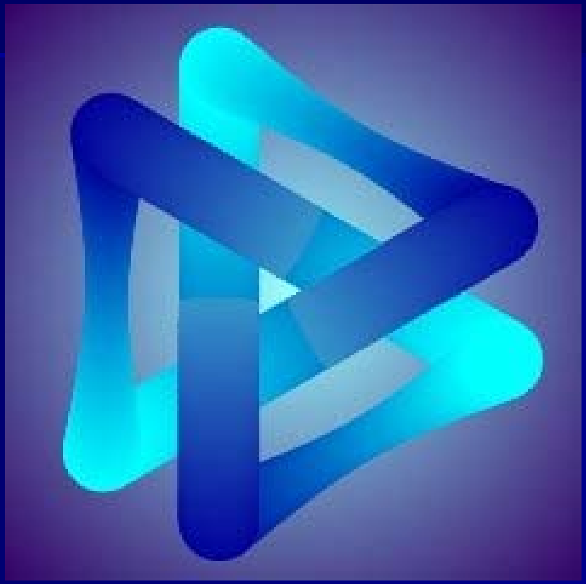
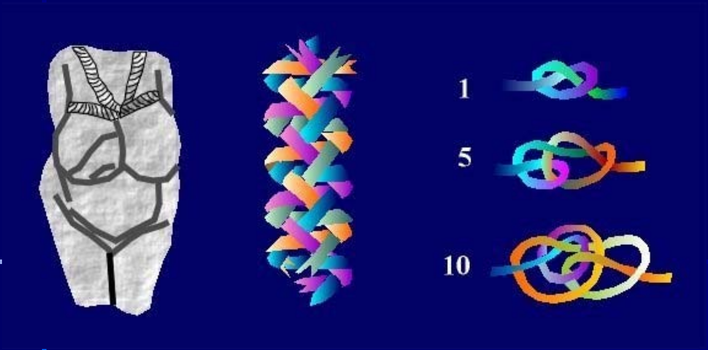
Knots, computers, conjectures



Slavik Jablan

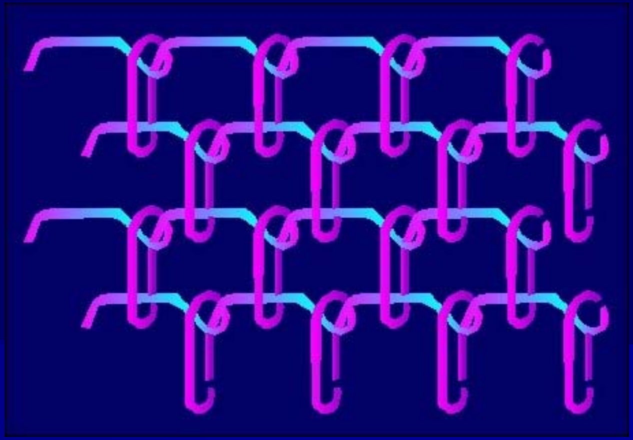
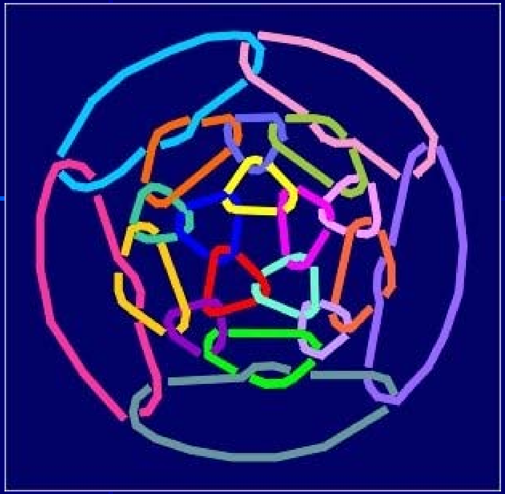
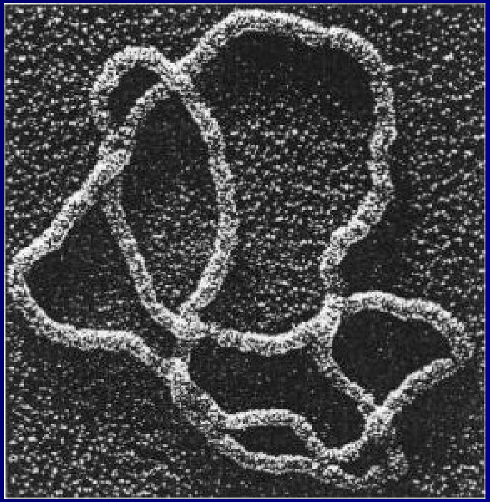
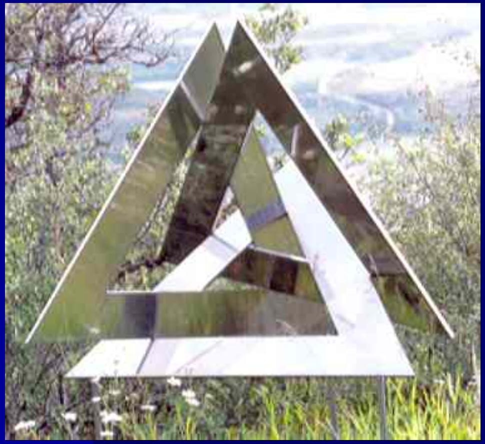
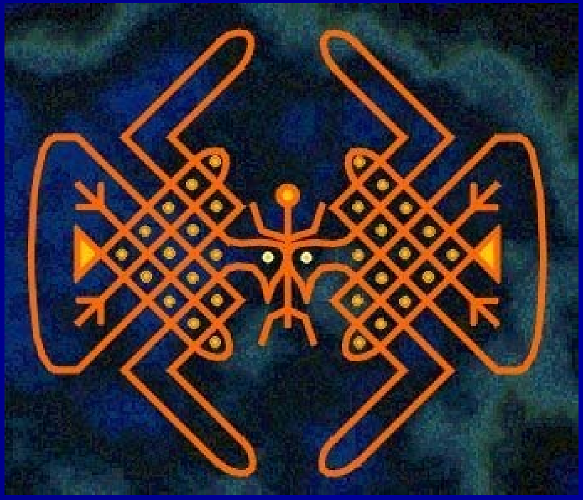


Kostenki,
10 000 B.C.

Peruvian quipu

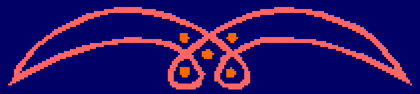
Chinese decorative knots

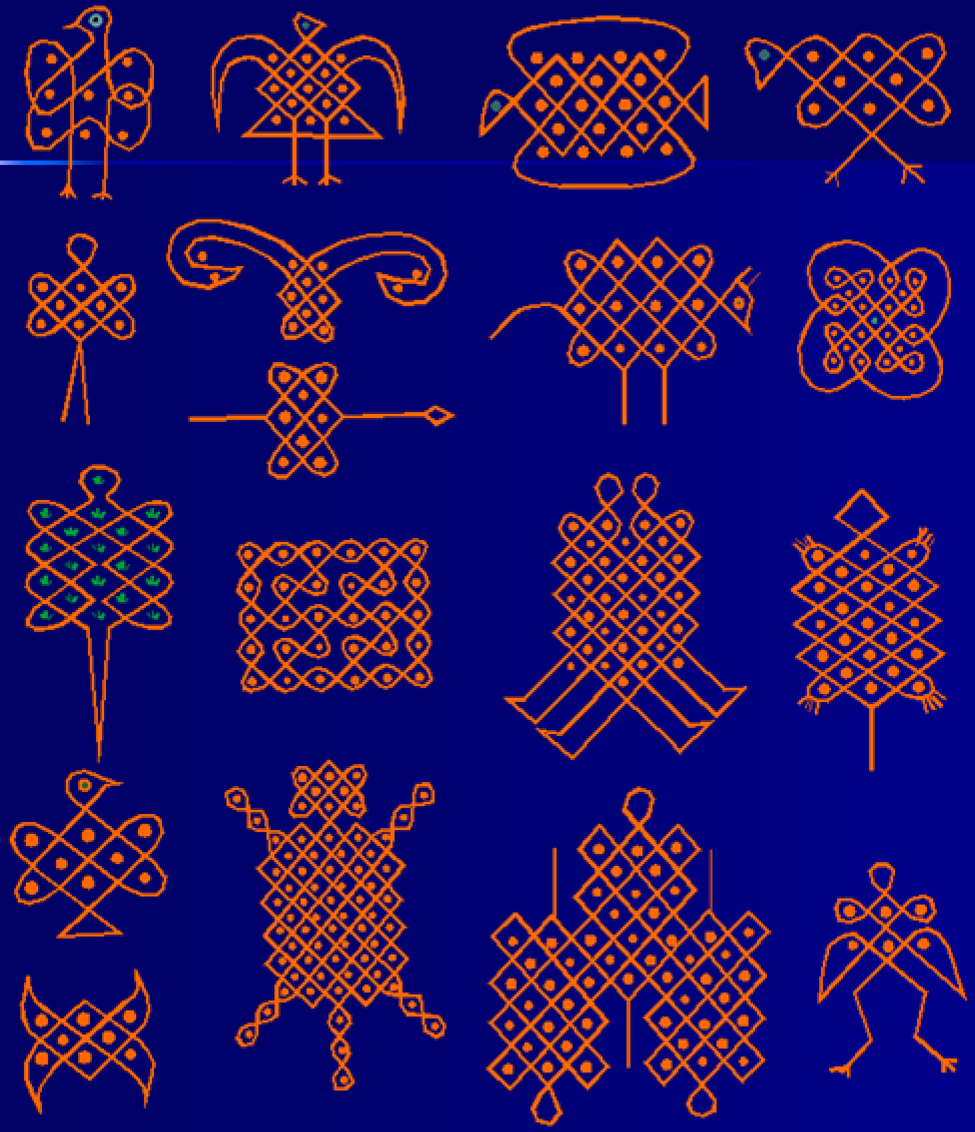
Celtic knots

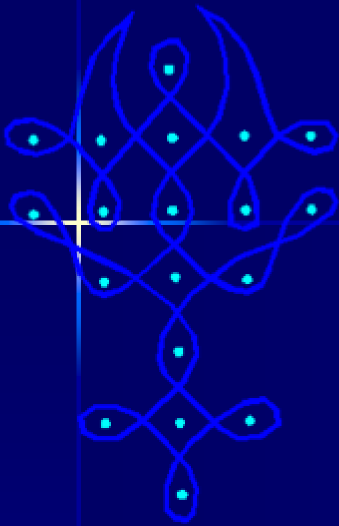






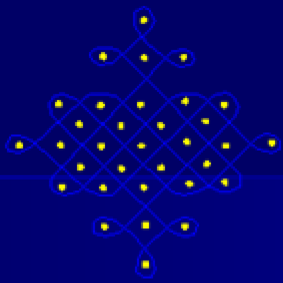
Hagfish practicing knot-theory

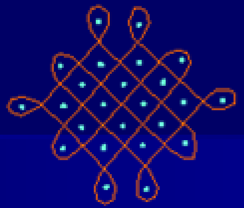
Tchokwe Sand Drawings

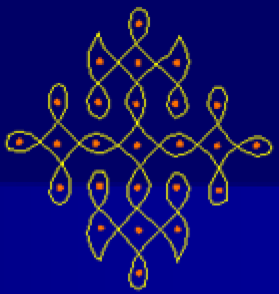


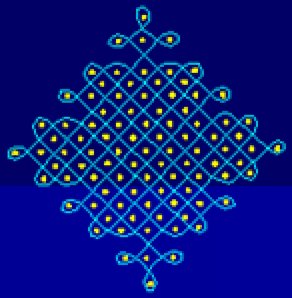


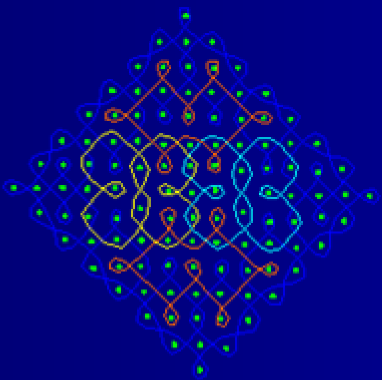


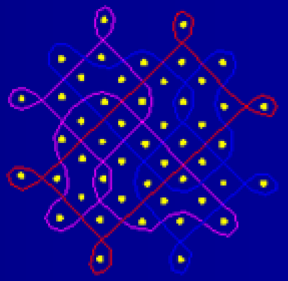


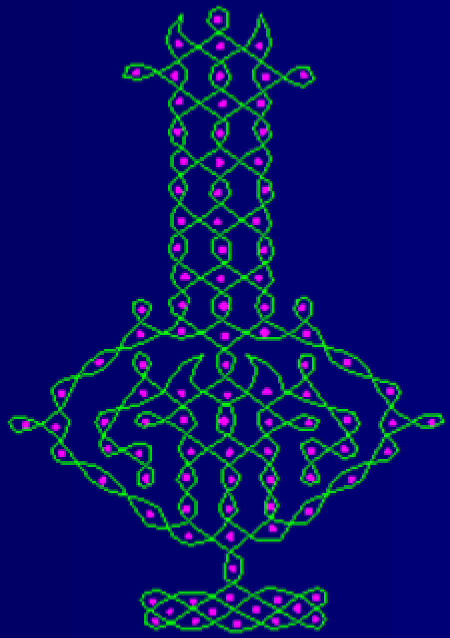






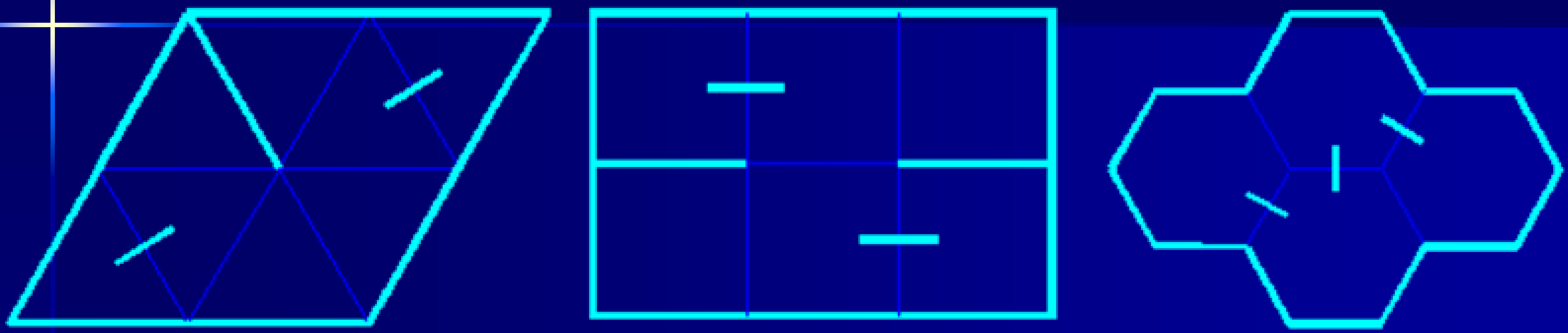


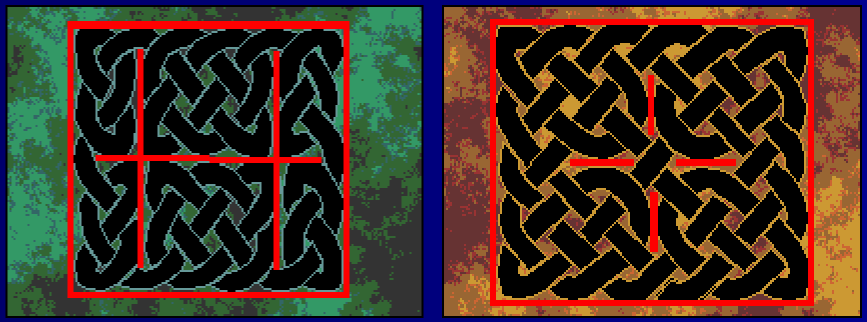




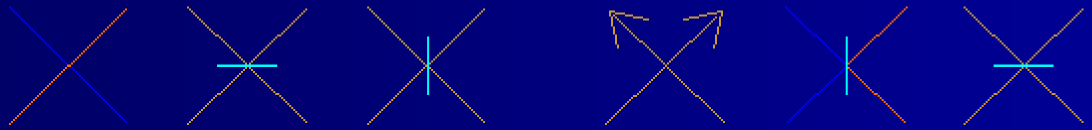
Tamil Knots (Pavitram)

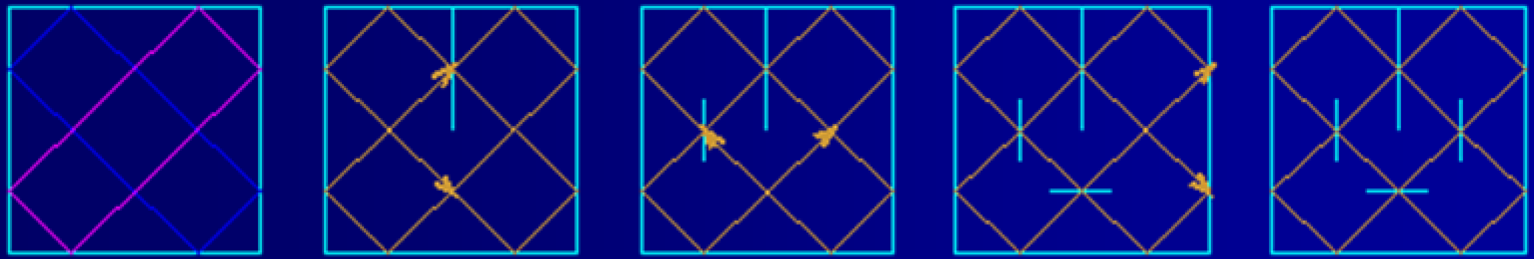
Mirror curves

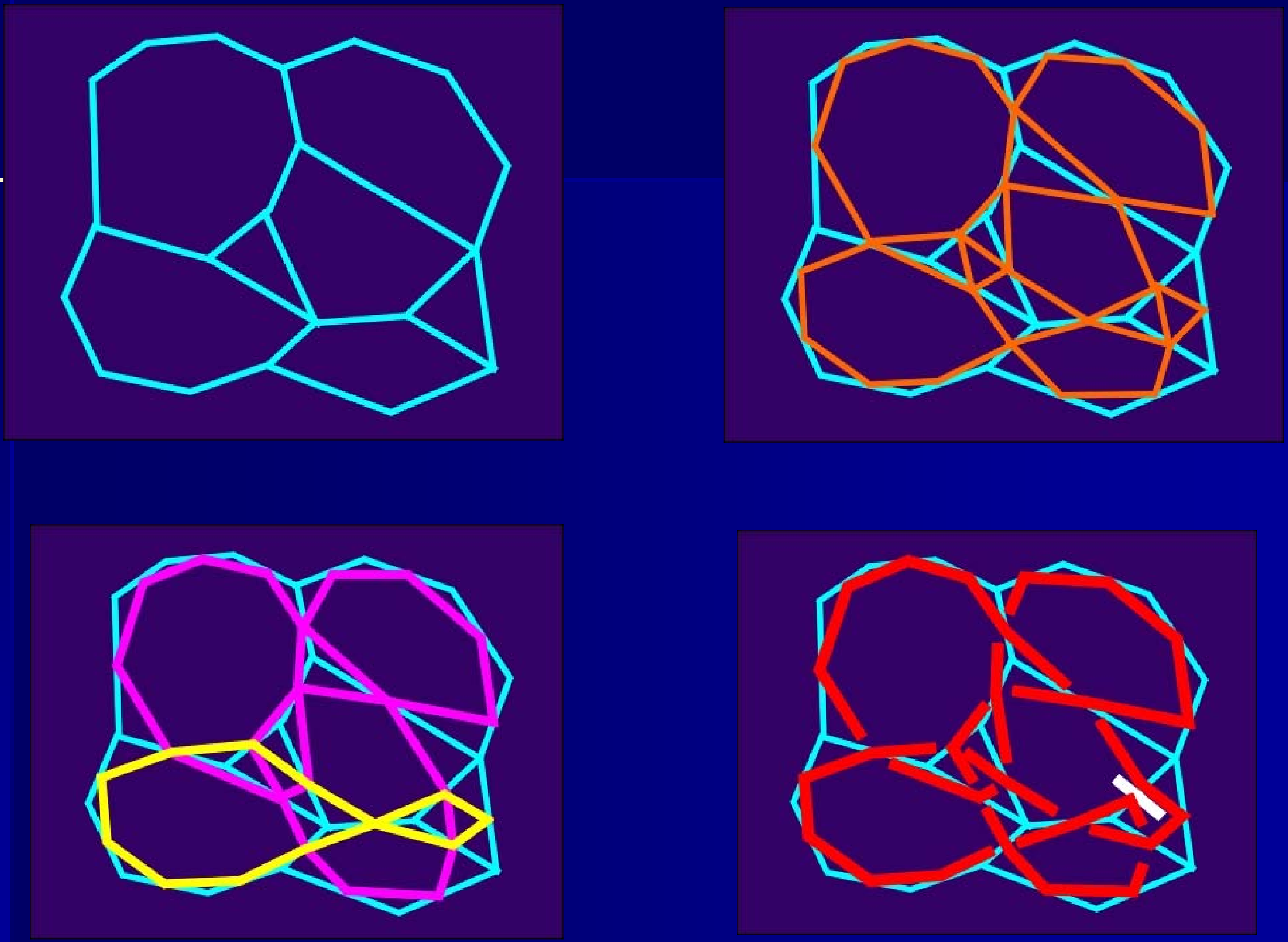




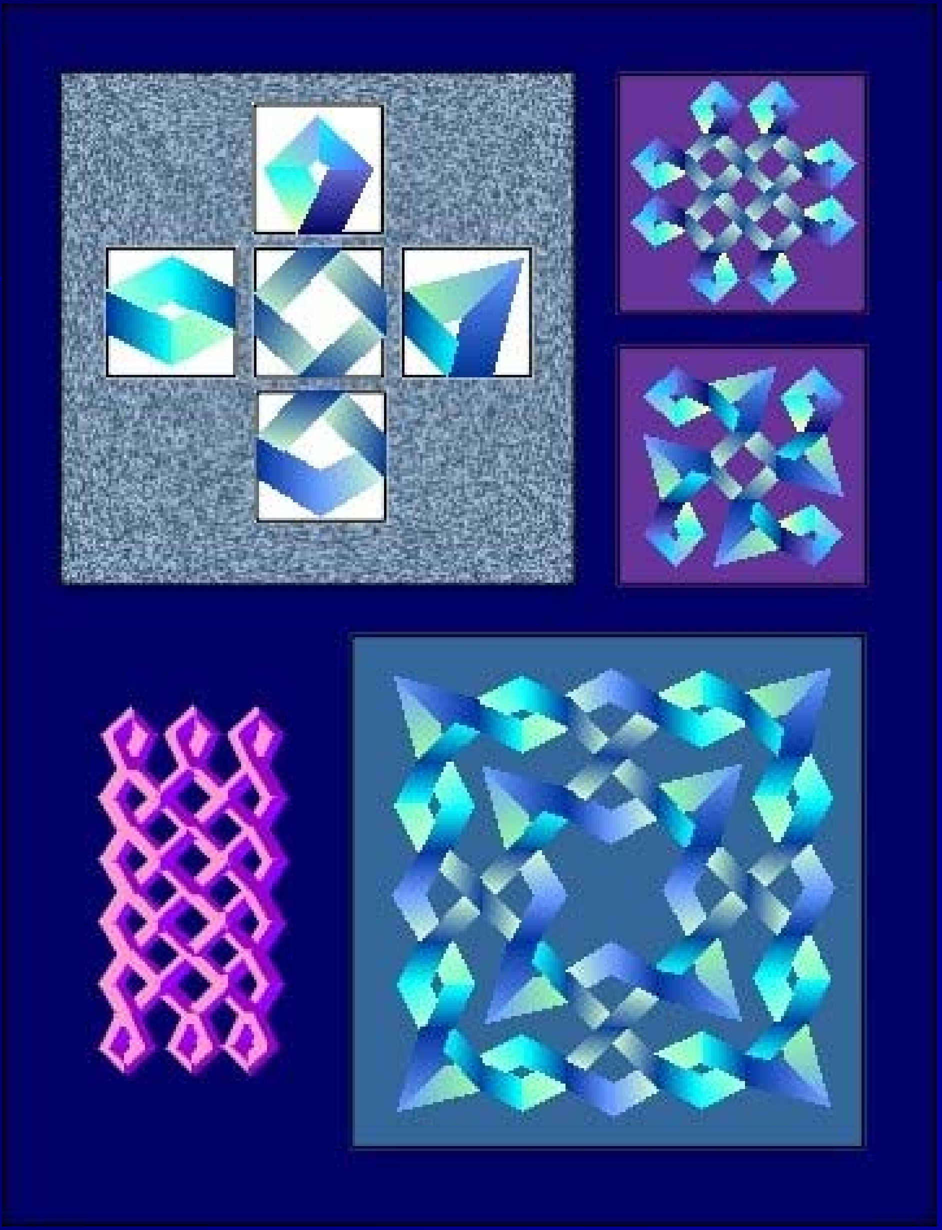
Construction rules







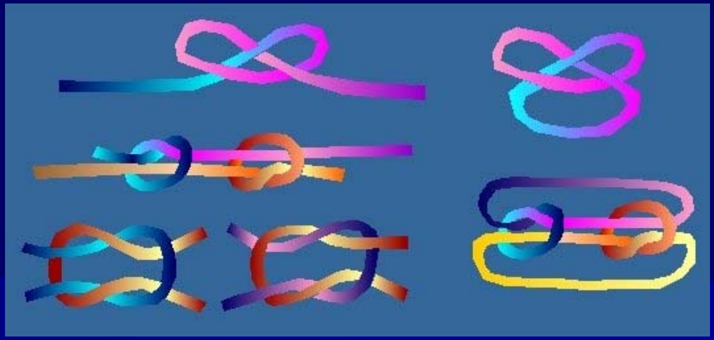
Modularity-
Knot Tiles



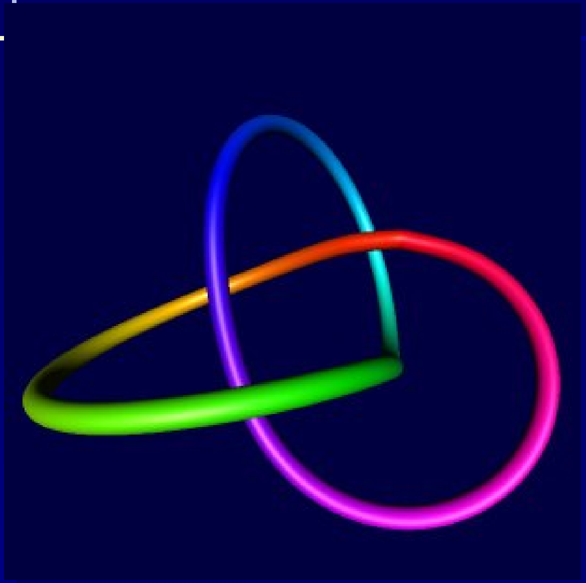
- Are atoms knots?
- Vortex theory: Kelvin
- Kirkman, Tait, Little

Fullerene C_{60}

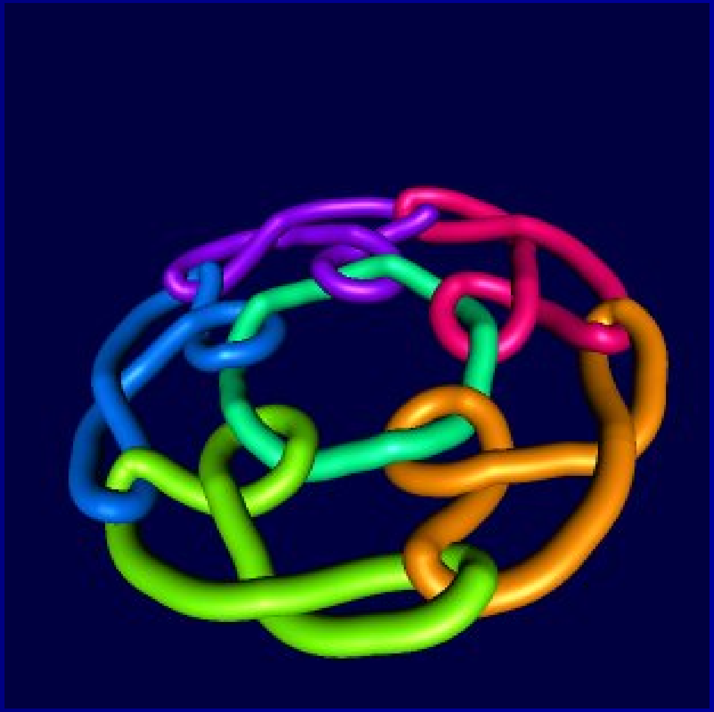
- Why LinKnot ?



