



**The Abdus Salam
International Centre for Theoretical Physics**



2037-16

Introduction to Optofluidics

1 - 5 June 2009

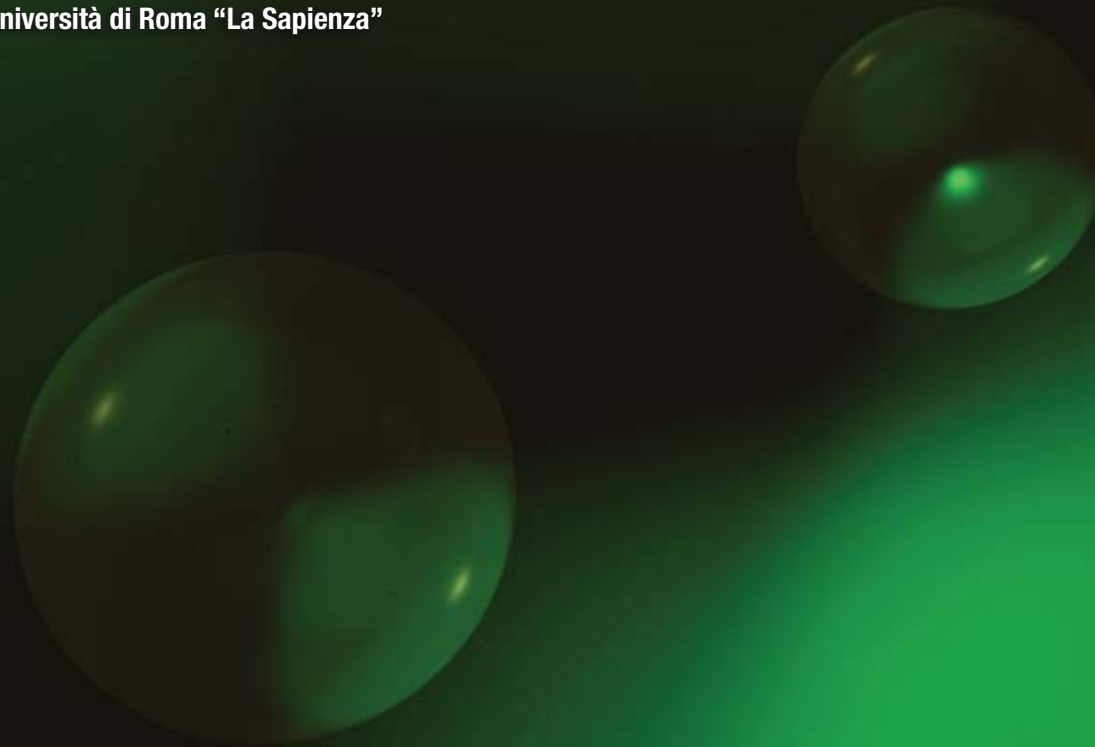
Statistical microhydrodynamics: fluid phenomena at the micron scale

R. Di Leonardo
*University "La Sapienza"
Roma
Italy*

Statistical microhydrodynamics: fluid phenomena at the micron scale

ROBERTO DI LEONARDO

CNR-INFM Dip. Fisica, Università di Roma "La Sapienza"

