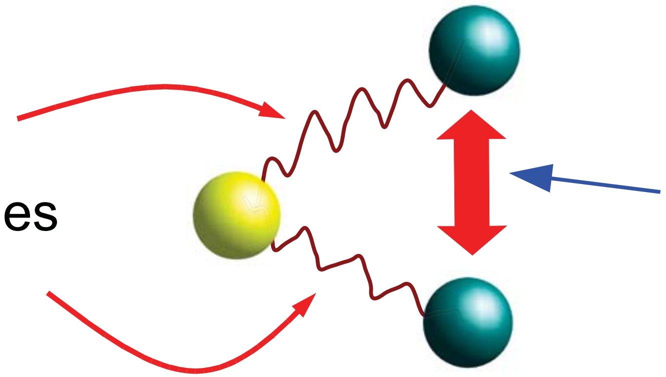
BCS-BEC crossover with mass imbalance

- Crystallization

Experiments on mixtures+Feshbach resonances: JILA, MIT, Hamburg, Florence, Munich, Innsbruck, Paris, Amsterdam, Tübingen...

Stability and Efimov effect in a heteronuclear mixture

resonant interspecies interaction



- centrifugal barrier for fermions
- no barrier for bosons

effective 3-body attraction $\propto R^{-2}$

- Efimov effect ☺
- fast relaxation ☹

Bose-Bose mixtures,
Fermi-Bose mixtures,
Fermi-Fermi mixtures with mass ratios > 13.6

centrifugal repulsive barrier $\propto R^{-2}$

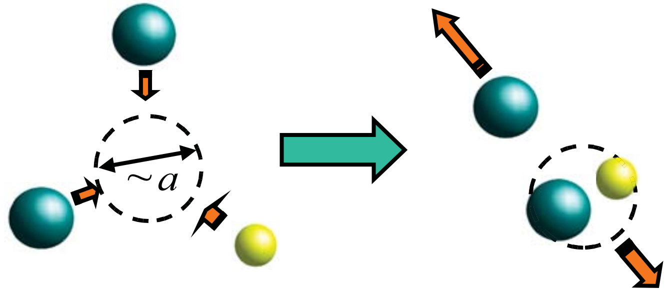
- no Efimov effect ☹
- suppressed relaxation ☺

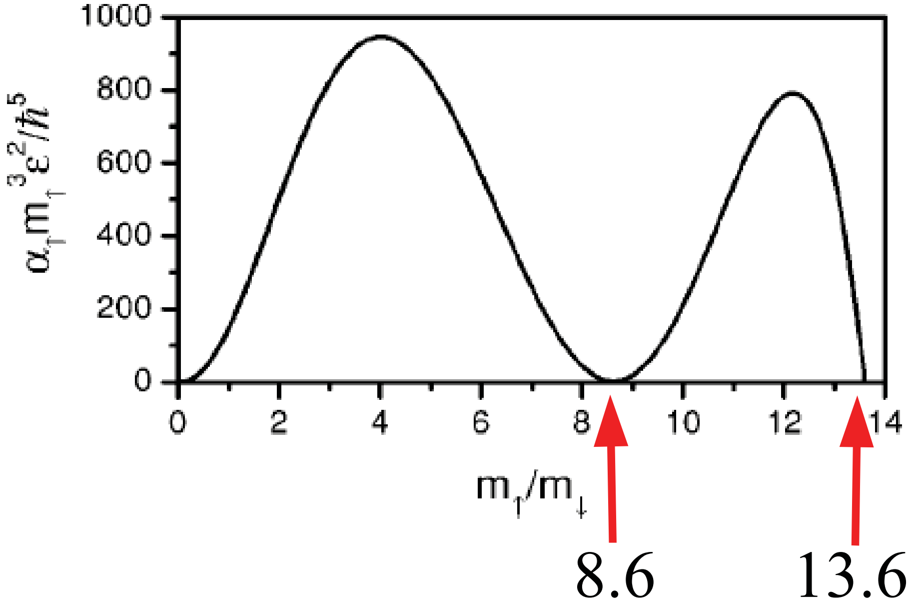
Fermi-Fermi mixtures with mass ratios < 13.6

We look for interesting physics & long lifetime!

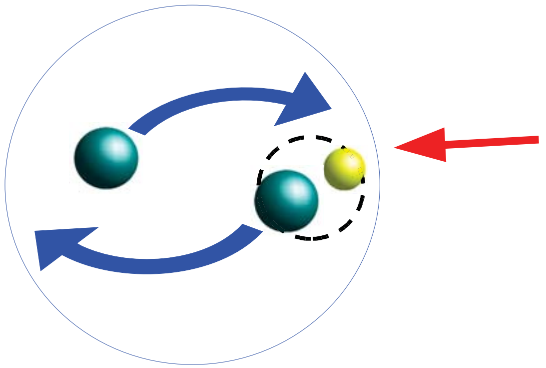
Fermi-Fermi mixtures, magic mass ratios

3-body recombination to a weakly bound level, DSP (2003)





Emergence of a trimer state for M/m>8.2, Kartavtsev & Malykh (2006)

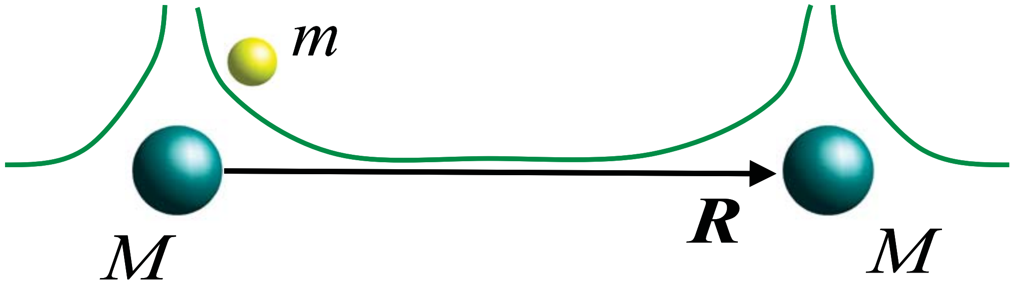


M/m<8.2 p-wave atom-dimer scattering resonance

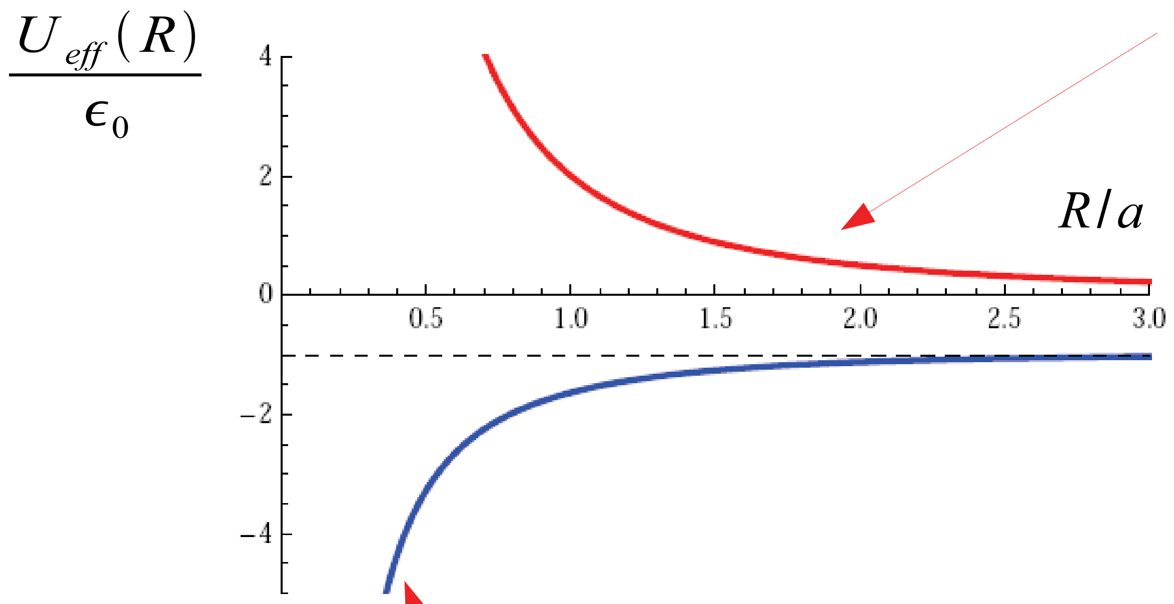
M/m>8.2 trimer state with L=1

Bound trimer state, NOT EFIMOV

3-body problem. Born-Oppenheimer approximation



Effective interaction between heavy atoms is provided by exchange of the fast light particle. Born-Oppenheimer approximation.