"Real" and "mathematical" knots

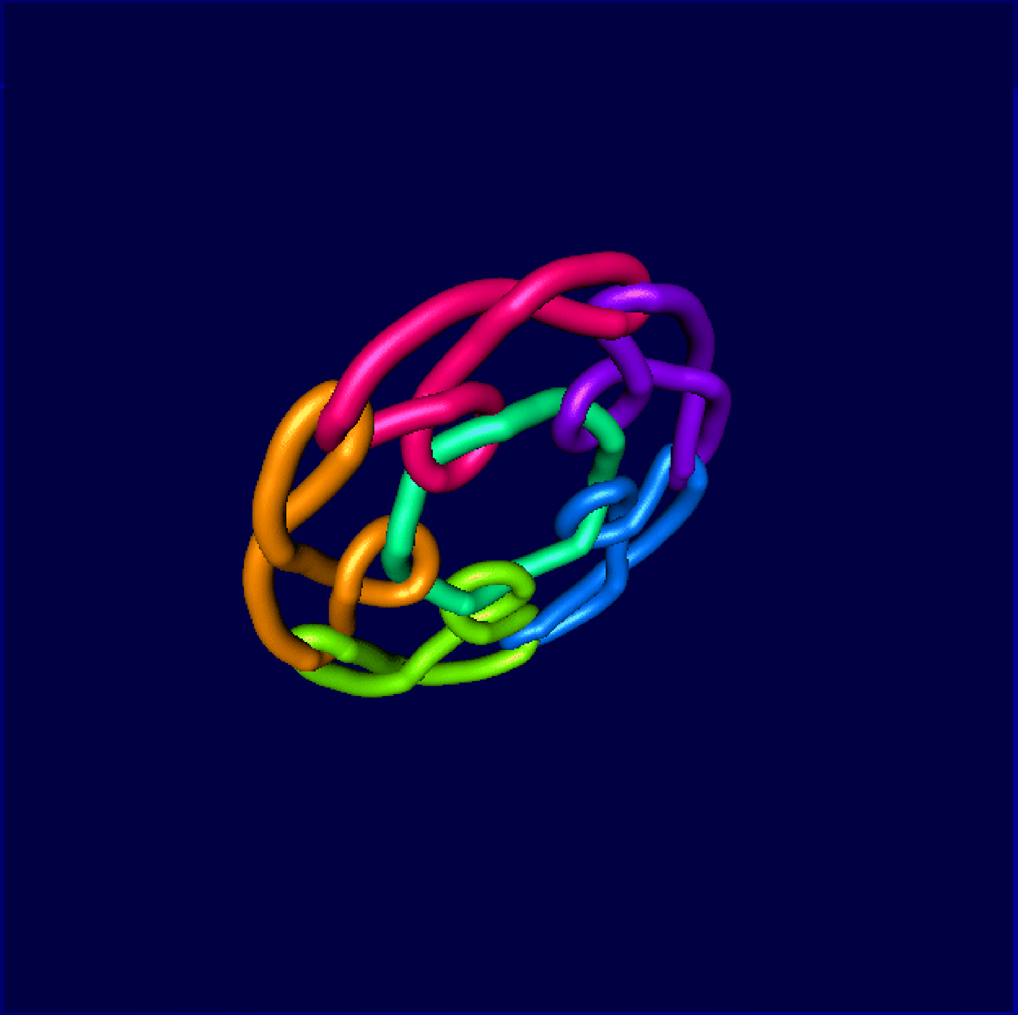


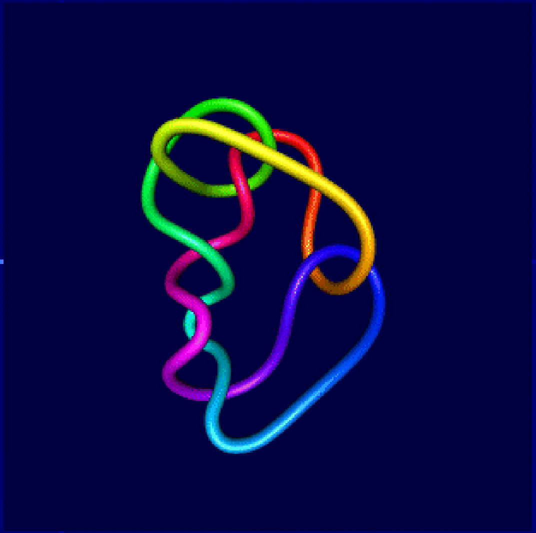
Knot

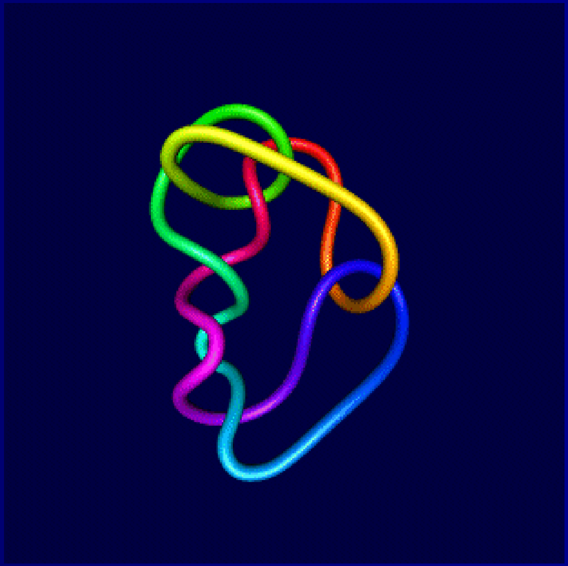


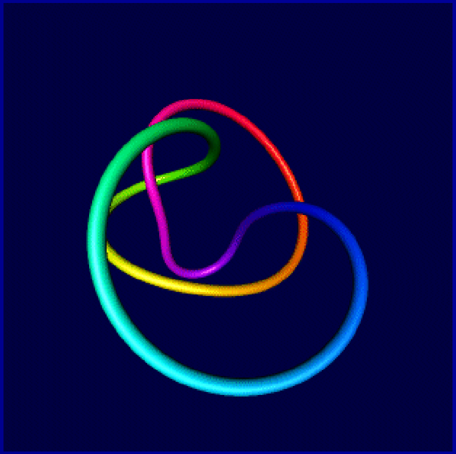
Link

Ambient isotopy





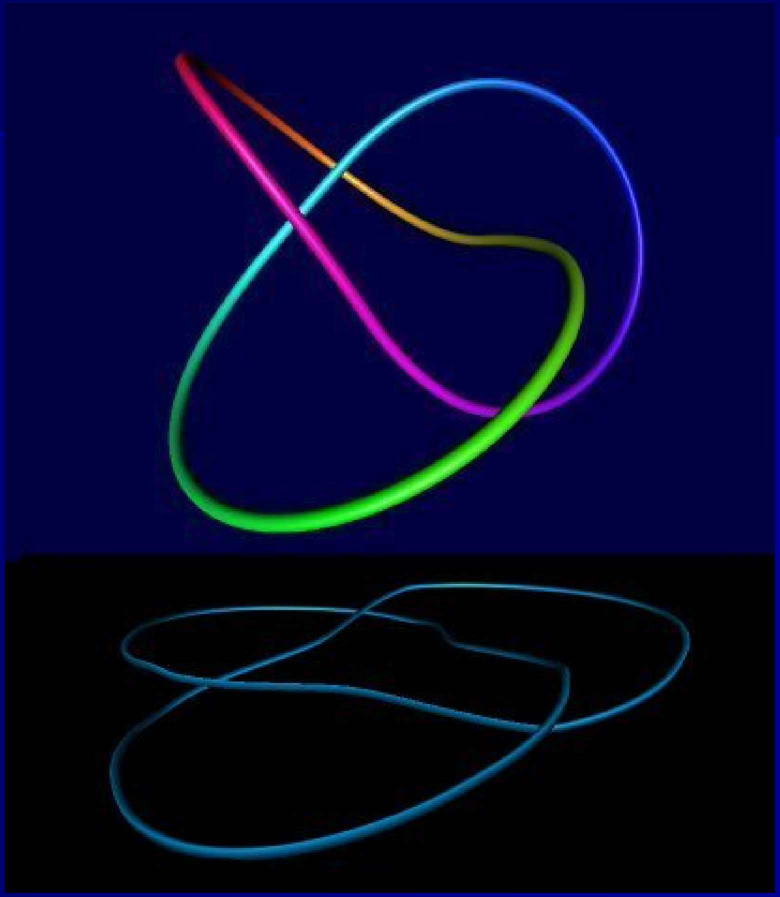


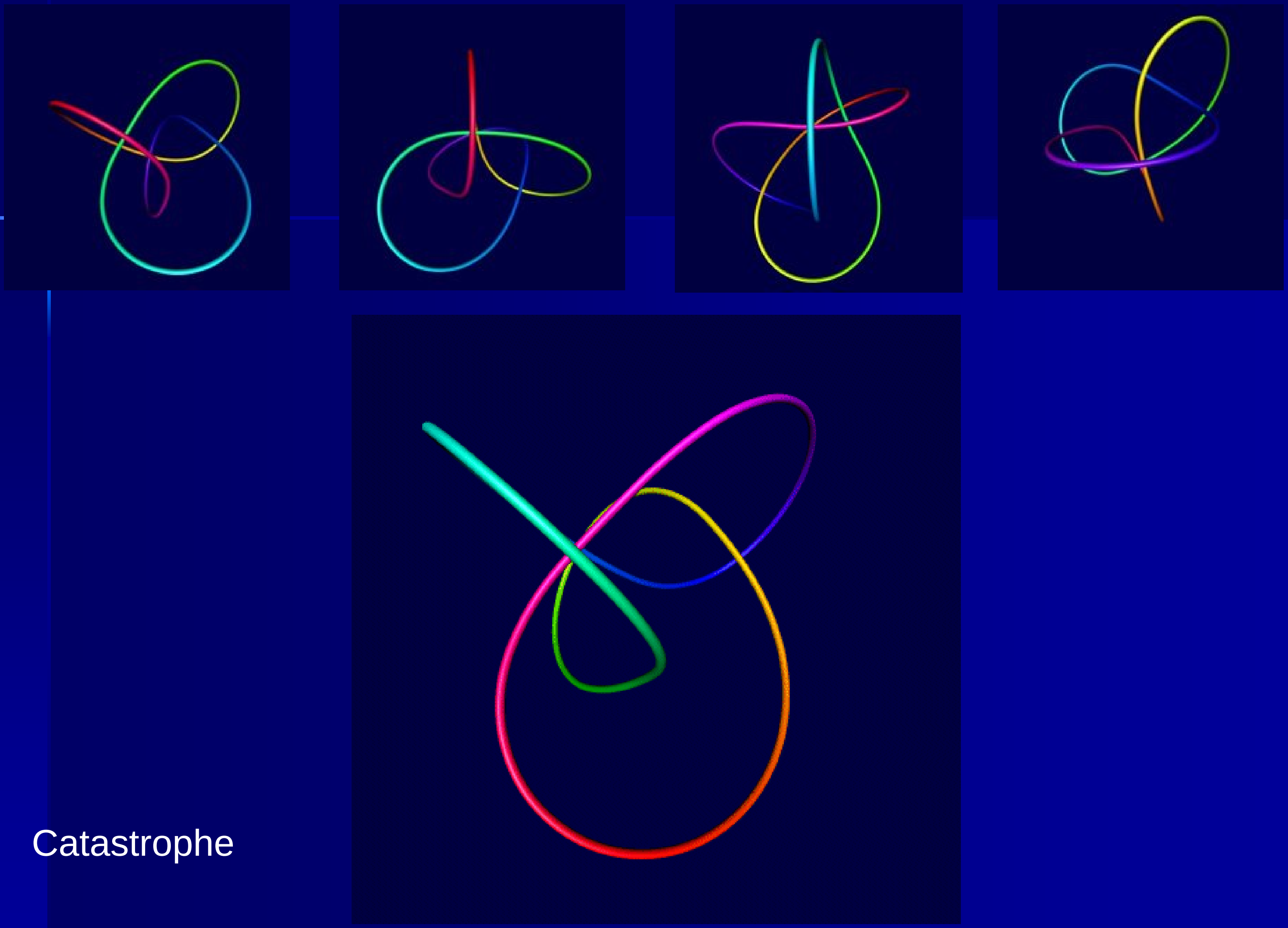


KL equality

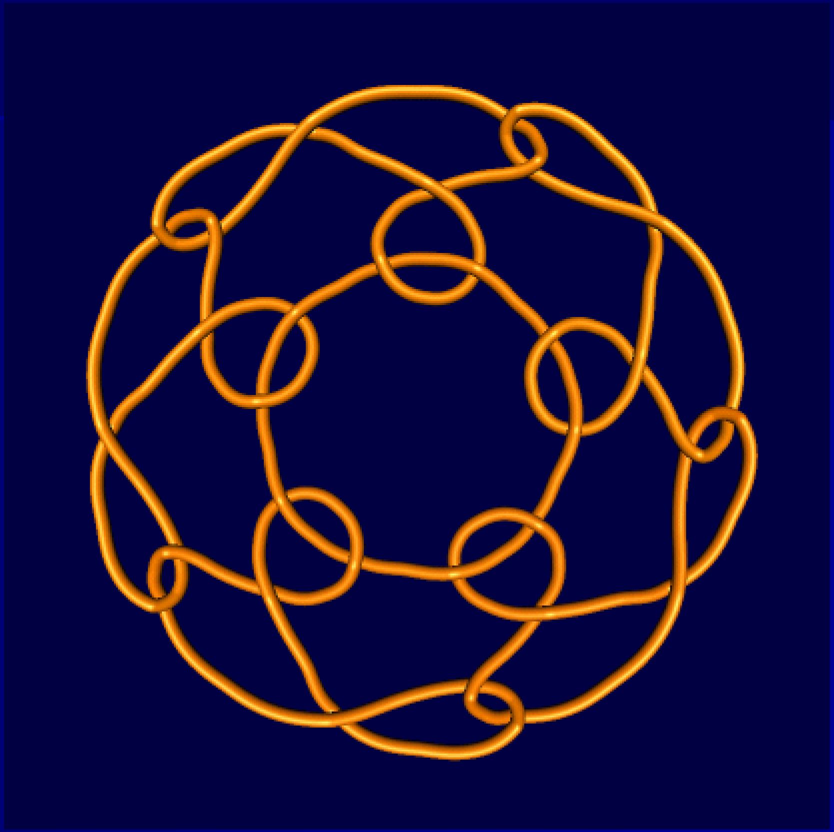
KL

shadows

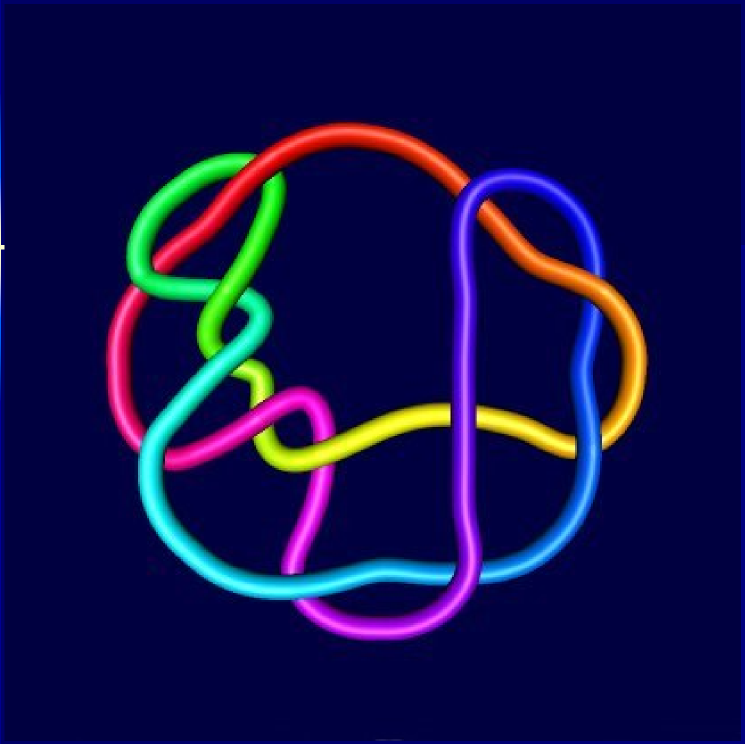




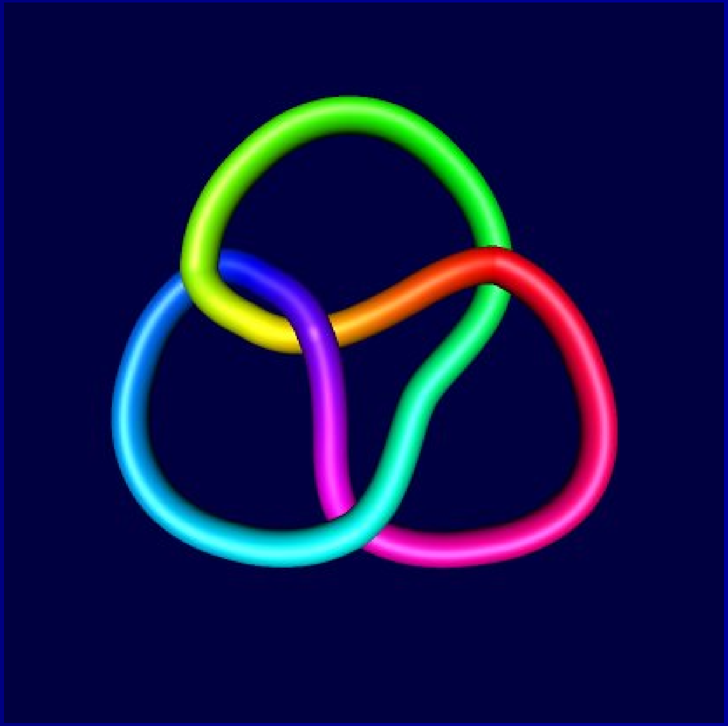
Catastrophe

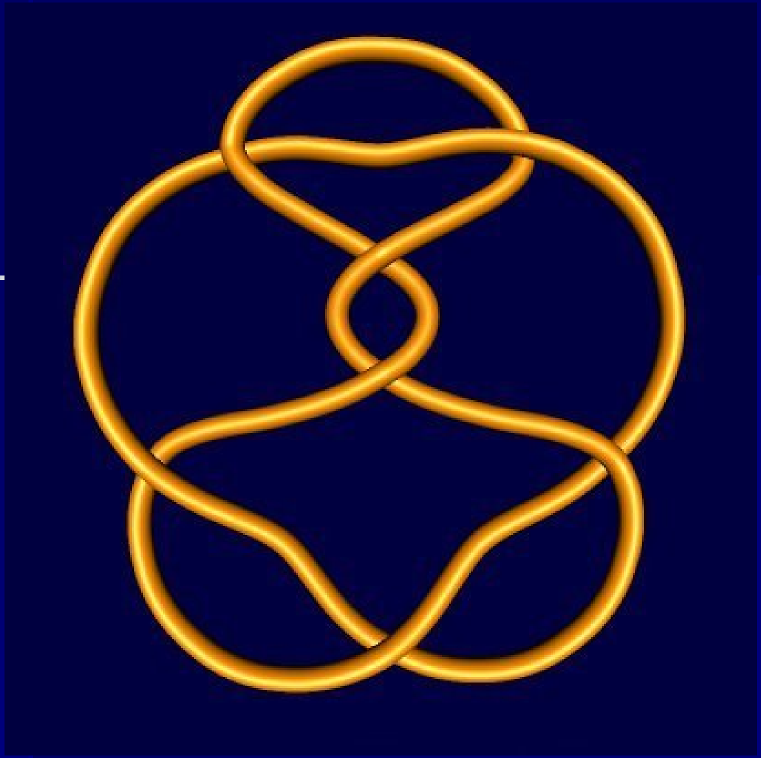


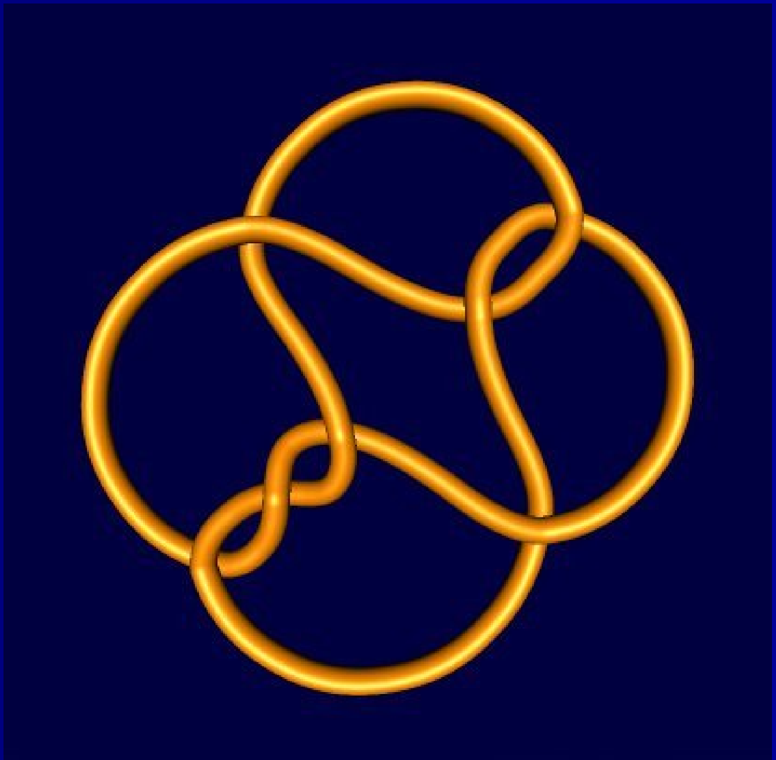
Number of components



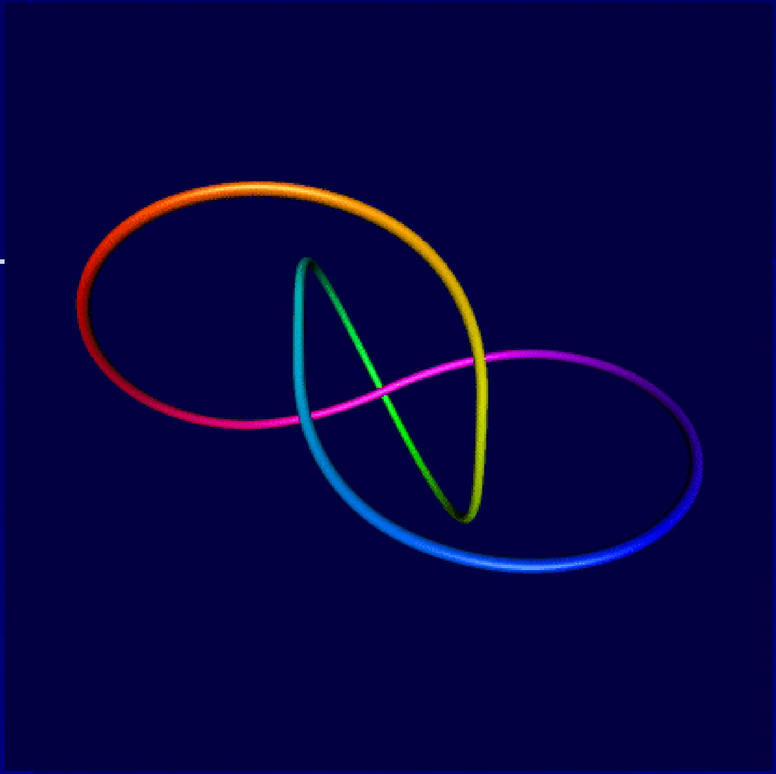
Non-minimal representation
of a trefoil



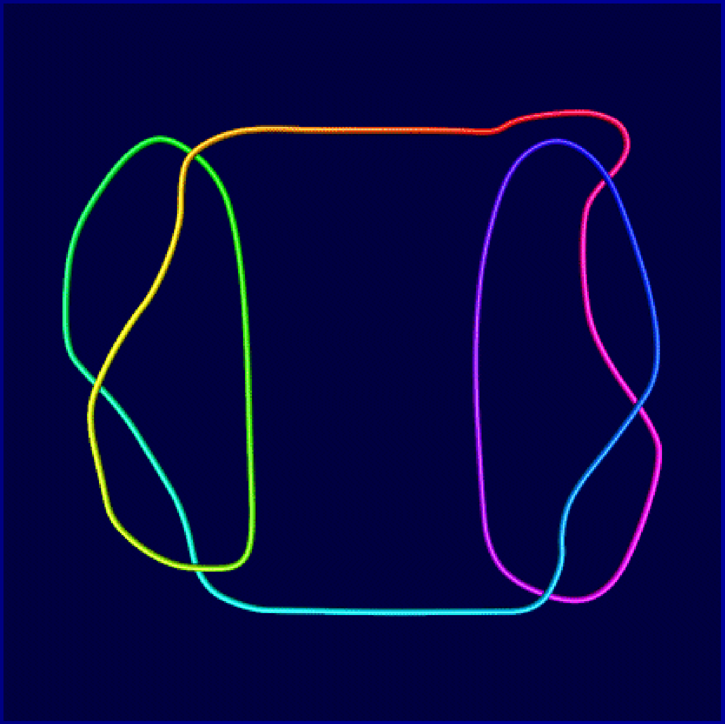




Non-isomorphic minimal projections of the knot 7_2

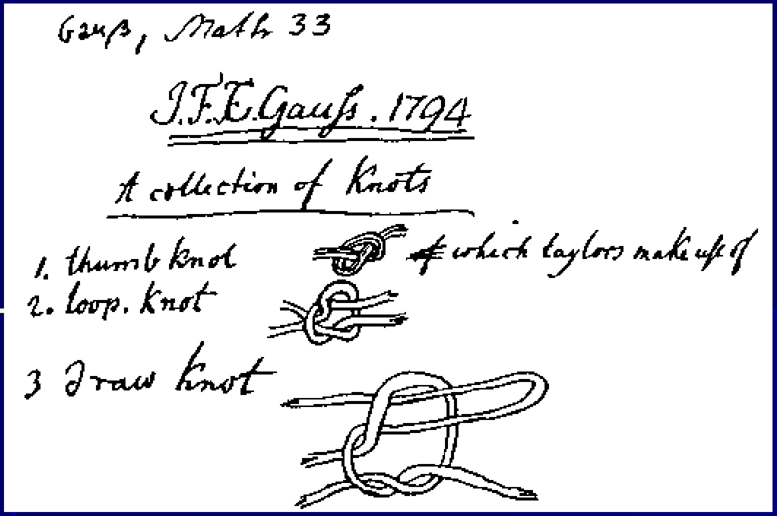


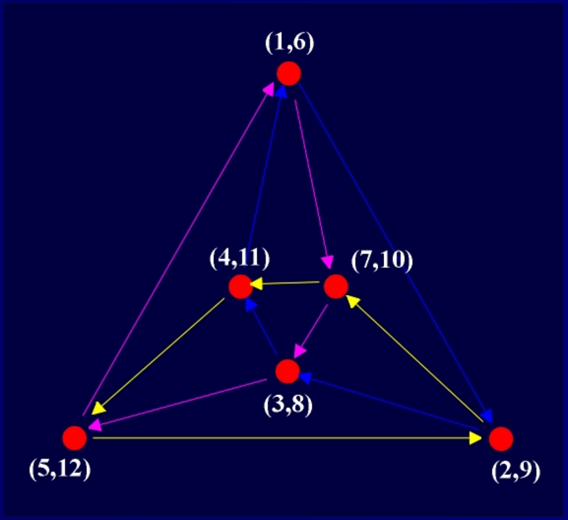
Composite knot- after deleting two edges, its graph disconnects.



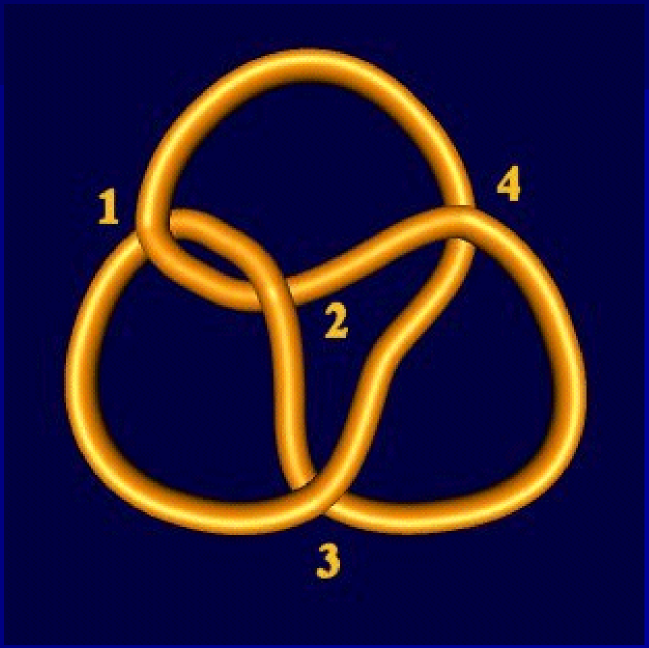
Prime knot- after deleting two edges, its graph remains connected.

Prime and composite KLs



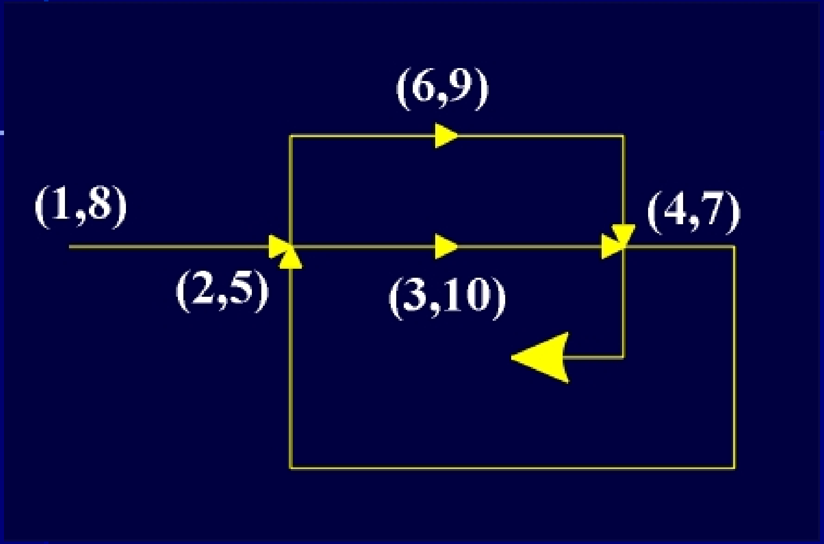


Dowker code of Borromean rings is
{{2,2,2},{6,8,12,10,2,4}}.



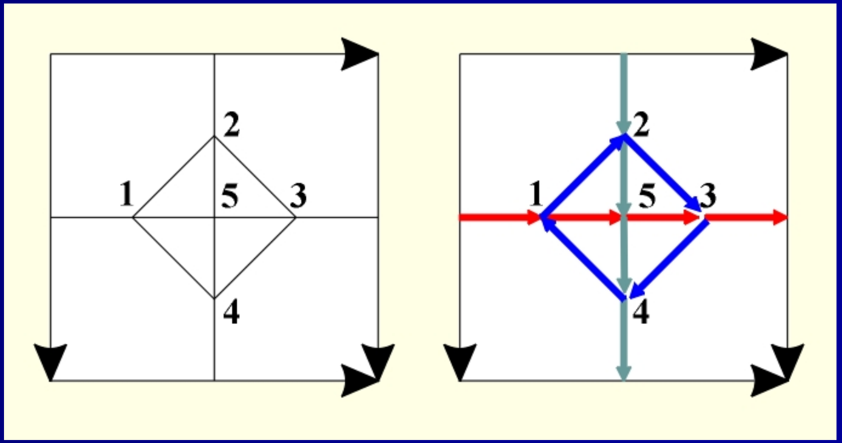
Gauss code of the figure-eight knot is {{1,2,4,3,2,1,3,4}}

Realizability

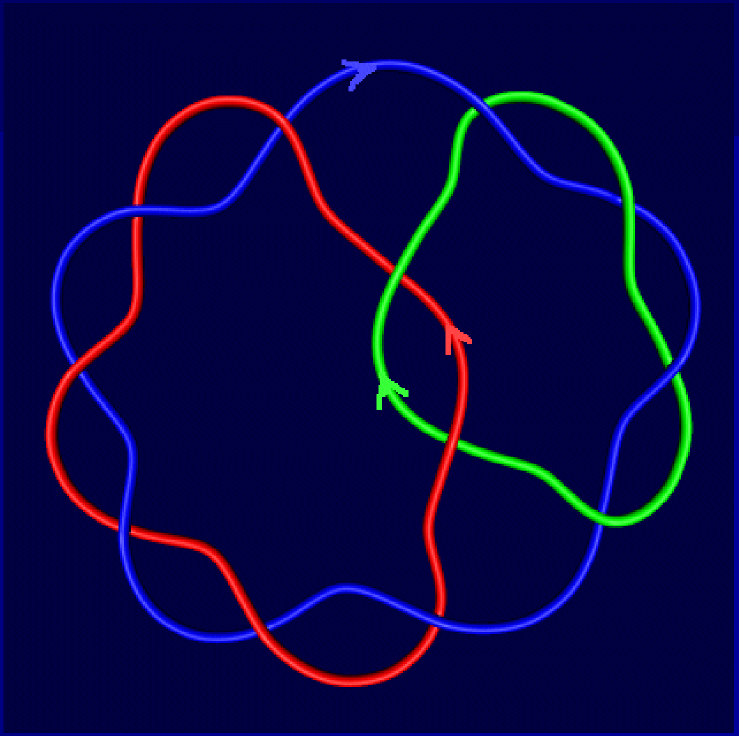


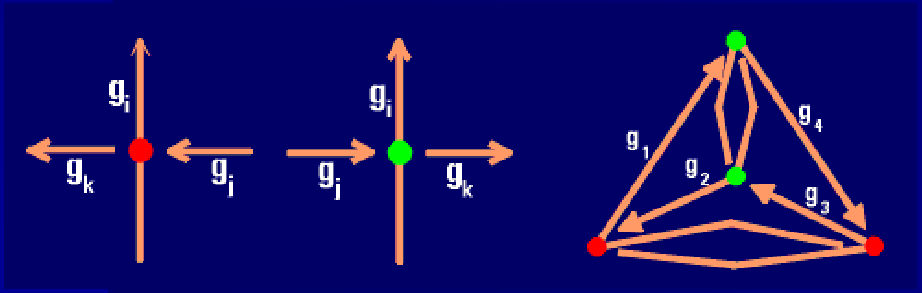
Non-realizable potential Dowker code {{5},{8,10,2,4,6}}.

