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Ultracold Fermi gases in quasi low dimensions

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Ultracold Fermi Gases in Quasi Low Dimensions

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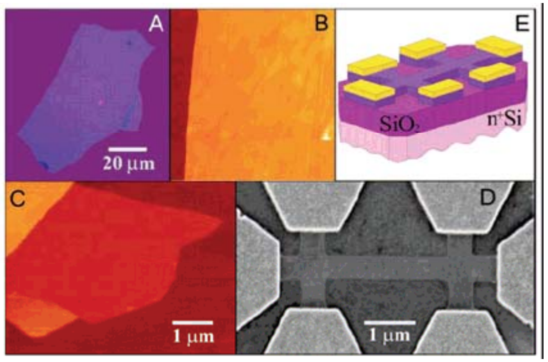
Outline

- **Introduction**
  - Ultracold Fermi gases in quasi low dimensions
  - Feshbach resonance
  - BCS-BEC crossover in 3D
- **Fermions in quasi-low dimensions: two-body problem**
  - Binding energy
  - DOF in strongly confined transverse directions
- **Fermions in quasi-low dimensions: many-body problem**
  - Effective low D Hamiltonian and dressed molecules
  - BCS-BEC crossover in Q2D: an example
  - The significance of dressed molecules
- **Summary**

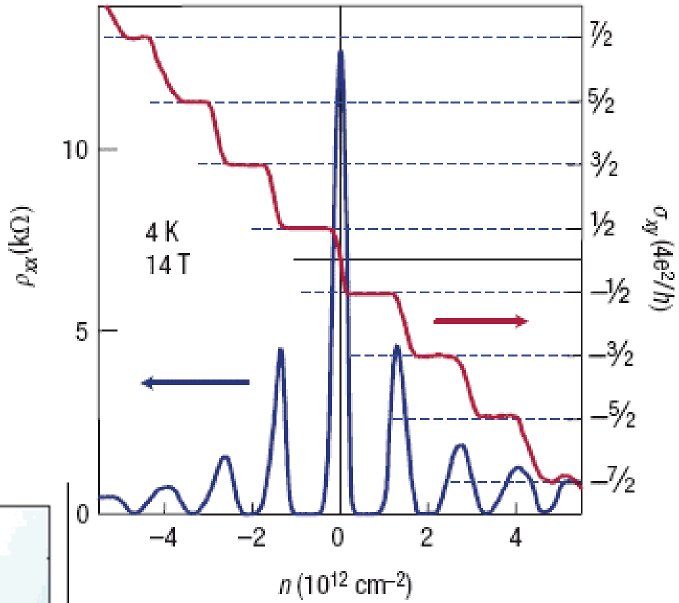
Fermi systems in low dimensions

QHE

Semiconductors

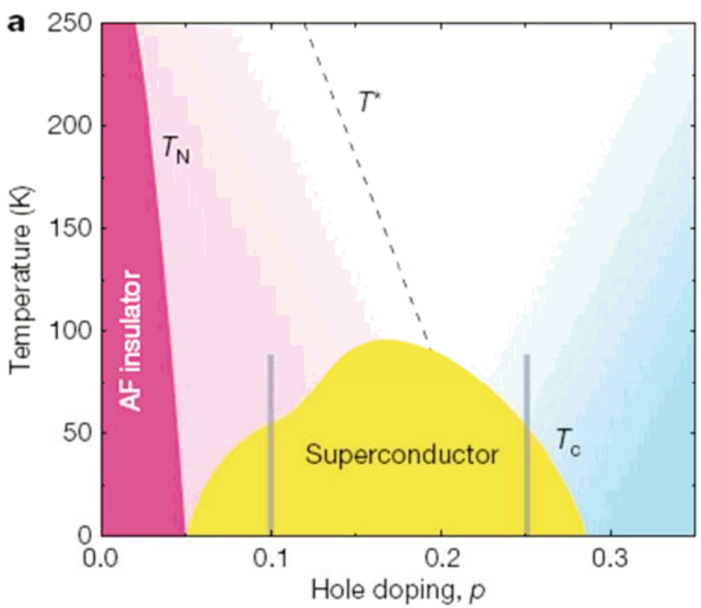


Science 306, 666 (2004)



Nat. Mater. 6, 183 (2007)

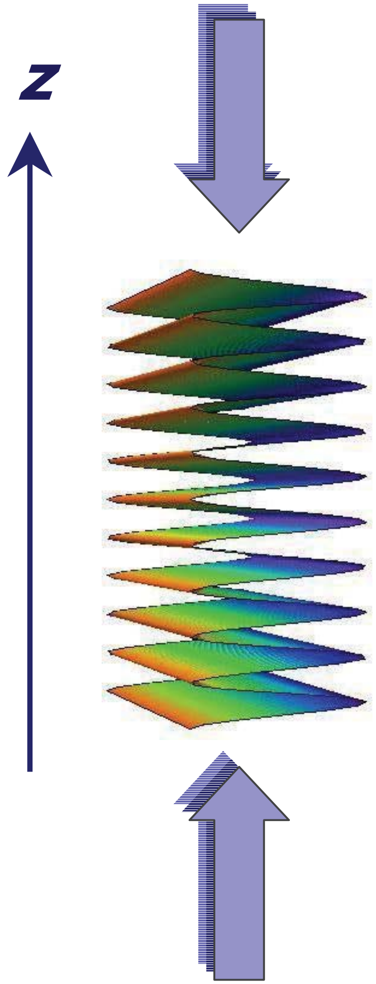
HTC



Nature. 447, 565 (2007)

Fermi gases in optical lattice

$$V_{\mathrm{OL}} = -V_0 \cos(k_z z) \times \exp[-2(x^2 + y^2)/w^2]$$



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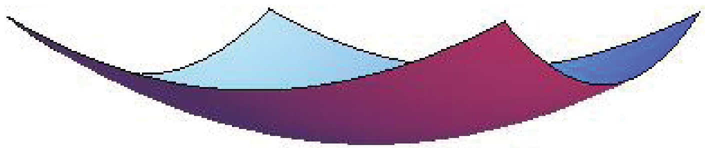

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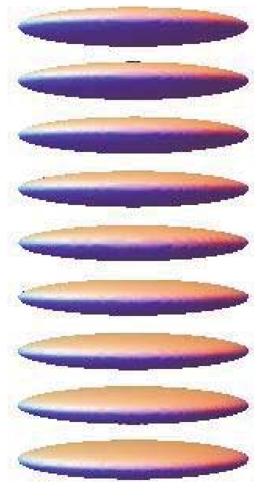
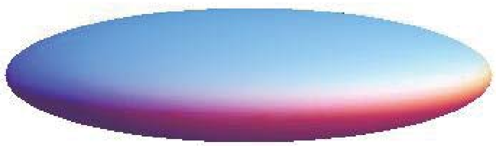


Quasi-low Dimensional Fermi system

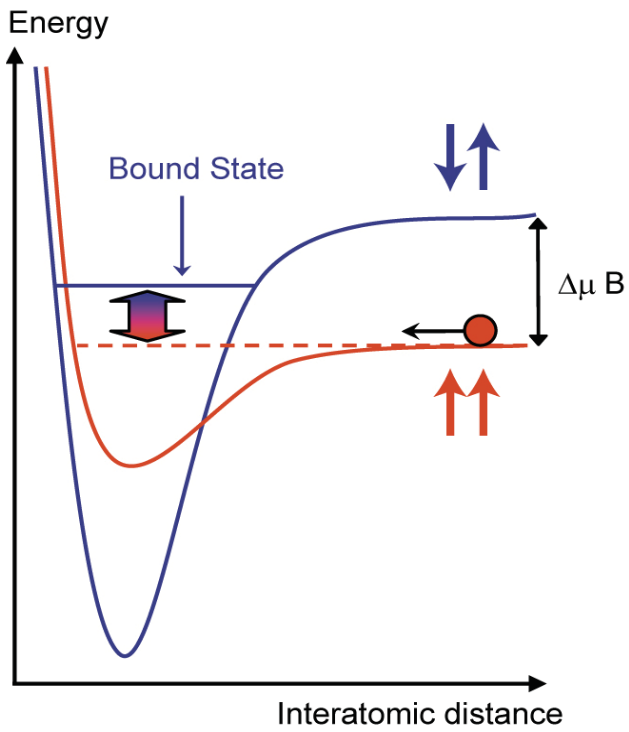


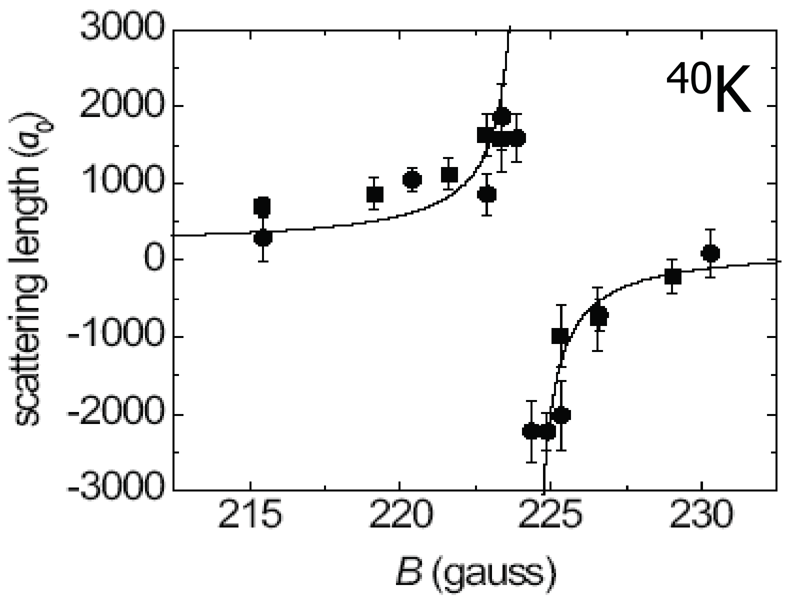
$$V_{\mathrm{OL}} \approx V_z + V_\perp = \frac{m}{2}(\omega_z^2 z^2 + \omega_\perp^2 r^2)$$

with $\omega_z >> \omega_\perp$

- **Quasi 2D Conditions:**
  - Trapping potential $\hbar\omega_z >> E_F, k_B T >> \hbar\omega_\perp$
  - 3D interaction $a_z = \sqrt{\hbar/(m\omega_z)} >> R_e$
    R_e is interaction potential range (~nm)
    a_z~um
- **Q1D geometry: 2D lattice**

