The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
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International Atomic Energy Agency

2030-26

Conference on Research Frontiers in Ultra-Cold Atoms

4 - 8 May 2009

The frontier of the few

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The frontier of the 'few'

Rich & unexpected behaviors in

Few-vortex dynamics

Few-particle dynamics

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Dresden, Germany

mpipks

THE FRONTIER OF THE 'FEW'

Few-vortex dynamics:

Two vortices in a 2D trap (Bose condensate)

Few-particle dynamics:

Few-boson localization at lattice edge (Bose-Hubbard model)

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The frontier of the 'few'

Few-component systems are severely under-appreciated.

$\longrightarrow$ A new frontier opened up by ultracold atom physics.

Masud Haque

Max-Planck Institute for Physics of Complex Systems (MPI-PKS)

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Nice people thank their collaborators

Vortex dipole dynamics

Weibin Li

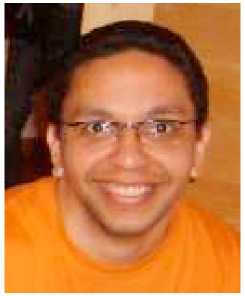
Dresden

Stavros Komineas

Dresden
↓
Crete

Phys. Rev. A **77**, 053610 (2008)

Edge localization



Ricardo Pinto

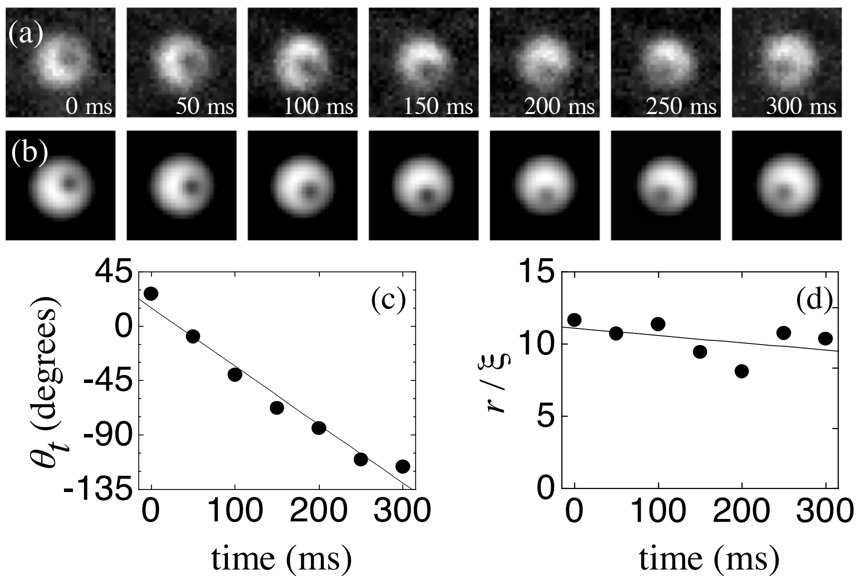
Dresden
↓
Riverside

Sergej Flach

Dresden

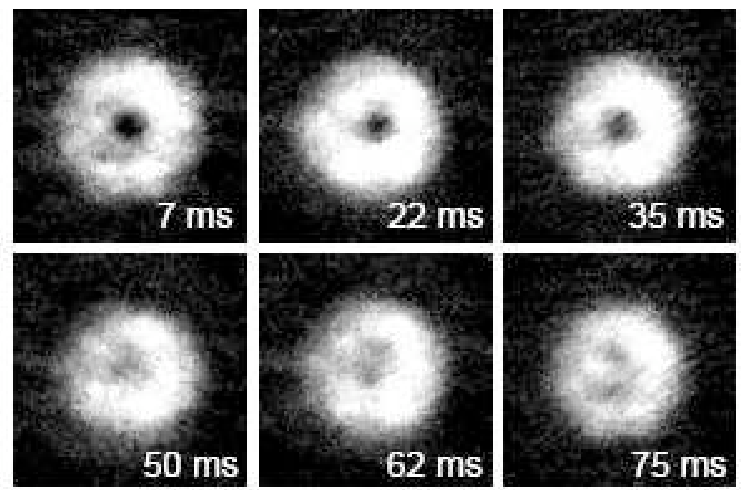
arXiv:0902.3249 (to appear, P.R.A)

