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**Anderson localization of a weakly interacting Bose-Einstein condensate**

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# ***Anderson localization of a weakly interacting Bose-Einstein condensate***

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Research Frontiers in Ultra-Cold Atoms, ICTP, Trieste, May 4, 2009

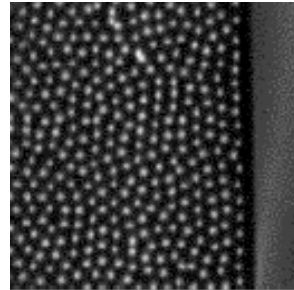


# Introduction

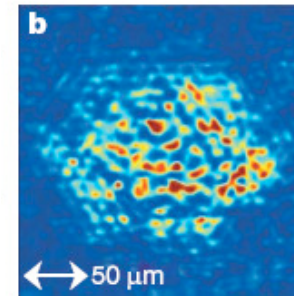
Disorder is ubiquitous in nature. Disorder, even if weak, tends to inhibit transport.



Superfluids  
in porous  
media



Superconducting  
thin films



Light propagation  
in random media

Still much has to be understood:

- localization of non-interacting particles (Anderson)
- interplay of disorder and interactions
- strongly correlated, disordered system

# Anderson localization

PHYSICAL REVIEW

VOLUME 109, NUMBER 5

MARCH 1, 1958

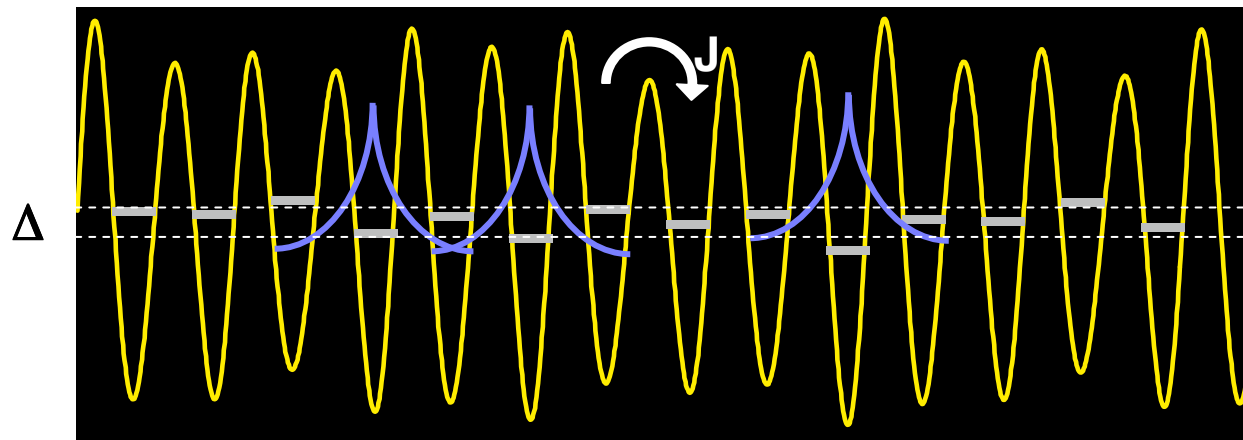
## Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON

*Bell Telephone Laboratories, Murray Hill, New Jersey*

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.



No transport can occur for  $\Delta > J$ , due to the destructive interference of many possible paths

# Anderson localization

An essential feature is the absence of interactions between particles:

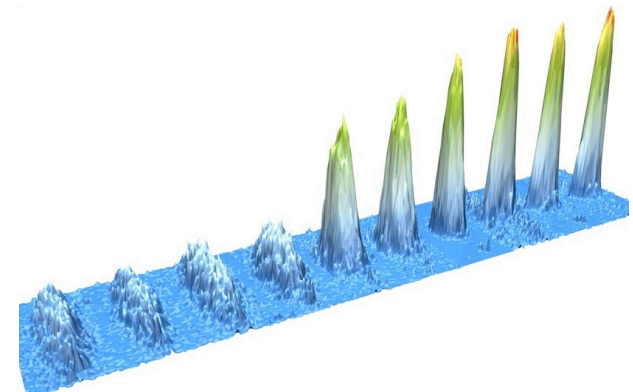
Light in powders and disordered photonic crystals: Van Albada & Lagendijk, *Phys. Rev. Lett.* **55**, 2692 (1985); Wiersma, et al. , *Nature* **390**, 671 (1997); Lahini, et al., *Phys. Rev. Lett.* **100**, 013906 (2008).

Microwaves: Dalichaouch, et al, *Nature* **354**, 53 (1991).

Ultrasounds: Weaver, *Wave Motion* **12**, 129-142 (1990).

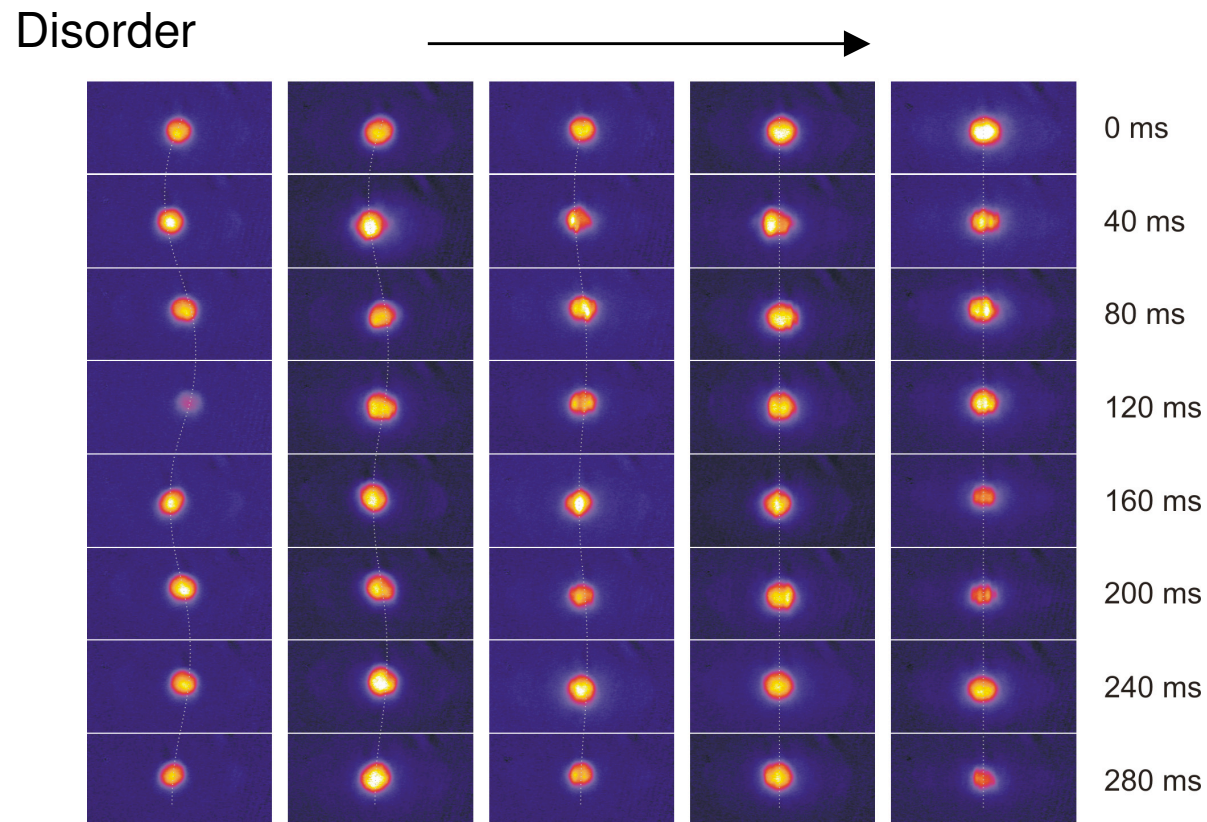
Disordered electronic systems: Akkermans & Montambaux *Mesoscopic Physics of electrons and photons* (Cambridge University Press,2006); Lee & Ramakrishnan, *Rev. Mod. Phys.* **57**, 287 (1985)

Our approach: use a Bose-Einstein condensate with a suitable Feshbach resonance and study separately **Anderson localization** and the **effects of interaction**



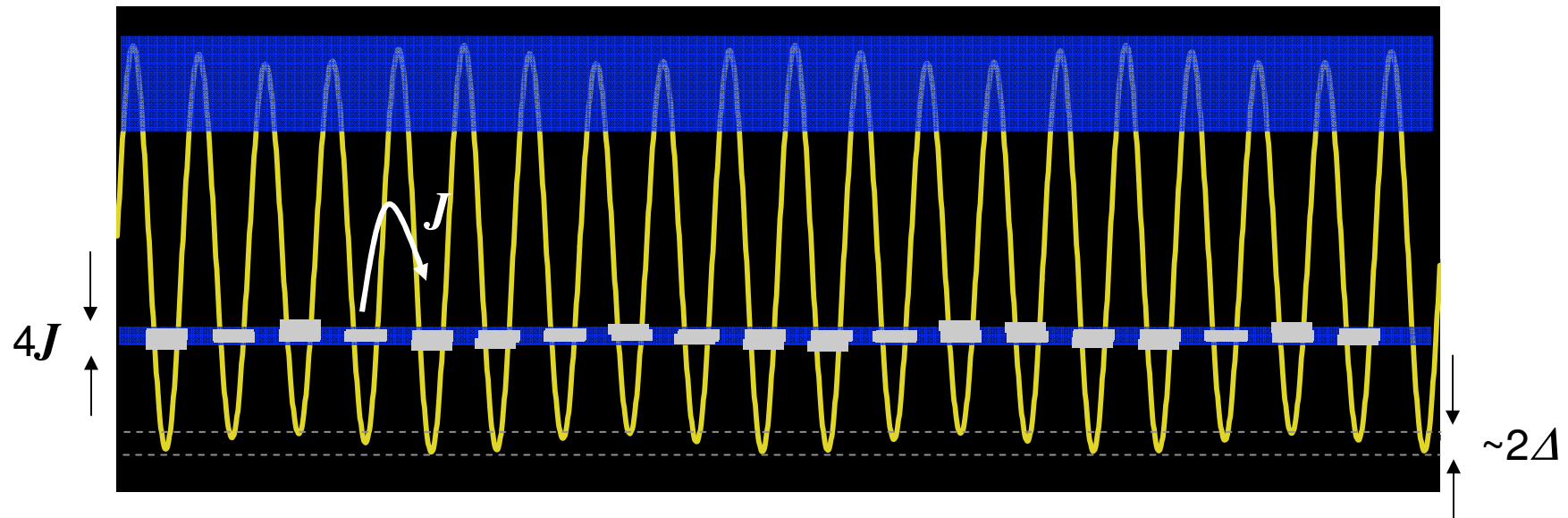
# Bose-Einstein condensates and disorder

Since 2004, experiments in Hannover + Barcelona, Orsay, Firenze, Austin, Urbana



Difficult to distinguish localization effects due to disorder from those due to the repulsive interaction.