$$U_{centrifugal}(R) = \frac{\hbar^2 l(l+1)}{MR^2}$$

$$U_{eff}(R) \approx -\frac{\hbar^2}{2ma^2} = -|\epsilon_0|$$

$$U_{eff}(R) \approx -0.16\frac{\hbar^2}{mR^2}$$

$$\boxed{\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\widetilde{U}_{eff}(R)\right]\chi(R)=E\chi(R)}$$

$$\widetilde{U}_{eff}(R)=U_{eff}(R)+|\epsilon_0|+\frac{\hbar^2 l(l+1)}{MR^2} \xrightarrow[R\ll a]{} \frac{\hbar^2}{MR^2}\underbrace{\left(l(l+1)-0.16\frac{M}{m}\right)}_{\beta}$$

$$R\ll a \Rightarrow \chi(R)\propto R^{1/2\pm\sqrt{\beta+1/4}}$$

$$\beta<-1/4,\quad M/m>13.6$$

$$\Downarrow$$

$$\chi(R)\propto\sqrt{R}\sin\left(\sqrt{-1/4-\beta}\ \log R/r_3\right)$$

$$\Downarrow$$

“Fall of a particle to the center in R^{-2} potential”. Infinite number of zeros of the wavefunction. Infinite number of trimer states. Efimov effect

$$\beta>-1/4,\quad M/m<13.6$$

$$\Downarrow$$

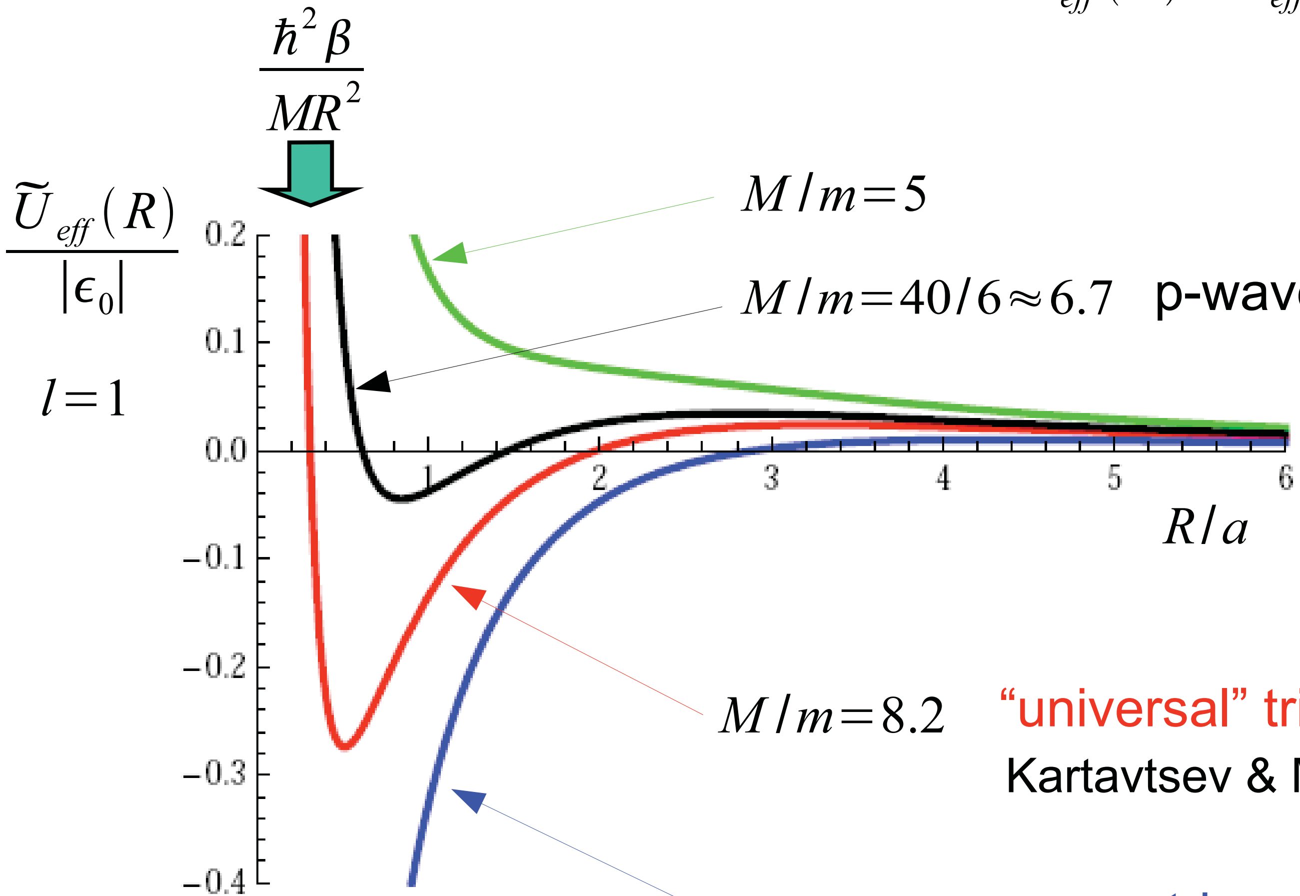
$$\chi(R)\propto R^{1/2+\sqrt{\beta+1/4}}$$

$$\Downarrow$$

“Universal” regime in the sense that one needs no three-body parameter. Fermi statistics wins over the induced attraction

$$\boxed{\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\tilde{U}_{eff}(R)\right]\chi(R)=E\,\chi(R)}$$

