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4 - 8 May 2009

A unitary Fermi supersolid: the Larkin Ovchinnikov state from a DFT

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A Unitary Fermi Supersolid: The Larkin Ovchinnikov State from a DFT

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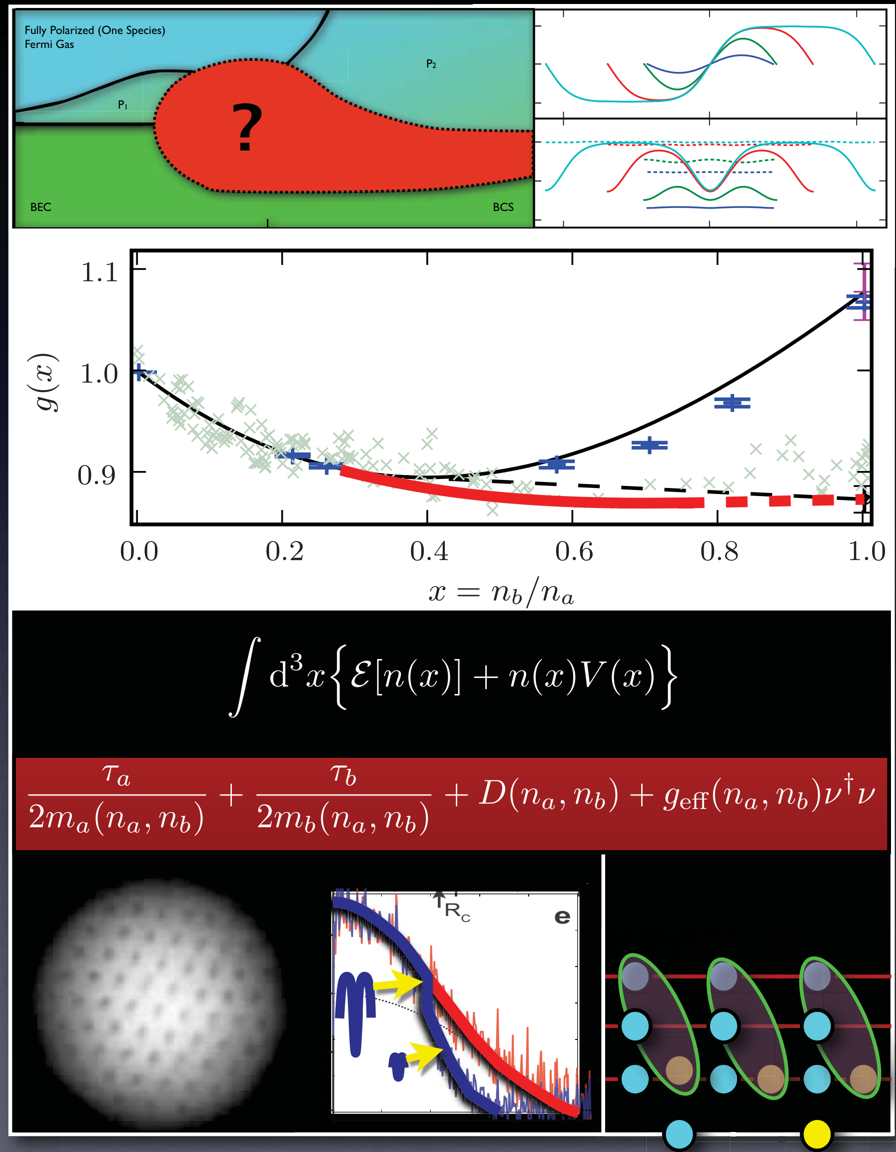
T-2 Division
Los Alamos National Laboratory
8 March 2009

Outline



- Unitary Phase Structure
- Fermi Supersolid (LOFF)
- Density Functionals
  - SLDA
  - ASLDA

Model

$$\widehat{\mathcal{H}} = \int \left(\widehat{\mathbf{a}}^\dagger \widehat{\mathbf{a}} E_a + \widehat{\mathbf{b}}^\dagger \widehat{\mathbf{b}} E_b \right) - g \int_\Lambda \widehat{\mathbf{a}}^\dagger \widehat{\mathbf{b}}^\dagger \widehat{\mathbf{b}} \widehat{\mathbf{a}}$$

$$E_{a,b} = \frac{p^2}{2m} - \mu_{a,b} \qquad \mu_\pm = \frac{\mu_a \pm \mu_b}{2}$$

Fix regulator and take $\Lambda \to \infty$ holding s-wave scattering length a fixed

$$a \sim \frac{1}{\Lambda - \frac{1}{g}}$$

Universal physics depends on the single parameter a

Only Parameters: $a, \ \mu_a, \ \mu_b, \ T$

Cold Symmetric Unitary Gas

$$\mu_a = \mu_b,\ T = 0,\ a = \infty$$

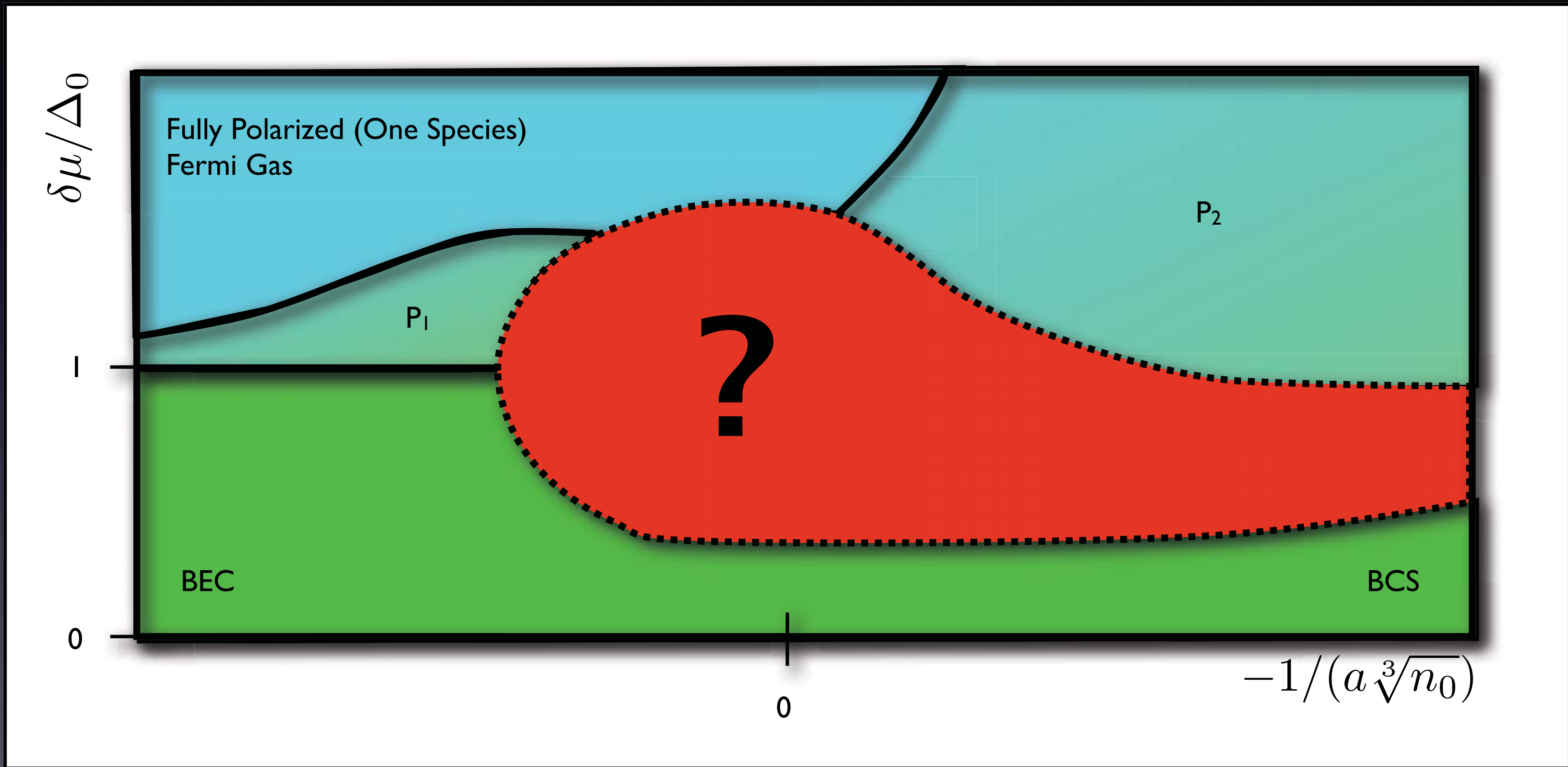
- Only one dimensionful parameter: μ
- All nontrivial thermodynamics in a single dimensionless parameter: $\xi \approx 0.40(1)$

$$\mathcal{P}(\mu) \propto \xi^{-3/2}\mu^{5/2} \qquad \mathcal{E}(n) \propto \xi n^{5/3}$$

$$n = \frac{\partial \mathcal{P}}{\partial \mu} \propto \frac{5}{2}\left(\frac{\mu}{\xi}\right)^{3/2}$$

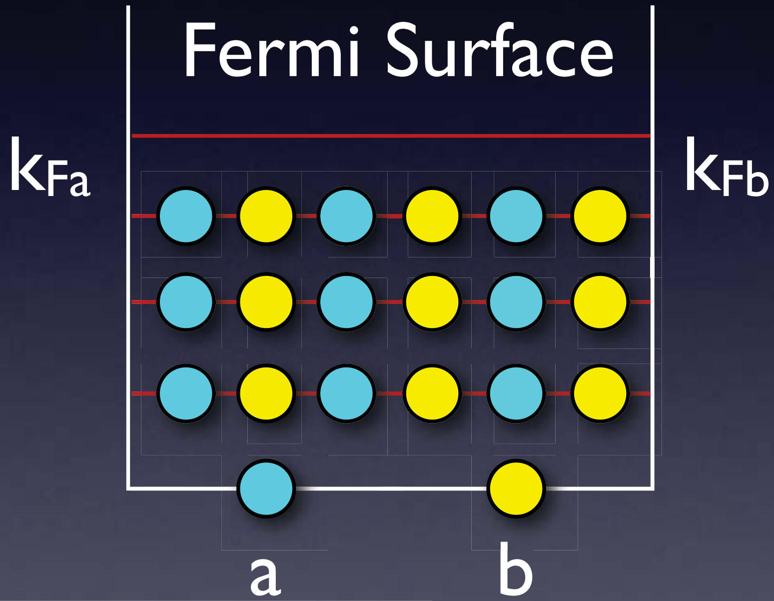
- Lack of scales greatly simplifies form of DFT
- Extensions including: $y = \frac{\mu_b}{\mu_a}$ $\quad \frac{\mu_{a,b}}{T}$ $\quad a^2\mu_{a,b}$

Phase Structure

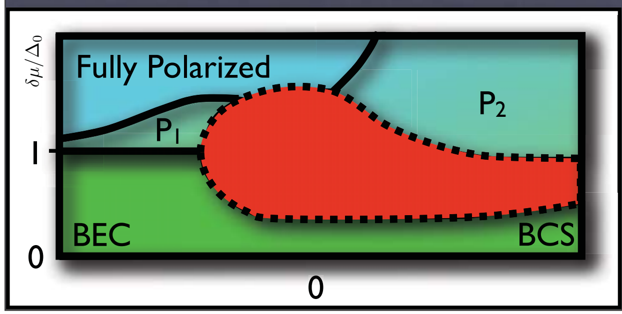


Based on D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk PRL 97 020402 (2006)

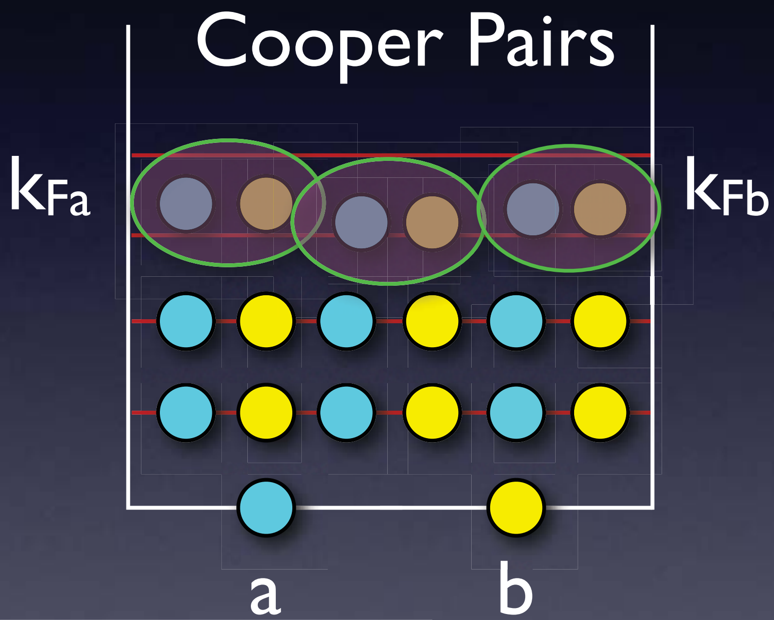
Degenerate Fermions



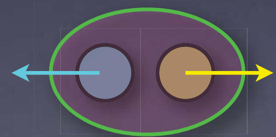
- Pauli Blocking.
- Filled Fermi Seas.
- Low energy states blocked.
- Excitations at Fermi Surface.



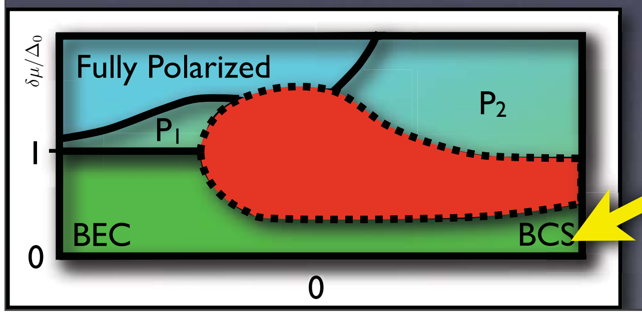
Weak Interactions: BCS



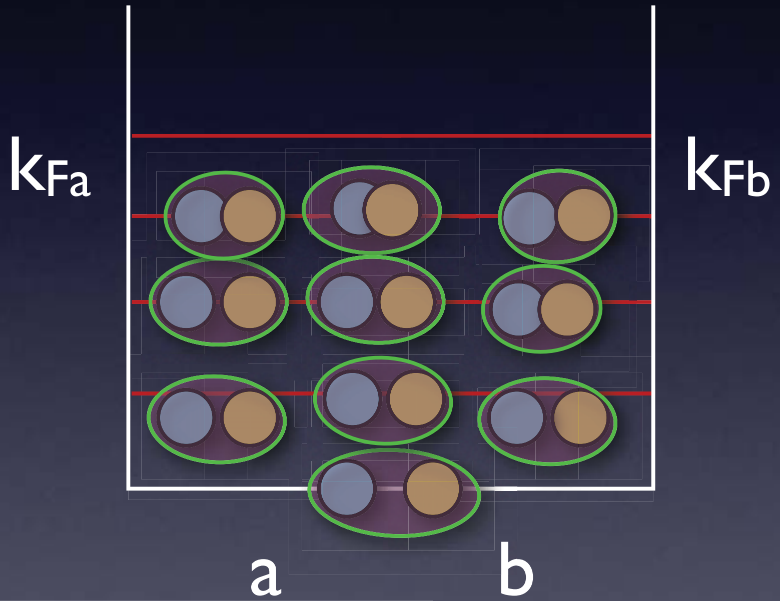
- Pairing at Fermi Surface
- Loosely bound Cooper Pairs
- Quantum coherence of pairs = BCS Superfluidity
- Energy gap Δ required to break pairs
- Pairs have zero momentum



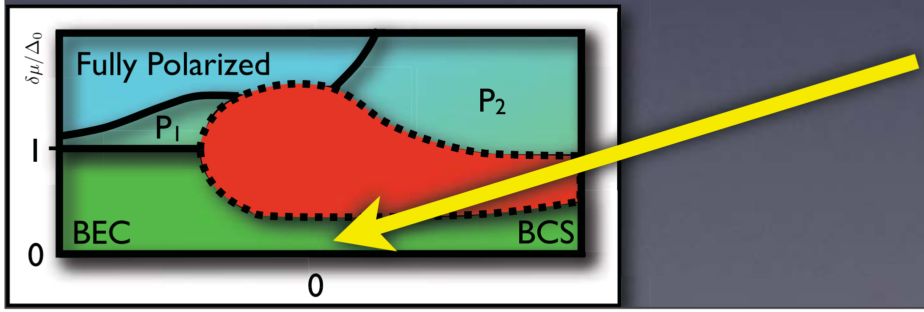
$p = p_a + p_b = 0$



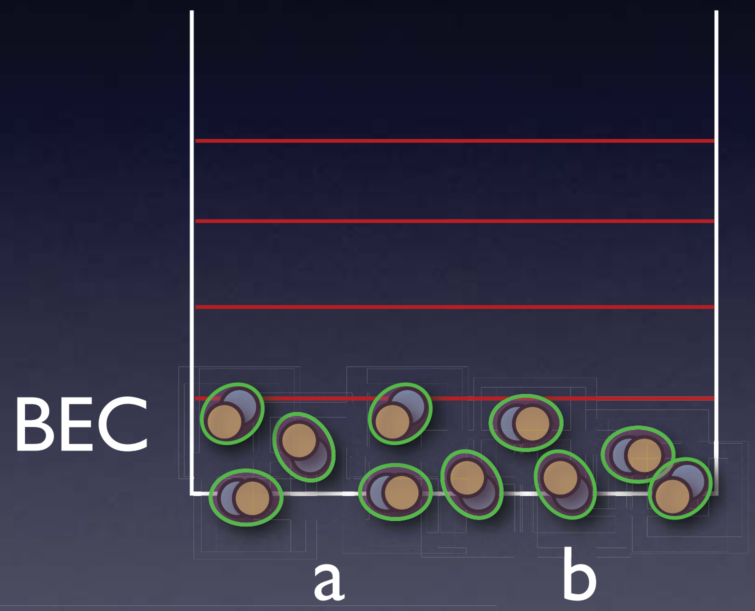
Stronger Interactions



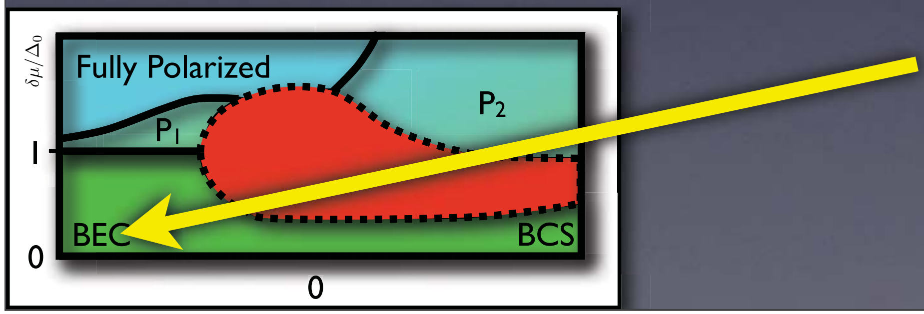
- Ubiquitous pairing (less pronounced Fermi Surface)
- More tightly bound Cooper Pairs
- Quantum coherence of pairs = Superfluidity
- Larger energy gap Δ required to break pairs