The 'few' frontier: Few-vortex experiments

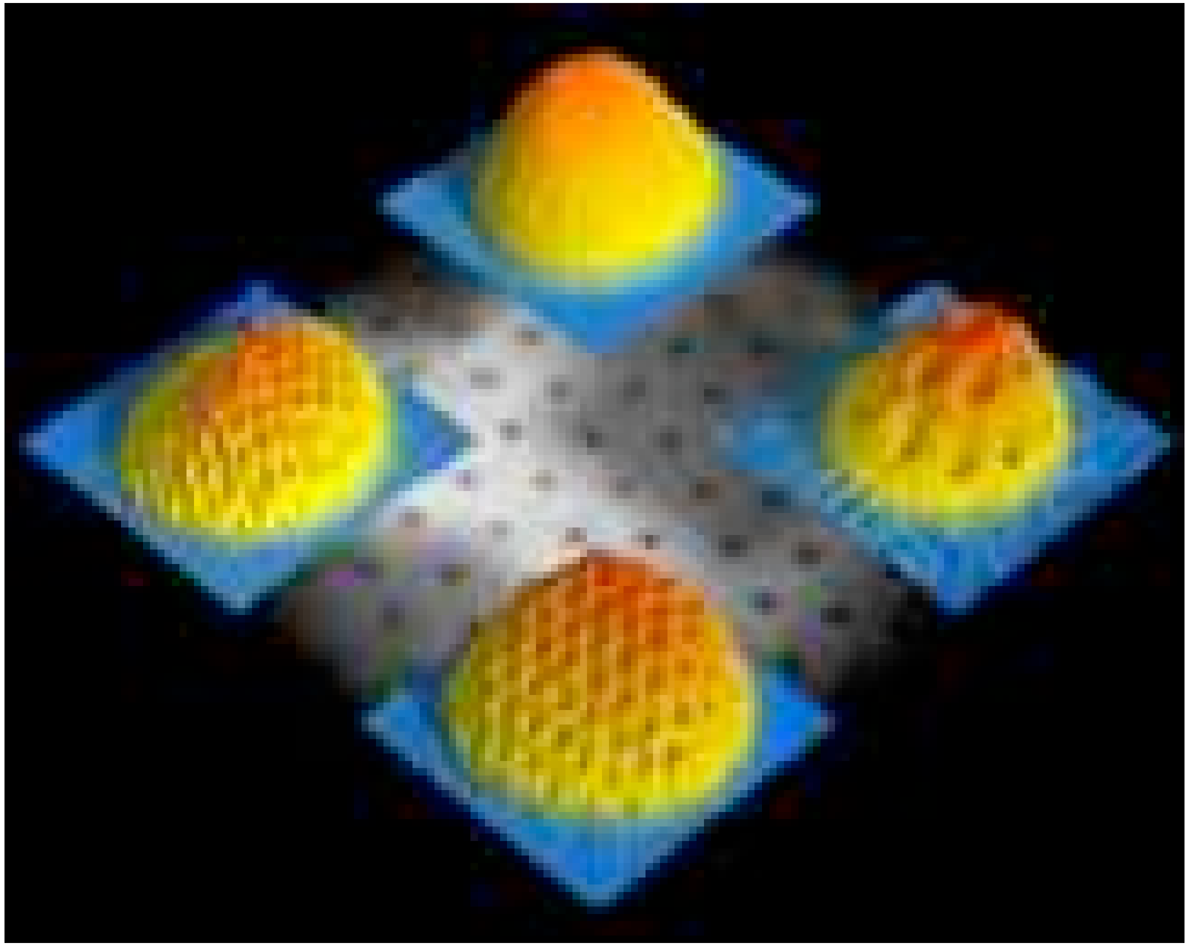


JILA, $\sim$ 2001

Prediction: Rokhsar 1997

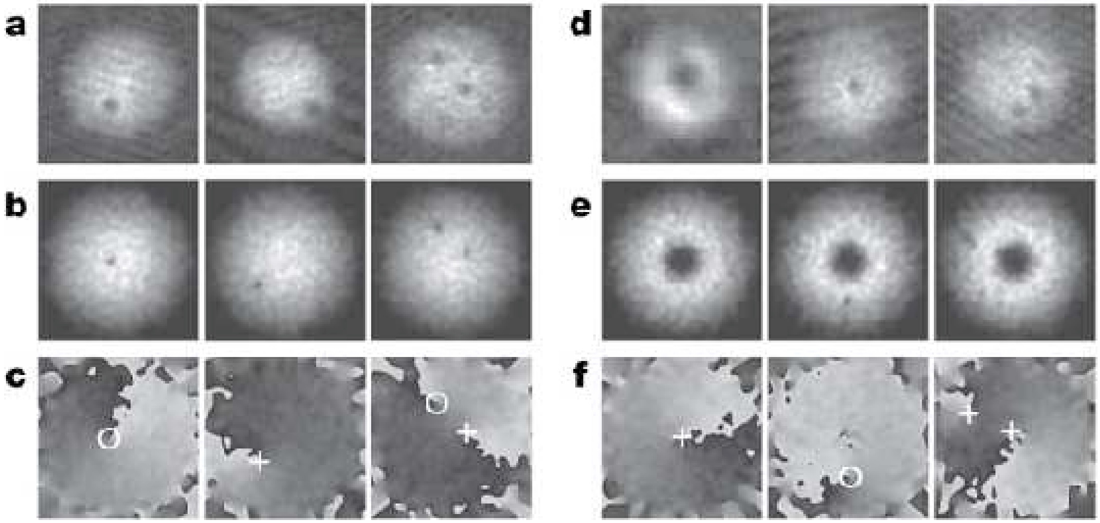


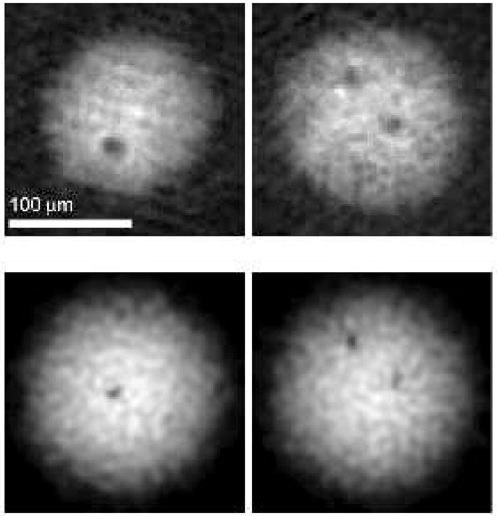
MIT, separation of double-quantized vortex

