The Abdus Salam International Centre for Theoretical Physics

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Conference on Research Frontiers in Ultra-Cold Atoms

4 - 8 May 2009

Using matter-waves and optical disorder to study coherent transport and Anderson localization

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COLD ATOMS AND OPTICAL DISORDER : A NEW TOOL TO STUDY QUANTUM TRANSPORT

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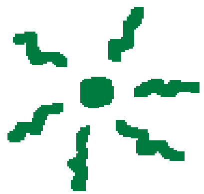
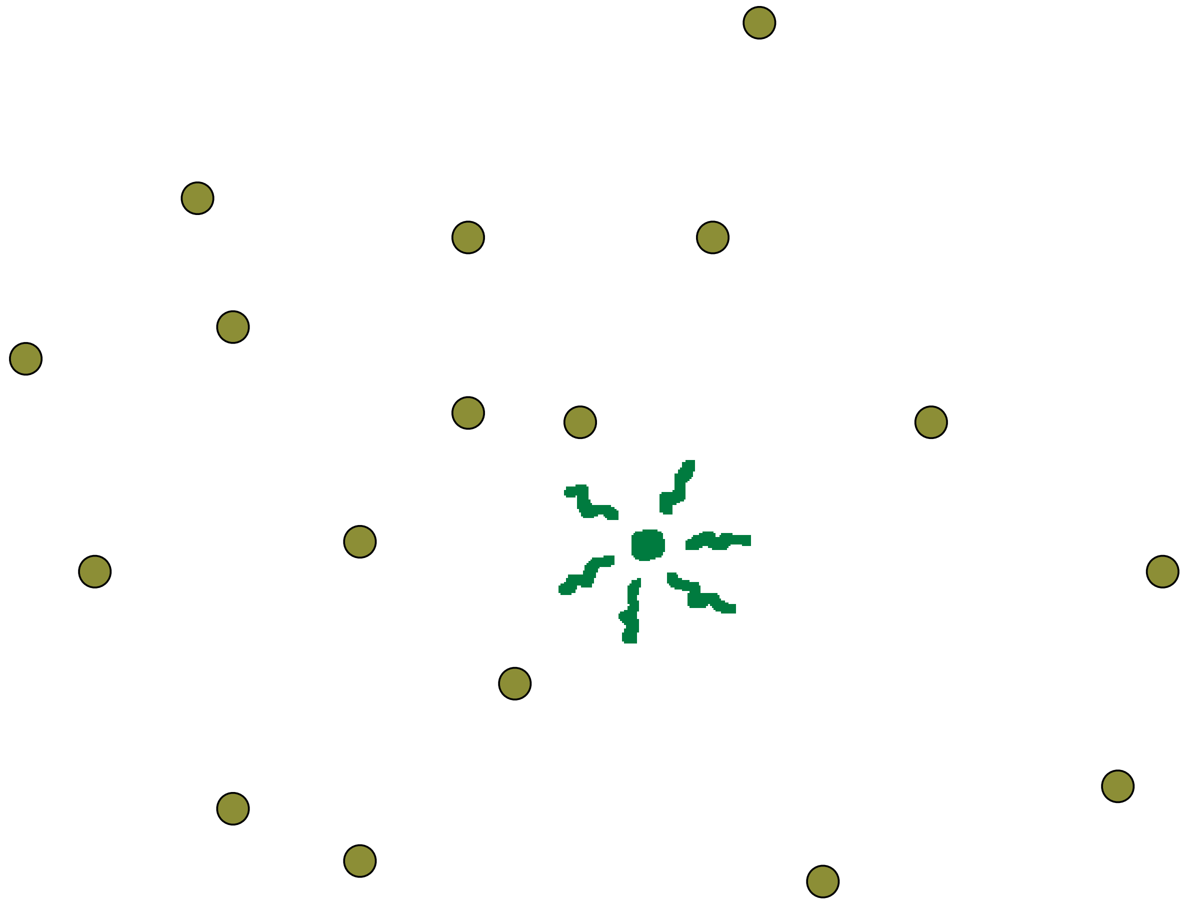
IFRAF

EFFECT OF DISORDER IN QUANTUM TRANSPORT

TRANSPORT



TRANSPORT THROUGH IMPURITIES



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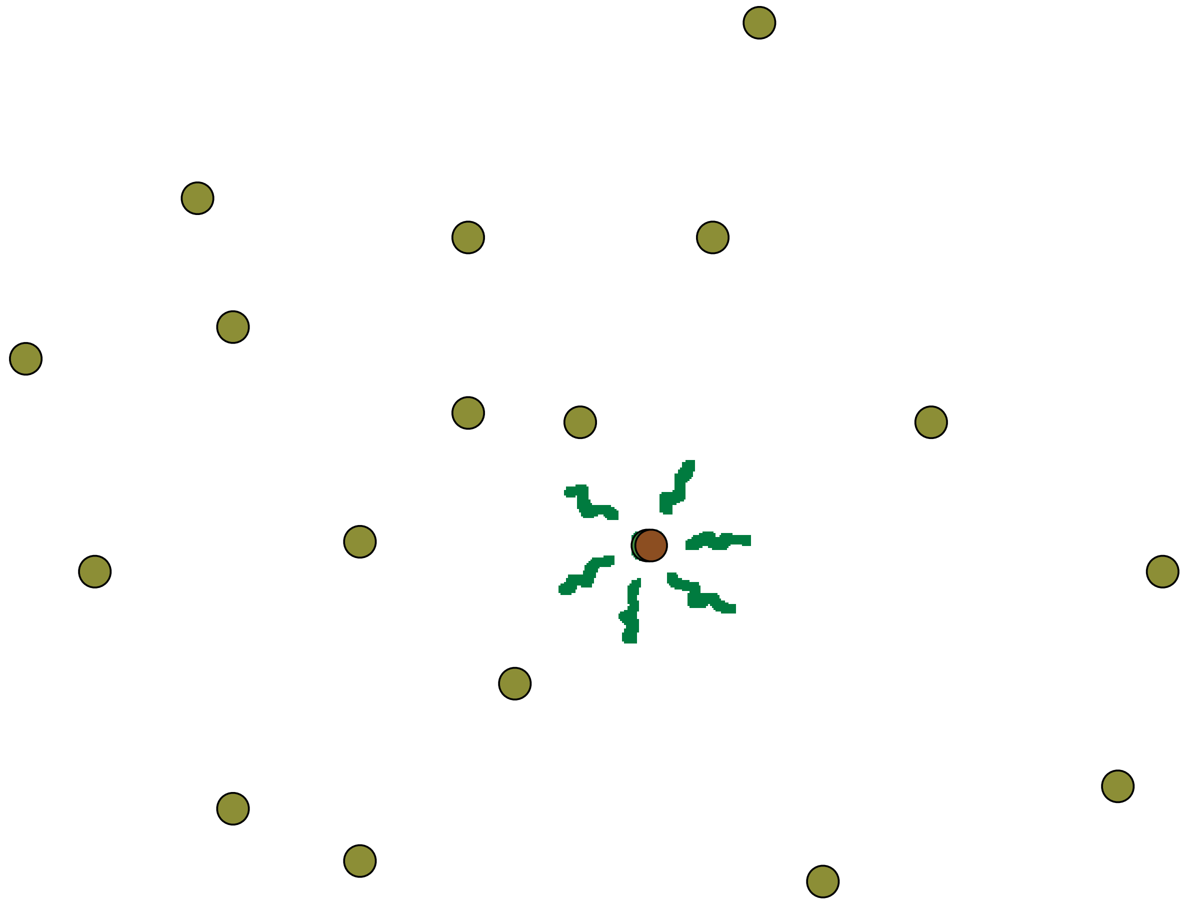
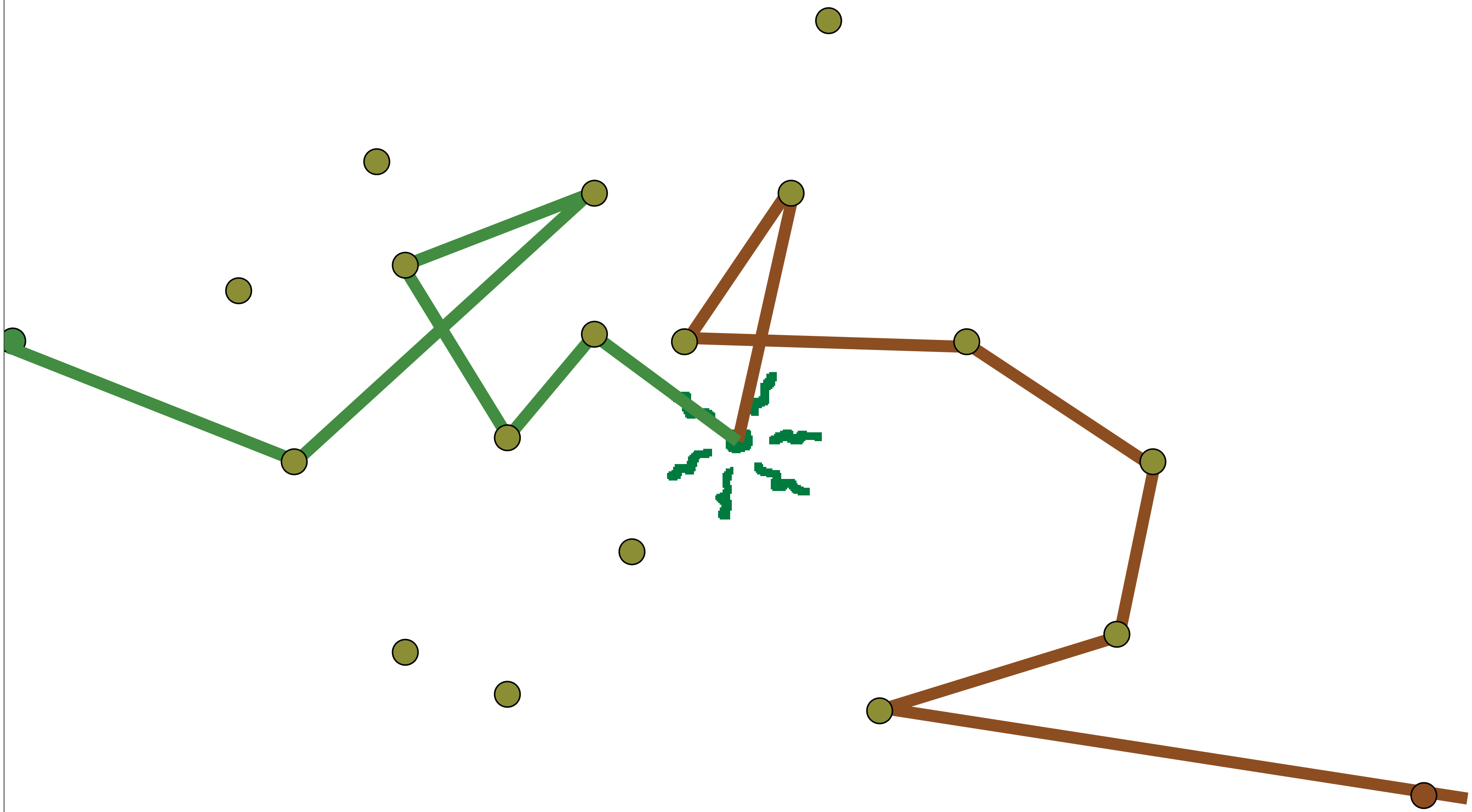
CLASSICAL TRANSPORT THROUGH IMPURITIES (MATTER)



Classical particles bouncing on impurities = diffusive transport (Drude)

CLASSICAL TRANSPORT THROUGH IMPURITIES (MATTER)



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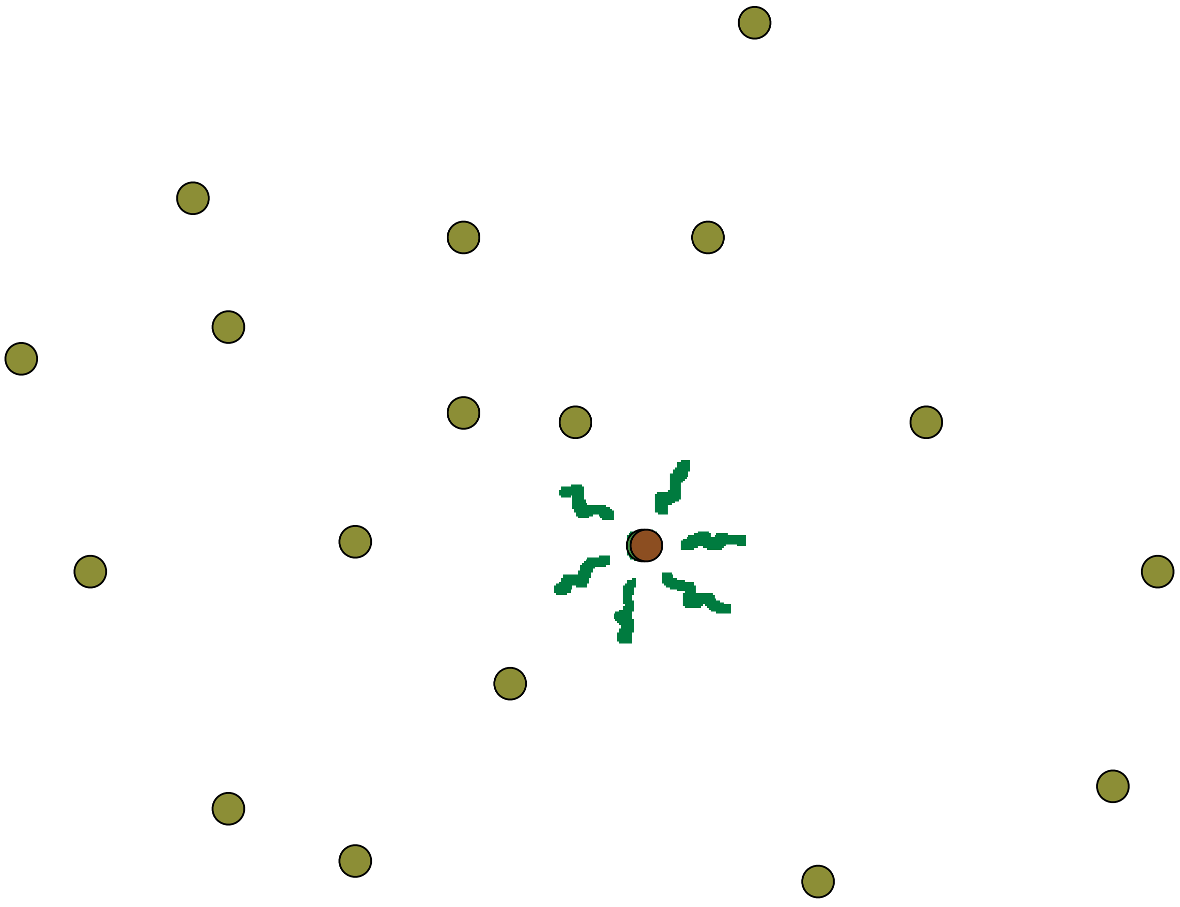
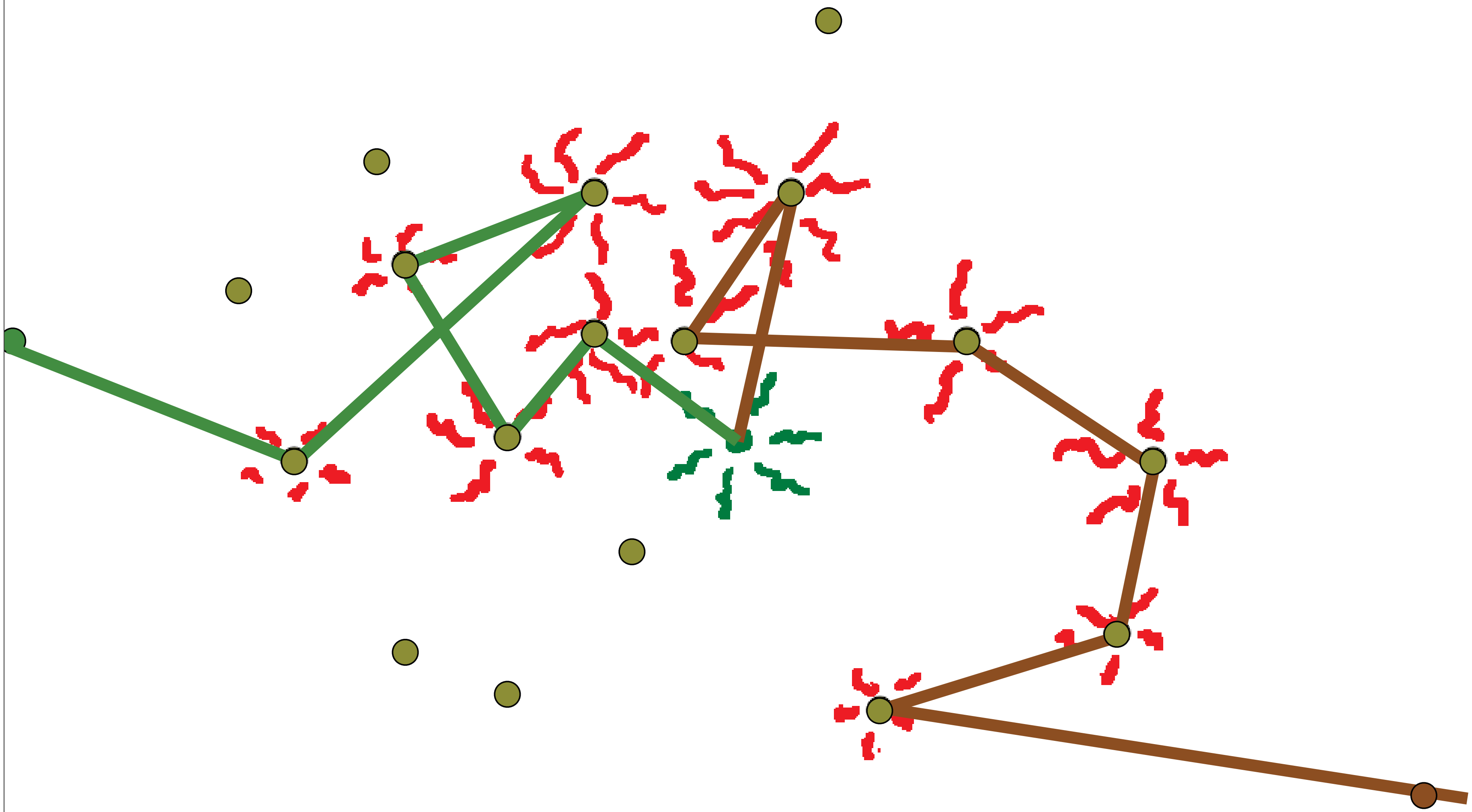
CLASSICAL TRANSPORT THROUGH IMPURITIES (WAVES)