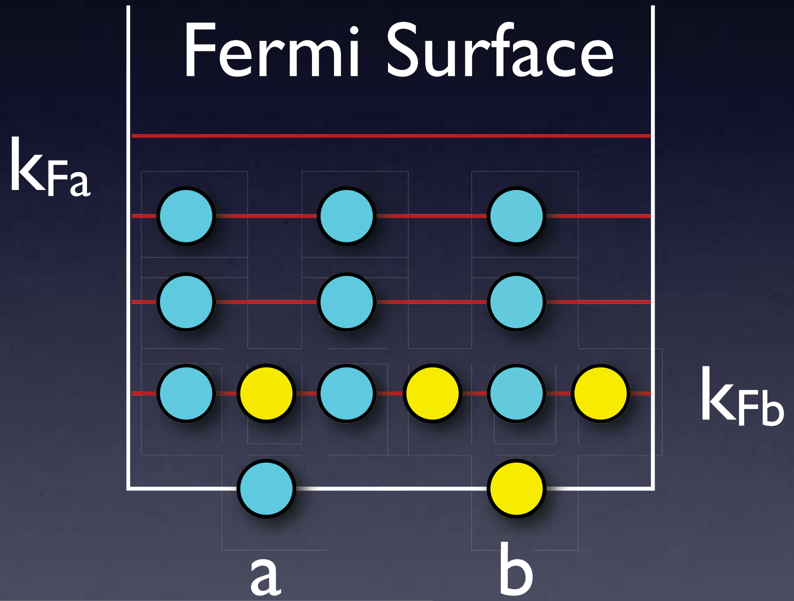
Strong Interactions: BEC



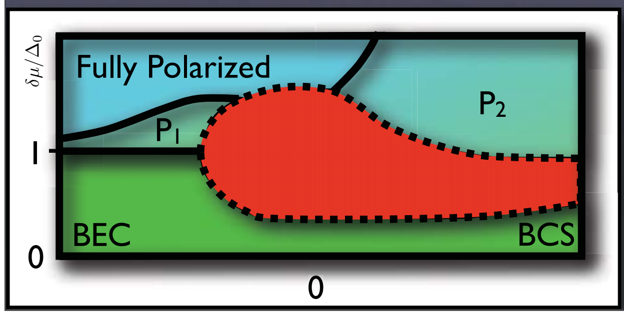
- Tightly bound pairs: Dimers
- Dimers Bose Condense: BEC
- Quantum coherence of pairs = BEC Superfluidity
- Very large energy gap Δ required to break pairs



Asymmetric Fermions

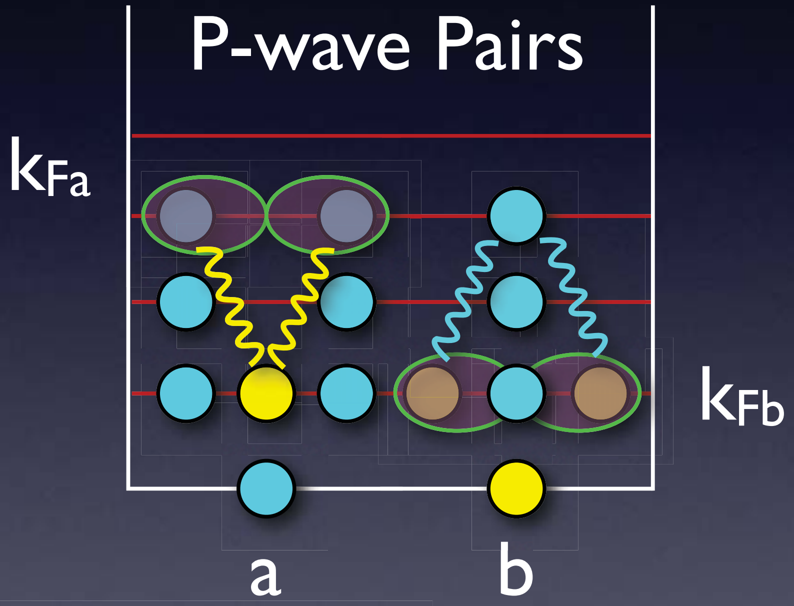


- Pauli Blocking.
- Two Filled Fermi Seas.
- Low energy states blocked.
- Excitations at Fermi Surface.

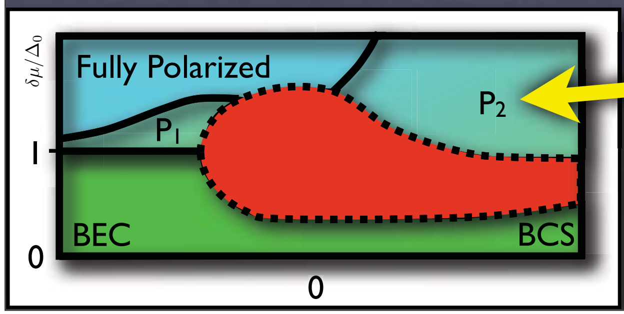


Weak Interactions:
P-wave Superfluid

Intra-species P-wave Pairs



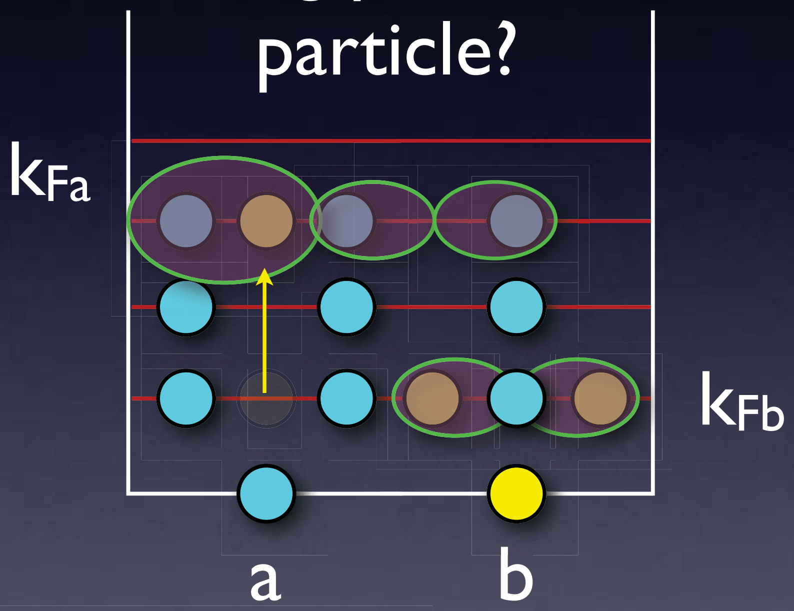
- Conventional Pairing Frustrated
- Induced Intra-species P-wave Pairing
- Slight Quantum coherence on top of Normal state
- Very small energy gap Δ to break p-wave pairs
- Two coexisting superfluids



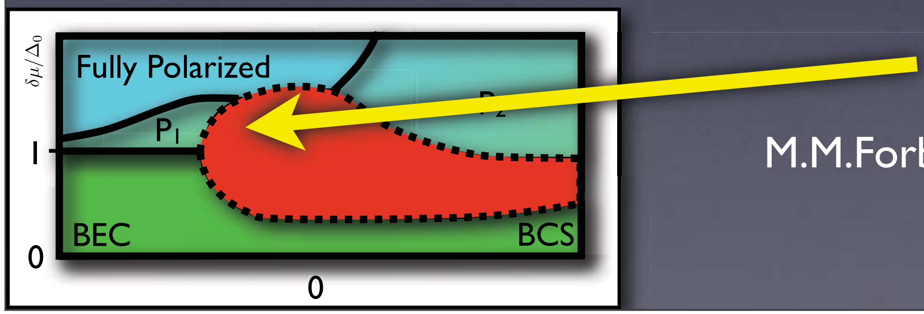
A.Bulgac, M.M.Forbes, A.Schwenk PRL 97 020402 (2006)

Stronger Interactions:
Gapless Superfluid?

Pairing promotes particle?

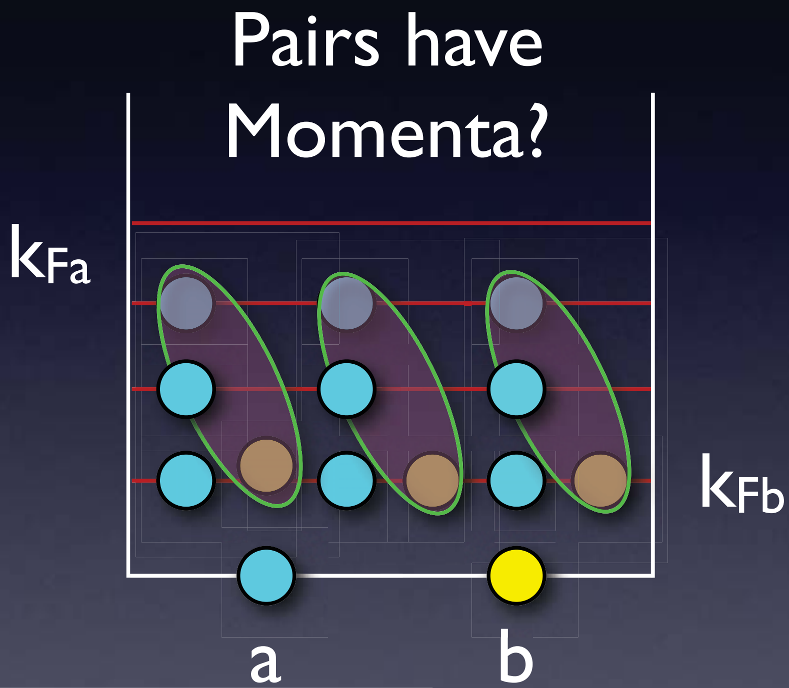


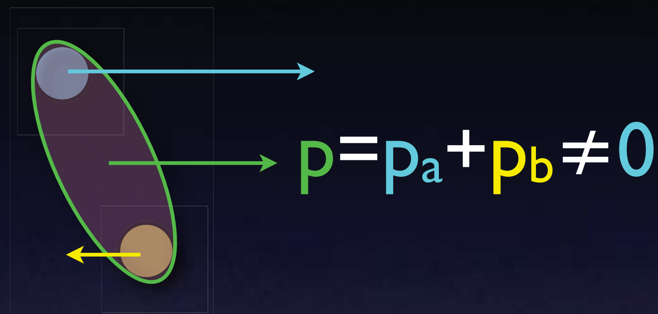
- May allow S-wave pairing
- "Breach" in pairing due to Pauli Blocking
- Still have induced Intra-species P-wave Pairing
- May need large mass ratio or structured interactions (not likely at weak coupling in cold atoms)



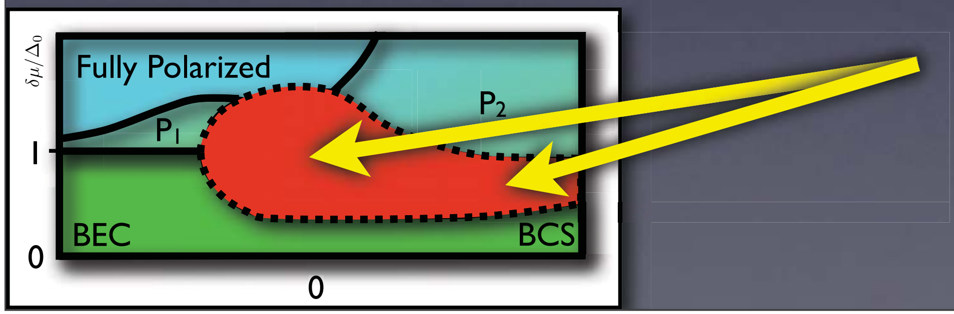
M.M.Forbes, E.Gubankova, W.V.Liu, F.Wilczek (PRL 2005)

Stronger Interactions:
Inhomogenous Superfluid?

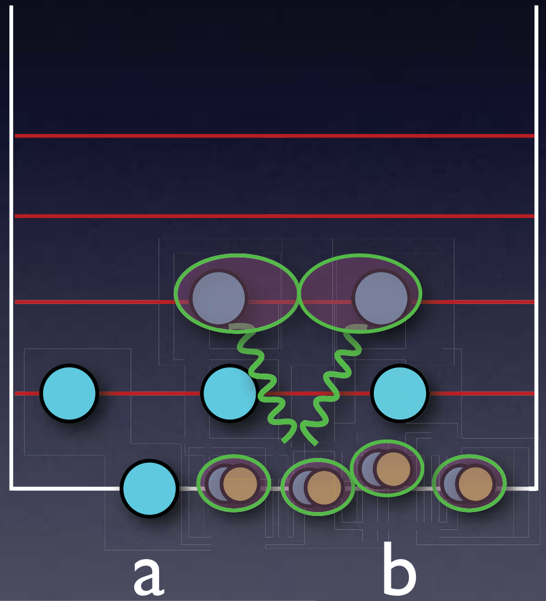




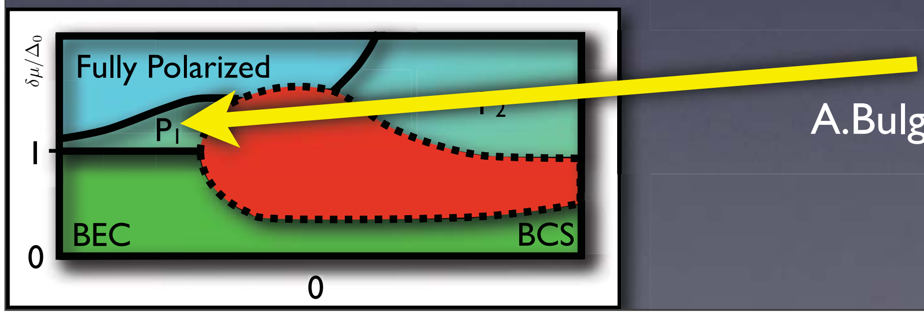
- Pairs have definite momentum
- State (FF) has definite direction
- State (LO) has crystal structure (superposition of momenta)



Strong Interactions:
P-wave superfluid in a BEC

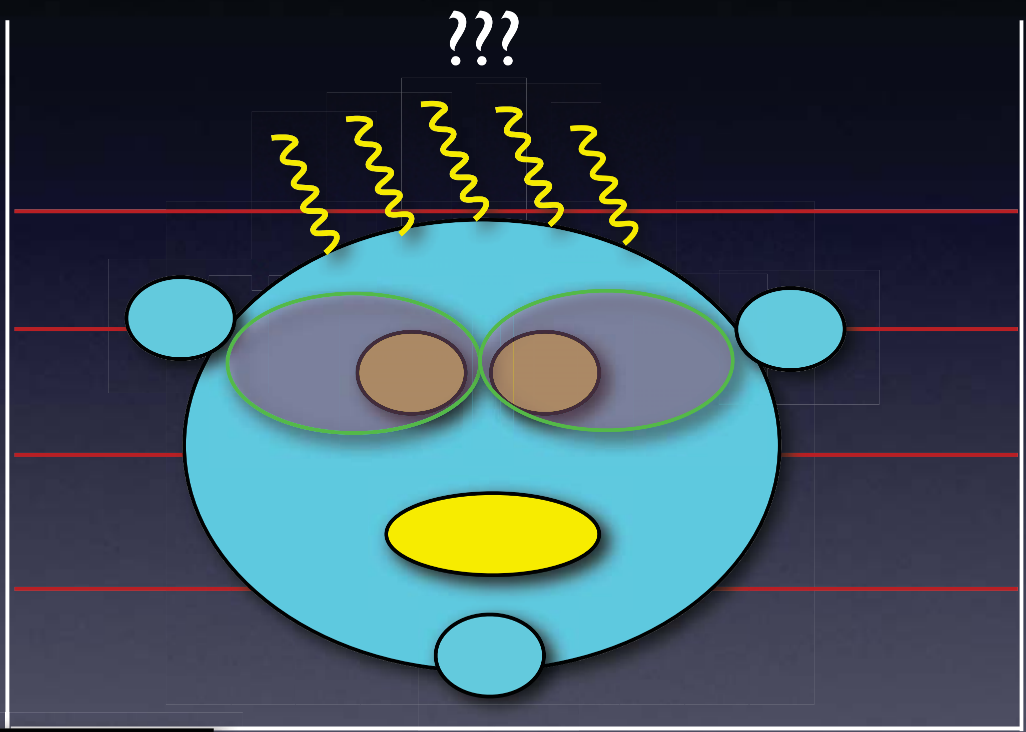


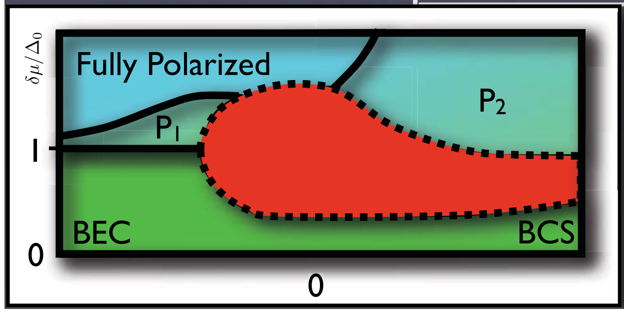
- Induced Intra-species P-wave Cooper Pairing
- BEC and P-wave superfluids coexist homogeneously
- P-wave gap scales as s-wave gap towards unitarity (may become large!)



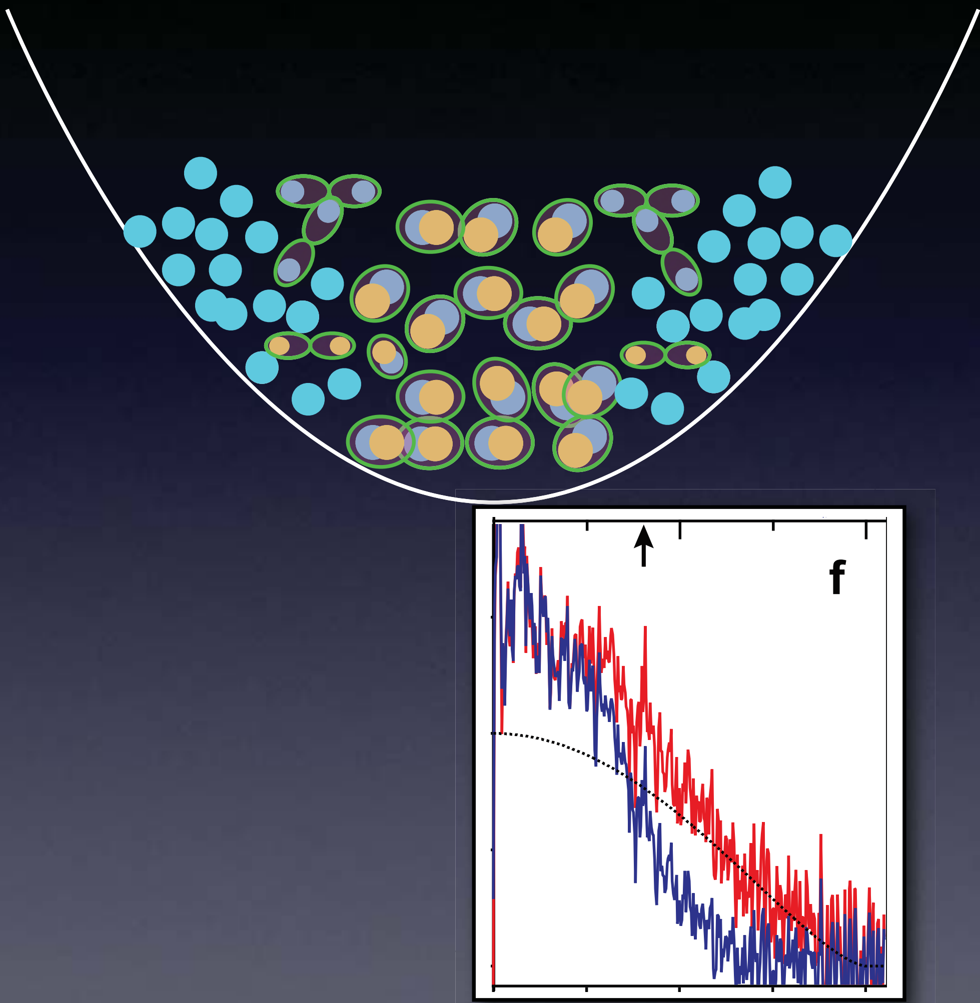
A.Bulgac, M.M.Forbes, A.Schwenk PRL 97 020402 (2006)

Something Totally Different?