Embedding of non-planar graph K_5 on a torus



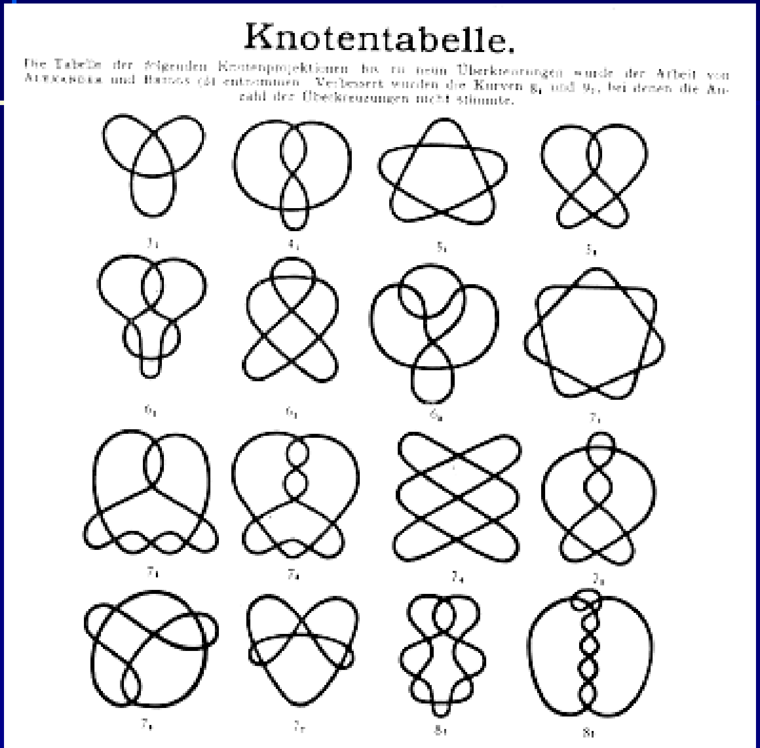
- Orientation





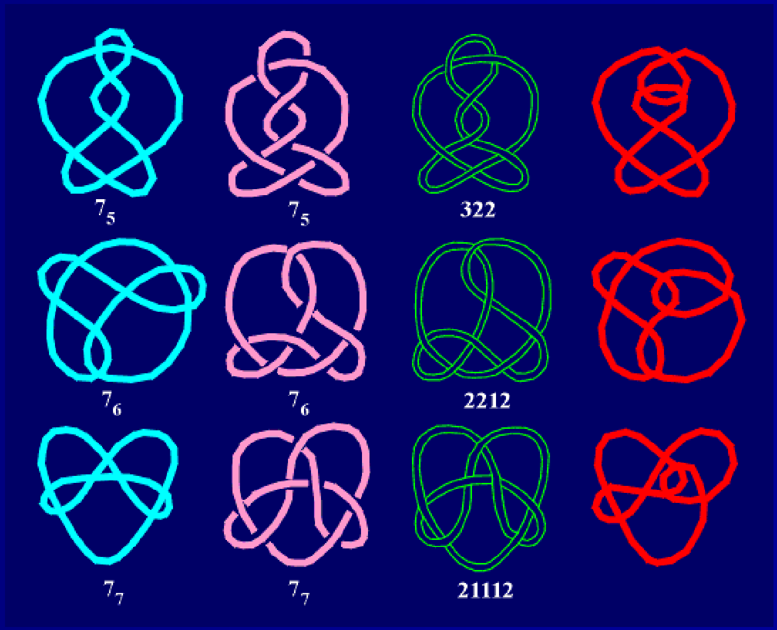
Eight oriented links 6,4,2

Knot tables

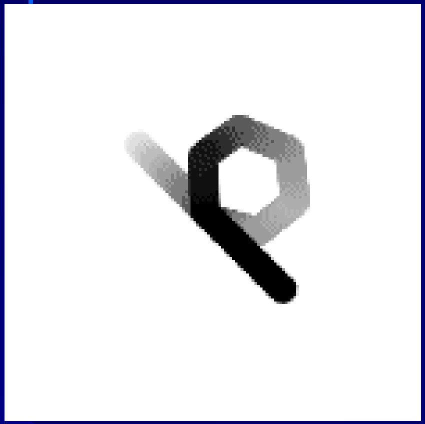


Part of Reidemeister's table of knots.

In the first column are given alternating knots 7_5, 7_6, 7_7 from K.Redmeister, in the second from the monograph *Knots* by G.Burde and H.Zieschang, in the third from D.Rolfsen *Knots and links*, but there in Conway notation. The fourt column contains nonisomorphic projections of these knots.

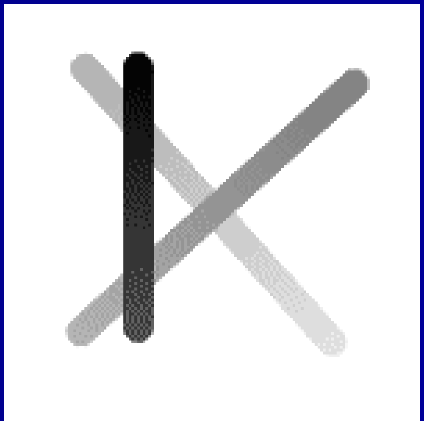


- Reidemeister moves



I

II

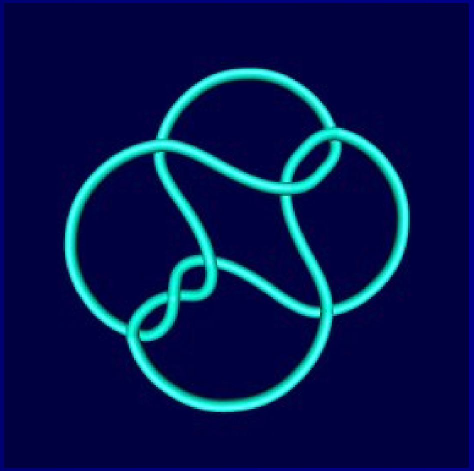


III

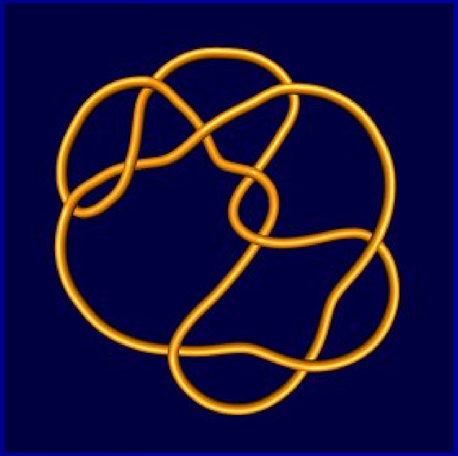
Moves I and II are decreasing number of crossings

Minimization

Goeritz's unknot



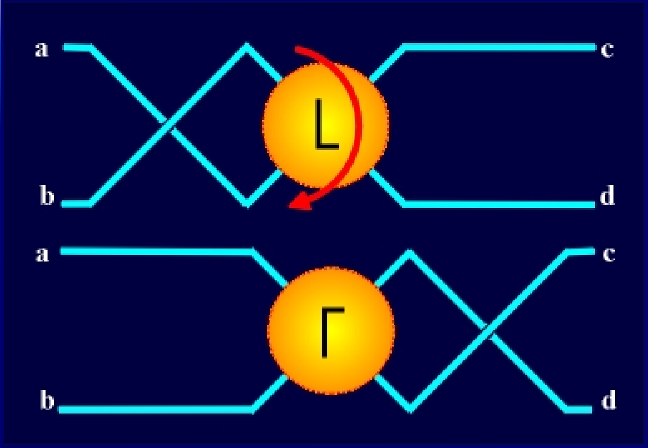
Nasty unknot

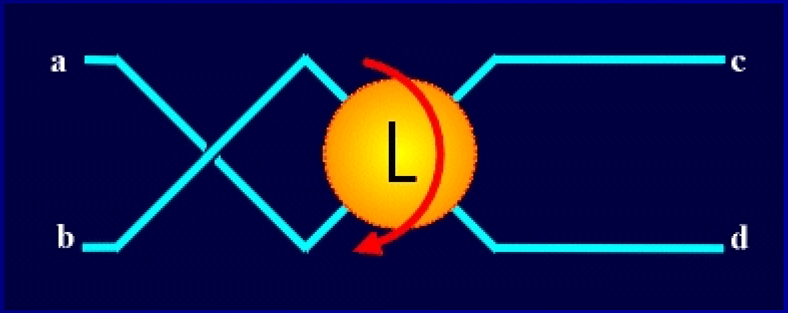


Monster unknot

Examples of (un)knots that cannot be minimized without increasing the number of crossings

- Flype



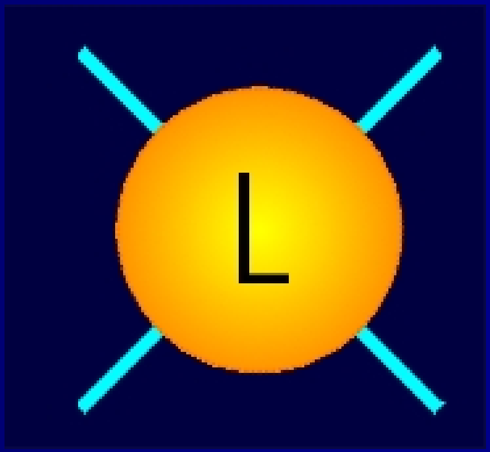


Tait Flyping Conjecture (Theorem)
Every minimal projection of an alternating *KL* can be obtained from another minimal projection by a series of flypes.

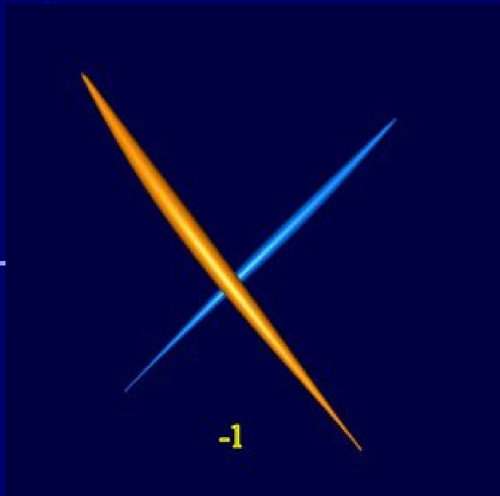
Knot theory programs

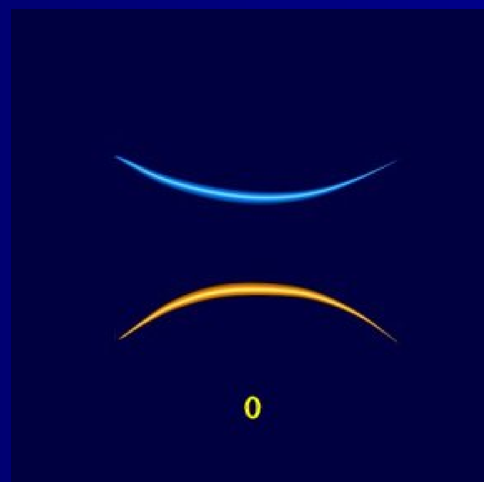
- Knotscape (J. Hoste and M. Thistlethwaite, with help from B. Ewing, K. Millett, K. Stephenson and J. Weeks)
- KnotPlot (Rob Scharein)
- K2K (M. Ochiai, N. Imafuji)
- Knot Atlas (Dror Bar Natan)
- KhoHo (A. Shumakovitch)
- SnapPea (J. Weeks)
- Knot Tables (M. Thisletwaite, C. Livingston)

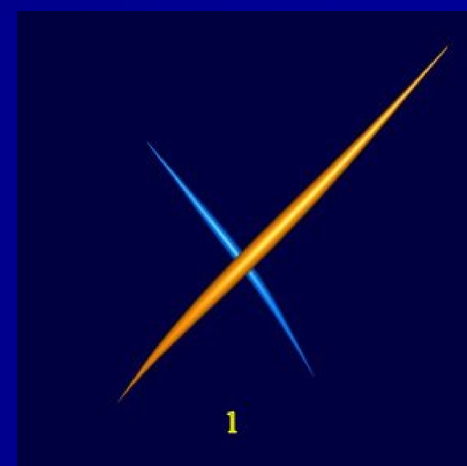
Conway notation



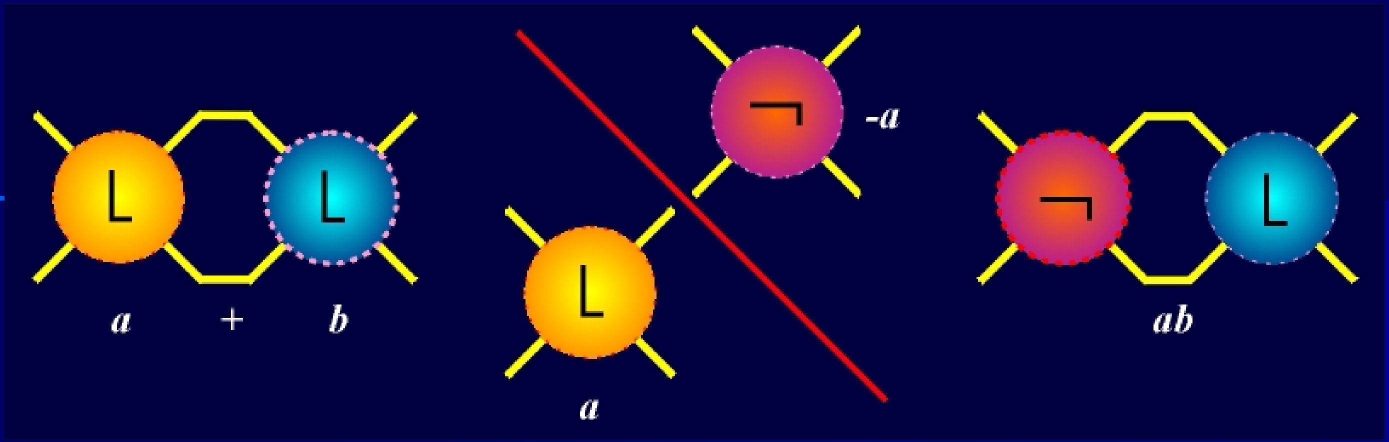
Tangle



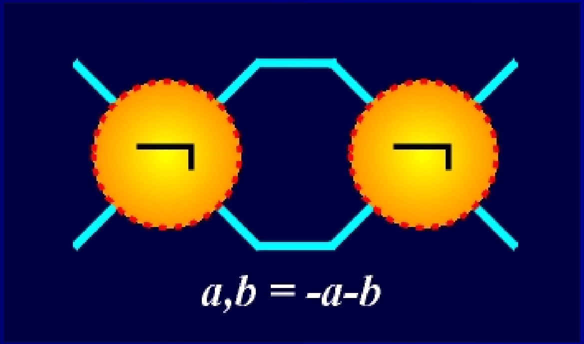




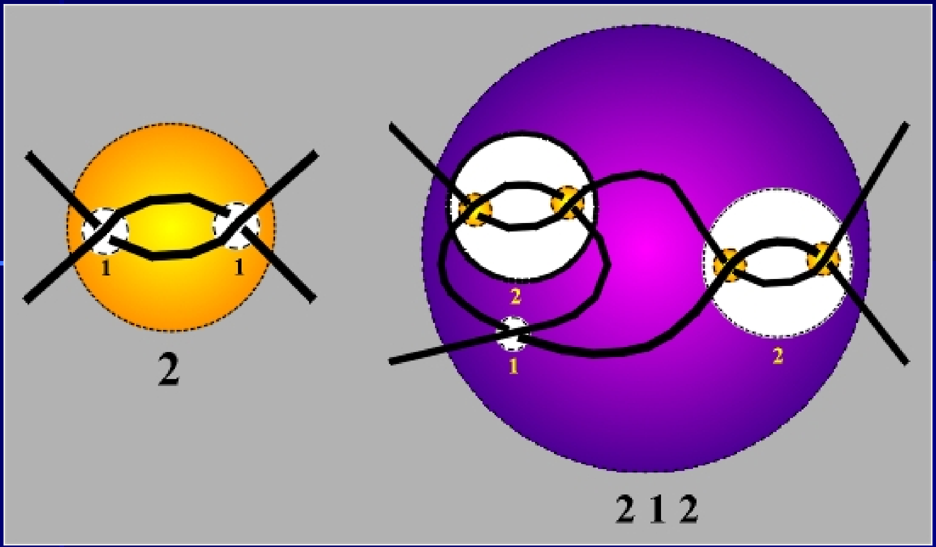
Elementary tangles



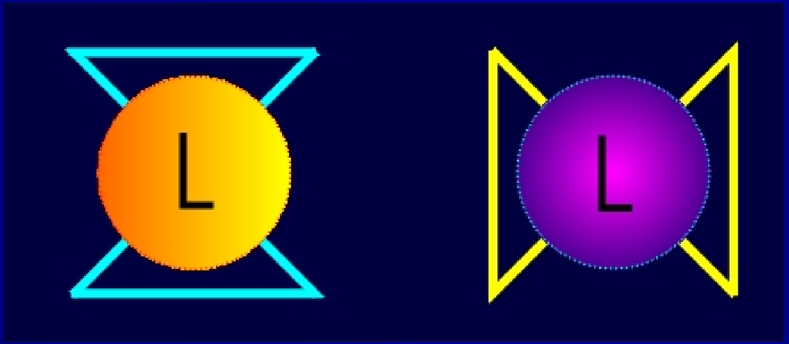
Sum and product of tangles



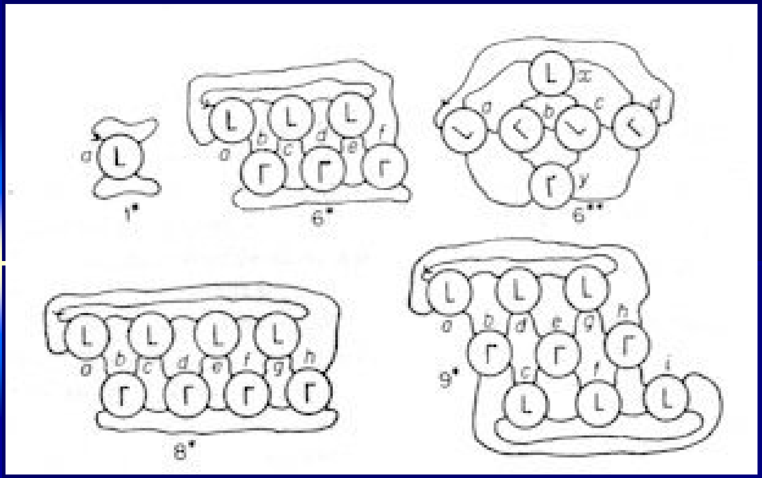
Ramification of tangles

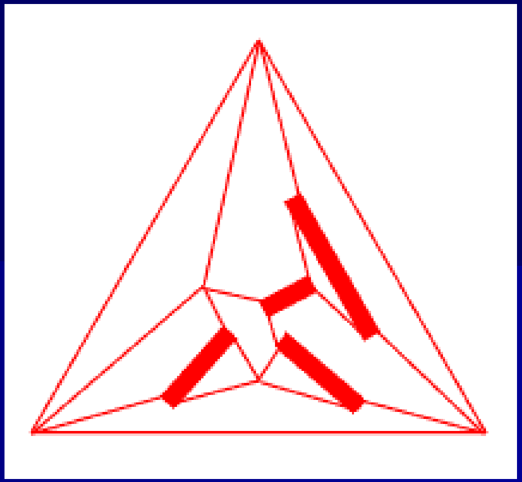


Some results of the operations with tangles



Numerator and denominator closure

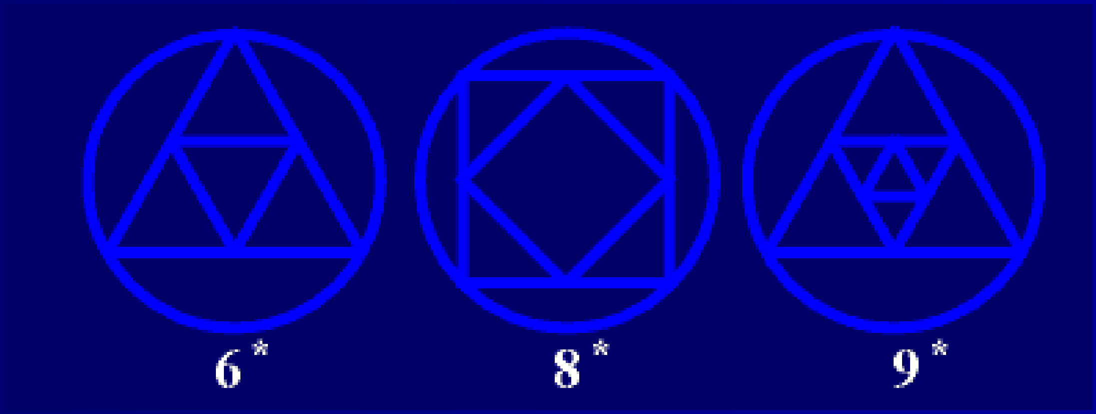




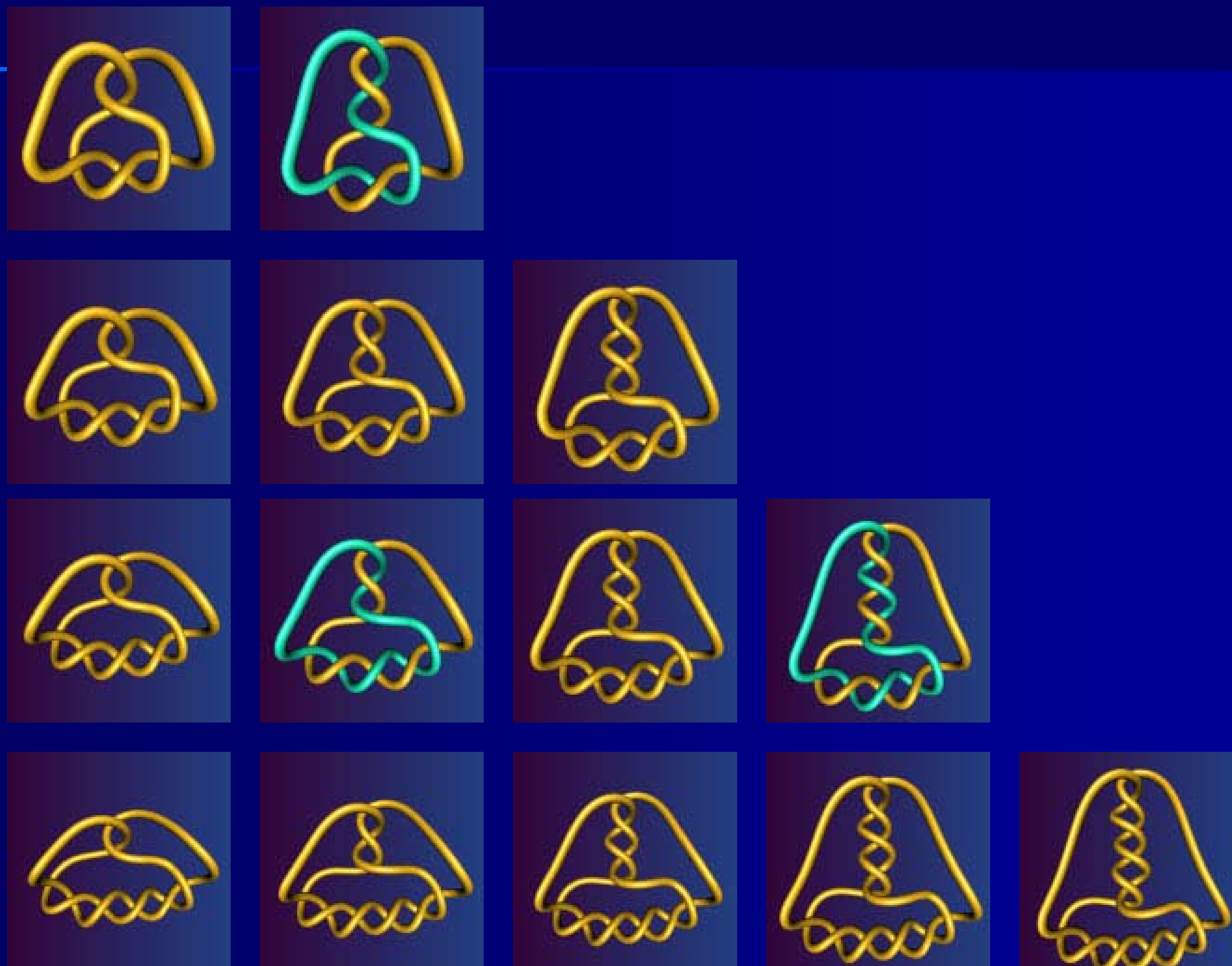
Basic polyhedra from J.Conway's paper

Digon collapse

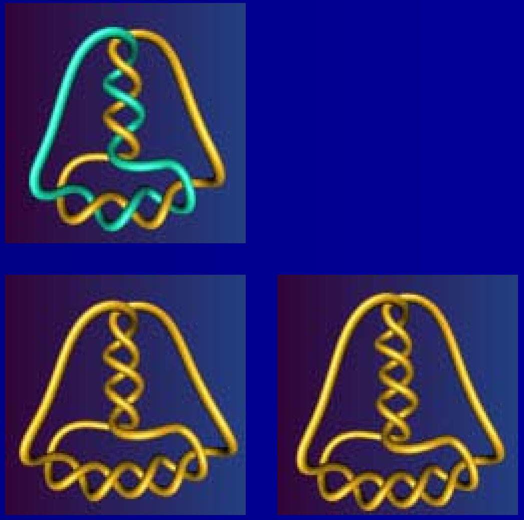
Basic polyhedra
6*, 8*, 9*.



KL families

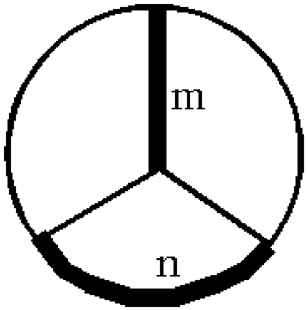


FAMILIES OF KNOTS AND LINKS



4_1 (m,n) $m>n>1$

p \ q	2	3	4	5	6	7	8	9
2	1 f 1-3 4_1							
3	1 2-3 5_2	2-3 6_2^2						
4	1 2-5 6_1	2 2-3+3 7_3	2 f 4-9 8_3					
5	1 3-5 7_2	3-5 8_2^2	2 3-5+5 9_4	3-5+5 10_4^2				
6	1 3-7 8_1	3 2-3+3-3 9_3	2 6-13 10_3	3 3-5+5-5 11_5	3 f 9-19 12_5			
7	1 4-7 9_2	4-7 10_2^2	2 4-7+7 11_4	4-7+7 12_4^2	3 4-7+7-7 13_6	4-7+7-7 14_6^2		
8	1 4-9 10_1	4 2-3+3-3+3 11_3	2 8-17 12_3	4 3-5+5-5+5 13_5	3 12-25 14_5	4 4-7+7-7+7 15_7	4 f 16-33 16_7	
9	1 5-9 11_2	5-9 12_2^2	2 5-9+9 13_4	5-9+9 14_4^2	3 5-9+9-9 15_6	5-9+9-9 16_6^2	4 5-9+9-9+9 17_8	5-9+9-9+9 18_8^2



Family n (n=2,3,4,…)

Jones polynomial

2 $1 + x^2$
3 $1 + x^2 - x^3$
4 $1 + x^2 - x^3 + x^4$
5 $1 + x^2 - x^3 + x^4 - x^5$
6 $1 + x^2 - x^3 + x^4 - x^5 + x^6$
7 $1 + x^2 - x^3 + x^4 - x^5 + x^6 - x^7$
8 $1 + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8$
9 $1 + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9$

HOMFLYPT polinom

2 $1 - x^2 - xy$
3 $1 - x^2 - 2xy$
4 $1 - x^2 - 3xy + x^3y + x^2y^2$
5 $1 - x^2 - 4xy + 2x^3y + 3x^2y^2$
6 $1 - x^2 - 5xy + 3x^3y + 6x^2y^2 - x^4y^2 - x^3y^3$
7 $1 - x^2 - 6xy + 4x^3y + 10x^2y^2 - 3x^4y^2 - 4x^3y^3$
8 $1 - x^2 - 7xy + 5x^3y + 15x^2y^2 - 6x^4y^2 - 10x^3y^3 + x^5y^3 + x^4y^4$
9 $1 - x^2 - 8xy + 6x^3y + 21x^2y^2 - 10x^4y^2 - 20x^3y^3 + 4x^5y^3 + 5x^4y^4$

HOMFLYPT polynomial reduced to one variable, link family *n* (*n*=2,3,4…)

	1	x^2	x^4	x^6	x^8	x^{10}	x^{12}	x^{14}	x^{16}	x^{18}	x^{20}
2	1	-2									
3	1	-3									
4	1	-4	2								
5	1	-5	5								
6	1	-6	9	-2							
7	1	-7	14	-7							
8	1	-8	20	-16	2						
9	1	-9	27	-30	9						
10	1	-10	35	-50	25	-2					
11	1	-11	44	-77	55	-11					
12	1	-12	54	-112	105	-36	2				
13	1	-13	65	-156	182	-91	13				
14	1	-14	77	-210	294	-196	49	-2			
15	1	-15	90	-275	450	-378	140	-15			
16	1	-16	104	-352	660	-672	336	-64	2		
17	1	-17	119	-442	935	-1122	714	-204	17		
18	1	-18	135	-546	1287	-1782	1386	-540	81	-2	
19	1	-19	152	-665	1729	-2717	2508	-1254	285	-19	
20	1	-20	170	-800	2275	-4004	4290	-2640	825	-100	2
21	1	-21	189	-952	2940	-5733	7007	-5148	2079	-385	21

A000096 $n(n+3)/2$

	1	x^2	x^4	x^6	x^8	x^{10}	x^{12}	x^{14}	x^{16}	x^{18}	x^{20}
2	1	-2									
3	1	-3									
4	1	-4	2								
5	1	-5	5								
6	1	-6	9	-2							
7	1	-7	14	-7							
8	1	-8	20	-16	2						
9	1	-9	27	-30	9						
10	1	-10	35	-50	25	-2					
11	1	-11	44	-77	55	-11					
12	1	-12	54	-112	105	-36	2				
13	1	-13	65	-156	182	-91	13				
14	1	-14	77	-210	294	-196	49	-2			
15	1	-15	90	-275	450	-378	140	-15			
16	1	-16	104	-352	660	-672	336	-64	2		
17	1	-17	119	-442	935	-1122	714	-204	17		
18	1	-18	135	-546	1287	-1782	1386	-540	81	-2	
19	1	-19	152	-665	1729	-2717	2508	-1254	285	-19	
20	1	-20	170	-800	2275	-4004	4290	-2640	825	-100	2
21	1	-21	189	-952	2940	-5733	7007	-5148	2079	-385	21

A005581 $(n\text{-}1)n(n+1)/6$

	1	x^2	x^4	x^6	x^8	x^{10}	x^{12}	x^{14}	x^{16}	x^{18}	x^{20}
2	1	-2									
3	1	-3									
4	1	-4	2								
5	1	-5	5								
6	1	-6	9	-2							
7	1	-7	14	-7							
8	1	-8	20	-16	2						
9	1	-9	27	-30	9						
10	1	-10	35	-50	25	-2					
11	1	-11	44	-77	55	-11					
12	1	-12	54	-112	105	-36	2				
13	1	-13	65	-156	182	-91	13				
14	1	-14	77	-210	294	-196	49	-2			
15	1	-15	90	-275	450	-378	140	-15			
16	1	-16	104	-352	660	-672	336	-64	2		
17	1	-17	119	-442	935	-1122	714	-204	17		
18	1	-18	135	-546	1287	-1782	1386	-540	81	-2	
19	1	-19	152	-665	1729	-2717	2508	-1254	285	-19	
20	1	-20	170	-800	2275	-4004	4290	-2640	825	-100	2
21	1	-21	189	-952	2940	-5733	7007	-5148	2079	-385	21

A005582 $n(n+1)(n+2)(n+7)/24$

	1	x^2	x^4	x^6	x^8	x^{10}	x^{12}	x^{14}	x^{16}	x^{18}	x^{20}
2	1	-2									
3	1	-3									
4	1	-4	2								
5	1	-5	5								
6	1	-6	9	-2							
7	1	-7	14	-7							
8	1	-8	20	-16	2						
9	1	-9	27	-30	9						
10	1	-10	35	-50	25	-2					
11	1	-11	44	-77	55	-11					
12	1	-12	54	-112	105	-36	2				
13	1	-13	65	-156	182	-91	13				
14	1	-14	77	-210	294	-196	49	-2			
15	1	-15	90	-275	450	-378	140	-15			
16	1	-16	104	-352	660	-672	336	-64	2		
17	1	-17	119	-442	935	-1122	714	-204	17		
18	1	-18	135	-546	1287	-1782	1386	-540	81	-2	
19	1	-19	152	-665	1729	-2717	2508	-1254	285	-19	
20	1	-20	170	-800	2275	-4004	4290	-2640	825	-100	2
21	1	-21	189	-952	2940	-5733	7007	-5148	2079	-385	21

A 005583 Coefficients of Chebyshev polynomial

	1	x^2	x^4	x^6	x^8	x^{10}	x^{12}	x^{14}	x^{16}	x^{18}	x^{20}
2	1	-2									
3	1	-3									
4	1	-4	2								
5	1	-5	5								
6	1	-6	9	-2							
7	1	-7	14	-7							
8	1	-8	20	-16	2						
9	1	-9	27	-30	9						
10	1	-10	35	-50	25	-2					
11	1	-11	44	-77	55	-11					
12	1	-12	54	-112	105	-36	2				
13	1	-13	65	-156	182	-91	13				
14	1	-14	77	-210	294	-196	49	-2			
15	1	-15	90	-275	450	-378	140	-15			
16	1	-16	104	-352	660	-672	336	-64	2		
17	1	-17	119	-442	935	-1122	714	-204	17		
18	1	-18	135	-546	1287	-1782	1386	-540	81	-2	
19	1	-19	152	-665	1729	-2717	2508	-1254	285	-19	
20	1	-20	170	-800	2275	-4004	4290	-2640	825	-100	2
21	1	-21	189	-952	2940	-5733	7007	-5148	2079	-385	21

A005584 Coefficients of Chebyshev polynomial

	1	x^2	x^4	x^6	x^8	x^{10}	x^{12}	x^{14}	x^{16}	x^{18}	x^{20}
2	1	-2									
3	1	-3									
4	1	-4	2								
5	1	-5	5								
6	1	-6	9	-2							
7	1	-7	14	-7							
8	1	-8	20	-16	2						
9	1	-9	27	-30	9						
10	1	-10	35	-50	25	-2					
11	1	-11	44	-77	55	-11					
12	1	-12	54	-112	105	-36	2				
13	1	-13	65	-156	182	-91	13				
14	1	-14	77	-210	294	-196	49	-2			
15	1	-15	90	-275	450	-378	140	-15			
16	1	-16	104	-352	660	-672	336	-64	2		
17	1	-17	119	-442	935	-1122	714	-204	17		
18	1	-18	135	-546	1287	-1782	1386	-540	81	-2	
19	1	-19	152	-665	1729	-2717	2508	-1254	285	-19	
20	1	-20	170	-800	2275	-4004	4290	-2640	825	-100	2
21	1	-21	189	-952	2940	-5733	7007	-5148	2079	-385	21

??? Congratulations from “Encyclopedia of Integer Sequences”

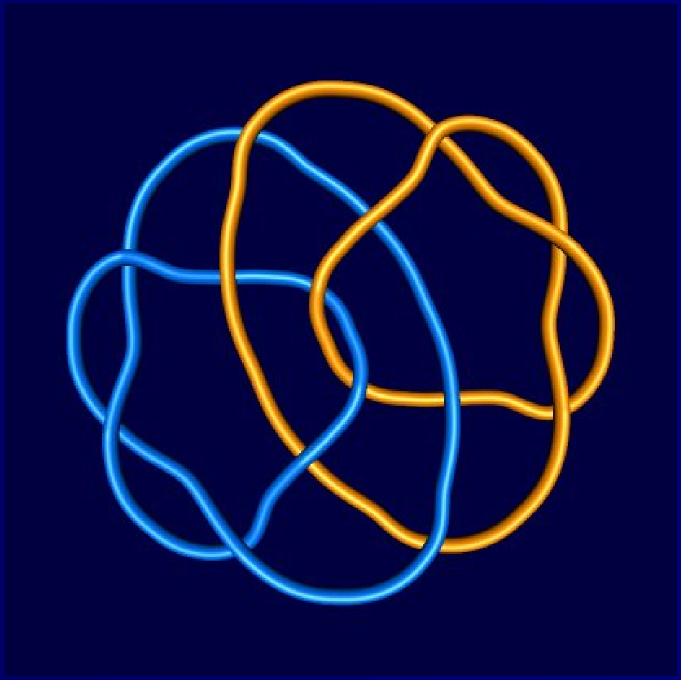
You discovered new sequence!