$$\tilde{U}_{eff}(R)=U_{eff}(R)+|\epsilon_0|+\frac{\hbar^2 l(l+1)}{MR^2}$$



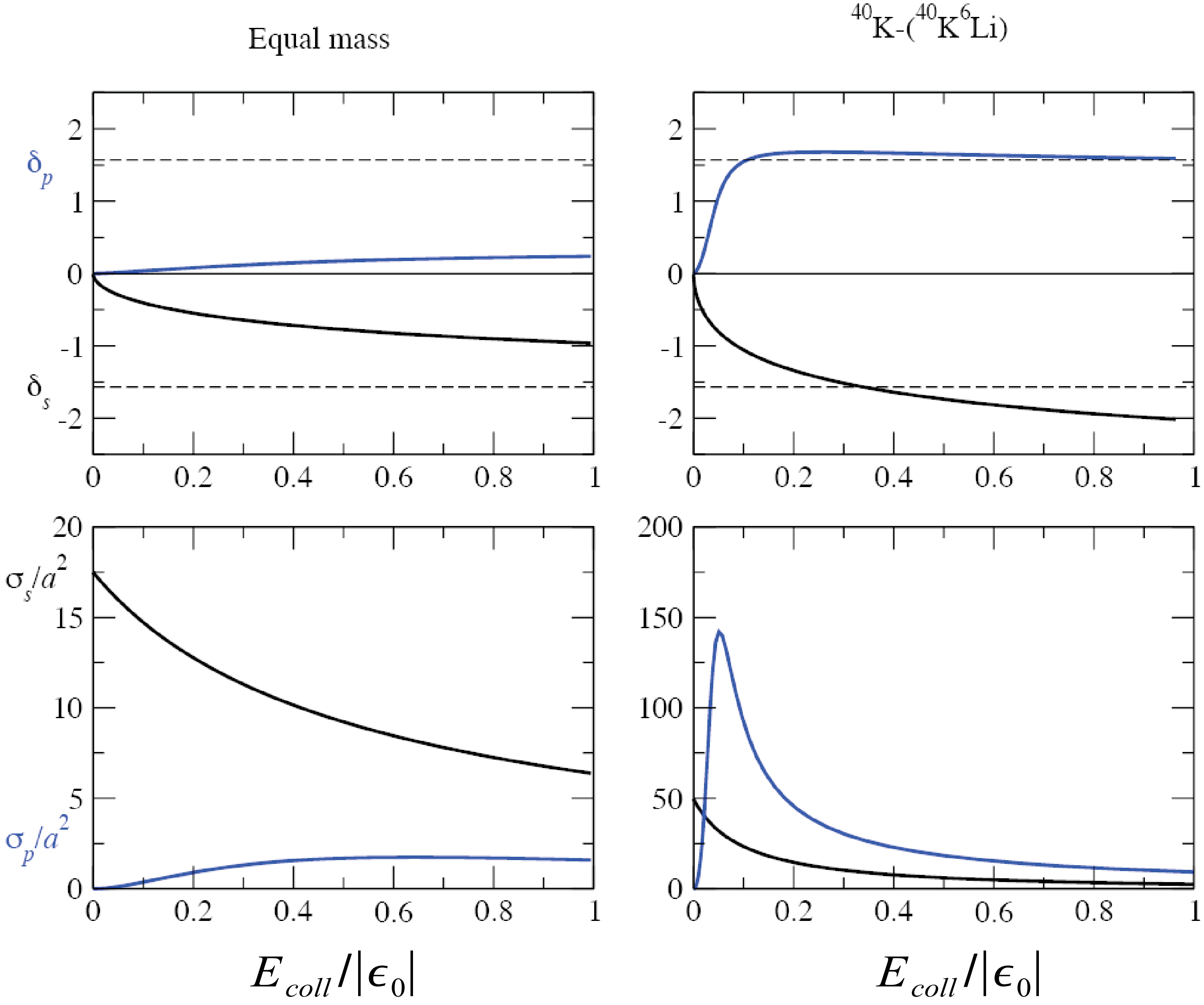
$M/m=5$

$M/m=40/6\approx 6.7$ p-wave scattering resonance ?

$M/m=8.2$ “universal” trimer state at threshold

Kartavtsev & Malykh (2006)

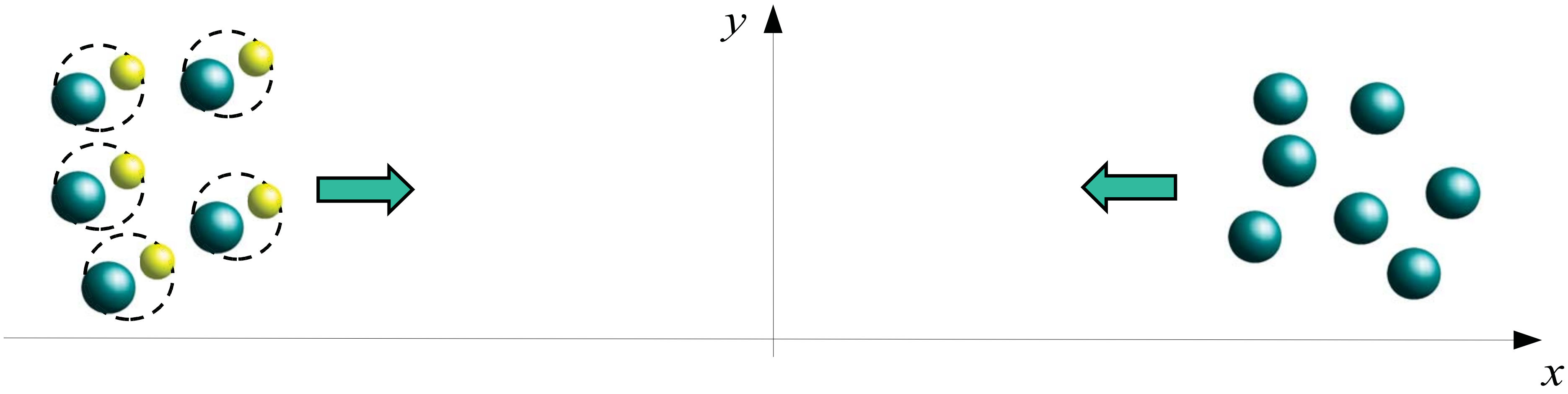
$M/m>13.6$ many trimer (Efimov) states

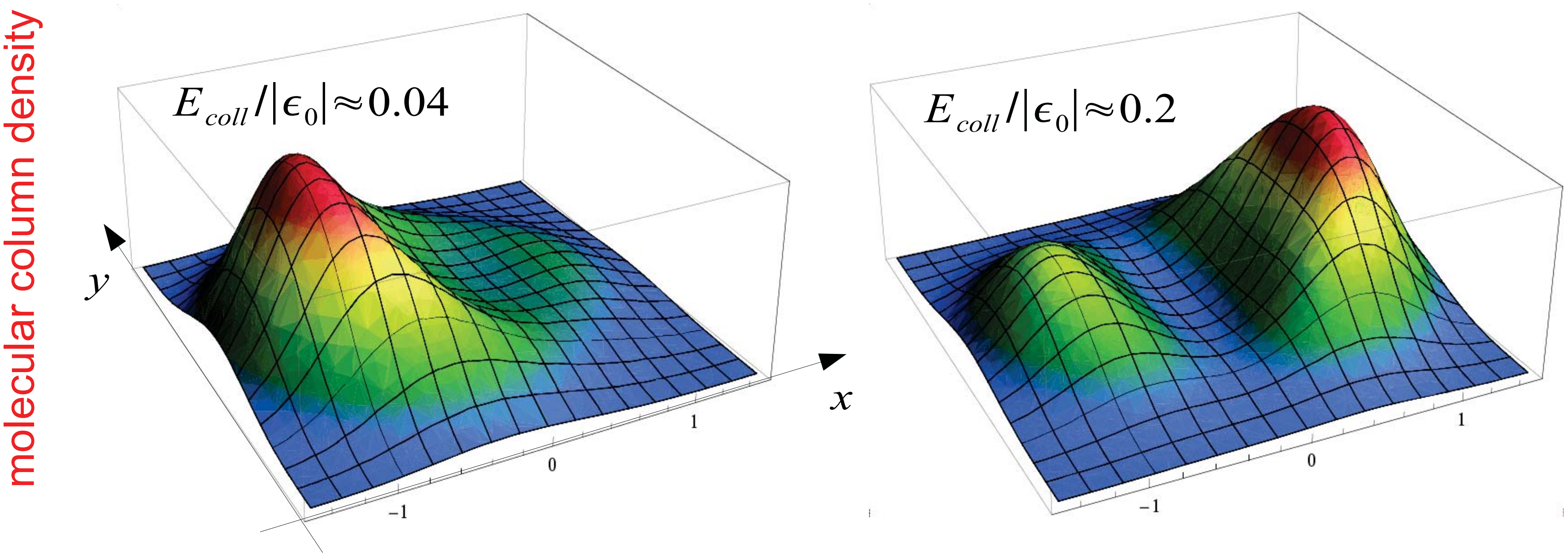


Equal mass

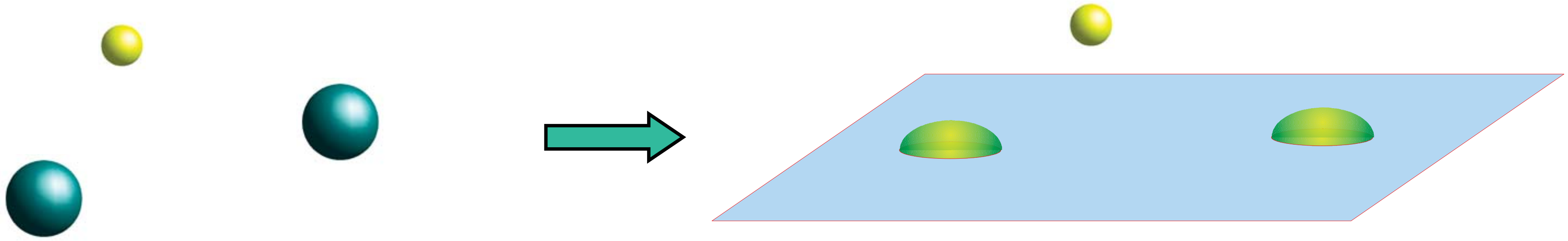
^{40}K-(^{40}K^{6}Li)