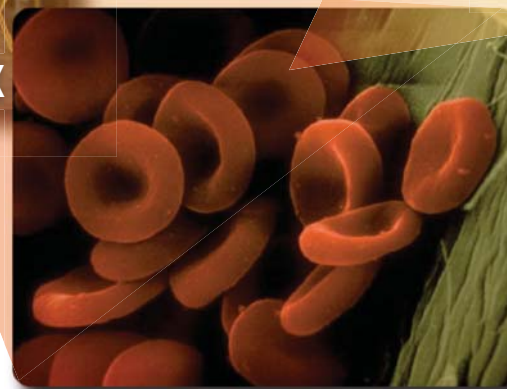


Mesoscopic world



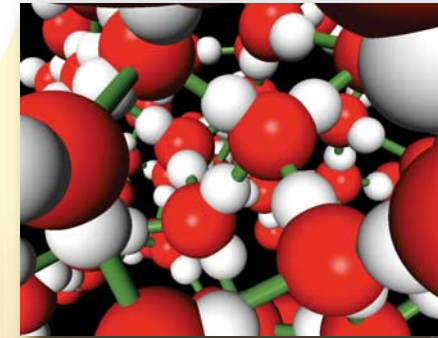
MACROSCOPIC

$10^6 \times$



MESOSCOPIC

- inertialess dynamics
- surface forces
- noisy environment
- light pushes



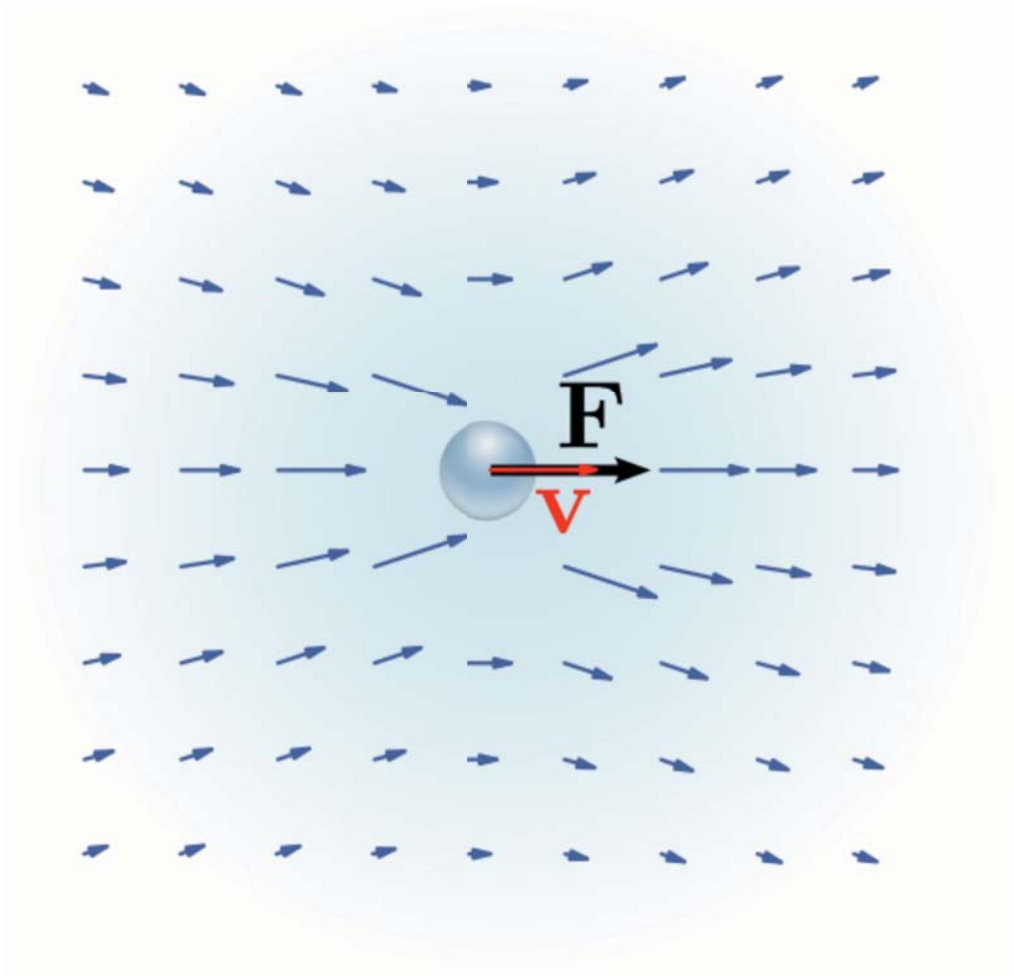
MICROSCOPIC

$10^4 \times$

INERTIALESS DYNAMICS

Hydrodynamic interactions

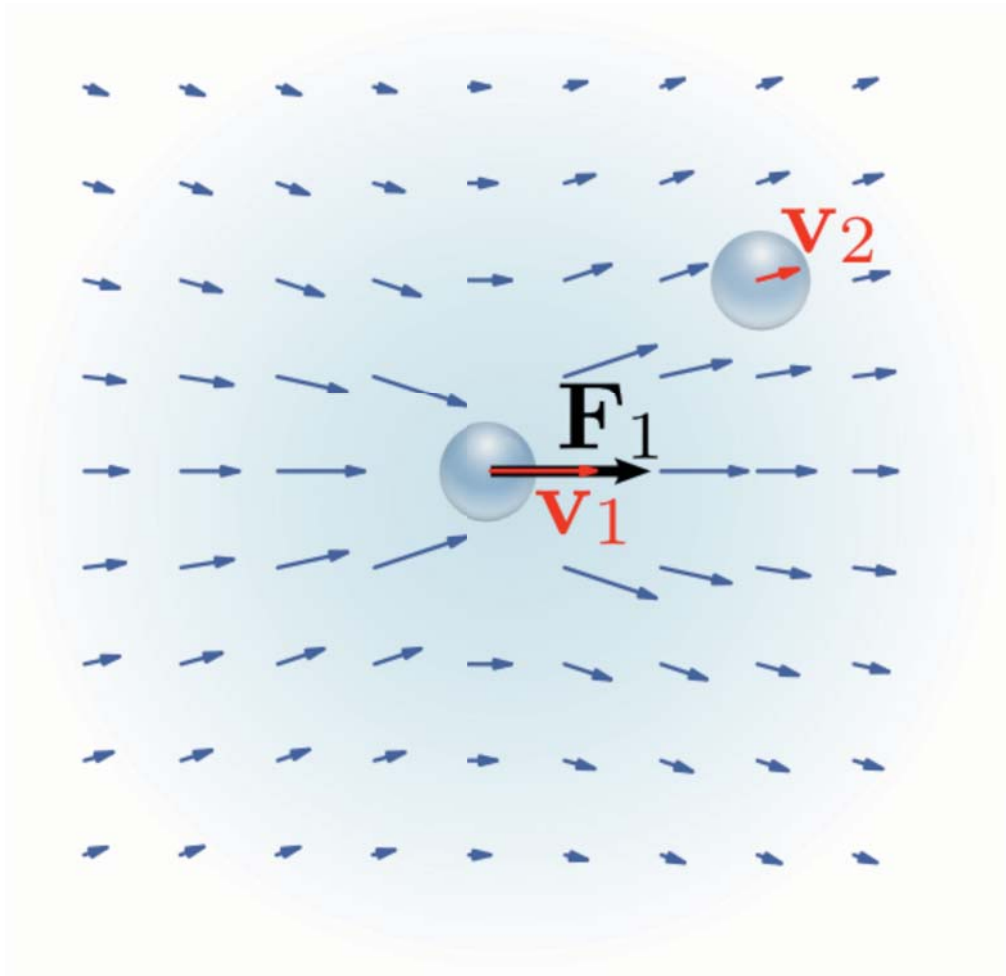
Hydrodynamics interactions