# Two incommensurate lattices: the Aubry-André model

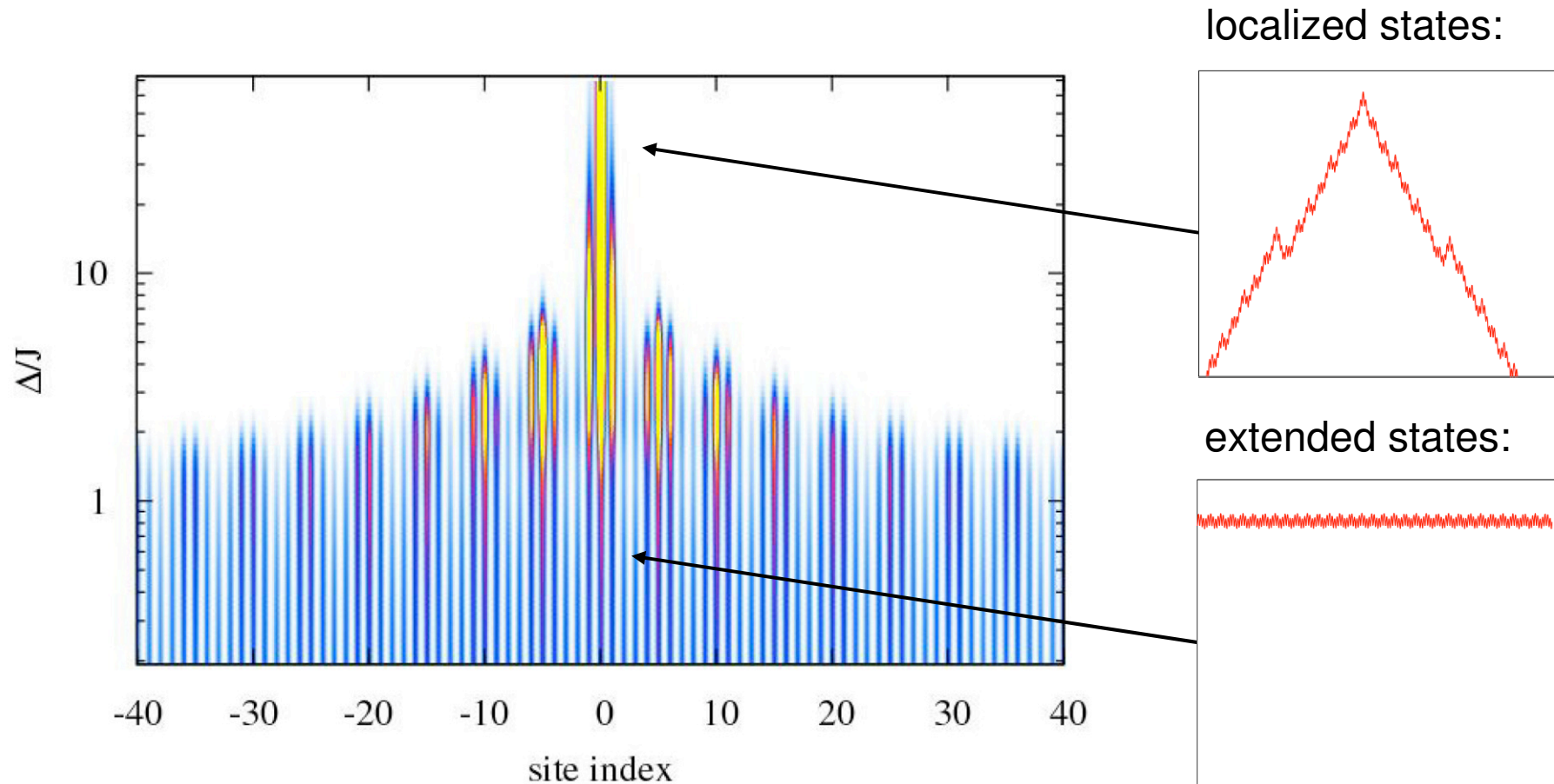


$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^+ \hat{b}_j + \Delta \sum_i \cos(2\pi\beta i) \hat{n}_i \quad \beta = \frac{k_2}{k_1}$$

$$\hat{H}_{\bar{k}} = -J \frac{\Delta}{2J} \sum_{\bar{k}} \hat{b}_{\bar{k}}^+ \hat{b}_{\bar{k}+1} + \Delta \frac{2J}{\Delta} \sum_i \cos(2\pi\beta \bar{k}) \hat{n}_{\bar{k}}$$

G. Harper, Proc. Phys. Soc. A 68, 674 (1965); S. Aubry and G. André, Ann. Israel Phys. Soc. 3, 133 (1980); D. R. Grempel, et al., Phys. Rev. Lett. 49, 833-836 (1982).

# The ground state of the A-A model



Solution of the A-A model for the experimental parameters:

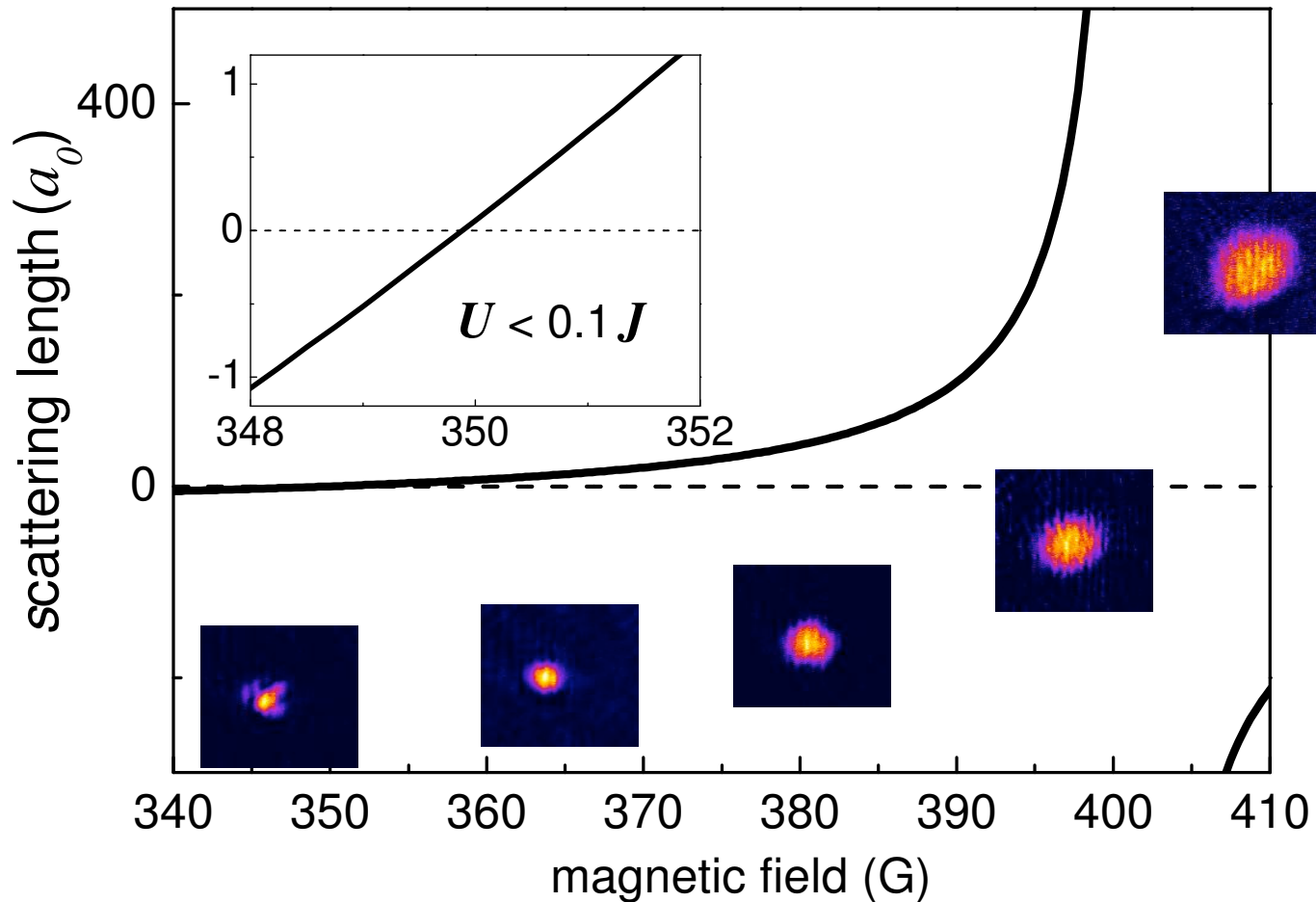
$$\beta = 1030 \text{ nm} / 860 \text{ nm} = 1.1972..$$

Average separation of localized states

$$\bar{d} = \frac{1}{\beta - 1} \approx 5.1 \text{ sites}$$



# 39K: a Bose gas with highly controllable interaction



Release energy:

G. Roati, et al., Phys. Rev. Lett. 99, 010403 (2007)

Feshbach spectroscopy:

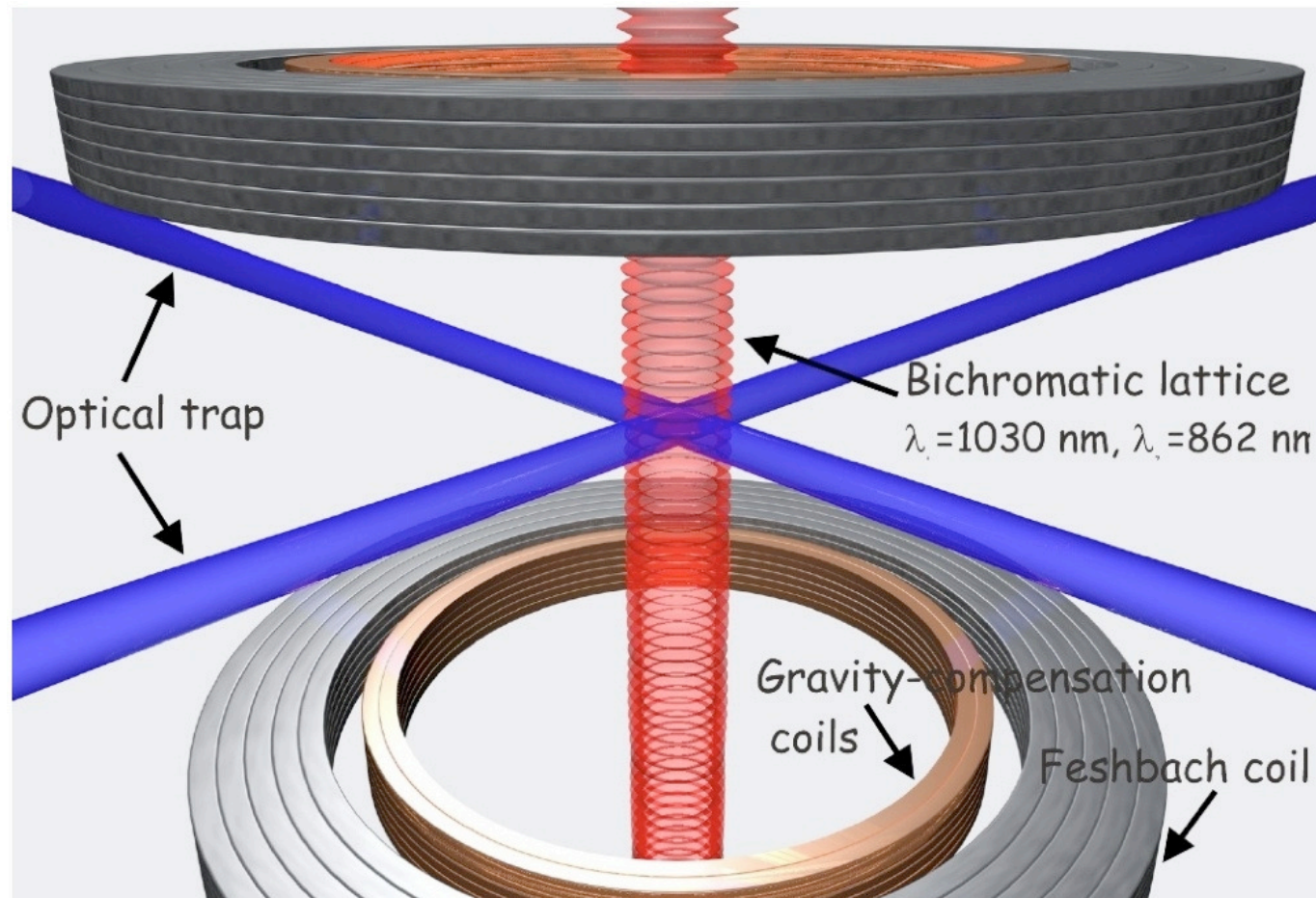
C. D'Errico et al. New J. Phys. 9, (2007)

Interplay of contact and dipolar interaction at  $a \sim 0$ :

M. Fattori et al. Phys. Rev. Lett. 100, 080405 (2008);

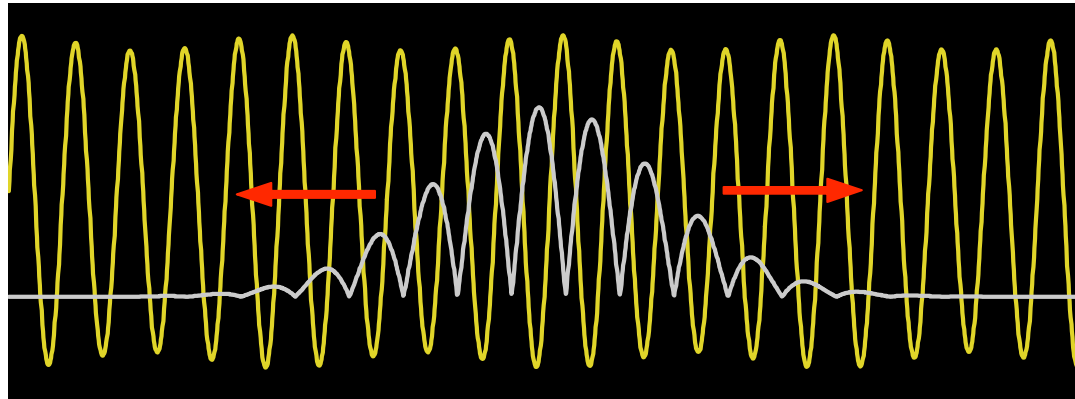
M. Fattori et al. Phys. Rev. Lett. 101, 190405 (2008).