Collision of atomic and molecular thermal clouds

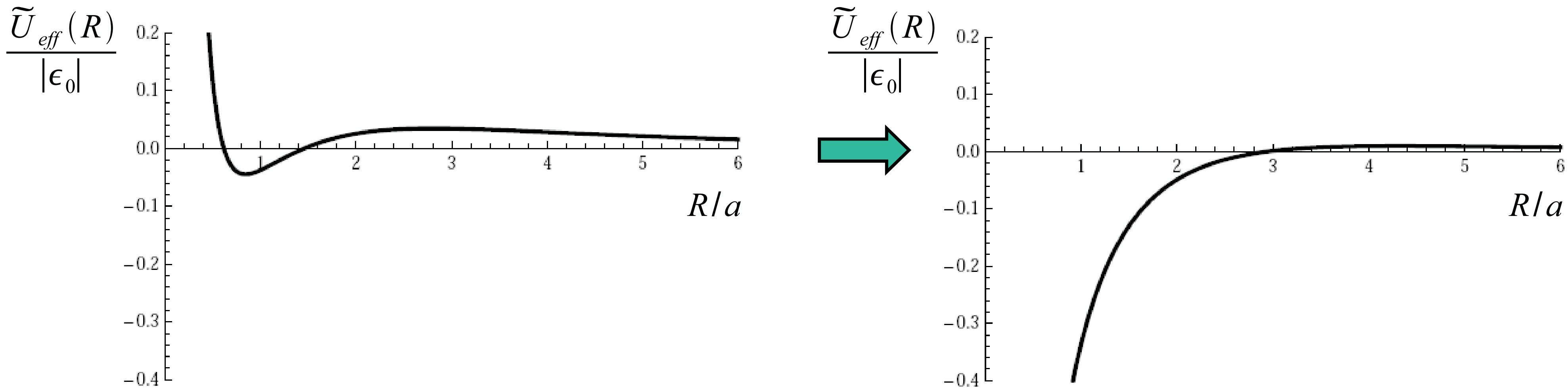


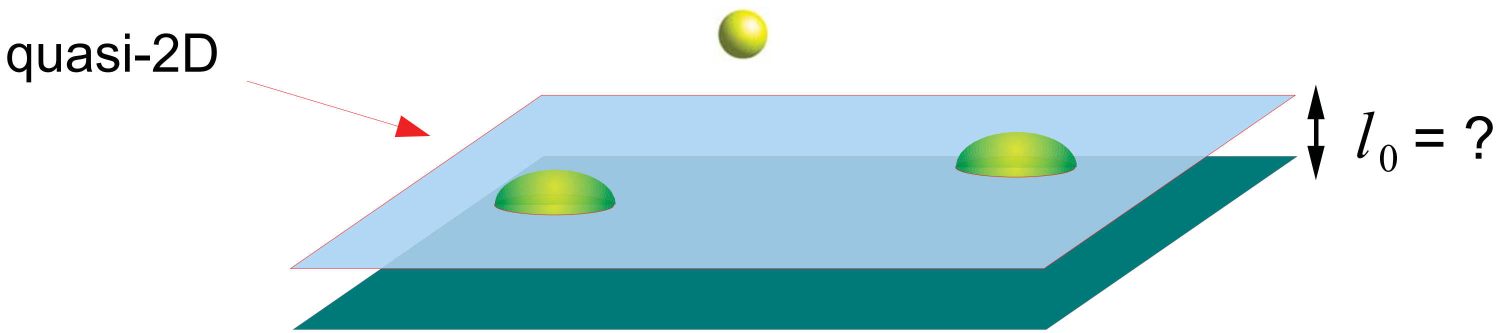


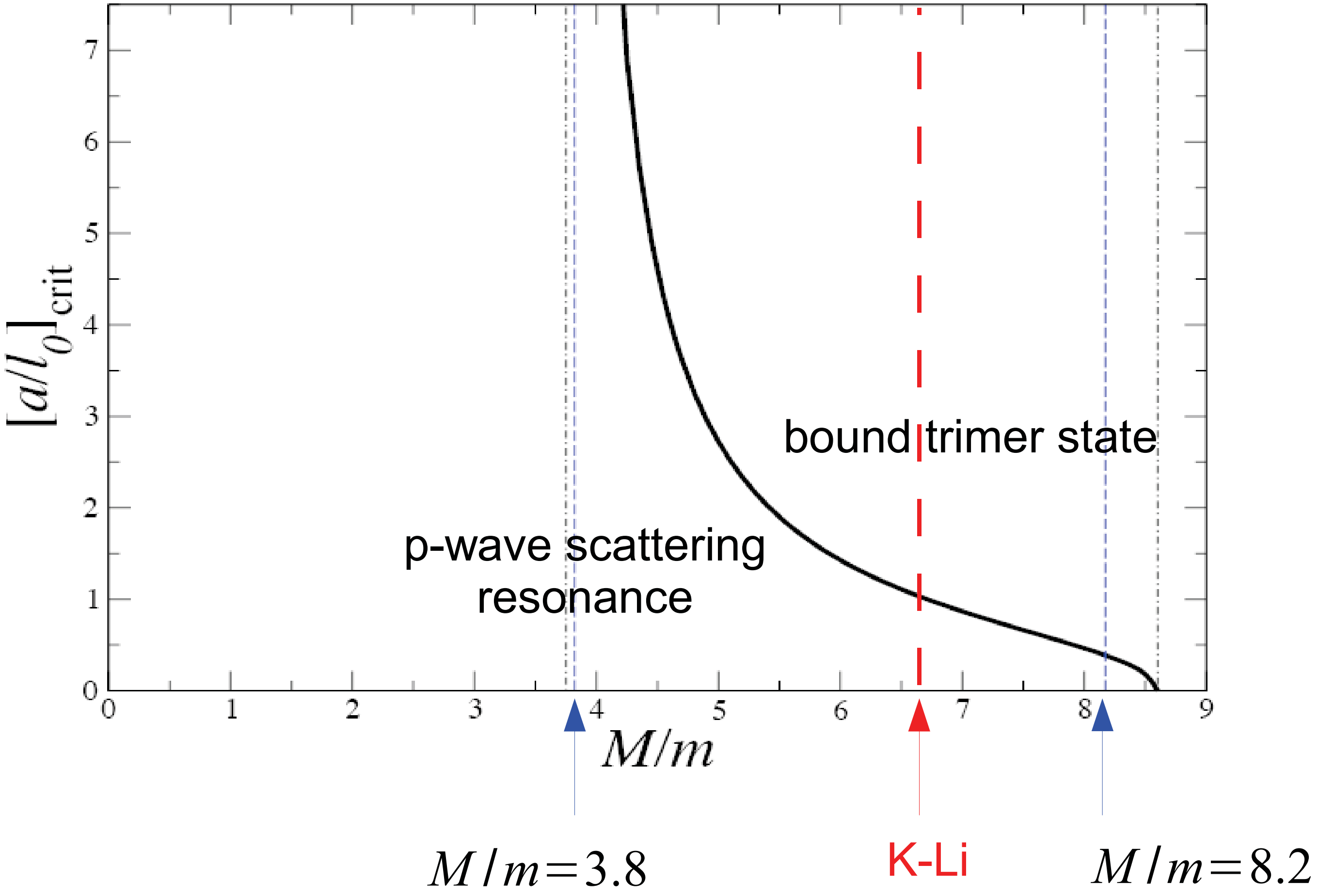
Can we make a bound Li-K-K trimer state?



$$U_{centrifugal}(R)=\frac{\hbar^2 l(l+1)}{MR^2} \quad \Longrightarrow \quad U_{centrifugal}(R)=\frac{\hbar^2 l^2}{MR^2}$$

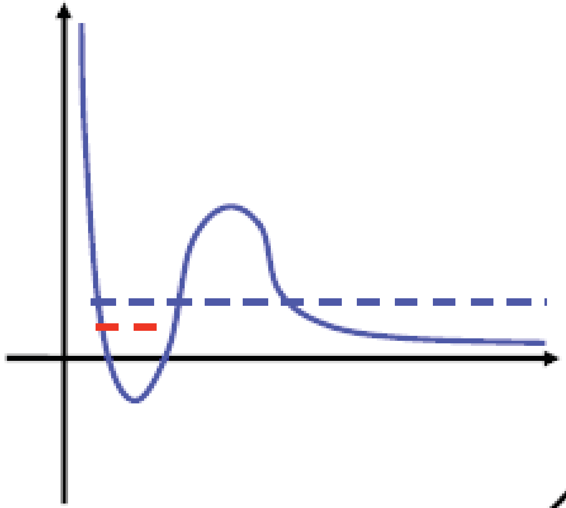



quasi-2D
l_0 = ?