Waves but no coherence bouncing on impurities = diffusion

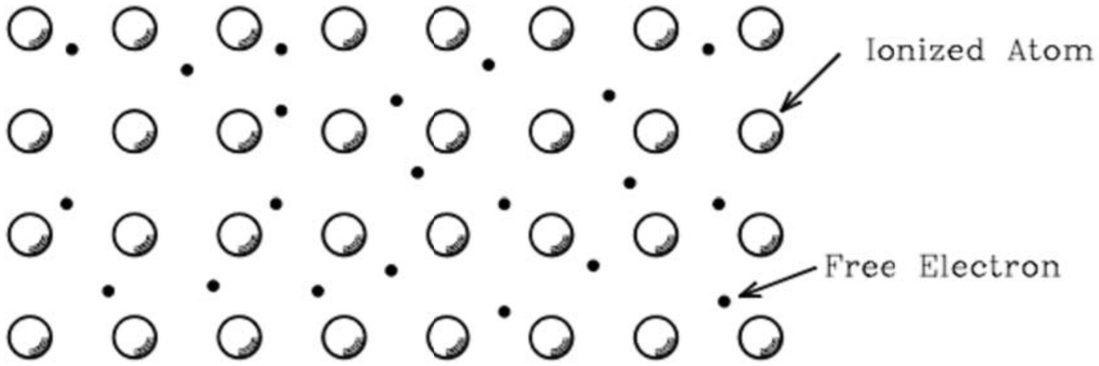
CLASSICAL TRANSPORT THROUGH IMPURITIES (WAVES)



Waves but no coherence bouncing on impurities = diffusion

DISORDER IS IMPORTANT FOR CONDUCTION

In a perfectly ordered metal



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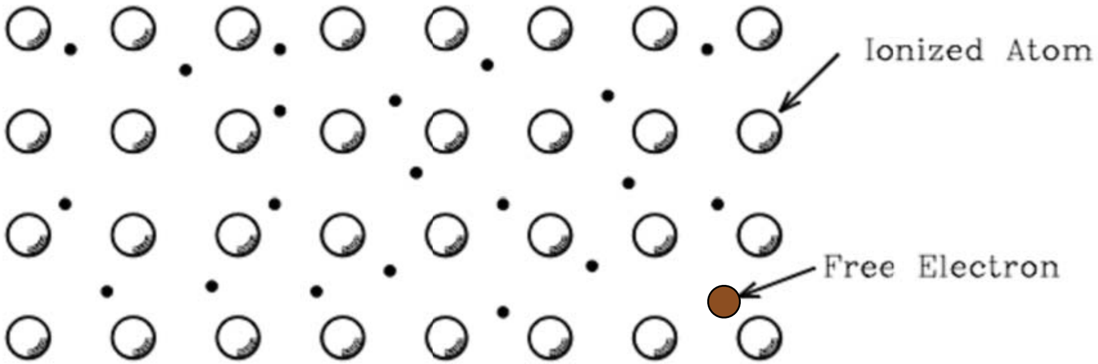

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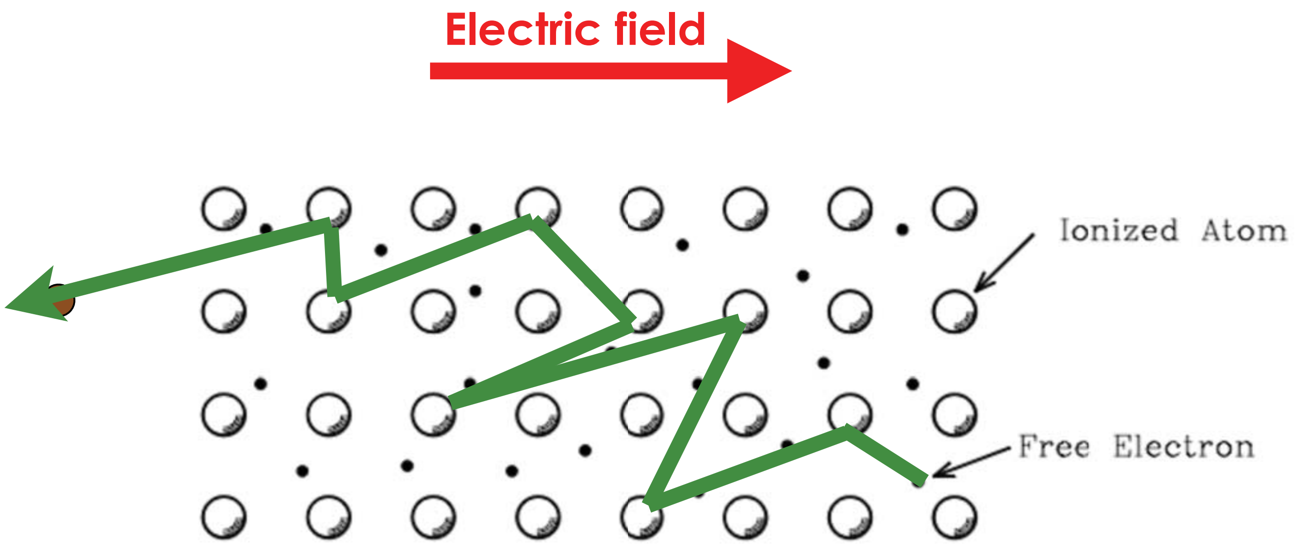
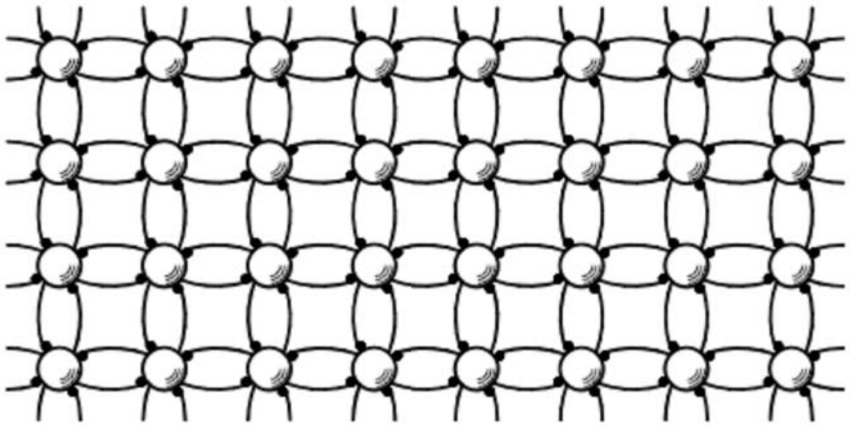


Diffusive transport caused by thermal excitation

DISORDER IS IMPORTANT FOR CONDUCTION

In a semiconductor



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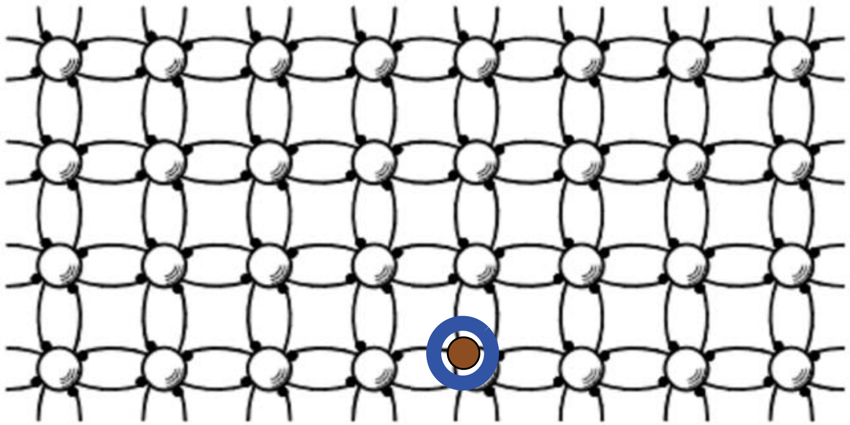
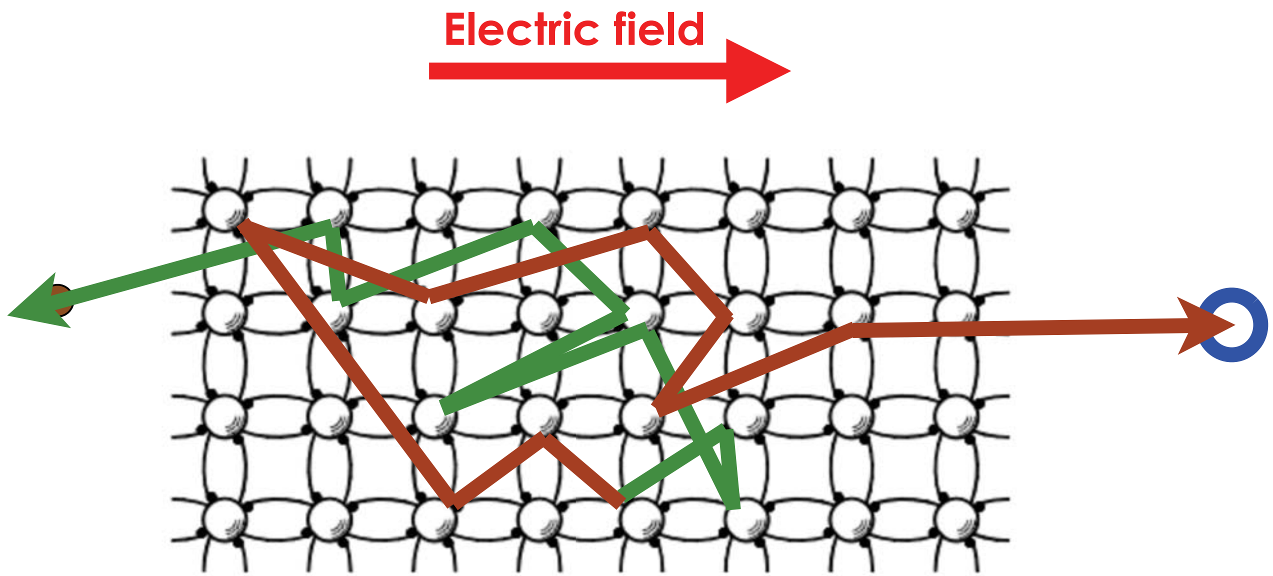
In a semiconductor



DISORDER IS IMPORTANT FOR CONDUCTION

In a semiconductor



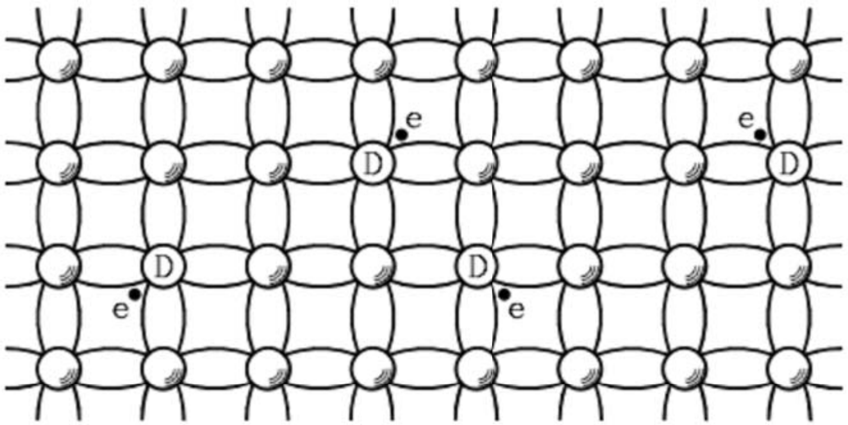
Electrons and holes diffuse in opposite direction

Electrons - holes recombination hinders transport

DISORDER IS IMPORTANT FOR CONDUCTION

In a semiconductor

Electric field



Electrons and holes diffuse in opposite direction

Electrons - holes recombination hinders transport

NEED TO ADD IMPURITIES - DISORDER

EFFECT OF DISORDER IN QUANTUM TRANSPORT : ANDERSON LOCALISATION

Disorder is important in semiconductor conduction

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

Disorder can induce anomalous transport or inhibition of transport

- Disruption of electron transport due to defects in a solid
- Suppression of superfluidity of ^{4}He in porous media with disorder
- Anomalous diffusion of photons in strongly scattering semiconductor powders