	1	x^2	x^4	x^6	x^8	x^{10}	x^{12}	x^{14}	x^{16}	x^{18}	x^{20}
2	1	-2									
3	1	-3									
4	1	-4	2								
5	1	-5	5								
6	1	-6	9	-2							
7	1	-7	14	-7							
8	1	-8	20	-16	2						
9	1	-9	27	-30	9						
10	1	-10	35	-50	25	-2					
11	1	-11	44	-77	55	-11					
12	1	-12	54	-112	105	-36	2				
13	1	-13	65	-156	182	-91	13				
14	1	-14	77	-210	294	-196	49	-2			
15	1	-15	90	-275	450	-378	140	-15			
16	1	-16	104	-352	660	-672	336	-64	2		
17	1	-17	119	-442	935	-1122	714	-204	17		
18	1	-18	135	-546	1287	-1782	1386	-540	81	-2	
19	1	-19	152	-665	1729	-2717	2508	-1254	285	-19	
20	1	-20	170	-800	2275	-4004	4290	-2640	825	-100	2
21	1	-21	189	-952	2940	-5733	7007	-5148	2079	-385	21

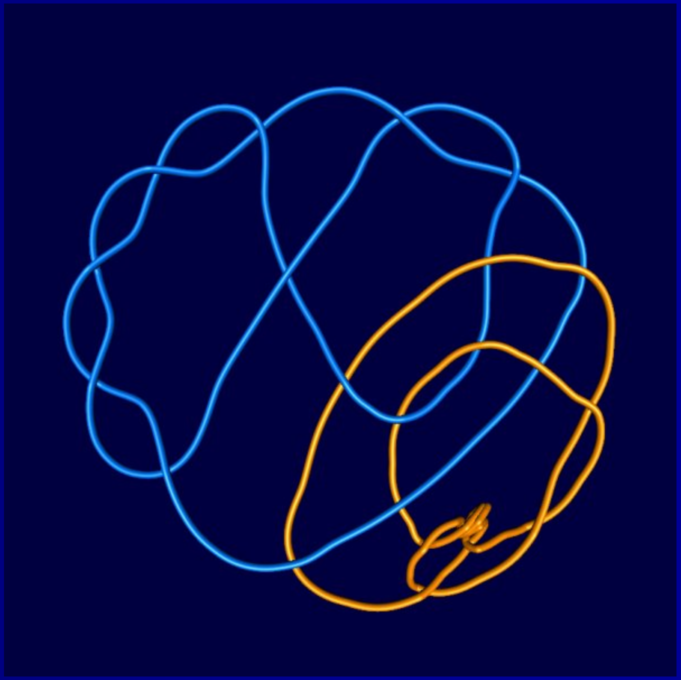
Links with trivial Jones polynomial

There are infinite families of links with trivial Jones polynomial (Eliahou, Kauffman, Thistlethwaite, 2003).

OPEN PROBLEM: “Jones unknot” (O. Dasbach)

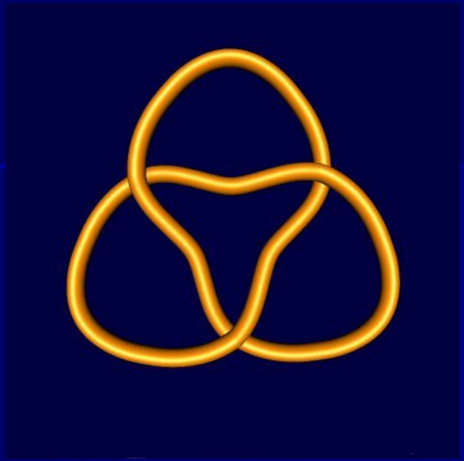
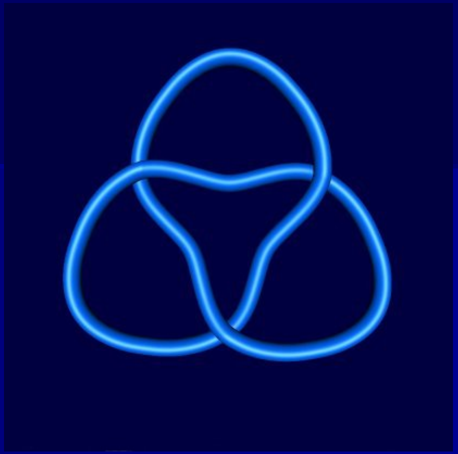


9*3:-1.-1.2.-1.-1:-3

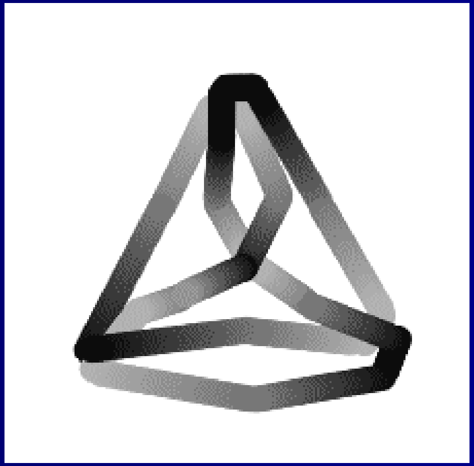
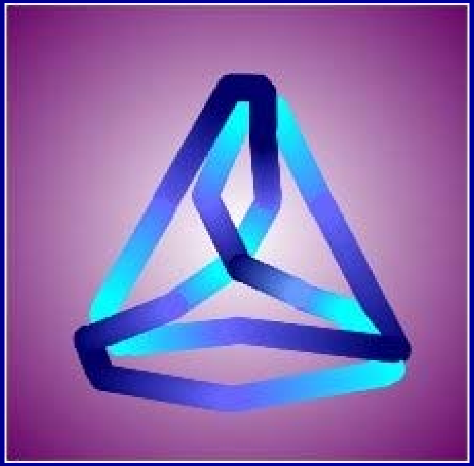


9*5 1 2:-1.-1.2.-1.-1:-5 -1 -2

- Chirality

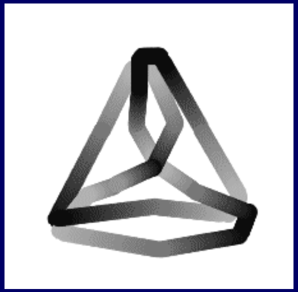



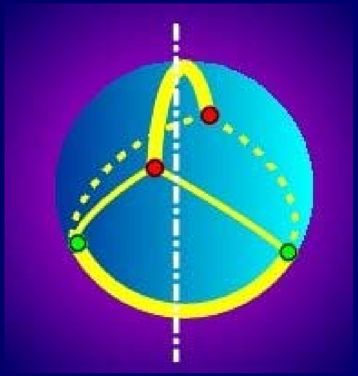
“Left” and “right” trefoil

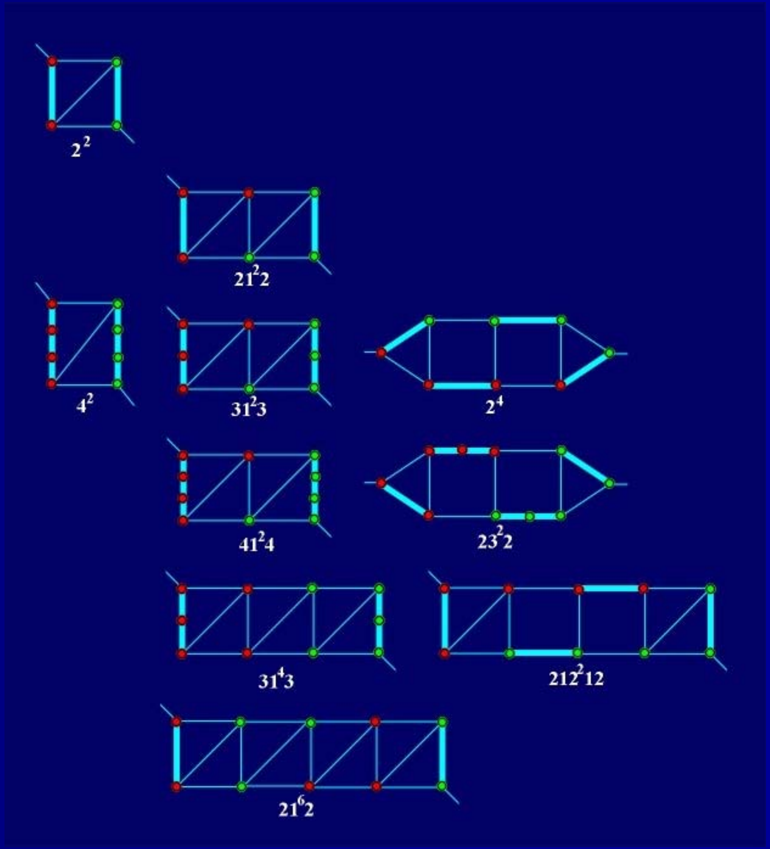
Achiral figure-eight knot 2 2

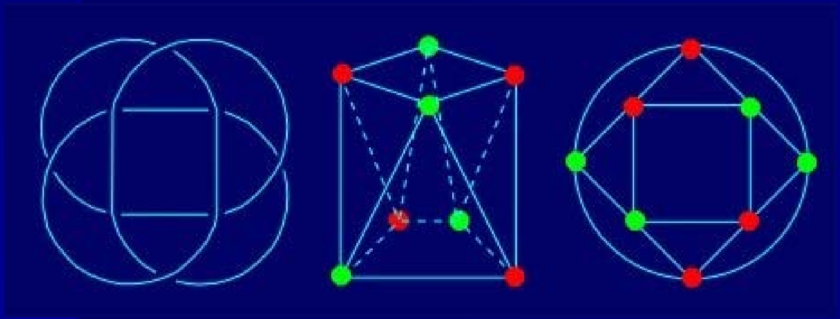
Amphicheirality





2 2



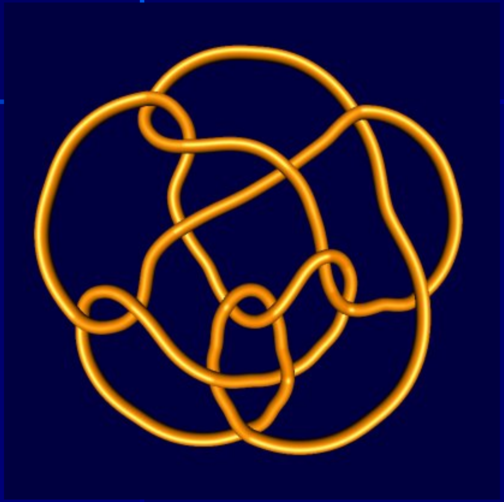


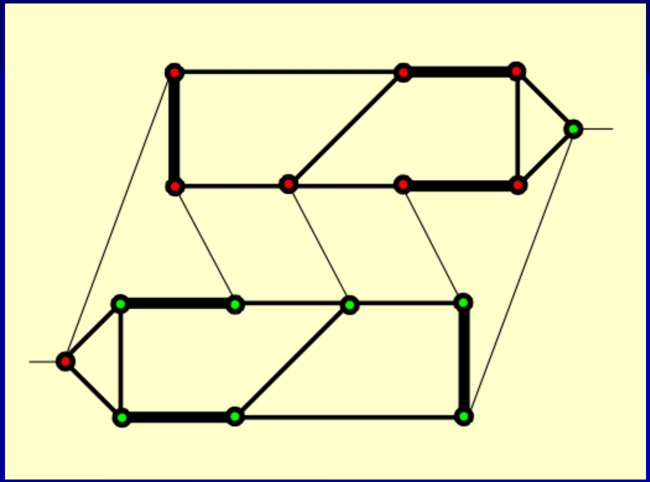
Rigid amphicheiral representation of 2 2

Tait Conjecture: Amphicheiral knot has an even number of crossings.

Amphicheirality

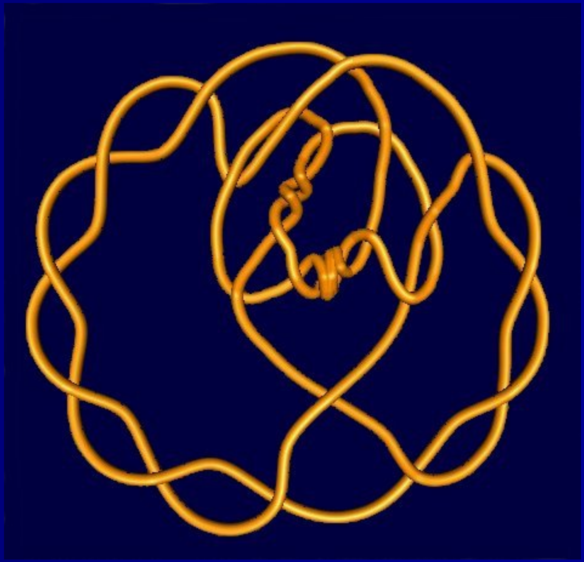
n=16





n=15

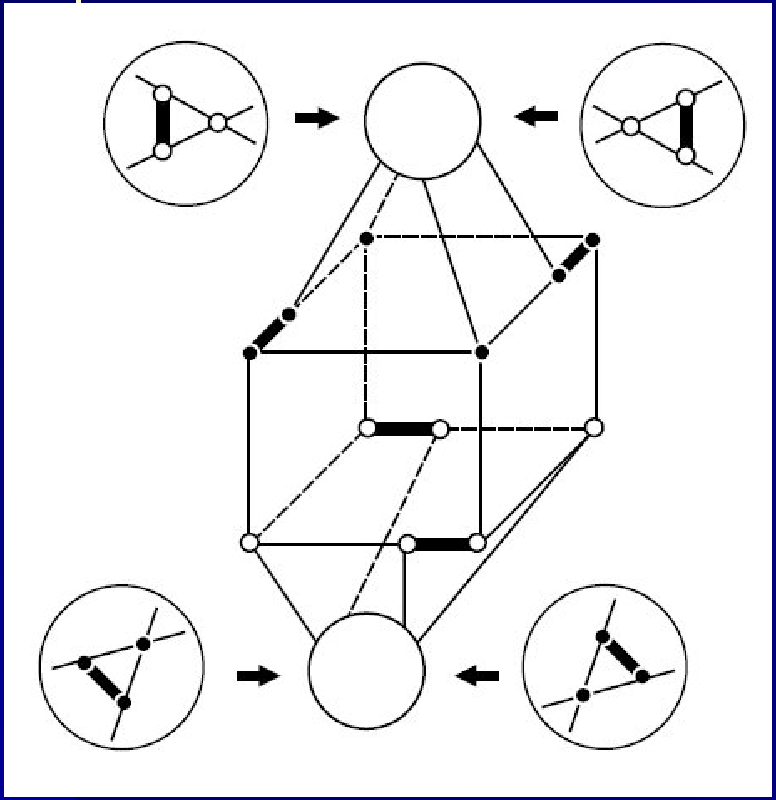
102 0.2..-2 0..2 0.-1.-1.-1.-2 0**

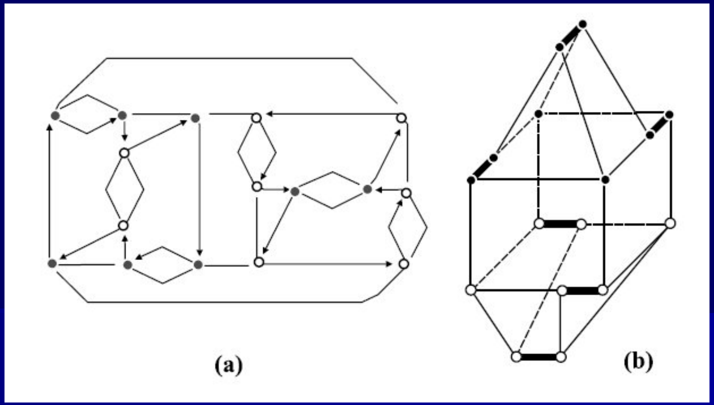


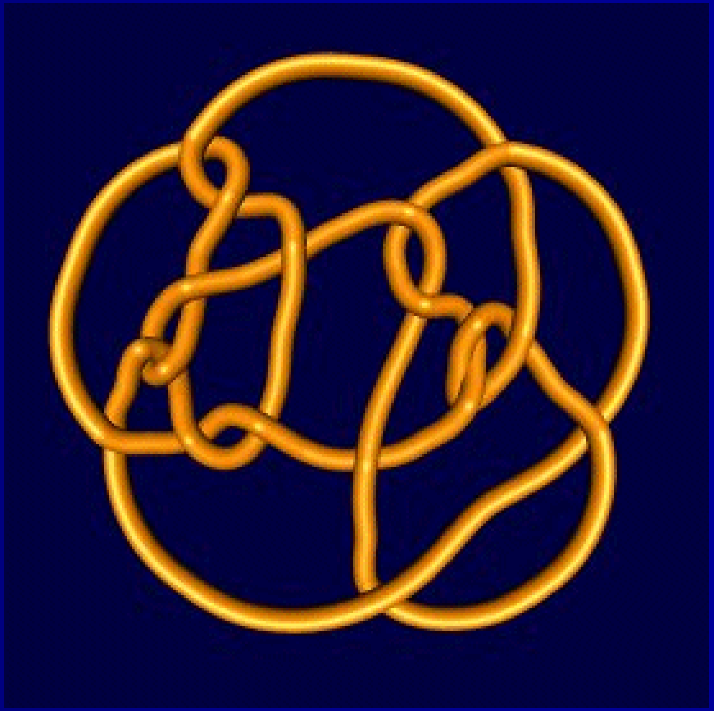
n=39

(Thistlethwaite)

Amphicheirality

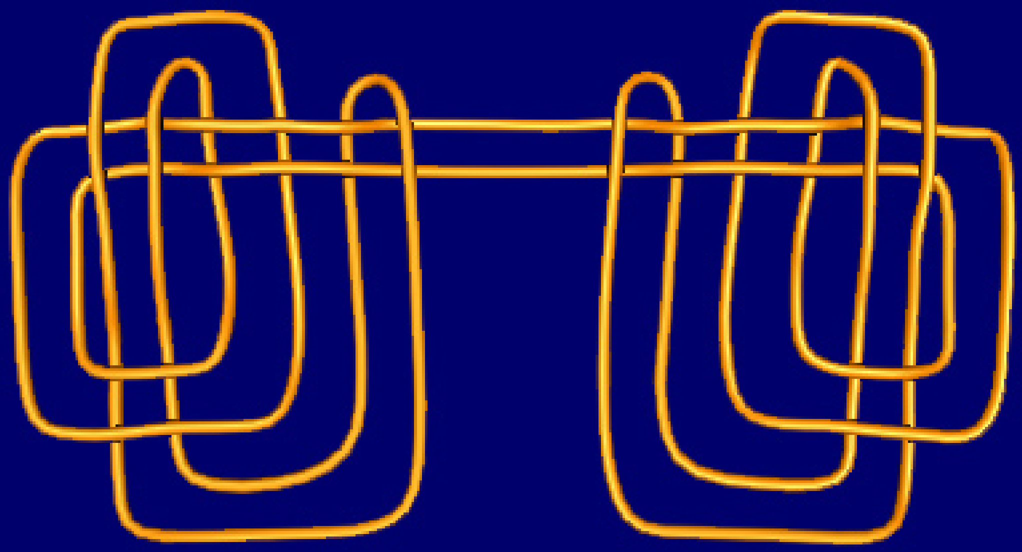






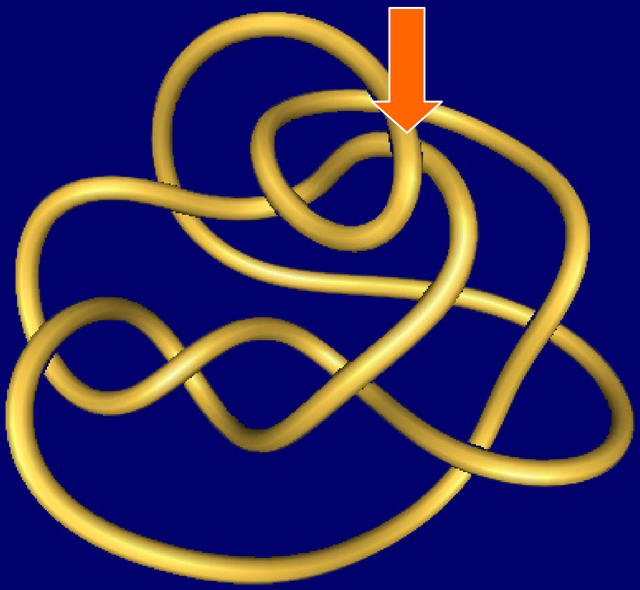
10*2:2:.2 0:2 0.2 1.2 1 0 – amphicheiral knot without antisymmetrical diagram**

NASTY UNKNOT

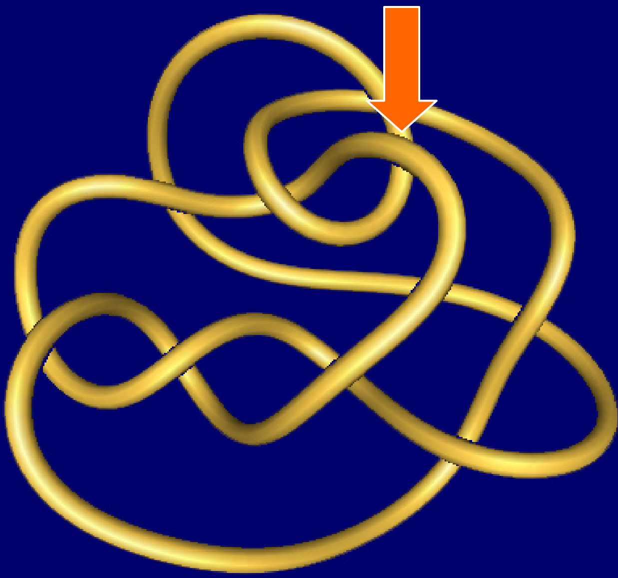


KNOTS WHICH ARE REALY KNOTTED

TREFOIL

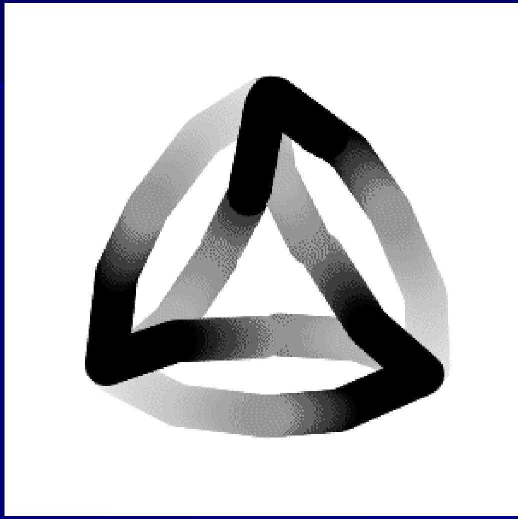


UNKNOT

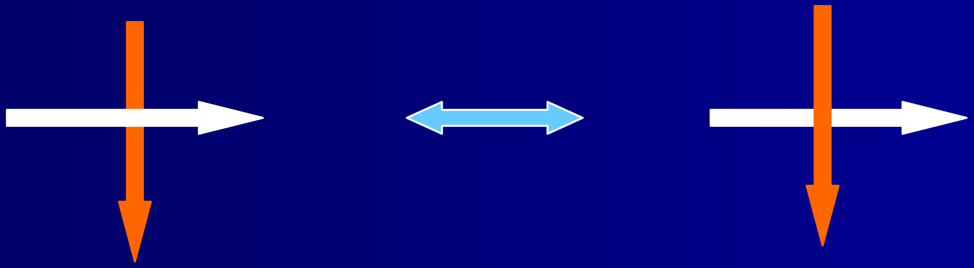


“Measure of knottednes” (“unlinking number”) is one from the most difficult problems in knot theory.

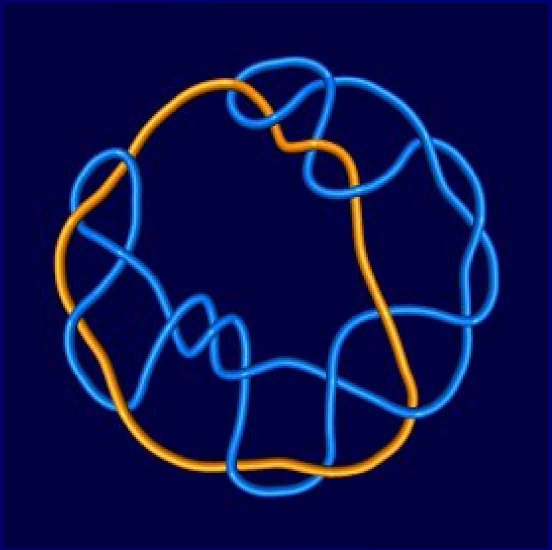
Unknotting number



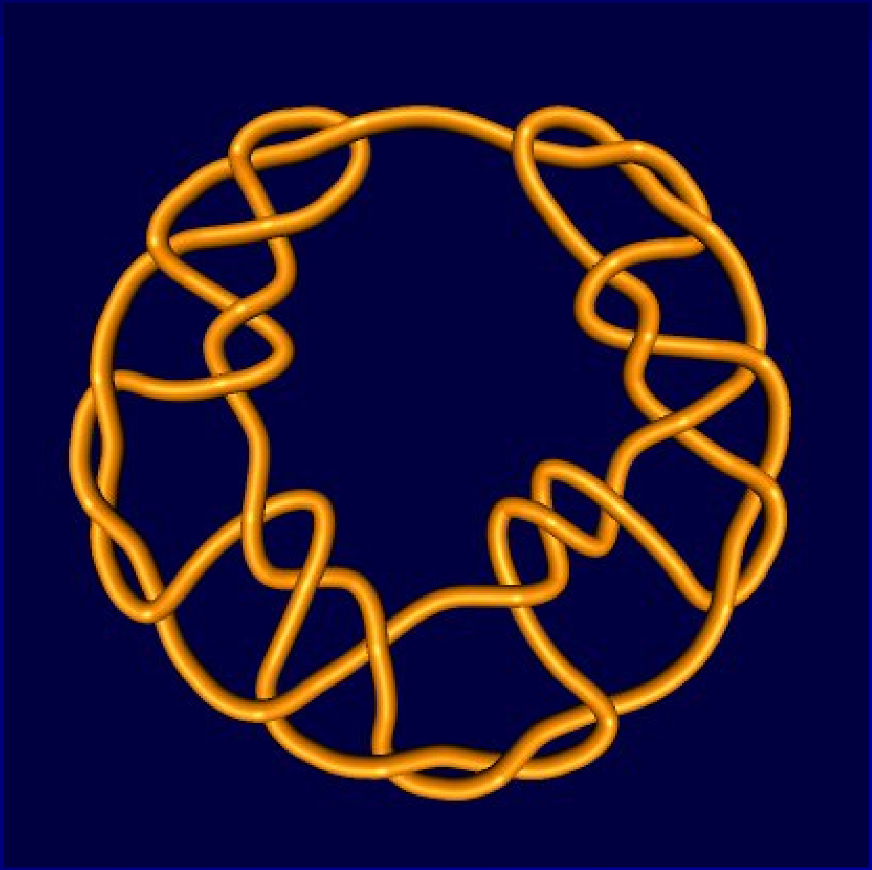
Minimal number of crossing changes necessary to obtain unknot (unlink).



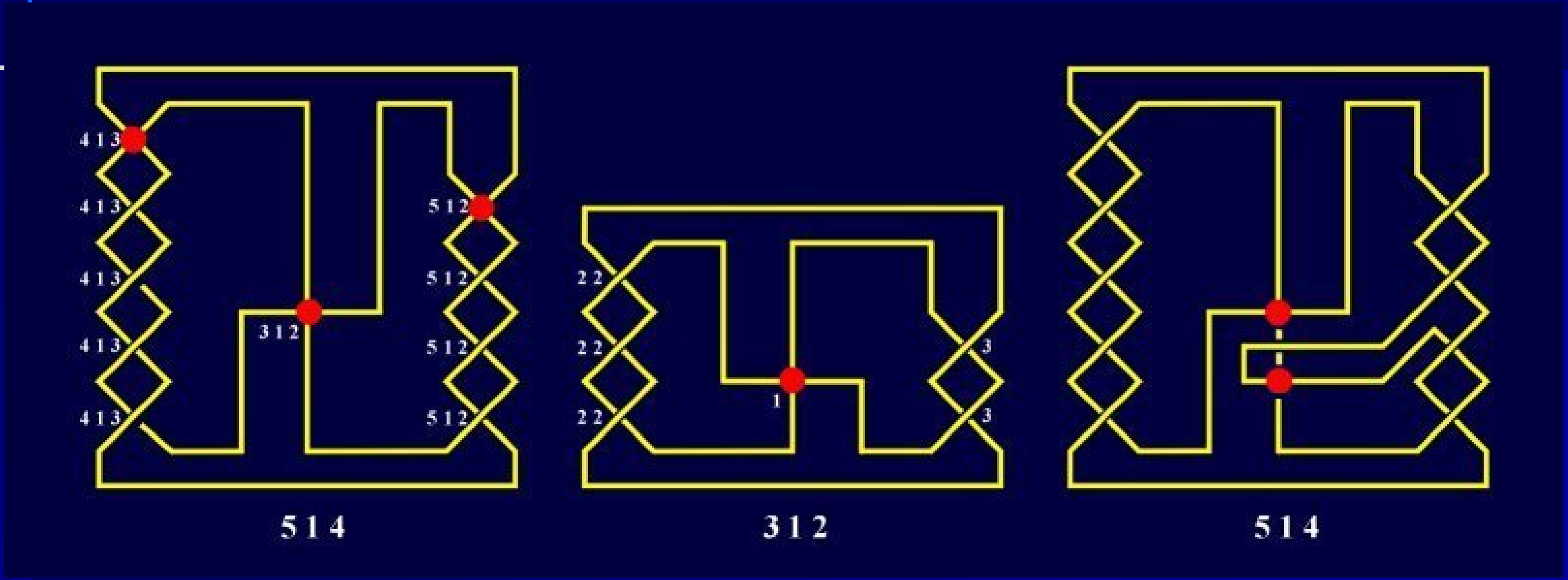
Unknotting number



u=1

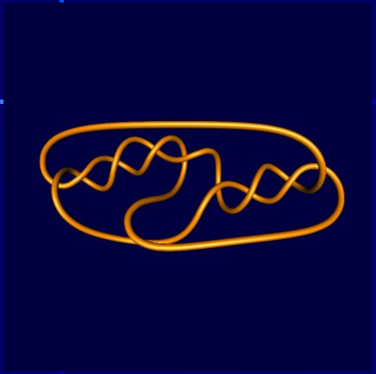


u=0

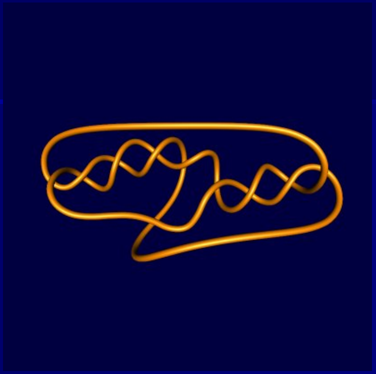


The Nakanishi-Bleiler example: (a) the minimal projection of the knot 5 1 4 that requires at least three crossing changes to be unknotted; (b) the minimal projection of the knot 3 1 2 with the unknotting number 1; (c) non-minimal projection of the knot 5 1 4 from which we obtain the correct unknotting number u(5 1 4) = 2.

