The Abdus Salam International Centre for Theoretical Physics

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2030-21

Conference on Research Frontiers in Ultra-Cold Atoms

4 - 8 May 2009

Superfluid transition in a Bose gas with correlated disorder

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Superfluid transition in a Bose gas with correlated disorder

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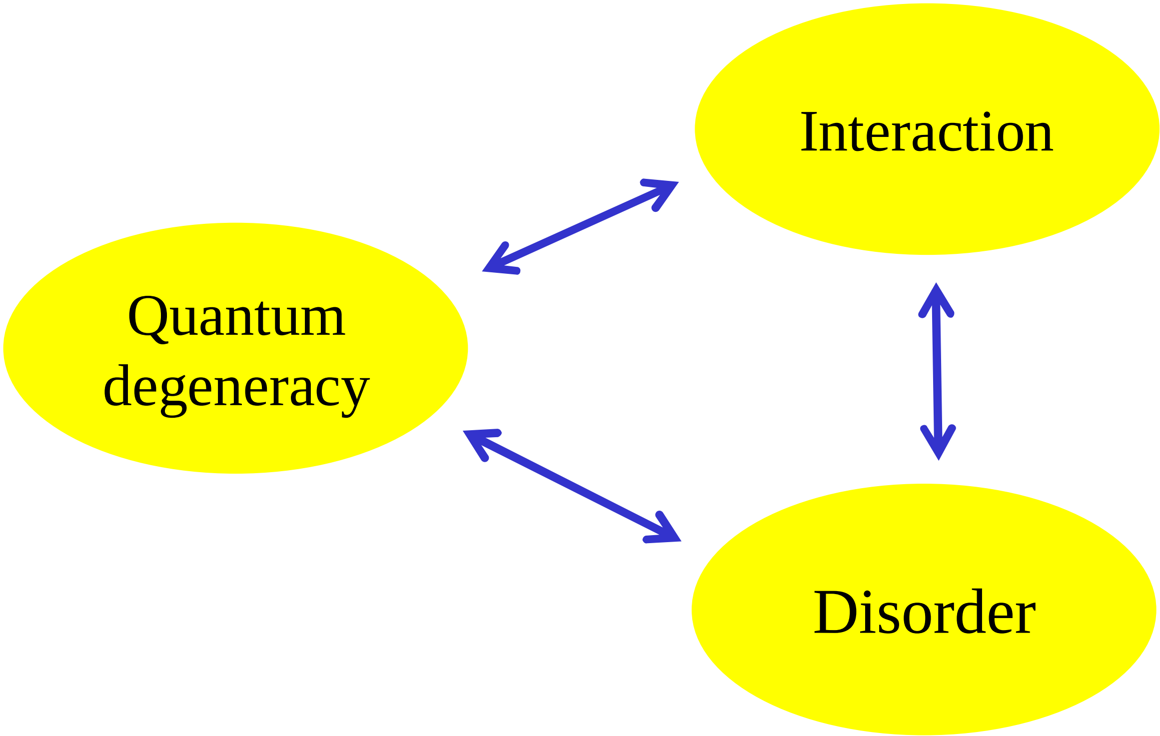
Research Frontiers in Ultracold Atoms
Trieste, May 4-8 2009

BOSE EINSTEIN CONDENSATION

Istituto Nazionale per la Fisica della Materia
Research and Development Center on Bose-Einstein Condensation
Dipartimento di Fisica – Università di Trento

Dirty boson problem (Fisher et al. 1989)

Central topic in condensed matter physics



Important differences compared to the metal-insulator transition in Fermi systems

- **Insulating – conducting (superfluid) (Quantum) phase transition**

  Natural order parameter ($\rho_s \neq 0$ $T<T_c$) in contrast to metals where true phase transition only at $T=0$ (otherwise a crossover).

- **Absence of Pauli exclusion**

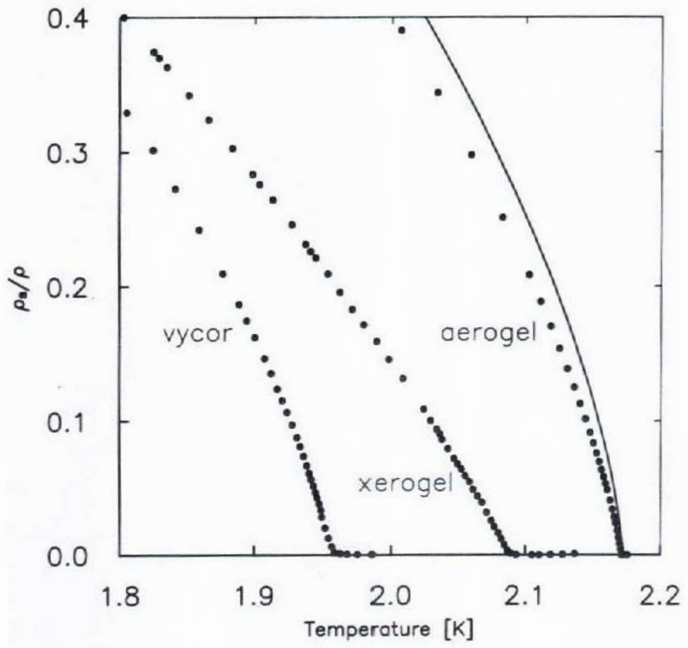
  Repulsive interactions crucial in preventing condensation in the lowest localized single-particle eigenstate of the random potential.