# Experimental scheme

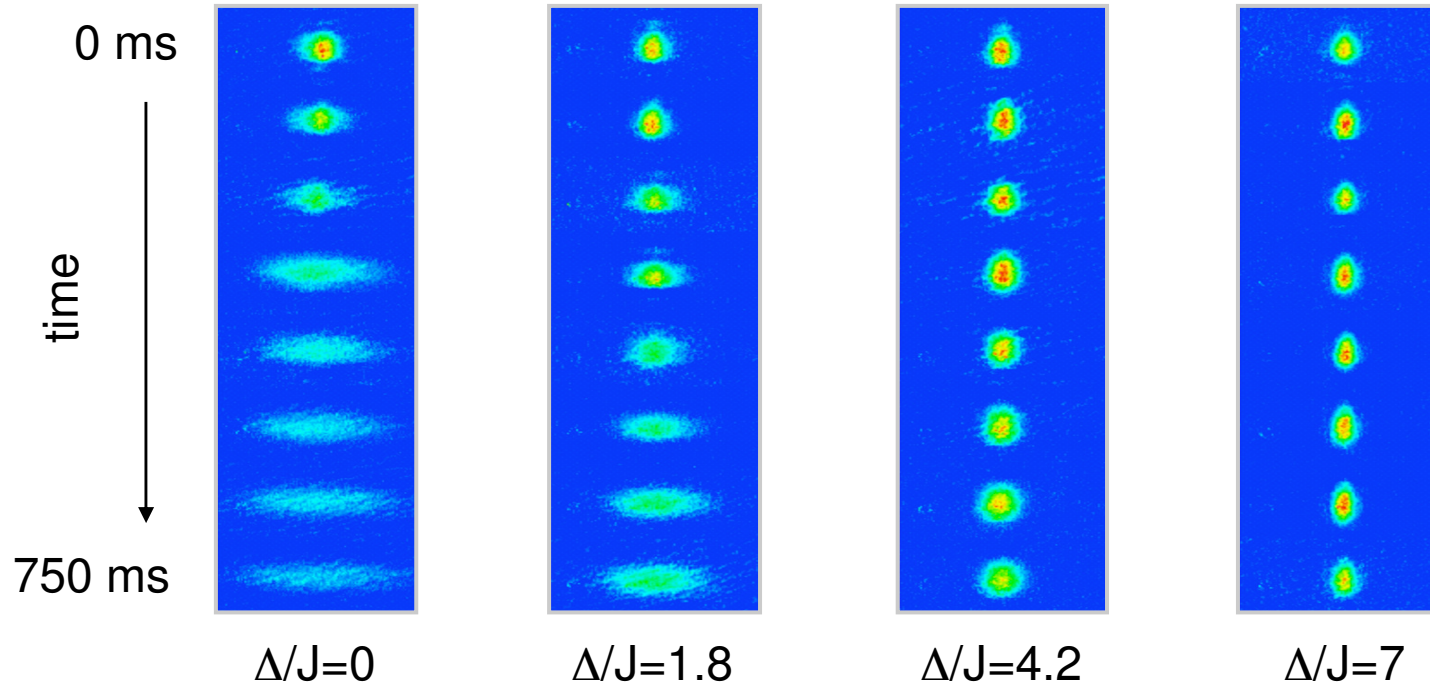


Fast tuning of the interaction:  $\tau \sim 300 \mu\text{s}$

# Probing the transport properties

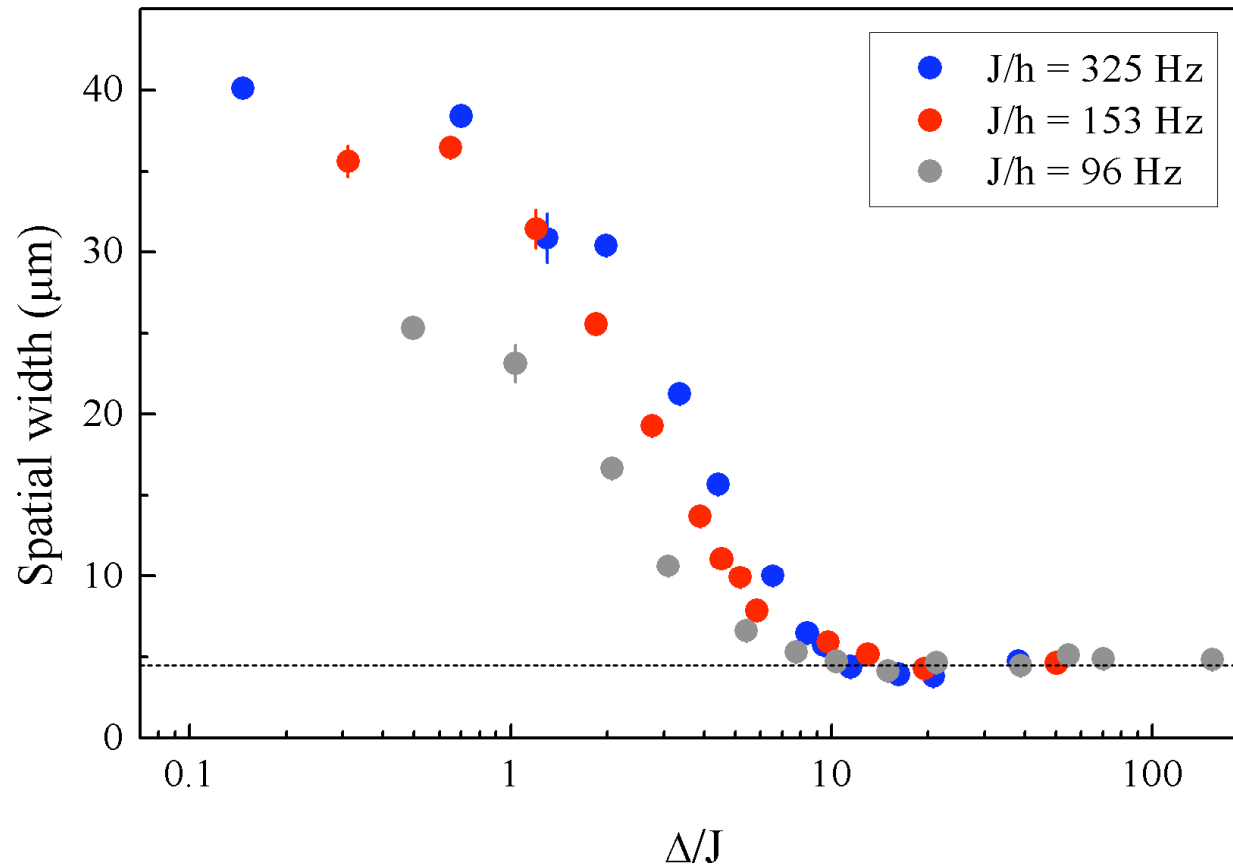


condensation ( $a=200 a_0$ )  $\xrightarrow{50 \text{ ms}}$   $a \sim 0.05 a_0$   $\xrightarrow{100 \text{ ms}}$  lattice loading



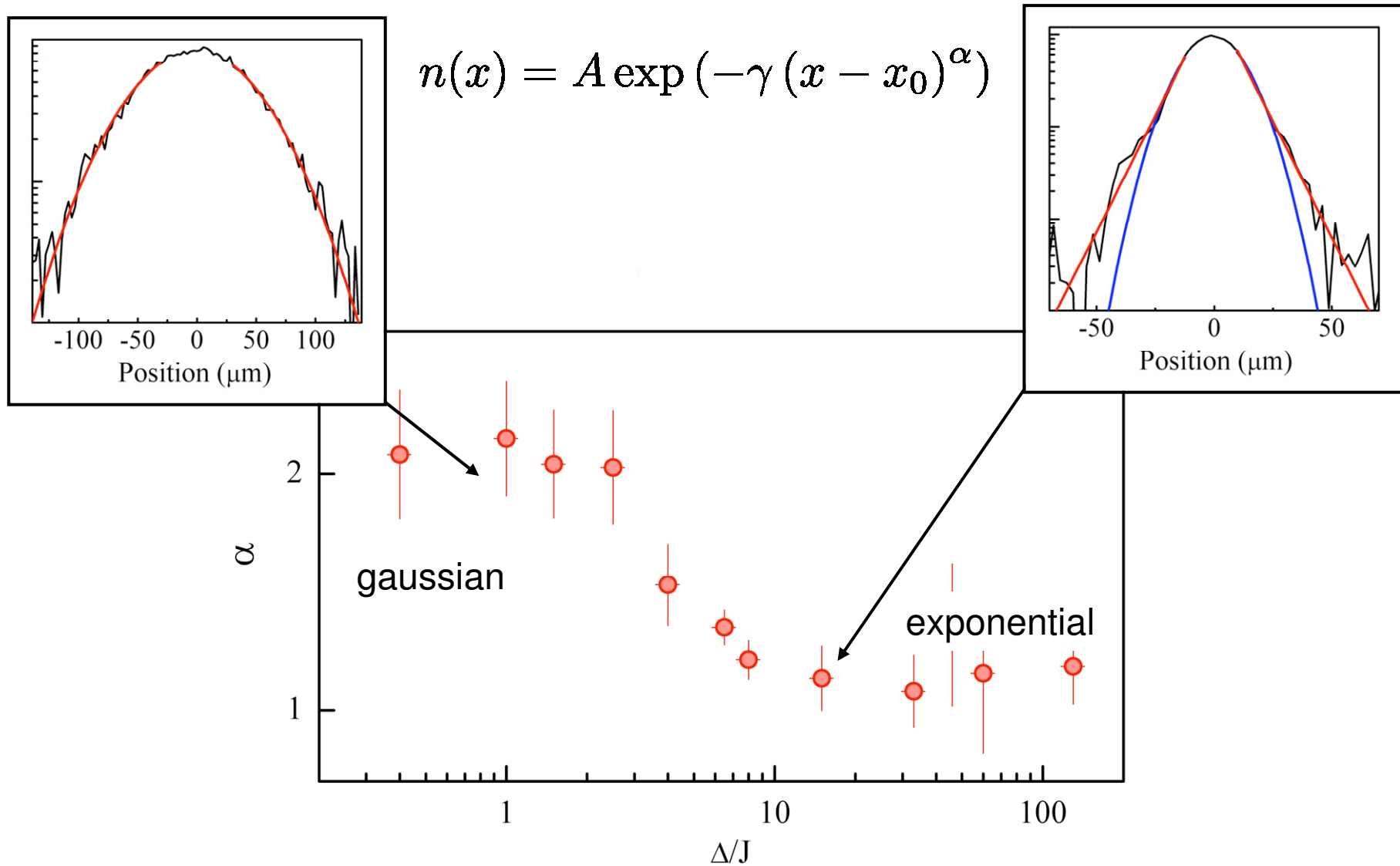
# Probing the transport properties

Size of the condensate after 750 ms of expansion in the quasi-periodic lattice:

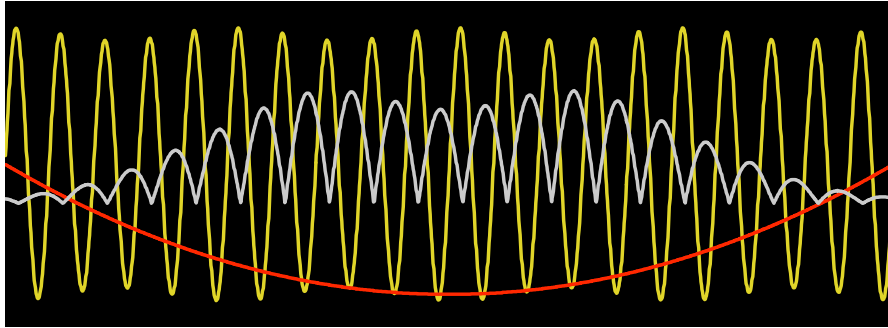


# Probing the spatial distribution

Fit of the density distribution with a generalized exponential function:

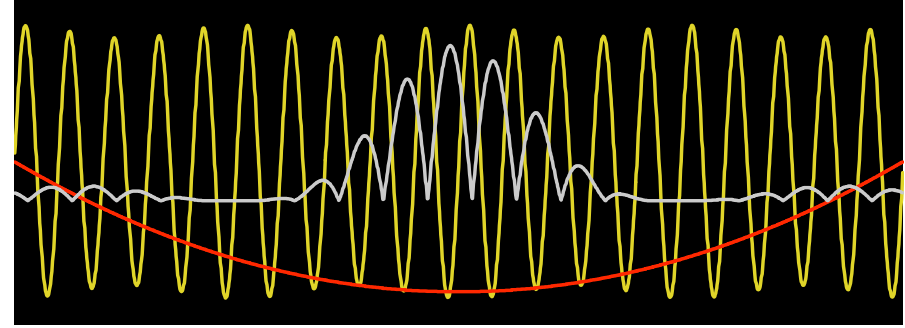


# Probing the momentum distribution



Extended state in the “superlattice”

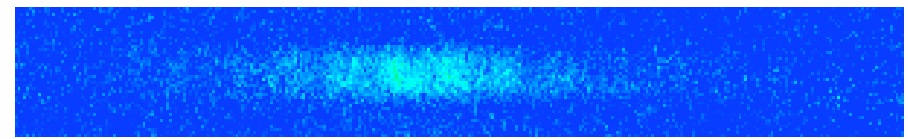
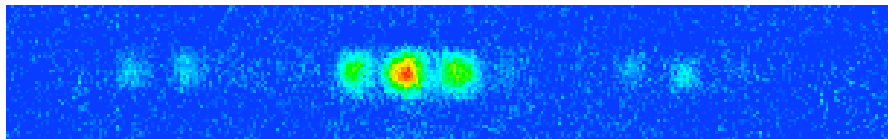
$$\Delta/J < 2$$