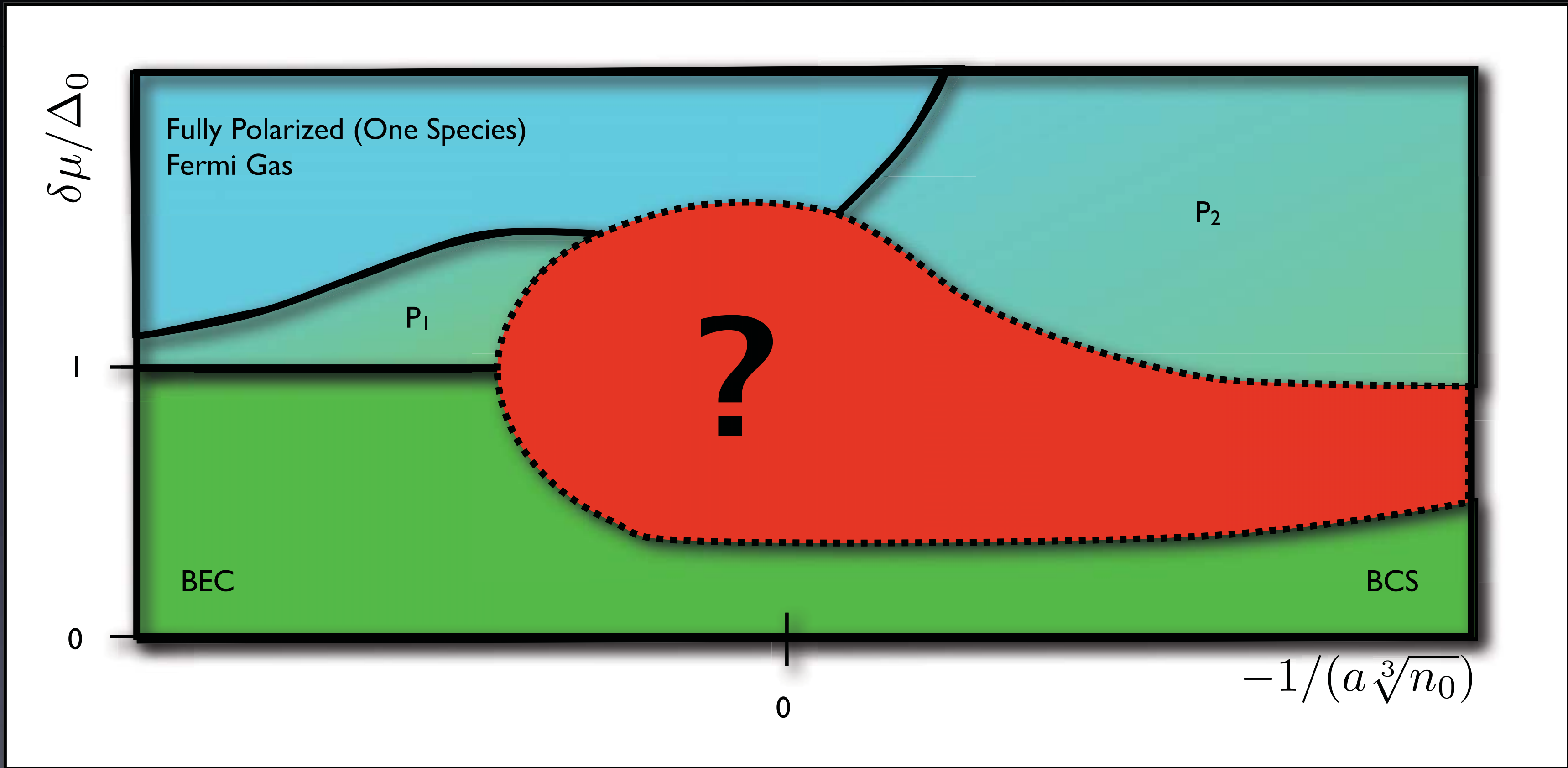
Optical Trap



- Trap separates phases
- Core paired
- Asymmetry at edge
- Large traps are well described by LDA with correct thermodynamics.

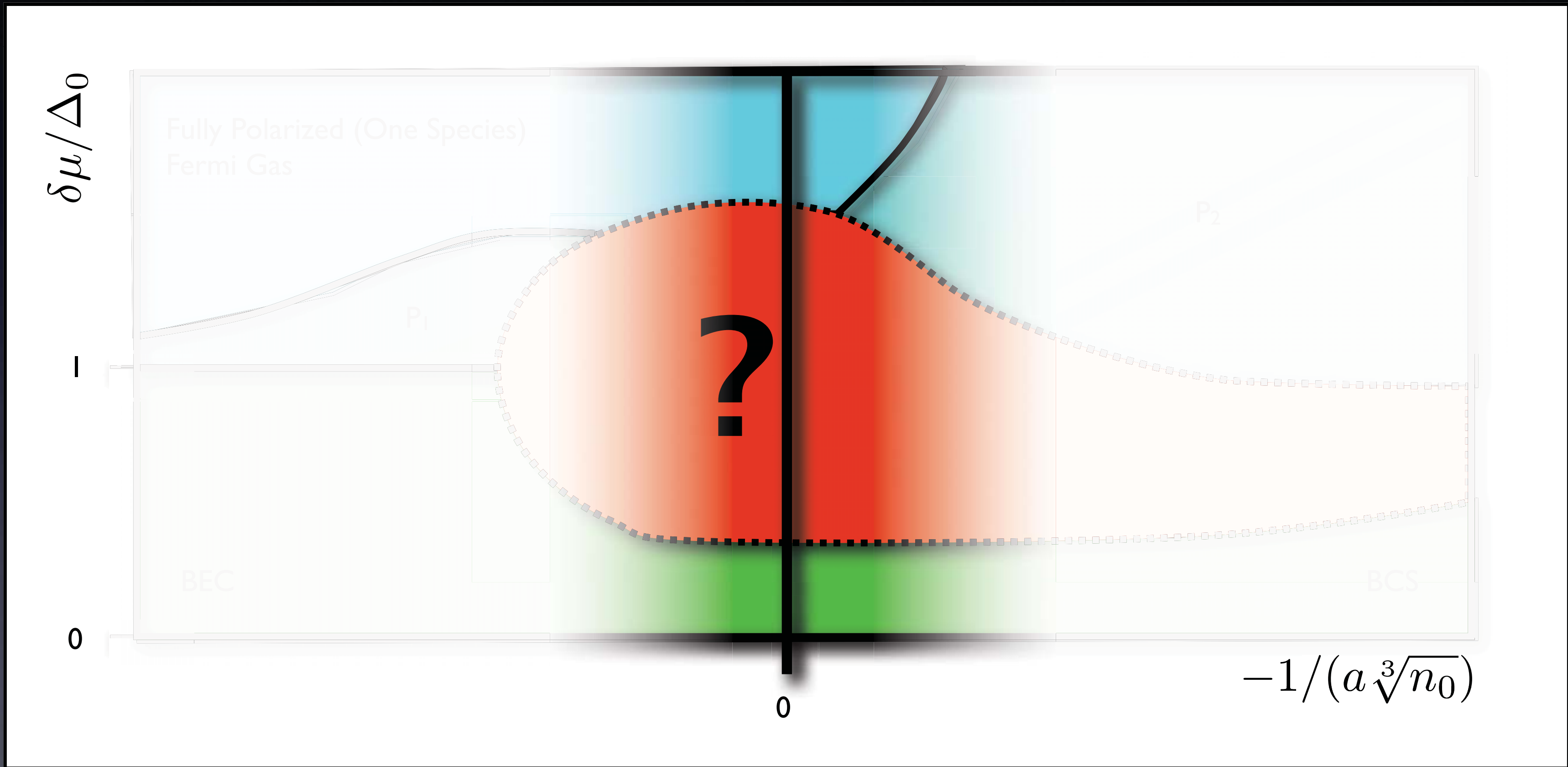
Data from Shin, Schunck, Schirotzek, and Ketterle (2008)

Phase Structure



Based on D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk PRL 97 020402 (2006)

Unitary Regime

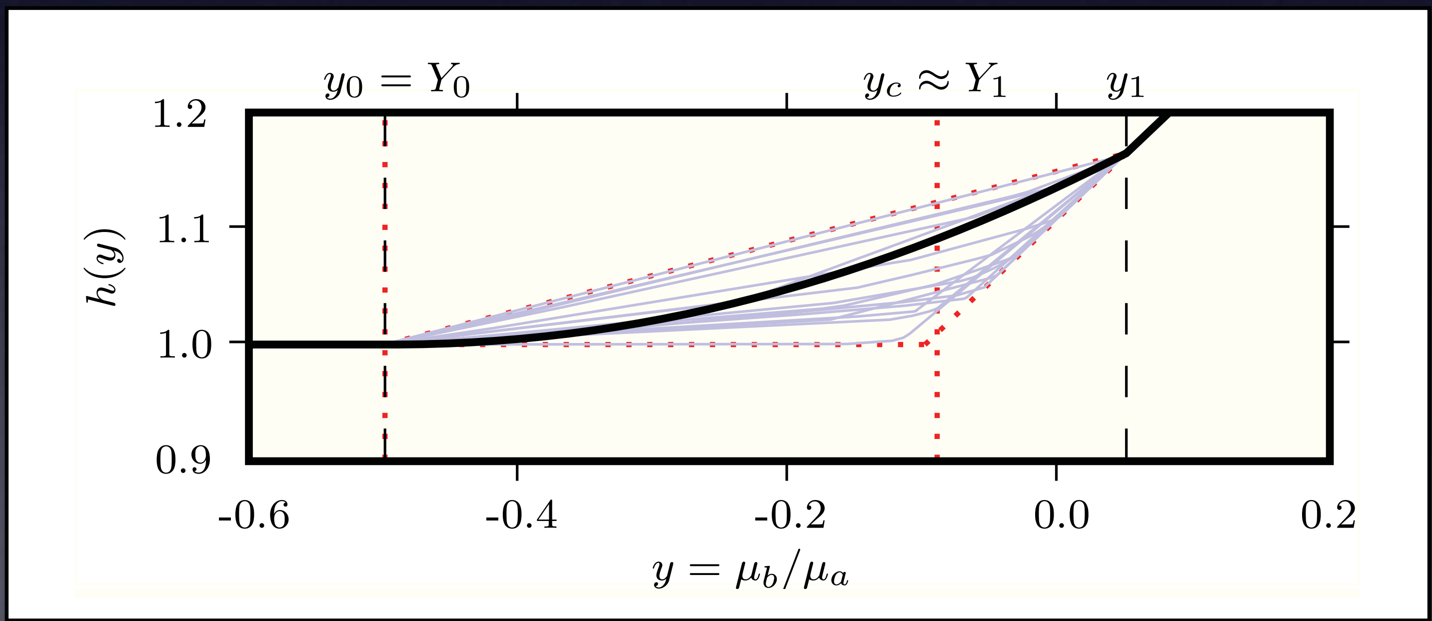


Cold Asymmetric Unitary Gas

$$T = 0,\ a = \infty$$

- Need a single dimensionless function $h(y)$:

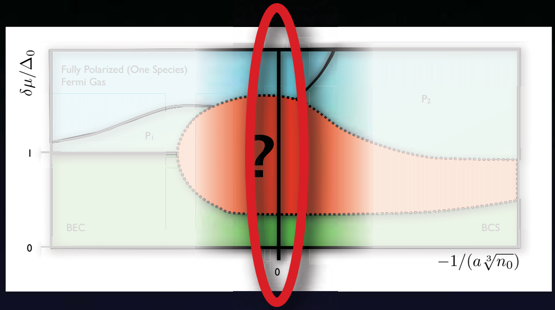
$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$



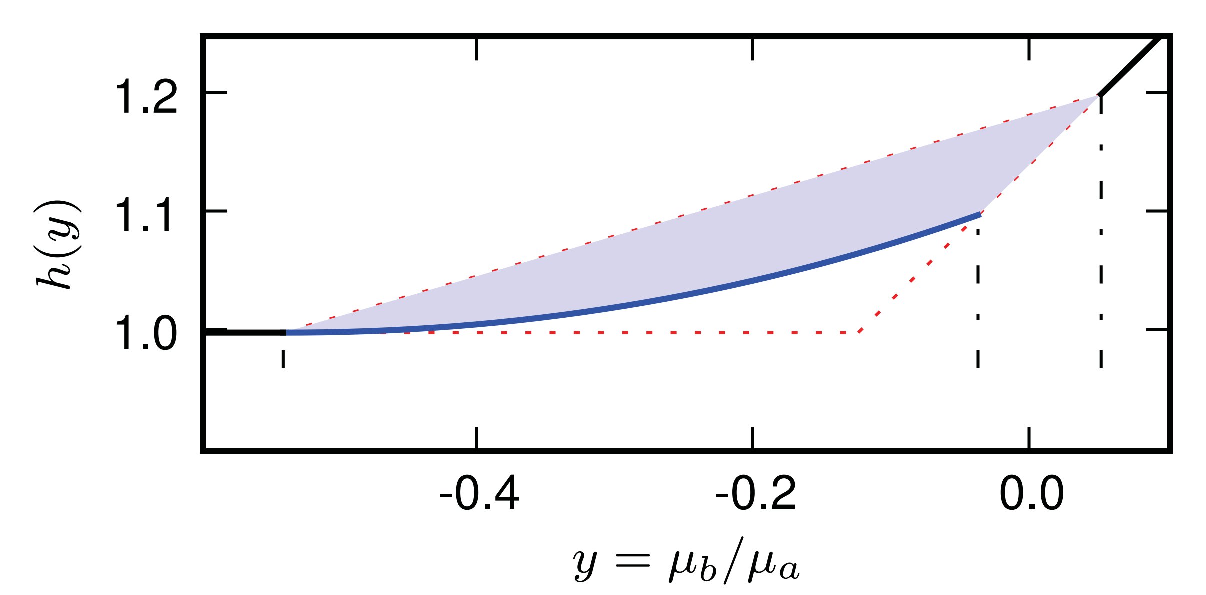
A. Bulgac and M.M. Forbes PRA 75, 031605(R), (2007)

Grand Canonical Ensemble

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$



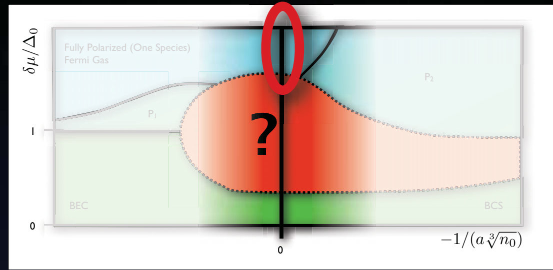
- Legendre transform.
- Only (and all) pure phases.