$$\mathbf{v} = m_0 \cdot \mathbf{F}$$

$$\text{mobility } m_0 = \frac{1}{6\pi\mu a}$$

Hydrodynamics interactions



$$\mathbf{v}_2 = \mathbf{G}(\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{F}_1$$

flow propagator
or Stokeslet

$$\mathbf{v} = \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{pmatrix}$$

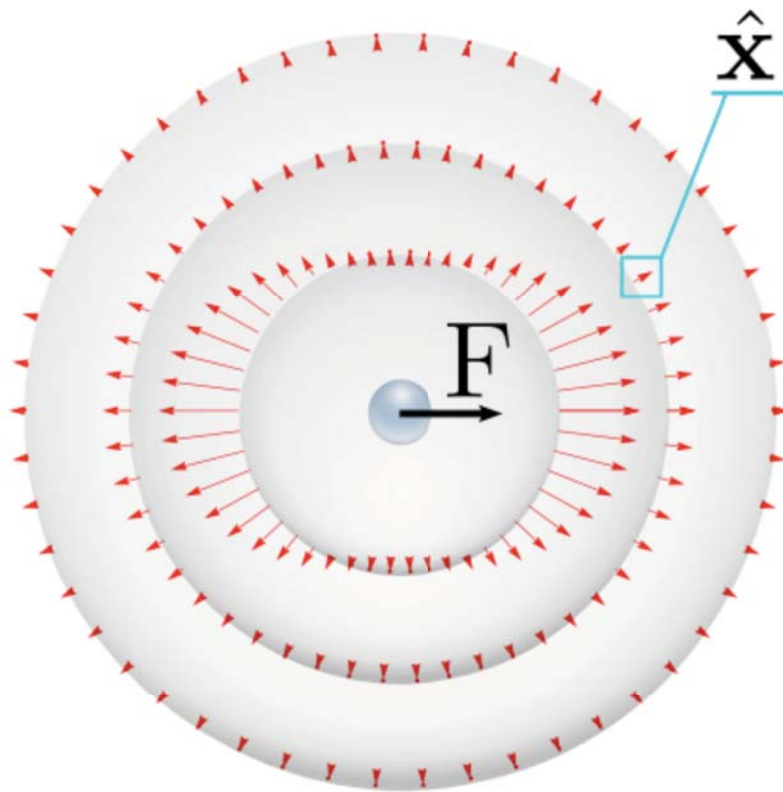
$$\mathbf{V} = \mathbf{M} \cdot \mathbf{F}$$