Feshbach Resonance





Regal, et. al., PRL, 2003

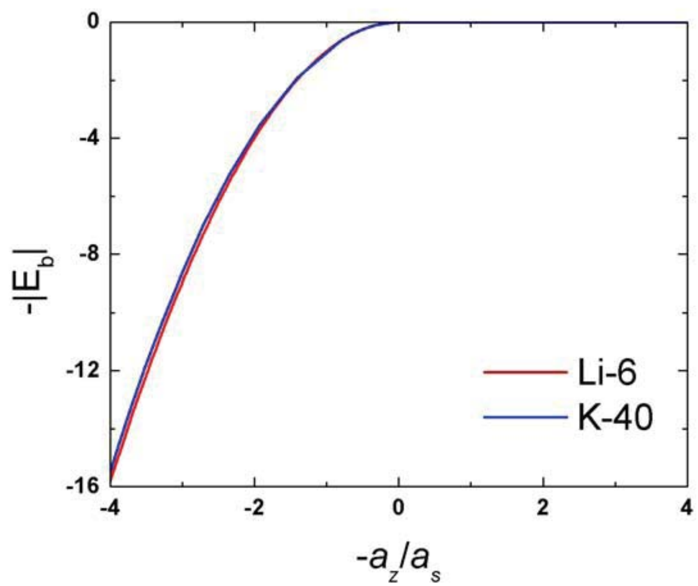
Two-body Problem

- Two-channel Model

$$H_{3D}=\sum_{\sigma=\uparrow,\downarrow}\int d^3\mathbf{x}\psi_\sigma^+\left(-\frac{\hbar^2\nabla^2}{2m}+V_{\rm ext}\right)\psi_\sigma+\int d^3\mathbf{x}\phi^+\left(-\frac{\hbar^2\nabla^2}{4m}+2V_{\rm ext}+\nu_b\right)\phi$$
$$+g_b\int d^3\mathbf{x}\left(\psi_\uparrow^+\psi_\downarrow^+\phi+\text{H.C.}\right)+U_b\int d^3\mathbf{x}\psi_\uparrow^+\psi_\downarrow^+\psi_\downarrow\psi_\uparrow$$

- Binding Energy: Wide FR

K:202G
Li:834G



$$a_s=a_{bg}\left(1-\frac{W}{B-B_0}\right)$$

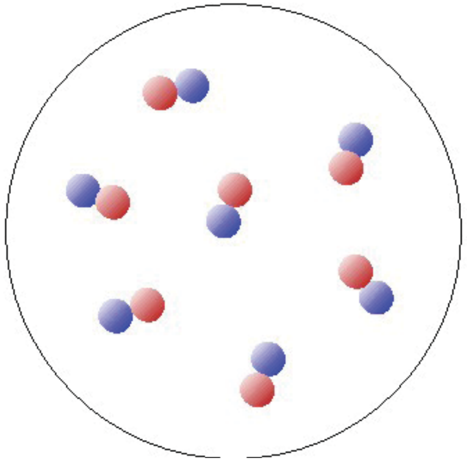
$$a_z=\sqrt{\hbar/(m\omega_z)}$$

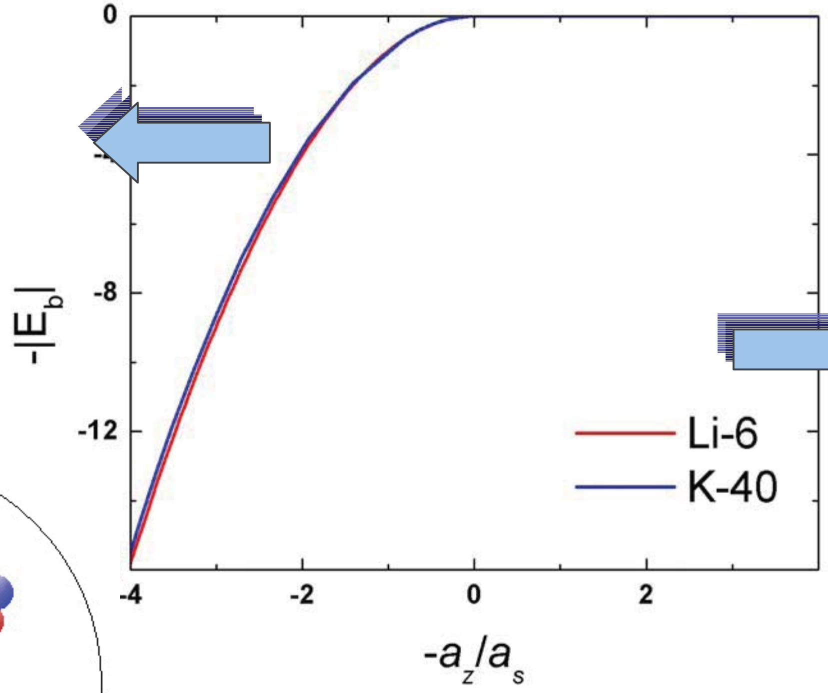
Universality

BCS-BEC Crossover in 3D

$a_s \to 0^+$

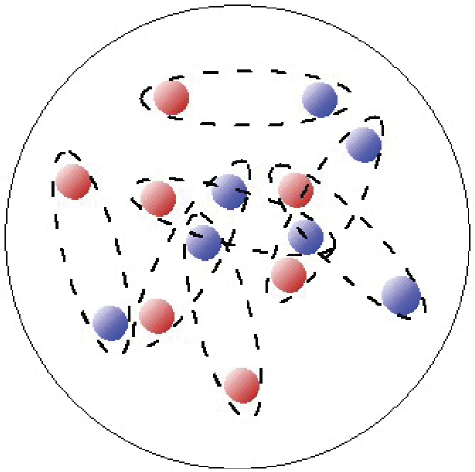
BEC limit- weakly int. Bose gas





$a_s \to 0^-$

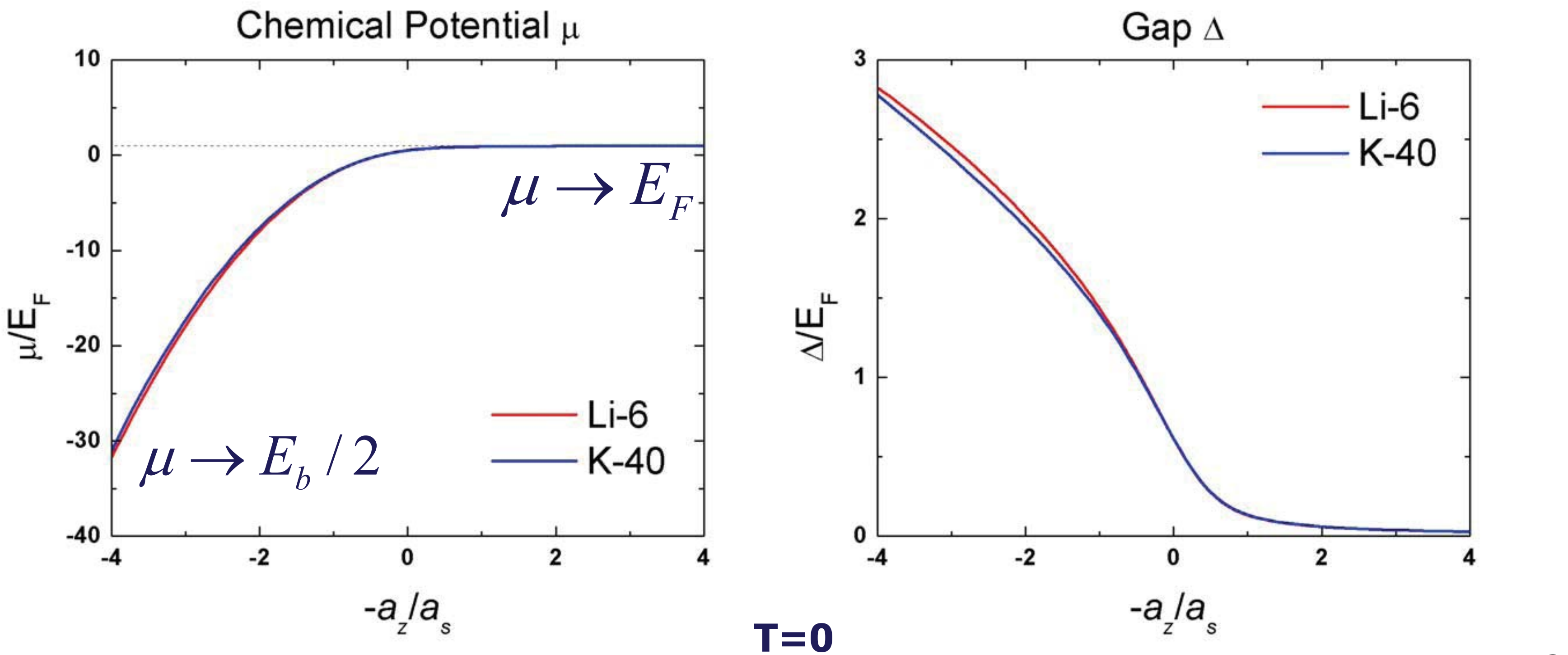
BCS limit- weakly int. Fermi gas



BCS-BEC Crossover in 3D

$$H=\sum_{\mathbf{k},\sigma} a^{+}_{\mathbf{k}\sigma}\left(\varepsilon_{\mathbf{k}}-\mu\right)a_{\mathbf{k}\sigma}+\sum_{\mathbf{q}} b^{+}_{\mathbf{q}}\left(\varepsilon_{\mathbf{q}}/2+\nu_b-2\mu\right)b^{+}_{\mathbf{q}}$$

