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Bose gas in atom-chip experiment: from ideal gas to quasi-condensate

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Density fluctuations in a very elongated Bose gas : from ideal gaz to quasi-condensate

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Trieste, May 2009

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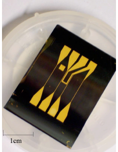
Physics in 1D gases

- 1D physics very different from 3D : no bec phenomena (no sturation of excited state population)
- Enhanced effect of interactions
- Special case of quantum correlated systems : exact solutions exist
- Chip experiment well suited to study 1D physics



Outline

1. Theoretical results on weakly interacting 1D gases
   - Homogeneous gases
   - harmonically trapped gas
2. Experimental study of the cross-over towards quasi-bec
   - Observation of bunching effect
   - Inhibition of bunching in the quasi-bec regime
   - Failure of Hartree-Fock theory to explain the transition towards a quasi-bec
3. Other results
4. Conclusion

Outline

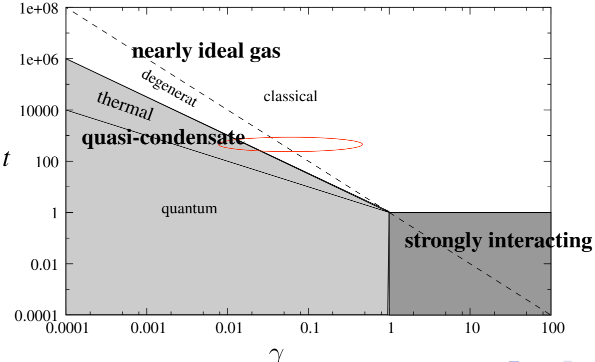
1D Bose gas with repulsive contact interaction

$$H = -\frac{\hbar^2}{2m}\int dz\psi^+\frac{\partial^2}{\partial_z^2}\psi + \frac{g}{2}\int dz\psi^+\psi^+\psi\psi,$$

Exact solution : Lieb-Liniger Thermodynamic : Yang-Yang (60') n, T

Length scale : $l_g = \hbar^2/mg$, Energy scale $E_g = \hbar^2/2ml_g^2$

Parameters : $t = T/E_g$, $\gamma = 1/nl_g = mg/\hbar^2 n$

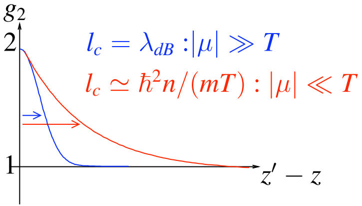


Nearly ideal gas regime : bunching phenomena

- For each quantum state, Boltzmann distribution

$\rightarrow$ particle number fluctuations : $\langle n_k^2 \rangle - \langle n_k \rangle^2 = \underbrace{\langle n_k \rangle}_{\text{shot noise}} + \underbrace{\langle n_k \rangle^2}_{\text{bunching}}$

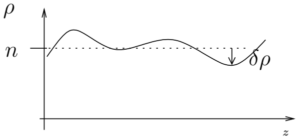
- Two-body correlation function $g_2(z)$



- Bunching effect $\rightarrow$ density fluctuations.
  $\langle n(z)\, n(z') \rangle = \langle n \rangle^2 g_2(z' - z) + \langle n \rangle \delta(z - z')$
- Bunching : correlation between particles. Quantum statistic
- Classical field $\psi = \sum a_k e^{ikz}$: speckle phenomena
  $n_k = |a_k|^2 \Rightarrow \langle n_k^2 \rangle - \langle n_k \rangle^2 = \langle n_k \rangle^2$

Crossover towards quasi-condensate

- Repulsive Interactions → Density fluctuations require energy



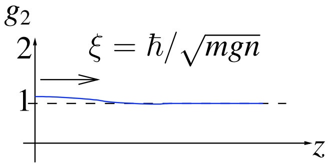
$$\int dz \delta\rho = 0 \Rightarrow$$
$$H_{int} = \frac{g}{2}\int dz \rho^2 = \frac{g}{2}\int dz \delta\rho^2 > 0$$