  No perturbation expansion around non-interacting limit

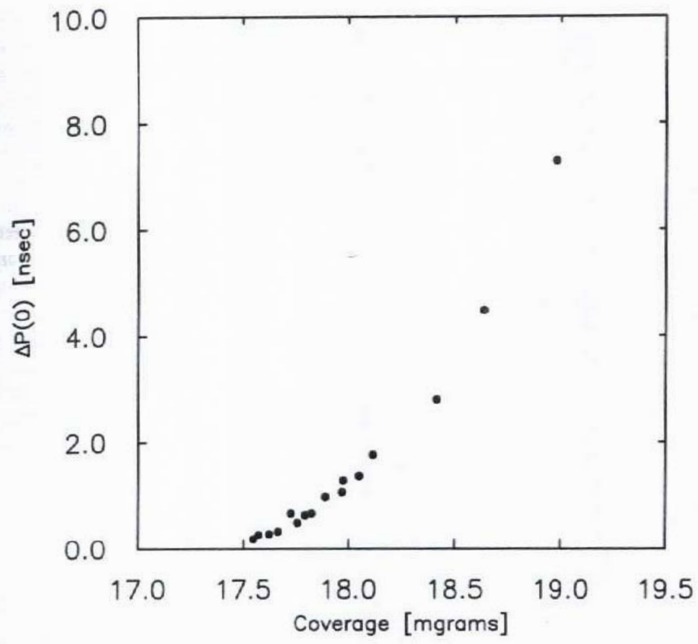
Liquid ^{4}He in porous media

Reppy and coworkers since 1983

Full pores



Films on vycor
T=0 extrapolation

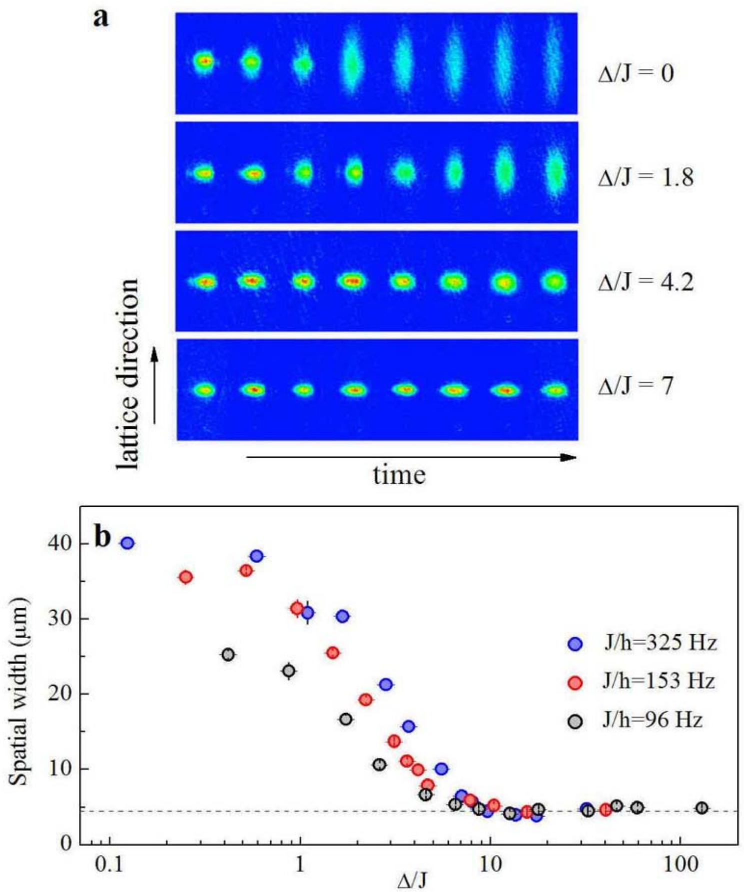


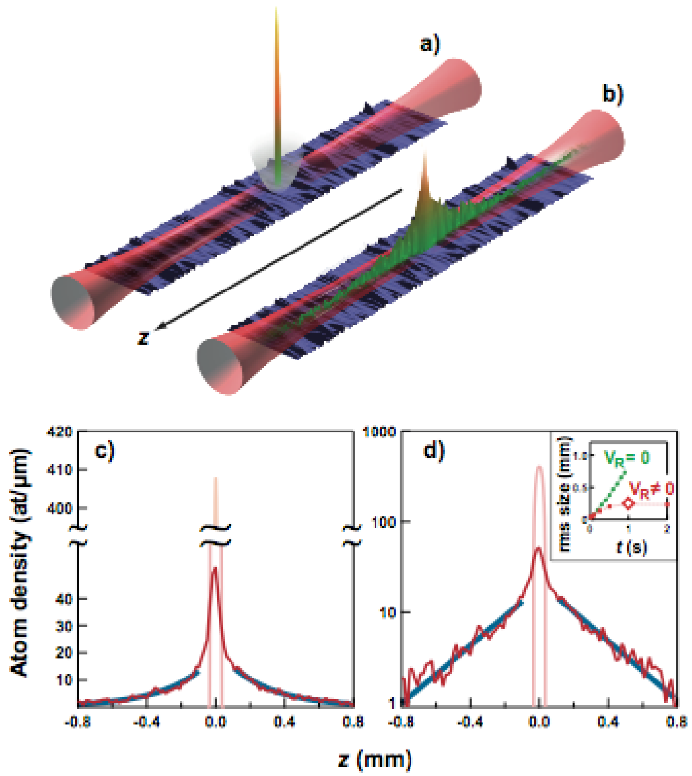
- T_c is reduced compared to bulk helium
- localization below a critical density

Ultracold gases **(Florence-Paris-Rice-Urbana)**

Roati et al. (2008)
Localization in 1D
quasi-periodic lattice

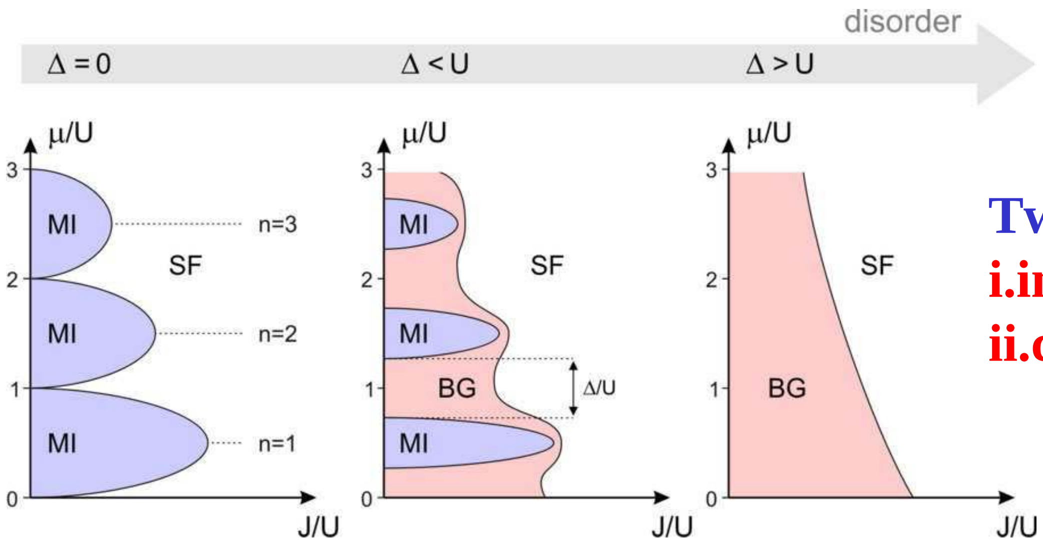
Billy et al. (2008)
Localization in 1D
speckle potential





Theory of disordered Bose-Hubbard lattice model

numerical studies: Krauth, Trivedi and Ceperley 1991, ….