$$+\left(g_b/L^{3/2}\right)\sum_{\mathbf{k},\mathbf{q}}\left(a^{+}_{\mathbf{k}+\mathbf{q},\uparrow}a^{+}_{-\mathbf{k},\downarrow}b_{\mathbf{q}}+\mathrm{H.C.}\right)+\left(U_b/L^{3}\right)\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} a^{+}_{\mathbf{k}+\mathbf{q},\uparrow}a^{+}_{-\mathbf{k},\downarrow}a_{-\mathbf{k}',\downarrow}a_{\mathbf{k}'+\mathbf{q},\uparrow}$$



T=0

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- **Introduction**
  - Fermi systems in low dimensions
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  - BCS-BEC crossover in 3D
- **Fermions in quasi-low dimensions: two-body problem**
  - Binding energy
  - Excited states in strongly confined directions
- **Fermions in quasi-low dimensions: many-body problem**
  - Effective Hamiltonian and dressed molecules
  - BCS-BEC crossover in Q2D: an example
  - The significance of dressed molecules
- **Summary**

Two-body Problem in Quasi-2D

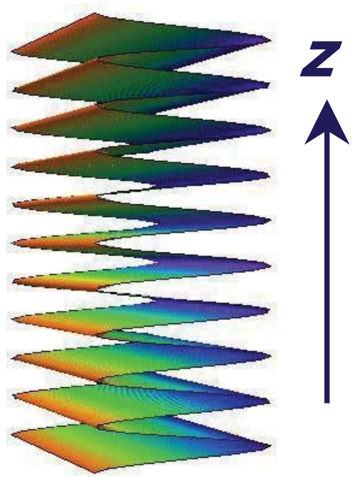
- Two-channel Model

$$H_{3D} = \sum_{\sigma=\uparrow,\downarrow} \int d^3\mathbf{x}\psi_\sigma^+ \left(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}\right)\psi_\sigma + \int d^3\mathbf{x}\phi^+ \left(-\frac{\hbar^2\nabla^2}{4m} + 2V_{\text{ext}} + \nu_b\right)\phi$$
$$+ g_b \int d^3\mathbf{x}\left(\psi_\uparrow^+\psi_\downarrow^+\phi + \text{H.C.}\right) + U_b \int d^3\mathbf{x}\psi_\uparrow^+\psi_\downarrow^+\psi_\downarrow\psi_\uparrow$$

- Strong confinement along z-direction

- Homogeneous in x-y plane

$$\omega_x = \omega_y = 0$$



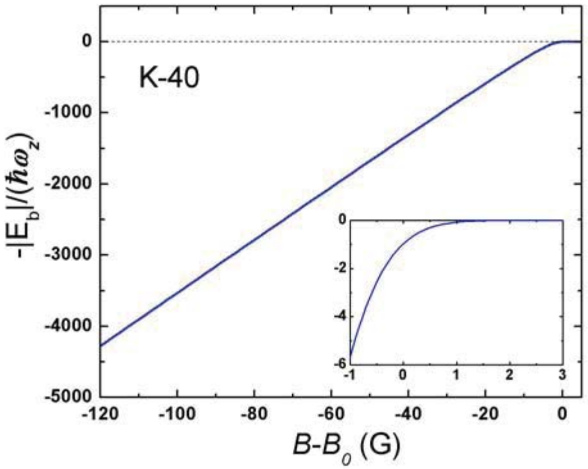
First Trial: Push to 2D

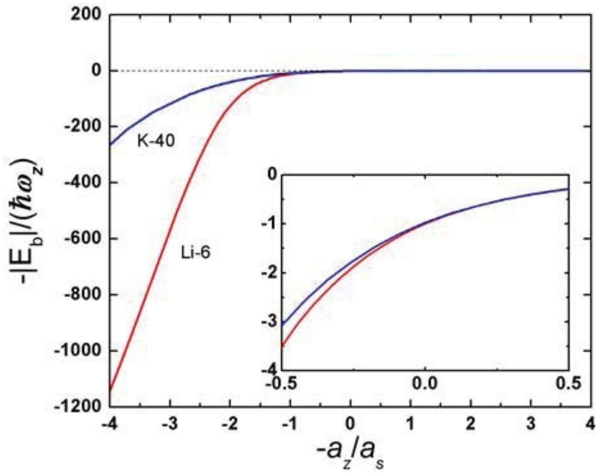
- Excited states along z direction are completely frozen.
  - fermions: $\psi_z = \left(\frac{m\omega_z}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega_z z^2}{2\hbar}\right)$
  - molecules: $\phi_z = \left(\frac{2m\omega_z}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega_z z^2}{\hbar}\right)$

$$H_{2\mathrm{D}} = \sum_{\sigma=\uparrow,\downarrow} \int d^2\mathbf{r}\psi^+_{\perp\sigma}\left(-\frac{\hbar^2\nabla^2_\perp}{2m} + \frac{\hbar\omega_z}{2}\right)\psi_{\perp\sigma} + \int d^2\mathbf{r}\phi^+_\perp\left(-\frac{\hbar^2\nabla^2_\perp}{4m} + \nu_b + \frac{\hbar\omega_z}{2}\right)\phi_\perp$$

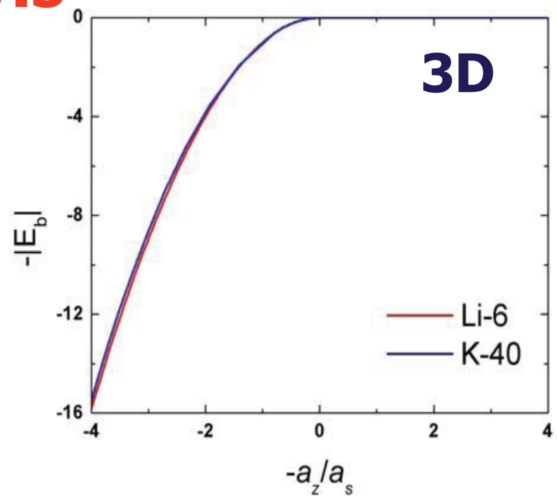
$$+ g_b\left(\frac{m\omega_z}{2\pi\hbar}\right)^{1/4} \int d^2\mathbf{r}\left(\psi^+_{\perp\uparrow}\psi^+_{\perp\downarrow}\phi_\perp + \mathrm{H.C.}\right) + U_b\left(\frac{m\omega_z}{2\pi\hbar}\right)^{1/2} \int d^2\mathbf{r}\psi^+_{\perp\uparrow}\psi^+_{\perp\downarrow}\psi_{\perp\downarrow}\psi_{\perp\uparrow}$$

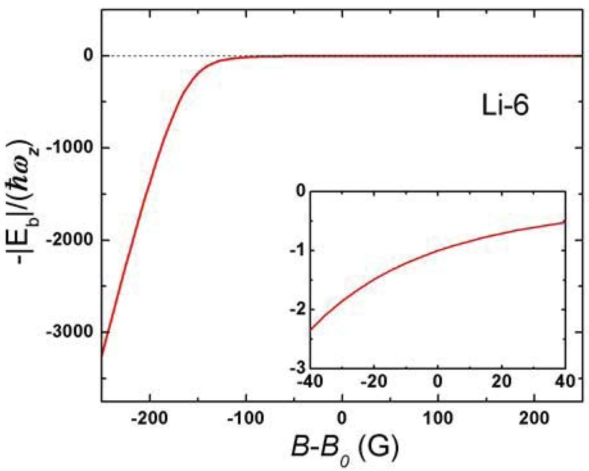
Results





v.s





- **Loss of Universality**
- Excitations along z direction are frozen ?