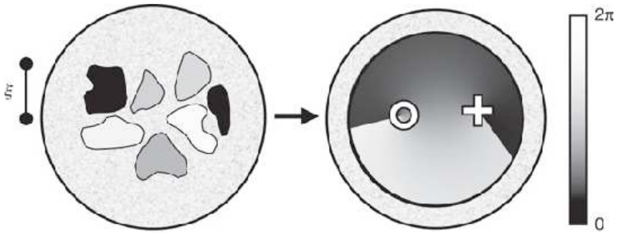


The 'few' frontier: Few-vortex experiments

Arizona, Brian Anderson's group

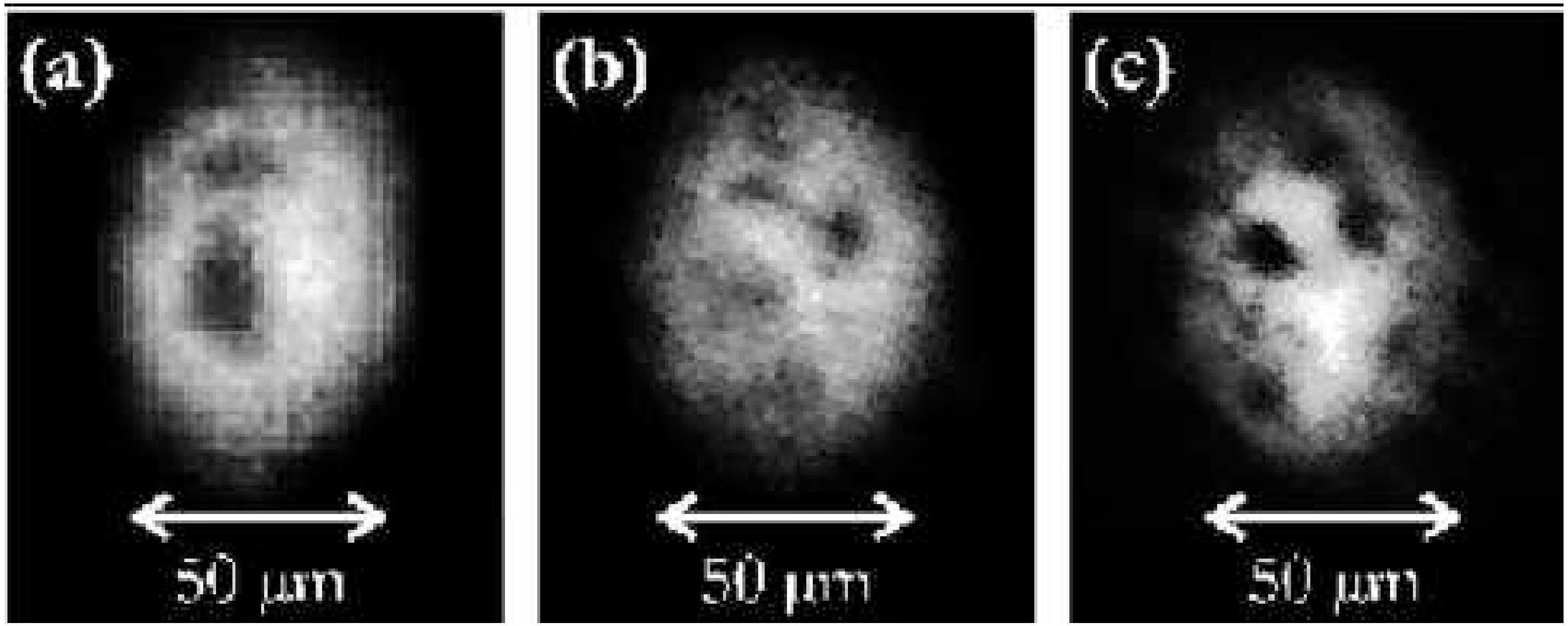




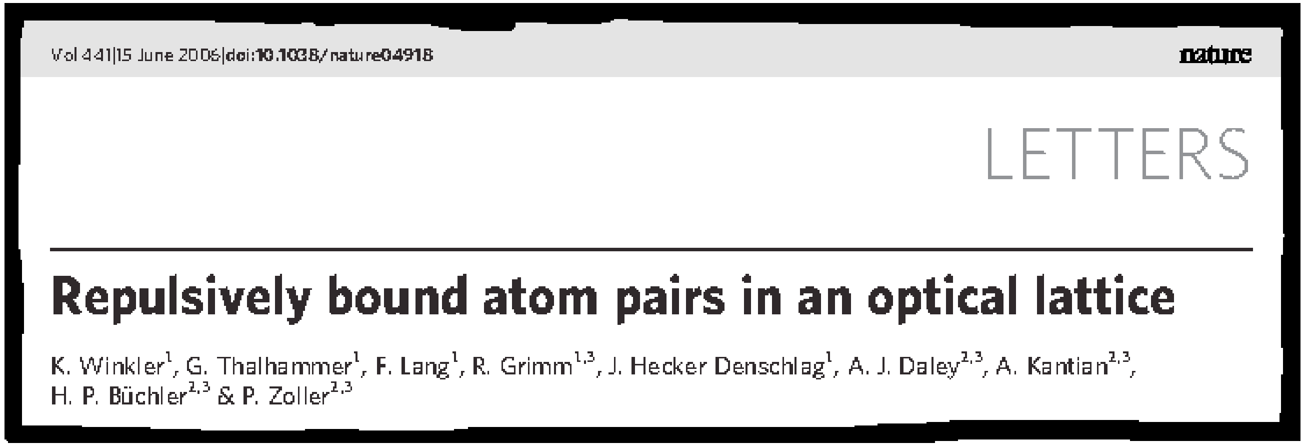


The 'few' frontier: Few-vortex experiments

Sao Paolo, V. Bagnato's group



The 'few' frontier: Few-particle experiments

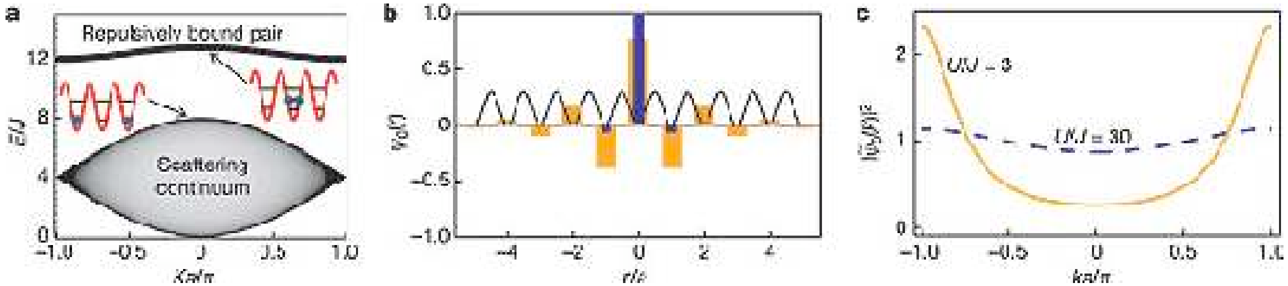

Vol 441|15 June 2006|doi:10.1038/nature04918

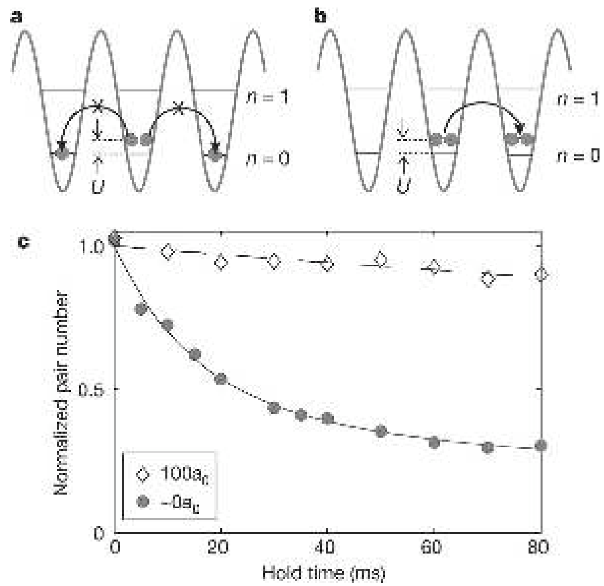
nature

LETTERS

Repulsively bound atom pairs in an optical lattice

K. Winkler[1], G. Thalhammer[1], F. Lang[1], R. Grimm[1,3], J. Hecker Denschlag[1], A. J. Daley[2,3], A. Kantian[2,3], H. P. Büchler[2,3] & P. Zoller[2,3]





Frontier of the 'few'

Experiment

Difficult to access in traditional condensed matter experiments

Theory

Nontrivial phenomena, surprisingly unexplored.
Traditional condensed-matter view: 'merely' finite-size effects.

For few-component systems, natural to study far-from-equilibium situations, or states far from the ground state.

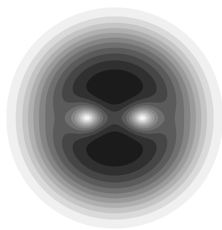
VORTEX DIPOLE OR VORTEX PAIR IN A TRAPPED 2D BOSE-EINSTEIN CONDENSATE

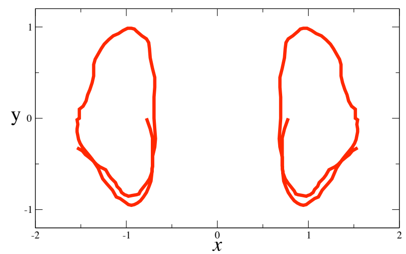
VORTEX DIPOLE

Stationary states.

Conservative dynamics – defect trajectories.

Many open questions





Phys. Rev. A **77**, 053610 (2008)

VORTEX PAIR

Precession with mutual orbiting

VORTEX TRIPLET

....

WHY VORTEX DIPOLES?

- VD's appear in many 2D situations.
  Normal fluids & superfluids; turbulent flow; flow over a sharp barrier, ...
- Vital for physics of Kosterlitz-Thouless transitions in 2D.
  Probed experimentally, ENS
- Fascinating analogies to vortex rings in 3D, solitons in 1D.
  Some of our results have analogs in 3D vortex ring physics.
- Created recently experimentally in BEC's by sudden perturbation.
- Can in principle be created & studied by phase imprinting.

TIME-DEPENDENT GPE

$$i\frac{\partial\psi(t)}{\partial t} = -\tfrac{1}{2}\nabla^2\psi + V_{\text{tr}}(\mathbf{r})\psi + g|\psi|^2\psi$$

- Trap units.
- Isotropic (CIRCULAR) 2D trap →

$$V_{\text{tr}}(x,y) = \tfrac{1}{2}(x^2+y^2)$$

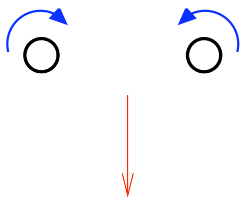
- g is an EFFECTIVE 2D interaction parameter.

$$g \propto g_{3\text{D}} \times N \times \sqrt{\omega_z}$$

- Conservative dynamics only.
  No dissipation.
  No temperature.
  No quantum depletion or fluctuations

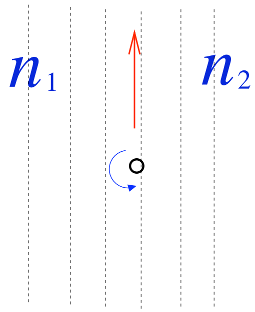
Two competing effects

Vortex dipole in uniform condensate is self-propelled.

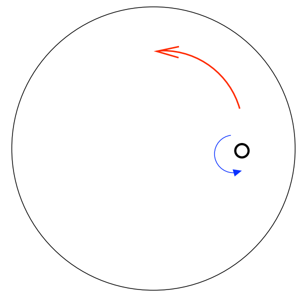


Fetter, Phys. Rev. 1965

Single vortex in non-uniform condensate is driven by inhomogeneity.



inhomogeneous BEC



precession in trap

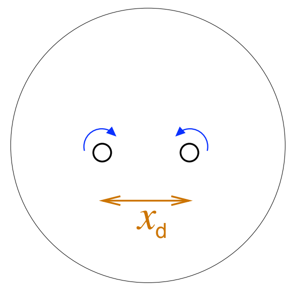
Rokhsar, PRL 1997

Vortex dipole in trap $\longrightarrow$

Small distance: mutually driven motion dominates.

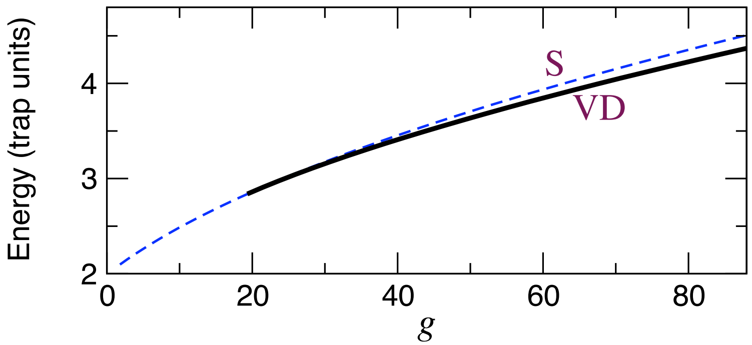
Large distance: inhomogeneity-driven motion dominates.

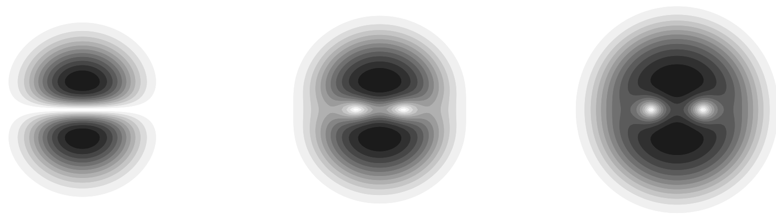
Balance $\rightarrow$ stationary solution.



Stationary solutions

- Stationary 'soliton'-like solution at small g. Bifurcation at $g \approx 18$.
- For $g \gtrsim 18$, one 'soliton' and one vortex-dipole branch.