generalized mobility tensor

$$\mathbf{M} = \begin{pmatrix} m_0 & \mathbf{G}(\mathbf{r}_{21}) \\ \mathbf{G}(\mathbf{r}_{12}) & m_0 \end{pmatrix}$$

hydrodynamic couplings
vanishing when $|\mathbf{r}_{12}| \rightarrow \infty$

Hydrodynamics interactions



$$\hat{\mathbf{x}} \cdot \mathbf{\Pi}$$

$$\mathbf{\Pi} = p\mathbf{1} - \boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} = \eta \left(\nabla \mathbf{v} + \nabla \mathbf{v}^T \right)$$



$$|\boldsymbol{\sigma}| \sim \frac{1}{r^2} \Rightarrow |\mathbf{v}| \sim \frac{1}{r}$$

hydrodynamic interactions are long ranged

$$|\mathbf{r}_{12}| \sim 100a \Rightarrow G/m_0 \sim 10^{-2}$$

Hydrodynamic interactions of trapped beads

VOLUME 82, NUMBER 10

PHYSICAL REVIEW LETTERS

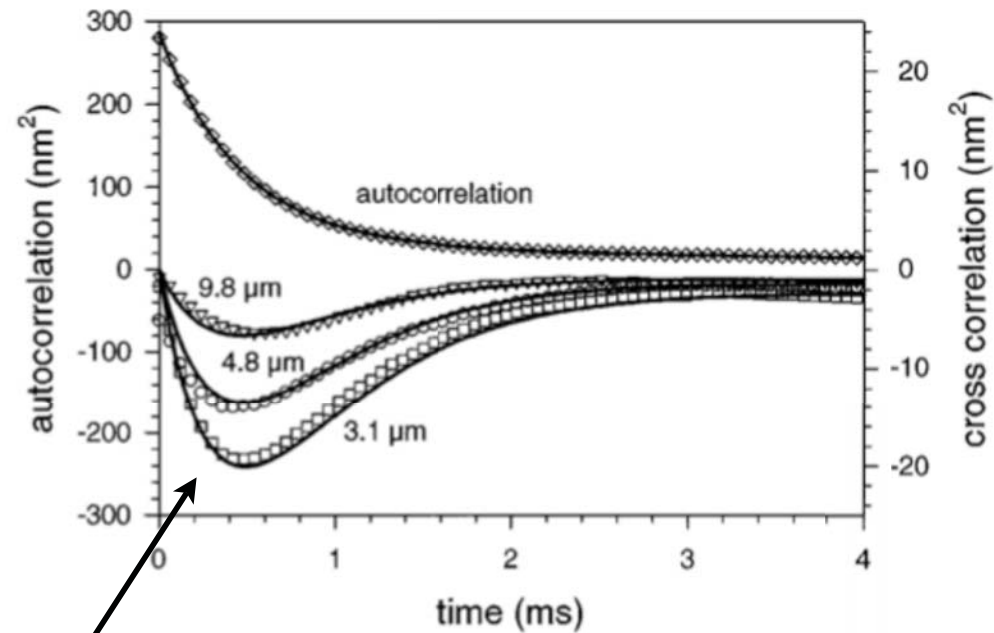
8 MARCH 1999

Direct Measurement of Hydrodynamic Cross Correlations between Two Particles in an External Potential