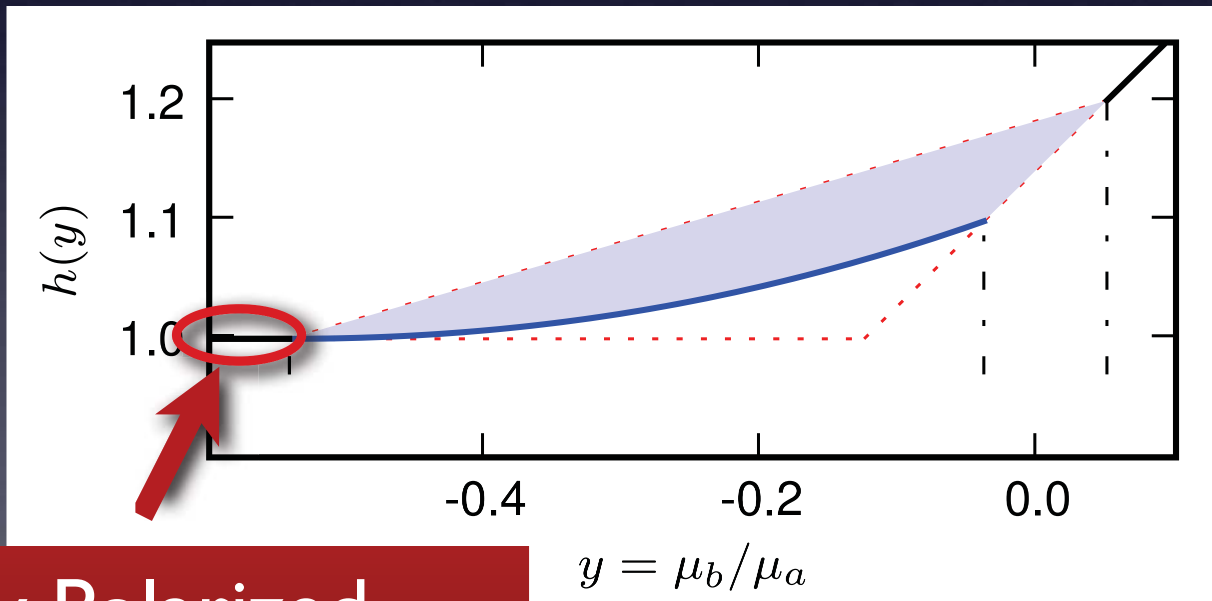


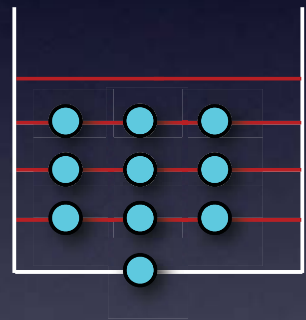
Fully Polarized Gas

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

- Free Fermi gas



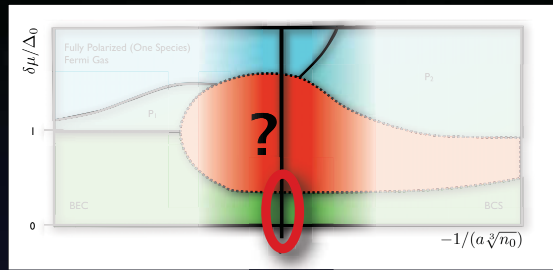




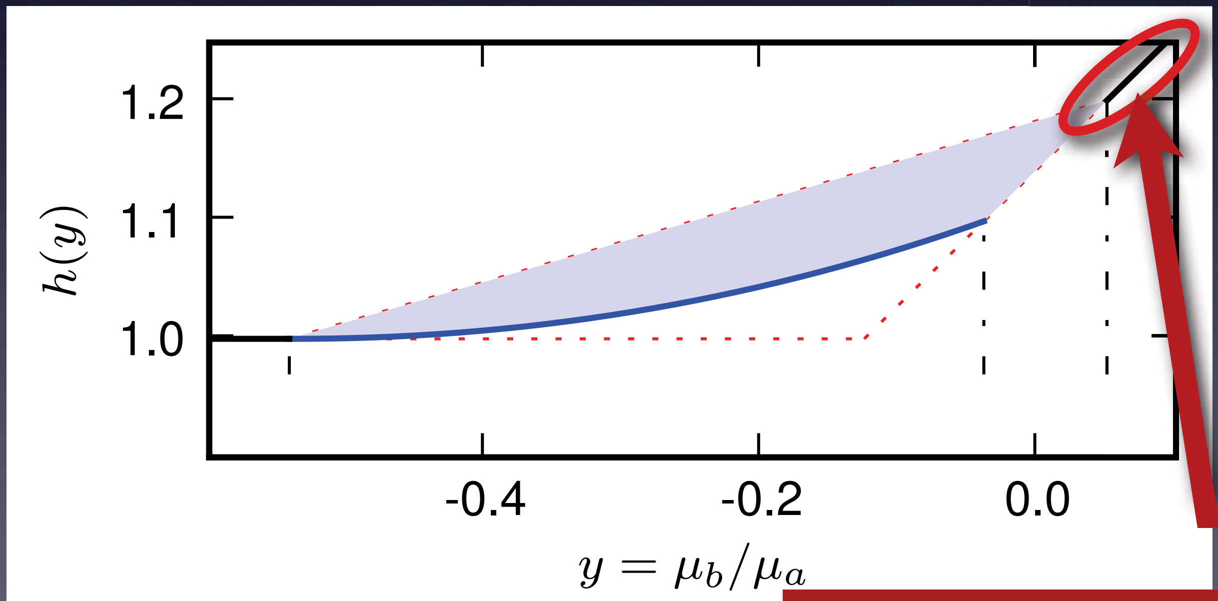
Fully Polarized

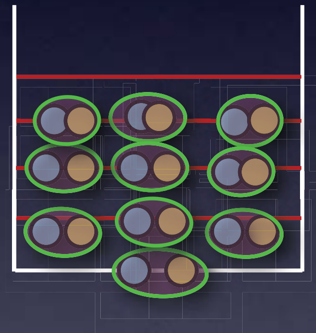
Fully Paired Superfluid

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$



- Well studied with Monte-Carlo
- One input into ASLDA DFT

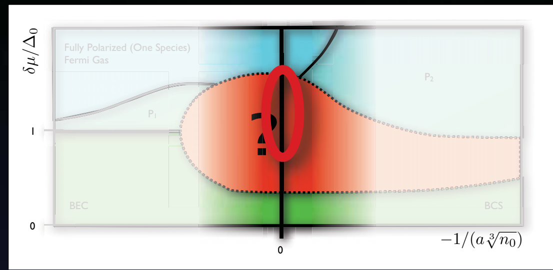




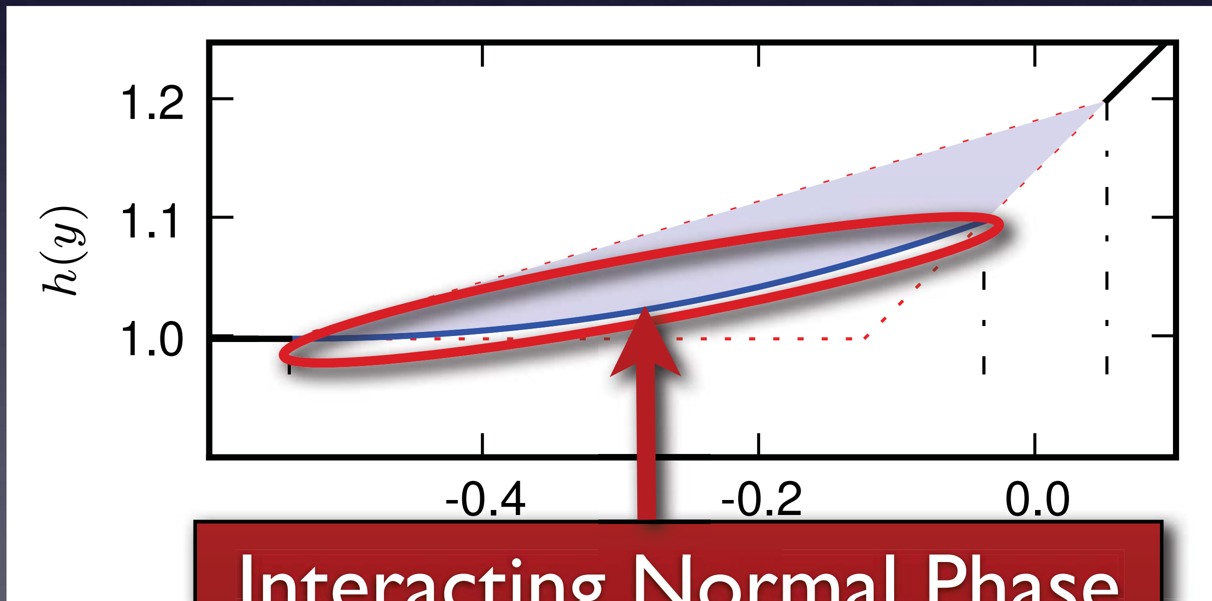
Fully Paired Superfluid

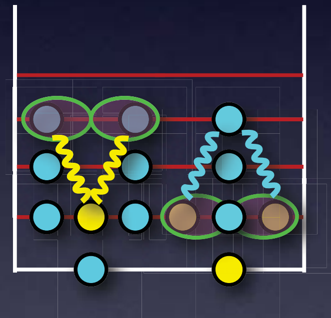
Partially Polarized Phase

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$



- Studied with Fixed Node MC
- Second input for ASLDA DFT

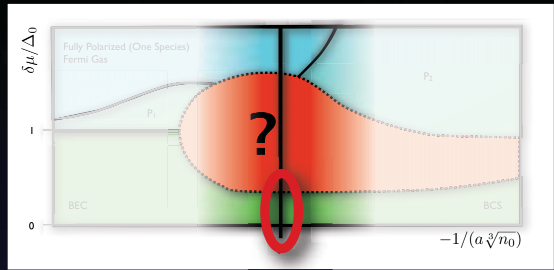




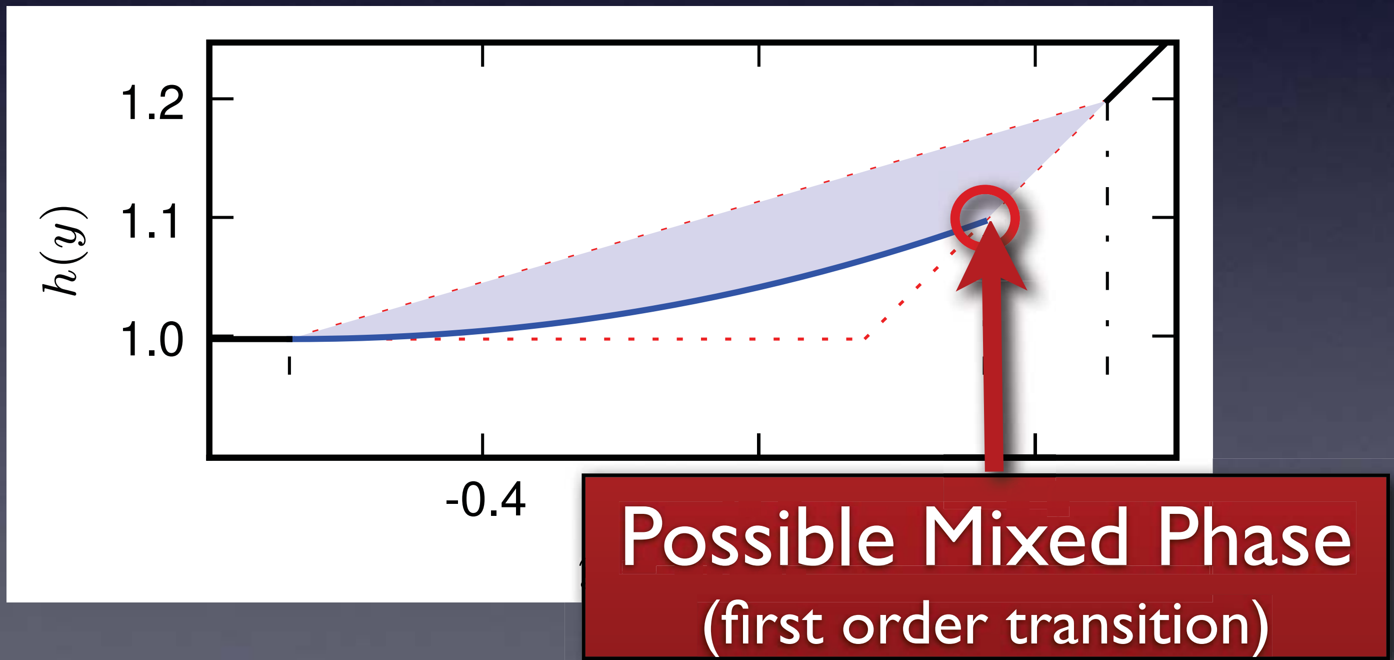
Interacting Normal Phase
(P-wave superconductor?)

Mixed Phase?

$$\mathcal{P}(\mu_a,\mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

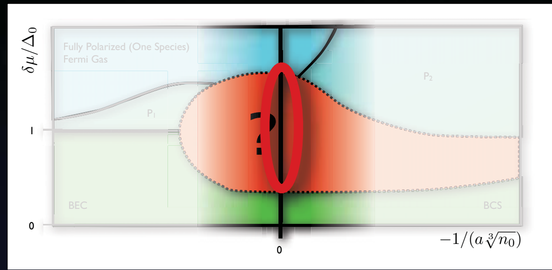


- Possible phase coexistence at kink (First order phase transition)

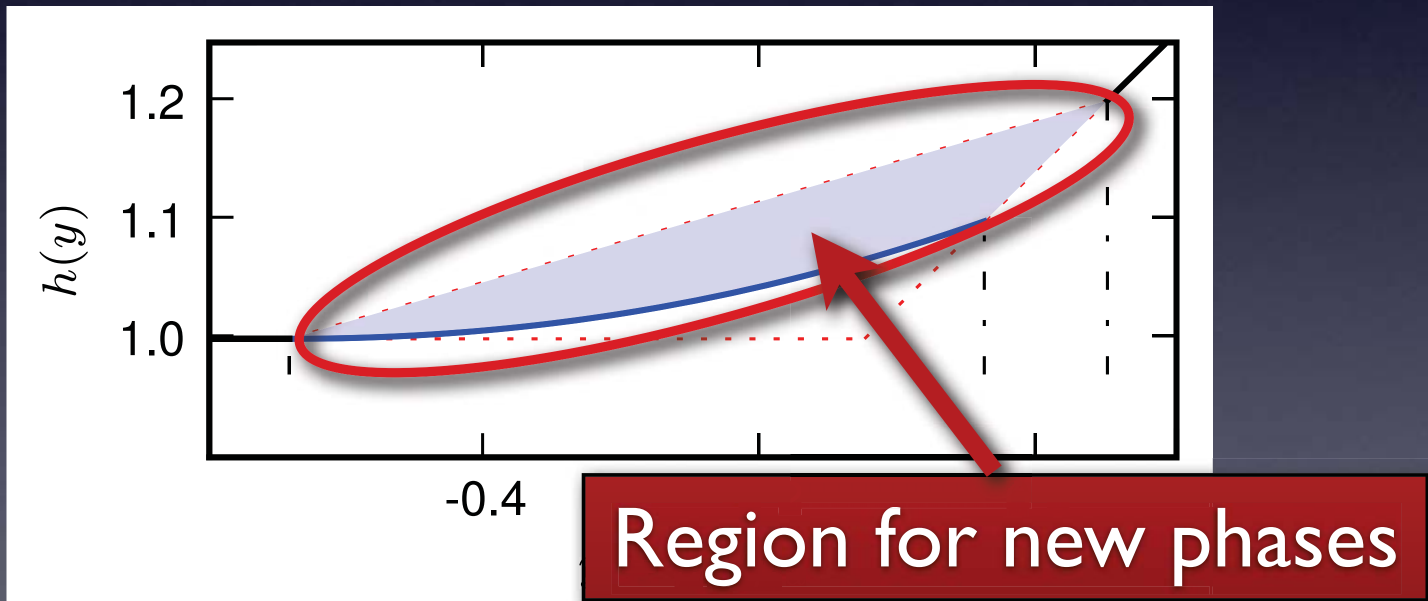


New Phases?

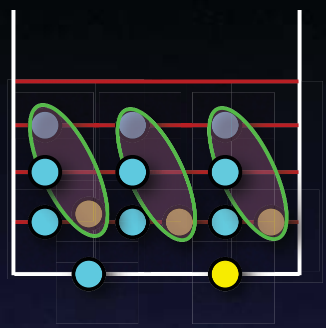
$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$



- Other phases could increase pressure (lower energy)



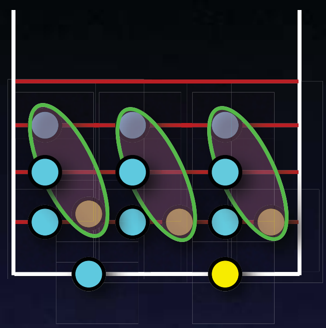
Possible Phases



- Symmetric Phases:
  - BCS crossover to BEC of dimers
- Asymmetric Phases:
  - Fermi gas with P-wave pairing
  - Inhomogeneous pairing (LOFF)
  - “Gapless superfluids” (with P-wave)
  - BEC coexisting with P-wave pairing
  - Exotica???

Fulde Ferrell (FF) State

$$\Delta(\vec{\mathbf{x}}) = \Delta_0 e^{i\vec{\mathbf{q}}\cdot\vec{\mathbf{x}}}$$

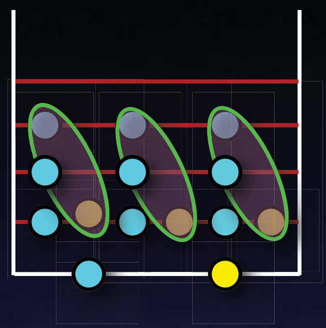


- Homogeneous
- Breaks rotational invariance
  - Carries super-current along $\vec{\mathrm{q}}$ countered by opposite normal current.

P. Fulde, R.A. Ferrell (1964)

Larkin Ovchinnikov (LO) State

$$\Delta(\vec{\mathbf{x}}) = \Delta_0 \sum_n c_n e^{i\vec{\mathbf{q}}_n \cdot \vec{\mathbf{x}}}$$



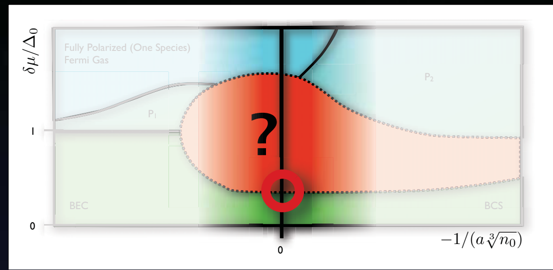
- Superposition of plane waves.
- Inhomogeneous.
- Gap has nodes.
- Density modulates (crystal structure).

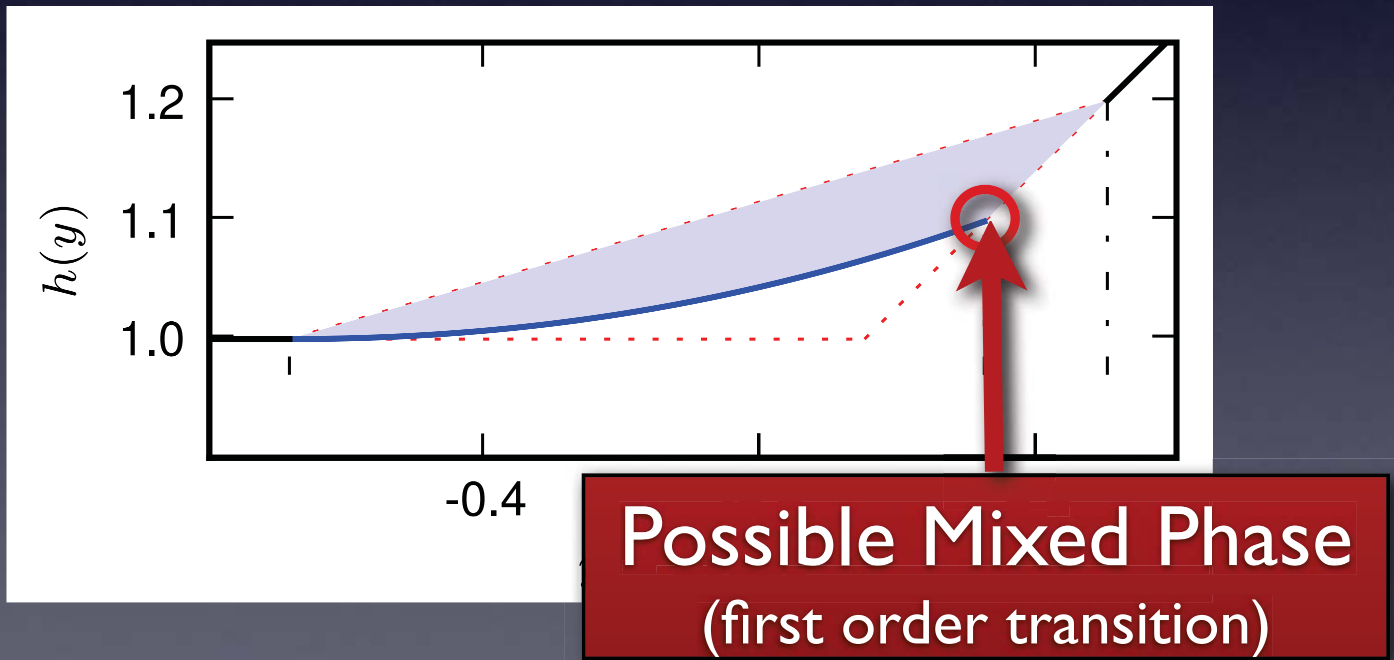
A.I. Larkin and Y.N. Ovchinnikov (1965)

Mixed Phase?