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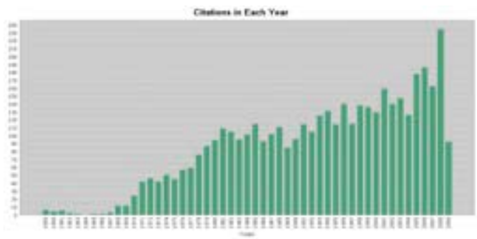
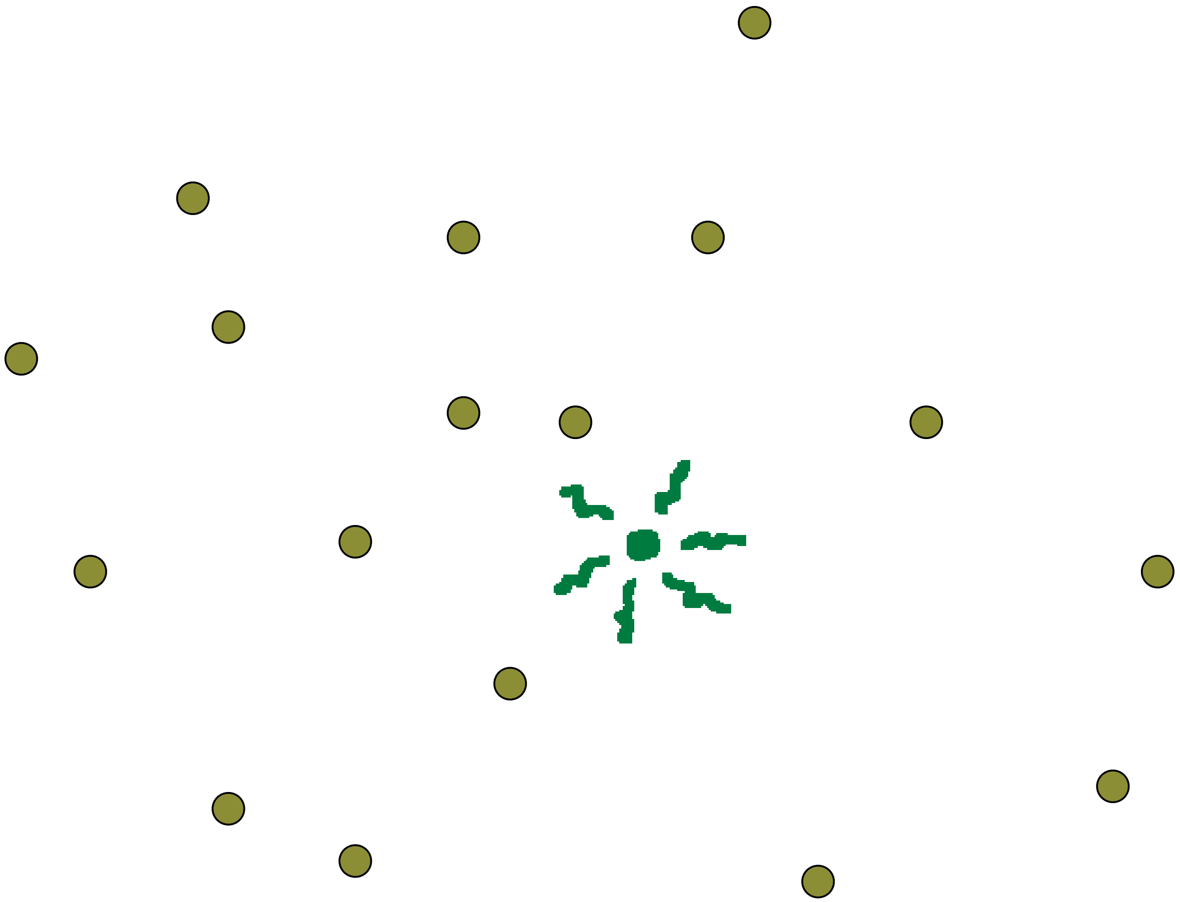


QUANTUM TRANSPORT THROUGH IMPURITIES



coherence effects can change the transport properties

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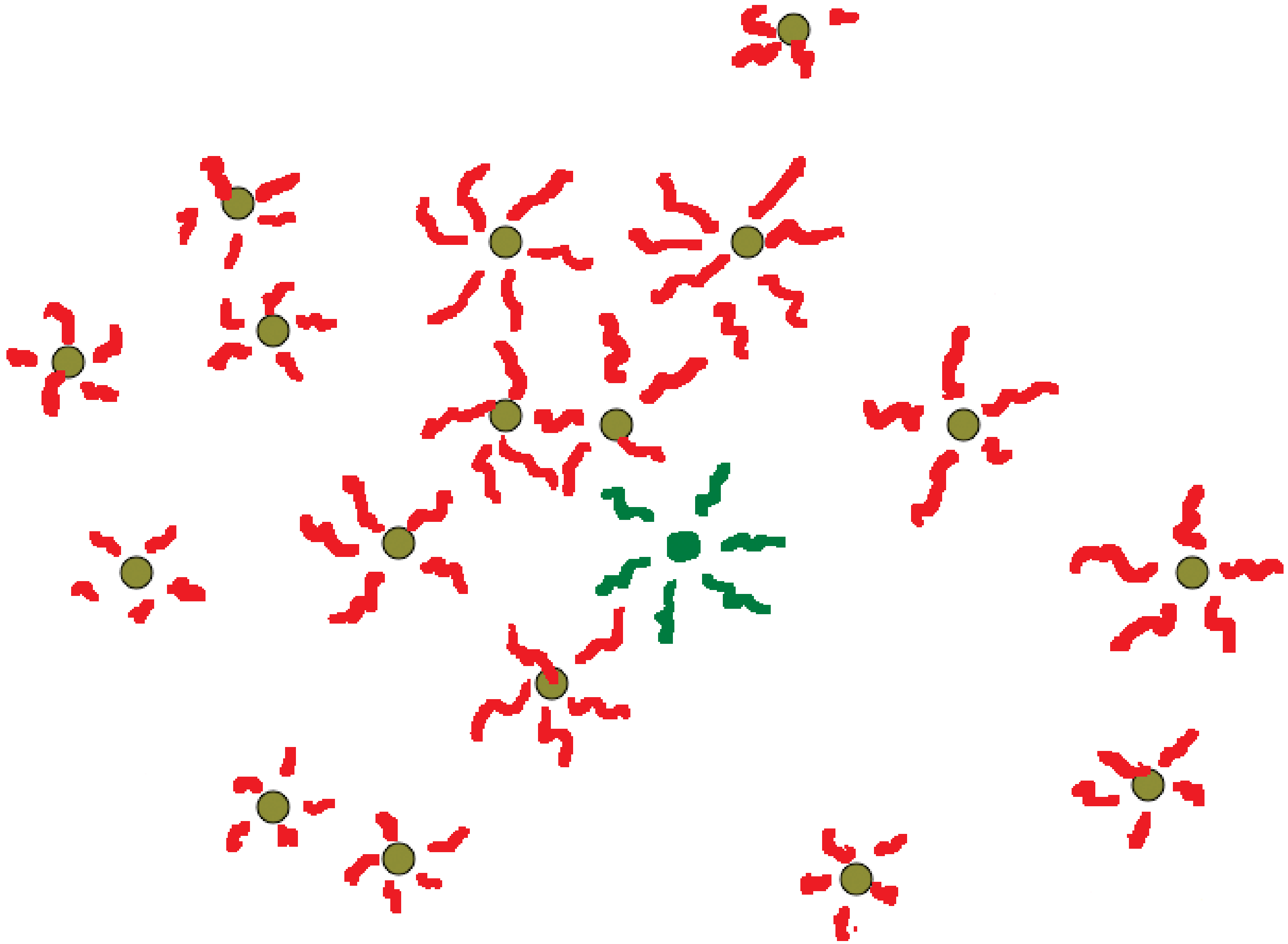

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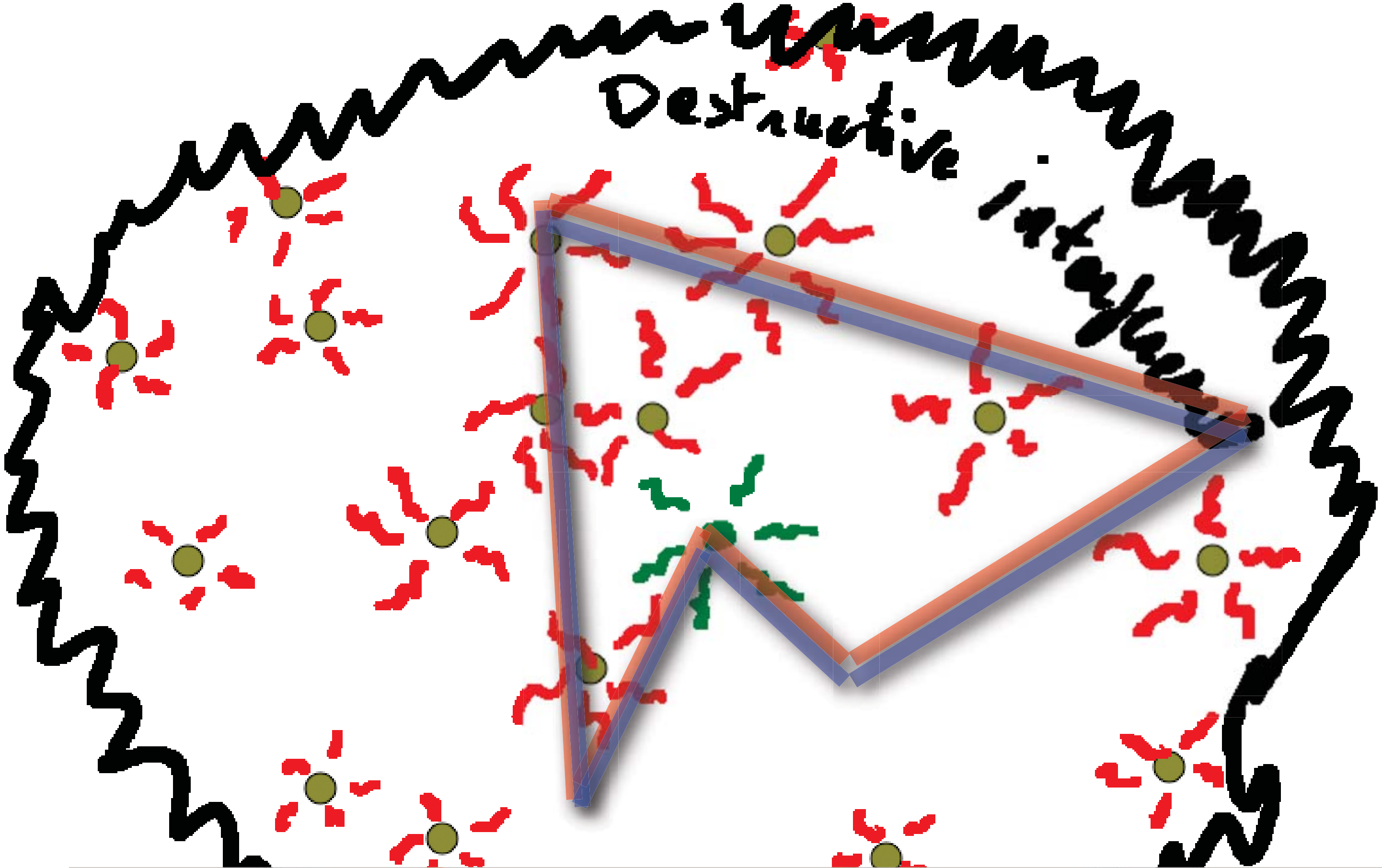

If mean free path smaller than de Broglie wavelength: constructive interference of trajectories returning to origin: localized states: insulator

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wind whistling in Alice's ears, and almost blowing her hair off her head, she fancied.

"Now! Now!" cried the Queen. "Faster!

"A slow sort of country!" said the Queen. "Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

The Queen propped her up against a tree, and said kindly, "You may rest a little, now."

EFFECT OF DISORDER ON TRANSPORT PROPERTIES

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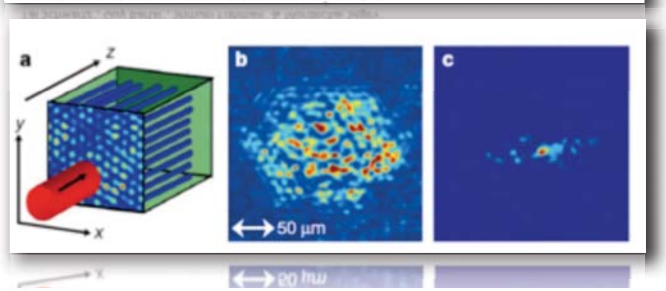
ANDERSON LOCALISATION

nature Vol 446 | 1 March 2007 | doi:10.1038/nature05623

LETTERS

Transport and Anderson localization in disordered two-dimensional photonic lattices

Tal Schwartz[1], Guy Bartal[1], Shmuel Fishman[1] & Mordechai Segev[1]

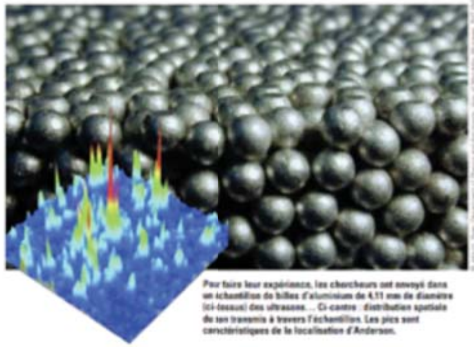


PHYSIQUE

Un phénomène quantique observé en 3D

Une collaboration franco-canadienne vient de démontrer l'existence de la « localisation d'Anderson » à trois dimensions, un phénomène quantique décrit pour la première fois il y a exactement cinquante ans.

Pourquoi, à basse température, certains métaux arrêtent-ils de conduire l'électricité ? L'explication théorique a été fournie en 1958 par l'Américain Philip W. Anderson, lauréat du prix Nobel en 1977 : un phénomène quantique dit de « localisation » bloque littéralement les électrons dans le matériau. Aujourd'hui, cinquante ans après qu'elle a été formulée, la théorie de la localisation d'Anderson vient enfin d'être vérifiée de façon convaincante en trois dimensions. Une véritable première réalisée par Bart Van Tiggelen et Sergey Skipetrov, du Laboratoire de physique et modélisation des milieux condensés (LPM2C), à Grenoble, en



Pour faire leur expérience, les chercheurs ont envoyé dans un échantillon de billes d'aluminium de 4,11 mm de diamètre (ci-dessus) des ultrasons... Ci-contre : distribution spatiale du son transmis à travers l'échantillon. Les pics sont caractéristiques de la localisation d'Anderson.