Two types of insulator:
i.incompressible Mott phase
ii.compressible Bose glass phase

Commensurability plays
an important role
e.g. disorder can favor superfluidity

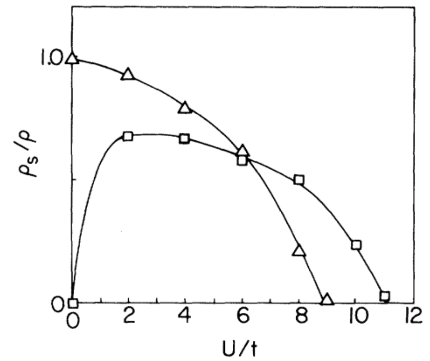
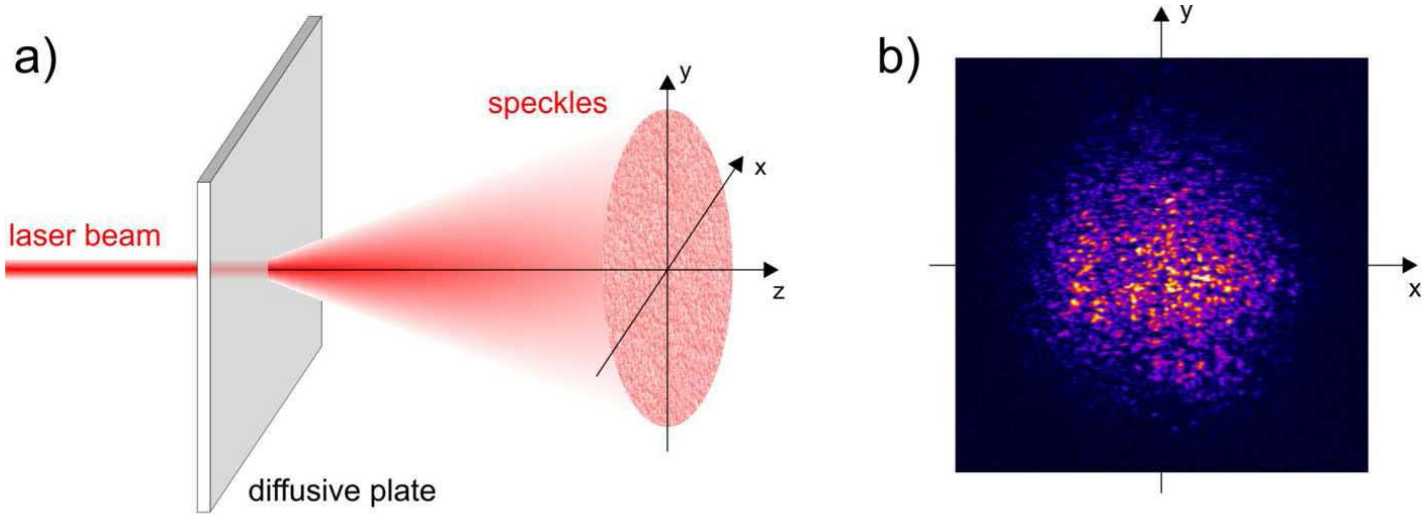


FIG. 3. Superfluid density ρ_s/ρ vs interaction strength U/t in a 10×10 system of density $\rho=1$ and $\beta t=4$. The values of the disorder parameter are $V/t=0$ (triangles) and 2 (squares).

Outline

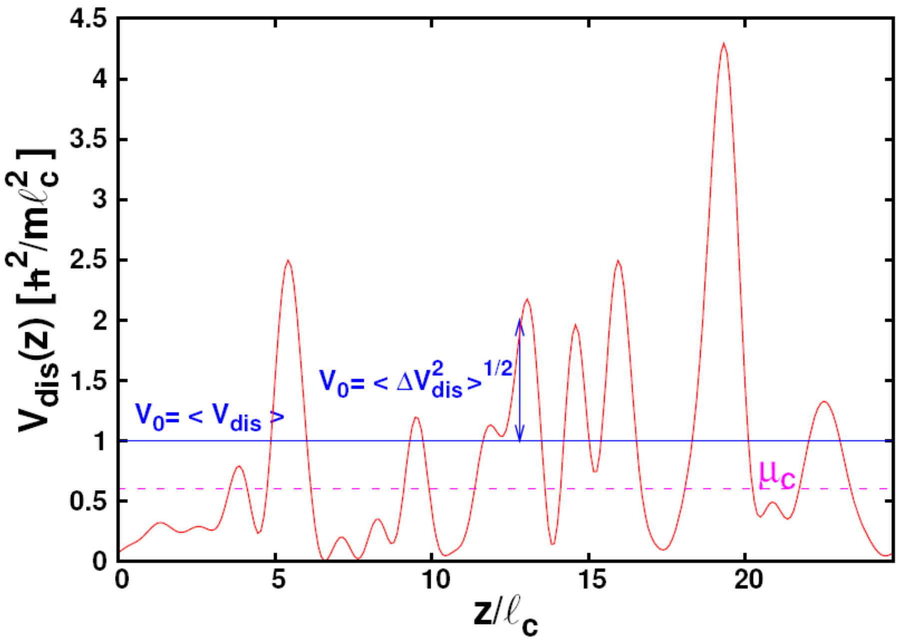
- Random potential created by optical speckles
- Superfluid transition in continuous 3D systems
- Thermodynamic behavior and evidence of Bose glass phase

Optical speckles



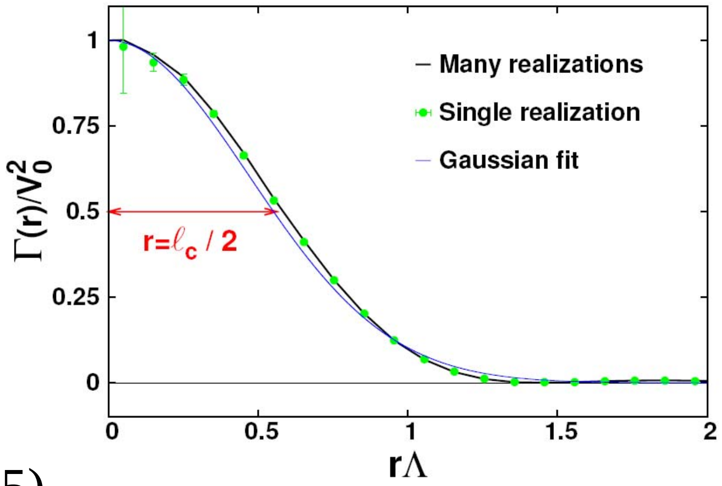
Probability distribution of intensity V

$$P(V) = \frac{e^{-V/V_0}}{V_0}$$



Autocorrelation function

$$\Gamma(r) = \langle V(\mathbf{r}')V(\mathbf{r}'+\mathbf{r})\rangle - V_0^2$$



ℓ_c = disorder correlation length

ℓ_c≈10μm (Fort et al. 2005-Clément et al. 2005)
ℓ_c≈0.3μm (Billy et al. 2008) (≈ $n^{-1/3}$)