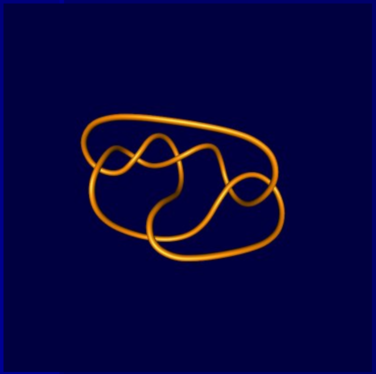
- **Unknotting 5 1 4**



5 1 4

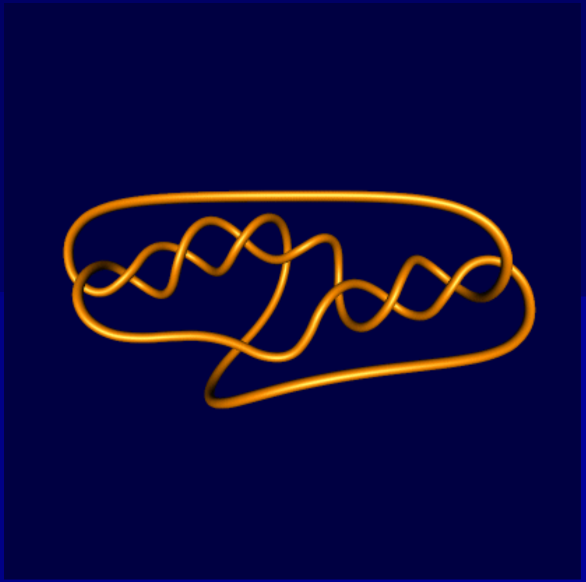


5 -1 4



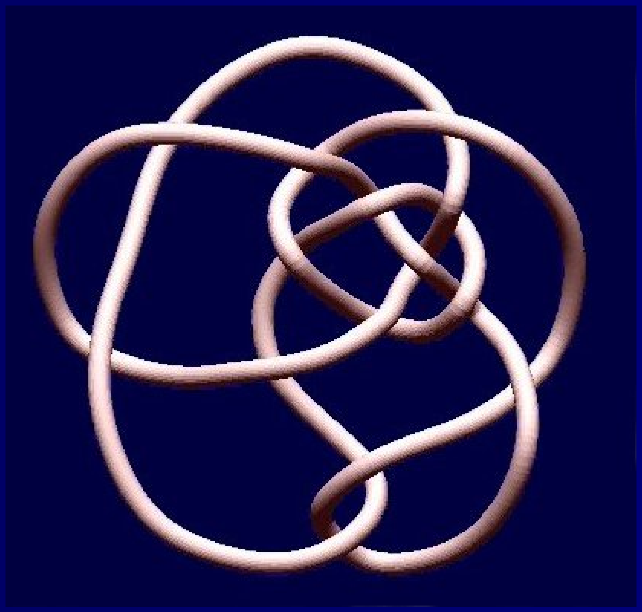
3 1 2

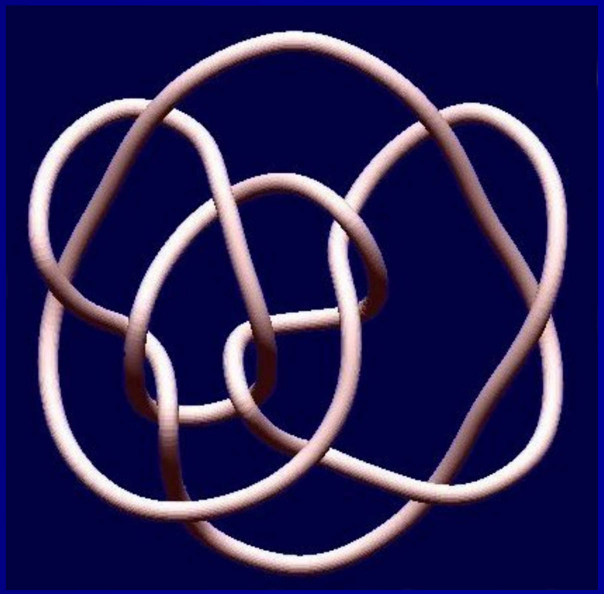
3 -1 2



- **Bernhard-Jablan Conjecture**
- ***u(L) = 0 for any unlink L;***
- ***u(L) = min u(L-)+1, where the minimum is taken over all minimal diagrams of links L-, obtained from a minimal diagram of L by one crossing change.***

- **Alternating *KL*s: one diagram is sufficient**
- **Non-alternating *KL*s: all diagrams are necessary**



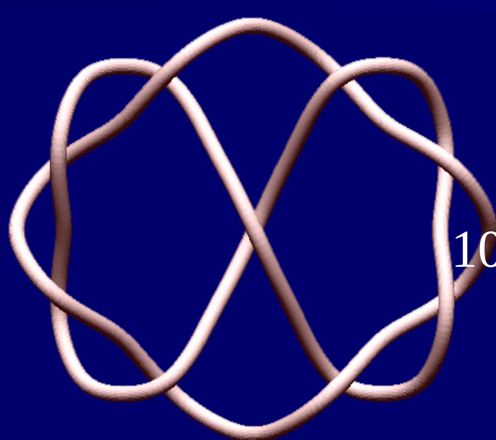


Two projections of the knot 14_{36750}, the first with *u*=1, and the other with *u*=2.

Confirmation: signature!

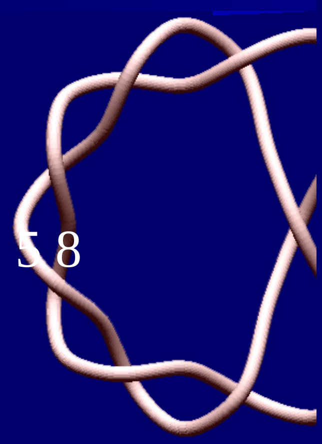
NEW MULTI-PARAMETER FAMILIES OF KLs WITH UNLINKING GAP

- **$(2k)(2l-1)(2m)$**
- **3-parameter family with arbitrarily large unlinking gap**
- **The first member is the link 4 1 4 with n=9 crossings**
- **Conditions: $k \geq 2$, $m \geq 2$, $2k \geq 2m \geq 2l-1$**

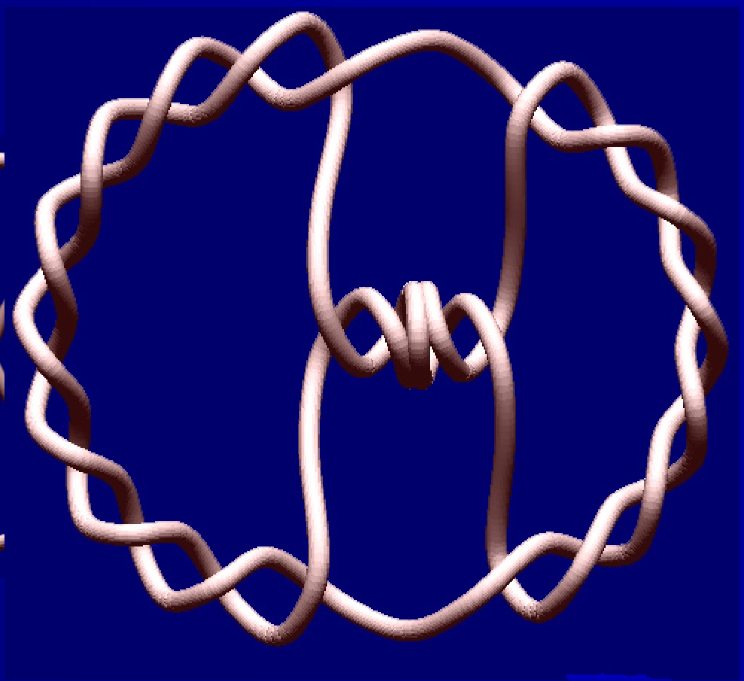


4 1 4

10 5 8

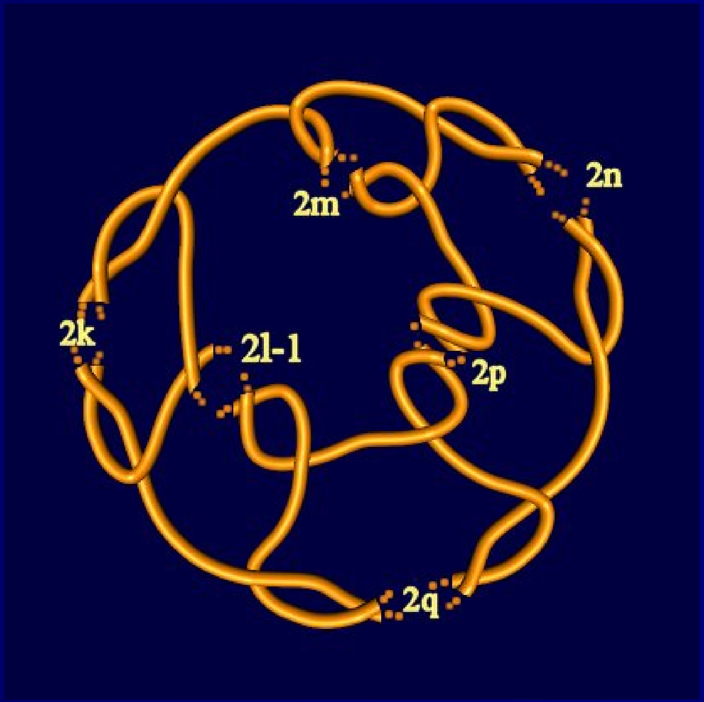


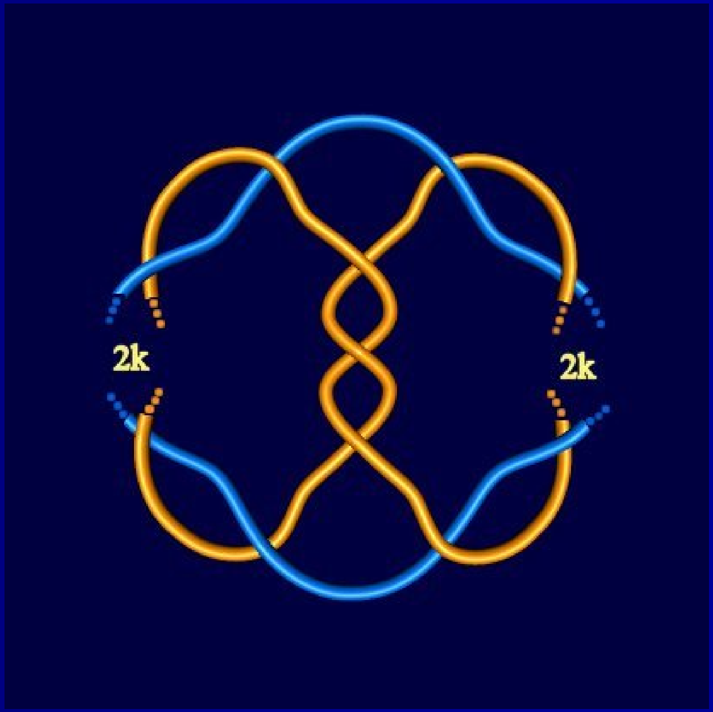
6 1 4



Unlinking gap

n =	9	10	11	12	13	14	15	16
	1	1	5	5	23	36	106	180



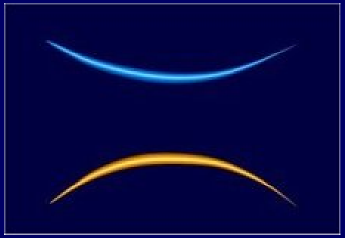


Alternating knot family with arbitrarily large unknotting gap

Non-alternating link family *2k*,3,-*2k* with arbitrarily large unknotting gap

S. Jablan and R. Sazdanovic: Unlinking number and unlinking gap, JKTR (2007)

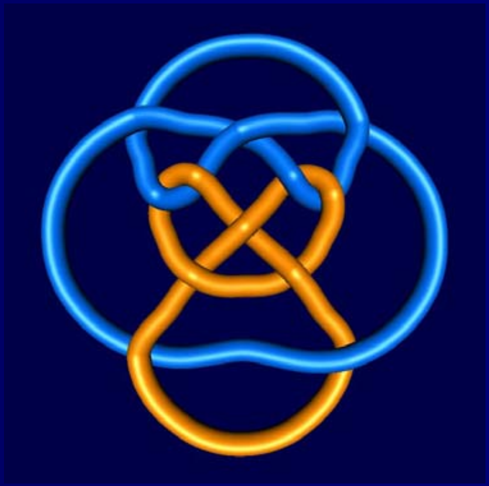
Four moves



Conjecture: Every knot is 4-move equivalent to the trivial knot (Nakanishi, 1979)

Proved for all knots up to *n*=12 crossings.

Open question: Is it true that every 2-component link is 4-move equivalent to the trivial link of two components or to the Hopf link?



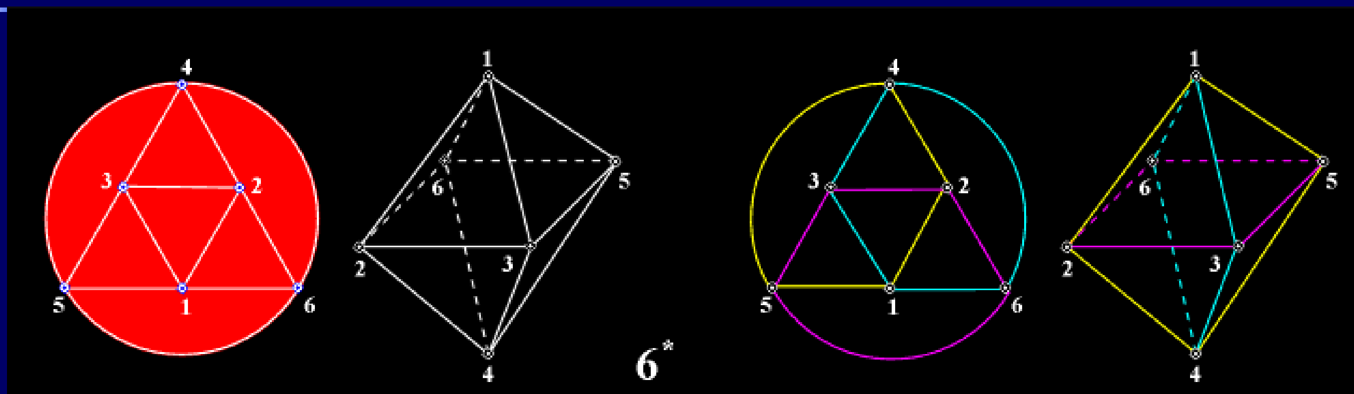
9*.2:.2:.2

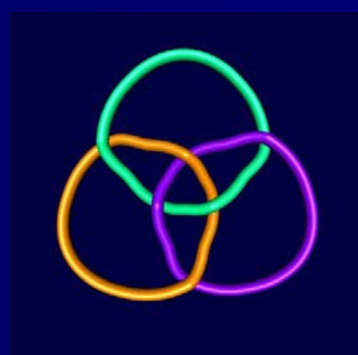
Potential counterexample: 9*.2:.2:.2

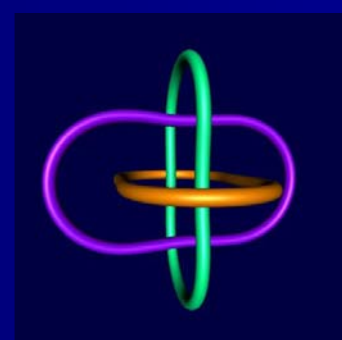
Open problem: 4-move unknotting numbers

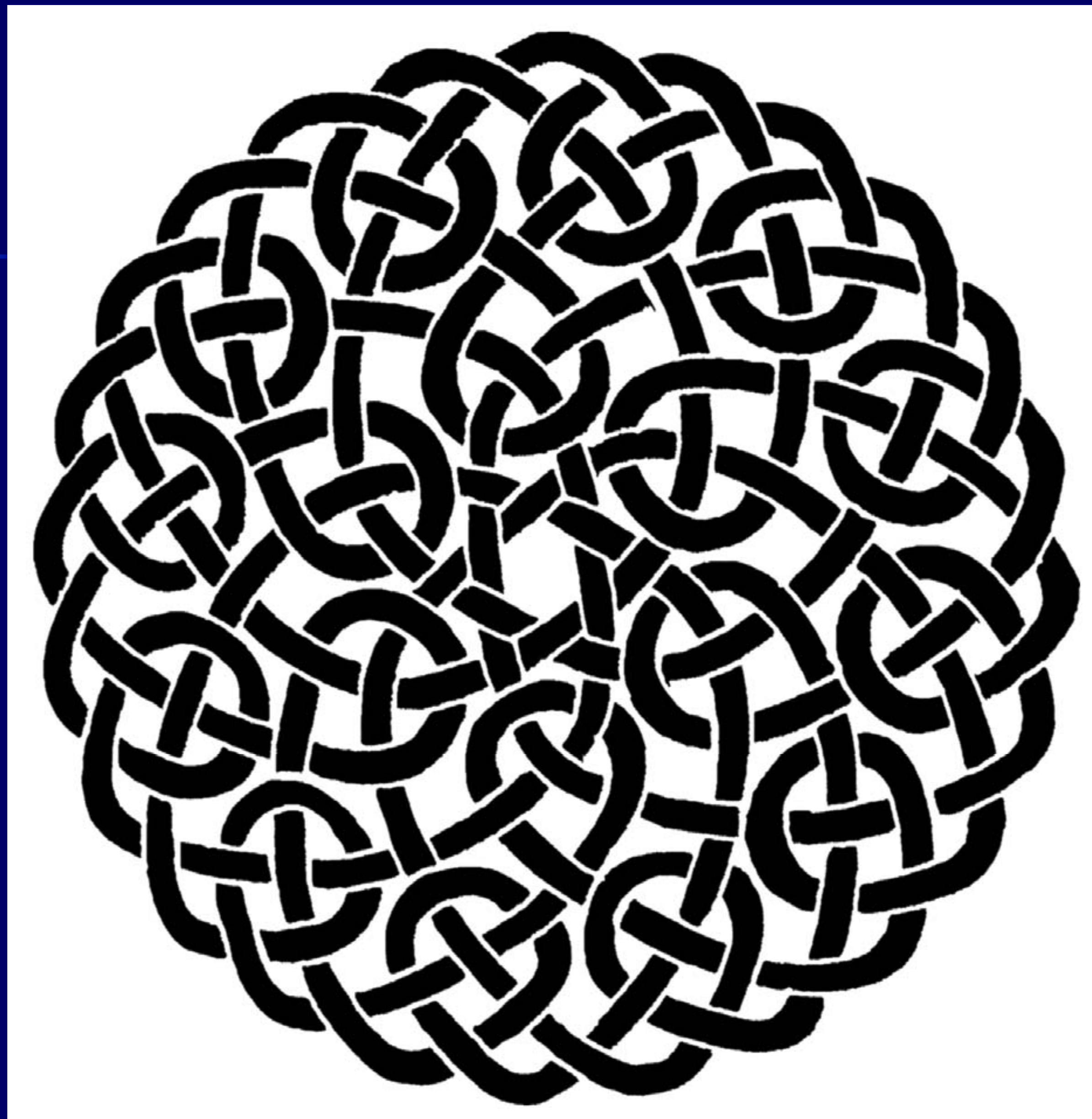
M.K. Dabkowski, S. Jablan, N.A. Khan, R.K. Sahi: On 4-move equivalence classes Of knots and links of two components (2009)

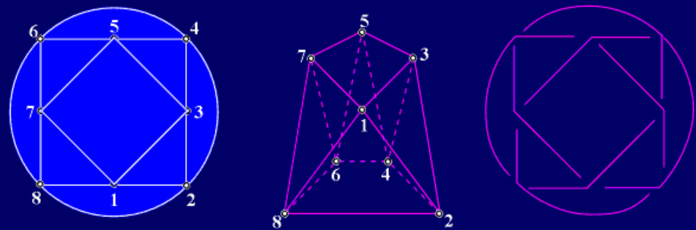
Basic Polyhedra



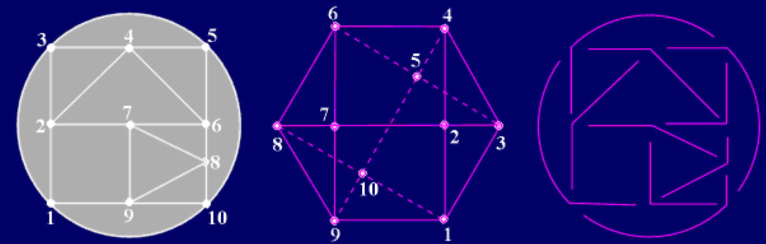




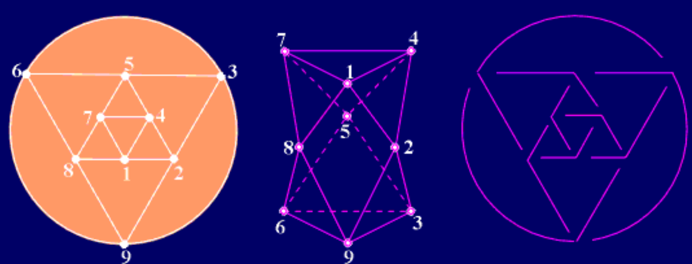




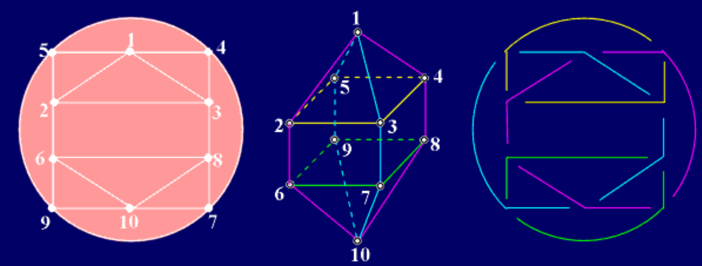
8^*



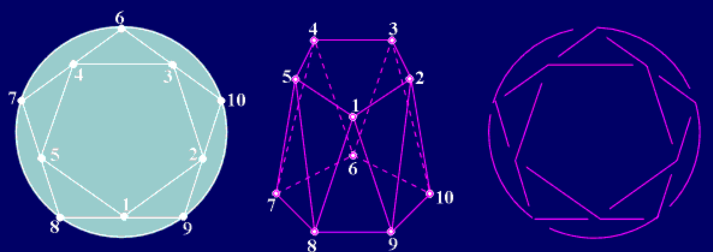
10^{**}



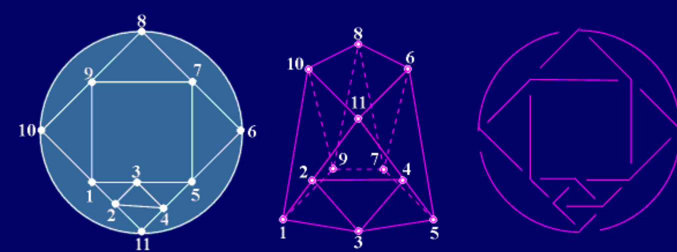
9^*



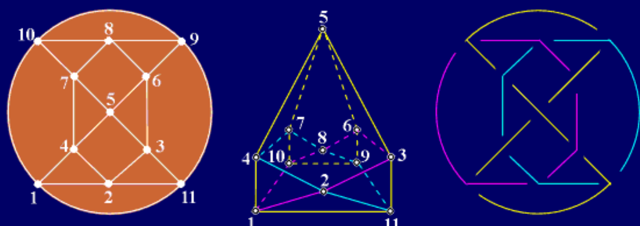
10^{***}



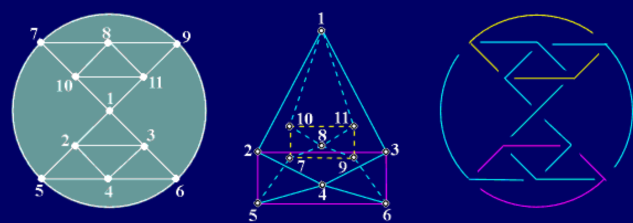
10^*



11^*



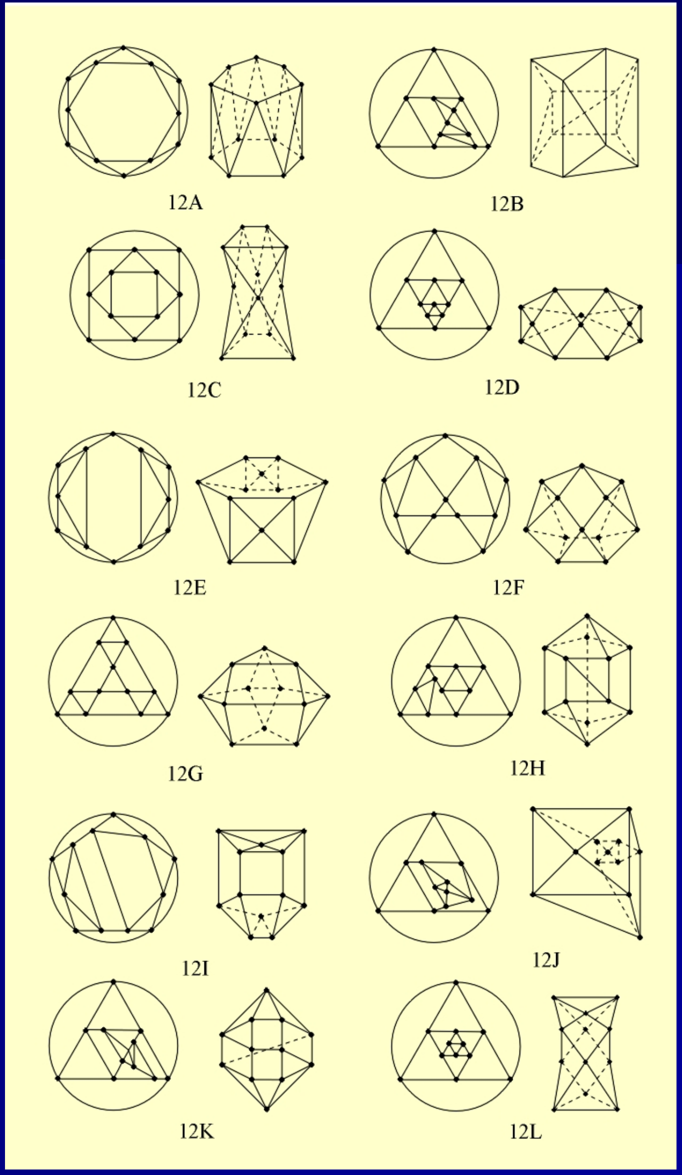
11**



11***

n	
13	19
14	64
15	155
16	510
17	1 514
18	5 146
19	16 966
20	58 782

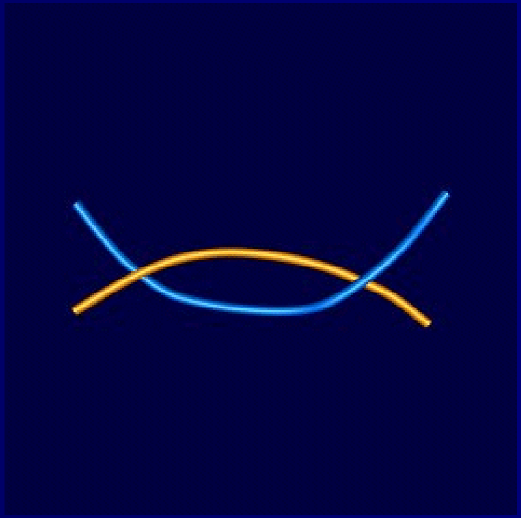
Acknowledgement to Brendan McKay



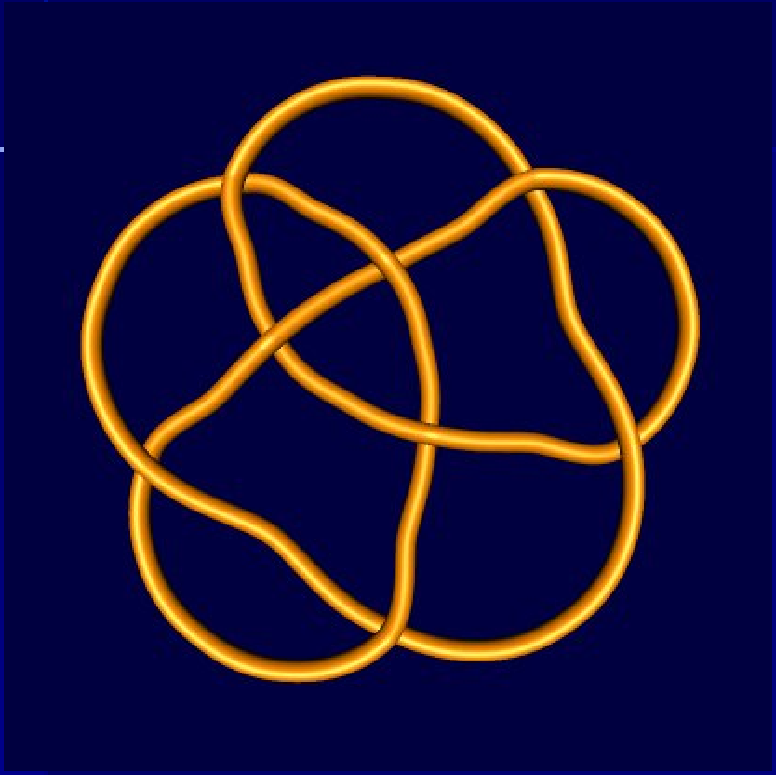
Graph-theoretical classification of KLs

- **Algebraic (rational, stellar, arborescent…)**
- **Non-algebraic (polyhedral)**

- **Bigon, bigon collapse**
- **Def. 1 *KL* is algebraic if it has a representation that collapses into a point (basic polyhedron 1*)**
- **Def. 2 *KL* is algebraic if it has a minimal representation that collapses into a point (basic polyhedron 1*)**



Algebraic or non-algebraic?