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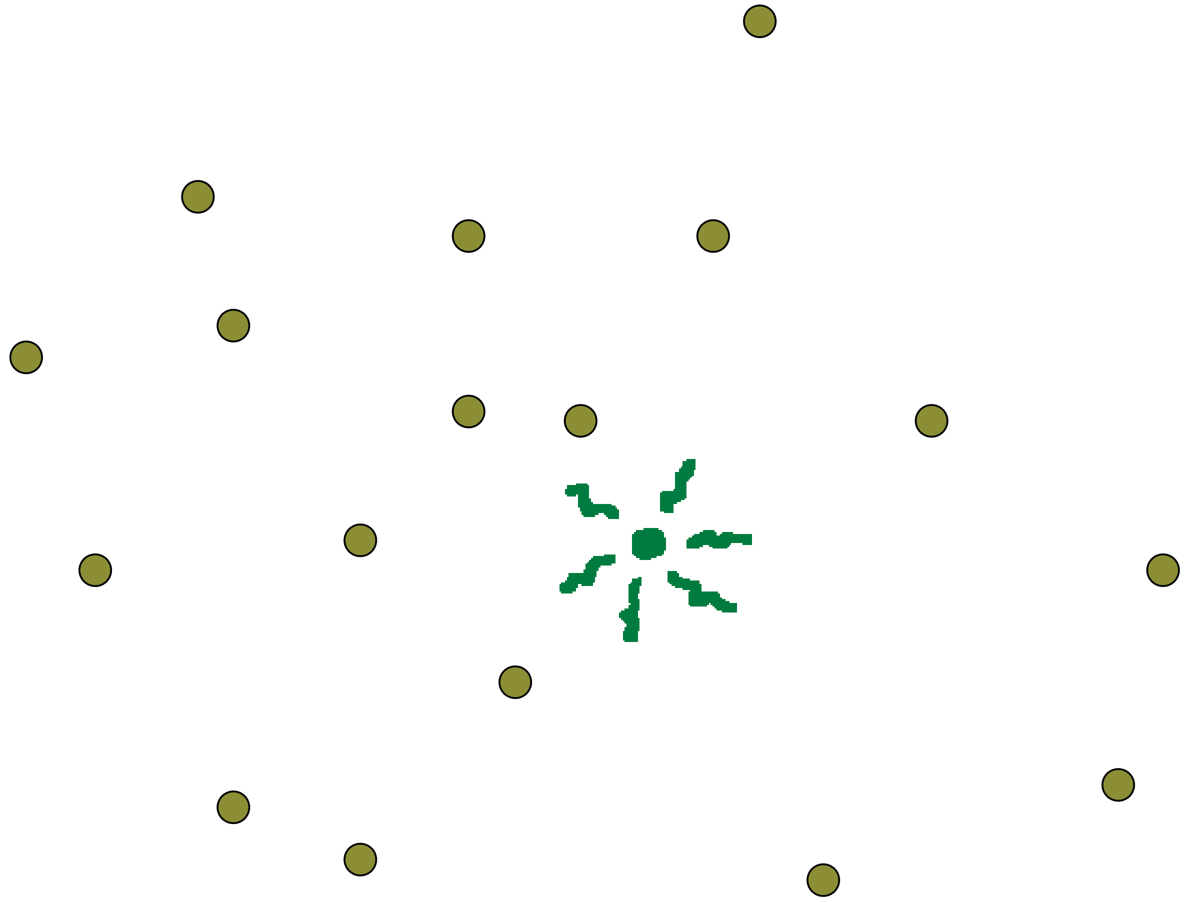
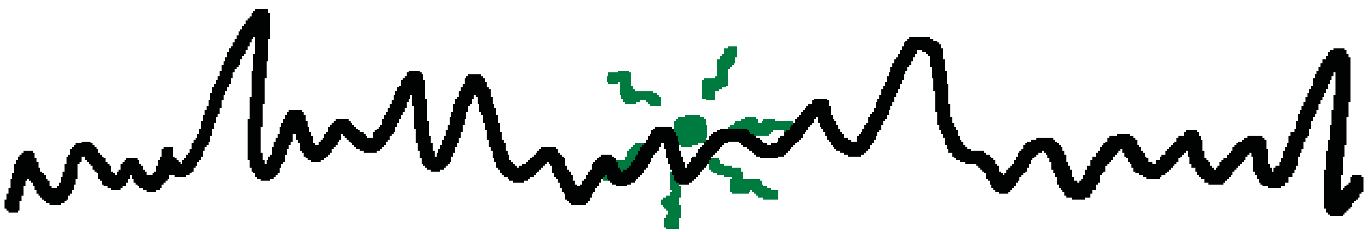
ANDERSON LOCALISATION FOR ATOMS

A.L. OF 1D BEC IN A 1D SPECKLE PATTERN



A.L. OF 1D BEC IN A 1D SPECKLE PATTERN

We consider matter waves in 1D in the presence of disorder.

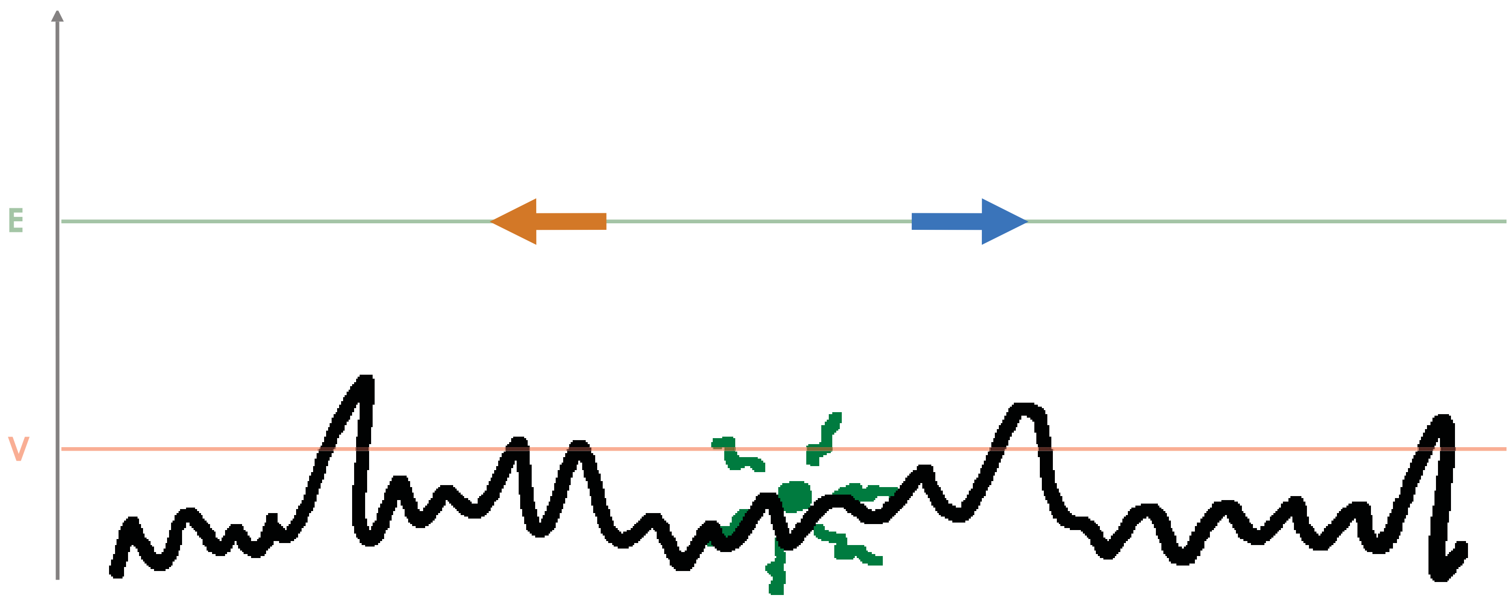


A.L. OF A 1D GAZ IN A 1D DISORDER

We consider matter waves in 1D in the presence of disorder.

Classically, if the energy of the particles is high enough, the reflection probability is small

propagation slightly affected.



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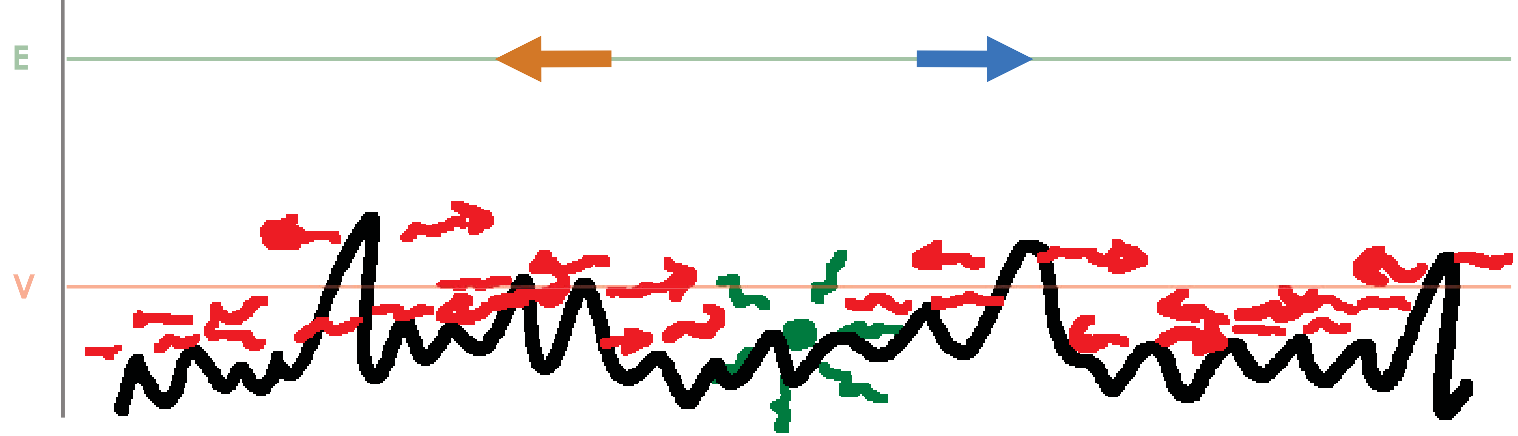
Anderson localisation



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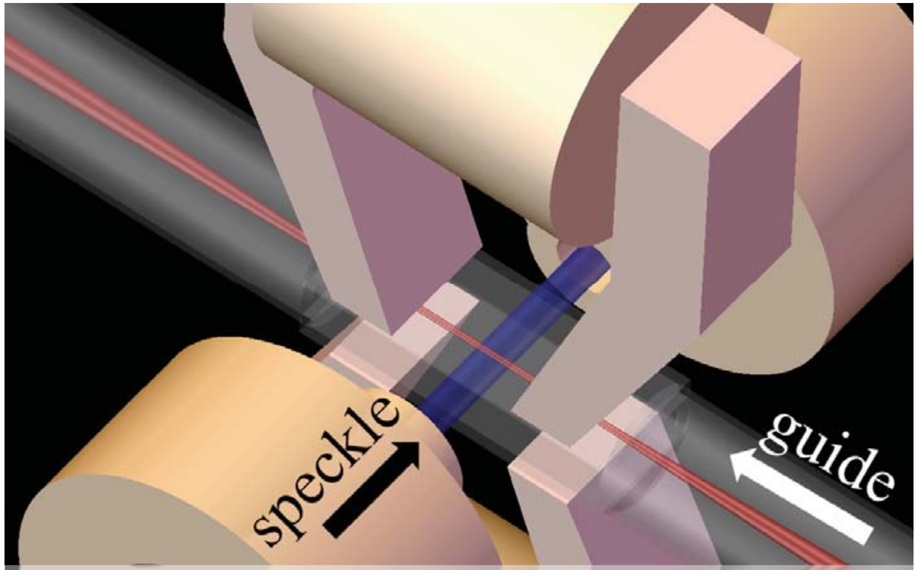
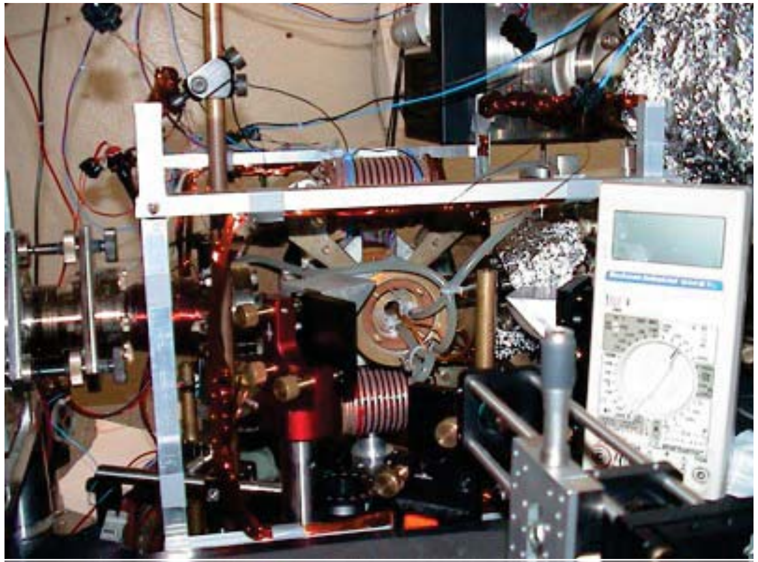
Anderson localisation

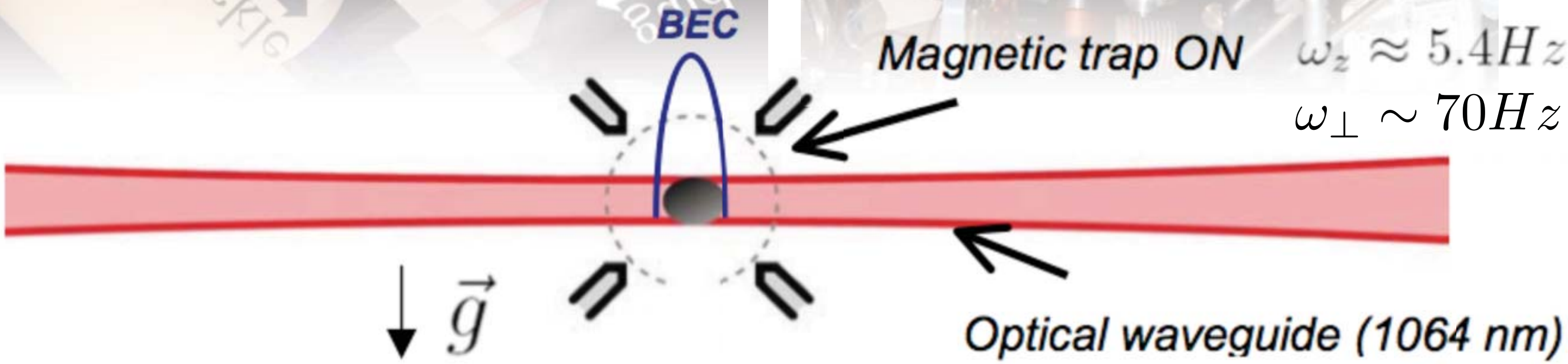
Naively (and also in the born approximation), localisation happens if you can find a Bragg condition $k_{\text{disorder}} = 2k_{\text{atoms}}$