Usual speckle patterns are 2D: $\ell_c^{\,z} >> \ell_c$
We consider a 3D pattern with the same ℓ_c in each direction

$V_0 >> \frac{\hbar^2}{m\ell_c^2}$	classical disorder	**many s.p. bound states in typical well of size ℓ_c**
$V_0 << \frac{\hbar^2}{m\ell_c^2}$	quantum disorder	**s.p. bound states only in rare wells of size >> ℓ_c or depth >> V_0**

- **Connection with δ-correlated disorder**

$$\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \gamma\delta(\mathbf{r}-\mathbf{r}')$$

assume Γ(r) gaussian

$$\gamma \propto V_0^2 \ell_c^3$$

Huang and Meng 1992
Lopatin and Vinokur 2002
Falco et al. 2007

- **Self-averaging**

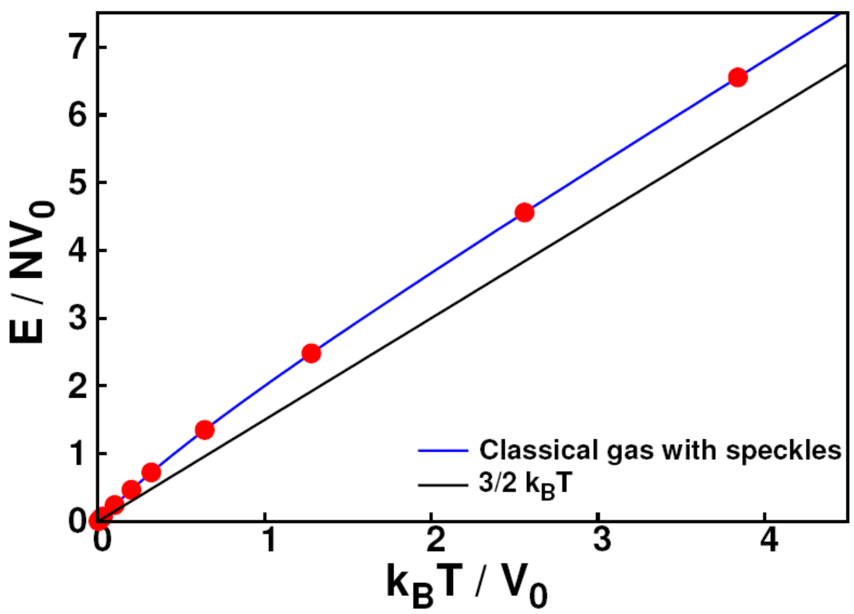
$$1/L^3 \int d^3r\, f[V(\mathbf{r})] = \int dV\, P(V) f(V)$$

valid for large volumes L^3

Example:
TD of a classical non-interacting gas

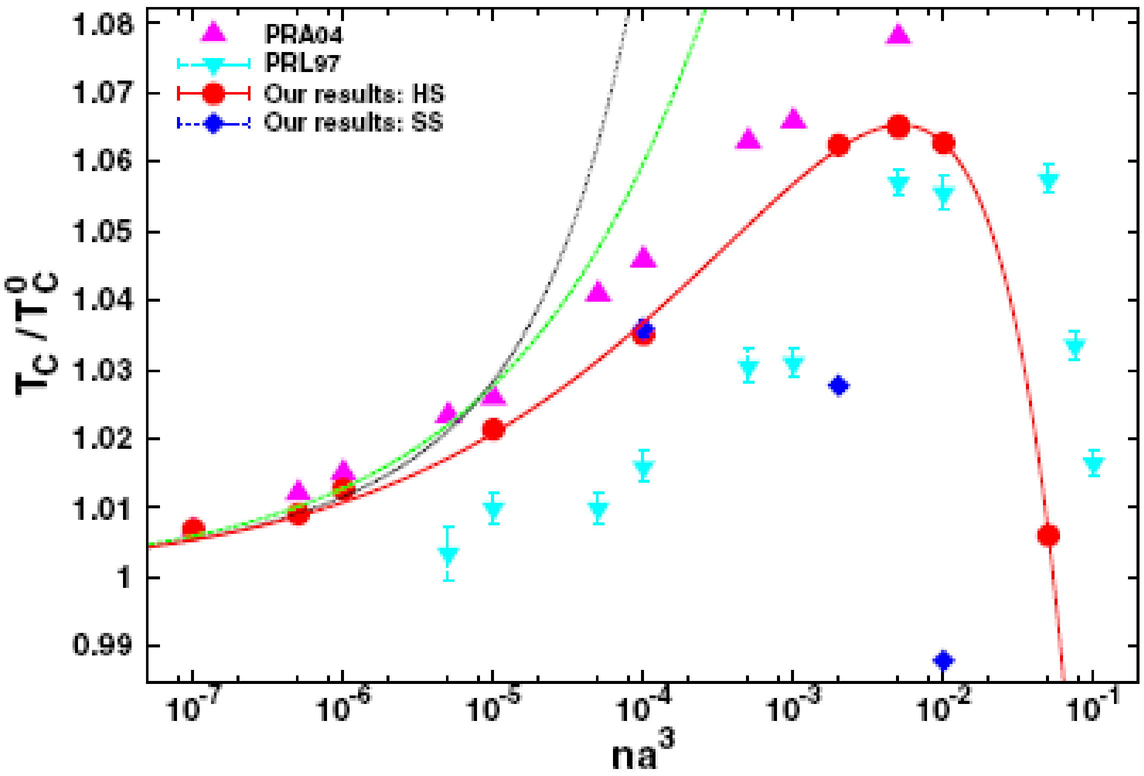
$$\frac{E}{N} = \frac{3}{2}k_BT + \frac{\int d^3r\, V(\mathbf{r}) e^{-V(\mathbf{r})/k_BT}}{\int d^3r\, e^{-V(\mathbf{r})/k_BT}} = \frac{3}{2}k_BT + \frac{V_0}{1+V_0/k_BT}$$

typical values $L \approx 20\text{-}50\,\ell_c$



Transition temperature: clean system (Pilati et al. 2008)

PIMC simulations for hard- and soft-spheres
finite-size scaling up to N=10^5 particles