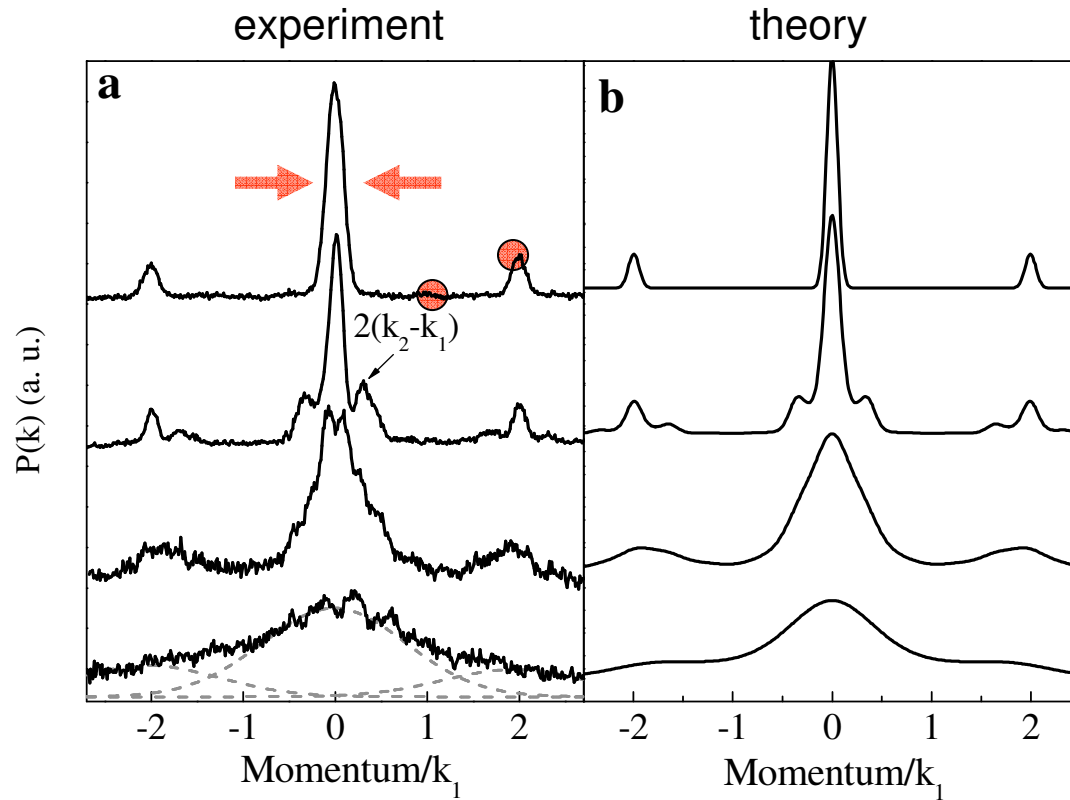
Localized states

$$\Delta/J > 2$$

Momentum distribution can be reconstructed  
after a long free expansion



# Probing the momentum distribution

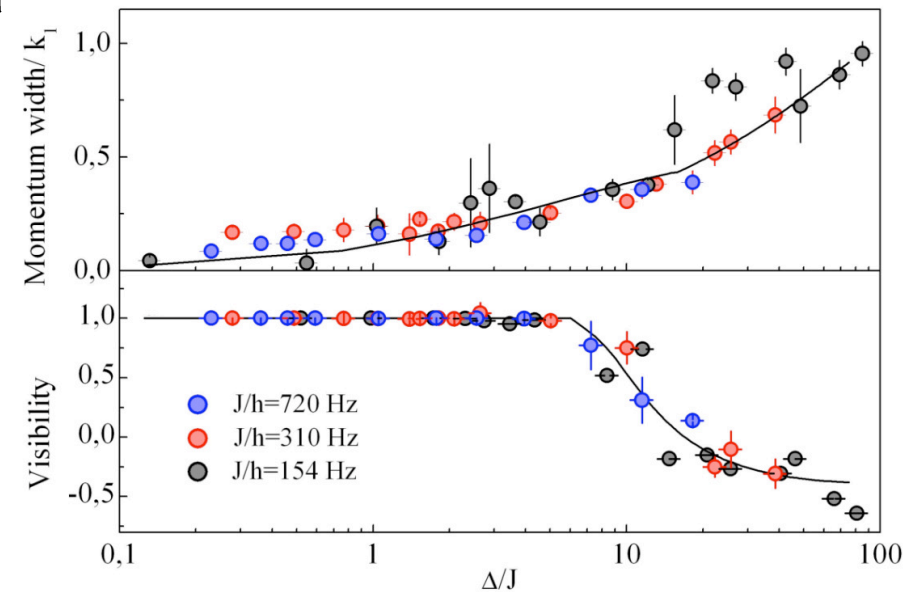


Width of the central peak

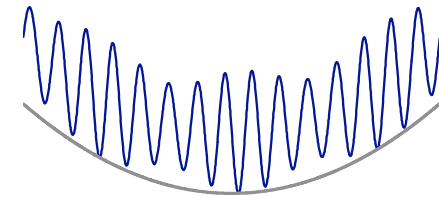
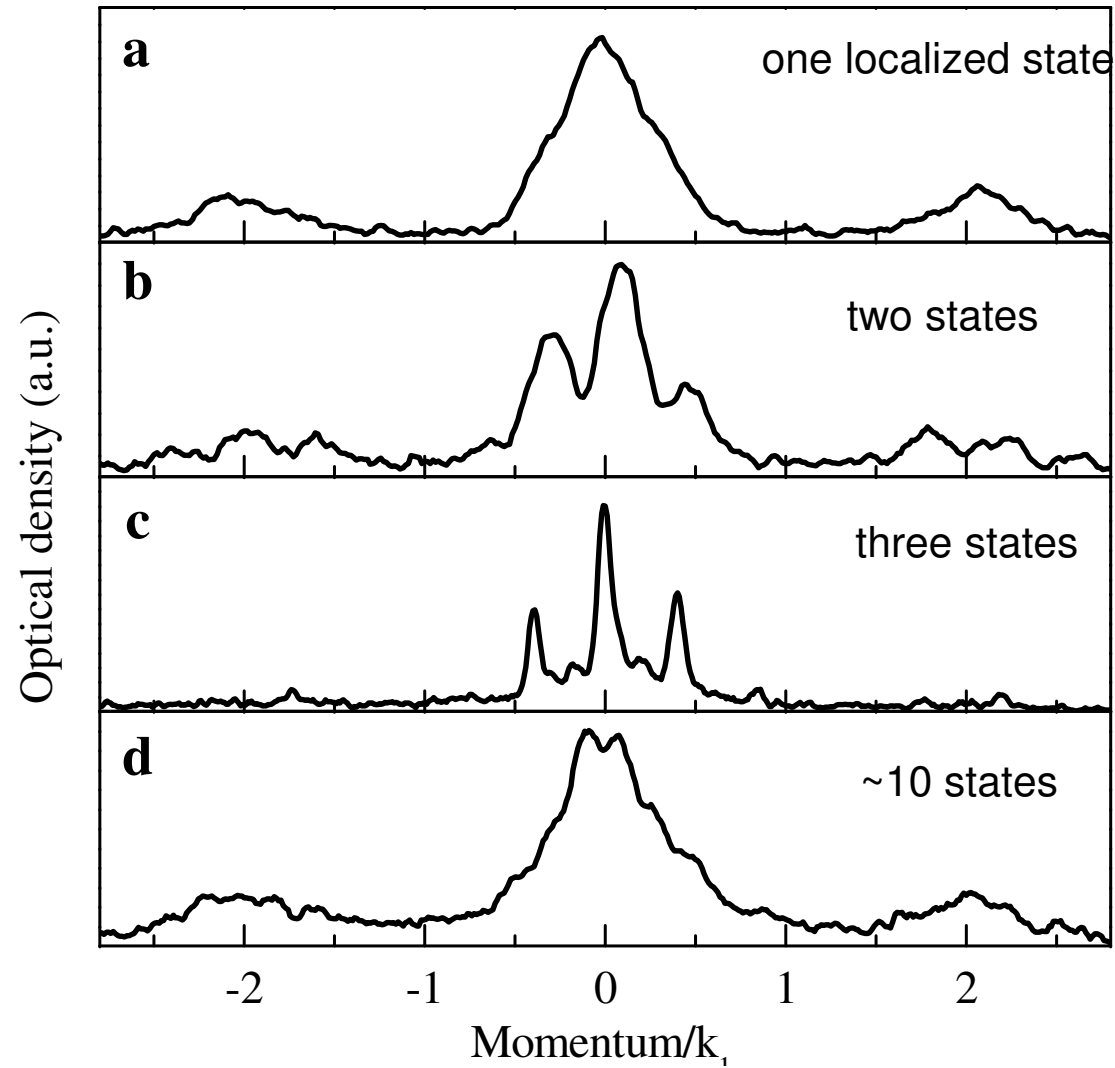
$$\text{Visibility} = \frac{P(2k_1) - P(k_1)}{P(2k_1) + P(k_1)}$$