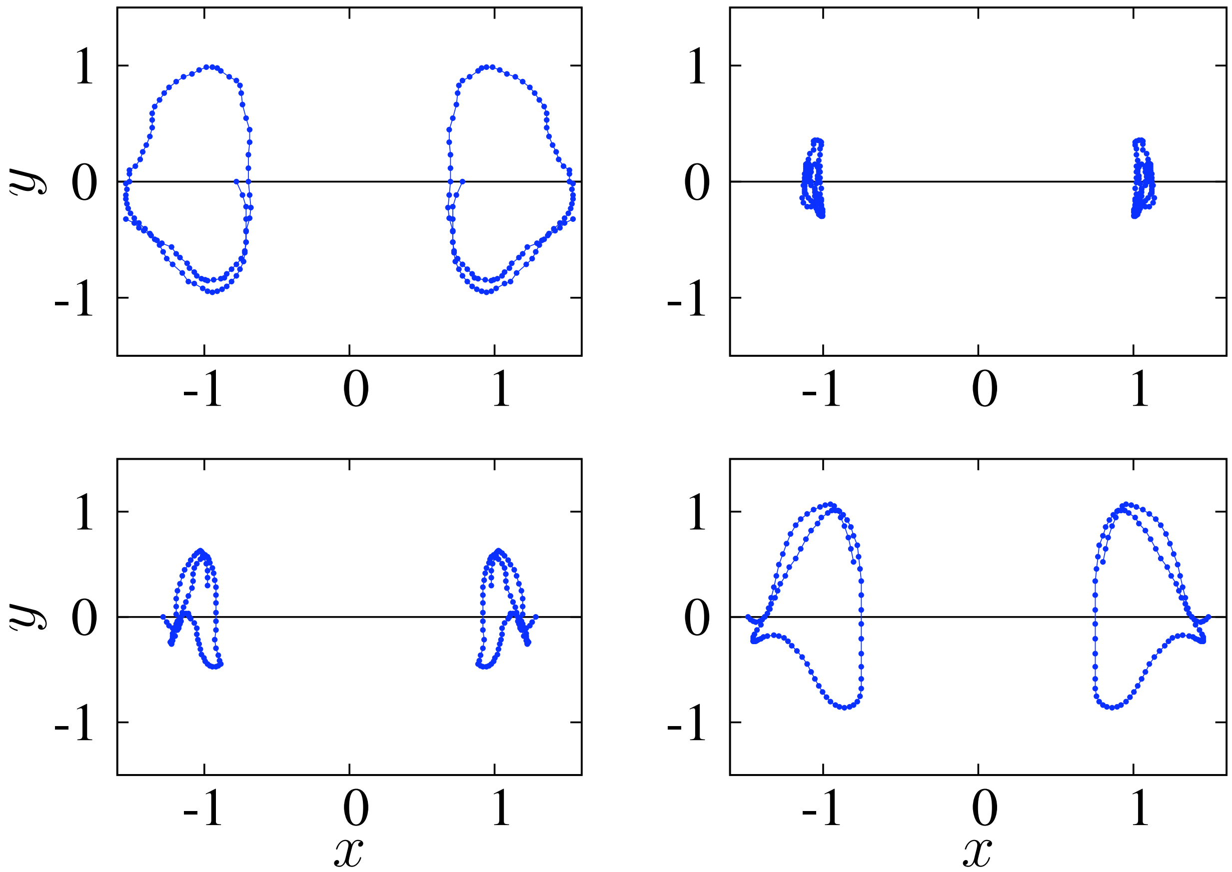




$g = 11$ $g = 25$ $g = 60$

Similar bifurcations: in 3D (vortex RING instead of vortex dipole); in elongated trap.

DYNAMICS (VORTEX TRAJECTORIES); LARGE g



- Simpler at large g.
- $g = 150$ shown here.
- Not periodic ('almost').
- Trajectories elongated in y direction.
  'Reflection' at edges clearer.
- Extra features.
  Curvatures at outer parts.
  Pointy feature at outer edge.
  Direction reversal for large initial $x_{\rm d}$).

Variational formulation

Lagrangian : $$L = \int dr \left[\frac{i}{2} \left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) + \tfrac{1}{2} \psi^* \nabla^2 \psi - V_{\mathrm{tr}}(\mathbf{r}) \, |\psi|^2 - \tfrac{1}{2} g \, |\psi|^4 \right]$$

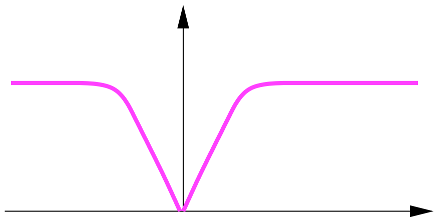
Trial w.f. : $$\psi = A(t) \, g_{\mathrm{v}}(u_1) e^{i\phi_1} \, g_{\mathrm{v}}(u_2) e^{-i\phi_2} \, f_{\mathrm{c}}(|z|^2) \qquad \begin{cases} u_i = |z - z_i|/\xi \\ \phi_i = \tan^{-1}(\frac{y-y_i}{x-x_i}) \end{cases}$$

2D coordinates bundled into complex $z = x + iy$. Vortex at z_1, antivortex at z_2.

Euler-Lagrange equations for $z_i = x_i + iy_i$: $$\frac{\partial L}{\partial x_1} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_1} \right)$$

$\longrightarrow$ equations of motion for x_1, y_1, x_2, y_2.

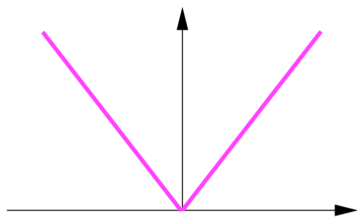
Vortex shape function $g_{\mathrm{v}}(u)$: ideally



We used $g_{\mathrm{v}}(u) = u \longrightarrow$

$$g_{\mathrm{v}}(u_1) \, e^{i\phi_1} = |z - z_1| e^{i\phi_1} = z - z_1$$

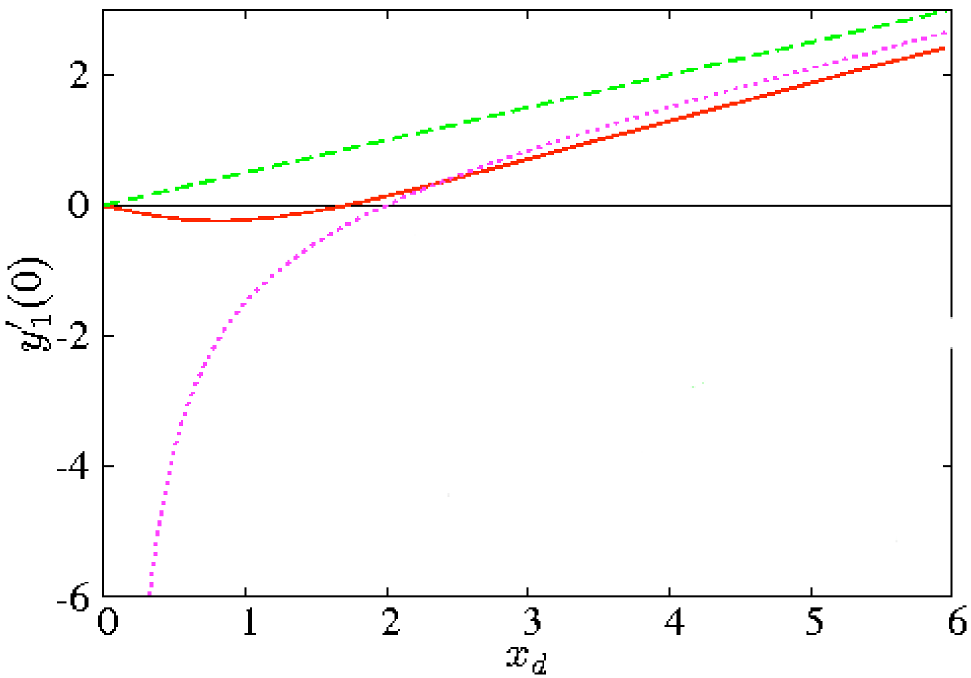
$$\psi = [z - z_1(t)] \left[z^* - z_2^*(t) \right] \, f_{\mathrm{c}}(|z|^2)$$



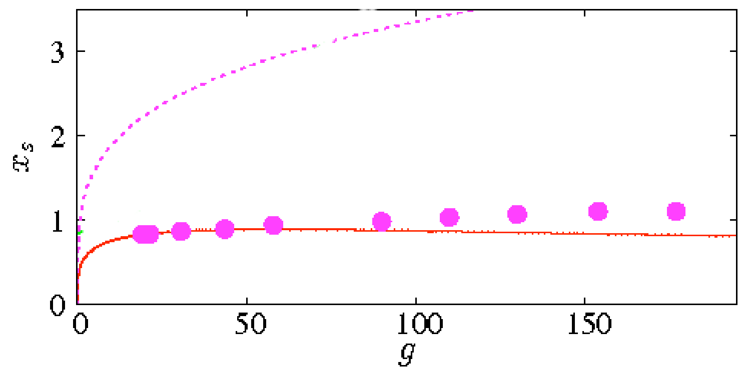
Variational Calculation (1): Stationary solution

$$\psi \; = \; [z - z_1(t)] \;\; [z^* - z_2^*(t)] \;\; f_{\mathrm{C}}(|z|^2) \qquad \begin{cases} z_1 = x_1 + iy_1 & \text{vortex} \\ z_2 = x_2 + iy_2 & \text{antivortex} \end{cases}$$