Reduction of density fluctuations at low temperature/high density

Cross-over temperature :

$$\frac{1}{N}H_{int} \propto gn \simeq |\mu| \quad \Rightarrow \quad \boxed{T_{c.o.} \simeq \frac{\hbar^2 n^2}{2m}\sqrt{\gamma}}$$

- For $T \ll T_{c.o.}$: quasi-bec regime
- bunching effect killed



$T > gn$

Crossover towards quasi-condensate

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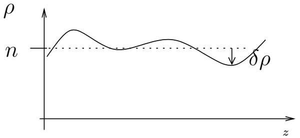

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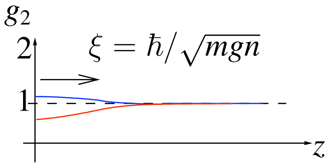


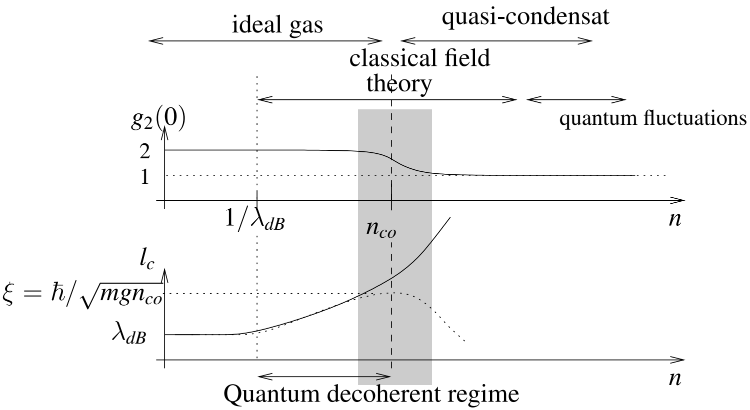
$T > gn$

$T < gn$

1D weakly interacting homogeneous Bose gas

$$t = \hbar^2 T/(mg^2)$$

$$n_{co} = \frac{1}{\lambda_{dB}} t^{1/6}$$



$\mu < 0$ $\mu \simeq gn > 0$

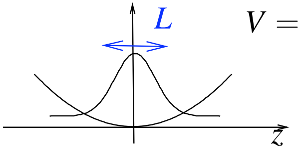
One-dimensional gas trapped in a harmonic potential

Competition : Interaction-induced cross-over / finite size BEC

- Cross over towards quasi-bec ω small

Local density approach

Cross-over : $n(z=0) = n_{c.o.}$



$V = 1/2 m\omega^2 z^2$

$$N_{c.o.} \simeq \frac{k_B T}{\hbar\omega} \ln\left(t^{1/3}\right)$$

Validity of LDA : $l_c \ll L$ At cross-over $\Rightarrow$ $\boxed{\hbar\omega \ll (mg^2T^2/\hbar^2)^{1/3}}$

- Finite size condensation phenomena at large ω

$$N_c = \frac{k_B T}{\hbar\omega} \ln\left(2T/\hbar\omega\right)$$

- LDA condition in 3D world : $g = 2\hbar\omega_\perp a$ for $a \ll l_\perp = \sqrt{\hbar/m\omega_\perp}$

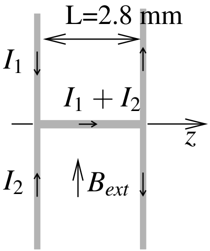
$\boxed{\omega \ll \omega_\perp (T/\hbar\omega_\perp)^{2/3}(a/l_\perp)^{2/3}}$ Easily satisfied

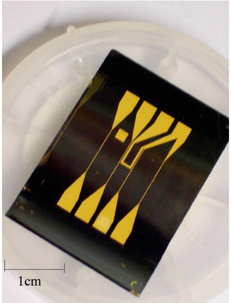
Bouhoule et al., PRA **75** *031606 (2007)*