Density distribution after time-of-flight of the initial stationary state

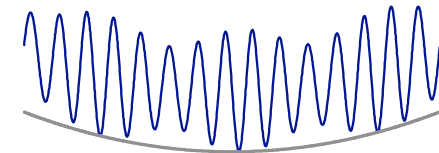
disorder



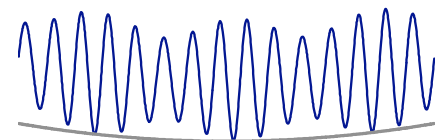
# Counting the localized states



$$a_{\text{ho}} = 1.2 \mu\text{m}$$



$$a_{\text{ho}} = 2.1 \mu\text{m}$$

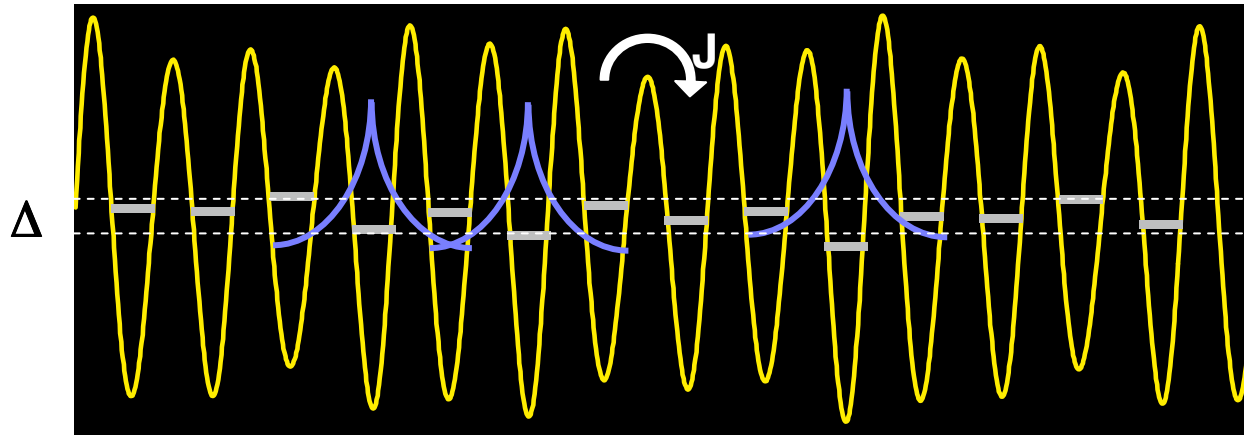


$$a_{\text{ho}} = 5.4 \mu\text{m}$$

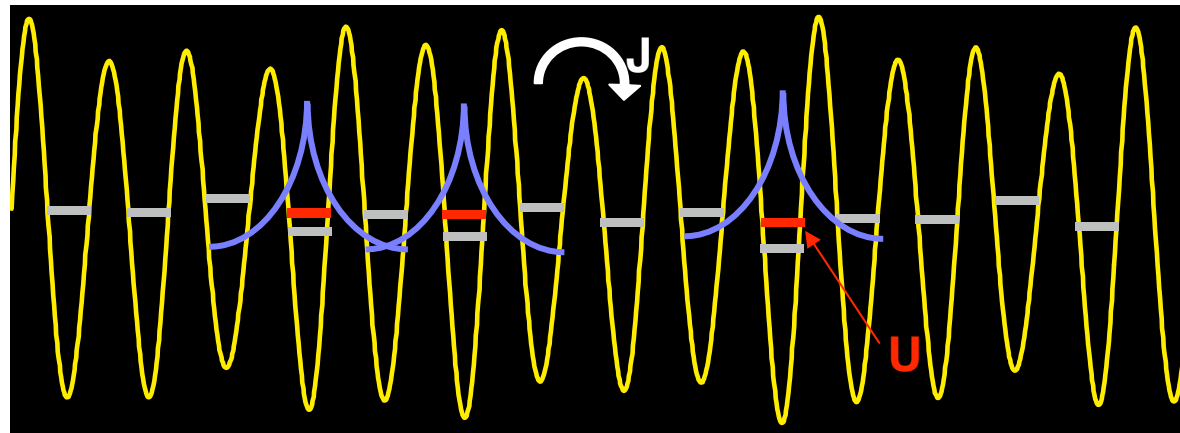


# Interaction and disorder: expectation

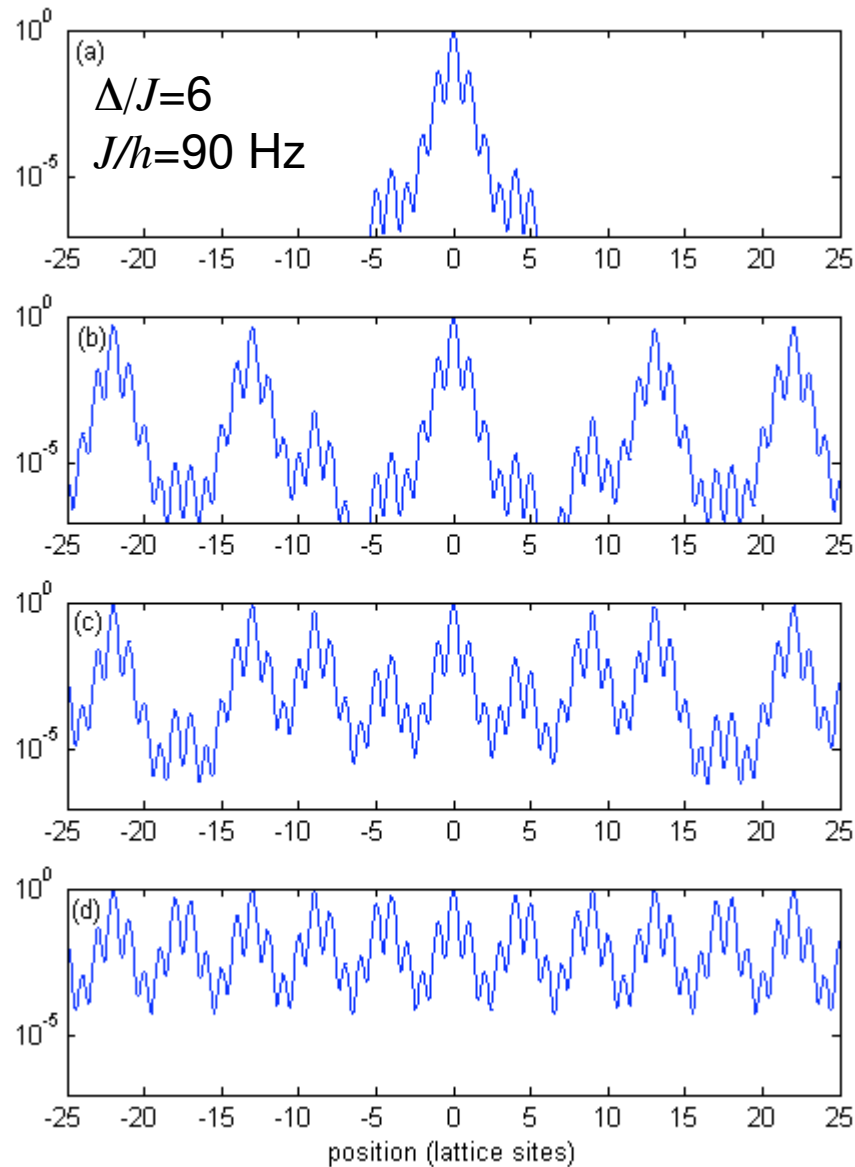
No interaction: several independent localized states. The relative phase is not constant.



With interaction: localized states get more extended and lock in phase. The system eventually condenses back into a single extended state.



# Interaction and disorder: ground state



Interaction energy:

$$U = \frac{4\pi \hbar^2}{m} a N \int |\varphi_i(r)|^4 d^3r$$

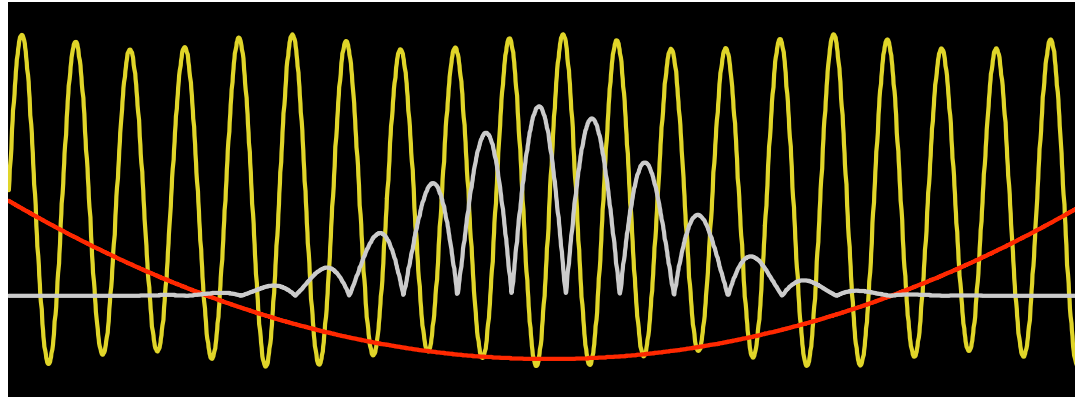
$$a = 0.15 a_0 \quad U = 0.25J$$

$$a = 1 a_0 \quad U = 2J$$

$$a = 20 a_0 \quad U = 42J$$

New wavelength ratio:  $\beta = 1064.4 \text{ nm} / 866.3 \text{ nm} = 1.228\dots$

# Interaction and disorder: experiment

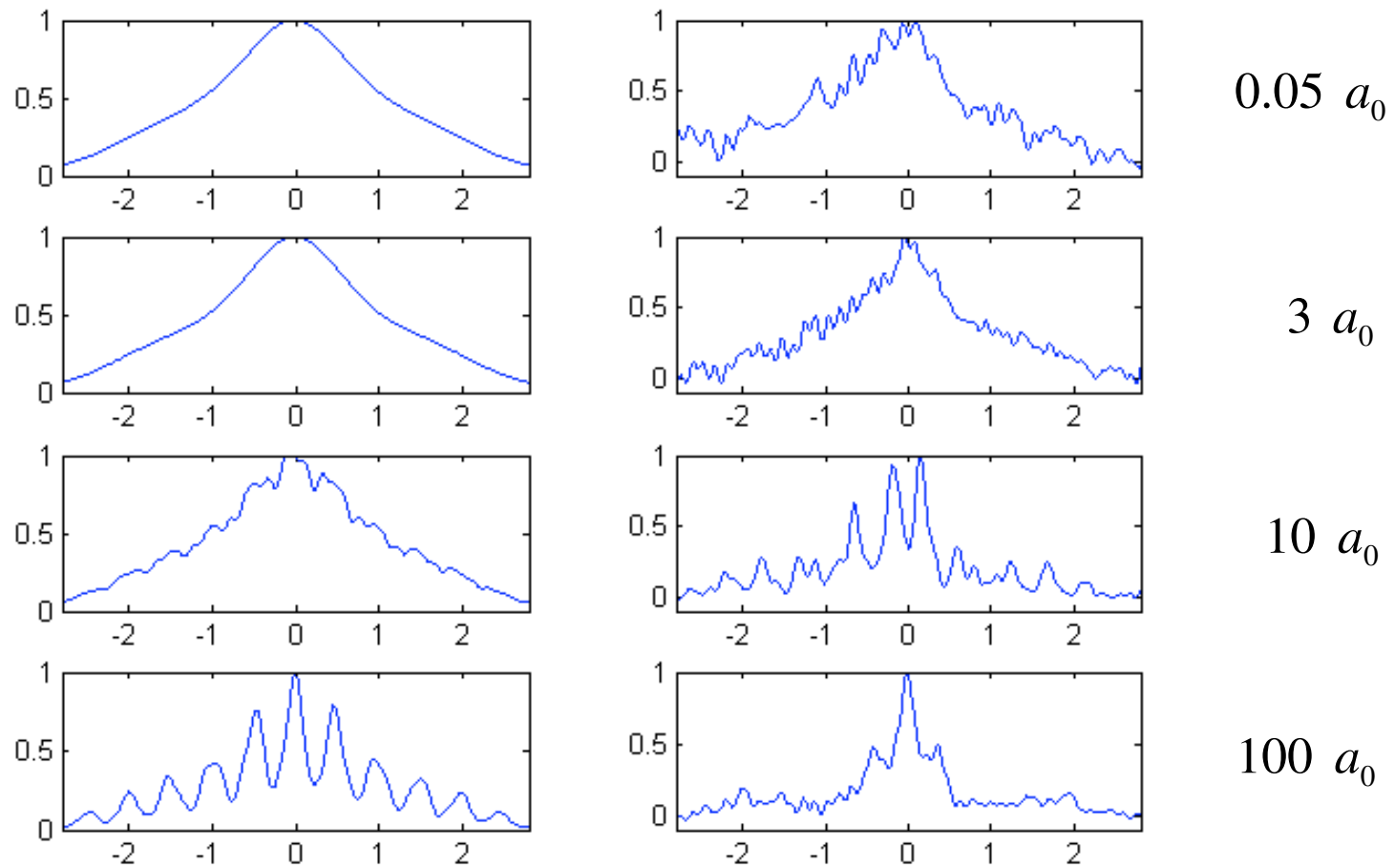


condensation ( $a=200 a_0$ )  $\xrightarrow{50 \text{ ms}}$   $a \sim 1-100 a_0$   $\xrightarrow{100 \text{ ms}}$  lattice loading