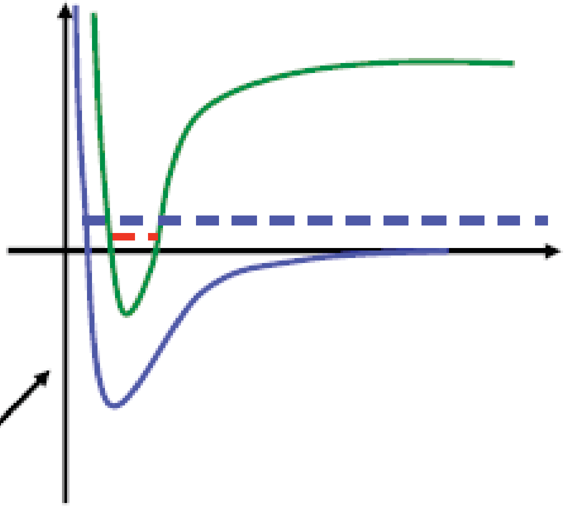

$[a/l_0]_{\mathrm{crit}}$
0
1
2
3
4
5
6
7
M/m
0
1
2
3
4
5
6
7
8
9
p-wave scattering resonance
bound trimer state
$M/m = 3.8$
K-Li
$M/m = 8.2$

Narrow Feshbach resonance?

Breit-Wigner



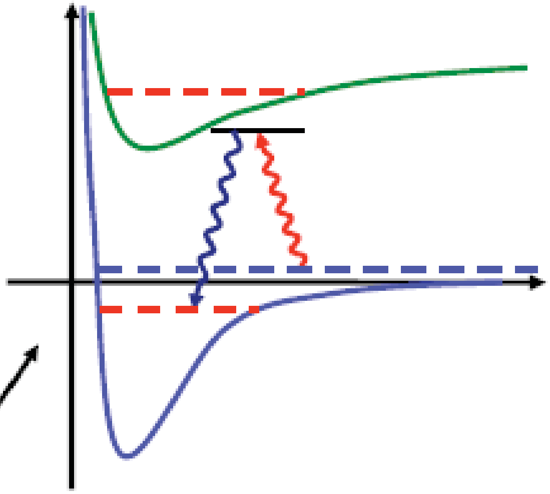
Feshbach



Stwalley (1976),

Tiesinga *et al.* (1993)

Optically induced



1 color:

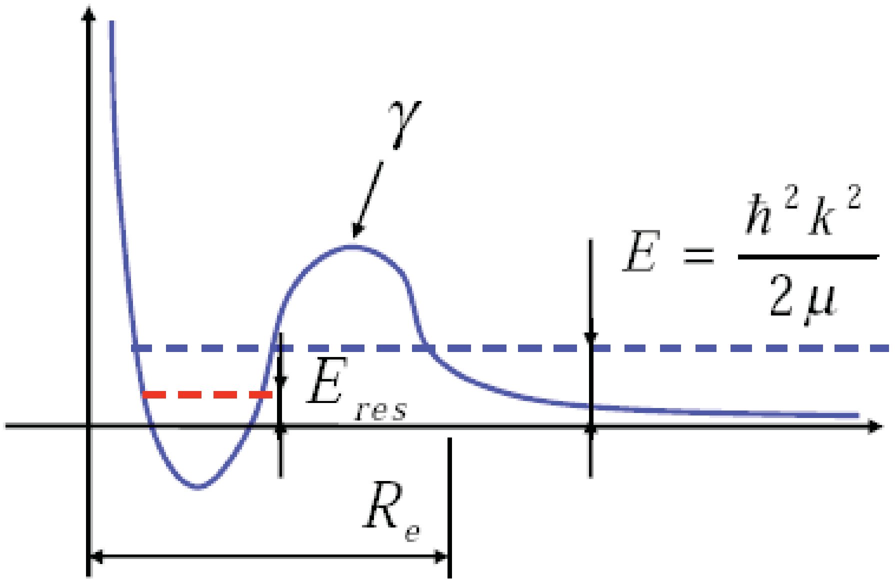
Fedichev *et al.* (1996)

2 color:

Bohn and Julienne (1999)

Resonance width is not tunable

Resonance width depends on the laser intensity



$$f(E) \approx -\frac{\hbar\gamma}{\sqrt{2\mu}\,(E - E_{res} + i\gamma\sqrt{E})}$$

$$\Updownarrow$$

$$f(k) \approx -\frac{1}{1/a + R^* k^2 + ik}$$

Effective range

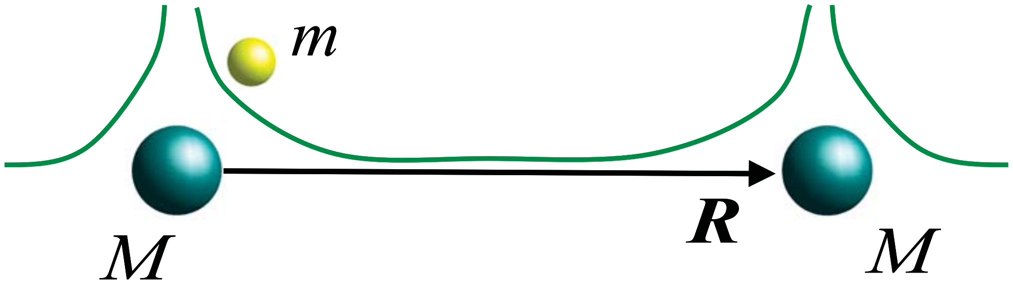
(strictly speaking, $r_0 = -2R^*$)

Feshbach resonance ⇒ $$R^* = \frac{\hbar^2}{2\mu\, a_{bg}\, (\partial E_{res}/\partial B)\Delta_B}$$

Li-K: $\Delta_B \approx 800\, mG,\ \ \partial B_{res}/\partial B \approx 1.7\, \mu_b,\ \ a_{bg} \approx 3\, nm$ ⇒ $R^* \approx 150\, nm$

Wille *et al.* (2008)