=

.2.2 0

n=8

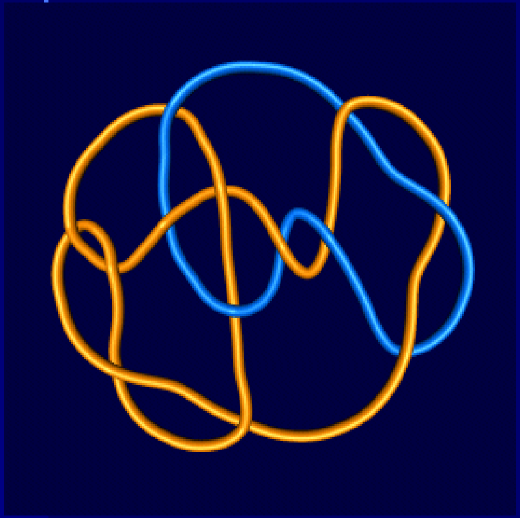
(-3,2) (2,-3)

n=10

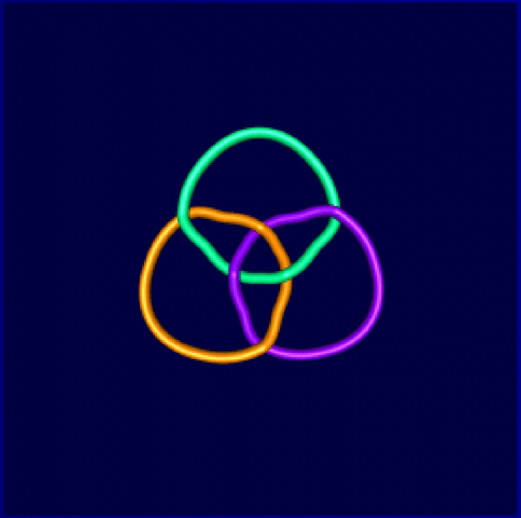
Basic polyhedron:

Def. 1 *KL*-diagram without bigons (bigon-free diagram)

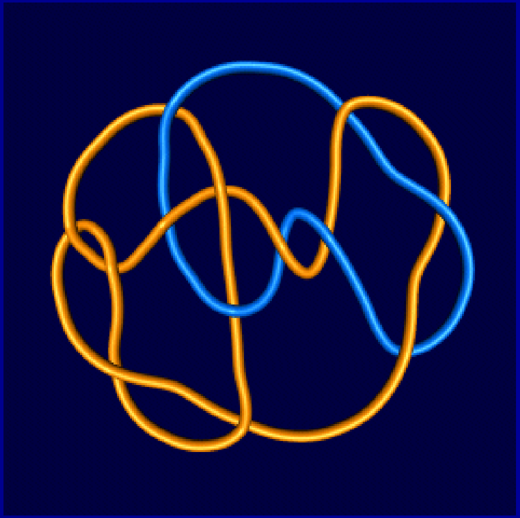
Def 2. *KL* that has at least one bigon-free diagram?



Collapse



Vertex substitution



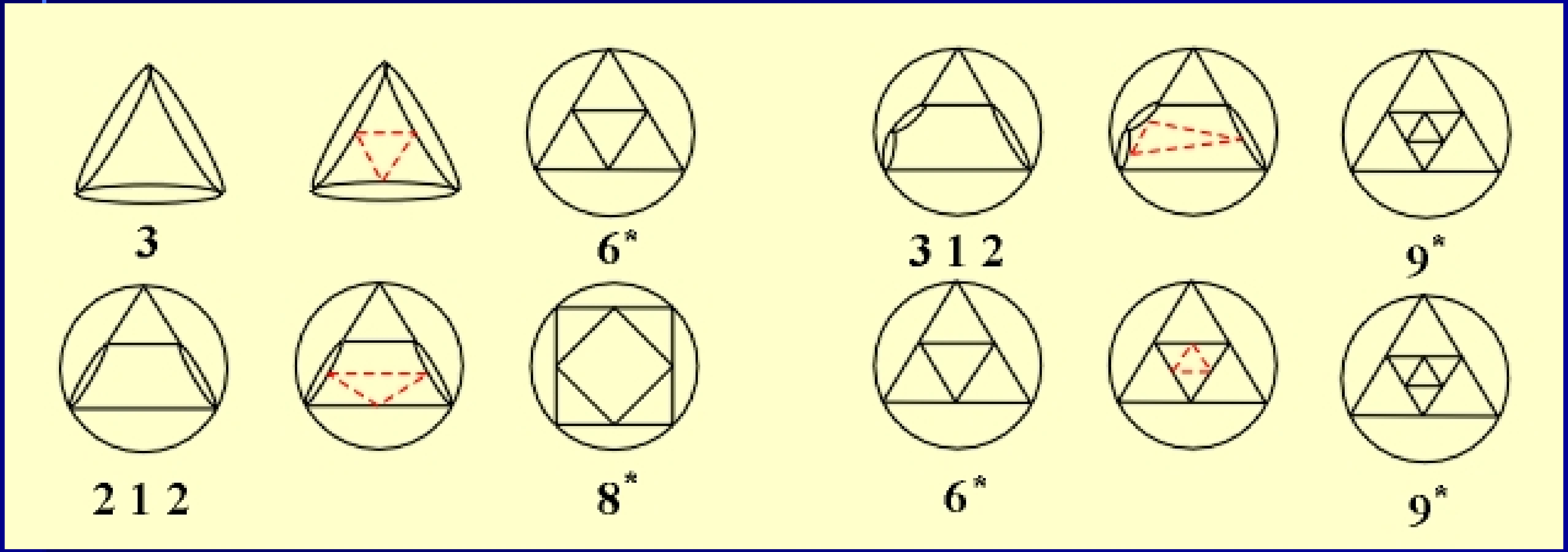
Immediate
("brute force")

Step by step

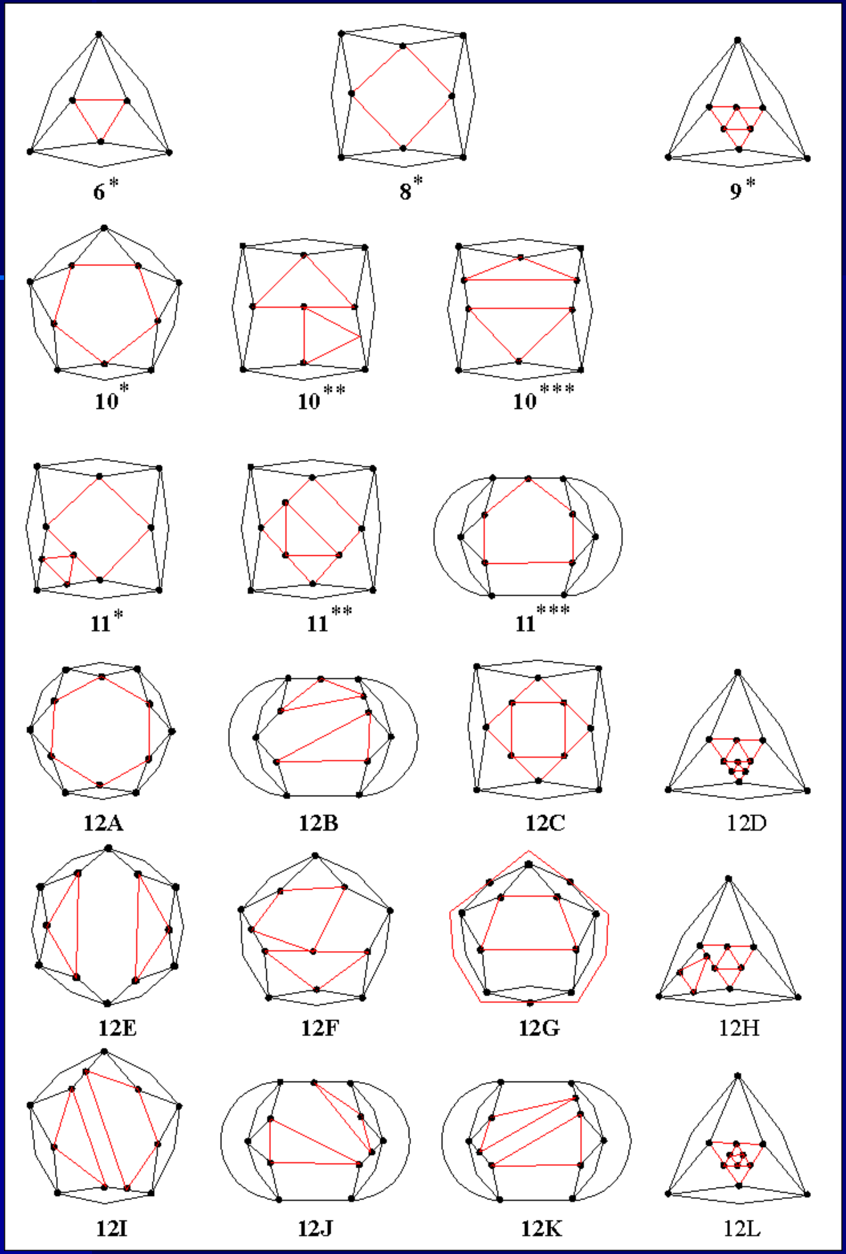
KL recognition

KL derivation

Derivation of basic polyhedra



Kirkman’s method: inscription of triangles

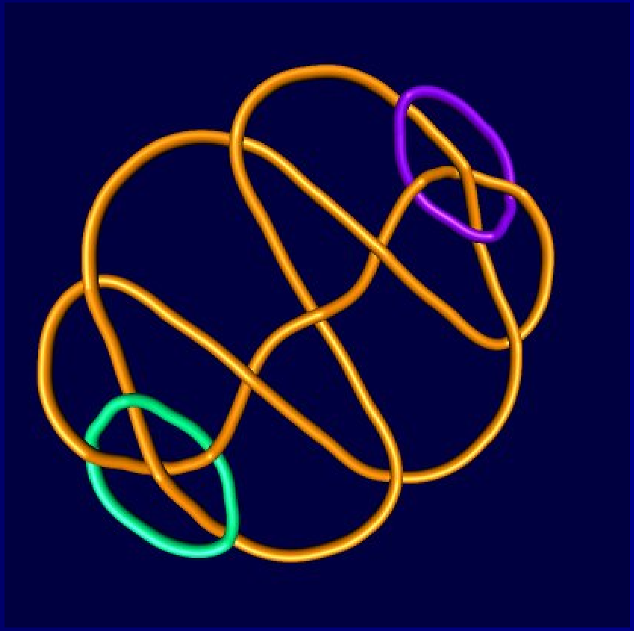


Generalized Kirkman's method: Inscription of polygons in *KL*s *n* (n=3,4,5,…) or their direct products (e.g., 3#3)

Open question: is this method exhaustive for basic polyhedra with more then 12 crossings?

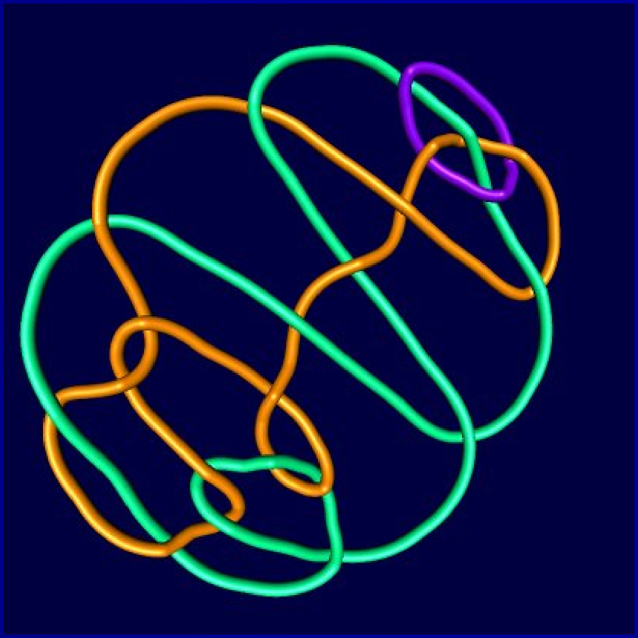
Graph-theory approach: enumeration of all different (non-isomorphic) 4-regular graphs (Brendan McKay) and elimination of non-prime *KL*s : *LinKnot*

n	
13	**19**
14	**64**
15	**155**
16	**510**
17	**1 514**
18	**5 146**
19	**16 966**
20	**58 782**

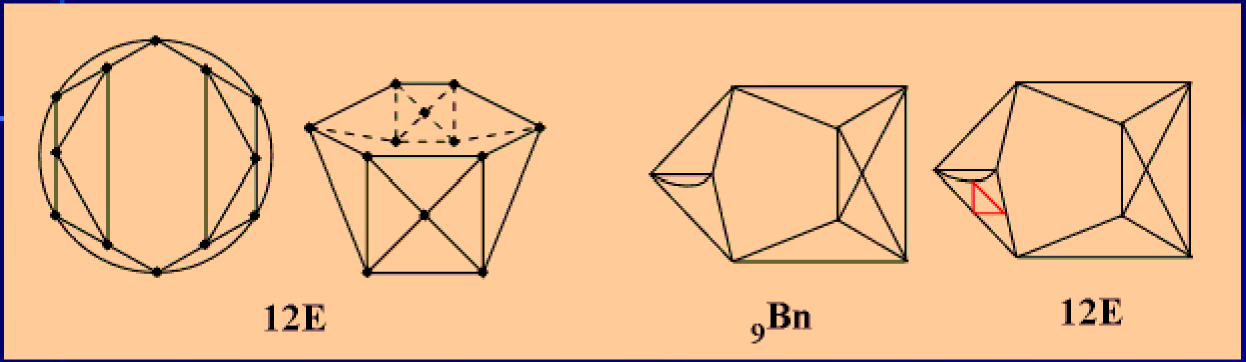


1720*

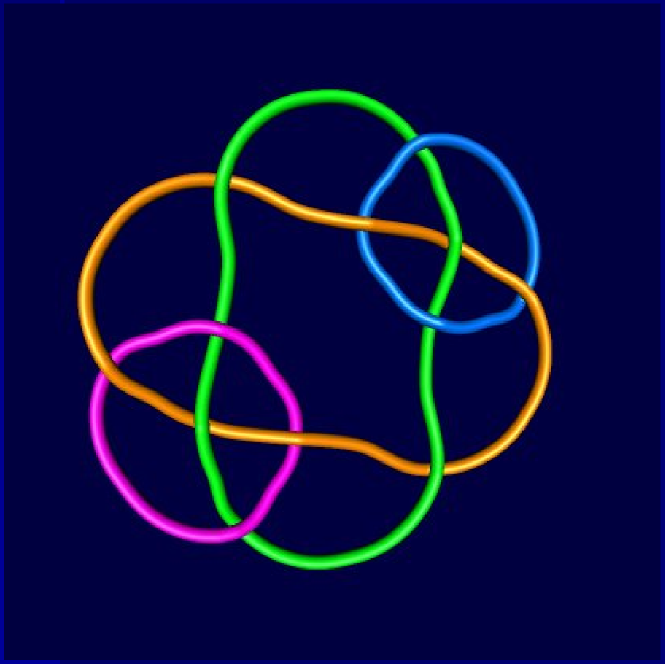
1720*2.2.2 0.2 1



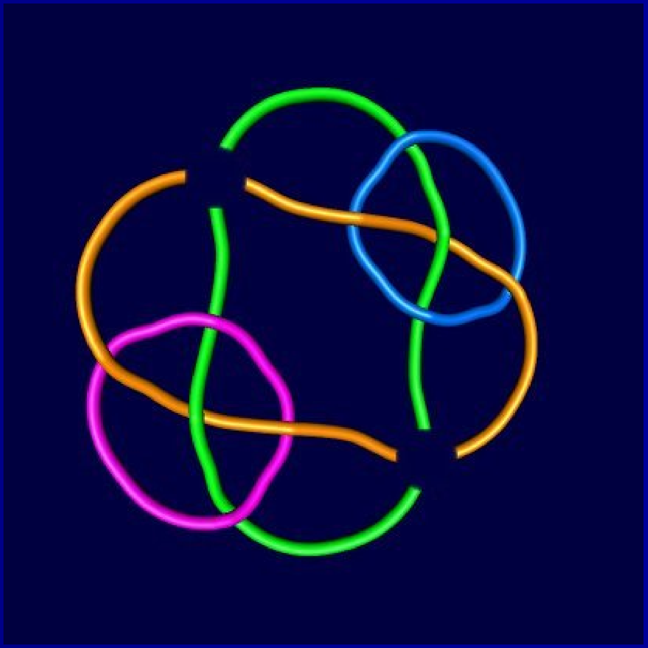
Missing basic polyhedron 12E (Caudron)



Derivation of 12E by Kirkman's method

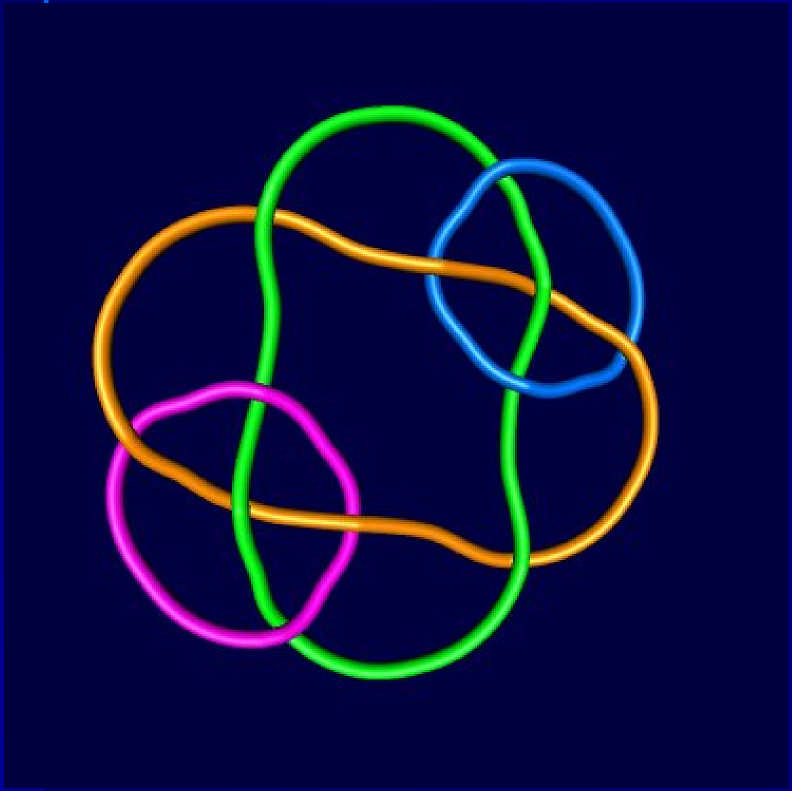


12E is the first 2-vertex connected basic polyhedron

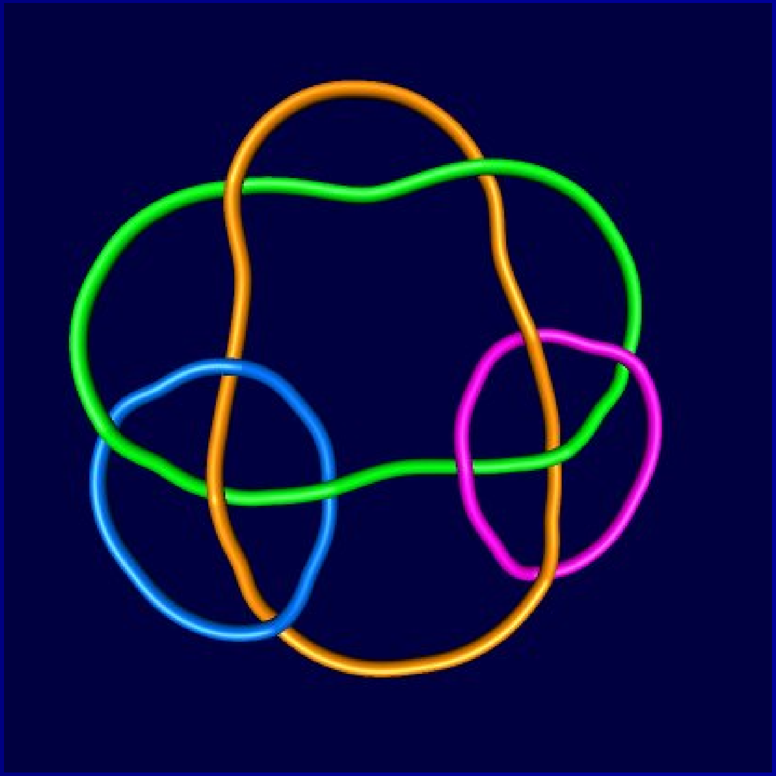


Are basic polyhedra ("solid knots") so solid?

All basic polyhedra with at most 11 crossings are "rigid bodies", i.e. *KLs* not changeable by flypes. First alternating basic polyhedron that has more then one projection is 12E. It's other projection, 11***2 is not a basic polyhedron.

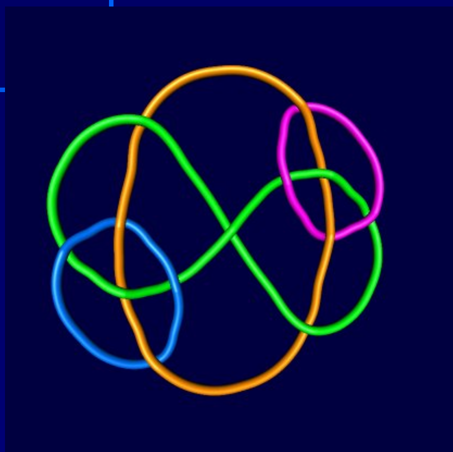


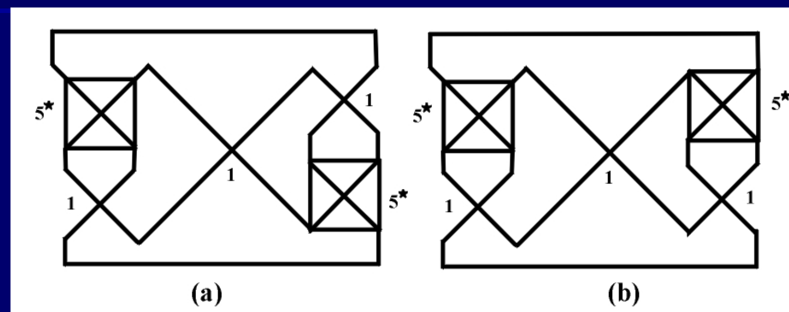
12E

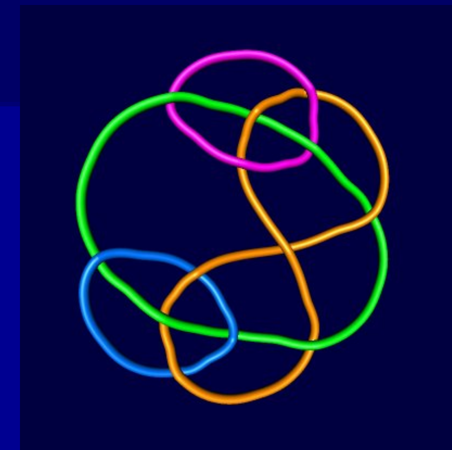


11***2

Basic polyhedra 136* and 1318* that are two different projections of the same link.





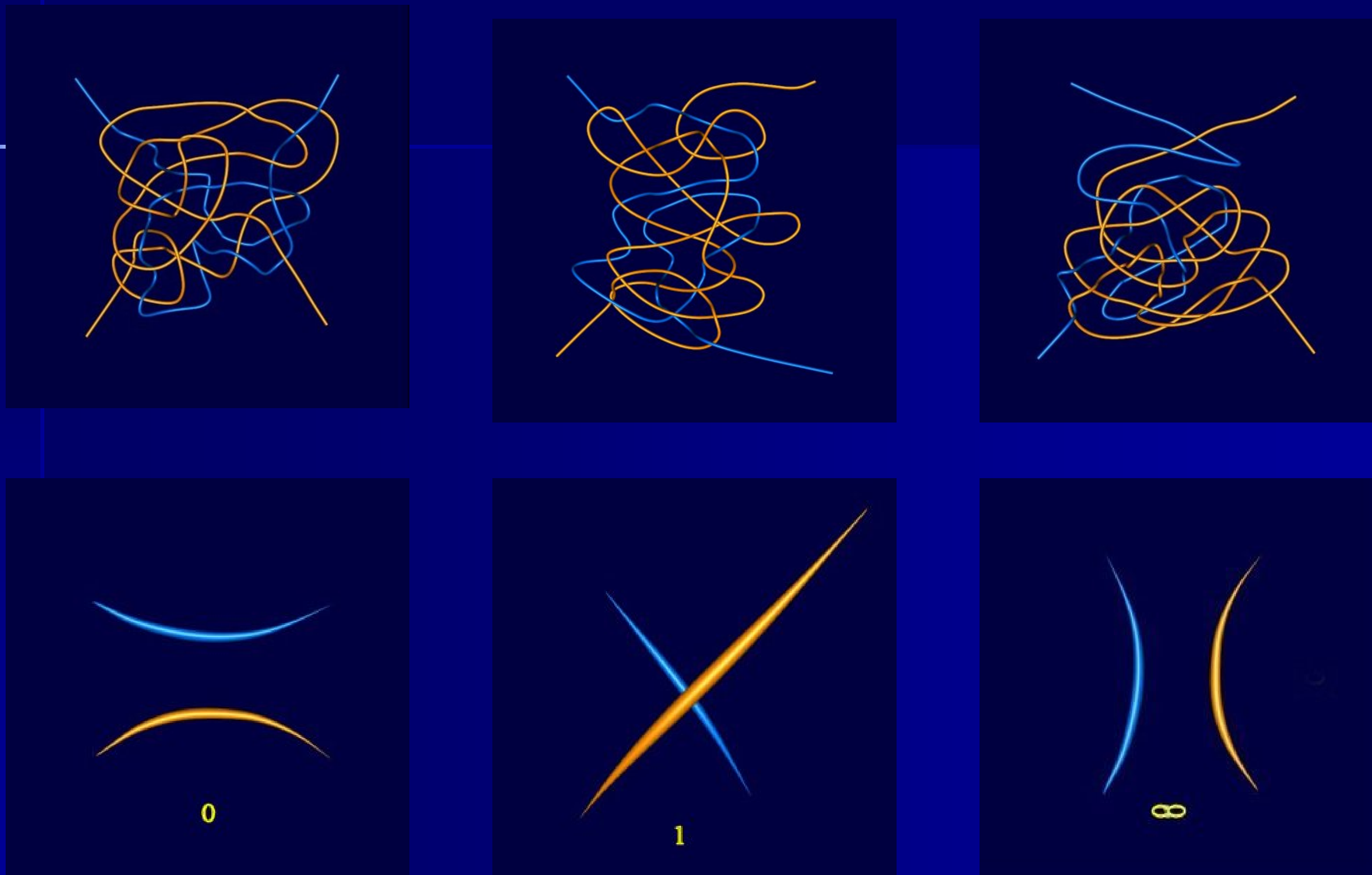


136*

1318*

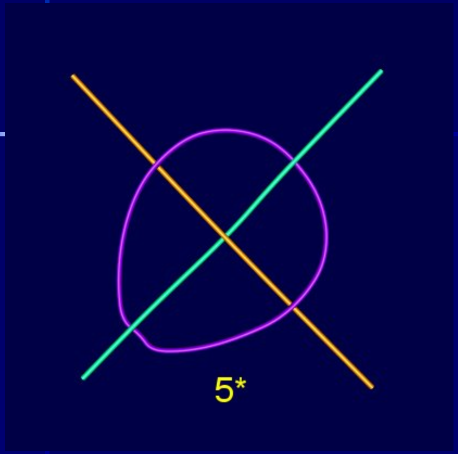
Consequence: the same *KL*s can be derived from different basic polyhedra substituting vertices by algebraic tangles (e.g., from 11*** and 12E, or from 136* and 1318*, but there are *KL*s that can be derived from one of such basic polyhedra, and not from the other!!!

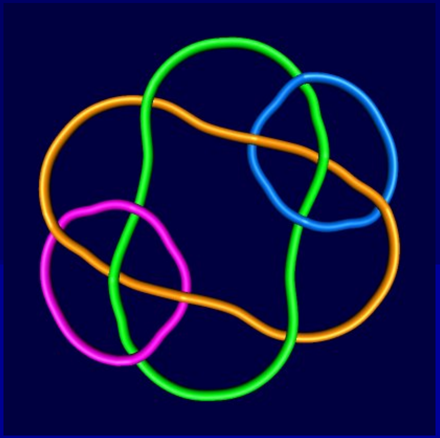
ALGEBRAIC TANGLES AND THEIR TYPES



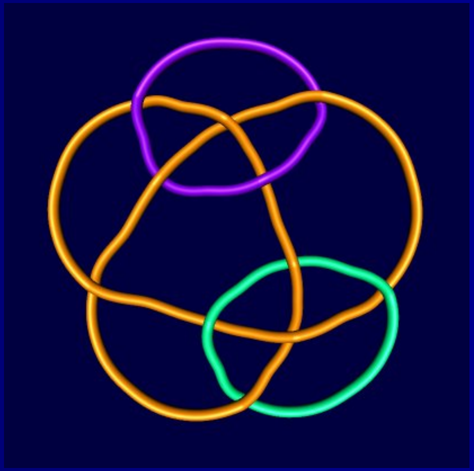
Types of elementary tangles: computation of the number of components

Non-algebraic 2-tangles

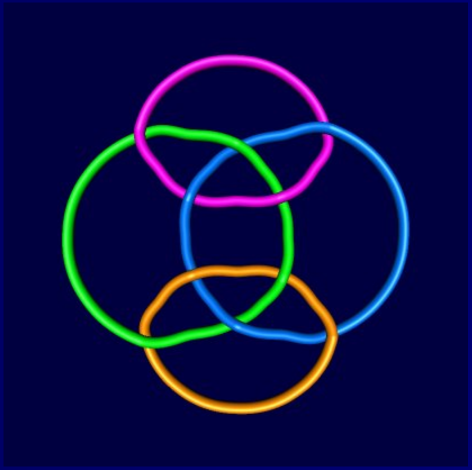




12E = 5* 2 5* =
11***2 !!!