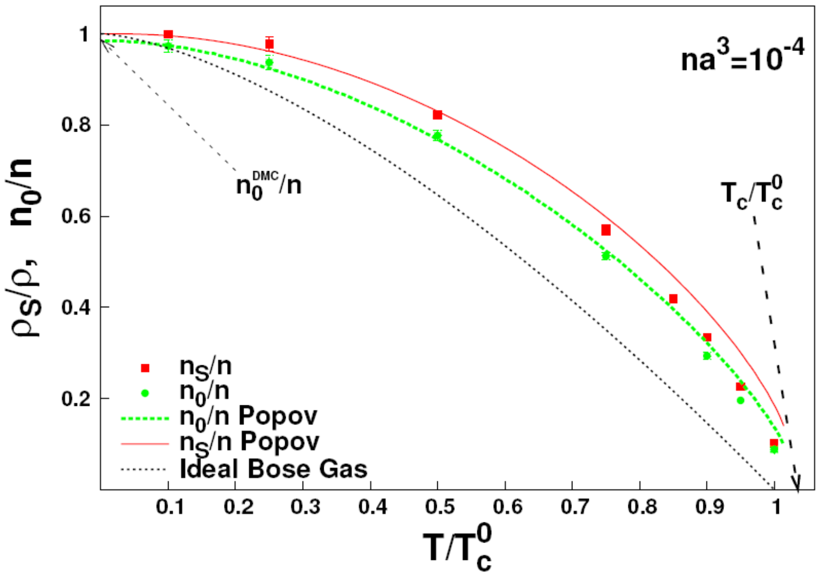
Low-density limit
Kashurnikov et al. 2001
Arnold and Moore 2001