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

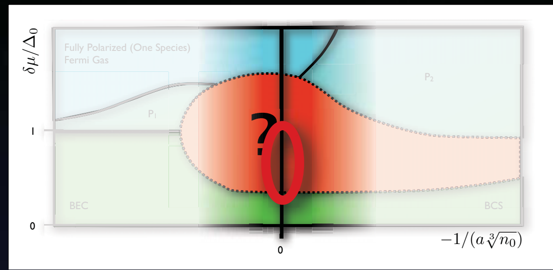
- Possible phase coexistence at kink



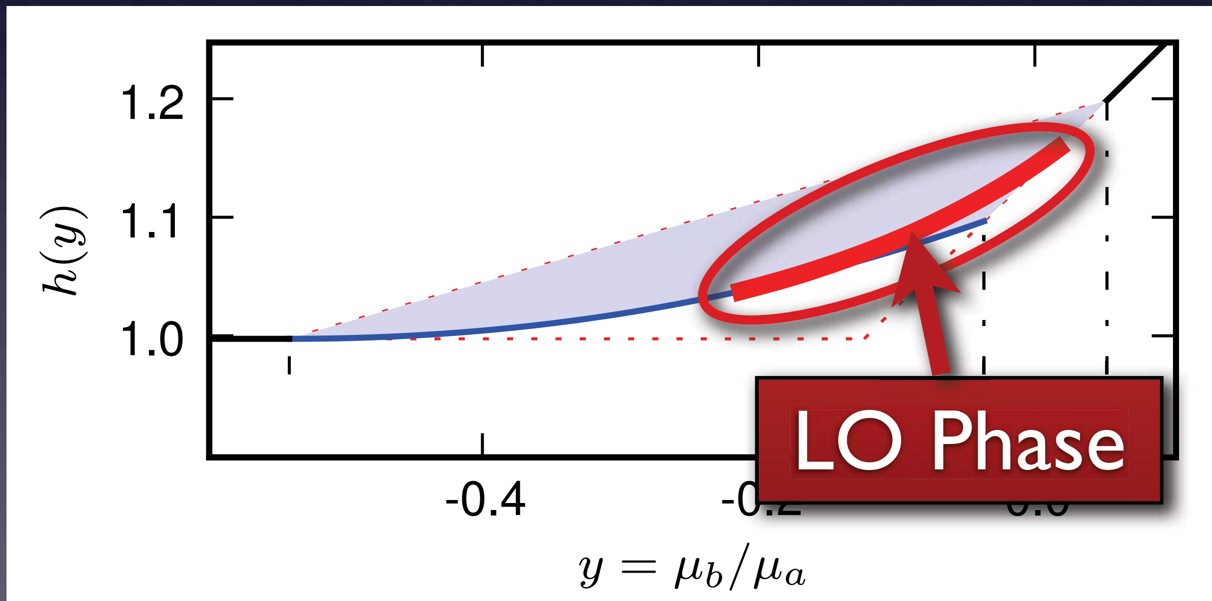


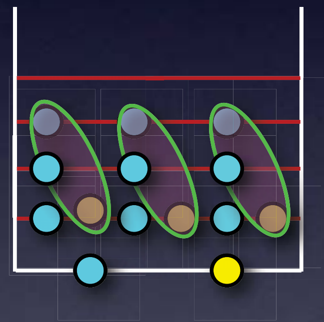
LOFF Phase

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$



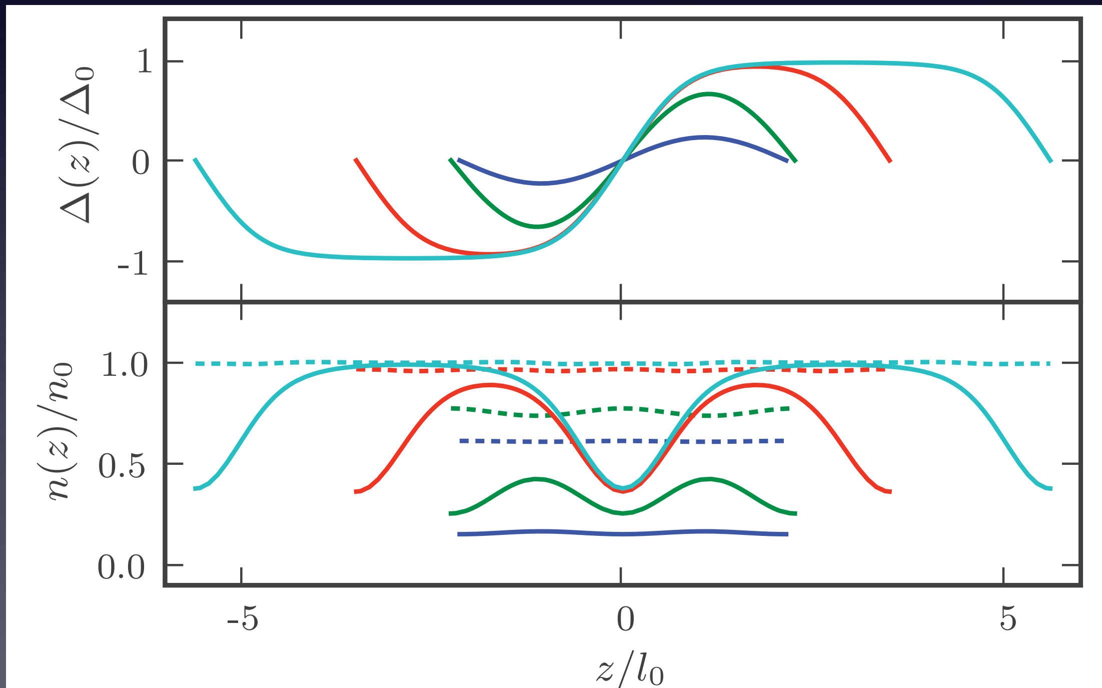
- In ASLDA DFT, LOFF phase beats mixed phase





Larkin Ovchinnikov (LO) State at Unitarity

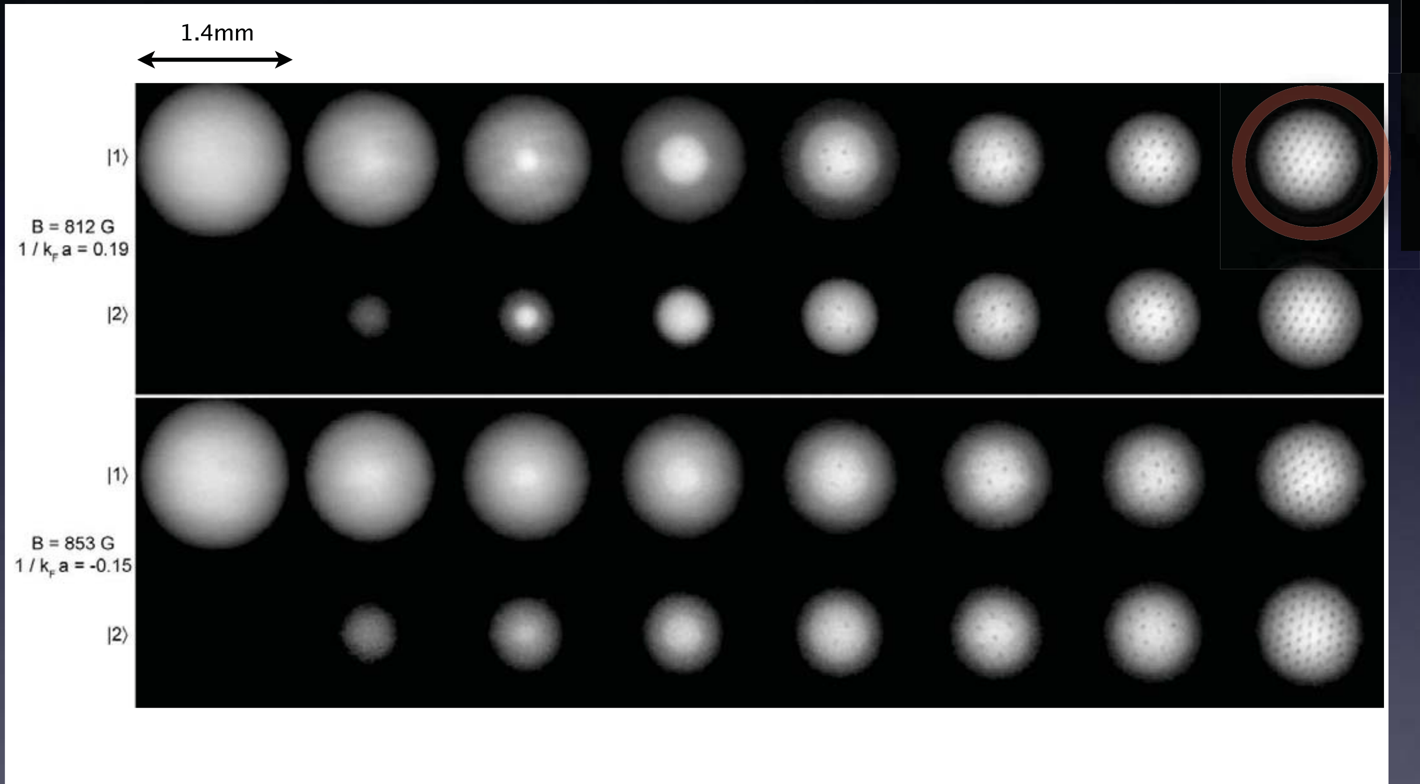
- Large density fluctuations (factor of 2).

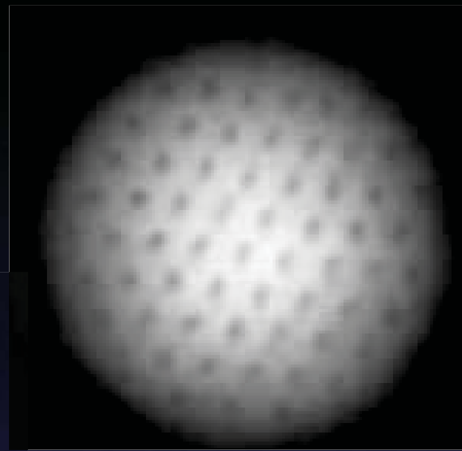


A. Bulgac, M.M. Forbes PRL 101 (2008) 215301

Density Fluctuations are Observable

MIT Vortex Data





M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle (2006)

Density Functional Theory (DFT)

- Ground state density in any external potential $V(x)$ can be found by minimizing the functional

$$\int \mathrm{d}^3x \Big\{ \mathcal{E}[n(x)] + n(x)V(x) \Big\}$$

- Functional may be complicated!
- Local Density Approximation (LDA)
- Kohn-Sham introduces kinetic term.

LDA:

Local Density Approximation

$$E = \int \mathrm{d}^3\vec{\mathbf{r}} \Big\{ \mathcal{E}\big(n(\vec{\mathbf{r}})\big) + n(\vec{\mathbf{r}}) V_{\text{ext}}(\vec{\mathbf{r}}) + \text{sources} \ldots \Big\}$$

- Universal local energy density:

$$\mathcal{E}(n) = \beta \frac{(3\pi^2 n)^{5/3}}{10 m \pi^2}$$

- Lack of scales completely constraints form!
- But... want to reproduce qualitative BCS behaviour
  - Quasi-particles, kinetic energy, etc.
  - Pairing, Superfluidity, Finite size effects

SLDA:

Superfluid Local Density Approximation

$$E = \int \mathrm{d}^3\vec{\mathbf{r}}\Big\{\mathcal{E}\big(n(\vec{\mathbf{r}}), \tau(\vec{\mathbf{r}}), \nu(\vec{\mathbf{r}})\big) + n(\vec{\mathbf{r}})V_{\text{ext}}(\vec{\mathbf{r}}) + \text{sources}\ldots\Big\}$$

- Universal local energy density:

$$\mathcal{E}(n, \tau, \nu) = \alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- Local Densities:
  - Standard:
  - Kinetic:
  - Anomalous (pairing):

$$n = \langle\widehat{\mathbf{a}}^\dagger\widehat{\mathbf{a}}\rangle + \langle\widehat{\mathbf{b}}^\dagger\widehat{\mathbf{b}}\rangle$$

$$\tau = \langle\nabla\widehat{\mathbf{a}}^\dagger\nabla\widehat{\mathbf{a}}\rangle + \langle\nabla\widehat{\mathbf{b}}^\dagger\nabla\widehat{\mathbf{b}}\rangle$$

$$\nu = \langle\widehat{\mathbf{a}}\widehat{\mathbf{b}}\rangle$$

BdG: A type of DFT

- BdG has no self-energy: $\beta = 0$
- BdG has unit effective mass: $\alpha = 1$

$$\mathcal{E}(n,\tau,\nu) = \alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

$$n = \langle \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} \rangle + \langle \hat{\mathbf{b}}^\dagger \hat{\mathbf{b}} \rangle$$

$$\tau = \langle \nabla\hat{\mathbf{a}}^\dagger \nabla\hat{\mathbf{a}} \rangle + \langle \nabla\hat{\mathbf{b}}^\dagger \nabla\hat{\mathbf{b}} \rangle$$

$$\nu = \langle \hat{\mathbf{a}}\hat{\mathbf{b}} \rangle$$

SLDA Parameters:

Superfluid Local Density Approximation

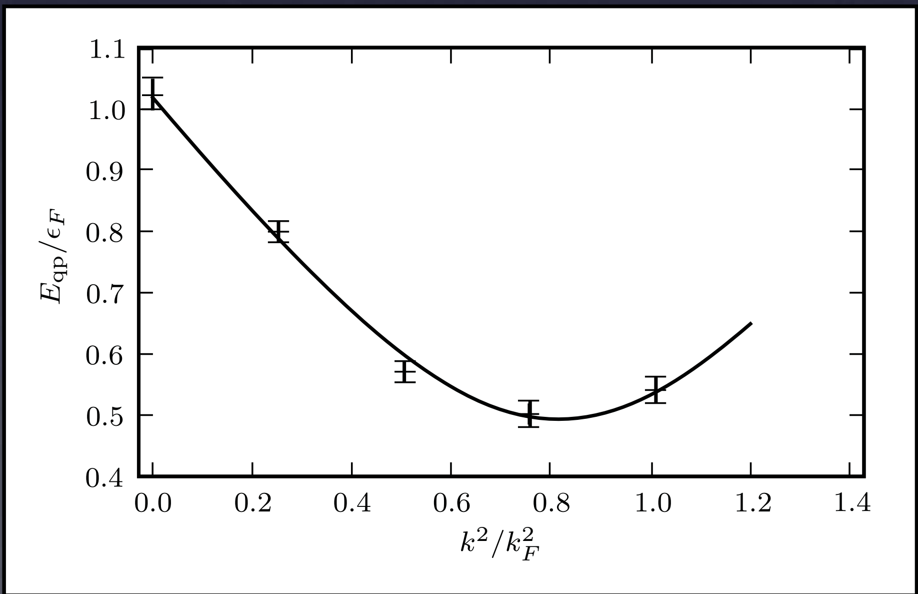
- Local energy density:

$$\alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- 3 Undetermined dimensionless parameters: α, β, γ
  - Effective mass: $m_{\text{eff}} = m/\alpha$
  - Self-energy: β
  - Pairing interaction: $g_{\text{eff}}^{-1} = n^{1/3}/\gamma + \Lambda$
- Fit parameters to Monte-Carlo data

Determining Parameters: Quasiparticle Dispersion

- Determine parameters by fitting Quasi-particle dispersion relations in superfluid and $\xi = 0.40(1)$
- <u>Bonus:</u> Form of single-particle dispersion is correct!



$$\alpha = 1.09(2)$$
$$\beta = -0.526(18)$$
$$\gamma = 11.0(9)$$

Data from J. Carlson, S. Reddy (2006)

Kohn-Sham Equations

$$\begin{pmatrix} \widehat{\mathbf{K}} + V(\vec{\mathbf{r}}) - \mu_a & \Delta(\vec{\mathbf{r}}) \\ \Delta^\dagger(\vec{\mathbf{r}}) & -\widehat{\mathbf{K}} - V(\vec{\mathbf{r}}) + \mu_b \end{pmatrix} \cdot \begin{pmatrix} u_n(\vec{\mathbf{r}}) \\ v_n(\vec{\mathbf{r}}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\vec{\mathbf{r}}) \\ v_n(\vec{\mathbf{r}}) \end{pmatrix}$$

- Minimize density functional.
- Fix particle number through chemical potentials $\mu_{a,b}$.

$$\frac{\delta}{\delta n, \tau, \cdots} E[n, \tau, \cdots] = 0$$

- Grand canonical ensemble where ground state maximizes pressure.
  - Only pure phases: (no phase coexistence).

$$\mathcal{P} = \mu_a n_a + \mu_b n_b - \frac{E}{V}$$

- BdG like form, but with self-energy interactions.

$$\widehat{\mathbf{K}} = -\tfrac{\hbar^2}{m}\alpha\frac{\nabla^2}{2}$$

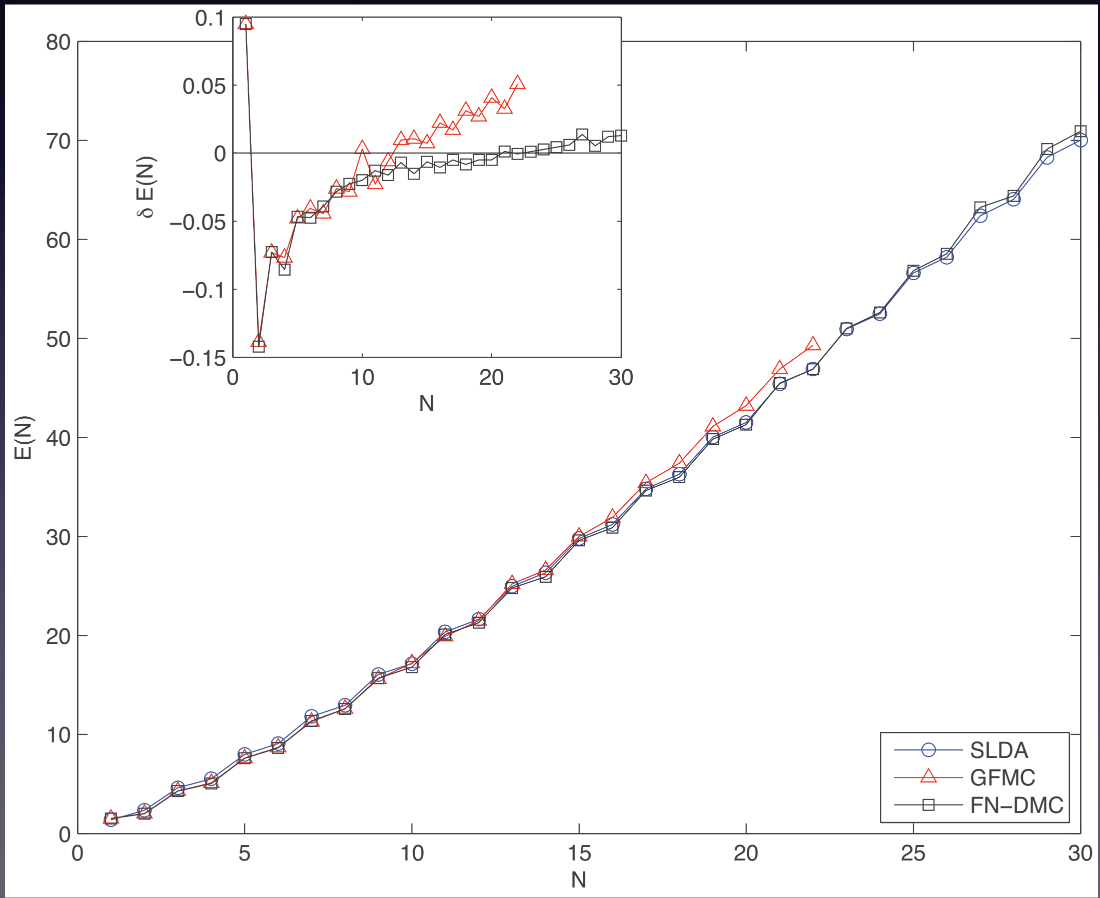
$$\tfrac{m}{\hbar^2}V = \beta\frac{(3\pi^2 n)^{2/3}}{2m} - \frac{\Delta^\dagger\Delta}{n^{2/3}\gamma}$$

SLDA:

- DFT (LDA) works well (5% level) in Chemistry for ground state properties
  - Have to work very hard to get excited state properties (Beyond LDA)
- How well does it work for Cold Atoms?
  - Fit to thermodynamic limit.
  - Makes "predictions" for finite systems.
  - Compare with MC data for trapped systems.