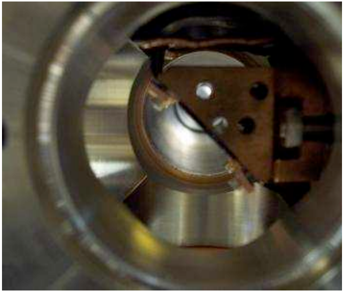
Outline

Realisation of very anisotropic traps on an atom chip

Use of a H shape trap





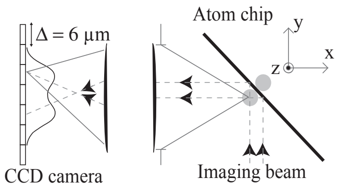


Transverse confinement :
$I_1 + I_2 = 3$A and $B_{ext} = 30$G :
$\omega_\perp = 2\pi \times 2800 Hz$
Longitudinal confinement :
$\omega_z \propto (I_1 - I_2)^2 : 6Hz \to 20Hz$

- ^{87}Rb atoms MOT transfered into the magnetic trap (3×10^6 atoms)
- Radio-frequency evaporative cooling $\to T \simeq 200nK \simeq 1.5\hbar\omega_\perp$, $N \simeq 5000$.

Absorption images

- Imaging geometry

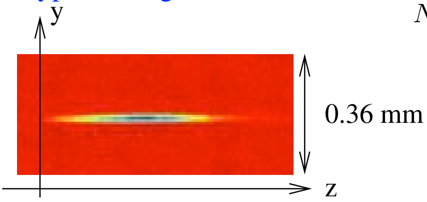


2 pictures taken :
* With atoms and trap still on
* Without atoms (delay of 200 ms)

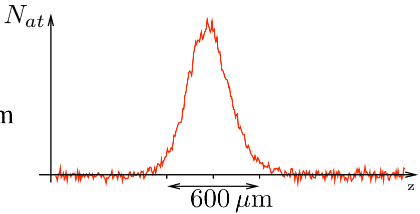
Atom per pixel : $n_{at} = \frac{\Delta^2}{\sigma} \ln(\frac{I_2}{I_1})$.

Error if variation of density on a scale smaller than pixel size and high optical density

- Typical image



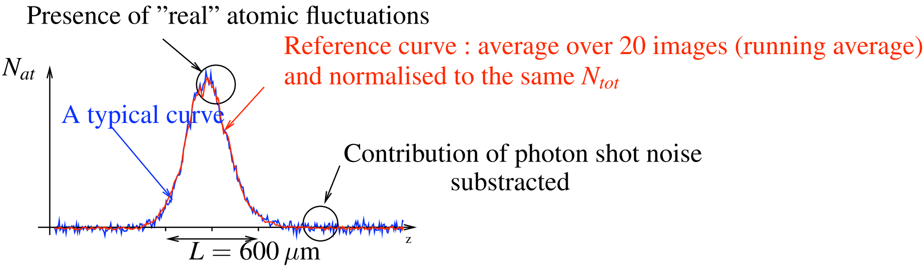
- Transverse integration : $N_{at} = \sum_y n_{at}$



Noise measurement

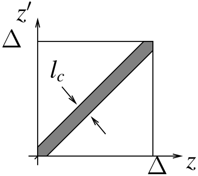
Inspired by *Föling et al. Nature* **434**, *481*, M. Greiner et al., PRL **94**, 110401

Statistical analysis over about 300 images taken in the same condition.