Jens-Christian Meiners and Stephen R. Quake

Department of Applied Physics, California Institute of Technology, Pasadena, California 91125

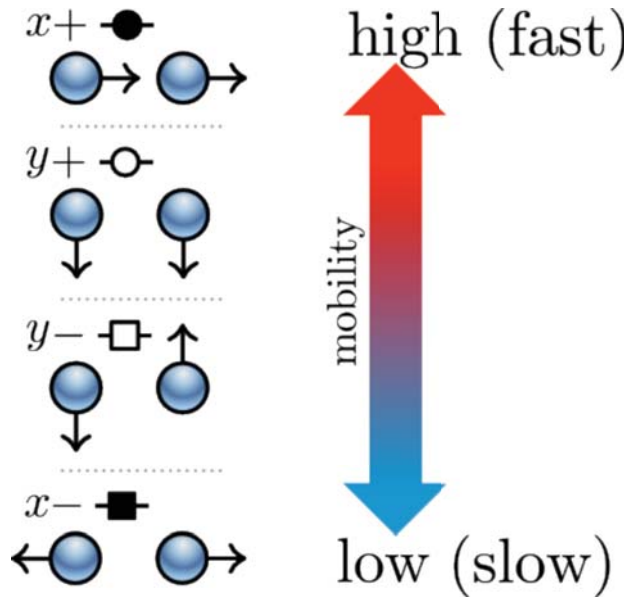
(Received 22 October 1998)