Expansion of a BEC in a 1D optical disorder for Atoms

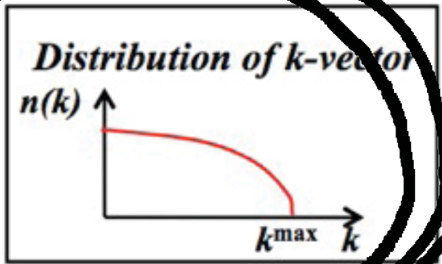
Expansion of a BEC in a 1D Guide







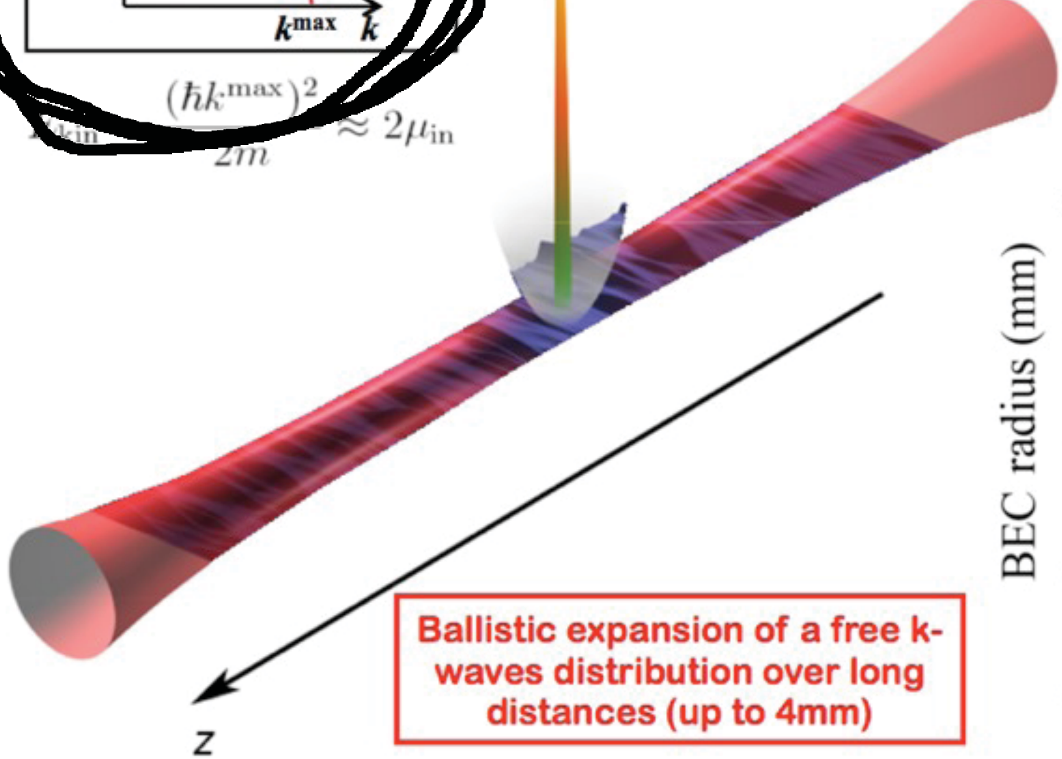
Expansion of a BEC in a 1D Guide



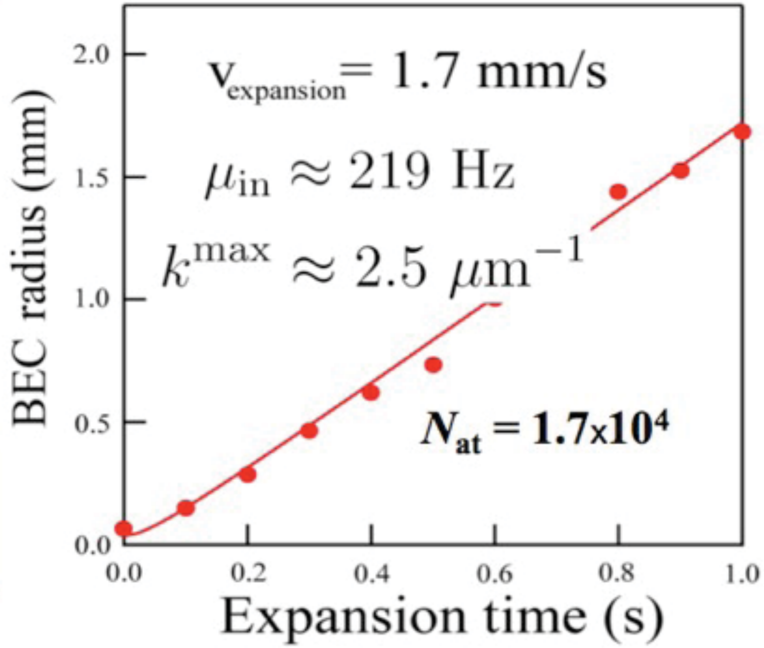
$$E_{kin} = \frac{(\hbar k^{max})^2}{2m} \approx 2\mu_{in}$$

To cancel any residual trapping, additional magnetic fields are accurately tuned.

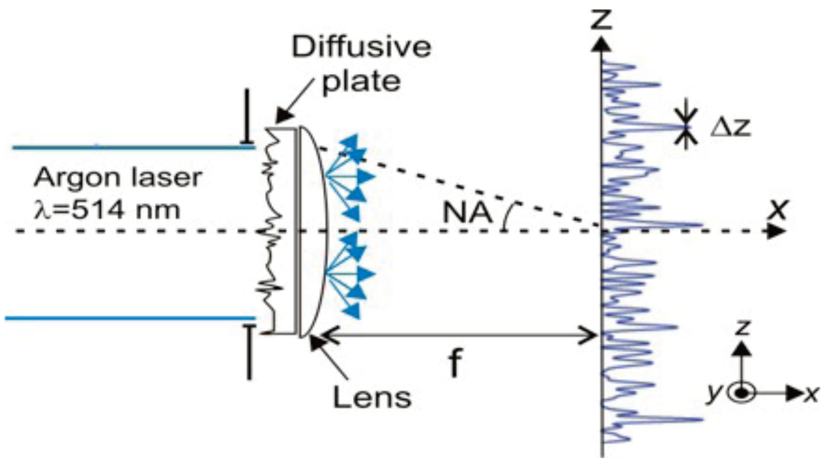
$f_{residual} < 0.05$ Hz

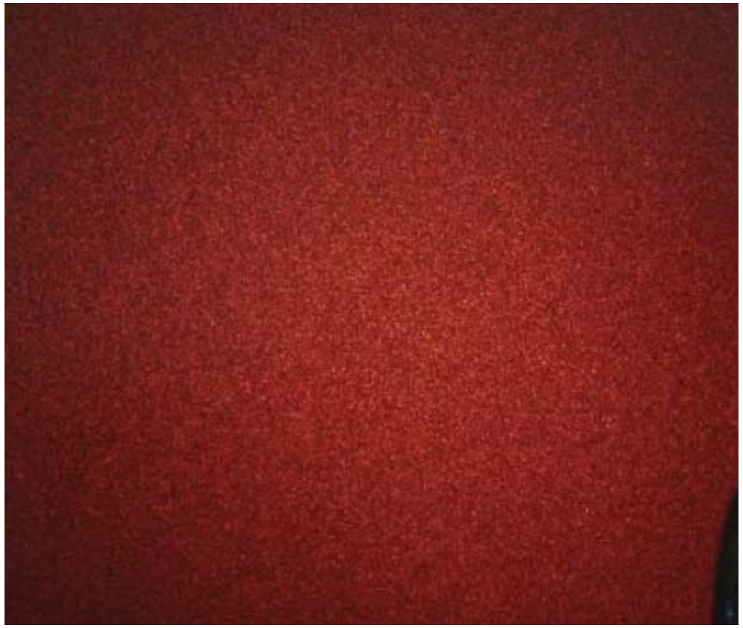


Ballistic expansion of a free k-waves distribution over long distances (up to 4mm)



Optical disorder for atoms : SPECKLE PATTERN





Depth of the optical wells

Indirect (CCD) and direct (using HF spectroscopy) measurements

Statistical properties verified $\sigma_I \sim < I >$

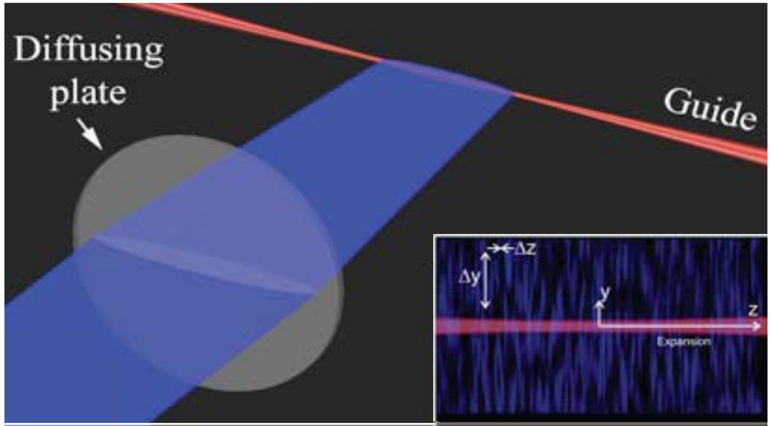
Potential depth σ_V calculated from σ_I

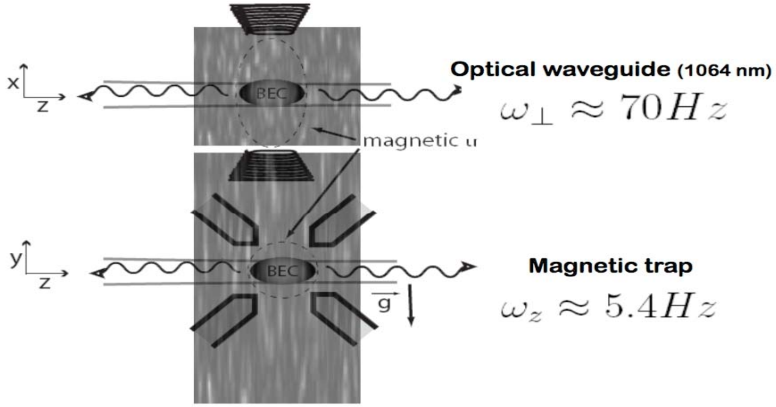
$$\sigma_V = \frac{2}{3}\frac{\Gamma^2}{2\delta}\frac{\sigma_I}{I_S}$$

Good agreement with HF measurements (calibration)

Disorder amplitude definition γ : $\gamma = \sigma_V/\mu$

A.L. OF 1D BEC IN A 1D SPECKLE PATTERN





Speckle size Δx, Δy, Δz

- $\Delta y, \Delta z \sim 1.22\lambda d/D$
- $\Delta z \leq 1\ \mu m$
- $\Delta y \simeq 80\ \mu m$
- $\Delta x = Rayleighlength \gg 10\ \mu m$

BEC elongated along z and confined (focused laser) transversely to z

$$2R_z^{TF} \simeq 300\ \mu m \gg \Delta z, R_\perp^{TF} \simeq 3\ \mu m \ll \Delta x, \Delta y$$

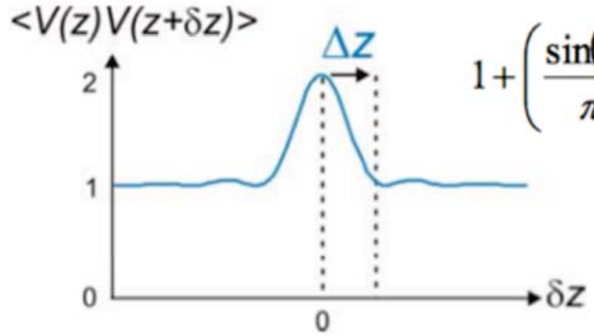
1 D situation for the elongated BEC.
Many speckle grains covered (self averaging system = ergodic)

1D situation: invariant transversely to z

A.L. OF 1D BEC IN A 1D SPECKLE PATTERN

High frequency cut off kc

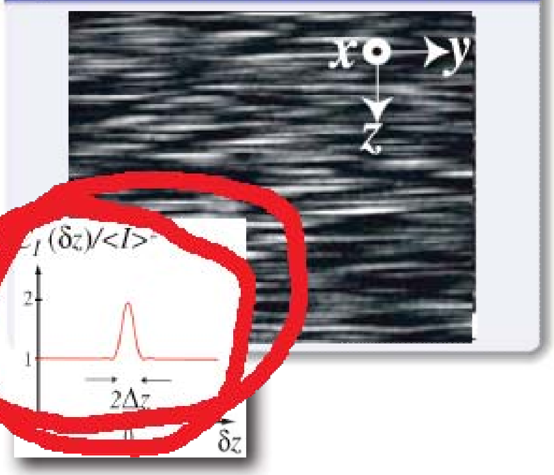
Spatial autocorrelation : C(δz)= <V(z)V(z+δz)>



$$1+\left(\frac{\sin(\pi\delta z/\Delta z)}{\pi\delta z/\Delta z}\right)^2$$

$$\Delta z = \frac{\lambda}{2(N.A.)} = 0.8\ \mu m$$

Speckle and autocorrelation



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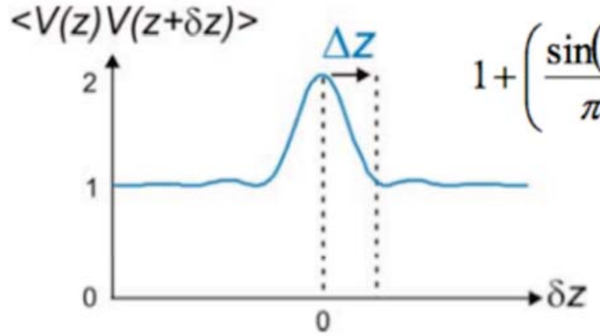

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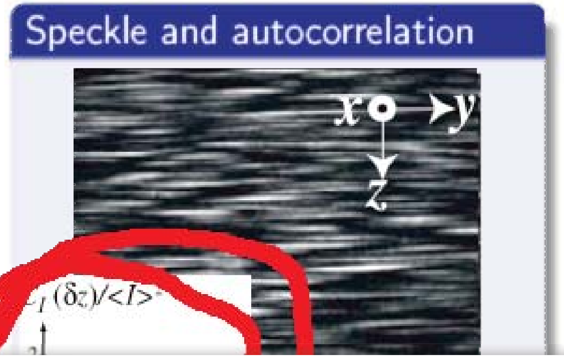
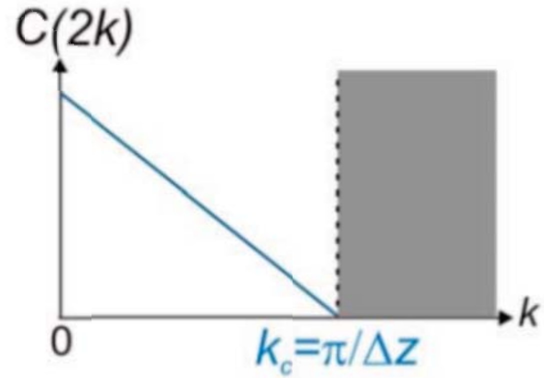


Spatial frequency distribution C(k)

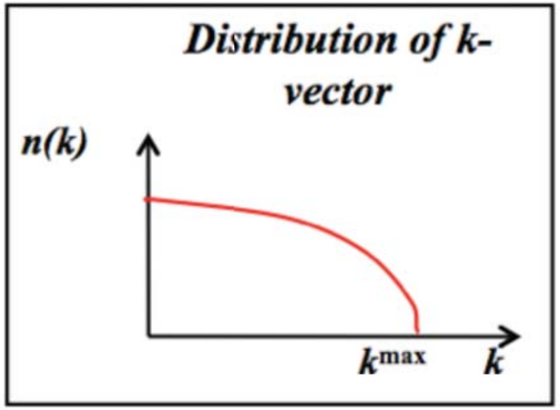


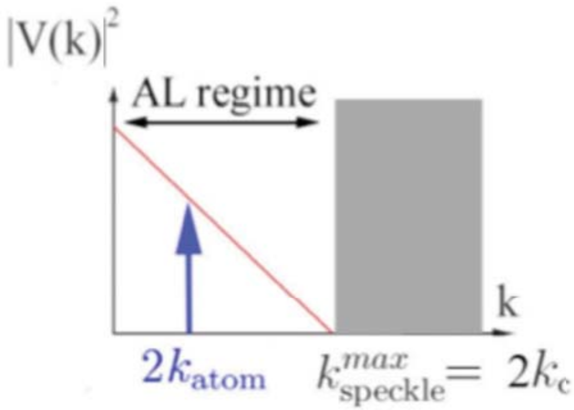
$$k_c=\frac{\pi}{\Delta z}=3.85\ \mu m^{-1}$$

speckle = correlated disorder: cut off in the spatial frequency spectrum

1D BEC EXPANDING IN AN OPTICAL GUIDE

- k_{max} : minimum λ_{dB} to be “reflected” by the speckle.
- k_c : minimum λ_{dB} that can be “reflected” by the speckle.