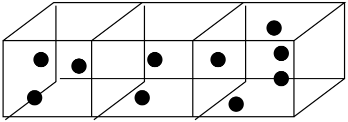
Pixel size : $l_c \ll \Delta \ll L \Rightarrow$ LDA valid

$$\begin{aligned}\langle N^2\rangle - \langle N\rangle^2 &= \int_\Delta \int_\Delta \left(\langle n(z)n(z')\rangle - \langle n\rangle^2\right) \\ &= \langle N\rangle + \langle n\rangle^2 \Delta \int dz (g_2(z) - 1)\end{aligned}$$



Atom-number fluctuations in an ideal Bose gas

- For each eigenstate : $\langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle + \langle n \rangle^2$
- Many occupied eigenstates



$$\langle N_t^2 \rangle - \langle N_t \rangle^2 = \langle N_t \rangle + \sum_i \langle n_i \rangle^2$$

For G equally occupied states ,

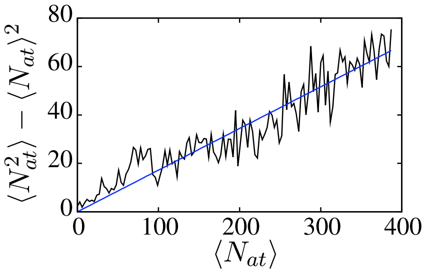
$$\langle N_t^2 \rangle - \langle N_t \rangle^2 = \langle N_t \rangle + \frac{1}{G} \langle N_t \rangle^2$$

- To observe bunching : G should not be too high
- Ratio bunching/shot noise : $\langle N_t \rangle / G = psd$

$G \simeq \left(\frac{T}{\hbar \omega_\perp} \right)^2 \frac{\hbar}{\Omega}$ for non degenerate gas. $\Omega =\Delta \sqrt{mk_B T}$

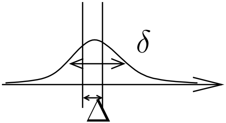
$G \downarrow$ when $\langle N \rangle \uparrow$ for a degenerate gas.

Observation of atomic shot noise



High temperature cloud

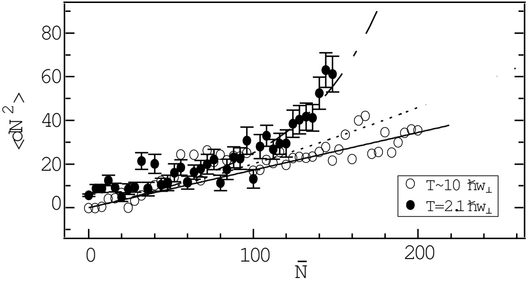
Slope smaller than 1 : due to finite optical resolution δ



Contribution of each atom on the absorption in a given pixel reduced by a factor $\propto \frac{\delta}{\Delta}$
$\Rightarrow$ slope of shot noise reduced

For Gaussian resolution of rms width δ, reduction by $\kappa = \frac{\Delta}{2\sqrt{\pi}\delta}$.
Measured reduction $\kappa = 0.17 \rightarrow \delta = 10\mu$m.
Measured resolution : 8μm

Observation of atomic bunching and evidence for quantum decoherent regime



Temperature deduced from longitudinal profile.

Dotted : non-degenerate gas approximation
Dashed-dotted : Exact formula

Expected atom number fluctuations in a one dimensional system in quasibec regime

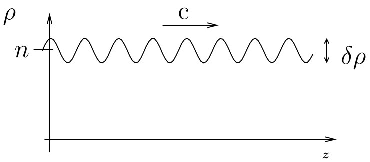
Pixel size Δ much larger than healing length $\xi = \hbar/\sqrt{mgn}$

$\Rightarrow$ Relevant excitations are phonons

Energy of a phonon of wave vector k :

$$H_k = \Delta\left(\frac{g}{2}\delta\rho_k^2 + n\frac{\hbar^2 k^2}{2m}\theta_k^2\right)$$

$$\psi = \sqrt{n+\delta\rho}\, e^{i\theta}$$



Thermodynamic equilibrium :

$$\frac{k_B T}{2} = \Delta\frac{g}{2}\langle\delta\rho_k^2\rangle \qquad (k_B T \gg gn)$$