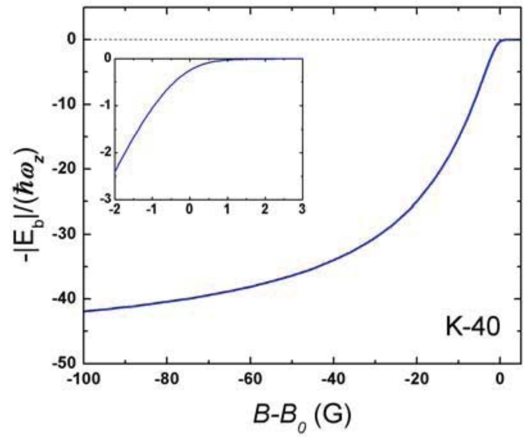
More General Description

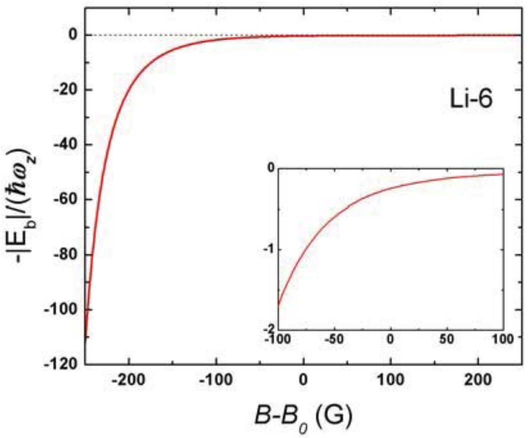
- Expanded by Harmonic Oscillators
  - fermions: $\psi_z = \sum_m a_m \Psi_m(z)$
  - molecules: $\phi_z = \sum_m b_m \Phi_m(z)$

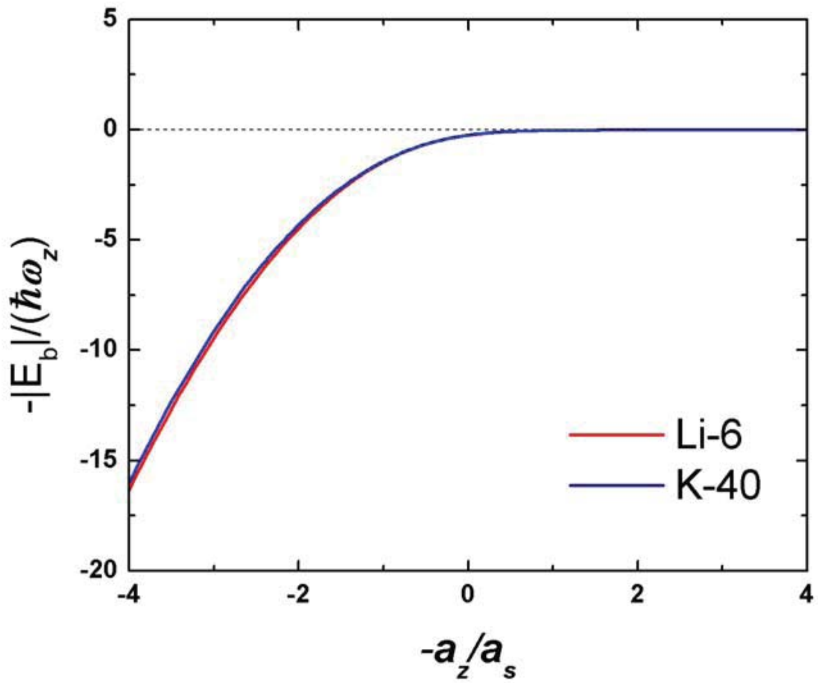
$$H_{2D} = \sum_{m,\mathbf{k},\sigma} a^+_{m\mathbf{k}\sigma}\left(\varepsilon_{m\mathbf{k}} + \frac{\hbar\omega_z}{2}\right) a_{m\mathbf{k}\sigma} + b^+_0\left(\frac{\varepsilon_{00}}{2} + \nu_b + \frac{\hbar\omega_z}{2}\right) b_0$$
$$+ \frac{g_b}{L}\left(\frac{m\omega_z}{\hbar}\right)^{1/4} \sum_{m,n,\mathbf{k}} \gamma_{mn}\left(a^+_{m,\mathbf{k},\uparrow} a^+_{n,-\mathbf{k},\downarrow} b_{00} + \mathrm{H.C.}\right)$$
$$+ \frac{U_b}{L^2}\left(\frac{m\omega_z}{\hbar}\right)^{1/2} \sum_{\substack{m,n,\mathbf{k},\\ m',n',\mathbf{k}'}} \gamma_{mn}\gamma_{m'n'} a^+_{m,\mathbf{k},\uparrow} a^+_{n,-\mathbf{k},\downarrow} a_{n',-\mathbf{k}',\downarrow} a_{m',\mathbf{k}',\uparrow}$$