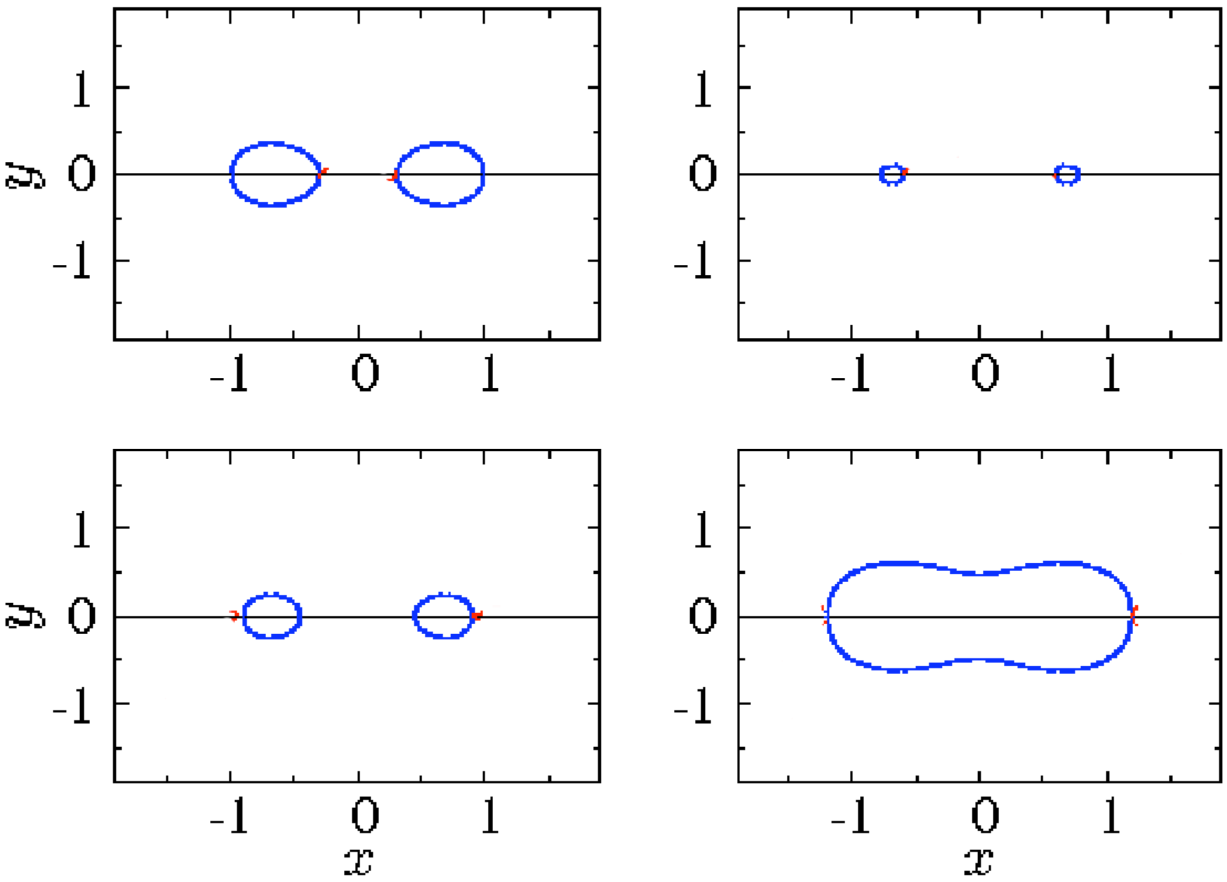
Good phase structure, unreliable (rigid) vortex size.



Stationary solution for
$x_1(0) = x_{\mathrm{s}}$, $x_{\mathrm{d}} = 2x_{\mathrm{s}}$

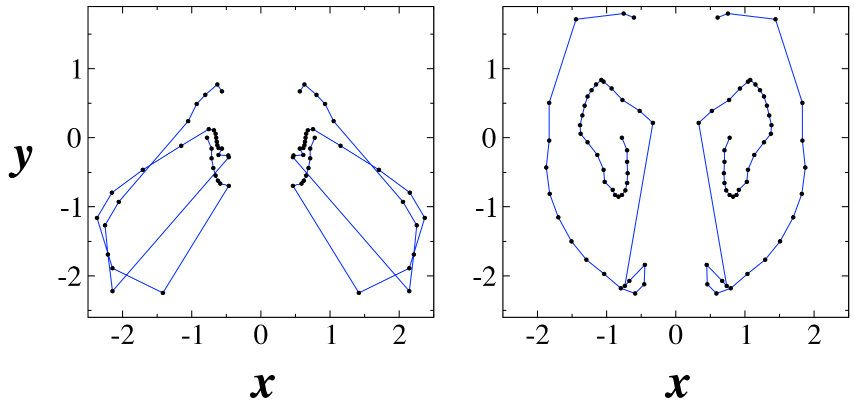


Insights from variational calculation (2) - dynamics



- Increasing initial distance $x_d = 2x_1(0)$.
- Each defect revolves around a stationary point.
- This trajectory type occurs in full GPE solutions. Not periodic, additional effects...
- Results qualitative only.
- Last trajectory is an artifact.

Trajectories; small g



$g = 10$ and $g = 50$.

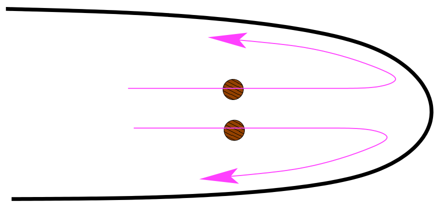
Many more unexplained features.

VORTEX POSITIONS alone are not sufficient description.
Additional dynamics possibilities!

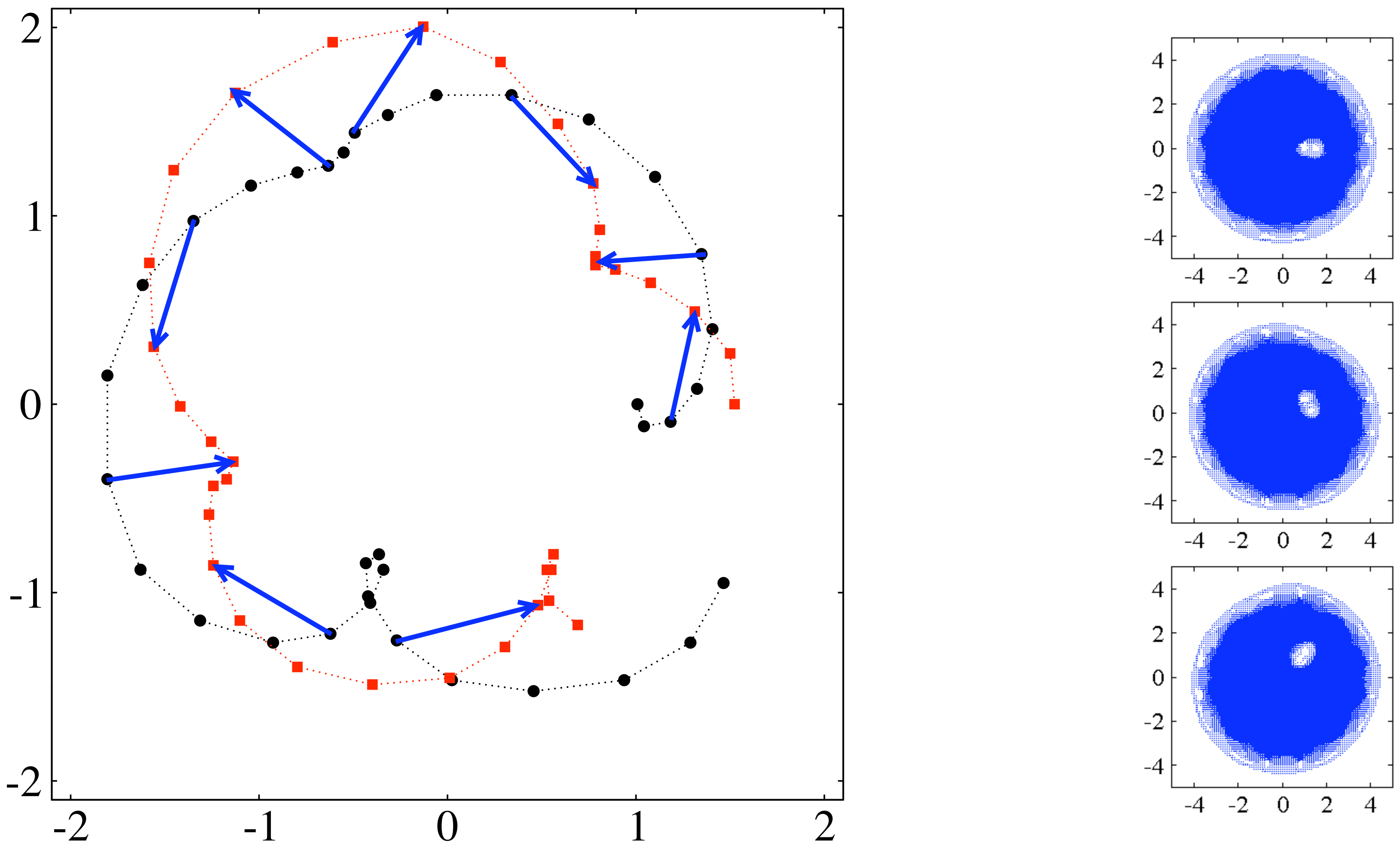
E.g.,
coupling to vortex shape dynamics;
extra defect pairs;
influence of 'nearby' soliton-like state.