$\xrightarrow{1 \mu\text{s}}$  potentials off  $\xrightarrow{300 \mu\text{s}}$   $a \sim 0.05 a_0$   $\xrightarrow{36 \text{ ms}}$  image

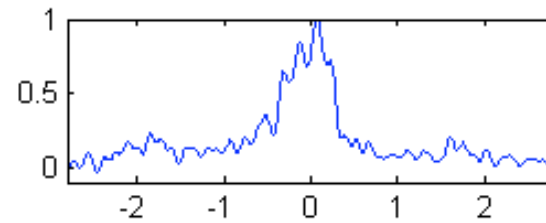
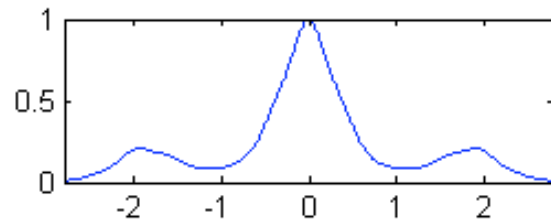
Momentum distribution of the interacting system

# Experiment: momentum distribution

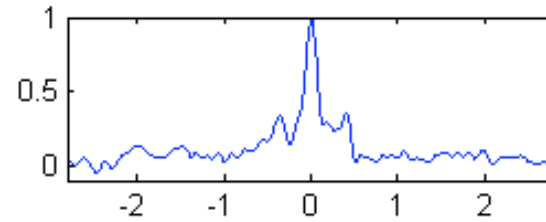
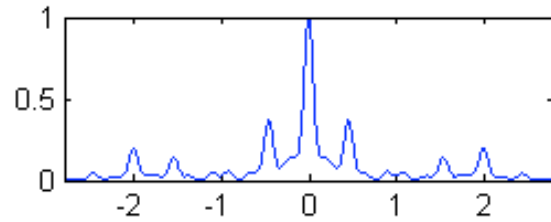


$\Delta/J=40$

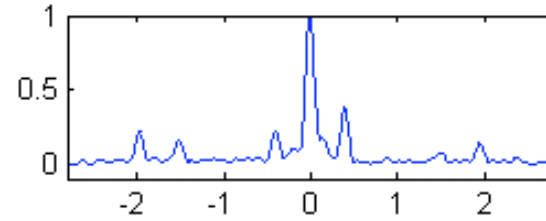
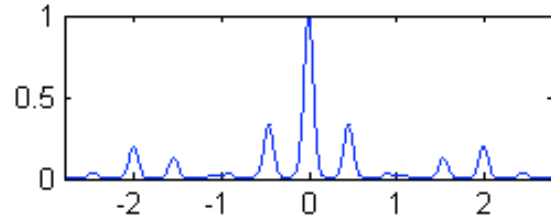
# Momentum distribution



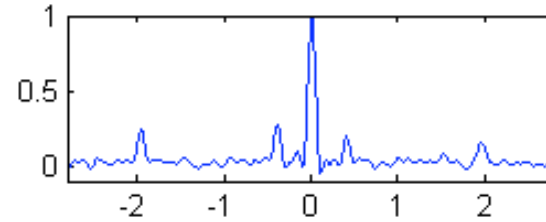
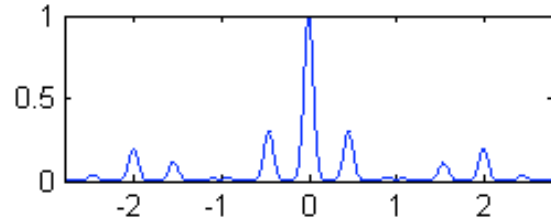
$0.05 a_0$