Two-body Problem Revisited

- **Binding Energy:**

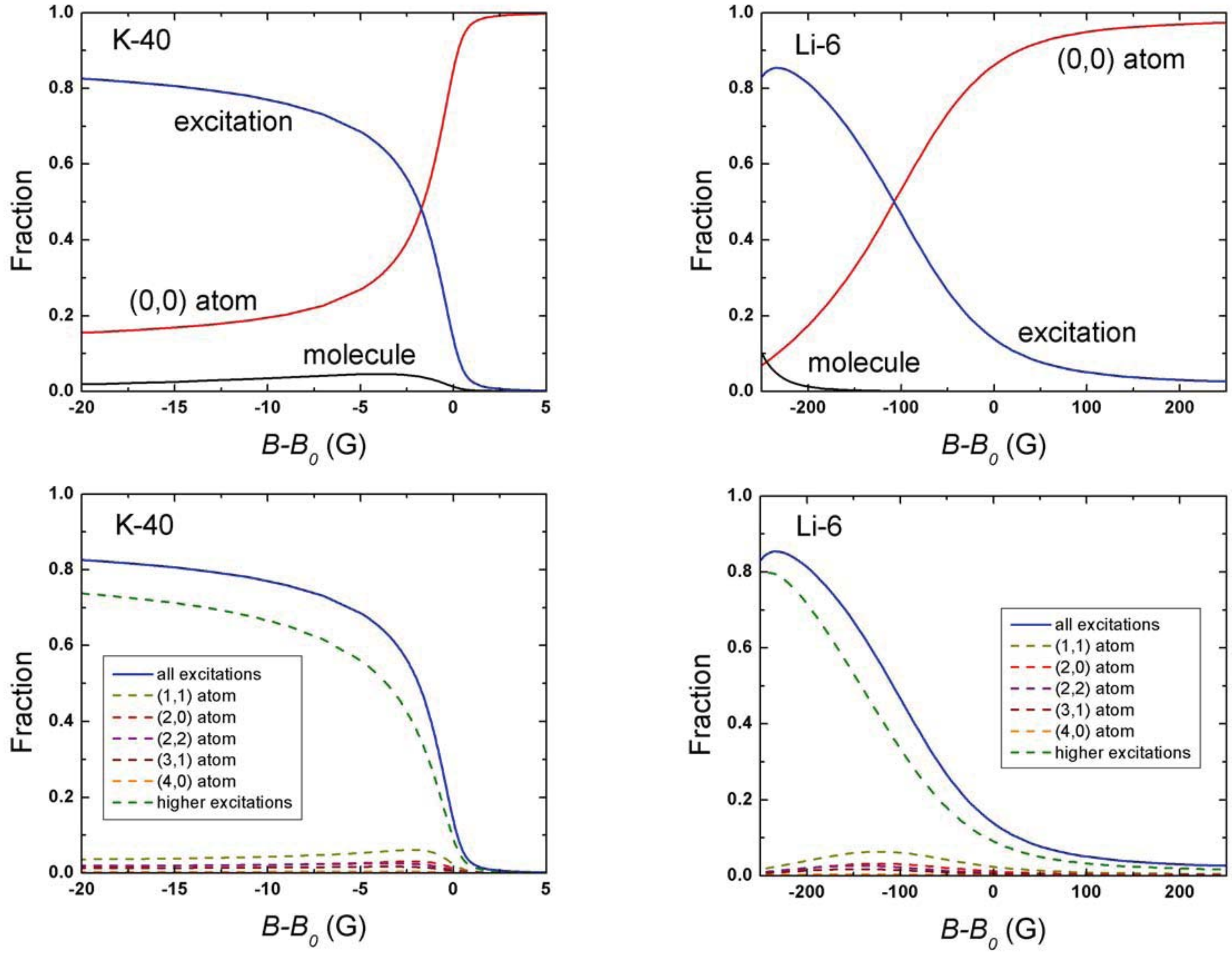




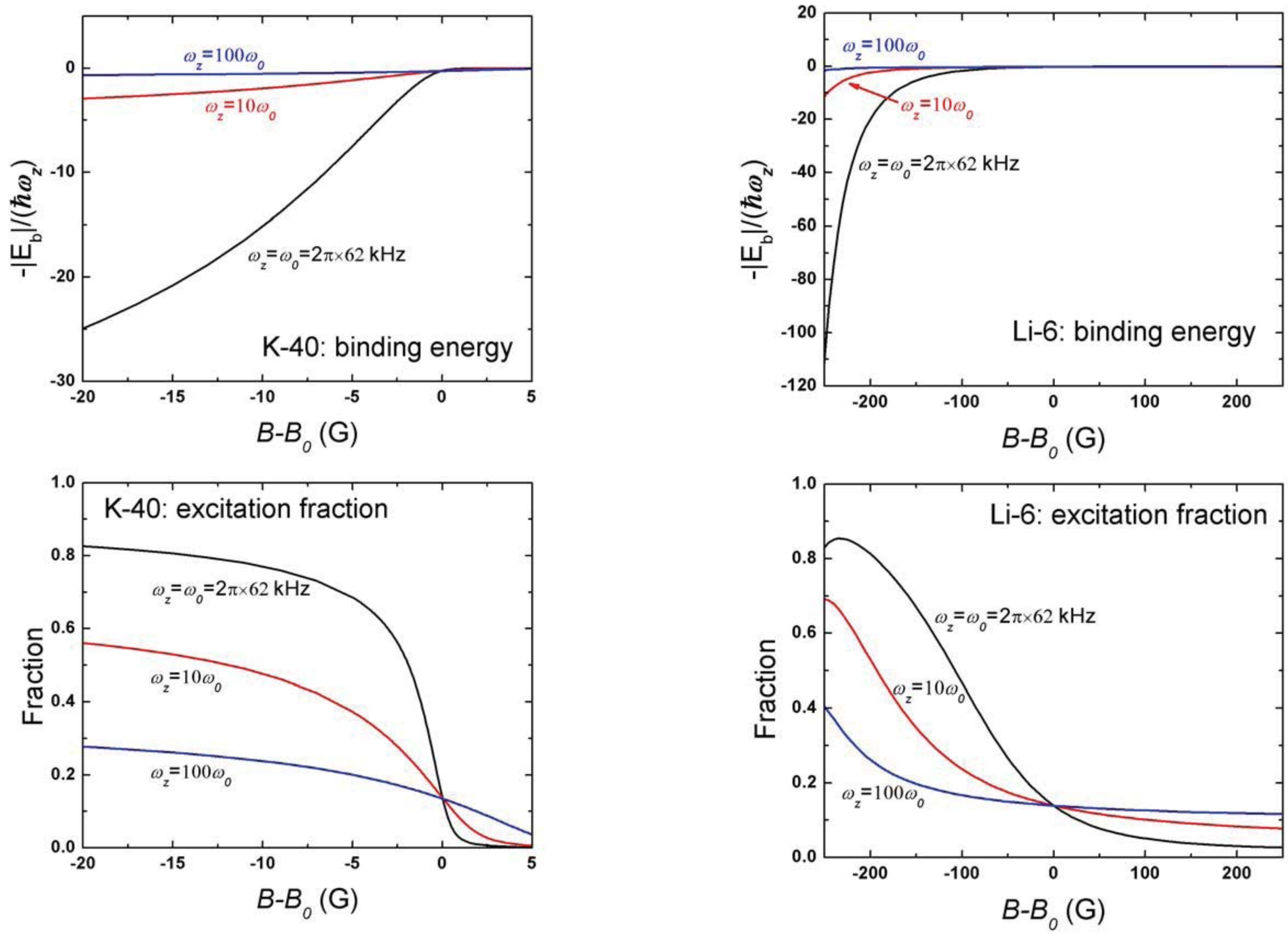


- Universality
- Excited states are important

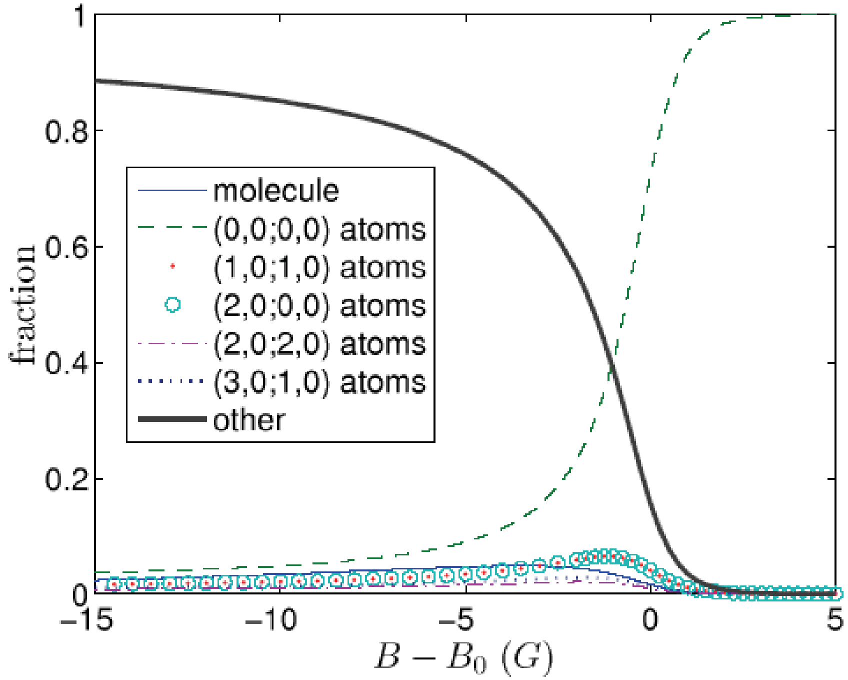
Population of Molecules and Excitations

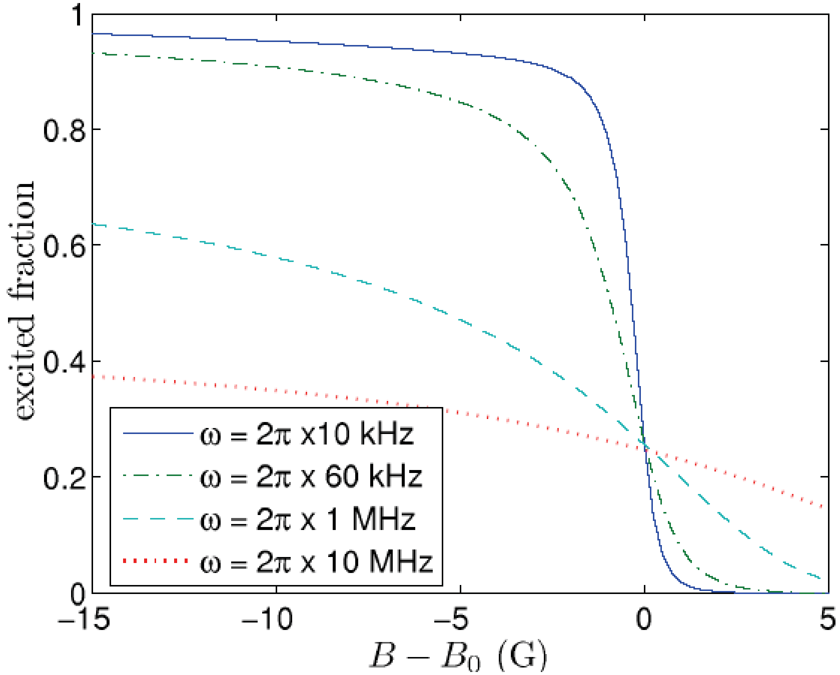


Excitation fraction cannot be suppressed



Two-body problem in Q1D

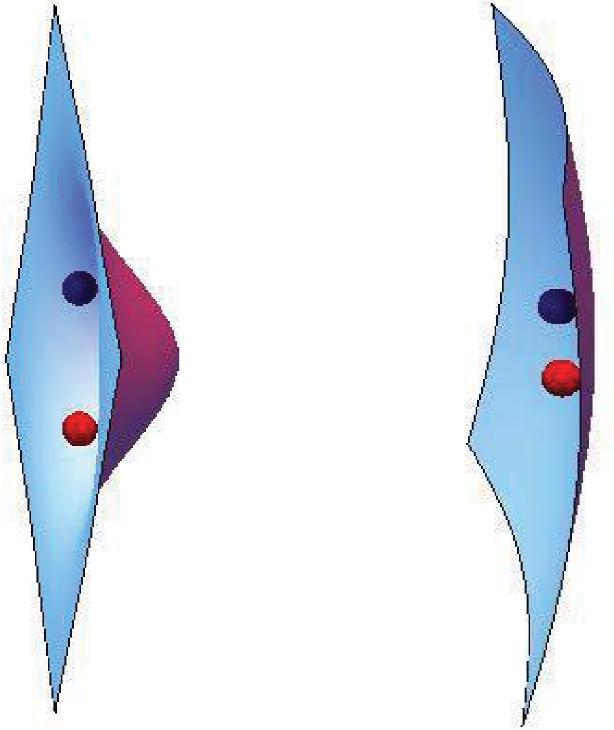


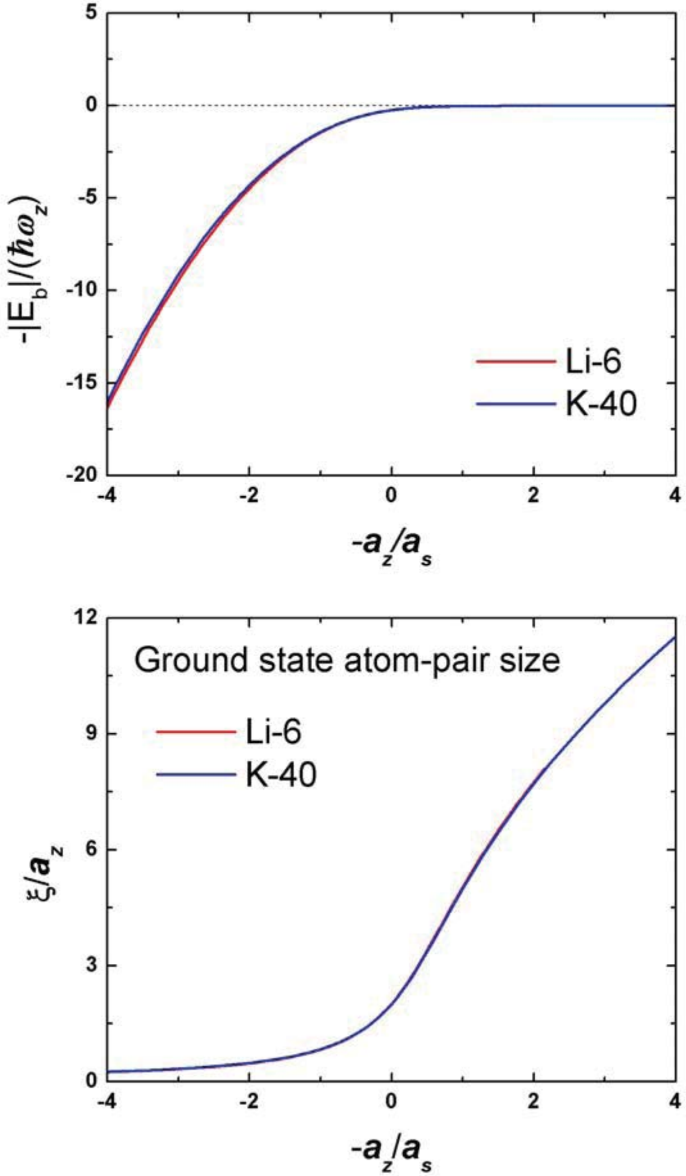


Kestner et. al., PRA, 74, 053606

Discussion

- BCS side $|E_b| << \hbar\omega_z,\ \xi >> a_z$





- BEC side $|E_b| >> \hbar\omega_z,\ \xi << a_z$

Excitation is important

- No matter how strong the confinement is, excited states can still be populated to a sizable value.