Atom number fluctuations :

$$\mathrm{Var}N = \int_\Delta\int_\Delta\langle\delta\rho(z)\delta\rho(z')\rangle = \int\int\sum_k\langle\delta\rho_k^2\rangle e^{ik(z-z')} = \Delta^2\langle\delta\rho_0^2\rangle$$

$$\Rightarrow \boxed{\mathrm{Var}N = \Delta\frac{k_B T}{g}}$$

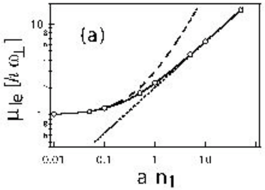
Expected atom number fluctuations in a nearly one dimensional system in quasibec regime

- $n \ll 1/a$: Purely one dimensional case recovered. ($g = 2\hbar\omega_\perp a$)
- n of the order or larger than $1/a$: Transverse breathing associated with a longitudinal phonon has to be taken into account.

Thermodynamic argument :

$$\mathrm{Var}(N) = k_B T \left.\frac{\partial N}{\partial \mu}\right)_T$$

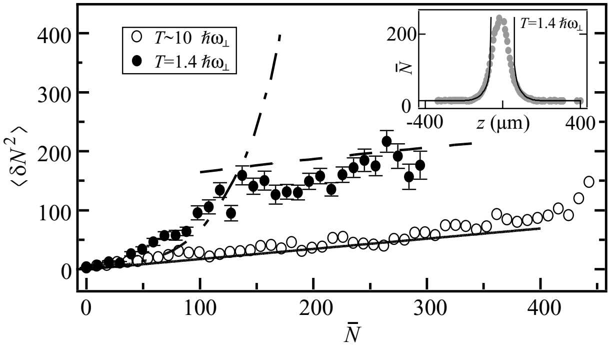
Analytic expression :
$\mu \simeq \hbar\omega_\perp\sqrt{1+4na}$



$$\Rightarrow \mathrm{Var}(N) = k_B T \Delta \frac{\sqrt{1+4na}}{2\hbar\omega_\perp a}$$

In good agreement with a 3D Bogoliubov calculation of $\mathrm{Var}(N)$.

Experimental results in the quasibec regime

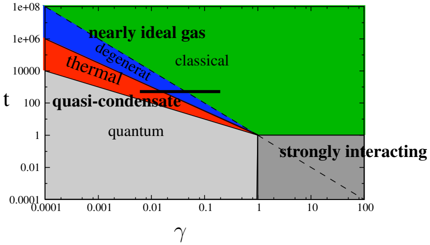


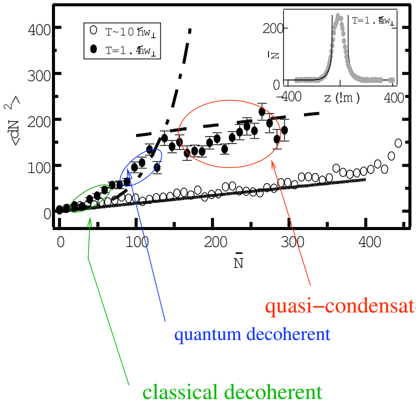
- Temperature fitted from the wings of the profile
- Good agreement with theory for low temperature

Estève et al. PRL **96**, *130403*

Conclusion on density fluctuations measurement

Most features of weakly interacting 1D Bose gases observed



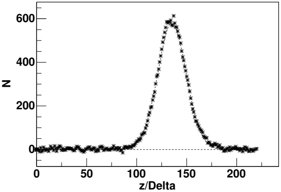


Density profile through the transition towards quasibec

Nature of the transition towards quasi-bec : driven by interactions

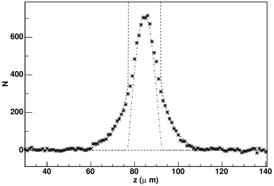
In situ density profile by absorption imaging