Testing the SLDA:

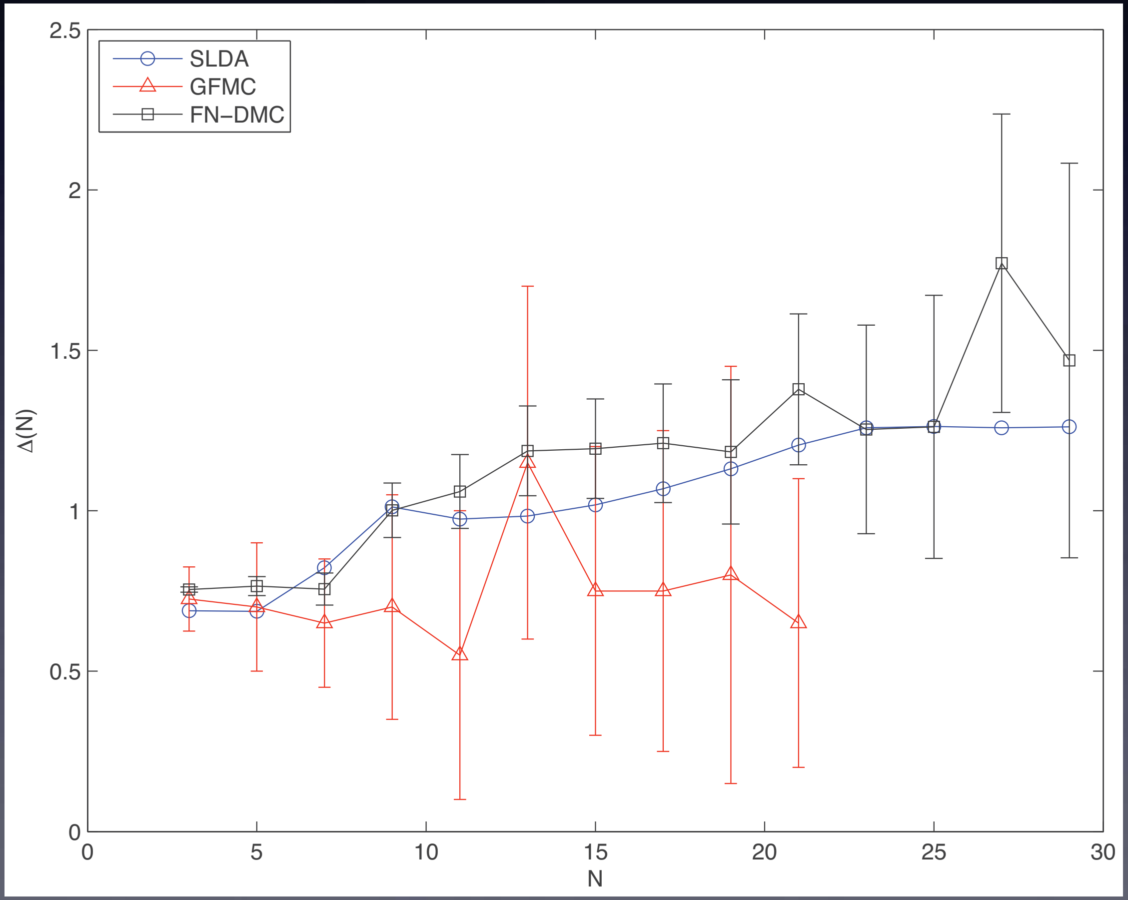
- Energies fit very well (few percent level) even for extremely small systems!



SLDA:
Bulgac (2007)
GFMC:
Chang and Bertsch (2007)
FN-DMC:
von Stecher, Greene, Blume (2007)

Testing the SLDA:

- Pairing gaps also fit very well!



SLDA:
Bulgac (2007)
GFMC:
Chang and Bertsch (2007)
FN-DMC:
von Stecher, Greene, Blume (2007)

SLDA

- Agreement at the few percent level.

- Bonus agreement with normal state:
  - SLDA functional predicts: ξ_N=0.567(24)
  - MC has computed this: ξ_N=0.55(2)

- Implies SLDA form good for near symmetric systems.

- Why not better?
  - Functional not unique
  - Gradients have been omitted...
    - but the good agreement implies that these are very small!

SLDA:

Superfluid Local Density Approximation

- Local energy density:

$$\alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- 3 Undetermined dimensionless parameters: α, β, γ
  - Effective mass: $m_{\text{eff}} = m/\alpha$
  - Self-energy: β
  - Pairing interaction: $g_{\text{eff}}^{-1} = n^{1/3}/\gamma + \Lambda$
- Fit parameters to Monte-Carlo data

$$x = \frac{n_b}{n_a}$$

ASLDA:

Asymmetric Superfluid Local Density Approximation

- Local energy density:

$$\frac{\tau_a}{2m_a(n_a,n_b)} + \frac{\tau_b}{2m_b(n_a,n_b)} + D(n_a,n_b) + g_{\text{eff}}(n_a,n_b)\nu^\dagger\nu$$

- Local Densities:
  - Standard:
  - Kinetic:
  - Anomalous (pairing):

$$n_a = \langle \widehat{\mathbf{a}}^\dagger \widehat{\mathbf{a}} \rangle \,, \qquad n_b = \langle \widehat{\mathbf{b}}^\dagger \widehat{\mathbf{b}} \rangle$$

$$\tau_a = \langle \nabla\widehat{\mathbf{a}}^\dagger \nabla\widehat{\mathbf{a}} \rangle \,, \qquad \tau_b = \langle \nabla\widehat{\mathbf{b}}^\dagger \nabla\widehat{\mathbf{b}} \rangle$$

$$\nu = \langle \widehat{\mathbf{a}}\widehat{\mathbf{b}} \rangle$$

$x = \frac{n_b}{n_a}$

ASLDA:

Asymmetric Superfluid Local Density Approximation

- Local energy density:

$$\frac{\tau_a}{2m_a(n_a, n_b)} + \frac{\tau_b}{2m_b(n_a, n_b)} + D(n_a, n_b) + g_{\text{eff}}(n_a, n_b)\nu^\dagger \nu$$

- 3 Undetermined functions: $\alpha(x), \quad \beta(x), \quad \gamma(x)$
  - Effective masses: $m_a = m/\alpha(x), \quad m_b = m/\alpha(x^{-1})$
  - Self-energy: $\beta(x)$
  - Pairing interaction: $g_{\text{eff}}^{-1} = (n_a + n_b)^{1/3}/\gamma(x) + \Lambda$
    - For simplicity, we take $\gamma(n_b/n_a) = \gamma$ (with value as in SLDA)

Kohn-Sham Equations

$$\begin{pmatrix} \widehat{\mathbf{K}}_a + V_a(\vec{\mathbf{r}}) - \mu_a & \Delta(\vec{\mathbf{r}}) \\ \Delta^\dagger(\vec{\mathbf{r}}) & -\widehat{\mathbf{K}}_b - V_b(\vec{\mathbf{r}}) + \mu_b \end{pmatrix} \cdot \begin{pmatrix} u_n(\vec{\mathbf{r}}) \\ v_n(\vec{\mathbf{r}}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\vec{\mathbf{r}}) \\ v_n(\vec{\mathbf{r}}) \end{pmatrix}$$

- Non-standard kinetic term due to varying effective mass.
- Different potential for each species.

$$\widehat{\mathbf{K}}_a u = -\tfrac{\hbar^2}{m}\nabla_i(\alpha_a(n_a,n_b)\nabla_i u)/2$$

$$\widehat{\mathbf{K}}_b u = -\tfrac{\hbar^2}{m}\nabla_i(\alpha_b(n_a,n_b)\nabla_i u)/2$$

$$\begin{aligned} \tfrac{m}{\hbar^2}V_a = {} & \alpha^-_{,a}(n_a,n_b)\frac{\tau_-}{2} + D_{,a}(n_a,n_b) \\ & - C_{,a}(n_a,n_b)\Delta^\dagger\Delta + \\ & - \frac{\alpha^+_{,a}(n_a,n_b)}{\alpha^+(n_a,n_b)}C(n_a,n_b)\Delta^\dagger\Delta + \\ & + \frac{\alpha^+_{,a}(n_a,n_b)}{\alpha^+(n_a,n_b)}\left(\alpha^+(n_a,n_b)\frac{\tau_+}{2} - \Delta^\dagger\nu\right) \end{aligned}$$

$$\alpha^\pm(n_a,n_b) = \tfrac{1}{2}[\alpha_a(n_a,n_b) \pm \alpha_b(n_a,n_b)],$$

$$\tau_\pm = \tau_a \pm \tau_b,$$

$$C(n_a,n_b) = (n_a+n_b)^{1/3}/\gamma(n_b/n_a)$$

Fitting Parameters

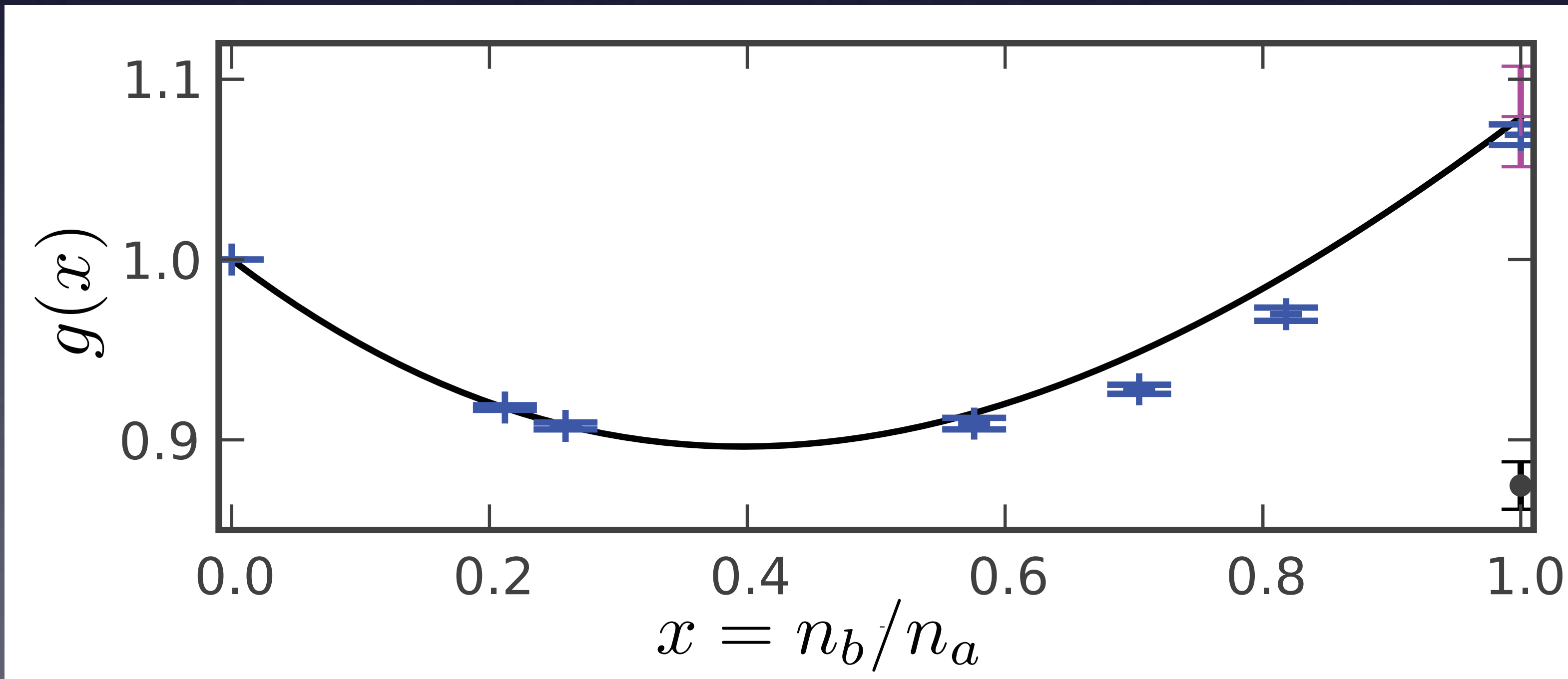
$$\alpha(x), \quad \beta(x), \quad \gamma(x)$$

- Fit the parameters to reproduce spatially uniform thermodynamic phases:
  - Normal polarized Fermi gas
  - Fully paired symmetric superfluid
- Use DFT to describe spatially varying phases:
  - Finite size systems (trapped gases)
  - Phases with density variations

Functional Forms: β(x)

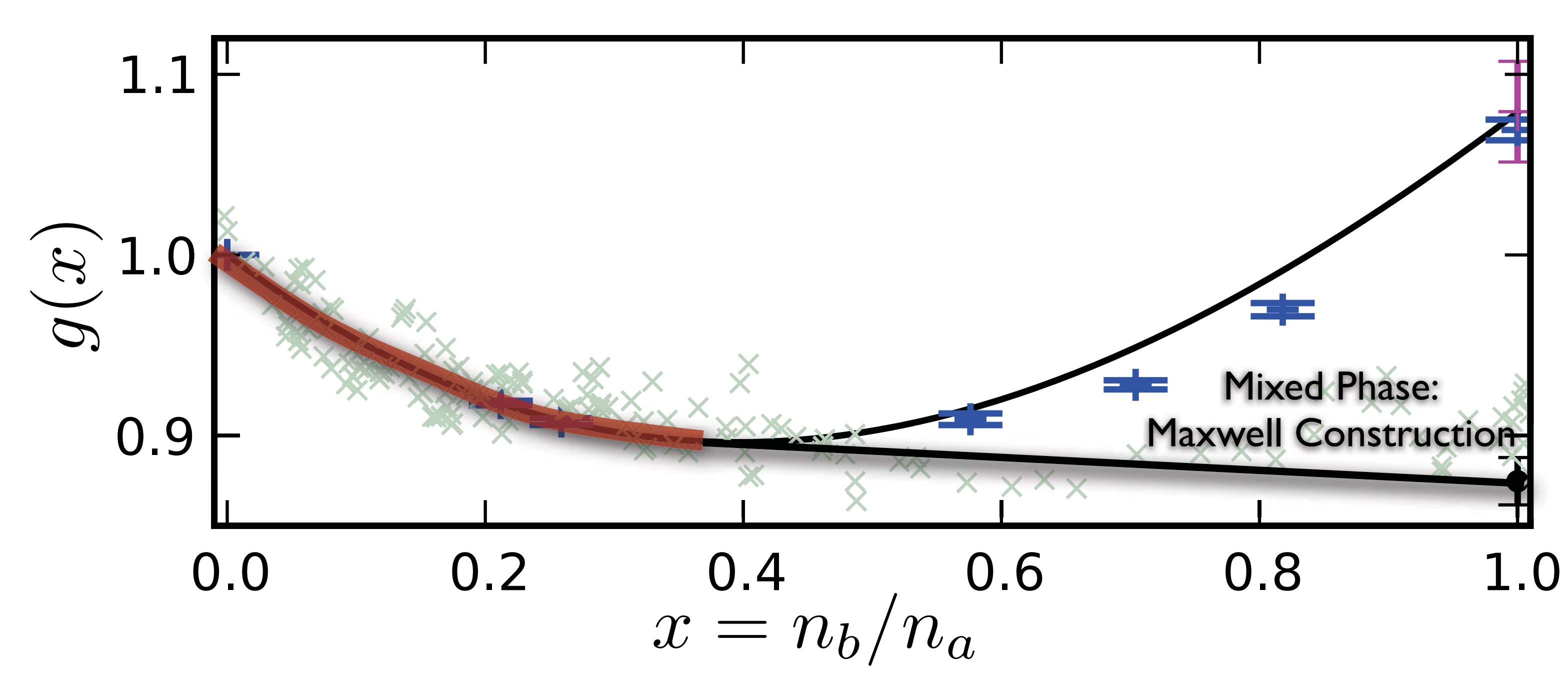
- Fit to energy of Normal polarized gas in thermodynamic limit.
  - Uniquely determines β(x) once mass is specified.

Data from Lobo et. al (2006)



Homogeneous States

- Canonical ensemble (fixed densities)
  - Interacting normal phase (P-wave superfluid?)
  - Fully paired superfluid
  - Mixed phase: (phase coexistence between these two phases.)



MIT Experimental data from Shin (2008), MC data from Lobo et. al (2006)

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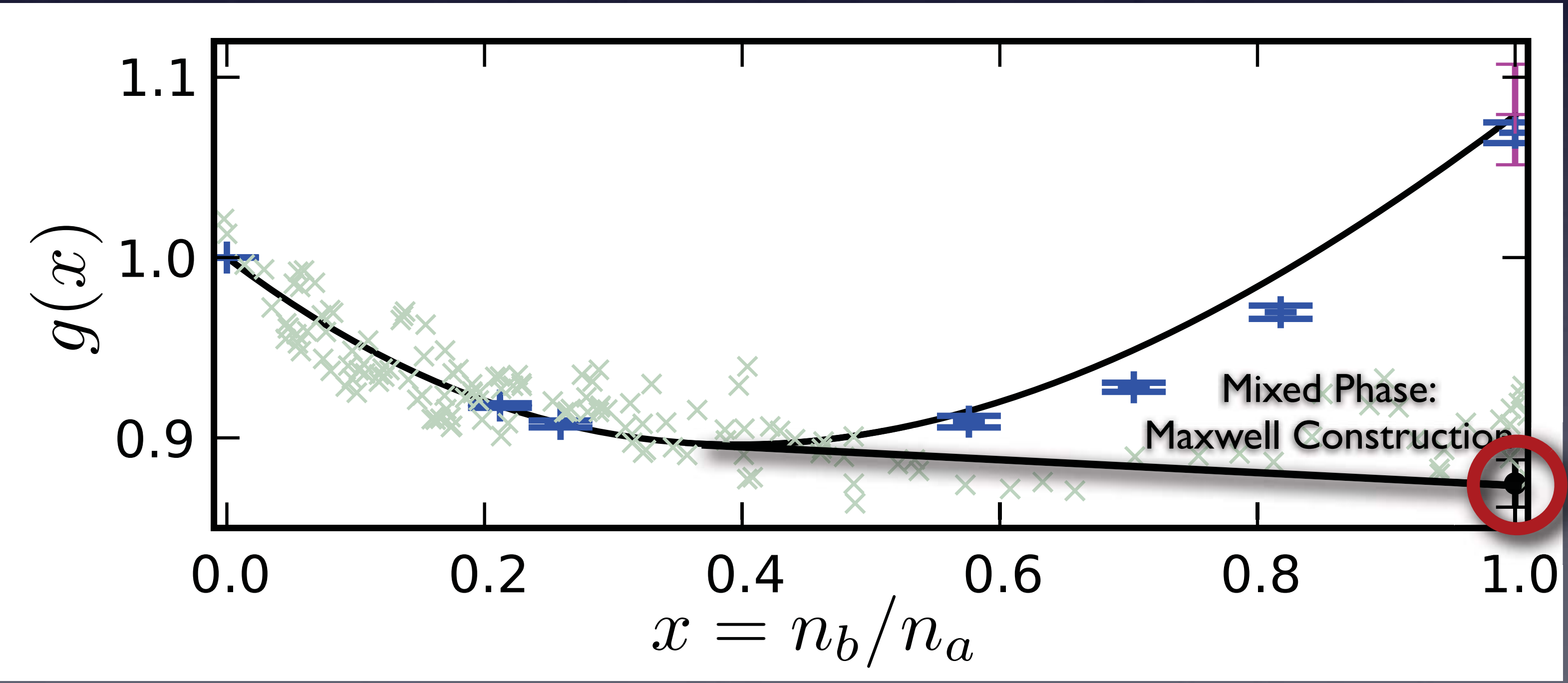