11*** = 5* 1 5*



10*** = 5* 5*

Non-algebraic 2-tangles with at most 9 crossings

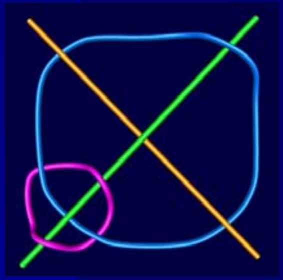
5_1*

7_1*

8_1*

8_2*

9_1*

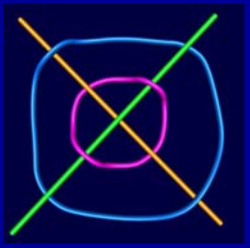


9_2*

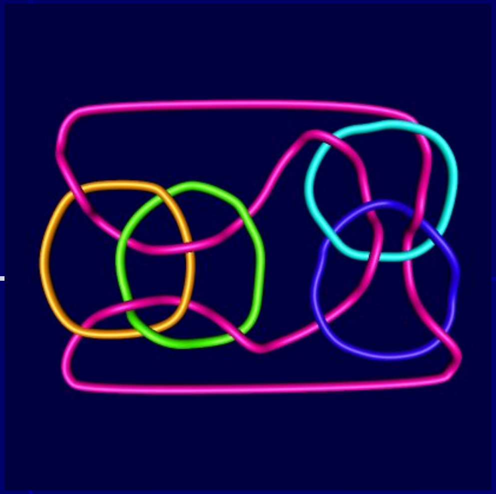
9_3*

9_4*

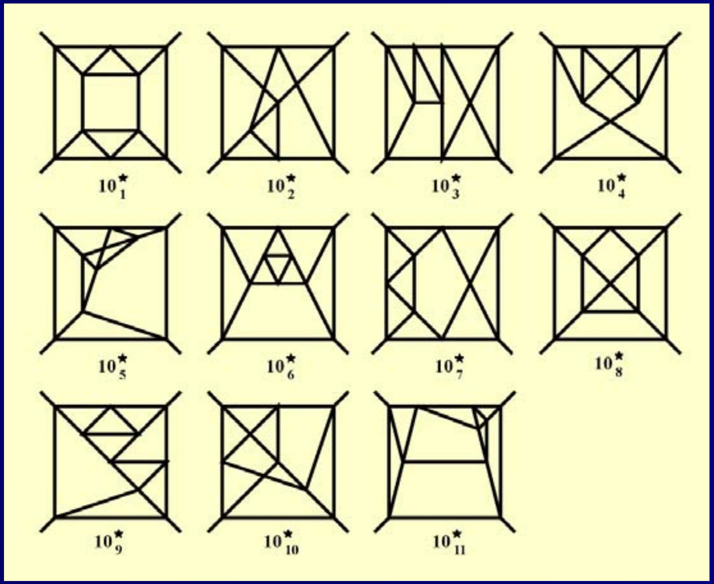
9_5*

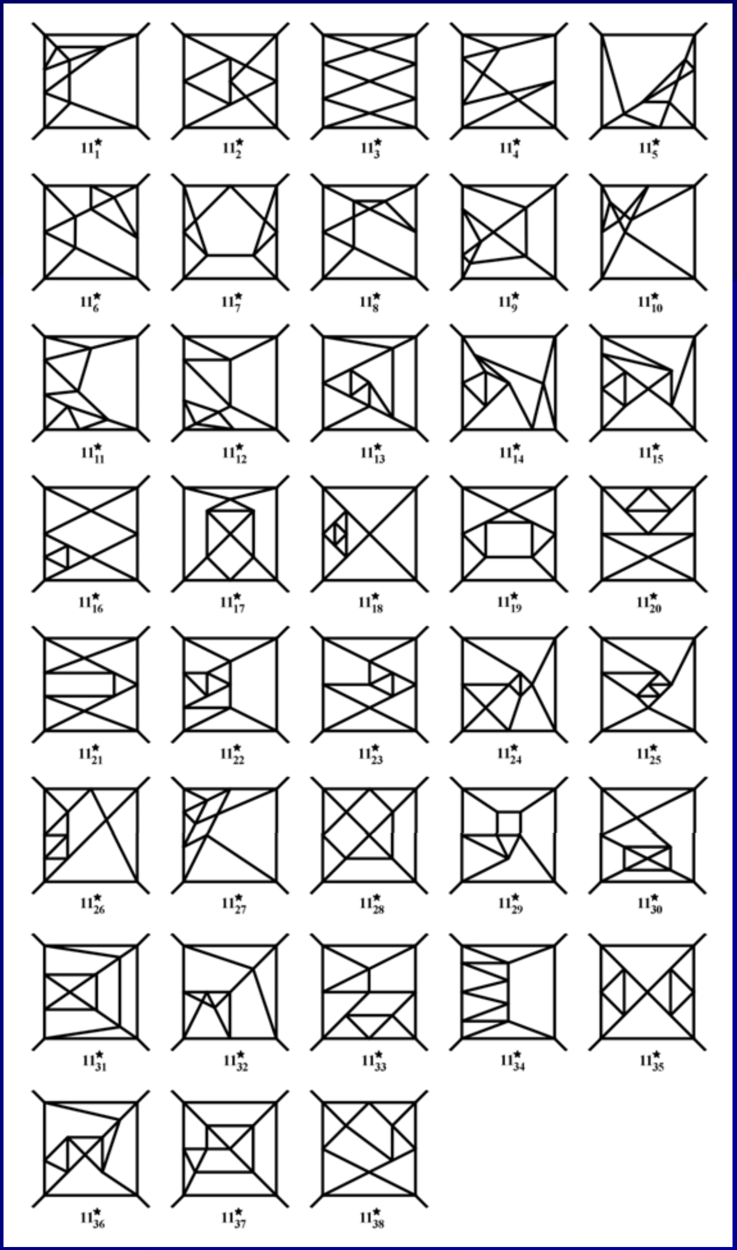


9_6*



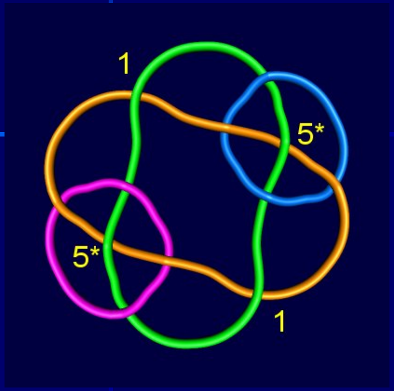
$10_1^* \; 10_1^*$





Non-algebraic tangles with *n* =10 and *n* =11 crossings

Composite basic polyhedra with *n*=12 crossings: 12E=5* 2 5*==11***2, 12I=7* 5*, and 12J=5* 1 1 5* (analogy: 12E with rational link 2 2 2, 12J with 2 1 1 2



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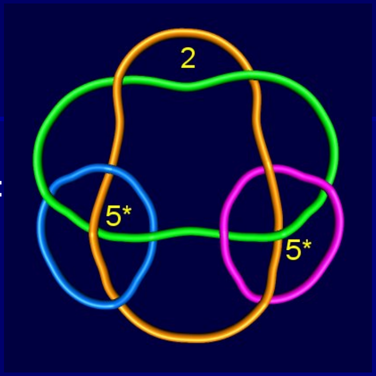
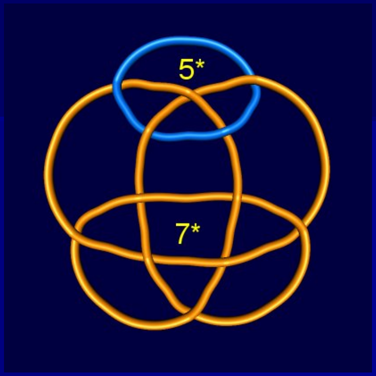
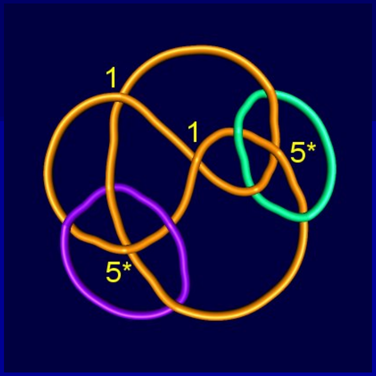


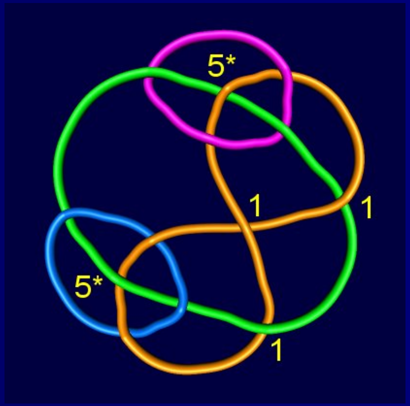




12E **11***2** **12I** **12J**

n=13: 131*=8_1* 5*, 135*=8_2*5*,1318*=5* 1 1 1 5*=136*, 139*=7* 1 5*, 1313*=7* 1 5*=1311*, 1319* =1210*2

1318*



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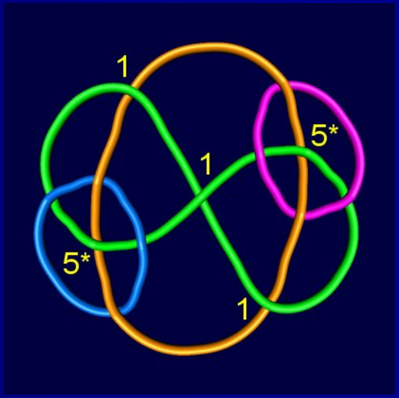


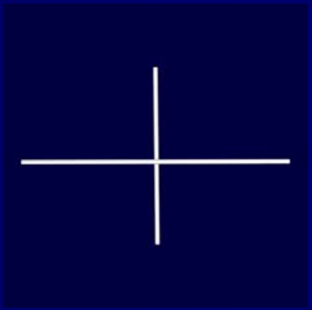
136*

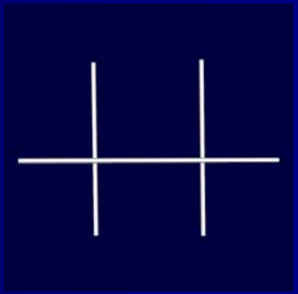
Are we satisfied ?

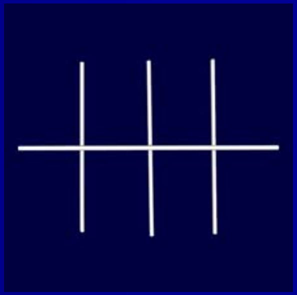
NO

Reason: we have again two data bases, the data base of basic polyhedra (very large) and the data base of non-algebraic tangles (much shorter), but we don't have generic description of basic polyhedra and the algorithm for their derivation

Elementary *n*-tangles (*n*=2,3,4,…)







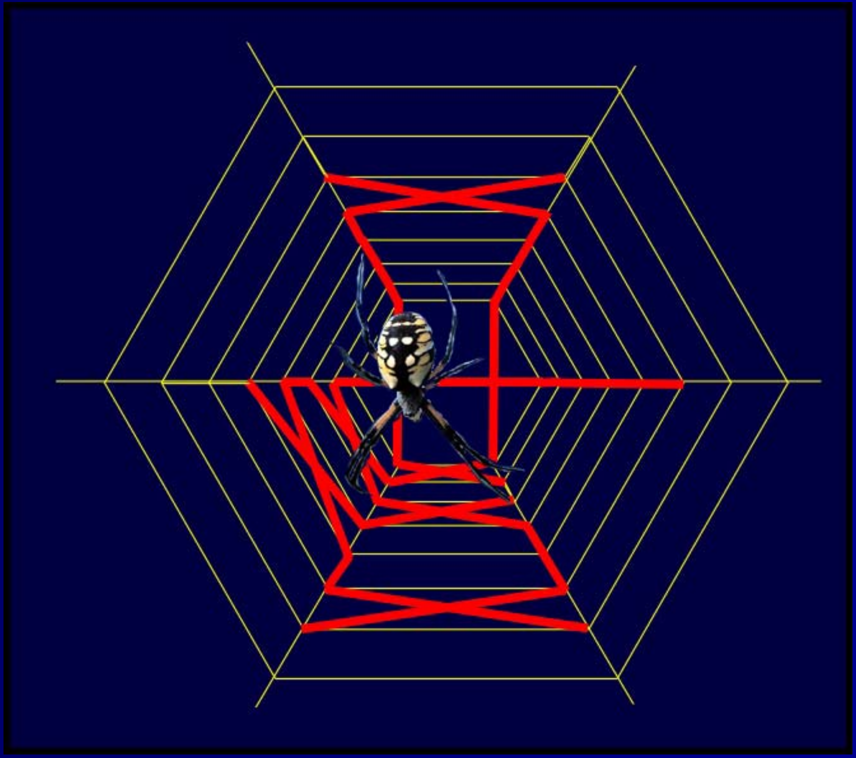
|1|

|2|

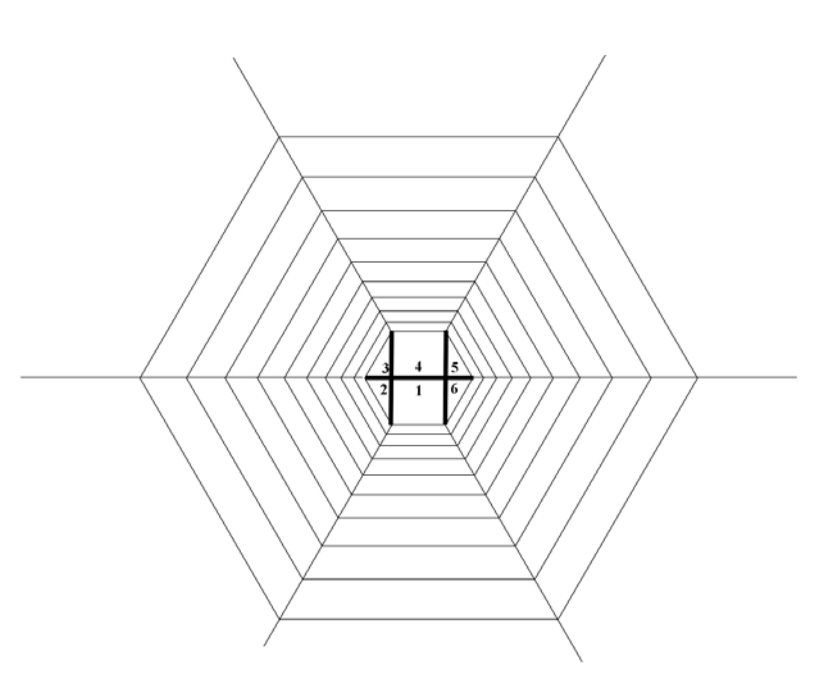
|3|

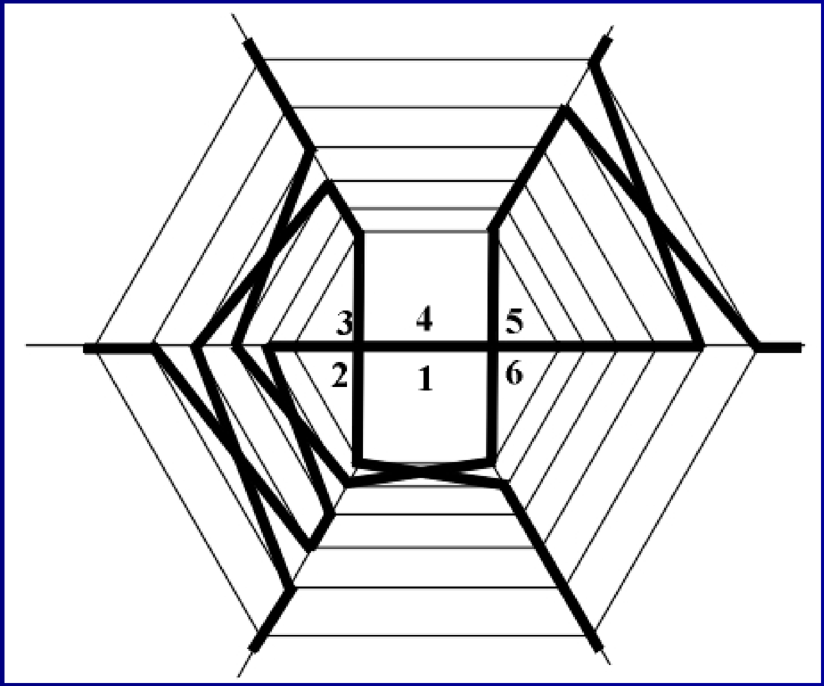
“Crazy Spider Algorithm”

Definition: Crazy spider is a spider weaving only basic polyhedra.



Crazy Spider Algorithm





Cooordinate sytem of the non-algebraic tangle |2|

Non-algebraic tangle |2| 1 2 3 2 5

Types of regions:

a) With one vertex - type 1

b) With the two vertices - type 2

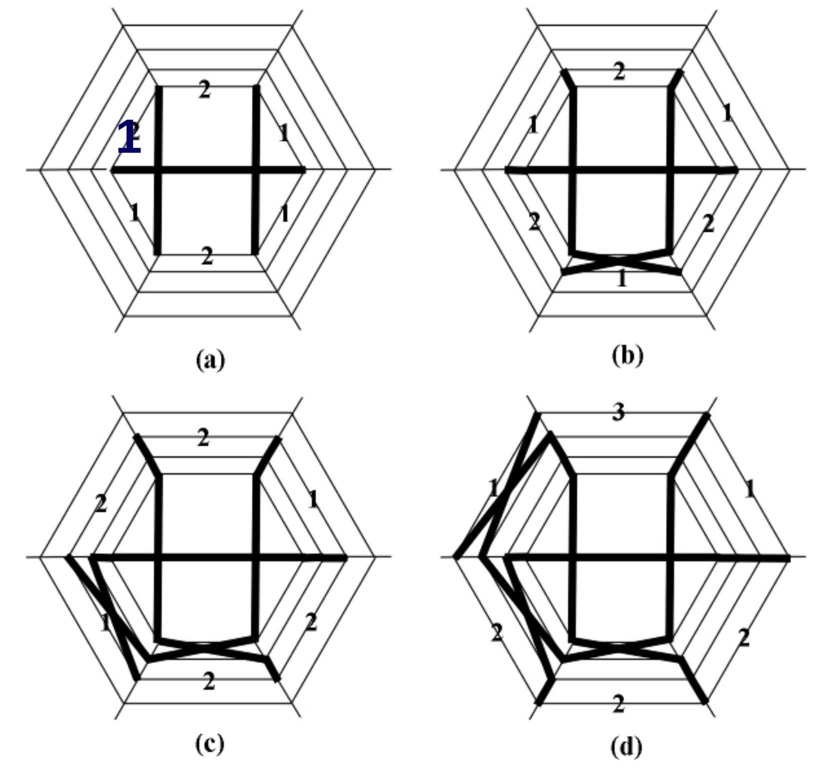
c) With three or more vertices - type 3

Rules for addition of 1-tangles:

Addition of a new 1-tangle in a open region changes the type of the region and the types of adjacent regions.

If the original type is 1, addition of a new 1-tangle is forbidden, because a bigon will be obtained.

If the type of the region is 2 or 3, it will be changed into 1, and to the types of the adjacent regions we add 1.



Closure rule:

Closure of the obtained *n*-tangle is basic polyhedron, if joining of outgoing strands does not result in the appearance of bigons. By joining free ends, one region can be closed, or two regions can be joined in one. This means that the type of the region which we are closing must be greater then 2 and that the sum of the types of regions joined into one must be greater then 2.

FAMILES OF BASIC POLYHEDRA

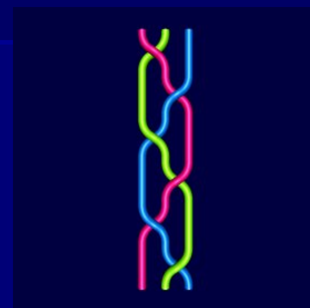
Basic polyhedra with n=12 crossings:

$\vert 2\vert$	1	2	1	2	1	2	1	2	1	2	12A
$\vert 2\vert$	1	2	1	2	1	2	3	2	1	2	12B
$\vert 2\vert$	1	2	1	2	3	2	1	2	1	2	12F
$\vert 2\vert$	1	2	1	2	3	4	3	2	1	2	12K
$\vert 2\vert$	1	2	1	3	2	1	3	2	1	2	12D
$\vert 2\vert$	1	2	1	3	2	4	3	2	1	2	12L
$\vert 2\vert$	1	2	1	6	1	2	3	2	1	2	12H
$\vert 3\vert$	1	2	1	2	3	2	1	2	3	12G	
$\vert 3\vert$	1	2	1	3	2	1	3	2	3	12C	
$\vert 3\vert$	1	2	3	2	1	3	2	1	2	12I	
$\vert 3\vert$	1	5	4	3	4	3	2	8	7	12E	
$\vert 3\vert$	1	6	7	6	5	4	3	2	3	12J	

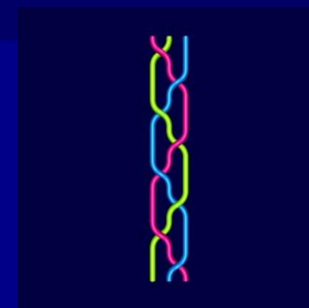
General form:
$|n\text{-}1|s_0^k\, s\, s_1^r$

The other minimality criterion:
minimalna length of *s*

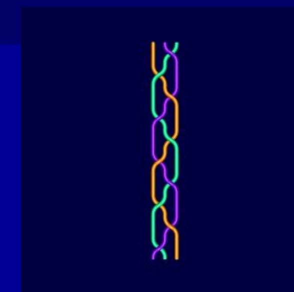
2n-antiprismatic basic polyhedra:
6*, 8*, 10*, 12A, … $|2|\,(1\ 2)^k$



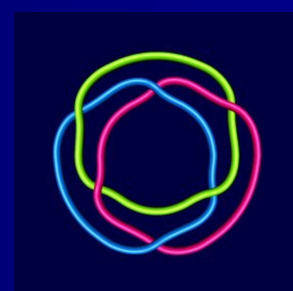
$(Ab)^3$



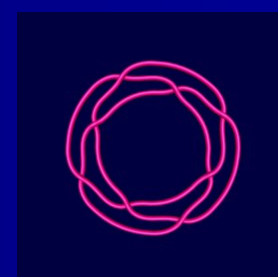
$(Ab)^4$



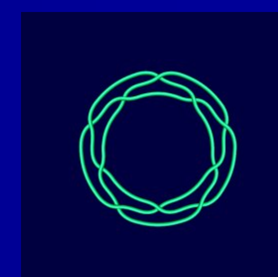
$(Ab)^5$



6*



8*

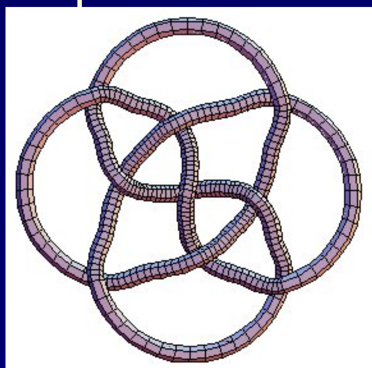


10*

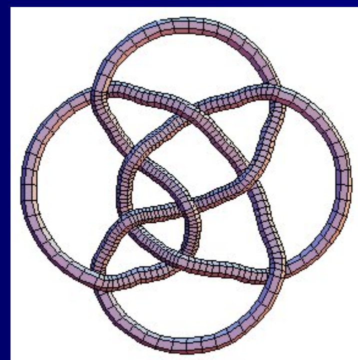
Analogy with *BFR*-s (*B*raid *F*amily *R*epresentatives) of the form $(Ab)^n$

Family of the basic polyhedron 9*: |2| s_0^k 3 2 1 2

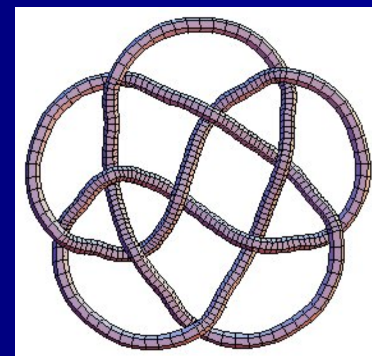
9*	|2| 1 2 1 3 2 1 2	133*	|2| 1 2 1 2 1 2 1 3 2 1 2
10**	|2| 1 2 1 2 3 2 1 2	148*	|2| 1 2 1 2 1 2 1 2 3 2 1 2
11*	|2| 1 2 1 2 1 3 2 1 2	1510*	|2| 1 2 1 2 1 2 1 2 1 3 2 1 2
12B	|2| 1 2 1 2 1 2 3 2 1 2	1625*	|2| 1 2 1 2 1 2 1 2 1 2 3 2 1 2



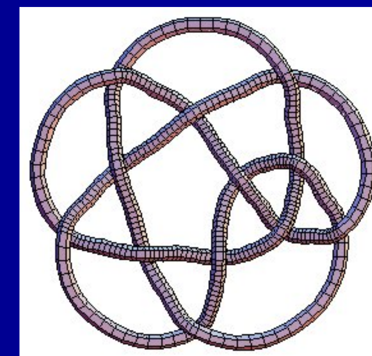
9*



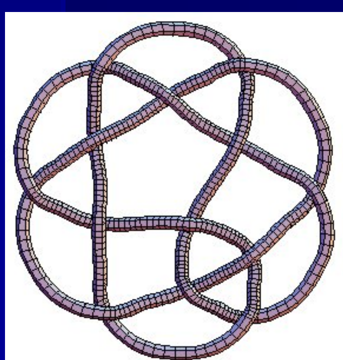
10**



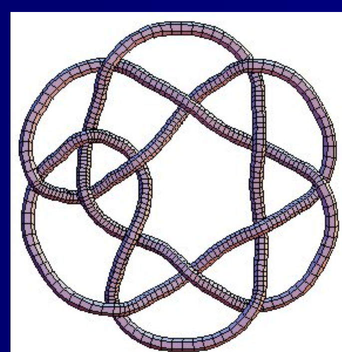
11*



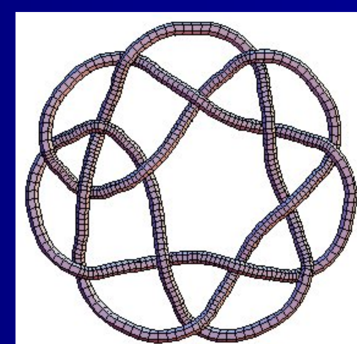
12B



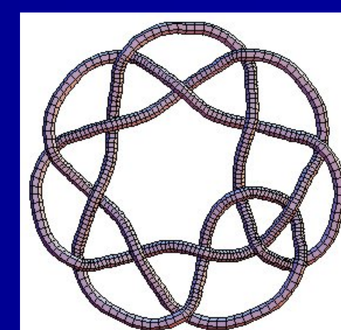
133*



148*



1510*

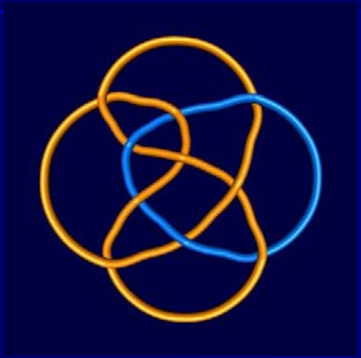


1625*

Number of components?

What will be the next basic polyhedron:
a) knot;
b) 2-component link;
c) something absolutely different

9*



10**

11*

12B

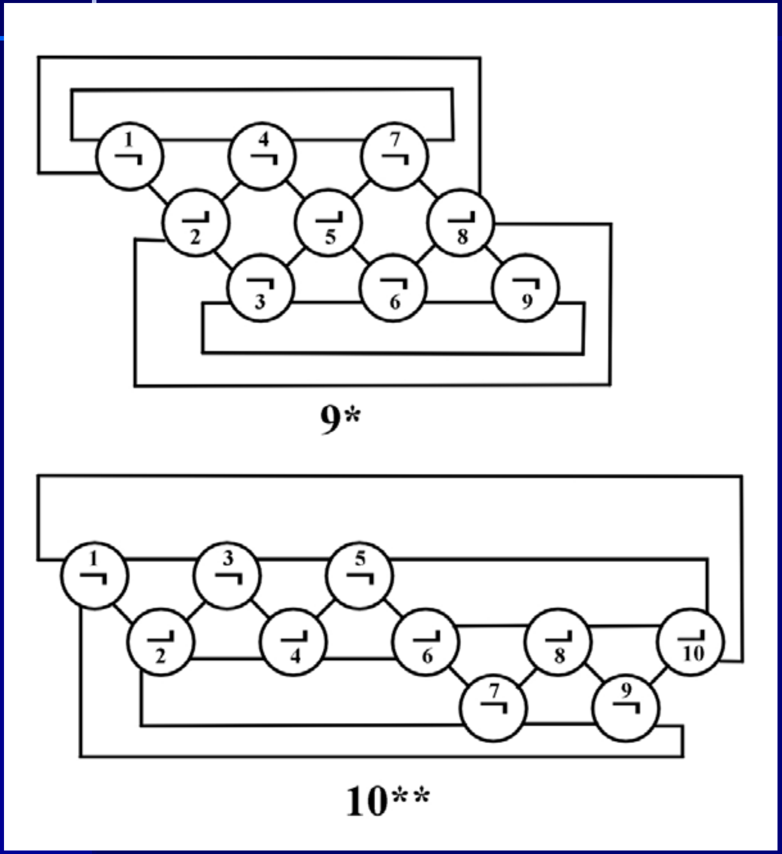
133*

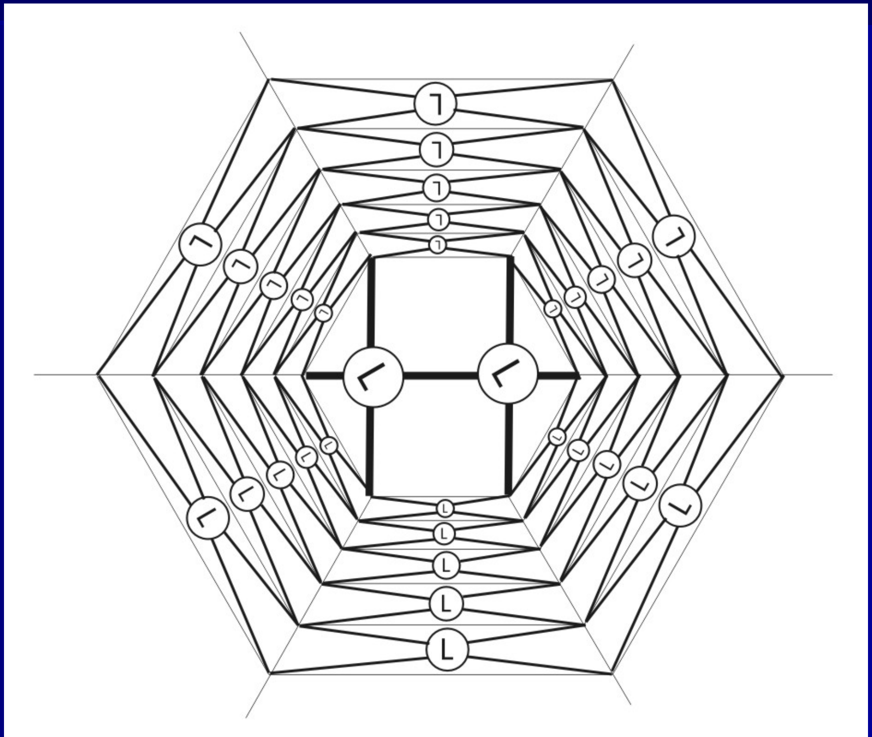
148*

1510*

1625*

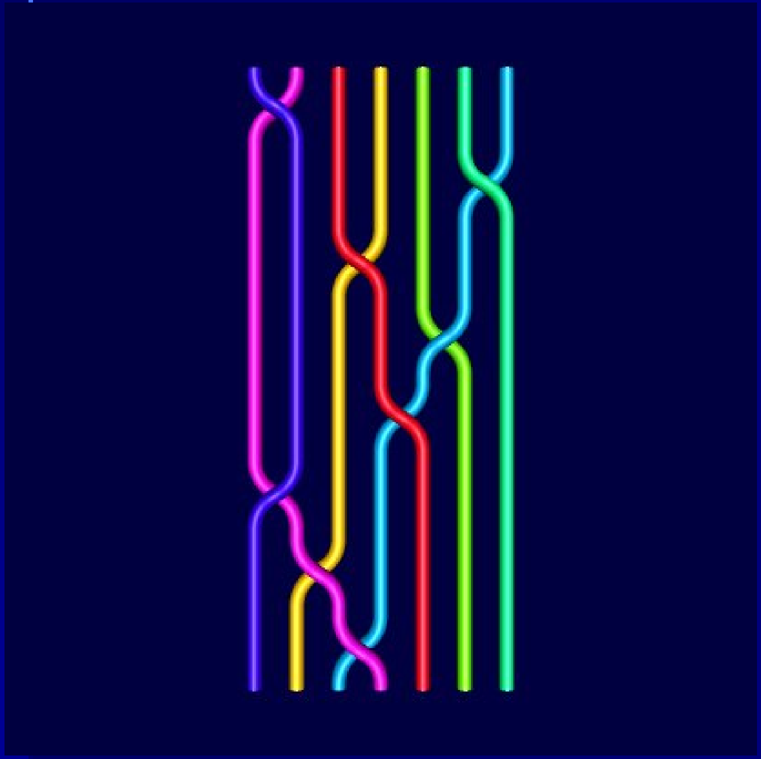
Orientation of algebraic tangles substituting vertices of basic polyhedra

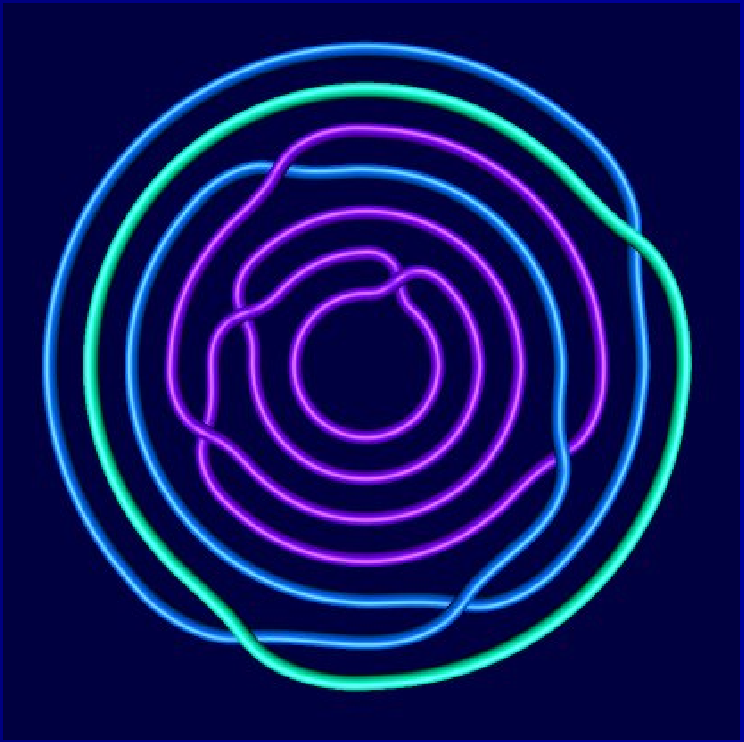


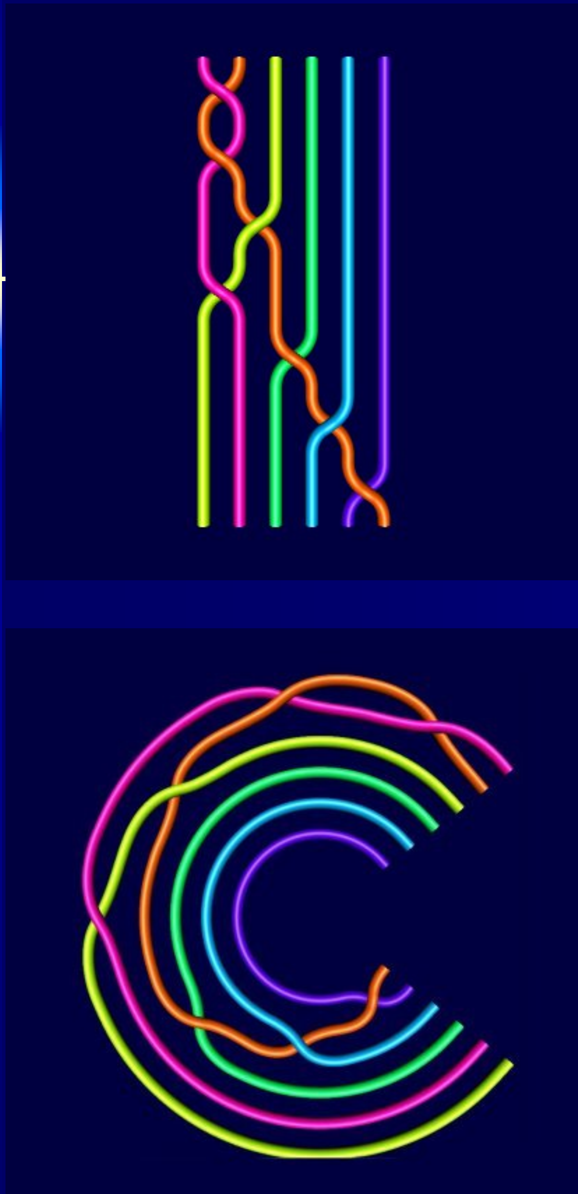


Canonical orientation of algebraic tangles in the coordinate system.