Open issues

- Don't understand all features of defect trajectories.
  Even at large g!!
- Do trajectories become periodic in $g \to \infty$ limit?
  Above some critical g value?
- Do trajectories lose features in $g \to \infty$ limit?
  i.e., become smoother?
- Details of reflection, *e.g.*, in elongated condensate.



Vortex pair



EDGE LOCALIZATION

R. Pinto, M. Haque, S. Flach,

arXiv:0902.3249 P.R.A, in press

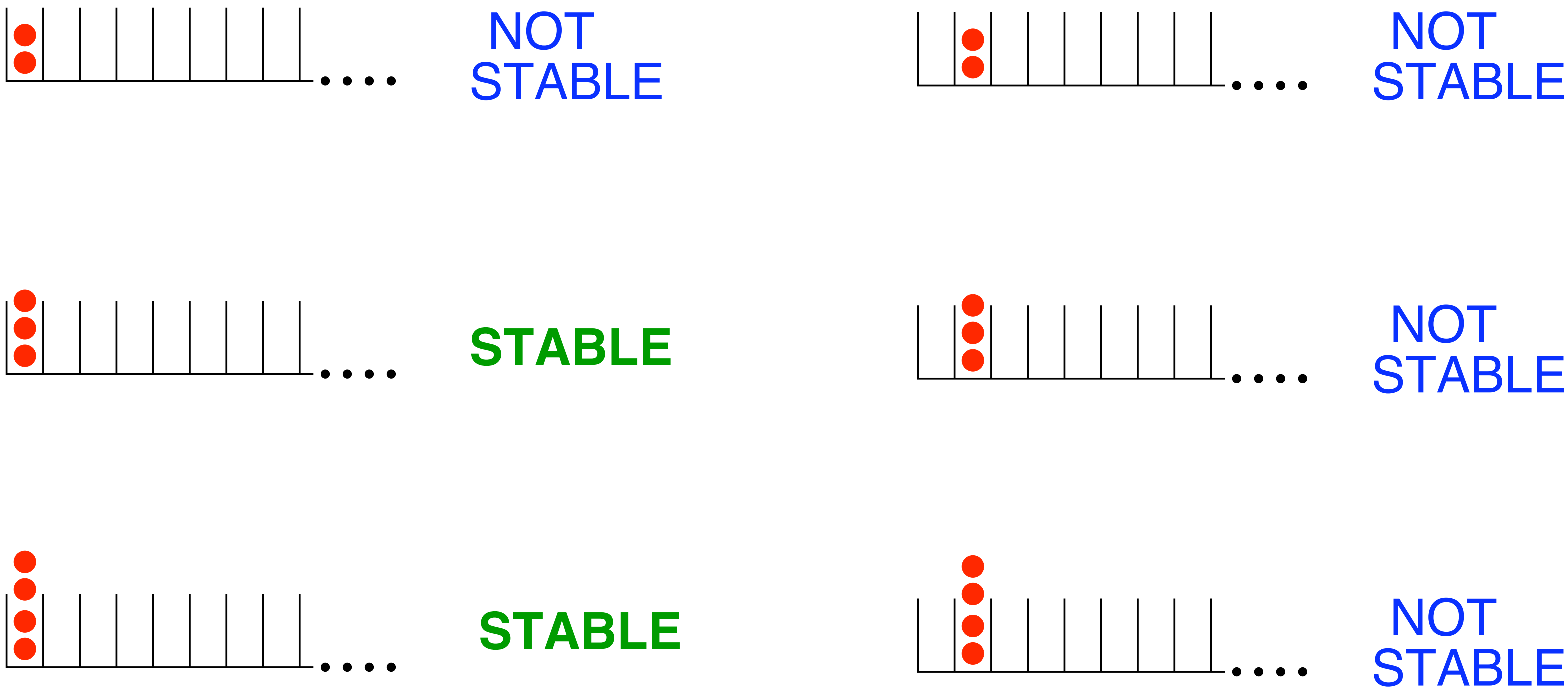
1D Bose-Hubbard model in an OPEN chain (has edges)

$$\hat{H} = - t \sum_{j=1}^{L-1} \left(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j \right) + \frac{U}{2} \sum_{j=1}^{L} a_j^\dagger a_j^\dagger a_j a_j$$

Consider $n = 2, 3, 4$ bosons.

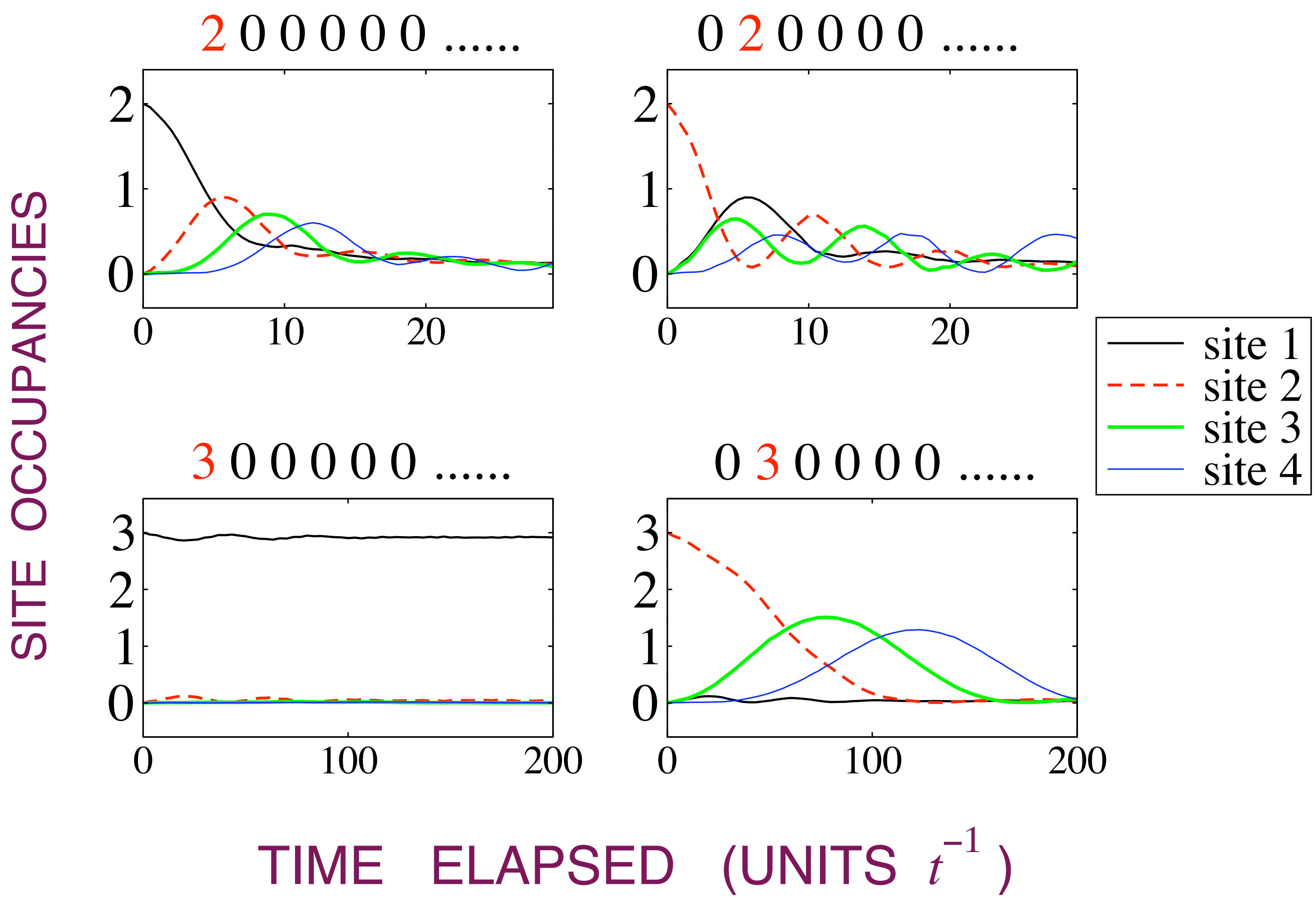
For $n \geq 3$ bosons, edge states are stable.