$$T_C = T_C^0\left[1+1.29(an^{1/3})\right]$$

PRL97 Grüter et al. 1997
PRA04 Nho and Landau 2004

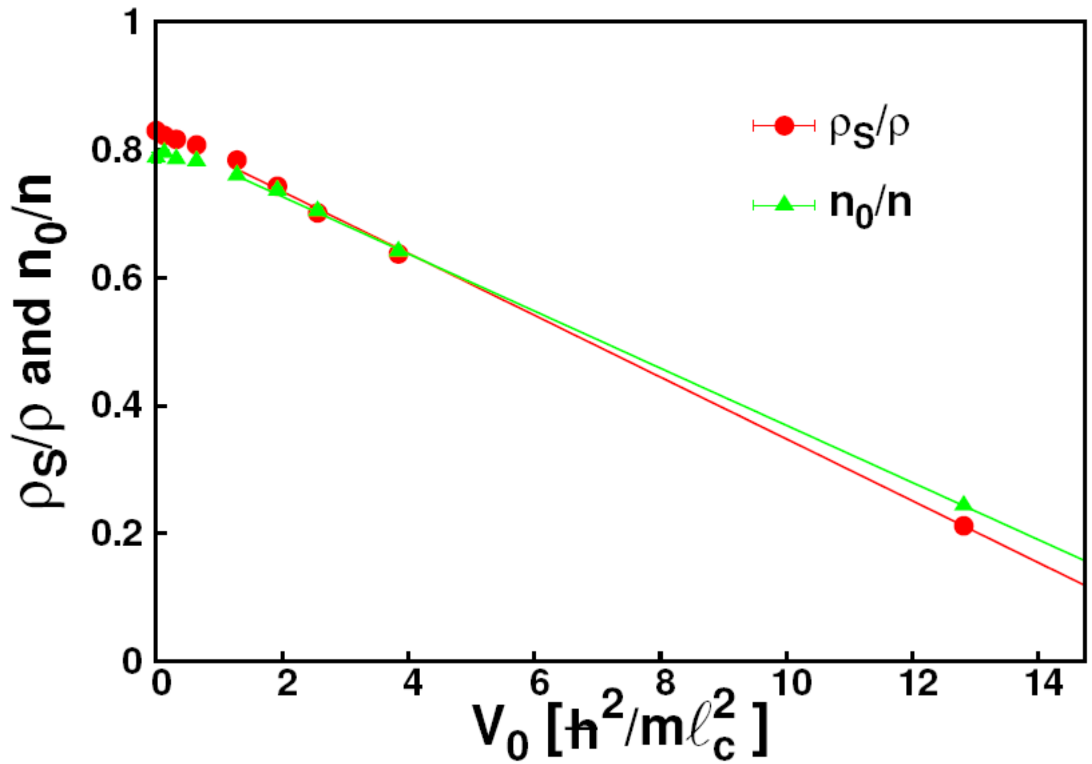
More details in Sebastaino's poster

Effect of disorder on the superfluid behavior



$V_0=0$

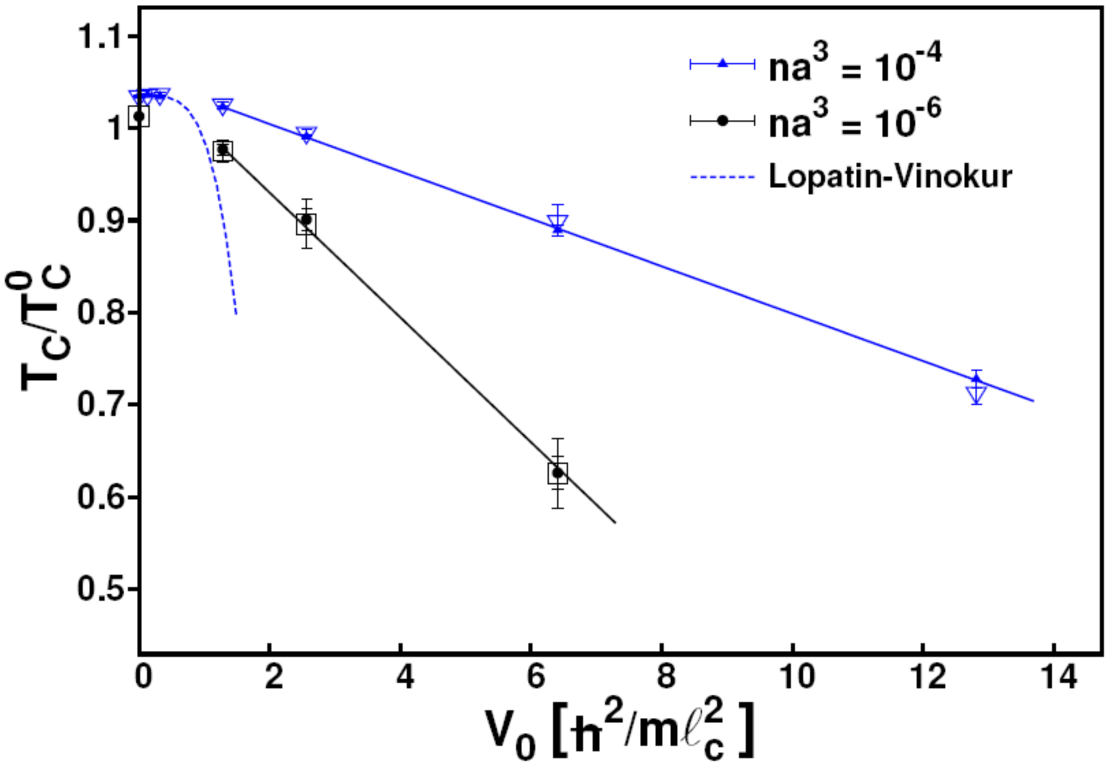
$T_c/T_c^0=0.5$



Transition temperature: disordered system

Canonical ensemble

ℓ_c is fixed : $n\ell_c^3=0.24$ such that $n4\pi\ell_c^3/3\approx 1$



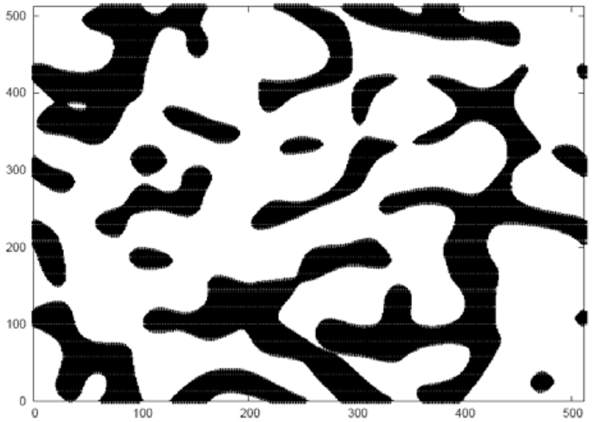
For larger values of V_0 the dependence on the realization becomes stronger we perform the calculation in the grand-canonical ensemble

Classical percolation threshold

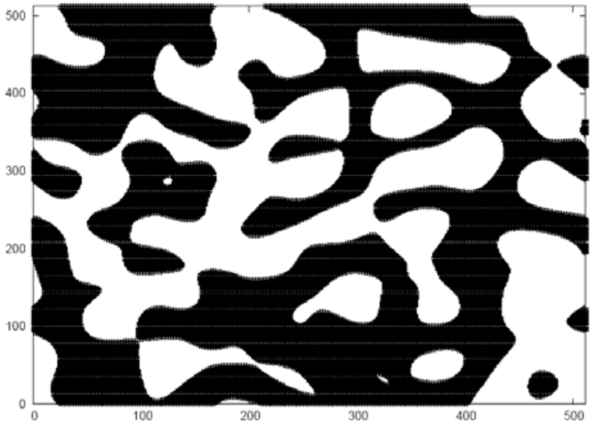
Accessible volume: $$\Phi(E) = \frac{1}{L^3} \int_{V(\mathbf{r})<E} d^3 r = 1 - e^{-E/V_0}$$