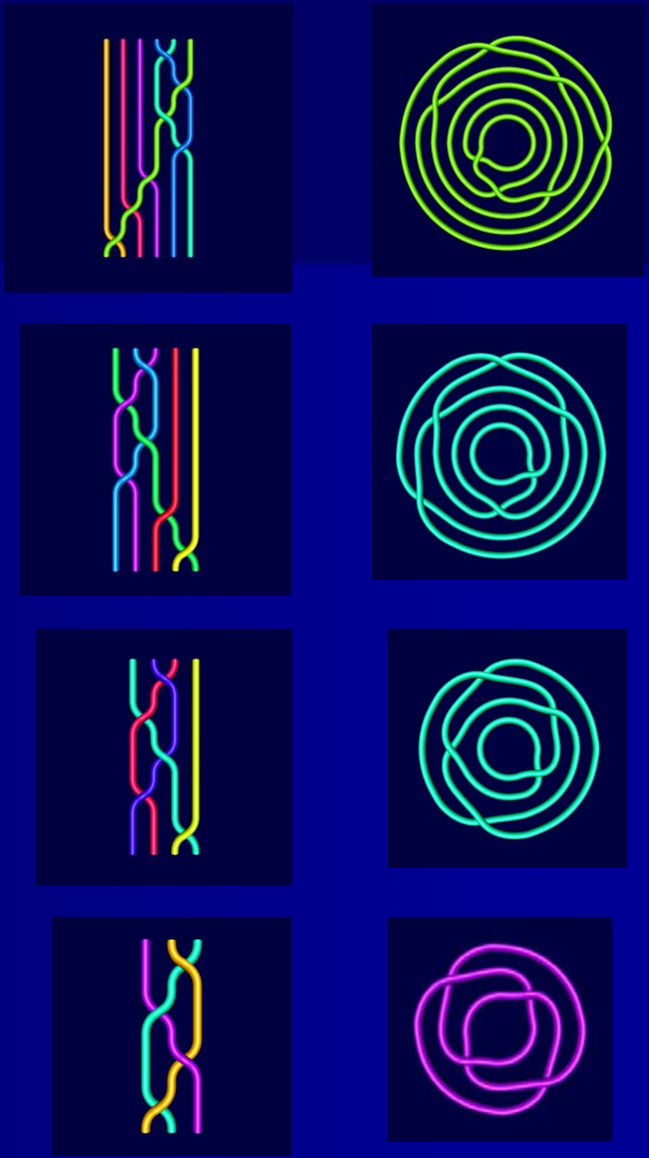
Braid Family Representatives





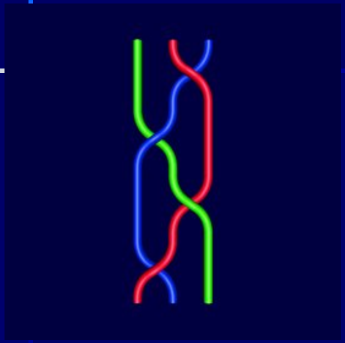
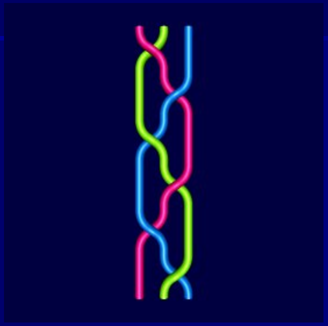
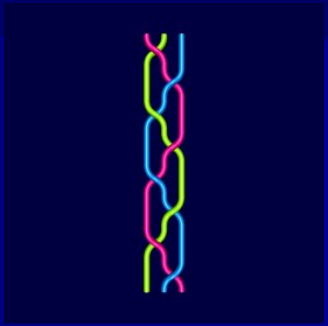
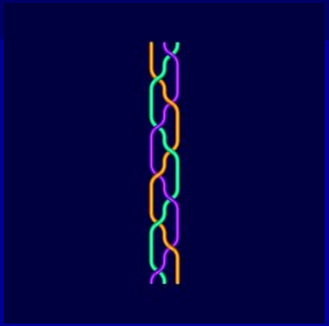


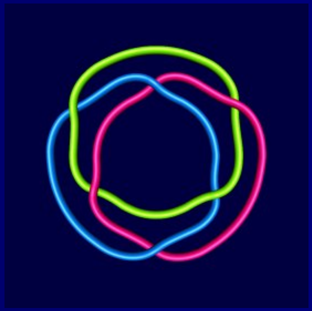
Minimum braids: T. Gittings

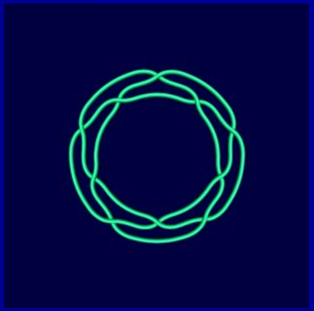


Different braids giving the same knot

- AbAbAbAbAb...

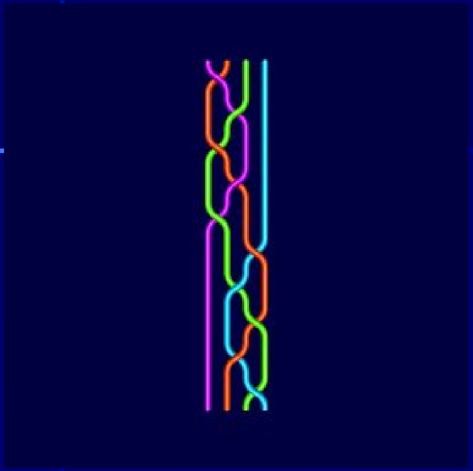




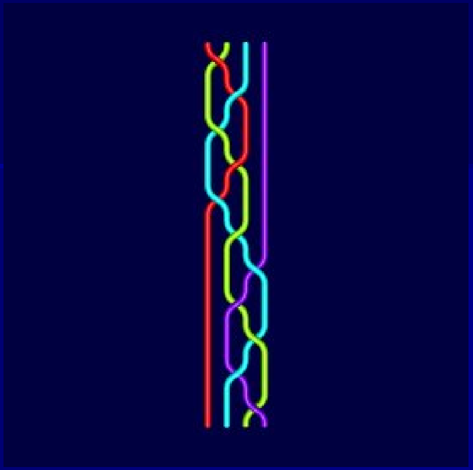
$(Ab)^2$=2 2 $(Ab)^3$=6* $(Ab)^4$=8* $(Ab)^5$=10*

***BFR*s of the family of antiprismatic basic polyhedra**

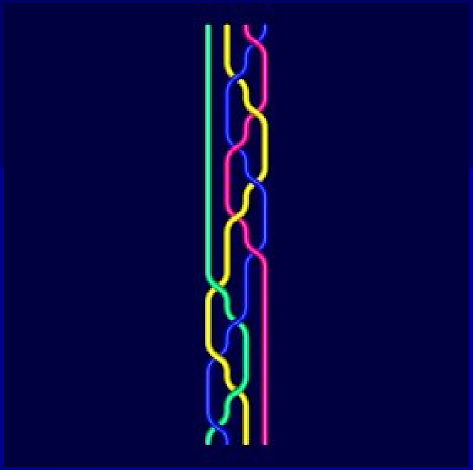
- AbAbAbAbAb…CbCbC



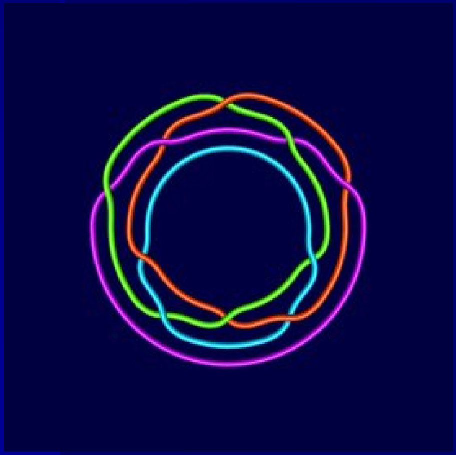
AbAbACbCbC



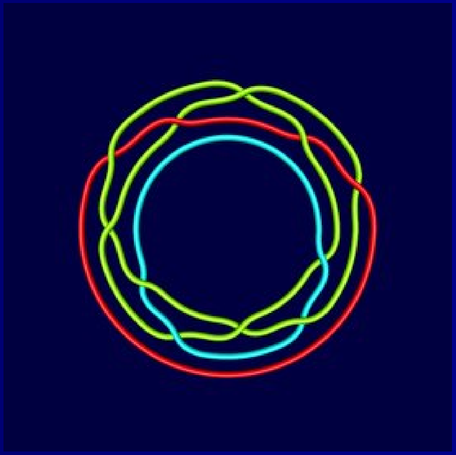
AbAbAbCbCbC



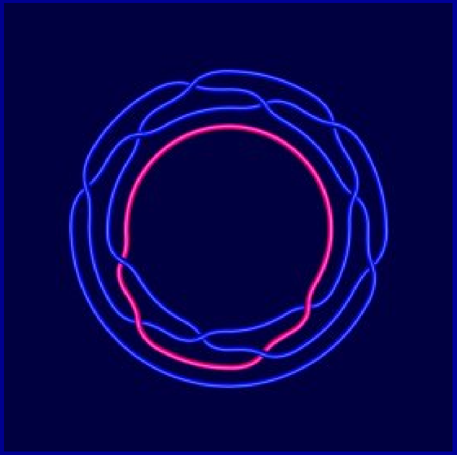
AbAbAbACbCbC



10***

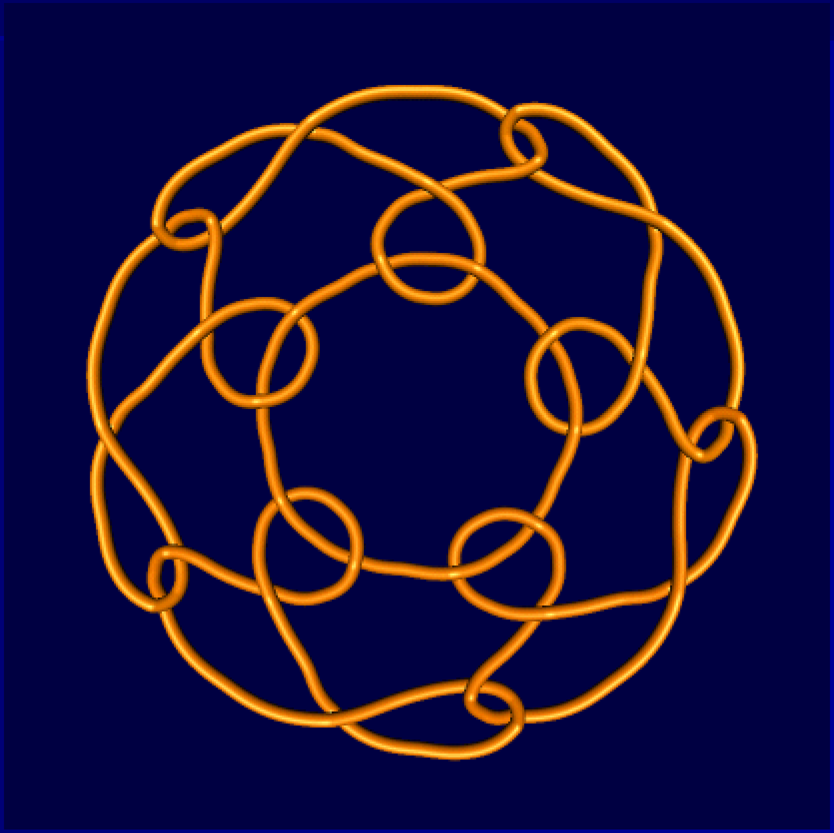


11***

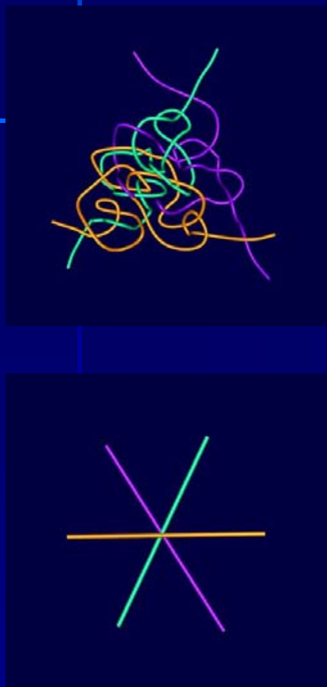


12l

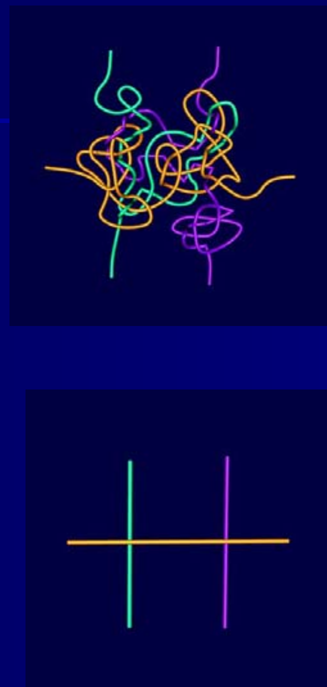
Number of components
of polyhedral *KL*s



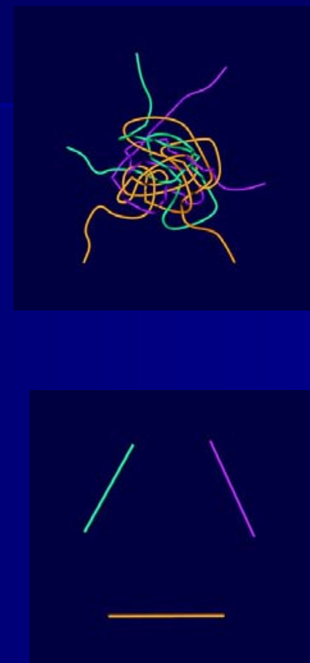
Chord diagrams



3.1

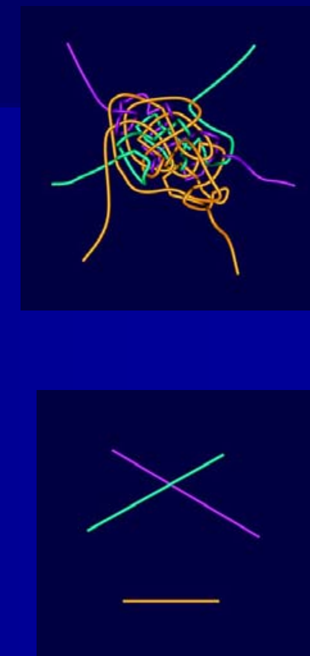


3.2



3.3

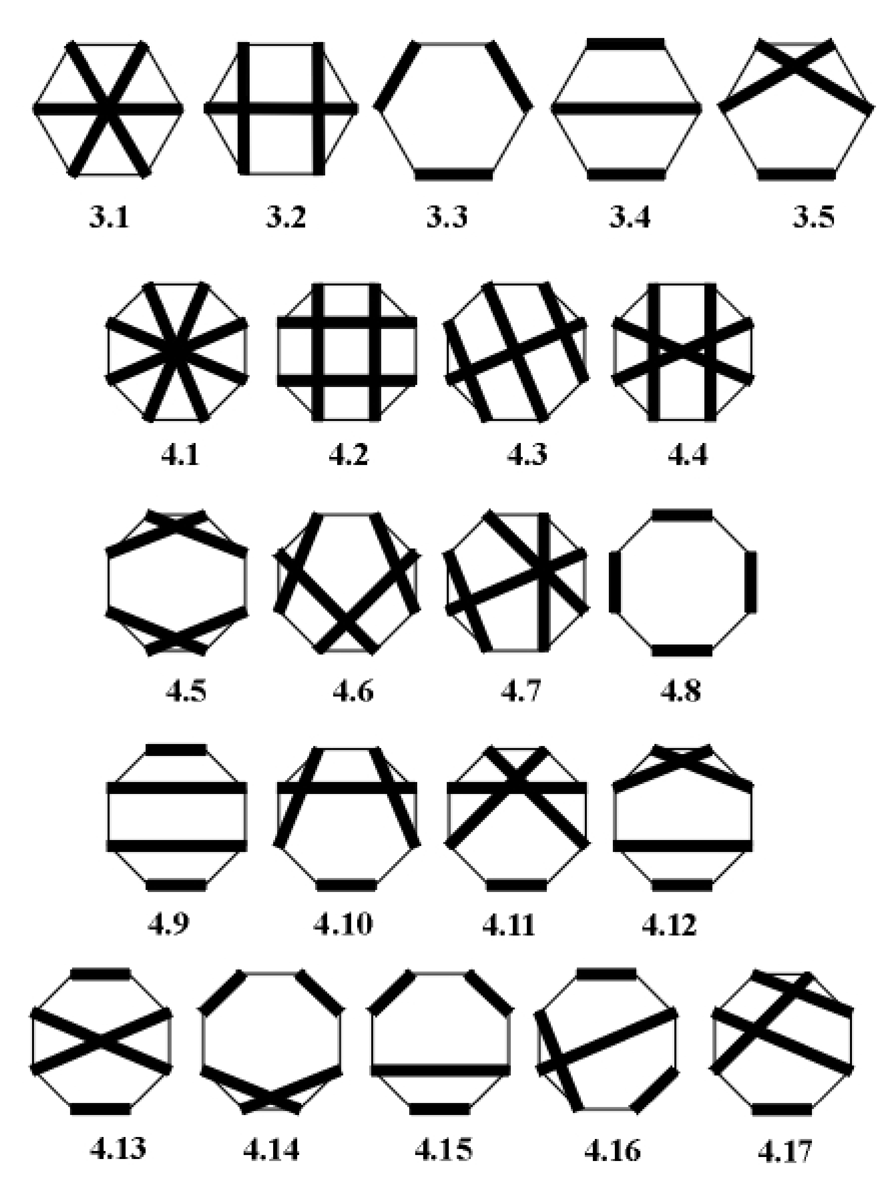
3.4



3.5

n	3	4	5	6	7	8	9	10	11
	5	17	79	554	5283	65346	966156	16411700	3127002217

(A. Khruzin - combinatorial result)



Basic *n*-diagrams =
chord diagrams without chords connecting adjacent vertices (e.g., 3.1-3.2; 4.1-4.7)

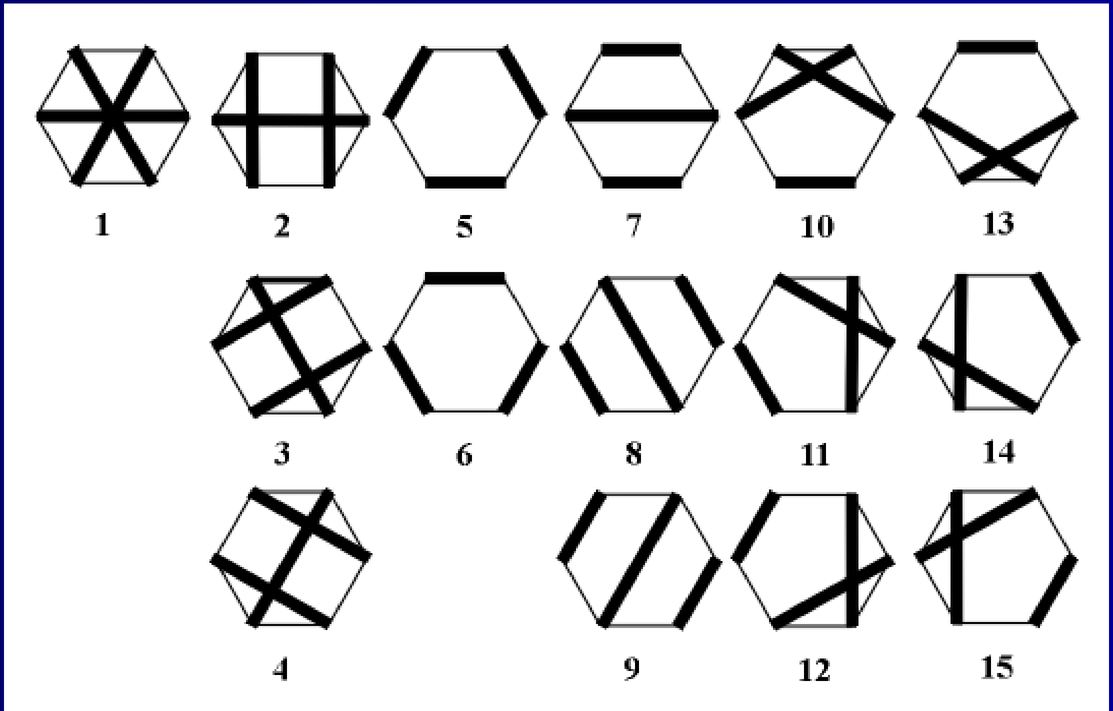
n	
2	**1**
3	**2**
4	**7**
5	**36**
6	**300**
7	**3218**
8	**42335**
9	**644808**

Dror Barnatan – effective derivation

Complete set of *n*-diagrams =
all positions of *n*-diagrams

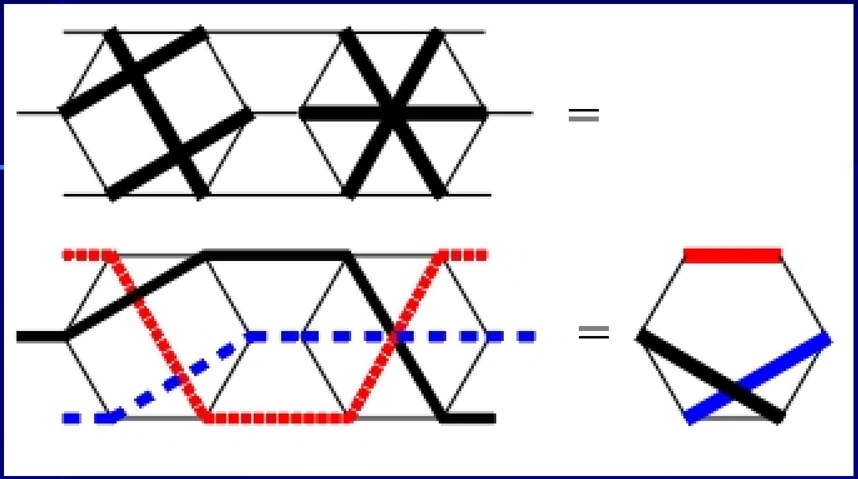
n	1	2	3	4	5	6	…
	1	3	15	105	945	10395	…

$(2n-1)!!=1\times3\times\ldots(2n-1)$

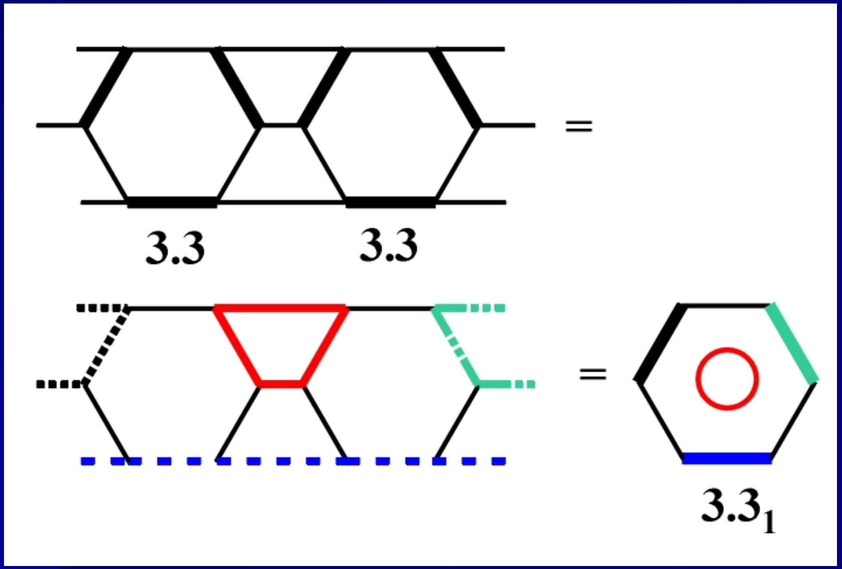


Complete set of 3-diagrams

Composition of *n*-tangles and composition of chord diagrams



Composition of two 3-diagrams

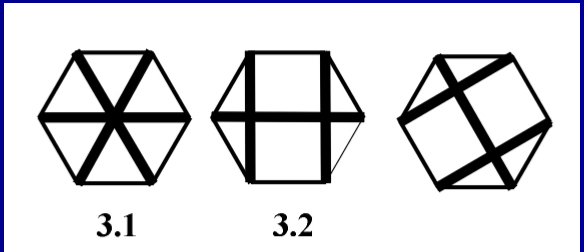


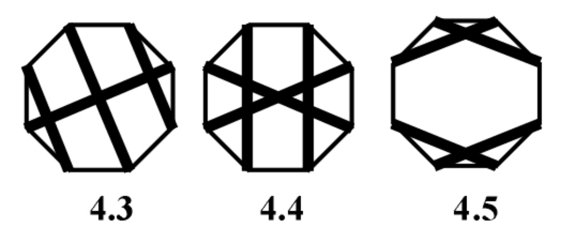
Occurrence of *closed internal component*

The set of *n*-diagrams is *closed* with regard to *n*-tangle compositions modulo internal closed components.

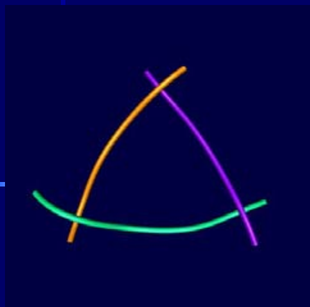
The complete set of *n*-diagrams with the operation of *n*-tangle composition is the *non-commutative monoid* - a *non-commutative semigroup* with *the neutral element*. The *neutral element* is the *n*-diagram with horizontal parallel chords (e.g., 3.4, 4.9). This set has (2*n*-1)!! elements, where (2*n*-1)!! is the odd factorial number (2*n*-1)!!=1×3×…(2*n*-1).

For *n*=3 the *minimal set of generators* of the complete set of 3-diagrams consists from three basic diagrams without connected adjacent vertices (e.g., the diagram 3.1, and two positions of the diagram 3.2 from Fig. 15). For *n*>3 *the set of generators* of the complete set of *n*-diagrams consists from basic *n*-diagrams without connected adjacent vertices (e.g., from the diagrams 4.1-4.7 for *n*=4, *etc*.)
From this set we can choose *six minimal sets of generators*, each consisting from three different diagrams: {4.3,4.4,4.5}, {4.3,4.4,4.6}, {4.3,4.5,4.7}, {4.3,4.6,4.7}, {4.4,4.5,4.7}, {4.4,4.6,4.7}, or make different sets consisting from two positions of the diagram 4.3 and one from the diagrams 4.4, 4.7.

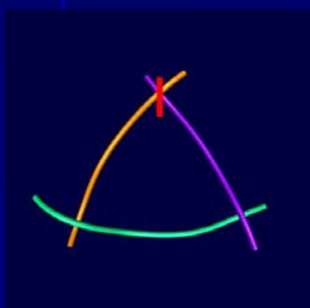


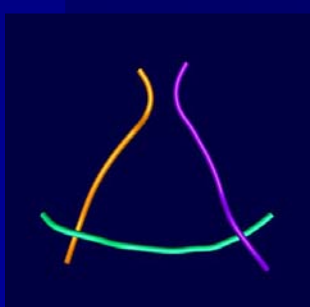


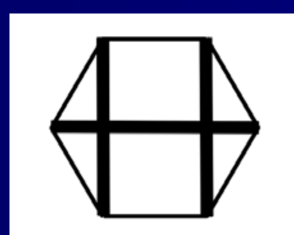
Number of components of polyhedral *KL*s



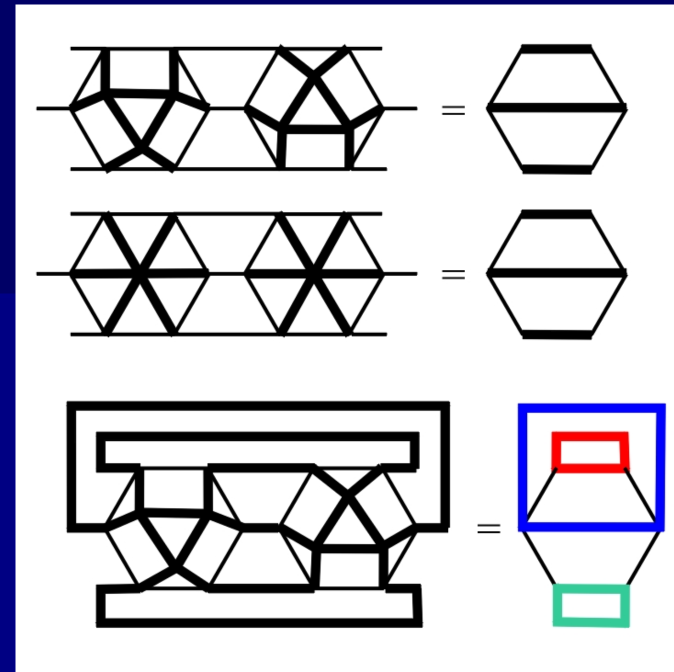
3.1







3.2



Borromean rings (basic polyhedron 6*) is the 3-component link.

Consequence: every *KL* derived from 6* with all algebraic tangles of the type [1] substituting its vertices is 3-component link.

Consequence: number of components is the property of the families of basic polyhedra and *KL*s derived from them, and not only the property of particular *KL*s.

Signature and unknotting (unlinking) numbers of families of basic polyhedra

	Basic polyhedron	Code	Comp. no.	Signature	Unkn (unl) no.
k=3	9*	|2| 1 2 1 3 2 1 2	1	2	2
k=4	10**	|2| 1 2 1 2 3 2 1 2	2	1	2
k=5	11*	|2| 1 2 1 2 1 3 2 1 2	1	0	1
k=6	12B	|2| 1 2 1 2 1 2 3 2 1 2	1	0	1
k=7	133*	|2| 1 2 1 2 1 2 1 3 2 1 2	2	1	3
k=8	148*	|2| 1 2 1 2 1 2 1 2 3 2 1 2	1	0	2
k=9	1510*	|2| 1 2 1 2 1 2 1 2 1 3 2 1 2	1	2	3
k=10	1625*	|2| 1 2 1 2 1 2 1 2 1 2 3 2 1 2…	2	1	3
k=11	17455*		1	0	2
k=12	182675*		1	0	2
k=13	195031*		2	1	3
k=14	2031002*		1	0	3
k=15			1	2	3
k=16			2	1	4
k=17			1	0	3
k=18			1	0	3
k=19			2	1	4
k=20			1	0	4
k=21			1	2	4