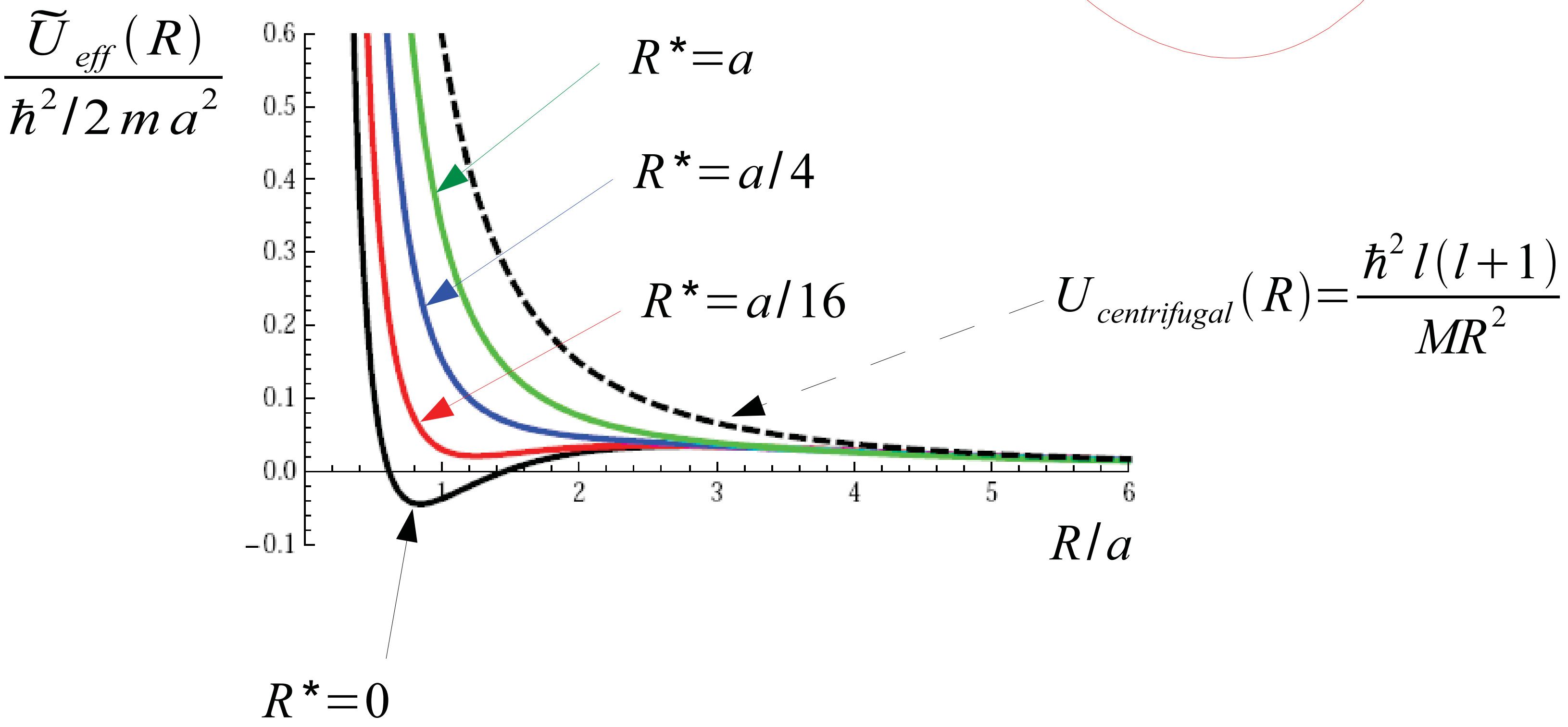
3-body problem near narrow resonance



Energy dependent scattering length!

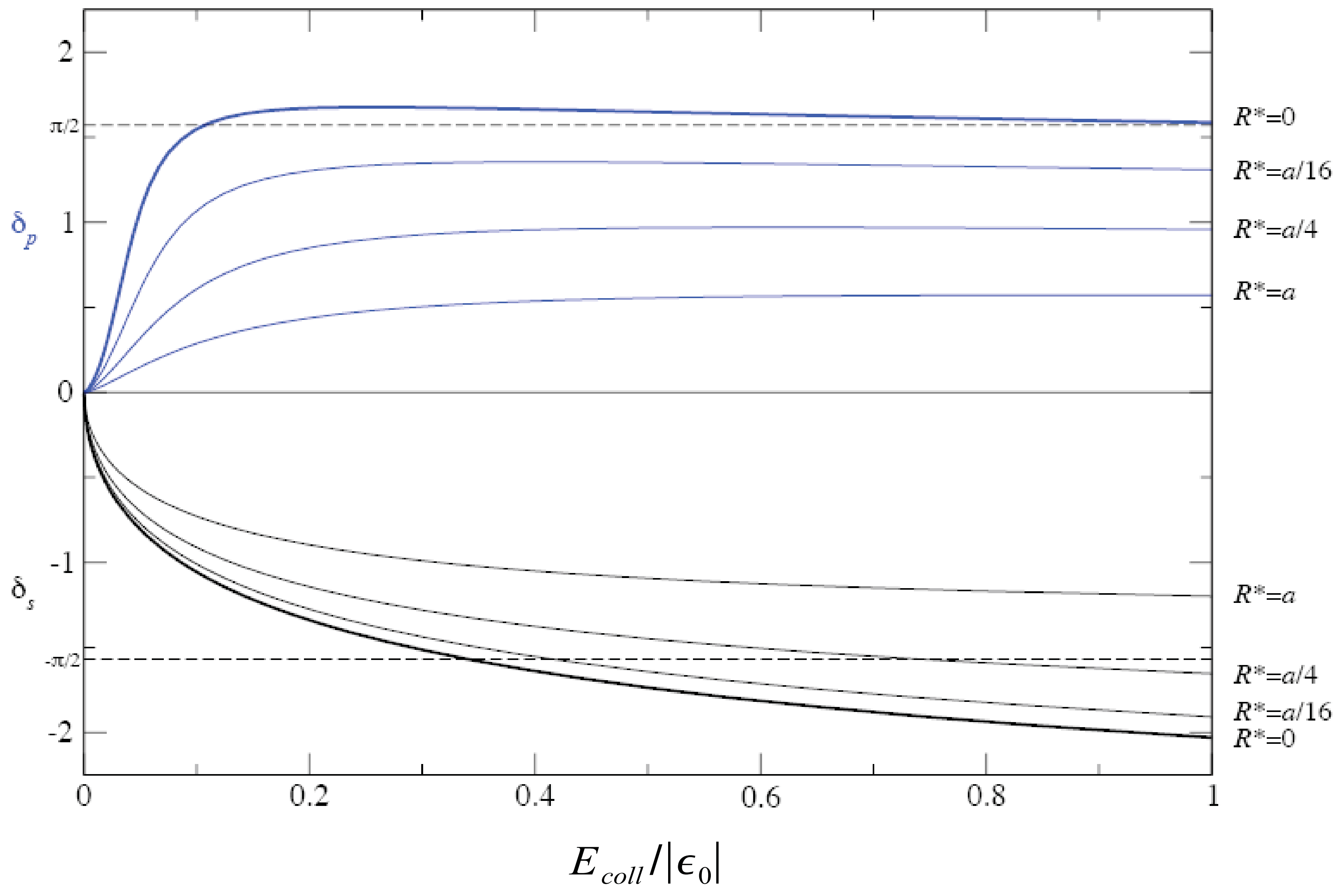
$$\boxed{\tilde{a}(E)=\frac{\hbar\gamma}{\sqrt{2\mu}\,(E-E_{res})}=\frac{1}{1/a+R^{*}k^{2}}}$$

$$\begin{tikzcd}
\tilde{a}(E) \arrow[r] & E=U_{eff}(R) \arrow[l, bend left]
\end{tikzcd}$$