Mobiliy edge for classical particles: $$\Phi(E_c) = \Phi_c$$

2D speckles



Φ=30%



Φ=50%

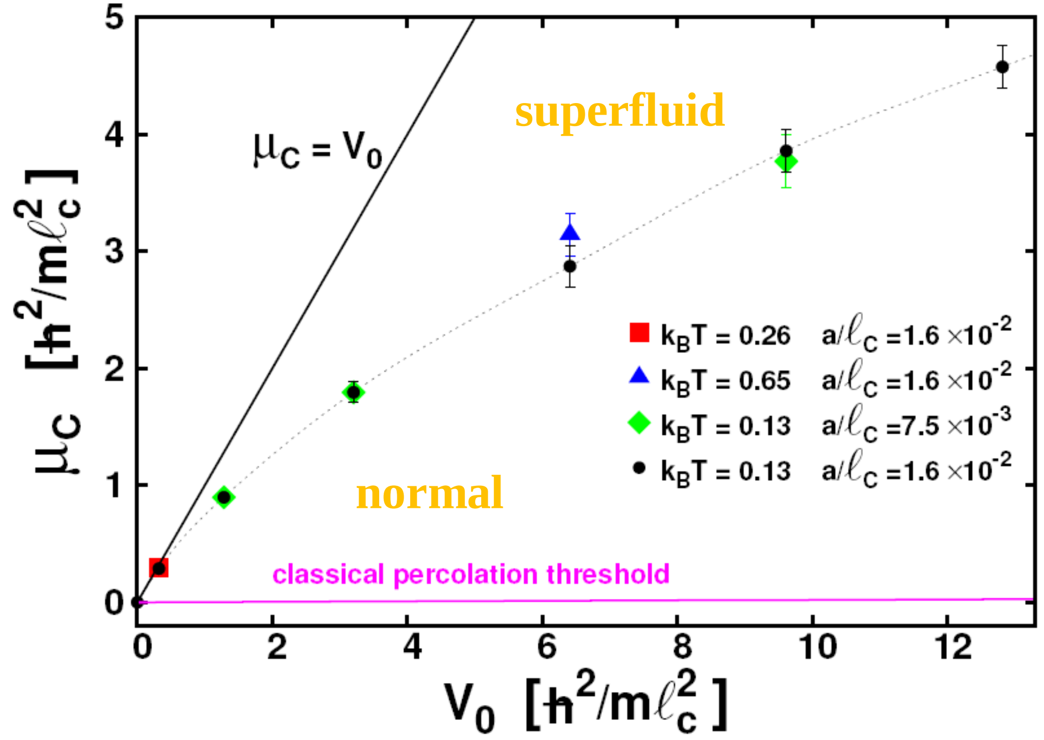
$\Phi_c \cong 40\%$

Experimental:
Smith and Lobb 1979
Numerical :
Weinrib 1982

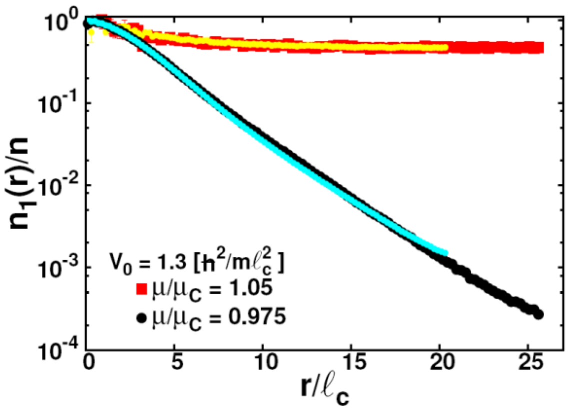
3D speckles

$\Phi_c \approx E_c \approx 0.002$ compare with Φ_c=0.03 of Swiss-cheese model

Critical chemical potential



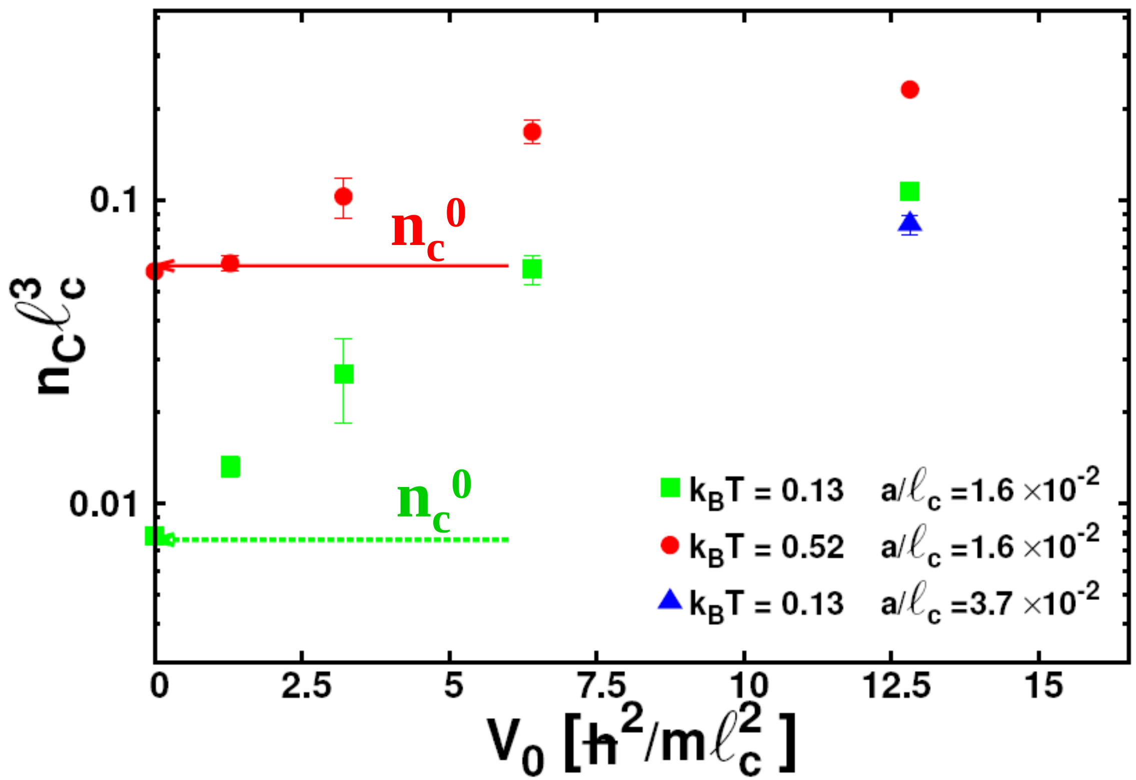
Behavior of OBDM

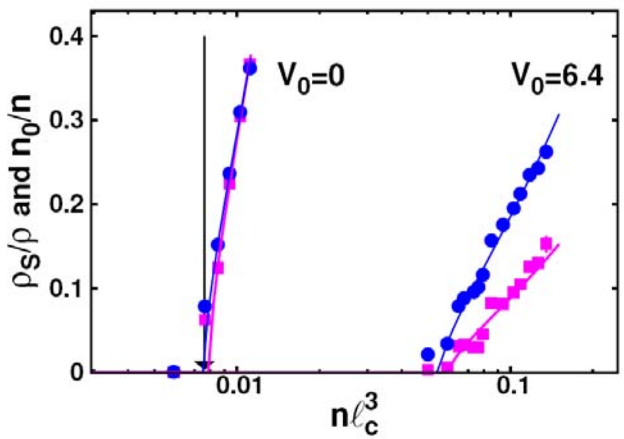


- **Scaling behavior: weak dependence on *T* and interaction (mobility edge)**

- **Large effects of quantum localization**

Critical density





$n_c^0 = 2.6(mk_BT/2\pi\hbar^2)^{3/2}$

For large disorder $n_c >> n_c^0$
$\rightarrow$ exotic normal phase (Bose glass?)

Thermodynamics for large disorder strength

- $\ell_c \ll n^{-1/3}$ <u>disorder is a small perturbation</u> $\left(V_0^2 \ell_c^3 \to 0\right)$

$$E(V_0) = E(V_0 = 0) + V_0$$

- $\ell_c \gg n^{-1/3}$ ($\xi = 1/\sqrt{8\pi n a}$ at low T) <u>LDA can be applied</u>

***T*=*0* GP equation** $$g n_0(\mathbf{r}) = \left(\mu - V(\mathbf{r})\right) \vartheta\left(\mu - V(\mathbf{r})\right)$$

from the normalization condition

$$\frac{\mu}{V_0} + e^{-\mu/V_0} = 1 + \frac{gn}{V_0}$$

- **if** $V_0 \gg gn$ $\quad \mu = \sqrt{2 g n V_0}$

- **if** $\mu > E_c$ $\quad n_0(\mathbf{r}) > 0$ along a percolation path

T≠*0* Hartree-Fock

1) elementary excitations

$$\varepsilon(\mathbf{p},\mathbf{r}) = \frac{p^2}{2m} + V(r) - \mu + 2g[n_0(\mathbf{r}) + n_T(\mathbf{r})]$$

2) thermal component

$$n_T(\mathbf{r}) = \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{1}{e^{\varepsilon(\mathbf{p},\mathbf{r})/k_B T} - 1}$$