  Quasi low D will never be low D

? Are these excited states important in a many-body problem?

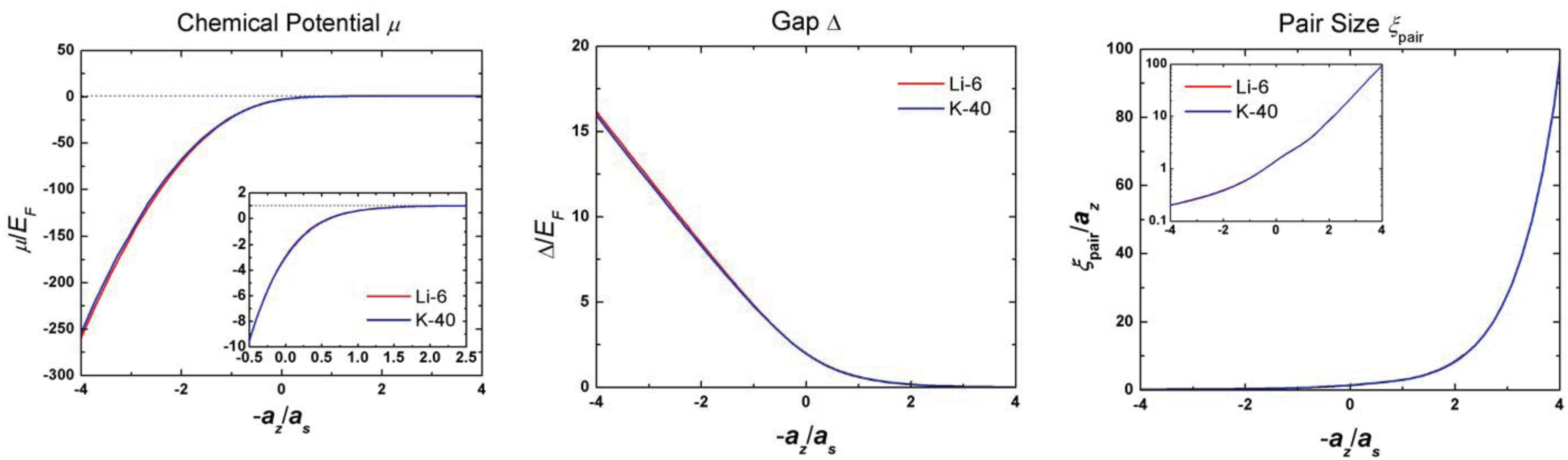
? If yes, how to deal with them?

BCS-BEC Crossover of a Q2D system: a 2D model w/o excited states

- Assume ground state along z-direction-> 2D model

$$H_{2D} = \sum_{\mathbf{k},\sigma} a^{+}_{0\mathbf{k}\sigma}\left(\varepsilon_{0\mathbf{k}} + \frac{\hbar\omega_z}{2} - \mu\right) a_{0\mathbf{k}\sigma} + \frac{U_{\mathrm{eff}}}{L^2}\left(\frac{m\omega_z}{2\pi\hbar}\right)^{1/2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} a^{+}_{0,\mathbf{k}+\mathbf{q},\uparrow} a^{+}_{0,-\mathbf{k},\downarrow} a_{0,-\mathbf{k}',\downarrow} a_{0,\mathbf{k}'+\mathbf{q},\uparrow}$$

- MF Approach: homogeneous case, T=0

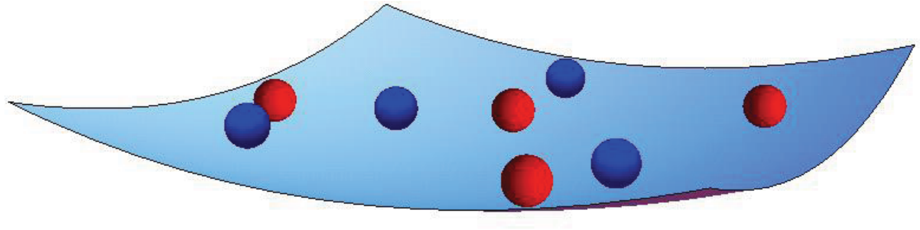


$$E_F / (\hbar\omega_z) = 0.01 \times \pi$$

Inhomogeneous case

- Harmonic trap

$$V_\perp(r) = \frac{1}{2} m\omega_\perp^2 r^2$$



- LDA

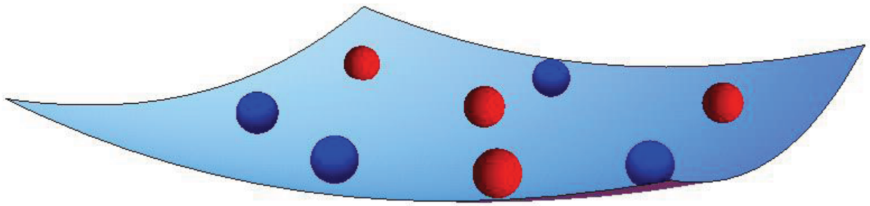
$$\mu(r) = \mu_0 - V_\perp(r) = \mu_0 - \frac{1}{2} m\omega_\perp^2 r^2$$

- Total Particle No. $N = 2\pi \int_0^{R_{\mathrm{TF}}} n(r) r dr$

- Thomas-Fermi Radius $n(R_{\mathrm{TF}}) = 0$

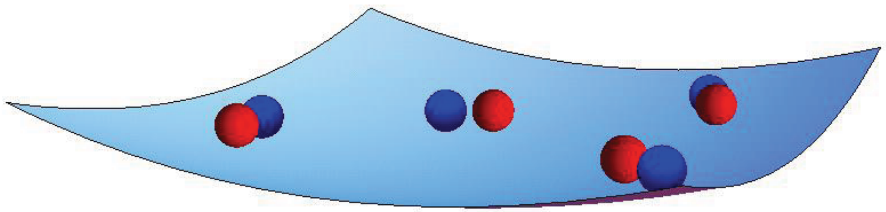
A constant TF Radius?

- BCS limit： Weakly Int. Fermi Gas

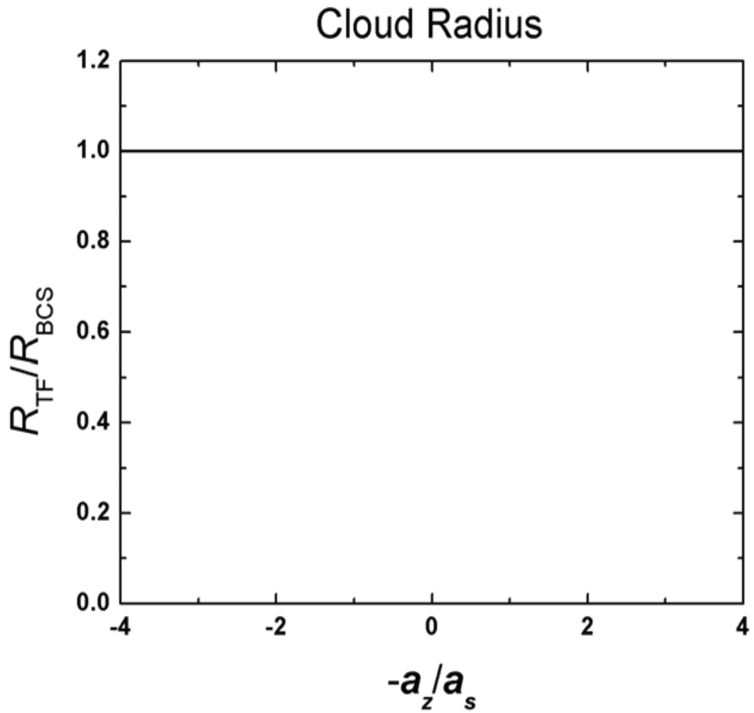


$$R_{\rm TF} \to R_{\rm BCS} > 0$$

- BEC limit: Weakly Int. Bose Gas



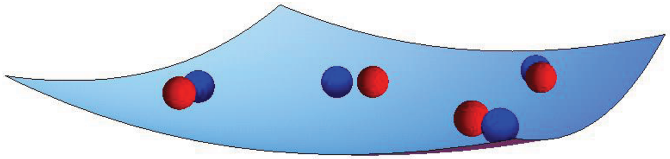
$$R_{\rm TF} \to 0$$



The Failure of the pure 2D model

- Equation of State $$n(r) = \frac{1}{\pi a_z^2}\left[\frac{F(a_s,a_z)}{2} + \mu(r)\right]$$

$$2\mu = \underline{2\pi a_z^2 n} - \underline{F(a_s,a_z)}$$



Chemical Pot. of Eff. Bosons

binding energy

- Constant interaction between effective bosons.
- The failure of the pure 2D model: cannot predict a weakly interacting Bose gas in the BEC limit.