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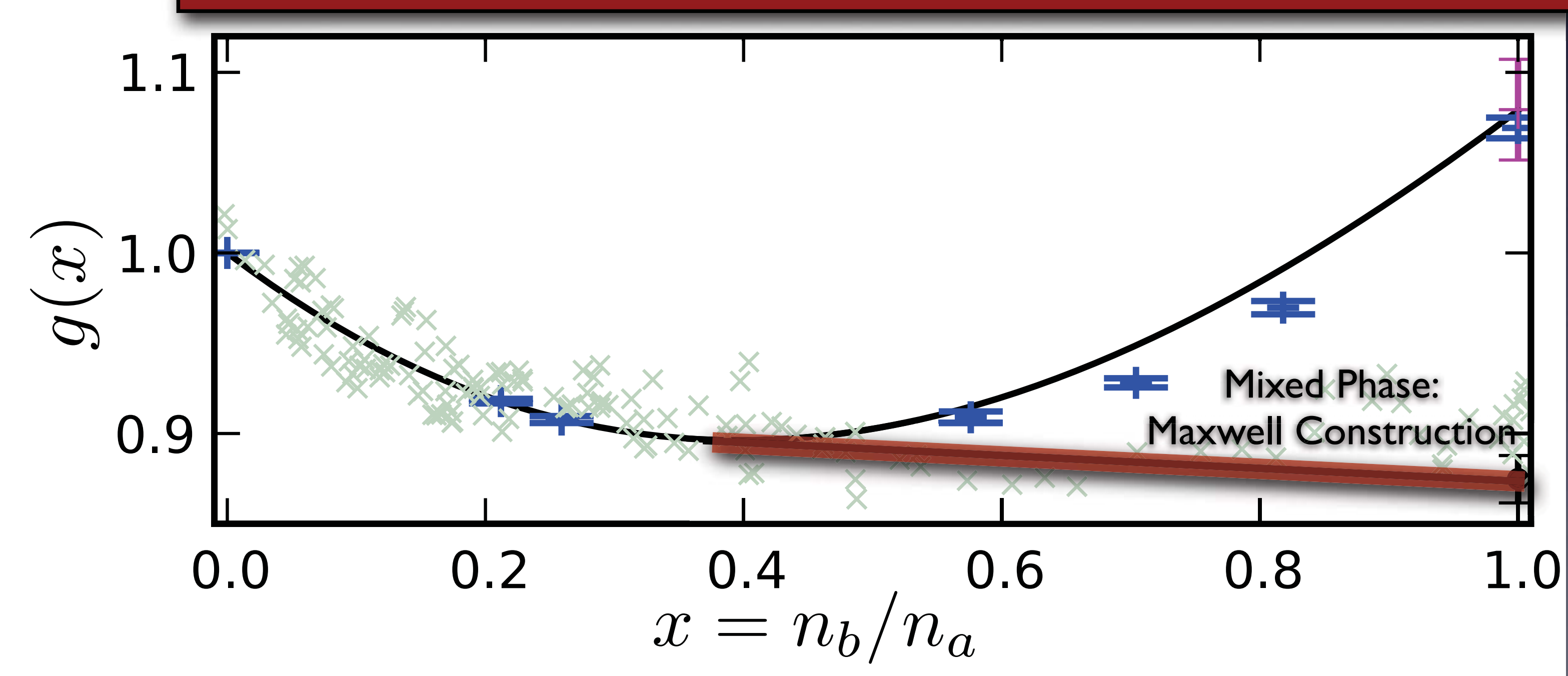
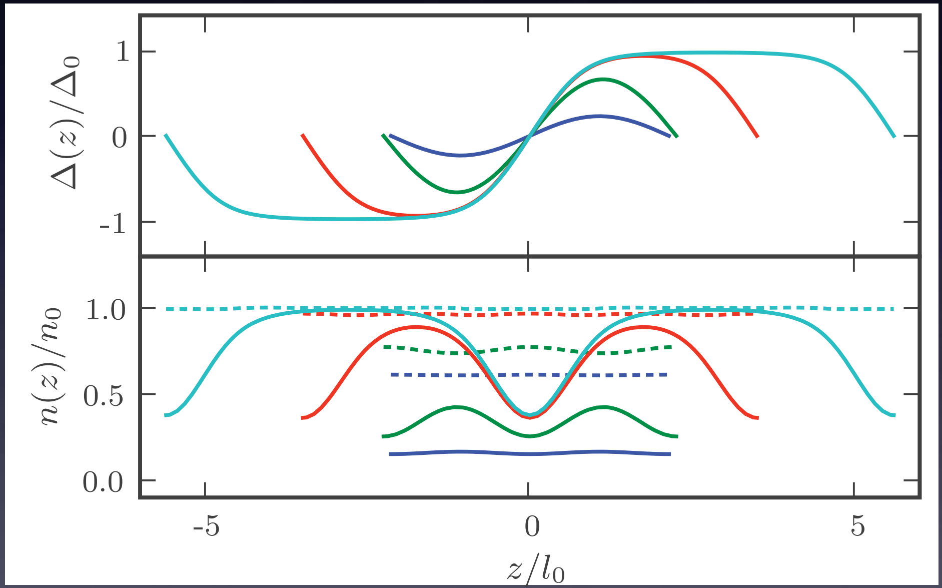


MIT Experimental data from Shin (2008), MC data from Lobo et. al (2006)

LO Supersolid Solutions

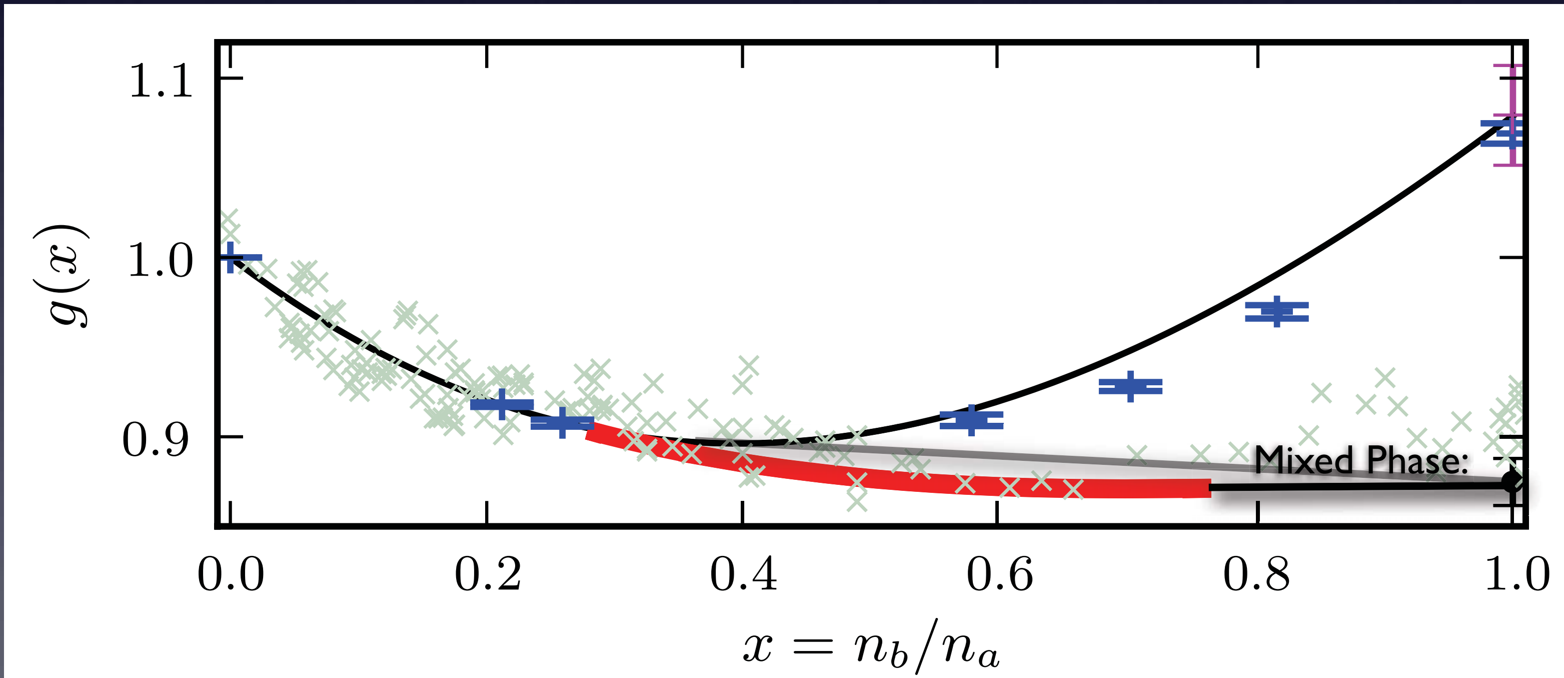
- Introduce ansatz $\Delta(z)$ with node.
- Periodic box of length L.
  - Determine L to maximize pressure: Gives natural LO period.
- Integrate over transverse and Bloch momenta.
- Large density fluctuations.
- Rigid crystal structure (solid) but with superfluid correlations.



A. Bulgac, M.M. Forbes 101 (2008) 215301

LO Supersolid wins

- Lower energy that homogeneous phases
  - Occurs for large range of asymmetries.
  - Small region of mixed LO and superfluid phase.



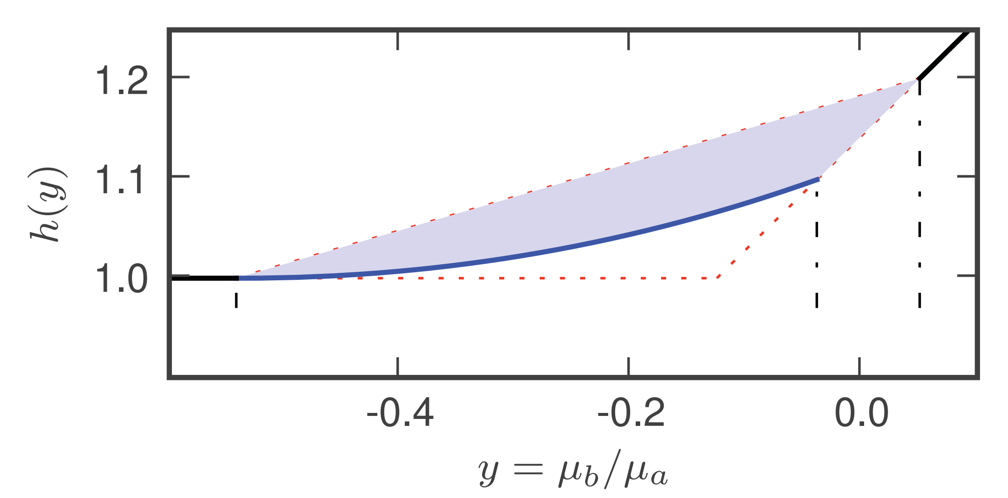
MIT Experimental data from Shin (2008), MC data from Lobo et. al (2006)

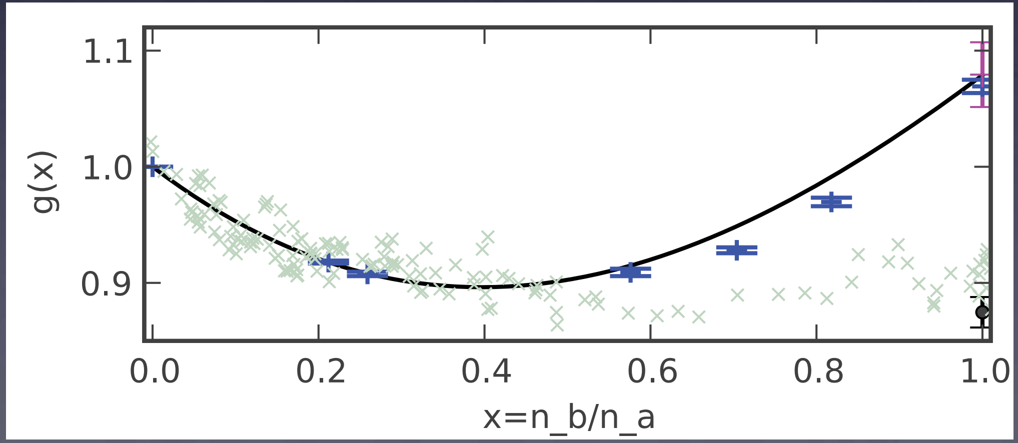
Fitting: Homogenous Phases

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

- Fully polarized phase
- Fully paired superfluid
- Interacting normal phase (P-wave superfluid?)
- Mixed phase: (phase coexistence between these two phases)

$$\mathcal{E}(n_a, n_b) \propto \left[n_a g\left(\frac{n_b}{n_b}\right)\right]^{5/3}$$





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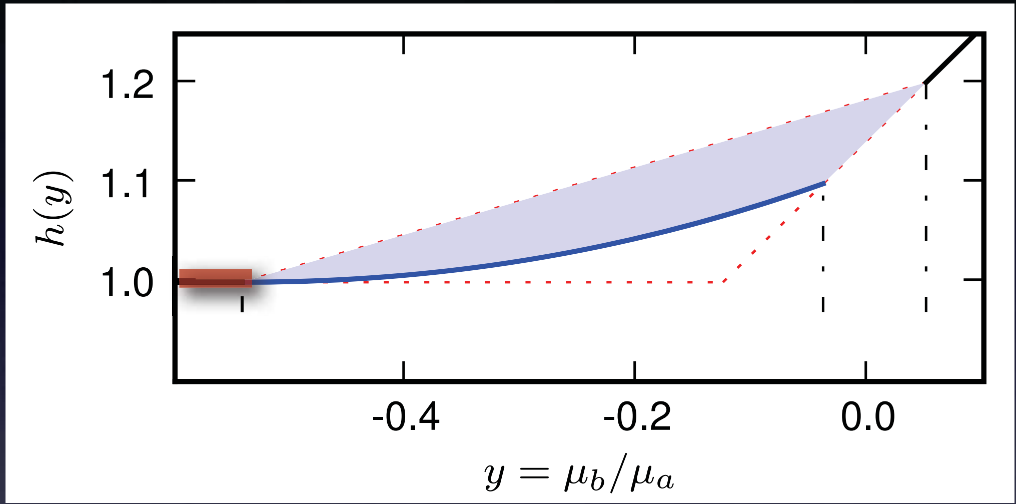

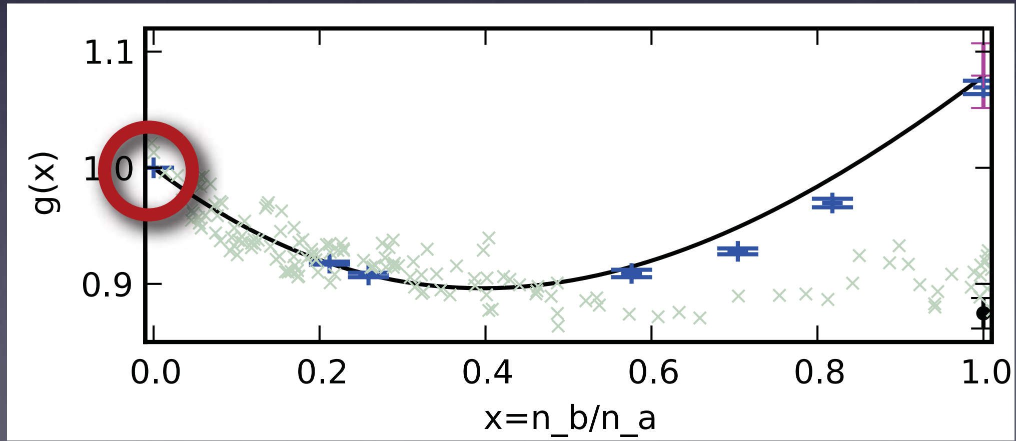

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- **Fully paired superfluid**
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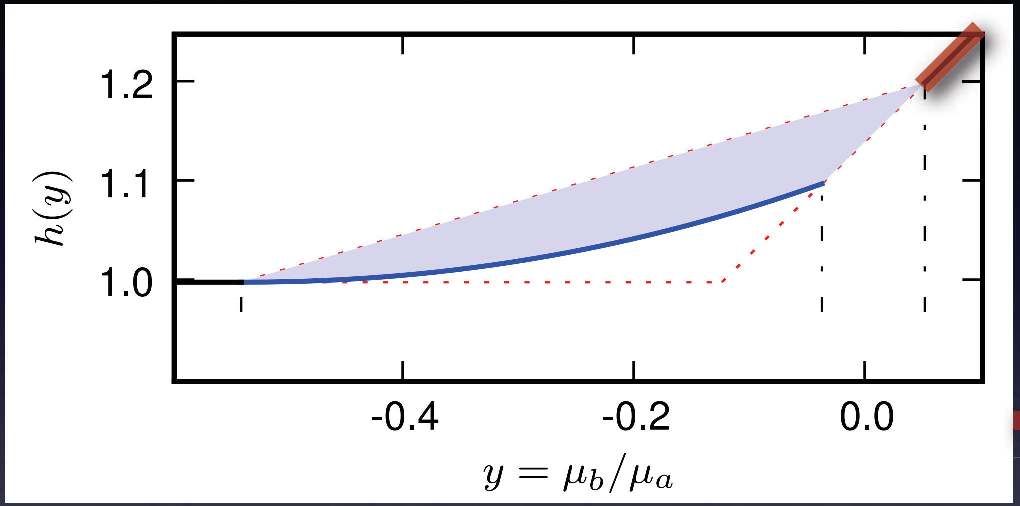

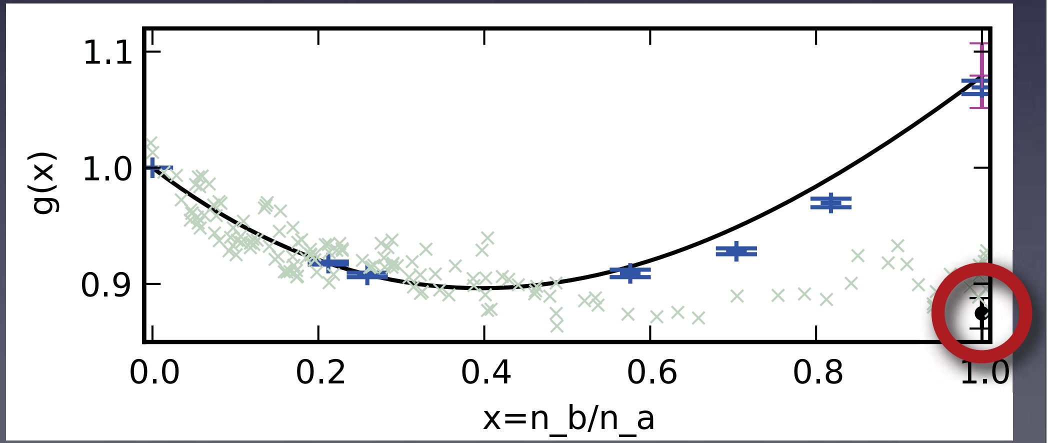

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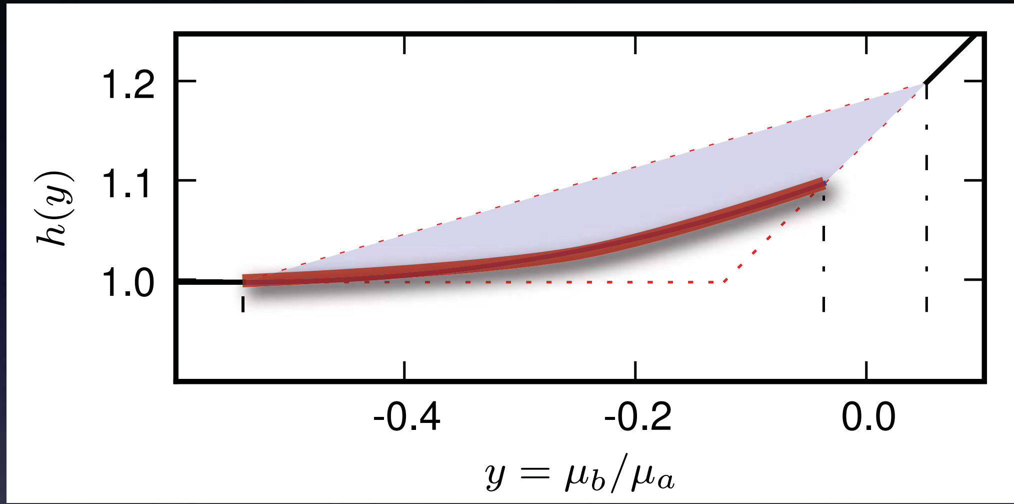