$$\langle x_1(0)x_2(t) \rangle$$

Hydrodynamic eigenmodes

EIGENMODES OF THE MOBILITY MATRIX



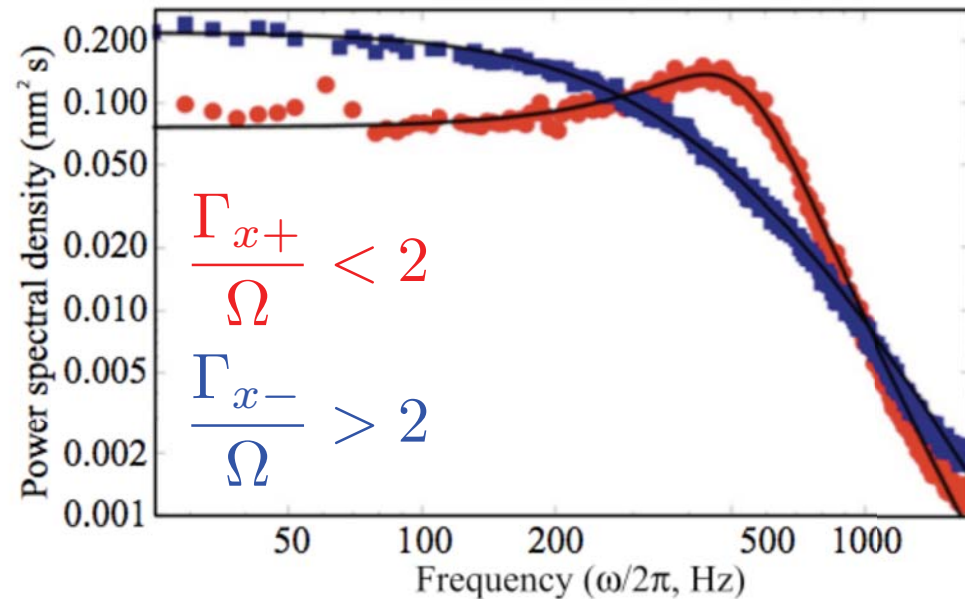
DECOUPLED LANGEVIN DYNAMICS

$$\ddot{Q}_j(t) + \Omega^2 Q_j(t) + \Gamma_j \dot{Q}(t) = \xi_j(t)$$

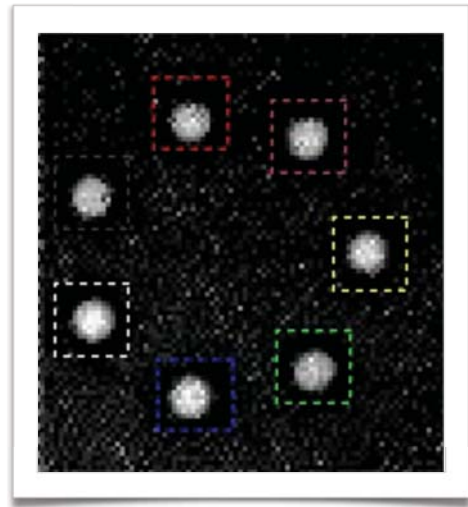
$$j = x+, y+, y-, x-$$

HYDRODYNAMICALLY COUPLED LIQUID DROPLETS

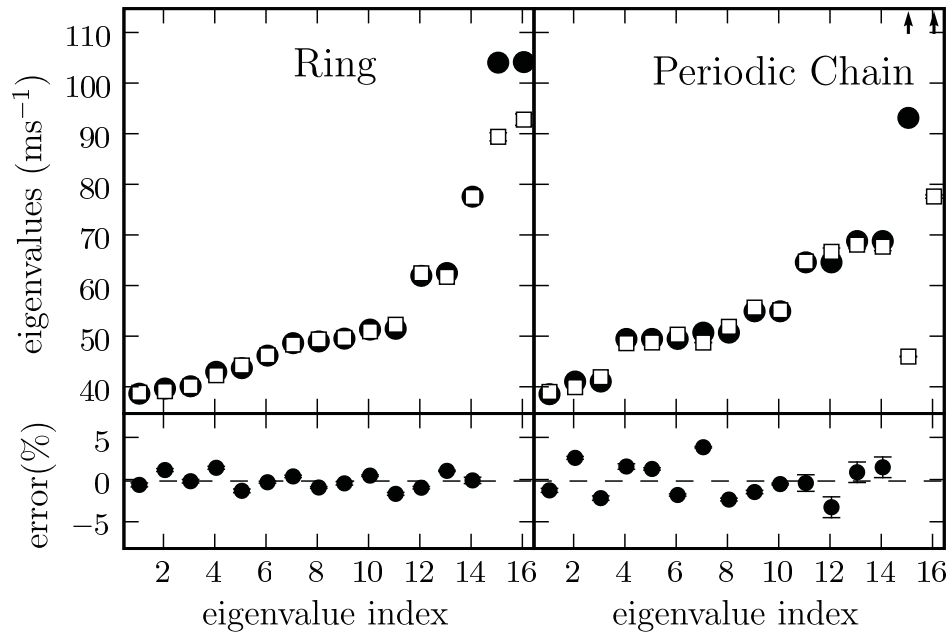
YAO et al. NJP (2009)



Hydrodynamic eigenmodes



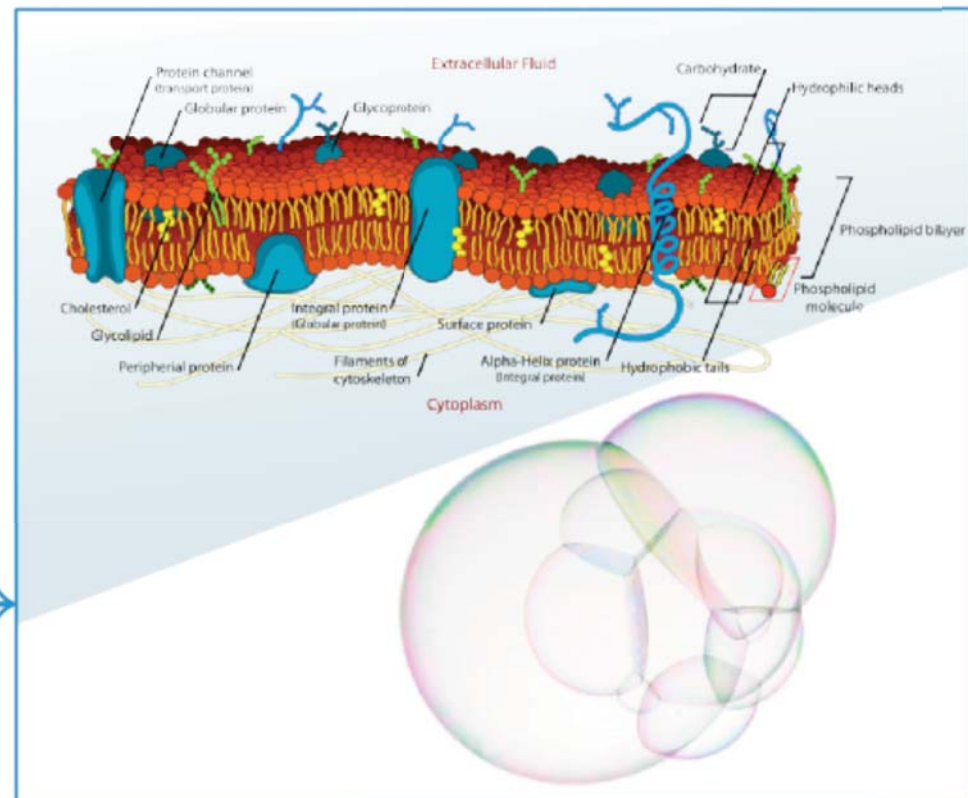
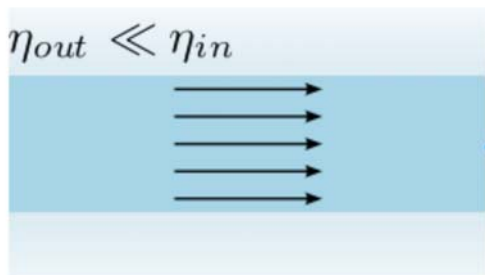
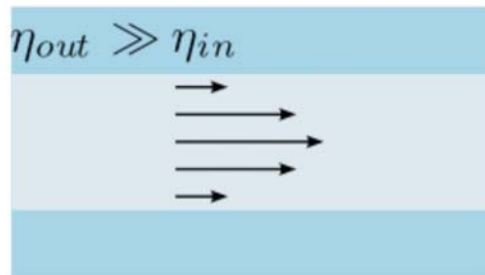
DI LEONARDO et al. PRE (2007)