$$k_{\text{atom}} < k_c$$

$$k^{\max} \approx 2.5\ \mu\text{m}^{-1}$$

$$k^{\max} < k_c \approx 3.85\ \mu\text{m}^{-1}$$

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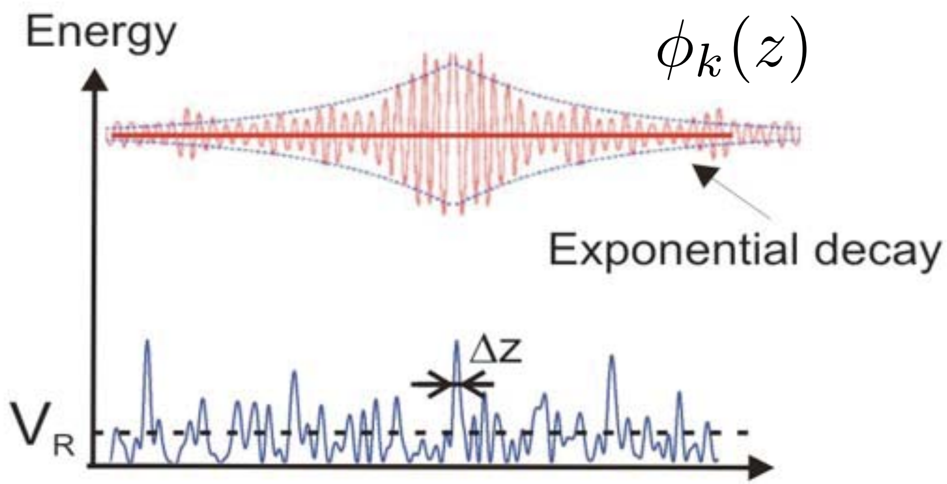
1D BEC EXPANDING IN AN OPTICAL GUIDE

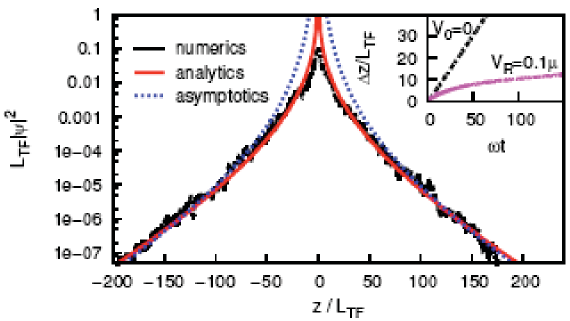
When $k_{max}<k_c$, each k (initial) plane wave will populate a localised wave function

$$\ln|\phi_k(z)| \simeq -\gamma(k)\,|z|$$

A. A. Gogolin *et al.*, Sov. Phys. JETP **42**, 168 (1976);
A. A. Gogolin, *ibid.* **44**, 1003 (1976).



$$\gamma(k) \simeq k_c\left(\frac{V_R}{E}\right)^2 (k/k_c)^2\, c(2k/k_c)$$



Each localised wave functions will add up to an exponentially decaying wave function.

L. Sanchez-Palencia et al., PRL 98, 210401 (2007)

OBSERVATION OF LOCALIZED PROFILES

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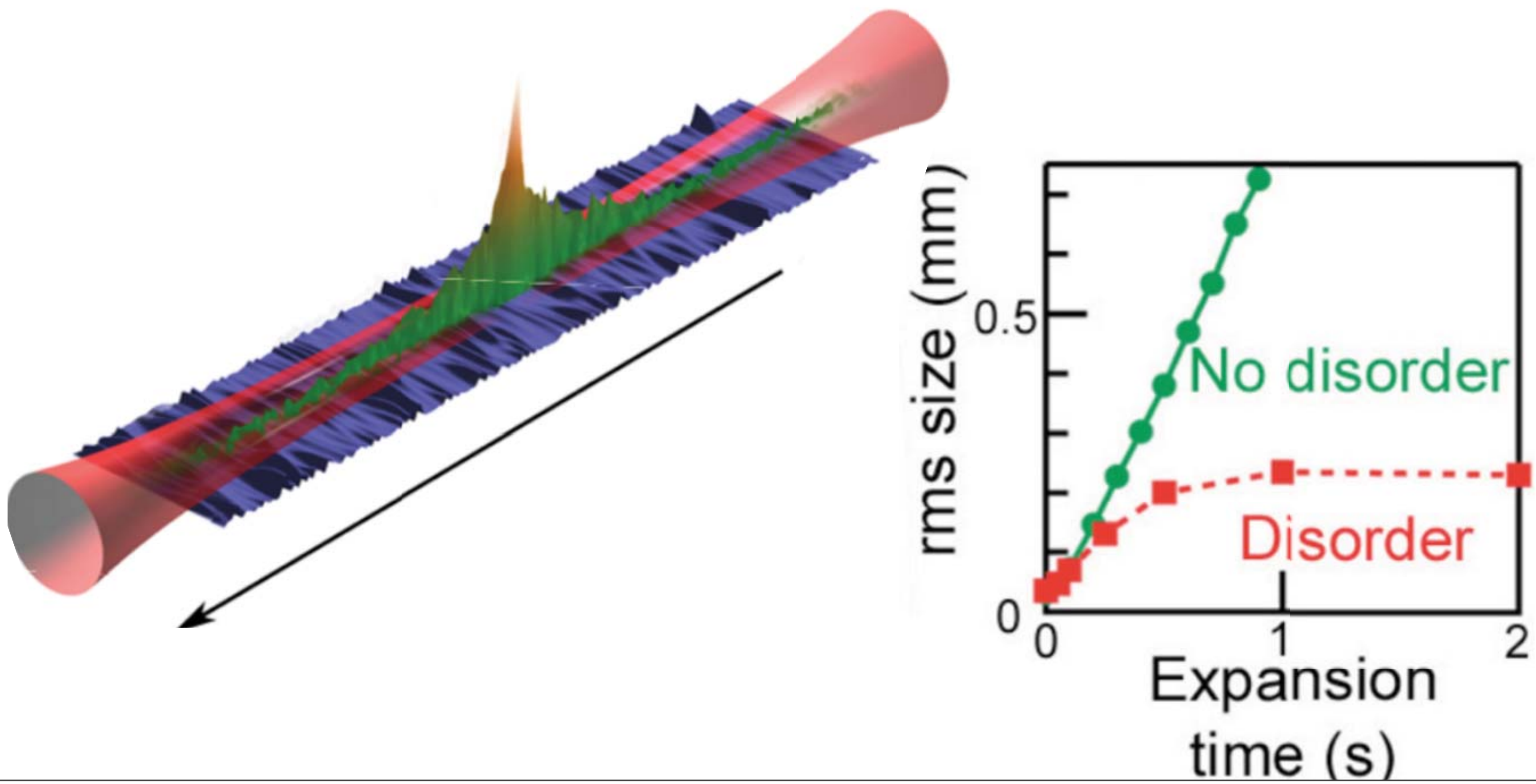
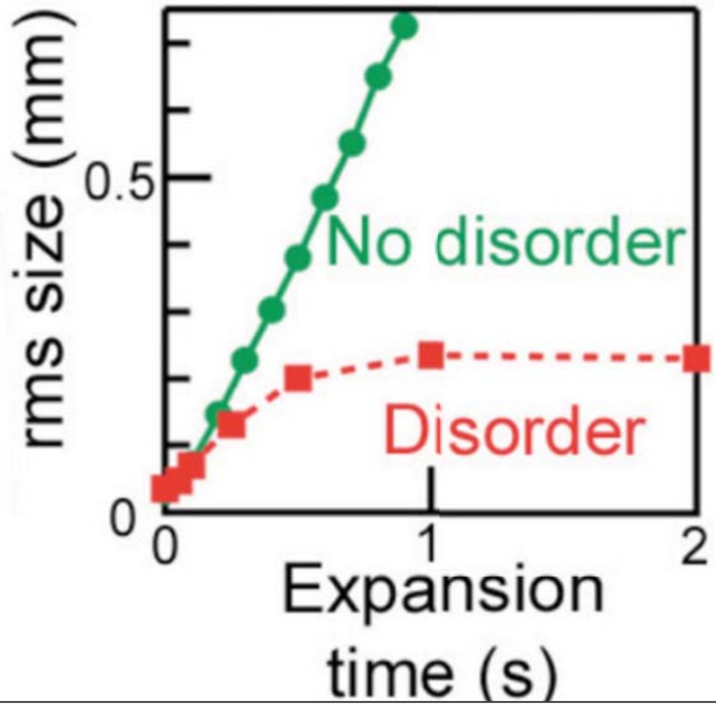
1D BEC EXPANDING IN AN OPTICAL GUIDE



OBSERVATION OF LOCALISED WAVEFUNCTION



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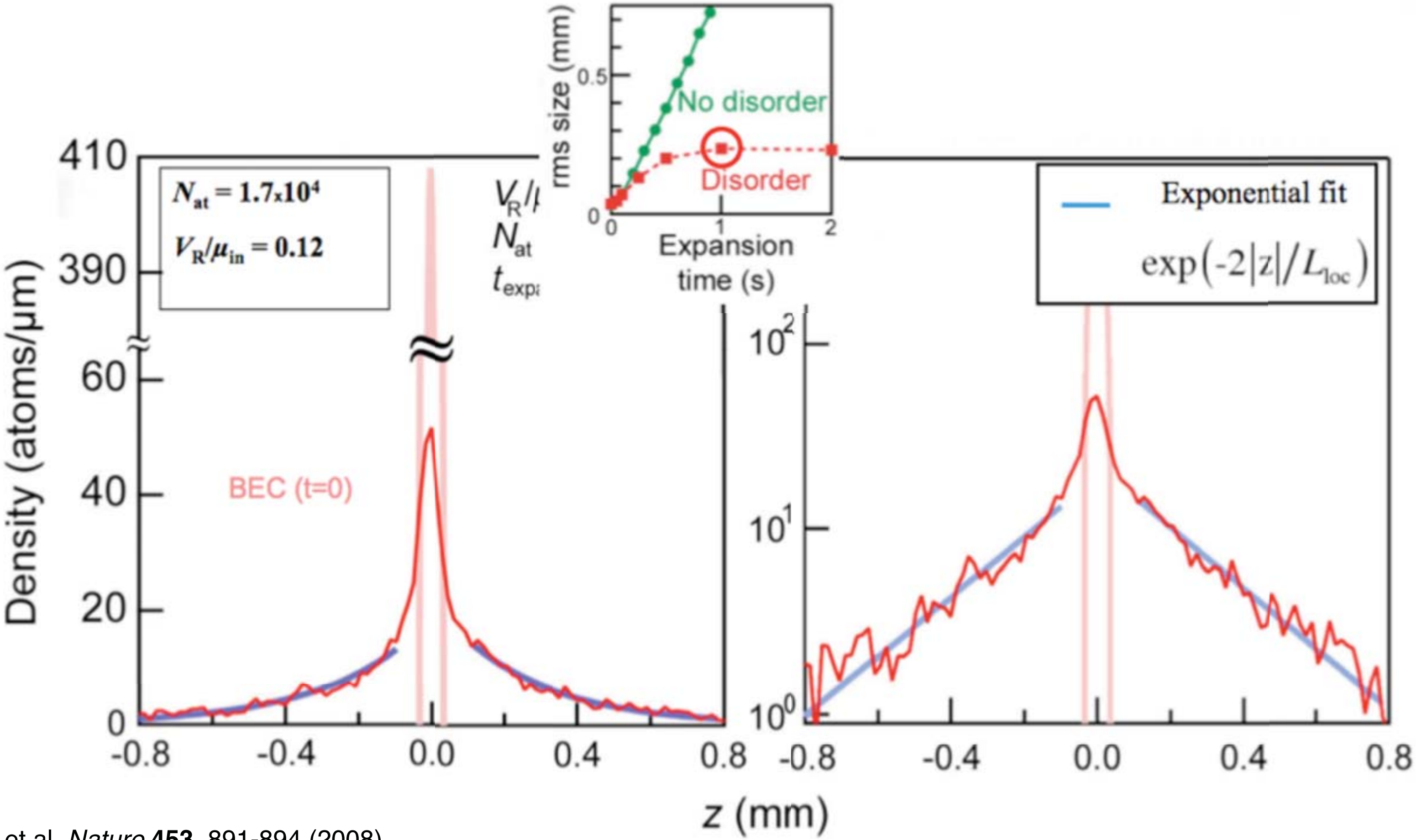
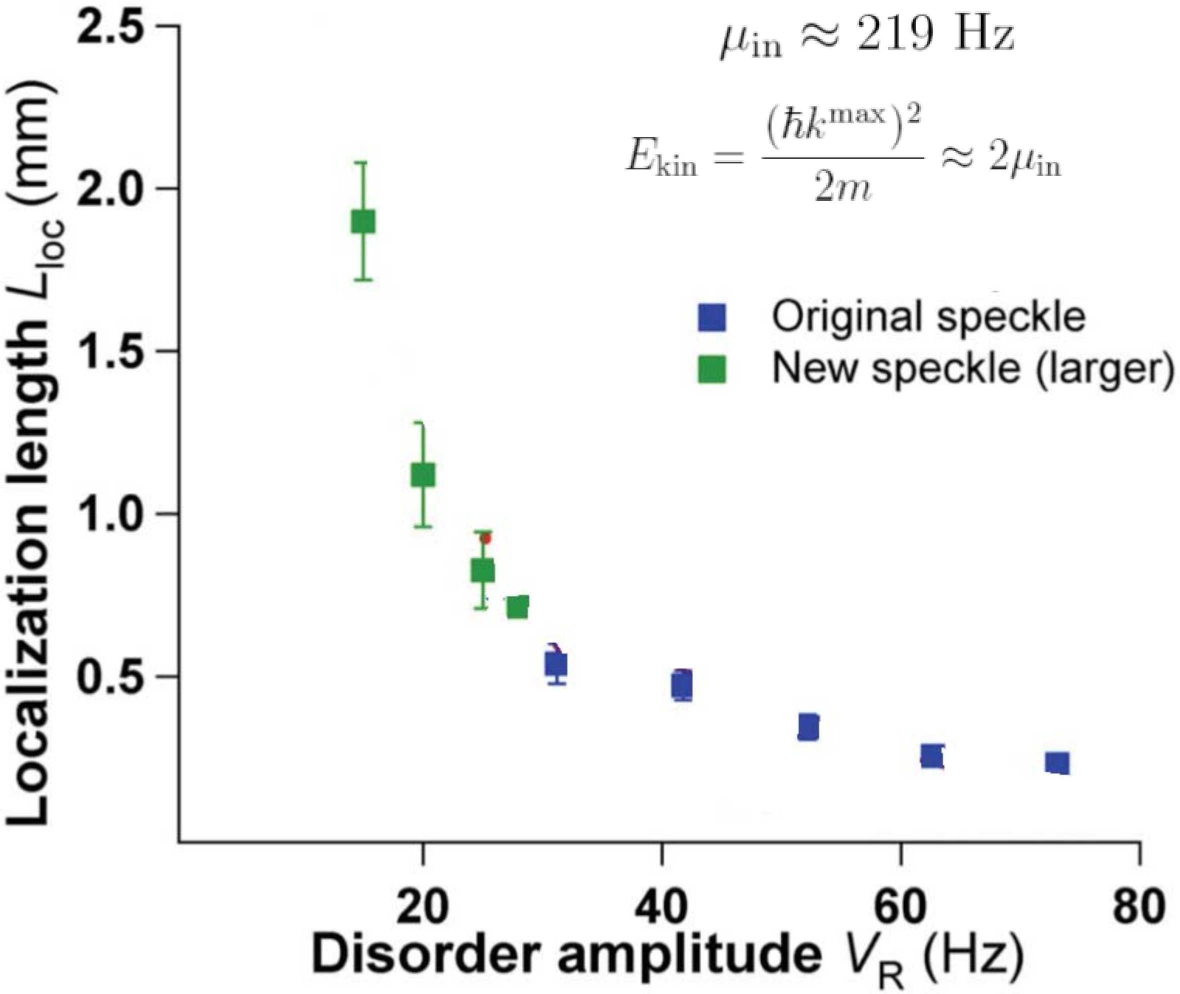


Billy, J. et al. *Nature* **453**, 891-894 (2008).