Edge-localized configurations

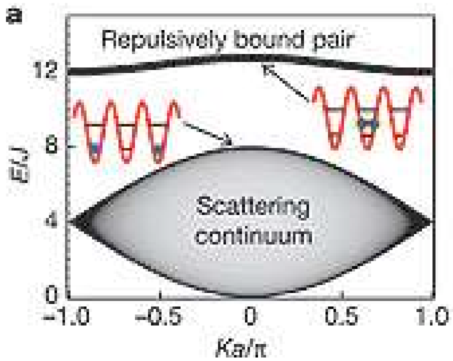


'Stable' means almost an eigenstate at large U/t.

TIME EVOLUTION



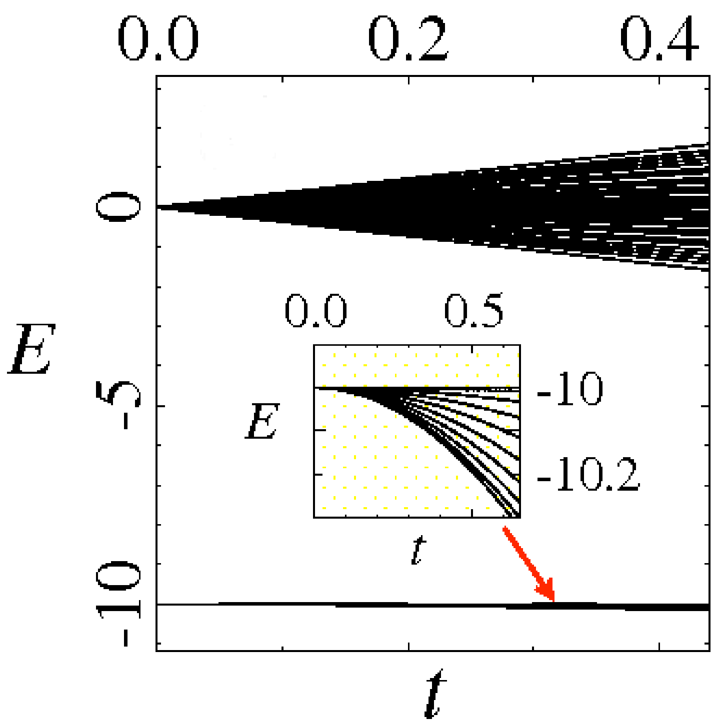
EDGE-LOCALIZATION: SPECTRAL PICTURE (2, 3 BOSONS)

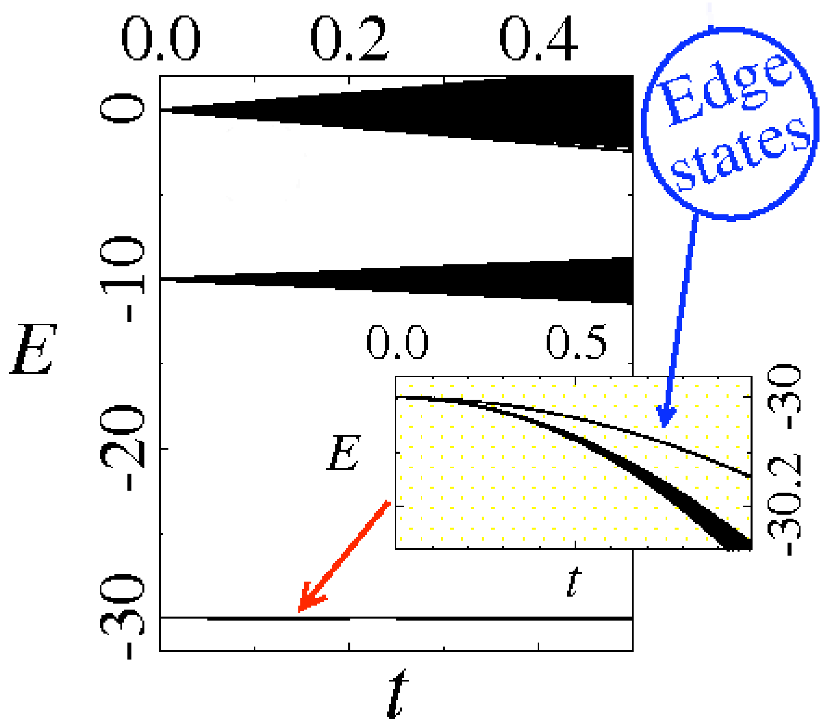


Bosons in 10-site chain.

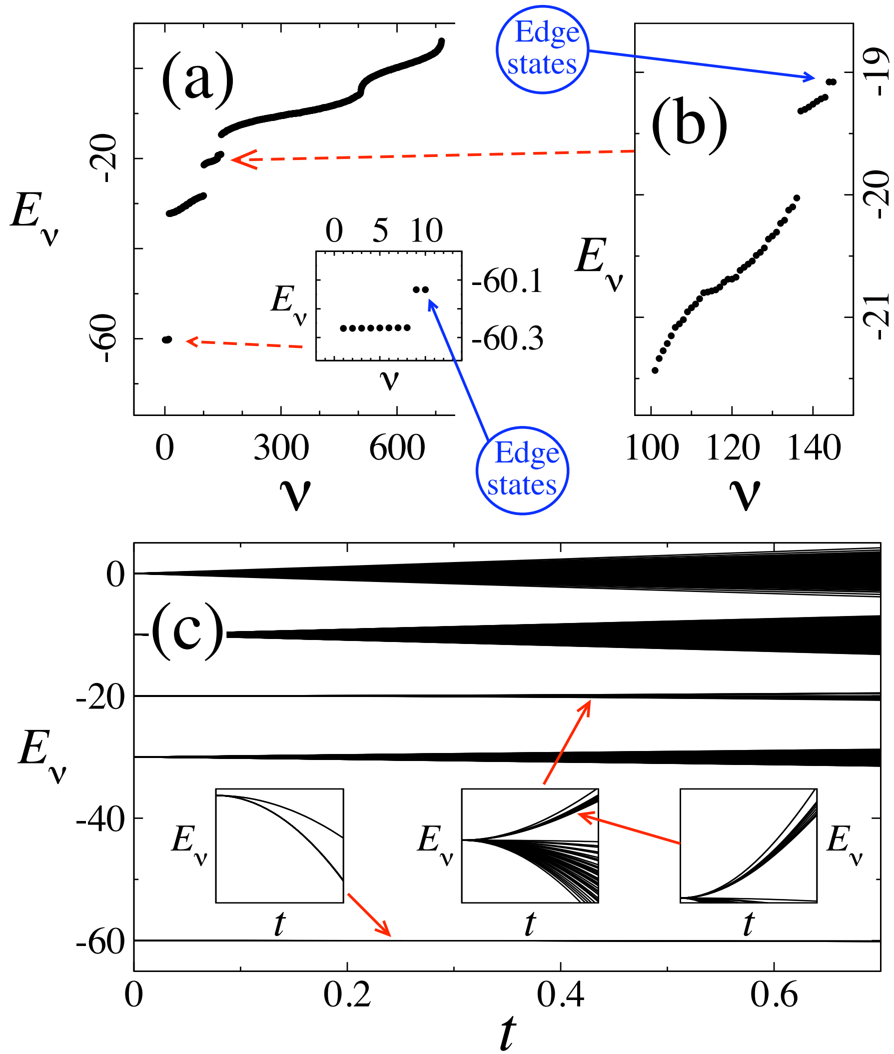
Negative-U spectra: $U = -10$

Left: 2 bosons. Right: 3 bosons.





Edge-localization: spectral picture (4 bosons)



EDGE-LOCALIZATION: PHYSICS

- Spectral separation.

- Degenerate perturbation theory:

  competition between energy shifts at $\mathcal{O}(t^2)$ and manifold mixing at $\mathcal{O}(t^n)$.