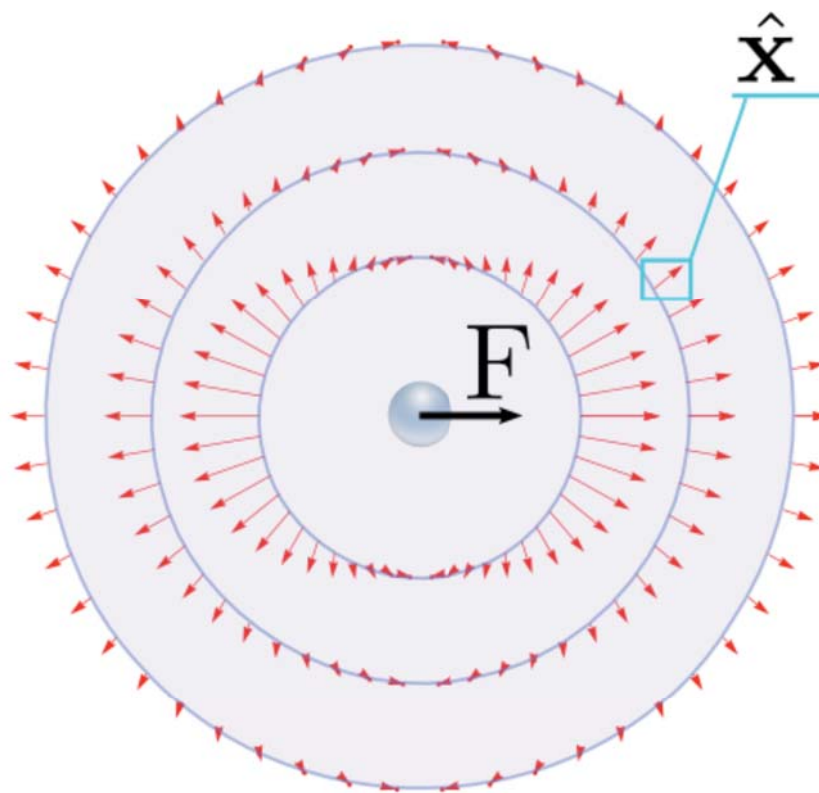
n.	ring	periodic chain	wrapped chain
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			

Hydrodynamics interactions in 2D

2D :one dimension is irrelevant $\Rightarrow \frac{\partial}{\partial z} [\dots] = 0$