MEASUREMENT OF LOCALISATION LENGTH

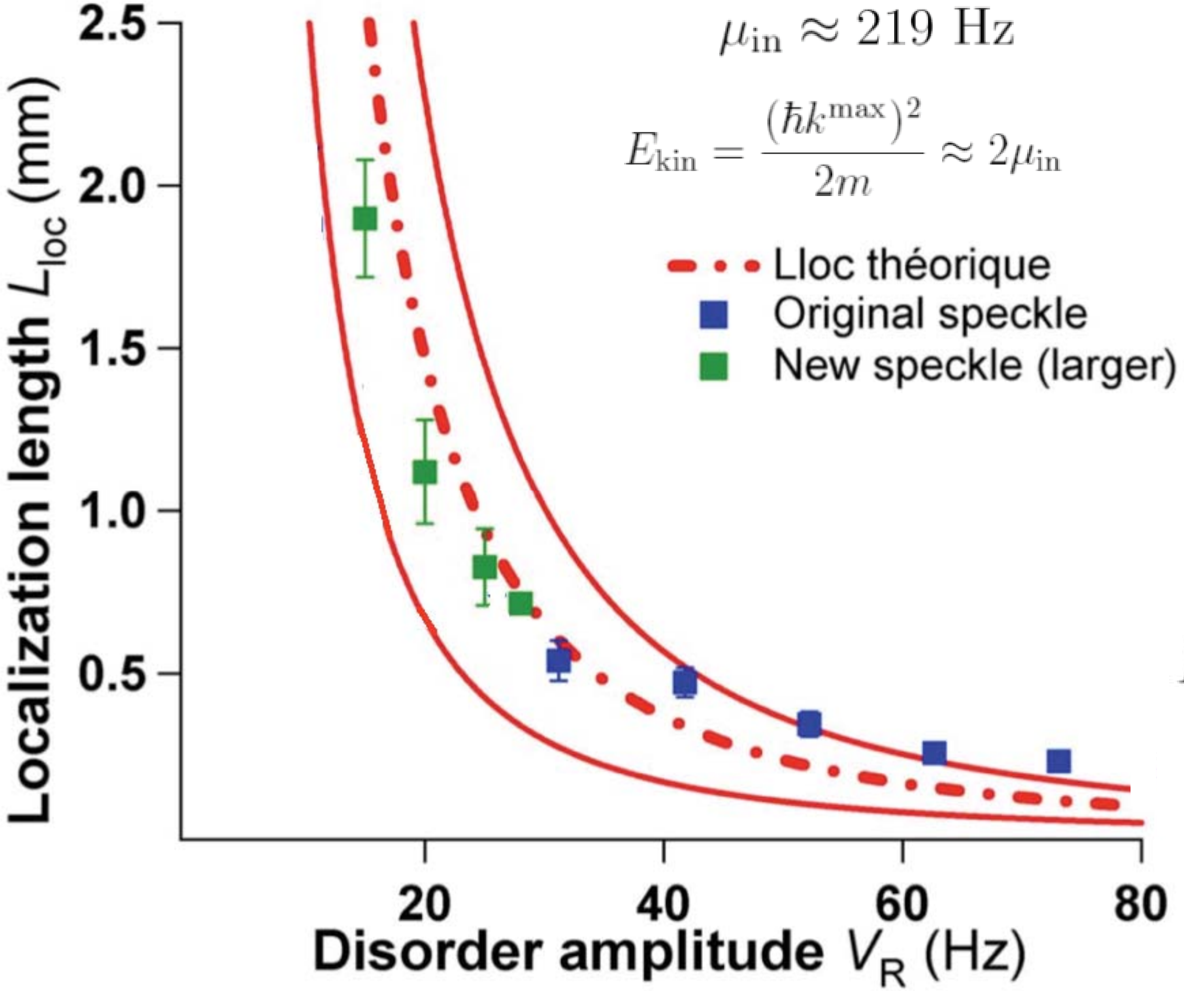
$$\mu_{\rm in} \approx 219\ \text{Hz}$$

$$E_{\rm kin} = \frac{(\hbar k^{\rm max})^2}{2m} \approx 2\mu_{\rm in}$$



$$\gamma(k) \simeq k_c \left(\frac{V_R}{E}\right)^2 (k/k_c)^2 \widehat{c}\,(2k\sigma_R)$$

MEASUREMENT OF LOCALISATION LENGTH



$$\mu_{in} \approx 219 \text{ Hz}$$

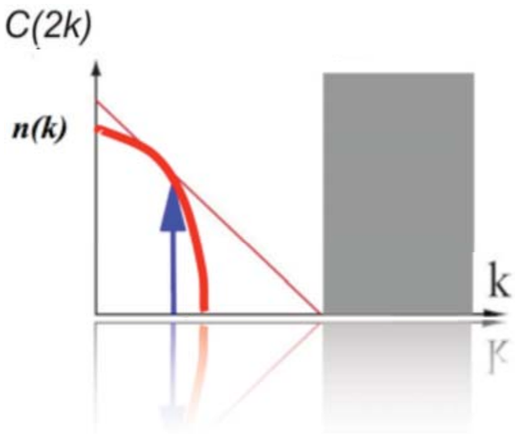
$$E_{kin} = \frac{(\hbar k^{max})^2}{2m} \approx 2\mu_{in}$$

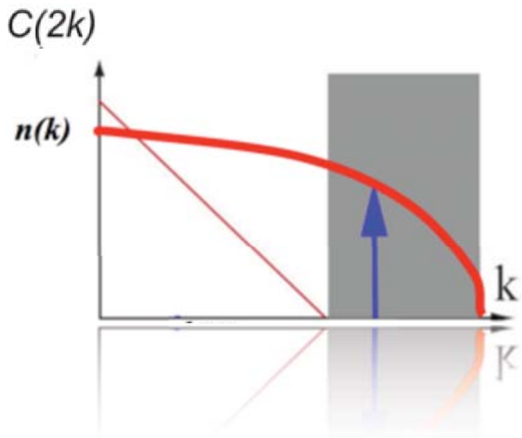
Sum of many "single k" localised states :

$$L_{loc} = \frac{2\hbar^4 k^{max\ 2} k_c}{\pi m^2 V_R^2 (1 - k^{max}/k_c)}$$

THE EFFECTIVE MOBILITY EDGE FOR CORRELATED DISORDER

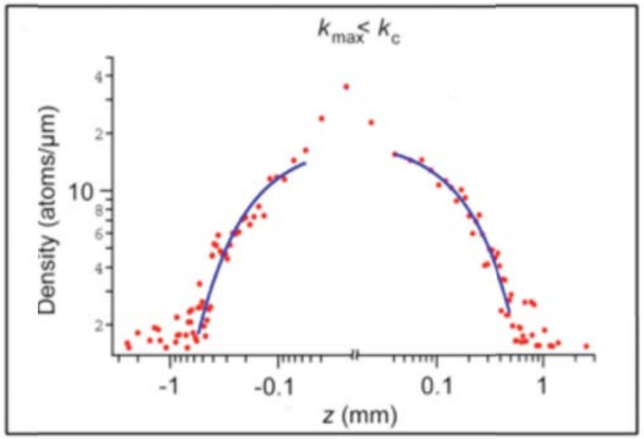
EFFECTIVE MOBILITY EDGE IN 1D-SPECKLE





$$N_{at} = 1.7 \times 10^4$$

$$k^{max}/k_c = 0.65 \pm 0.09$$

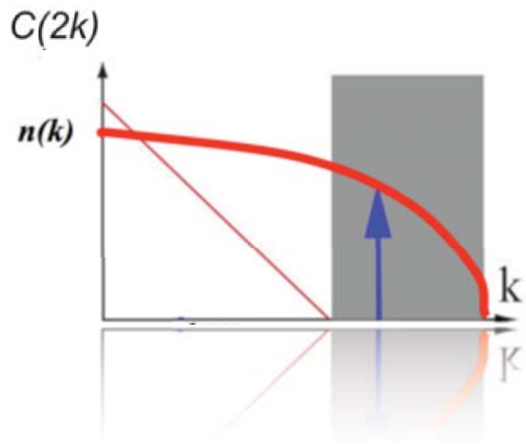


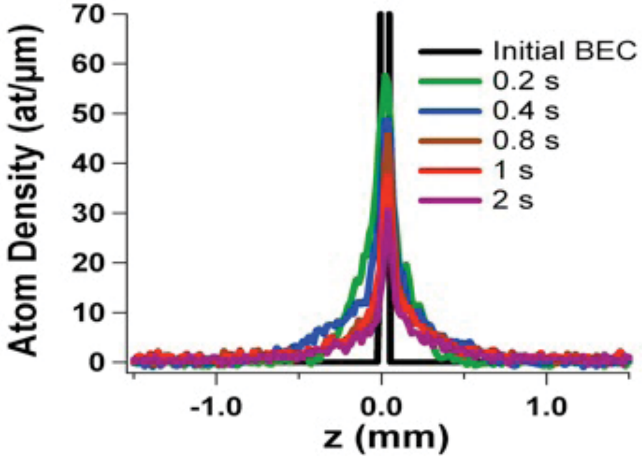
What happens if $k_{max} > k_c$?

Can we still see localisation ?

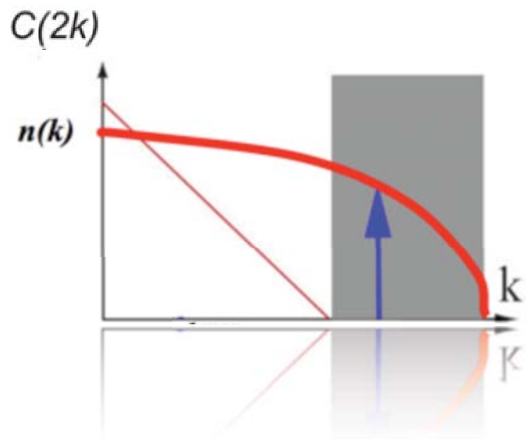
EFFECTIVE MOBILITY EDGE IN 1D-SPECKLE

Our naïve understanding (also rigorously a first order born approximation) tells that partial waves with $k>k_c$ cannot localise.





EFFECTIVE MOBILITY EDGE IN 1D-SPECKLE



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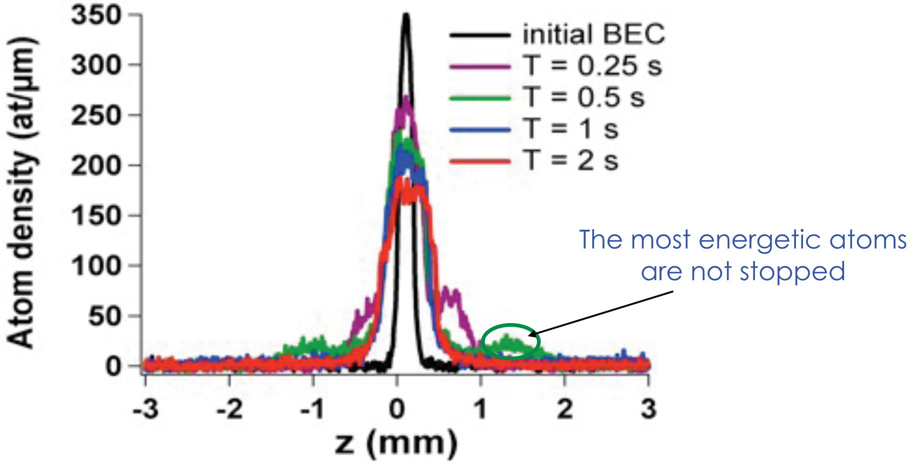
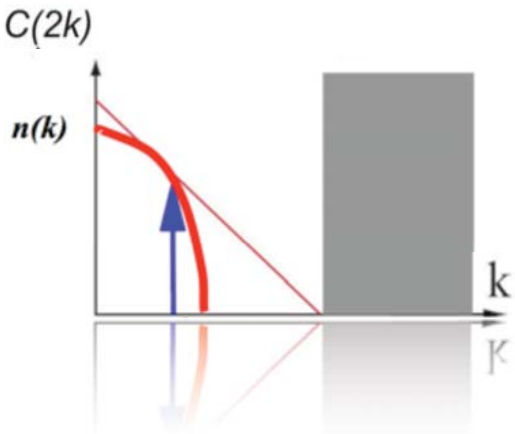


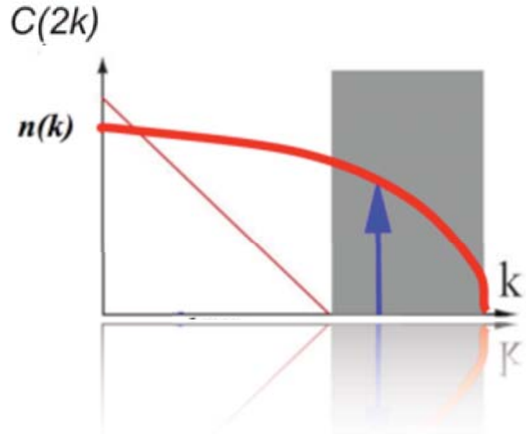
No more exponential localisation (effective mobility edge)

$$\gamma(k) \simeq k_c \left(\frac{V_R}{E}\right)^2 (k/k_c)^2 \, C(2k/k_c)$$

Wavefunction with algebraic wings (power law decay)

EFFECTIVE MOBILITY EDGE IN 1D-SPECKLE



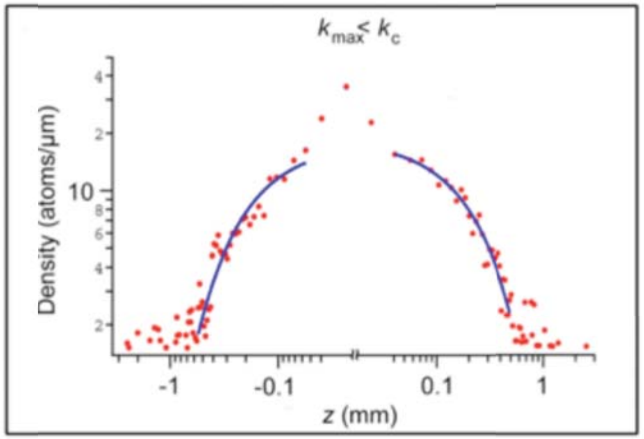


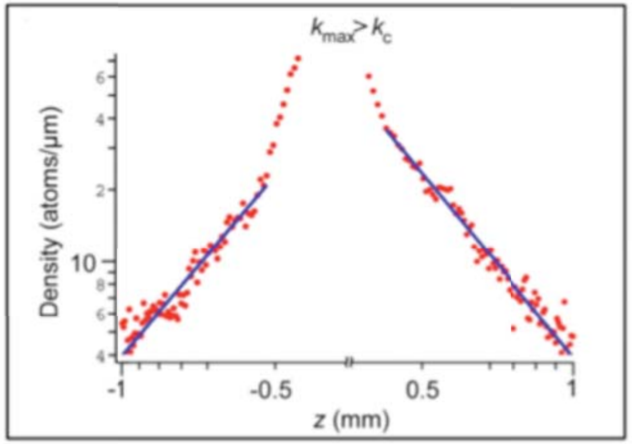
$N_{at} = 1.7 \times 10^4$

$k^{max}/k_c = 0.65 \pm 0.09$

$N_{at} = 1.7 \times 10^5$

$k^{max}/k_c = 1.16 \pm 0.14$





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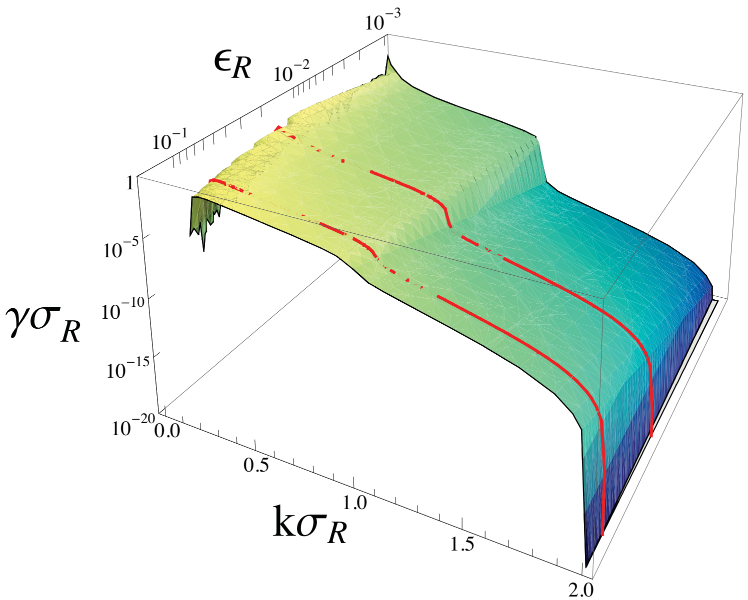
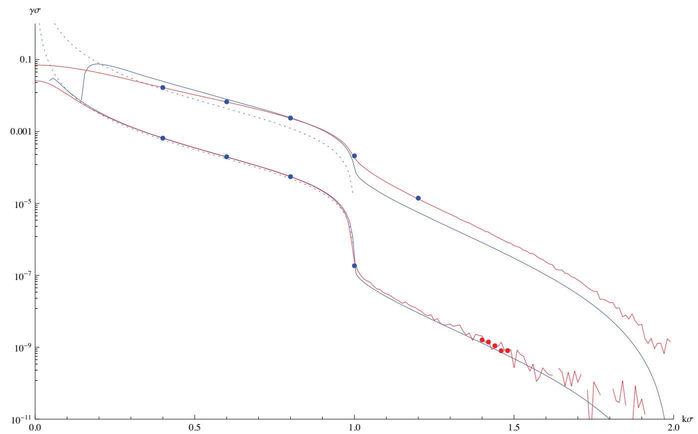
EFFECTIVE MOBILITY EDGE IN 1D-SPECKLE