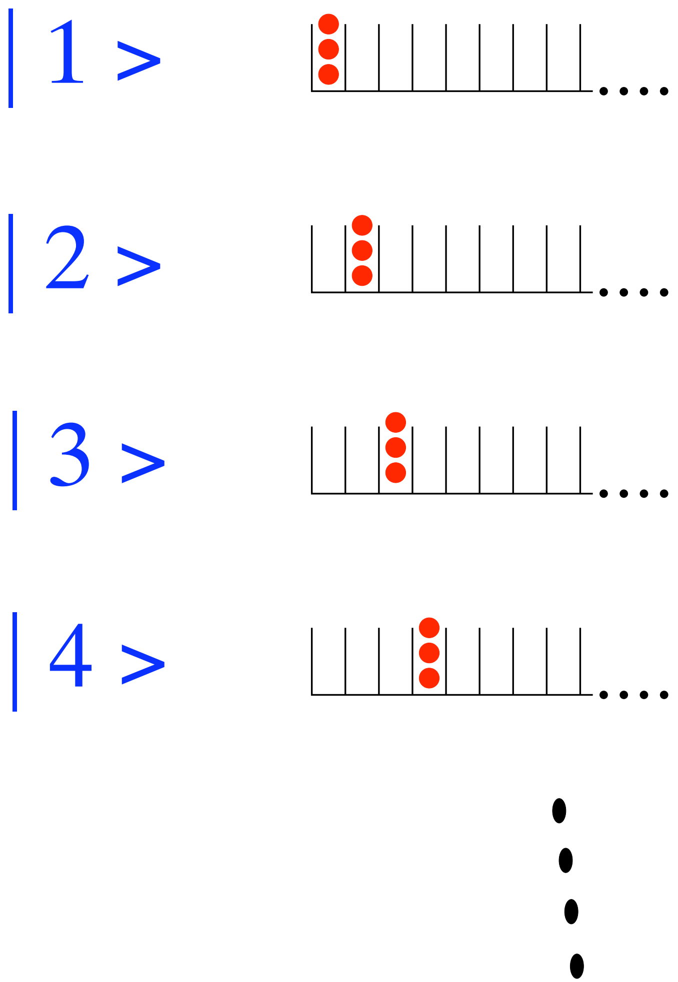
  arXiv:0902.3249

- 'Collective' phenomenon, even with $n = 3$ bosons.

- Experimentally observable?

  Don't know yet. (realizing edge, trap effects...)

Degenerate perturbation theory

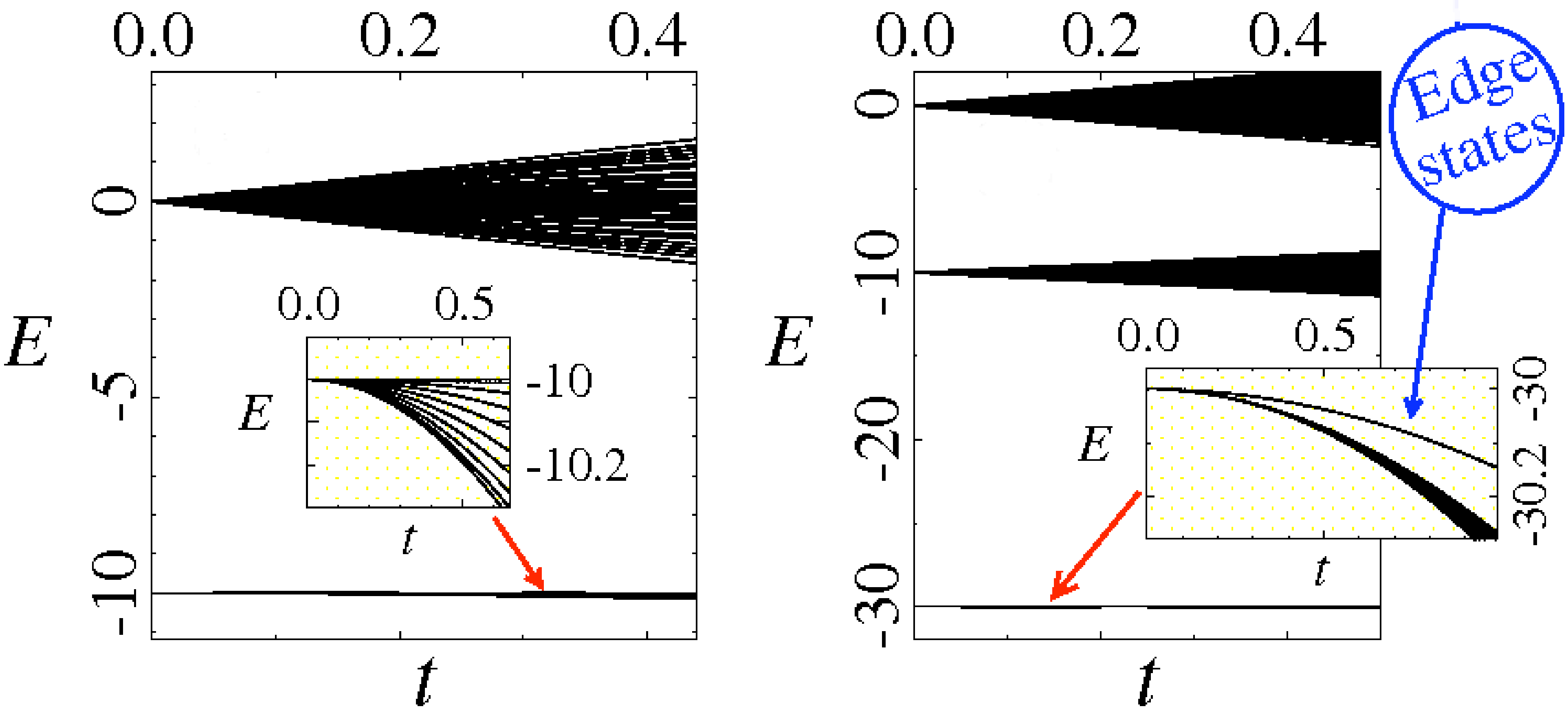


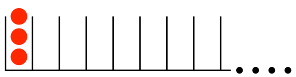
Degenerate manifold at $t/U = 0$.

States $|j\rangle$ and $|j+1\rangle$ connect at $\mathcal{O}(t^n)$.

State $|1\rangle$ acquires different shift at $\mathcal{O}(t^2)$.

State $|2\rangle$ acquires different shift at $\mathcal{O}(t^4)$.



$| L >$ = 

$| R >$ = 

Edge eigenstates: $| L > + | R >$ and $| L > + | R >$

Edge localization: more bosons

For more bosons, a hierarchy of localization patterns.

$n \geq 5$ bosons $\longrightarrow$ can also be bound in site 2

$n \geq 7$ bosons $\longrightarrow$ can also be bound in site 3

... ...etc

Actually, several hierarchies, with other localization patterns:

2 2 0 0 0 0 ......

EDGE LOCALIZATION: OTHER MODELS

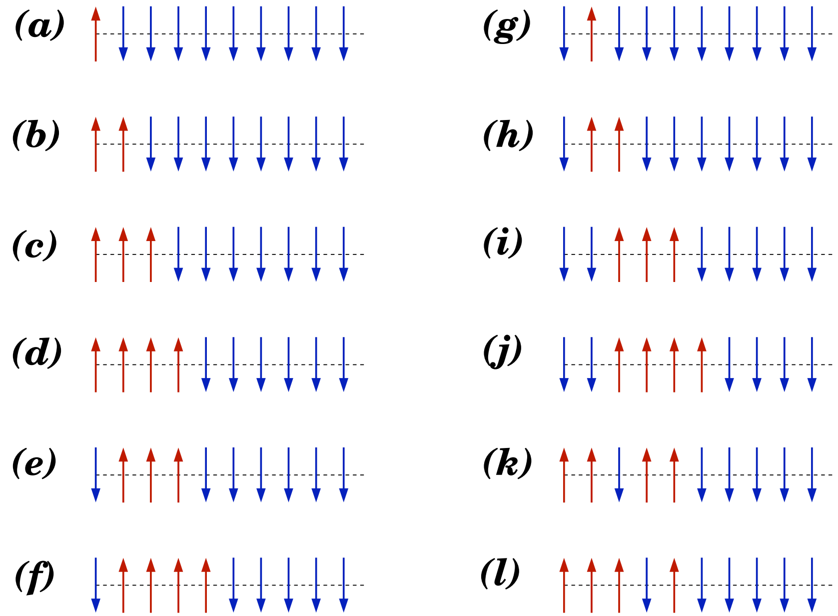
Similar localization phenomenon in spinless fermion models:

$$\widehat{H} \;=\; -\;t\sum_{j=1}^{L-1}\left(c_j^{\dagger}c_{j+1} \;+\; c_{j+1}^{\dagger}c\right) \;+\; V\sum_{j=1}^{L-1} c_j^{\dagger}c_{j+1}^{\dagger}c_{j+1}c_j$$

And therefore also in the XXZ spin chain:

$$H \;=\; J_x\sum_{j=1}^{L-1}\left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y \;+\; \Delta S_j^z S_{j+1}^z\right]$$

(M. Haque, to appear soon)



THE FRONTIER OF THE 'FEW'

Few-vortex dynamics: Two vortices in a 2D trapped condensate

Few-particle dynamics: localization at lattice edge

Few-component systems are severely under-appreciated.

$\longrightarrow$ A new frontier opened up by ultracold atom physics.

Masud Haque

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