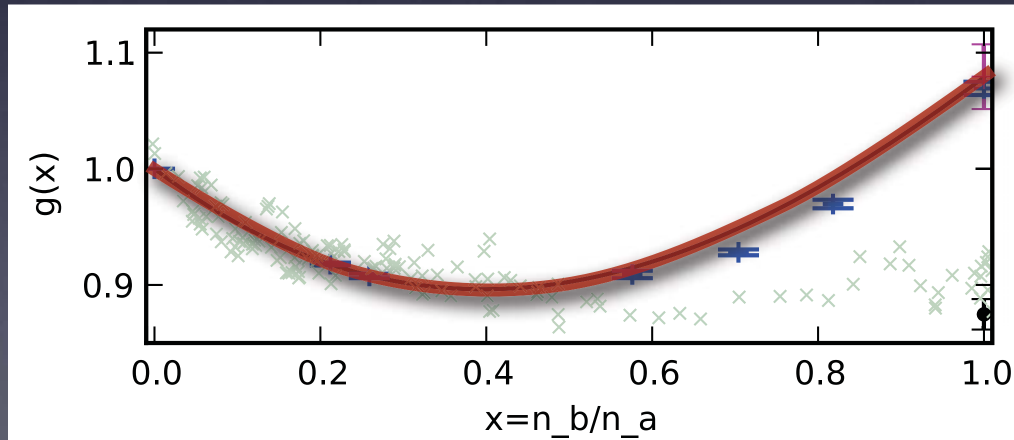

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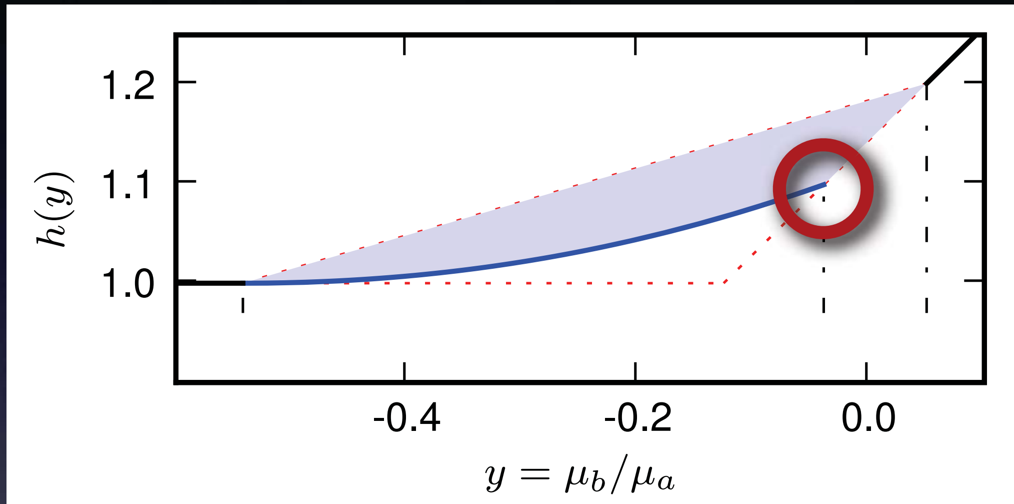
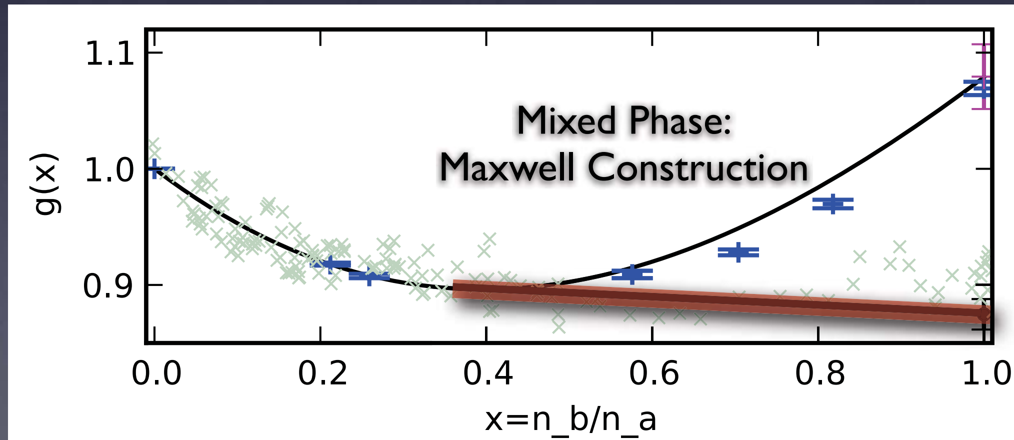


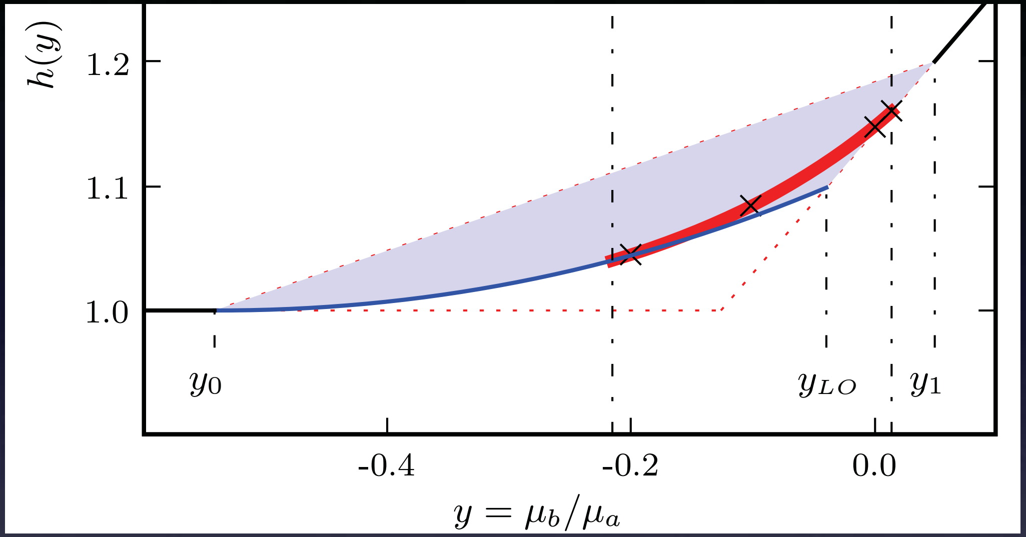


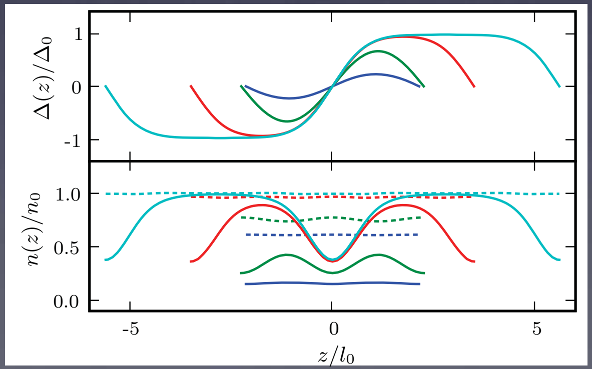
MIT Experimental data from Shin (2008), MC data from Lobo et. al (2006)

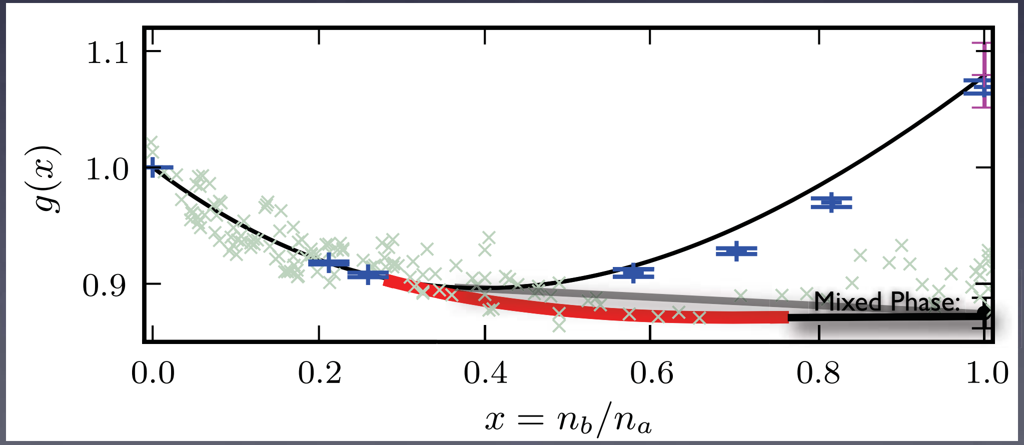
Grand Canonical Ensemble

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

- Legendre transform.
- Only (and all) pure phases.
- 2nd order transition to normal phase
- Weak 1st order transition to fully paired superfluid.







Phase Transitions

$$\mathcal{P}(\mu_a, \mu_b) \propto \left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

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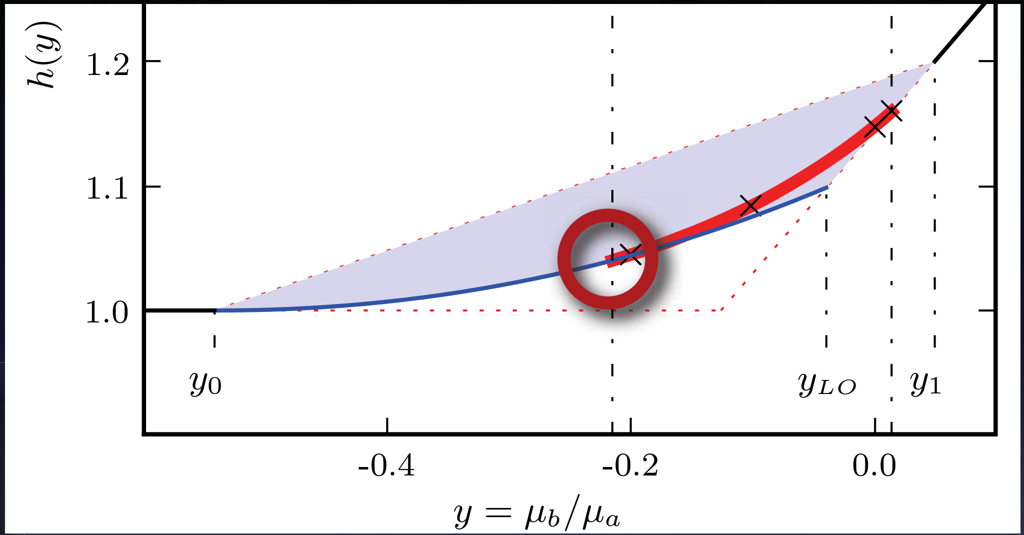

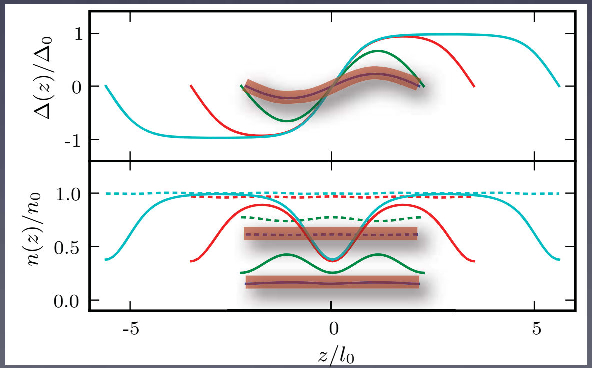

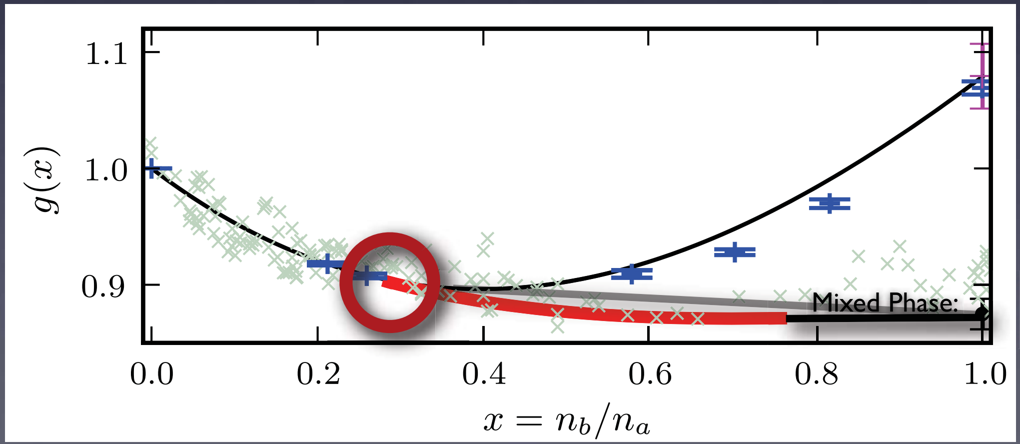

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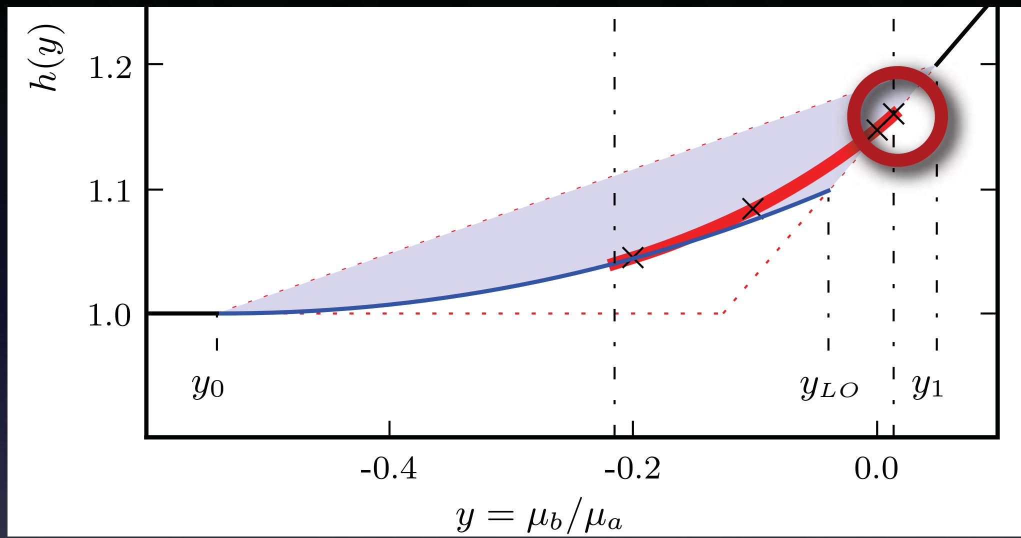

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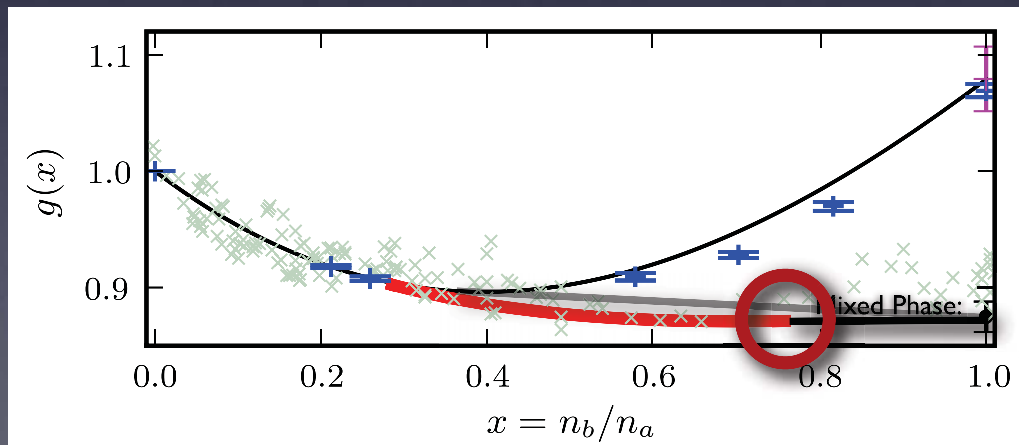
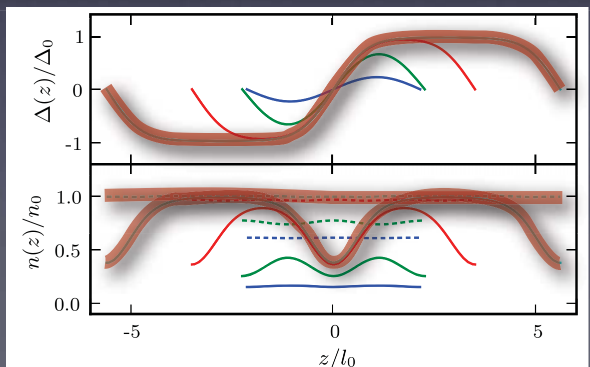
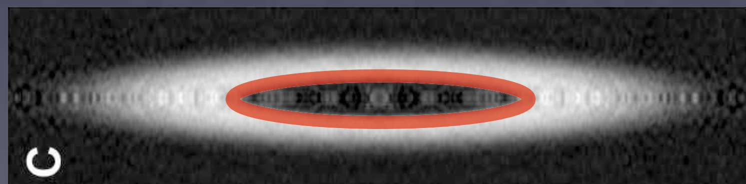




Comments

- LOFF not seen in BdG:
  - Need to include self-energy (Hartree) interaction.
- Still need to validate ASLDA
  - Need to validate with ab-initio methods or experiment...
  - Explore gradient corrections.
  - But... result that LO beats homogeneous phases is quite robust.
- Other phases may be better:
  - More complex crystal structure?
  - P-wave superfluid?

Why not seen yet?



Not enough space for oscillations

Conclusions

- Time to move beyond Mean-Field and BdG
  - SLDA works well for symmetric systems
  - Allows inclusion of MC data for quantitative analysis
  - Qualitatively new physics
- Unitary Fermi gas may contain a Fermi Supersolid (LOFF) state!