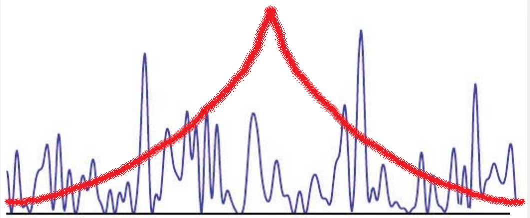
Is it compatible with our common understanding that in 1D, there is always localisation ?

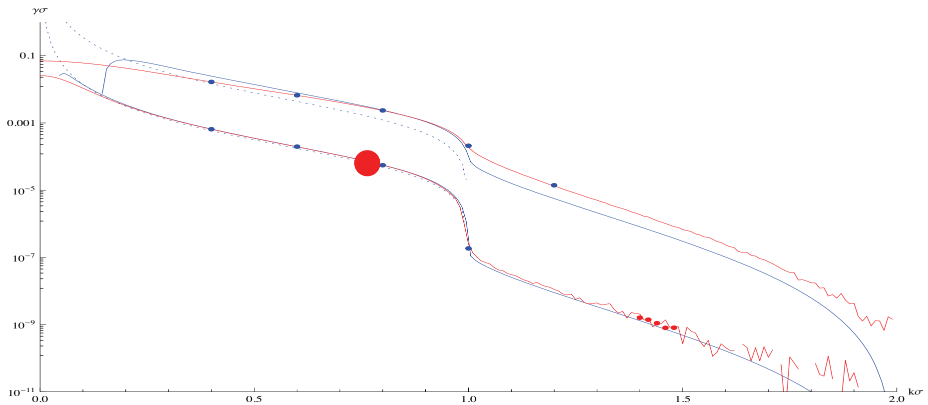
Yes : a more complete calculation (Collaboration between L. Sanchez Palencia, D. Delande, C. Miniatura and C. Muller) shows that the localisation length has a change in magnitude.





If $k_{max}>k_c$, the localisation lenght "explodes" beyond any observation scale for the experiment.





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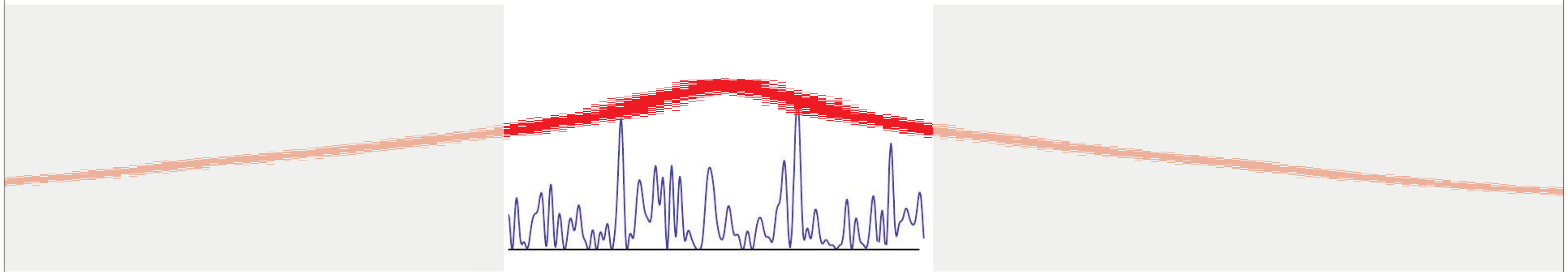
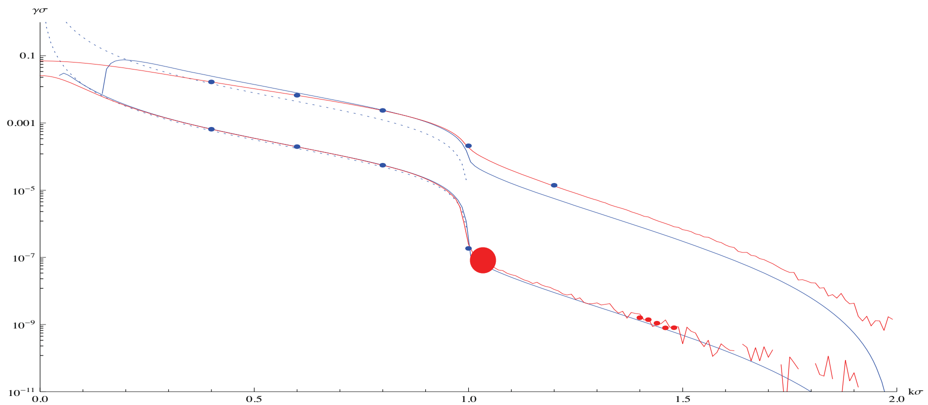
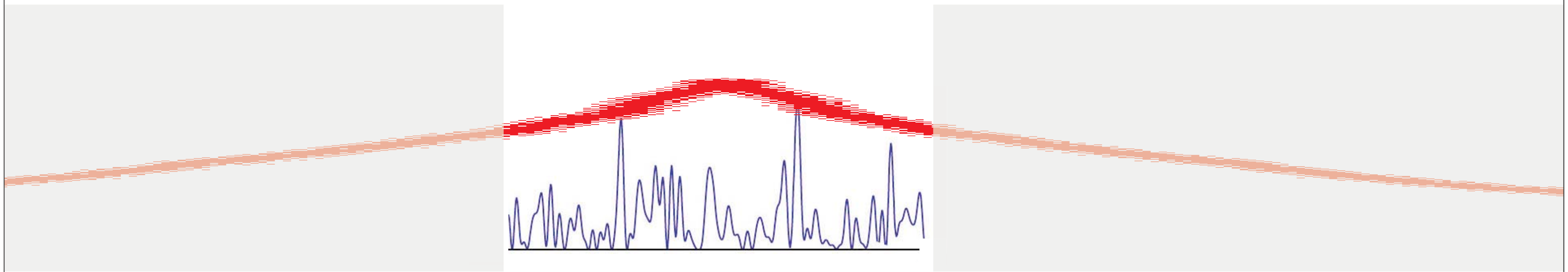


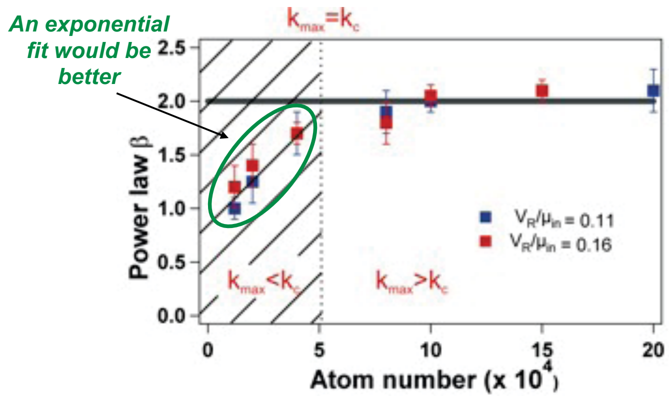


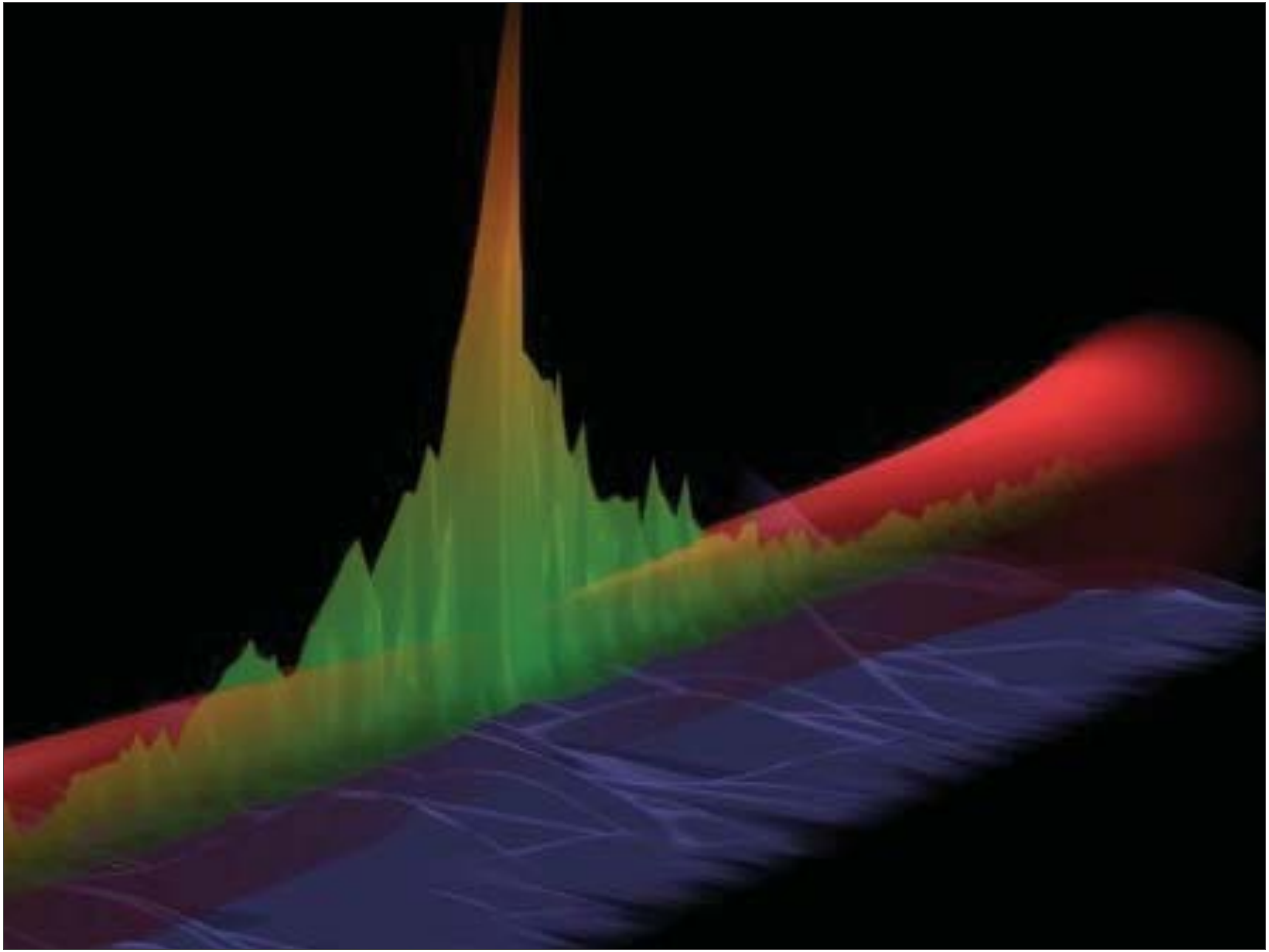
If $k_{max}>k_c$, the localisation lenght “explodes” beyond any observation scale for the experiment.



We cannot resolve the exponential decay anymore

The wavefunction decays as a power law (z^{-2} for our parameters).





We have directly observed Anderson Localised atomic wave functions

This experiment shows that we can now handle atoms and disorder :

- Possibility to control all matter-wave parameter (velocity, density, interactions ...)
- Very good control and understanding on the disorder model (Speckle pattern well characterized ...)

Direct observation of wavefunction, correlation functions ...

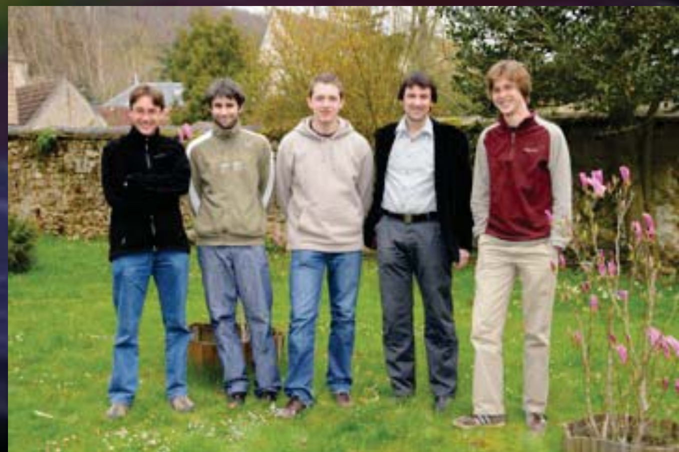
Understanding the rôle of disorder is fundamental in many field :

- Optics (wave propagation, random lasers ...), electrons, ... *Storzer et al. Phys. Rev. Lett. 96, 063904 (2006), Schwartz et al. Nature 446, 52 (2007), Lahini, Y. et al. Phys. Rev. Lett. 100, 013906 (2008) ...*
- On cold atoms, many experiment using the “mapping” from quantum chaos (dynamical localisation) has been used : *Moore et al. PRL (1994), Chabé et al. (2007) ...*
- There are now many experiments studying BEC and disorder, adressing various aspects (propagation, lattice ...) : *Billy et al. Nature 453, 891(2008), Roati et al., Nature 453, 896 (2008), Chen et al. Physical Review A 77, 033632 (2008), Schulte et al., PRL 95, 170411(2005), White et al arxiv:0807.0446 (2008)*

This systems offers a tool to understand and explore AL physics

- Effect at higher dimension : real quantum phase transition at 3D
- Rôle of interactions, localisation of excitations ...

Anderson localisation in the atom optics group (A. Aspect, www.atomoptic.fr)



Theory : L. Sanchez Palencia,

P. Lugan, B. Hambrecht, L. Pezze

Experiments : P. Bouyer

1D and 3D : V. Josse

J. Billy, A. Bernard, P. Cheinet, Z. Zuo

2D and controllable interactions : T. Bourdel

J-P Brantut, M. Robert de Saint Vincent, JF Clément