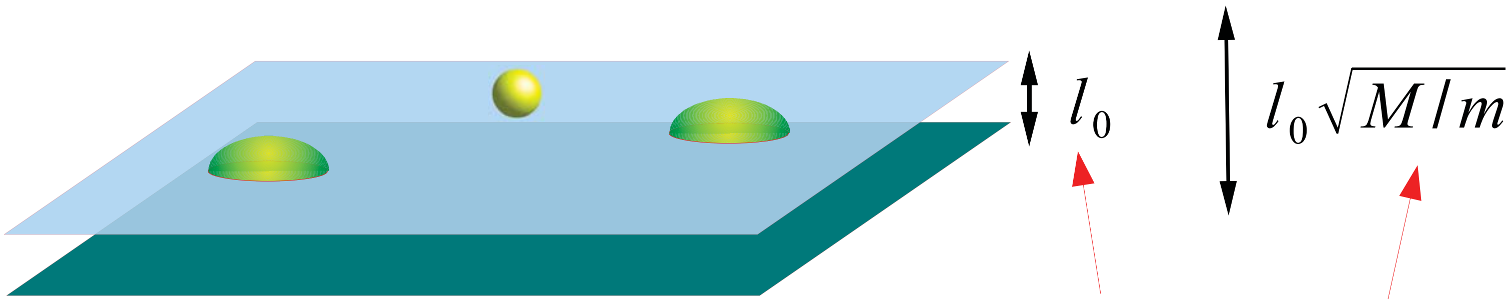


$$U_{centrifugal}(R)=\frac{\hbar^2 l(l+1)}{MR^2}$$

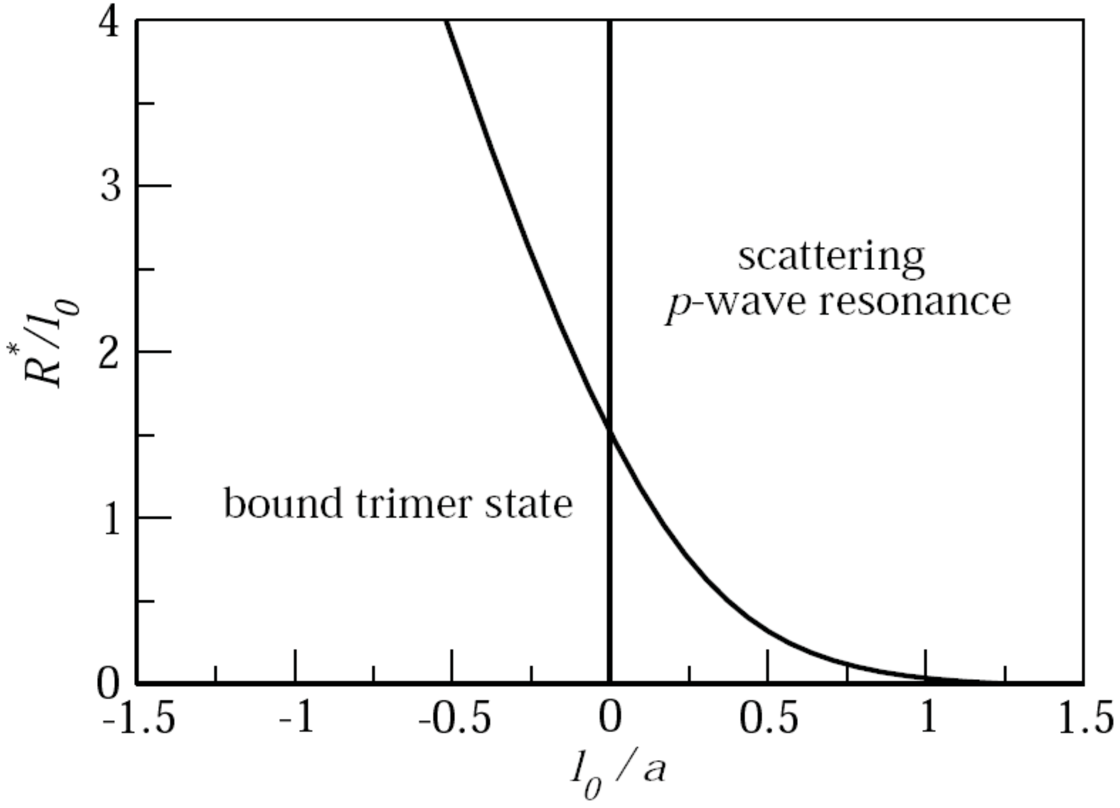
Exact (not BO) phase shifts



Quasi-2D – quasi-2D case $\omega_{Li}=\omega_K$

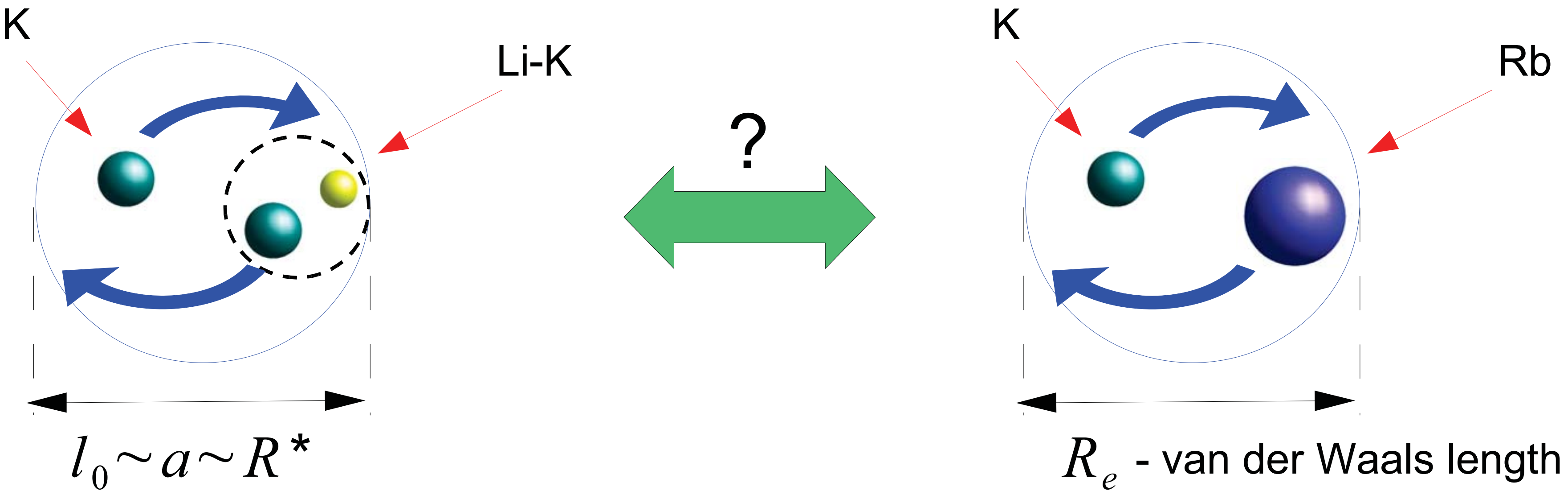


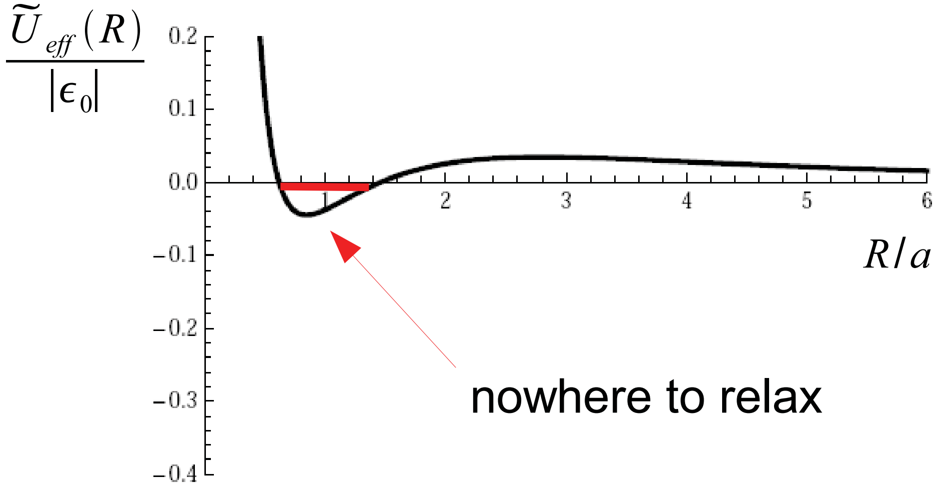
Oscillator length for K, for Li

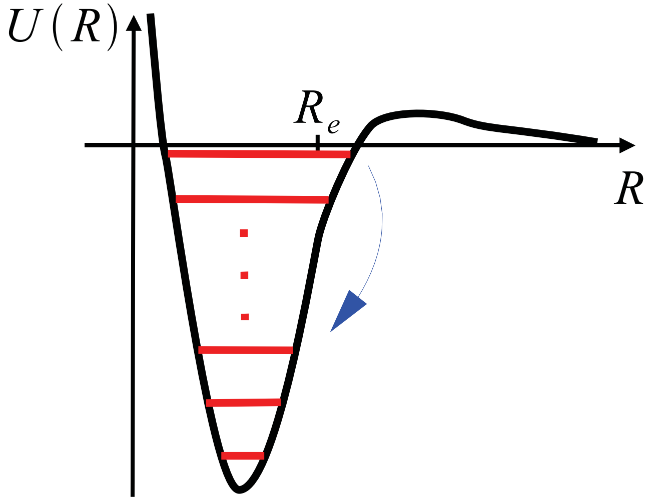


Lifetime

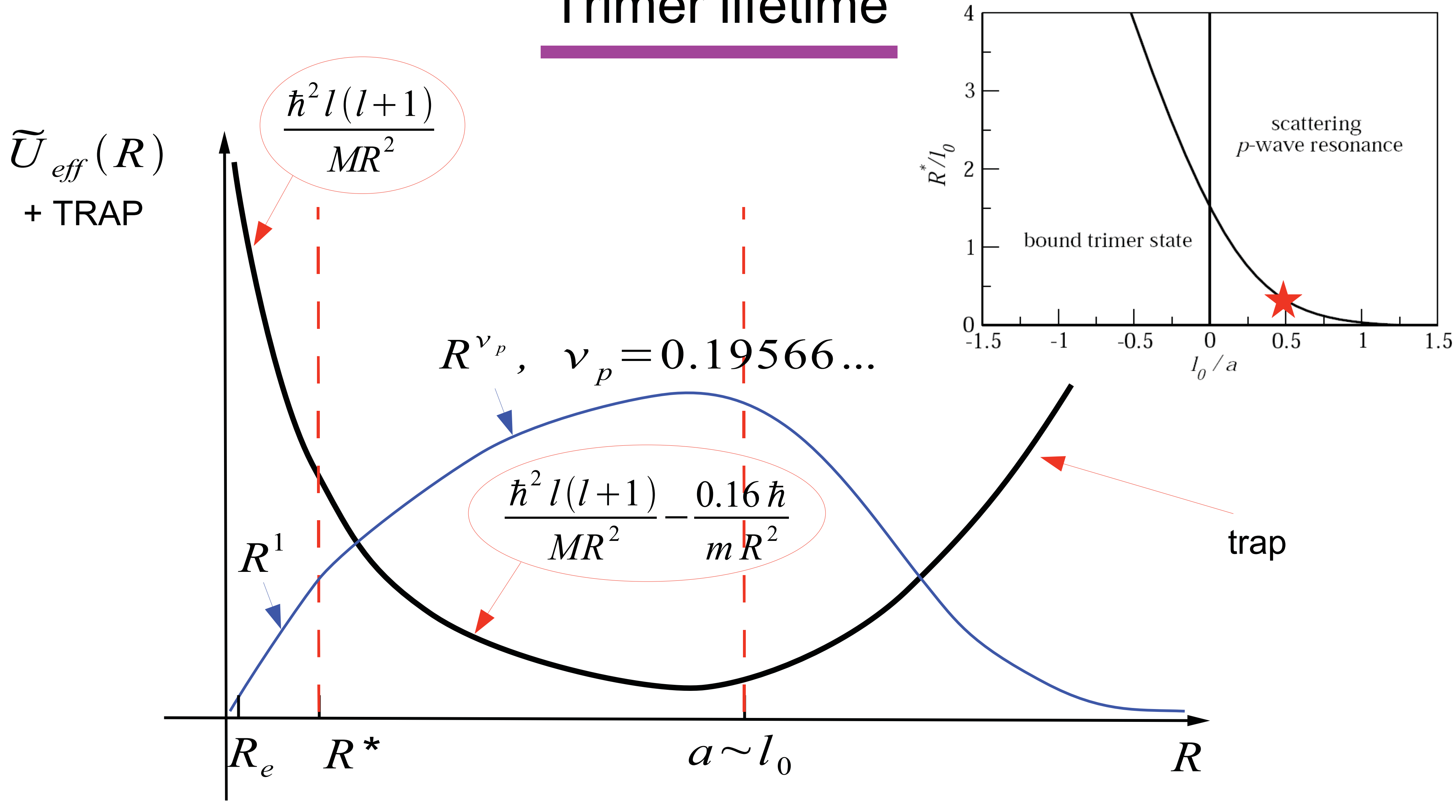
Why do we need atom-molecule mixture? Why don't we just take an atomic Bose-Fermi mixture with p-wave Feshbach resonance?







Trimer lifetime



$$\frac{1}{\tau_{tr}} \sim \frac{\hbar}{m R_e^2} \quad R_e^3 \; |\Psi(R_e)|^2 \sim \frac{\hbar}{m l_0^2} \left(\frac{R^*}{a}\right)^{2\nu_p} \frac{R_e^3}{a R^{*2}} \sim \frac{\hbar}{m l_0^2} 10^{-5}$$

Other decay channels

S-wave atom-molecule collisions

$$\alpha_{rel} \sim \frac{\hbar}{m} R_e \boxed{\left(\frac{R^*}{a}\right)^{1+2\nu_s}}$$

$$\nu_s = 2.0217\ldots$$

S-wave molecule-molecule collisions

$$\alpha_{rel} \sim \frac{\hbar}{m} R_e \boxed{\left(\frac{R^*}{a}\right)^{6+2\nu_{4body}}}$$

$$\left.\begin{array}{l} \nu_{4body}(M/m=1) \approx -1 \\ \nu_{4body}(4) \approx -0.7 \\ \nu_{4body}(8) \approx -0.5 \end{array}\right\}$$