- The excited states seem to be important for many-body problems

? How to incorporate them?

$$H_{2\mathrm{D}} = \sum_{m,\mathbf{k},\sigma} a^+_{m\mathbf{k}\sigma}\left(\varepsilon_{m\mathbf{k}} + \frac{\hbar\omega_z}{2} - \mu\right) a_{m\mathbf{k}\sigma} + \sum_{m,\mathbf{q}} b^+_{m\mathbf{q}}\left(\frac{\varepsilon_{m\mathbf{q}}}{2} + \nu_b + \frac{\hbar\omega_z}{2} - 2\mu\right) b^+_{m\mathbf{q}}$$

$$+ \frac{g_b}{L}\left(\frac{m\omega_z}{\hbar}\right)^{1/4} \sum_{m,n,l,\mathbf{k},\mathbf{q}} \gamma_{mn}\left(a^+_{m,\mathbf{k}+\mathbf{q},\uparrow} a^+_{n,-\mathbf{k},\downarrow} b_{l\mathbf{q}} + \mathrm{H.C.}\right)$$

$$+ \frac{U_b}{L^2}\left(\frac{m\omega_z}{\hbar}\right)^{1/2} \sum_{\substack{m,n,\mathbf{k},\\ m',n',\mathbf{k}',\mathbf{q}}} \gamma_{mn}\gamma_{m'n'} a^+_{m,\mathbf{k}+\mathbf{q},\uparrow} a^+_{n,-\mathbf{k},\downarrow} a_{n',-\mathbf{k}',\downarrow} a_{m',\mathbf{k}'+\mathbf{q},\uparrow}$$

- Hard to solve, even in MF level
- A detour is needed……

- The detout turns out to be an effective Hamiltonian

  - Can mimic the original Hamiltonian, at least around the regime of interest.

  - Easy to solve.

The Effective Hamiltonian

Feshbach Mol. Excited Fermion

Fermions in ground state

dressed molecule: $d_{\mathbf{q}}$

$$H_{2D}^{\text{eff}} = \sum_{\mathbf{k},\sigma} a_{0,\mathbf{k},\sigma}^{+} \left(\varepsilon_{\mathbf{k}} + \frac{\hbar\omega_z}{2} - \mu \right) a_{0,\mathbf{k},\sigma} + \sum_{\mathbf{q}} d_{\mathbf{q}}^{+} \left(\frac{\varepsilon_{\mathbf{q}}}{2} + \lambda_b + \frac{\hbar\omega_z}{2} - 2\mu \right) d_{\mathbf{q}}$$

$$+ \frac{\alpha_b}{L} \sum_{\mathbf{kq}} \left(a_{0,\mathbf{k+q},\uparrow}^{+} a_{0,-\mathbf{k},\downarrow}^{+} d_{\mathbf{q}} + \text{H.C.} \right)$$

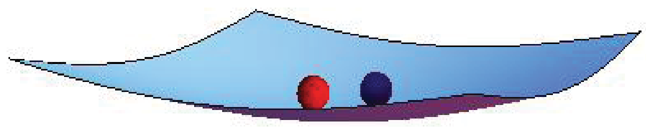
$$+ \frac{V_b}{L^2} \sum_{\mathbf{kk'q}} a_{0,\mathbf{k+q},\uparrow}^{+} a_{0,-\mathbf{k},\downarrow}^{+} a_{0,-\mathbf{k'},\downarrow} a_{0,\mathbf{k'+q},\uparrow}$$

Kestner et. al., PRA, 76, 063610
WZ et. al., PRA, 77, 063613

Matching parameters

- How to deal with excited states?
  - Excited states is populated when pair size is small
  - Short-range physics is dominated by two-body process due to diluteness condition

Ground state



Excited state

Matching cond.	Original H	Effective H
Bg. Scattering	U_b	V_b
Binding E.	E_b	E_b
Population	excitation+ Feshbach mol.	dressed mol.

Validity

- In the T-matrix representation, the matching conditions actually match two quantities:
  - The position of singular points of T(x)
  - The first derivative of 1/T(x) around singular points

- Thus, the effective H is approximately identical to the original H around the binding energy, with error to the order of

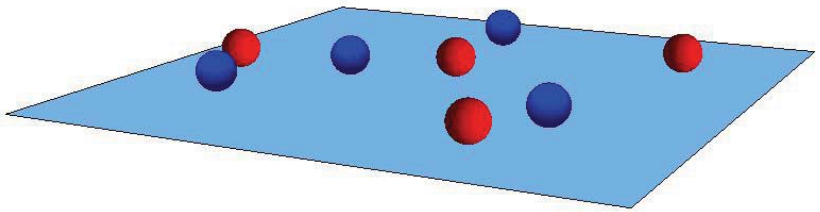
$$\Delta V = O\left(\frac{\mu - E_b/2}{\hbar\omega_z}\right)^2$$

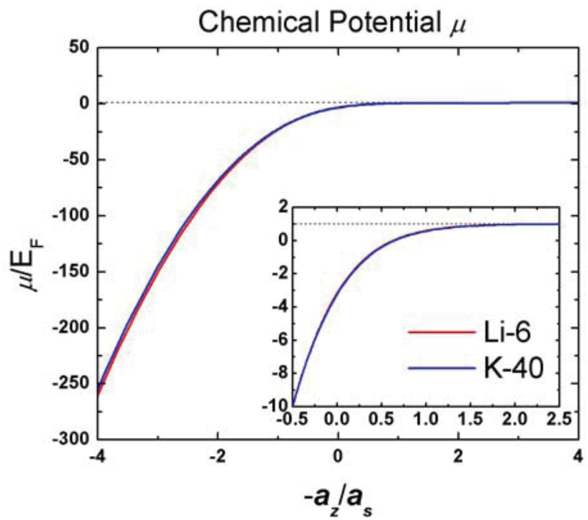
- Since $\mu - E_b/2 \le E_F$, the validity of this effective H is guaranteed by the quasi-low-D or the diluteness conditions

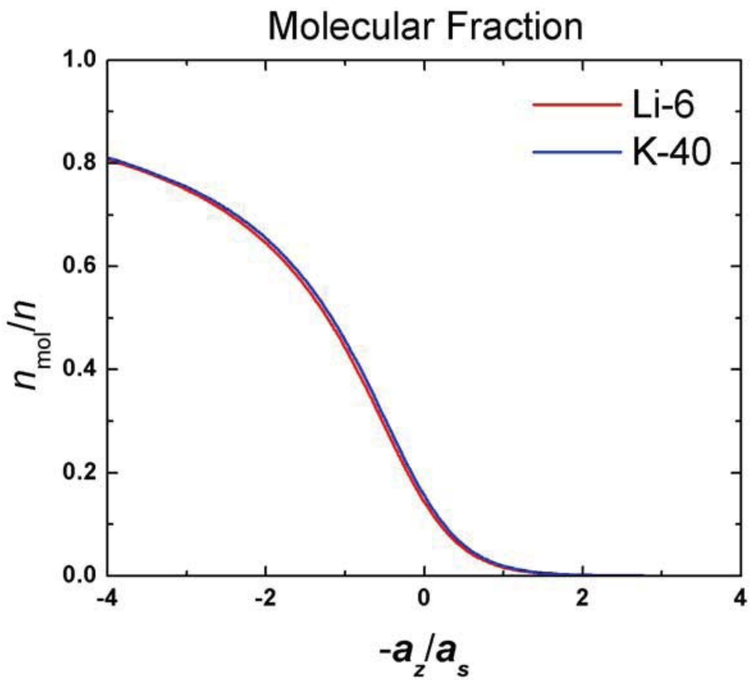
$$\hbar\omega_z >> E_F; \quad na_z^2 << 1$$