$k_BT = 5.7\hbar\omega_\perp$
$\mu = -3.6\hbar\omega_\perp$



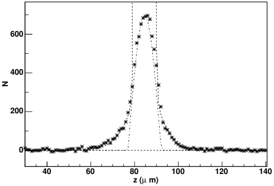
Good agreement with ideal Bose gas

$k_BT = 2.75\hbar\omega_\perp$
$\mu = 1.6\hbar\omega_\perp$



Appearance of quasi-bec at the center

$k_BT = 2.3\hbar\omega_\perp$
$\mu = 1.4\hbar\omega_\perp$



Good agreement with quasibec shape

Wings : ideal Bose gas equation of state
Quasi-bec shape : $\mu_{loc} = \mu - m\omega^2 z^2/2 = \hbar\omega_\perp\sqrt{1 + 4na}$

Success of Hartree-Fock theory in 3D Bose gases

3D ideal Bose gases : BEC for $\rho_c = 2.612.../\lambda_{dB}^3$

For weak interactions ($\rho a^3 \ll 1$), Mean-field theories accurate.

•For $\rho < \rho_c$: Hartree-Fock theory

variational method : non interacting Bosons that experienced V_{eff}.

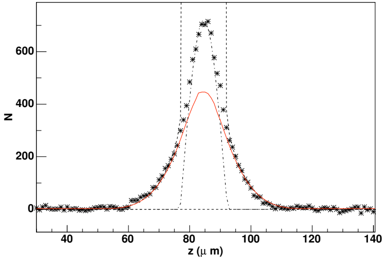
$$\Rightarrow \boxed{V_{eff}(r) = 2g\rho(r)}, \qquad g = 4\pi\hbar^2 a/m$$

- ρ_c unchanged
- for a gas trapped in harmonic potential, small shift of N_c (*Gerbier et al., Phys. Rev. Lett. 92, 030405 (2004)*)

• Beyond mean-field

- Validity of Mean-Field (Landau-Ginzburg criteria) : $|T - T_c|/T_c > a\rho^{1/3}$.
- Beyond mean-field effects :
  - small shift of T_c,
  - change of critical exponent (*Donner et al., Sience 315 :1556 (2007)*)

Failure of Hartree-Fock theory : a quasibec without condensation



Hartree-Fock calculation

*Population in the ground state $N_0/N_{tot} \simeq 3 \times 10^{-3} \ll 1$.

The appearance of quasi-bec is not explained by Hartree-Fock theory.

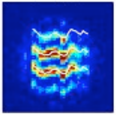
First failure of mean field theory in a weakly interacting regime.

J.-B. Trebbia et al. PRL **97**, *250403 (2006)*

Outline

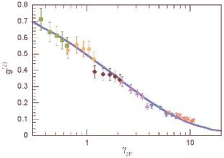
Other experimental results

- Phase fluctuations measurement in weakly interacting gases.
  $\langle \psi^+(z)\psi(0)\rangle = ne^{-mTz/2n\hbar^2}$
  Dettmer et al. PRL **87**, 160406 (2001), Richard et al. PRL **91**, 010405 (2003)
- Quantum phase fluctuations in weakly interacting 1D gas



S. Hofferberth et eal., Nature Physics **4**, 489 (2008)

- Strongly interacting 1D gases : fermionization



Kinoshita et al. PRL **95**, 190406 (2005)

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Conclusion

Prospects

- Study of 1D gases in the strong interaction regime.
  expected : $\omega_\perp = 40$ kHz, $n = 1$ at/μm
- Better imaging system $\Rightarrow$ Higher order of the distribution $\rightarrow g_3$
- Reaching quantum fluctuations
- Study of correlation length of density fluctuations in 1D gases.
  - Correlation between pixels
  - Tomography method
- Study of density fluctuations in 2D gases.
  Use of rf dressed potentials

Collaborators

Members of the chip experiment

- Chris Westbrook
- Jérôme Estève
- Thorsten Schumm
- Jean-Baptiste Trebbia
- Carlos Garrido-Alzar
- Julien Armijo

Theoreticians collaborators

- Karen Kheruntsyan
- Gora Shlyapnikov

Micro-fabrication

- LPN laboratory
- Dominique Mailly