(MC calc.)
Blume *et al.*
(2007-2008)

$$\nu_{4body}(6.7) \approx -0.6$$

Petrov's guess
(2009)

In both cases we need

$$a \gg R^*$$

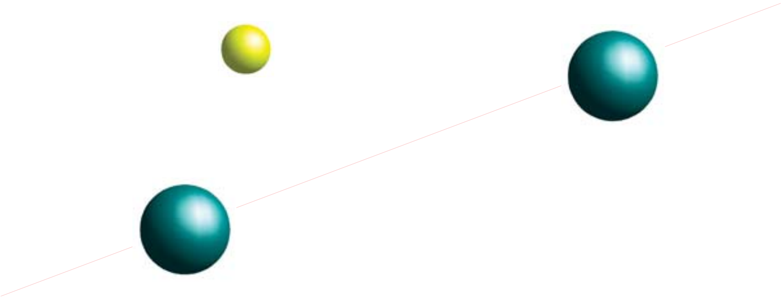
$$a = R^* \Updownarrow \Delta B = 16\, mG$$

Need very stable magnetic field !

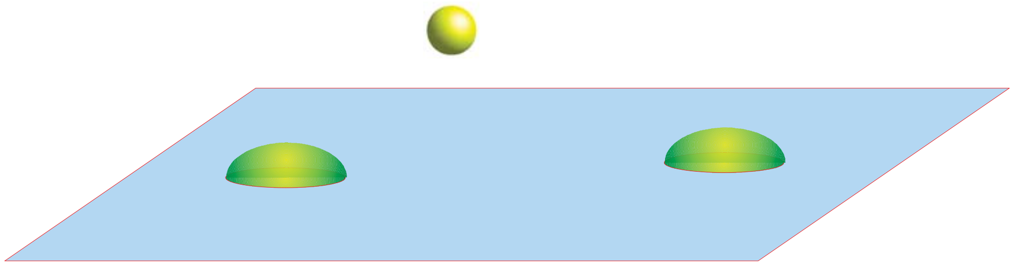
Related work

Fermi mixtures and Efimov physics in mixed dimensions, Nishida & Tan (2008 - 2009)

1D-3D

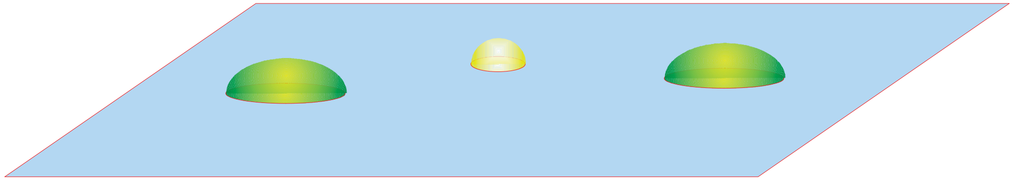


2D-3D

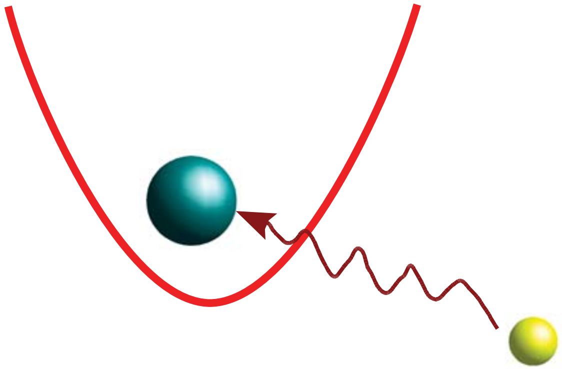


Trimer states in 2D, Pricoupenko & Pedri (2008)

2D-2D



Massignan & Castin (2006)



Summary

- p-wave scattering resonance in atom-dimer (K-(K-Li)) collisions. Interference between s- and p-waves

- Position of the resonance can be tuned by an external quasi-2D confinement. For sufficiently strong confinement it can be transformed into bound trimer state

- The trimer state (not a Efimov-type trimer) lives long and happy life. To avoid relaxation in atom-dimer and dimer-dimer collisions one needs to tune magnetic field very close to the resonance

- Degenerate Bose-Fermi mixture with p-wave interspecies resonance – any idea?