3) normalization

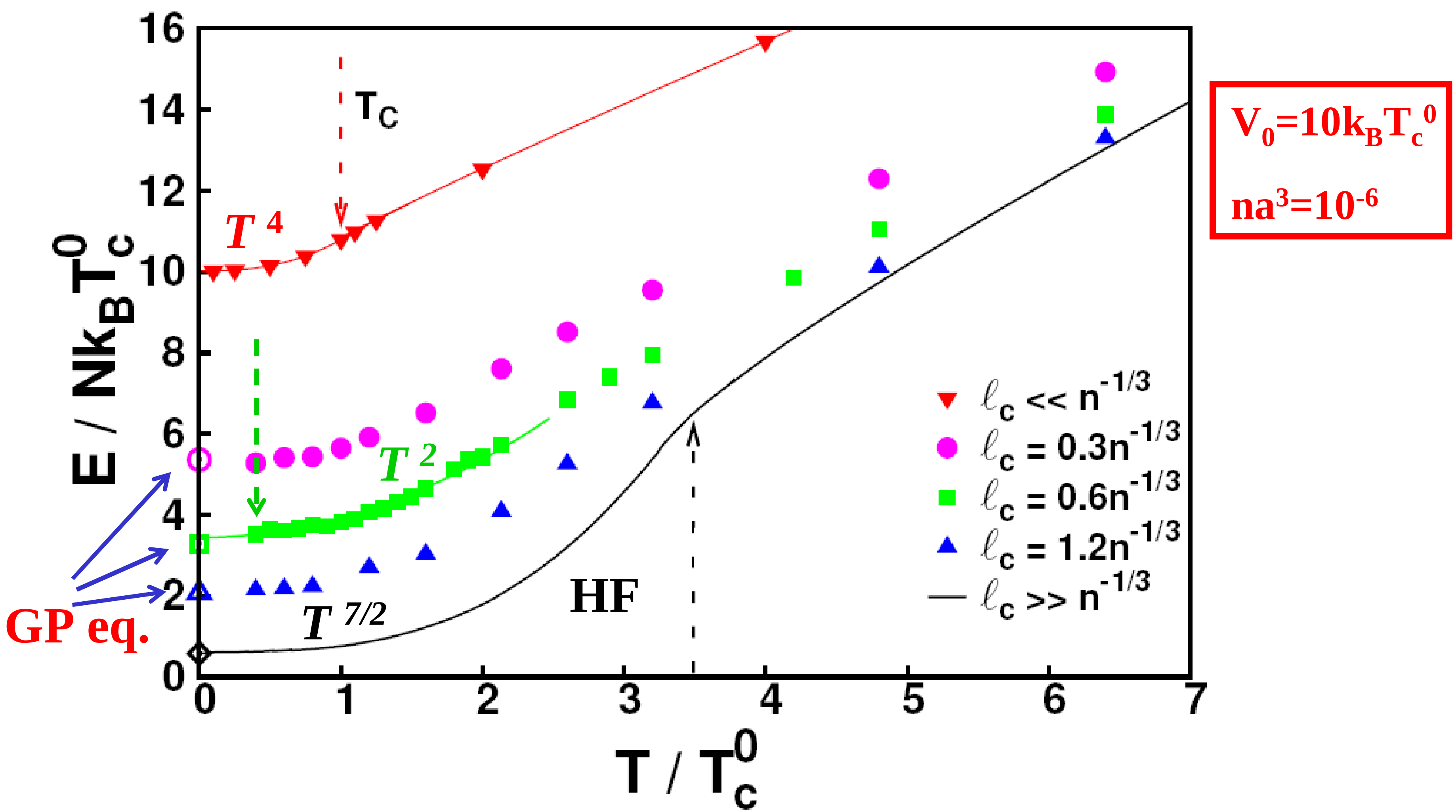
$$n = \int d^3 r \left[n_0(\mathbf{r}) + n_T(\mathbf{r})\right]$$

At low *T* lowest excitations live at the border of condensate lakes

$$\varepsilon(\mathbf{p},\mathbf{r}) = \frac{p^2}{2m} + \left|V(r) - \mu\right|$$

their density of states as ε→0

$$g(\varepsilon) \propto \varepsilon^{3/2} \quad \Longrightarrow \quad E \propto T^{7/2} \text{ at low } T$$



Evidence of T^2 behavior at low T

Bose glass phase

Conclusions

Critical behavior at the superfluid-normal transition with correlated disorder

if $n\ell_c^3 \approx 1$ disorder has a large effect

- **suppression of T_c**
- **quantum localization**
- **Bose glass phase**

Thank you for your attention!

The Trento team

Come to visit us!

PHYSICAL REVIEW B VOLUME 20, NUMBER 9 1 NOVEMBER 1979

Percolation in two-dimensional conductor-insulator networks with controllable anisotropy

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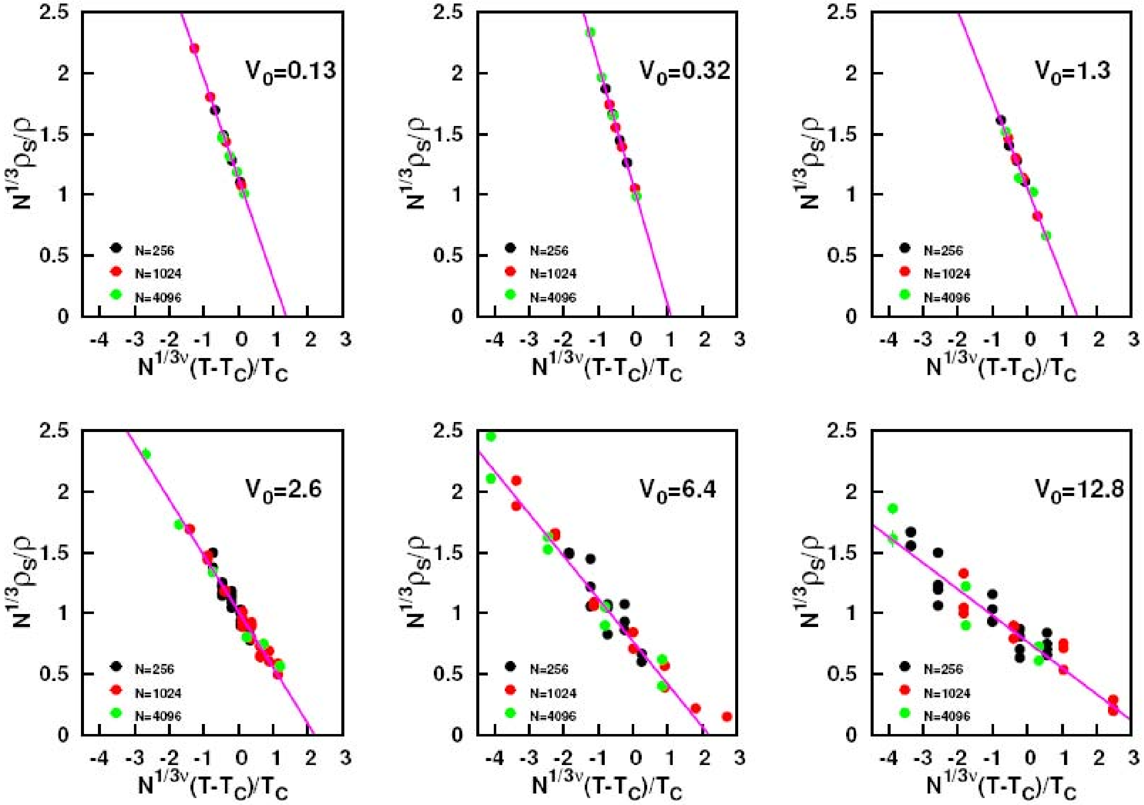
Department of Physics and Division of Applied Sciences, Harvard University, Cambridge, Massachusetts 02138
(Received 18 January 1979; revised manuscript received 22 May 1979)

The conductivities of two-dimensional conductor-insulator networks generated photolithographically from laser speckle patterns have been measured. Isotropic networks with $\approx$450 000 statistically independent units show a percolation threshold $f_c \approx 41\%$ conductor and a critical exponent $t \approx 1.30$. Measurements on anisotropic networks and numerical simulations indicate that either f_c, t, or the size of the "asymptotic region" must vary with the degree of anisotropy.

Finite-size scaling



$$N^{1/3}\frac{\rho_s(t,N)}{\rho}=f\left(tN^{1/3\nu}\right)$$

Density profiles