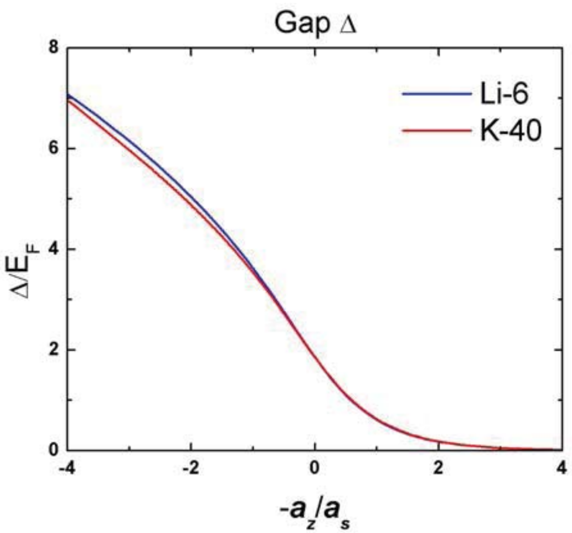
BCS-BEC Crossover in Q2D revisited

- Homogeneous case







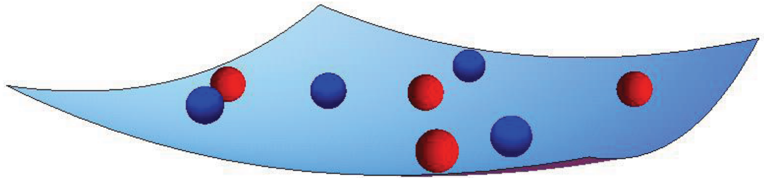


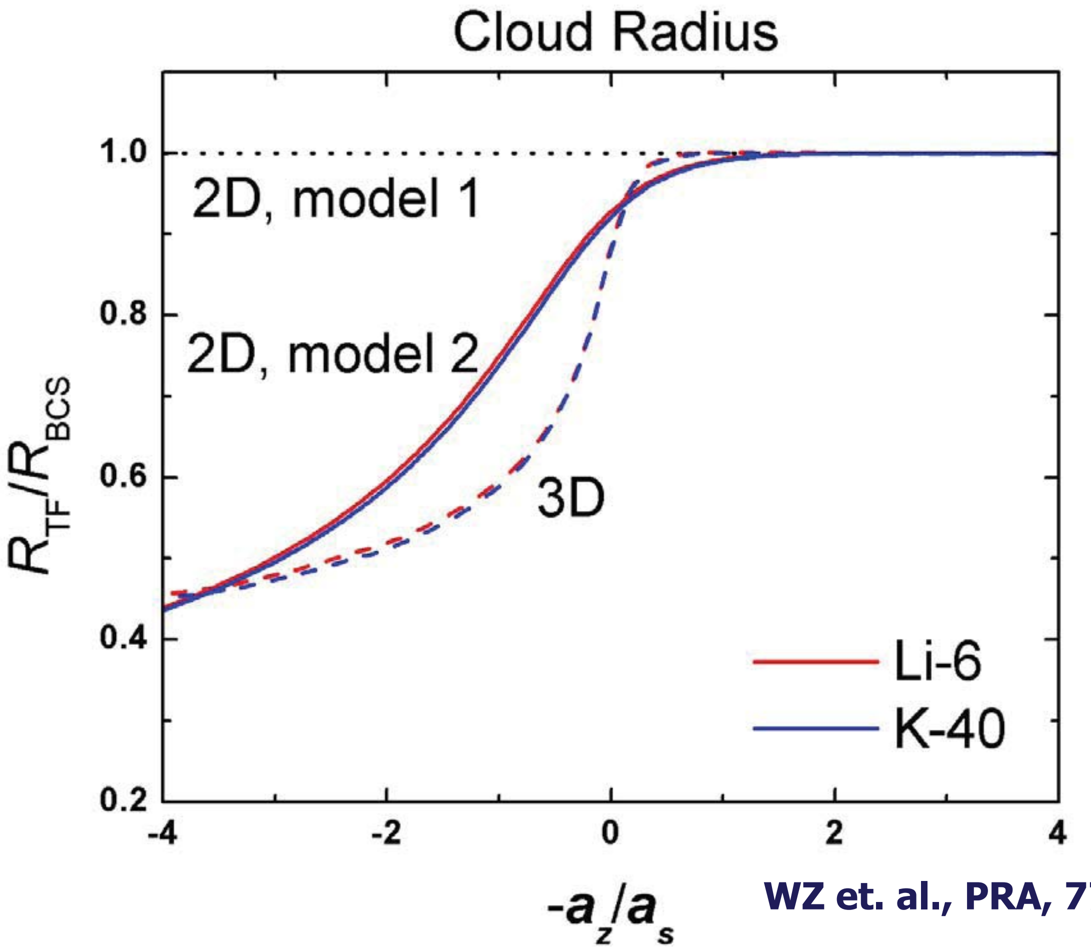
$$E_F / \left(\hbar\omega_z\right) = 0.01 \times \pi$$

WZ et. al., PRA, 77, 063613

TF radius

- In a harmonic trap





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BKT transition

- MF+phase fluctuation

$$\Delta(r,t) = \Delta_0 \exp\left[i\theta(r,t)\right]$$

$$S_{\text{eff}} = S_0 + S_{\text{fluc}}$$

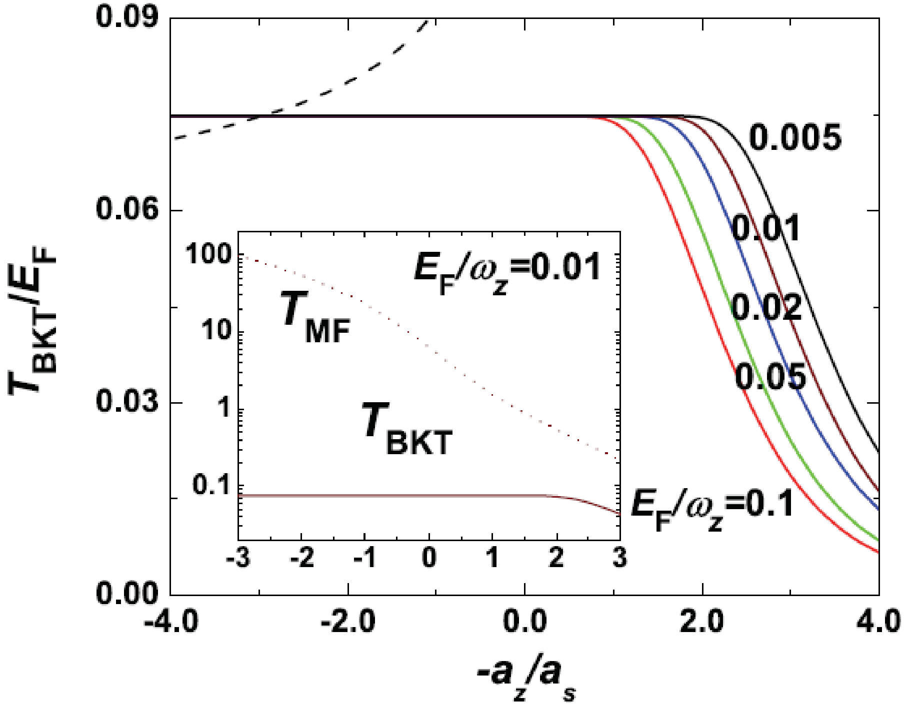
$$S_{\text{fluc}} \approx \frac{1}{2}\int_0^{\hbar/k_B T} d\tau \int d^2\mathbf{r}\left[iJ\partial_\tau\theta + K(\partial_\tau\theta)^2 + \rho_s(\nabla\theta)^2\right]$$

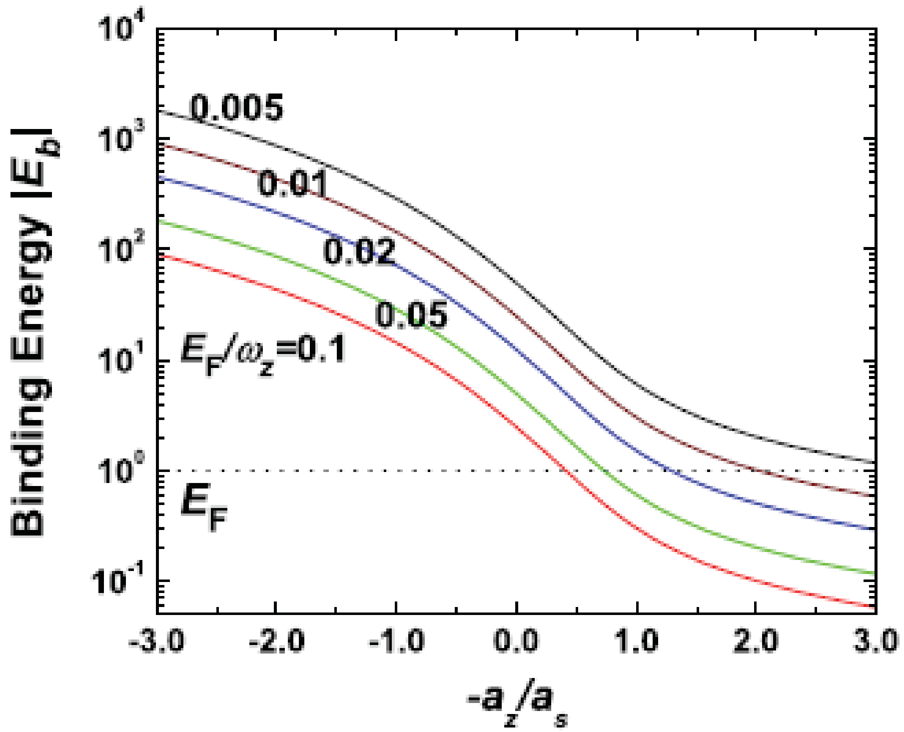
- BKT transition temperature

$$T_{BKT} = \frac{\pi}{2}\rho_s^R$$

$$\rho_s = \frac{1}{4m}\left\{n_{\text{mol}} + \frac{1}{L^2}\sum_{\mathbf{k}}\left[1 - \frac{\varepsilon_{\mathbf{k}}}{E_{\mathbf{k}}}\tanh\left(\frac{E_{\mathbf{k}}}{2k_B T}\right)\right] - \frac{\hbar^2}{4mk_B T L^2}\sum_{\mathbf{k}} k^2 \text{sech}^2\left(\frac{E_{\mathbf{k}}}{2k_B T}\right)\right\}$$

BKT transition





WZ et. al., PRA, 78, 043617

Summary

- In quasi low D conditions, no matter how strong the transverse confinement is, fermions can still be populated to excited states, and these populations do matter.

- These degrees of freedom can be incorporated by introducing an effective Hamiltonian.
  - The excited states are described by dressed molecules.
  - The effective H takes the form of a 2-channel model, with parameters given by matching 2-body physics.
  - As an example, this model has been used to analyze BCS-BEC crossover in Q2D, and predicted the trend of TF radius as expected.