$1 a_0$



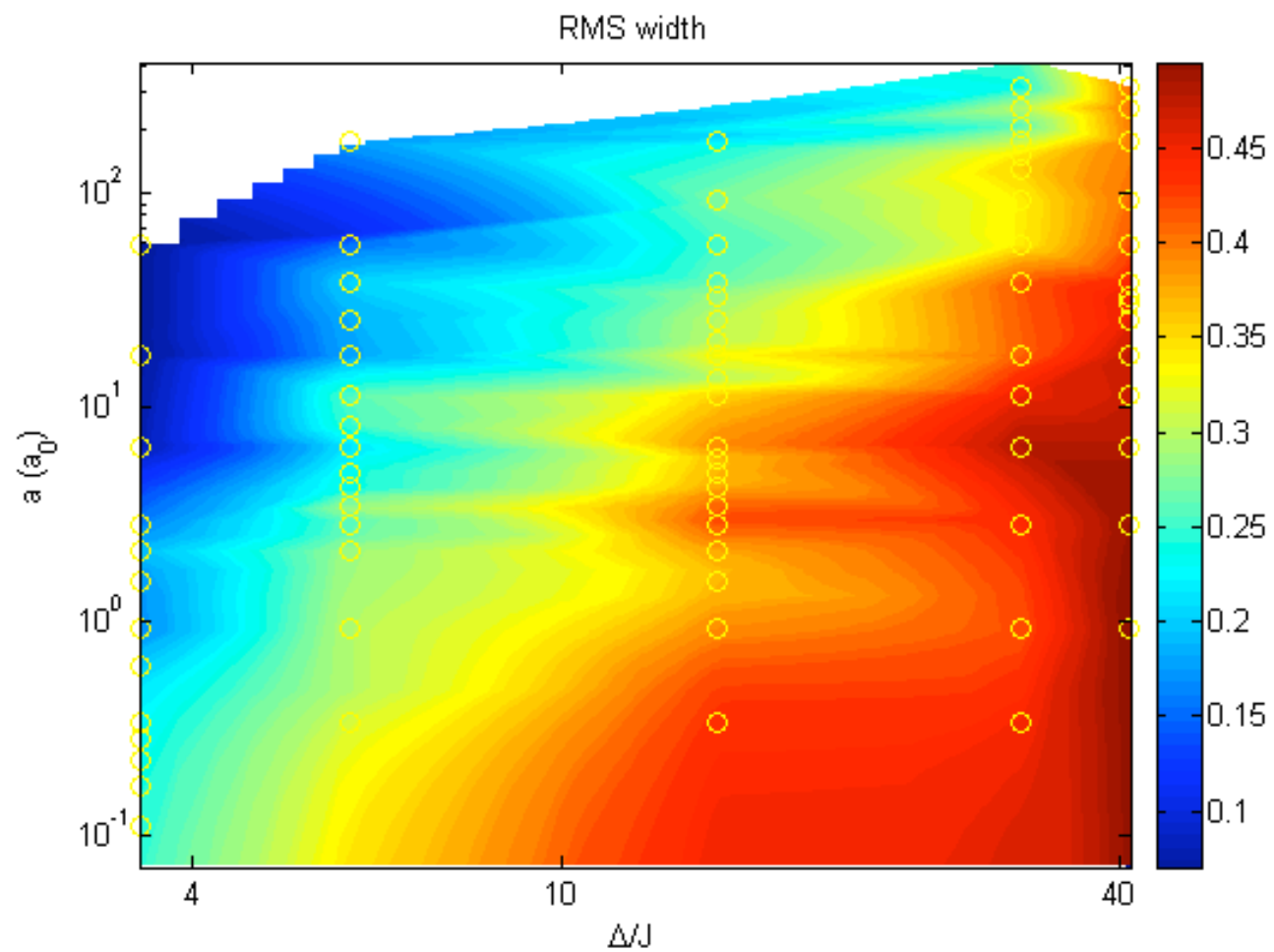
$5 a_0$



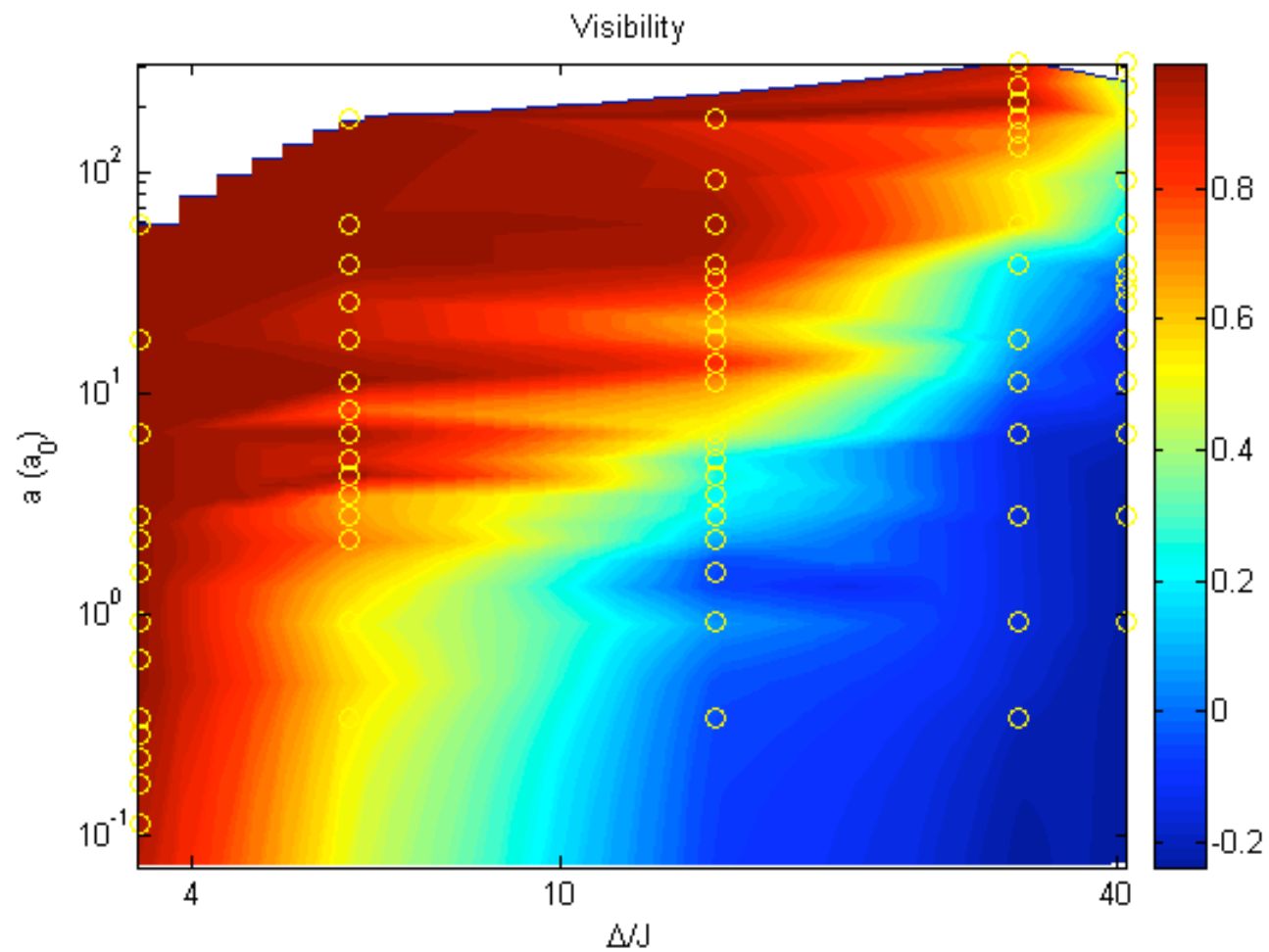
$20 a_0$

$\Delta/J=5$

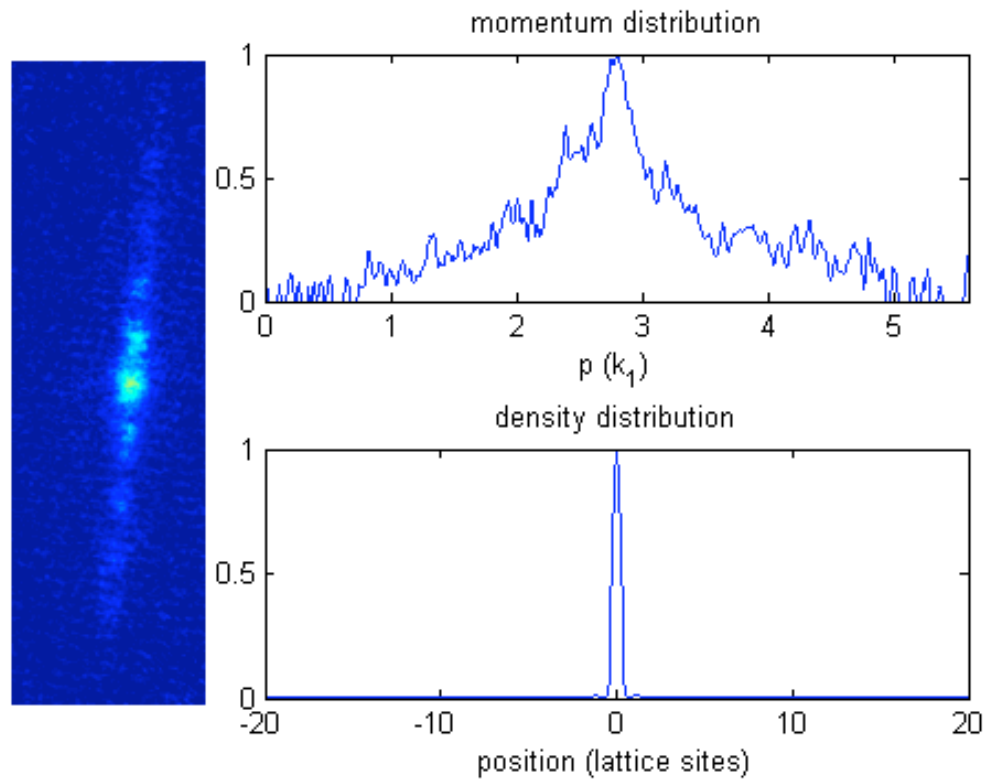
# Momentum distribution



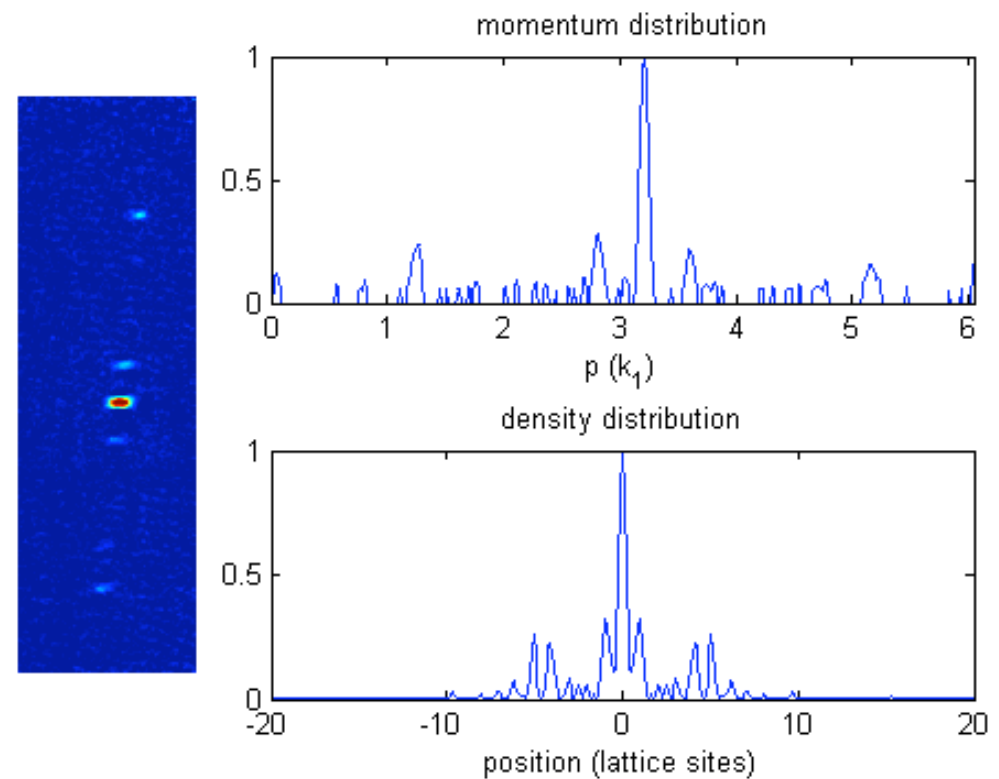
# Momentum distribution



# FFT of momentum distribution

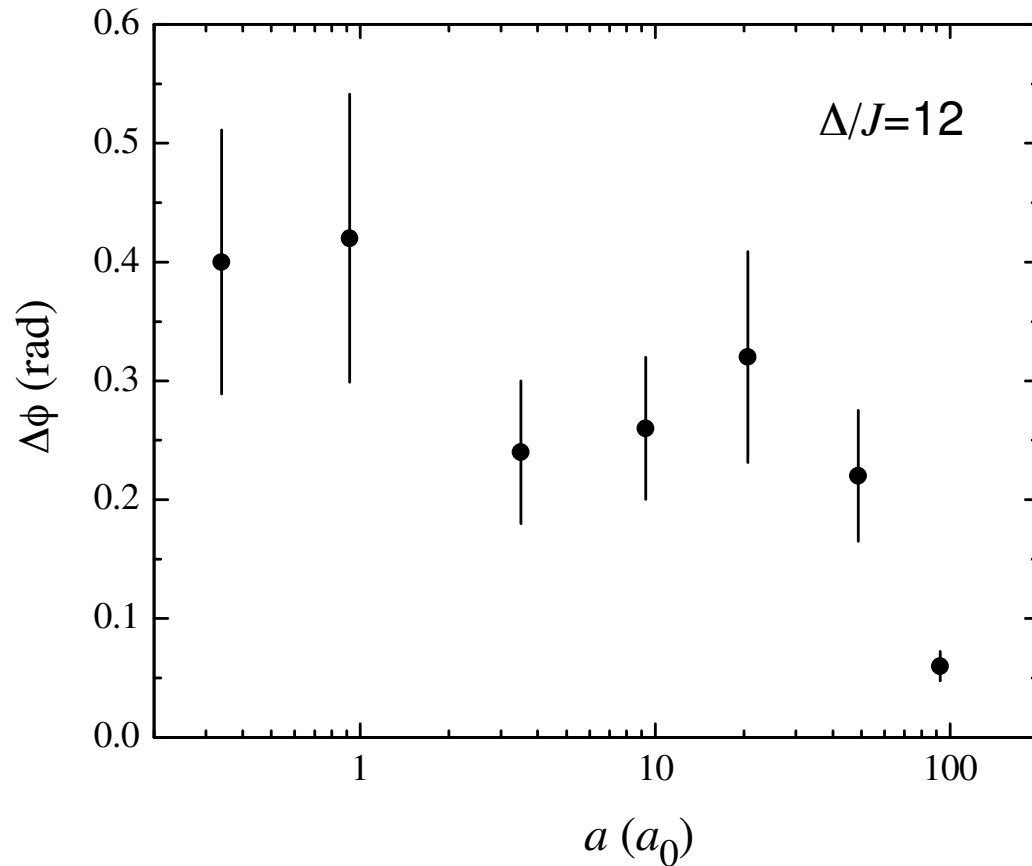


$$|\Psi(k)|^2$$





# Phase fluctuations



Phase fluctuations measured in both momentum space and real space tend to vanish for increasing interaction

B. Deissler, M. Zaccanti et al, in preparation.

# Outlook

We observe a crossover from the Anderson “glass” to a an extended system with phase coherence: is the latter **superfluid**?

Work in progress: **transport properties** of the interacting system (diffusion, dipolar oscillations, in collaboration with theorists Trento)

Can we further characterize the **intermediate phase**? (better resolution in momentum measurements, interferometry, transport...)

Does an **attractive interaction** enhance localization?

**Future work:** Extend the tuning of the interaction and the measurement of momentum distribution to the strongly correlated regime.

# The team



Chiara D'Errico  
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Giacomo Roati

Matteo Zaccanti  
Michele Modugno

Leonardo Fallani  
Franco Dalfovo  
Stefan Mueller (Stuttgart)

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Marco Larcher (Trento)  
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