Hydrodynamics interactions



$$\hat{\mathbf{x}} \cdot \boldsymbol{\Pi}$$

$$\boldsymbol{\Pi} = p\mathbf{1} - \boldsymbol{\sigma}$$

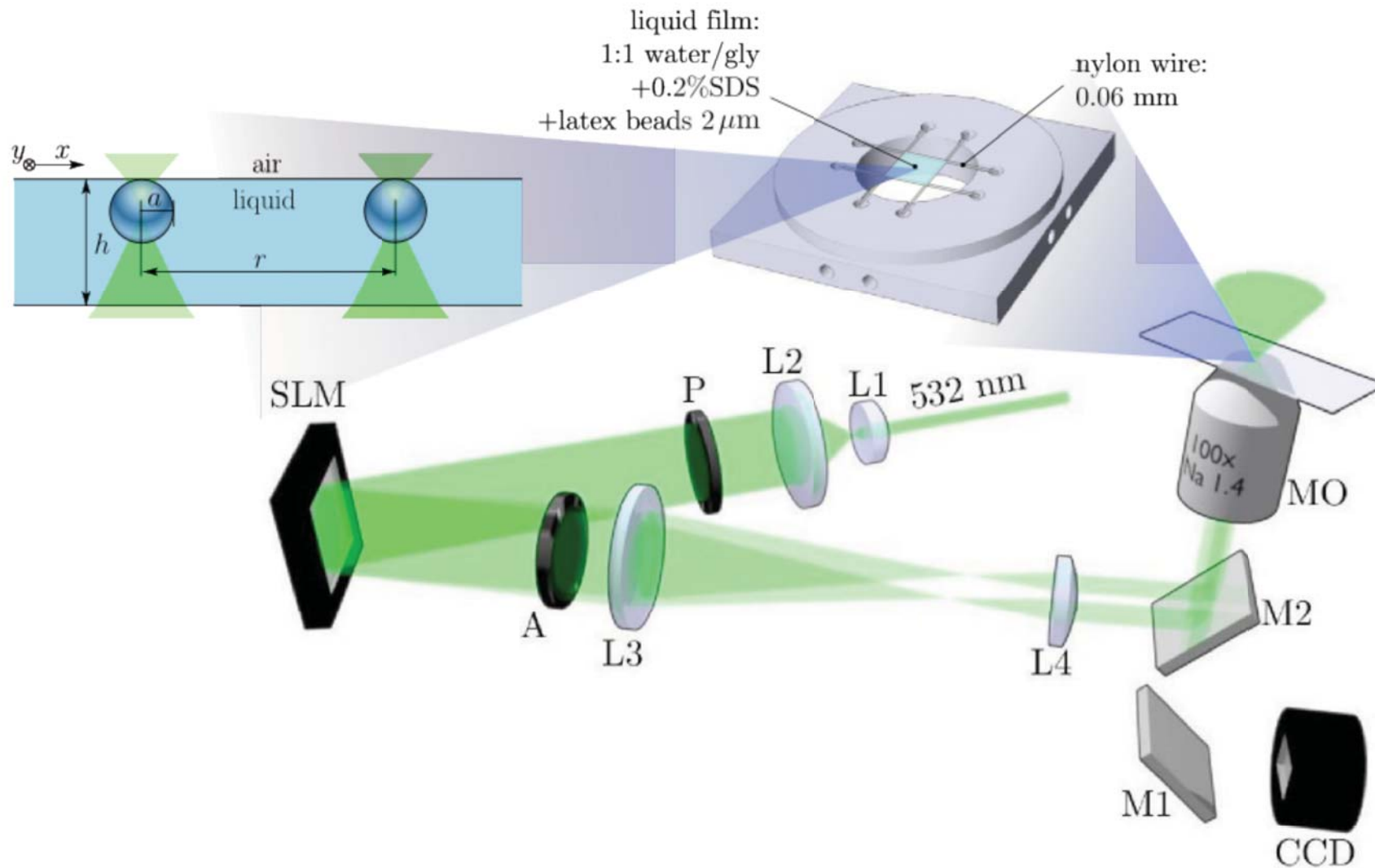
$$\boldsymbol{\sigma} = \eta \left(\nabla \mathbf{v} + \nabla \mathbf{v}^T \right)$$

$$|\boldsymbol{\sigma}| \sim \frac{1}{r} \Rightarrow |\mathbf{v}| \sim \log(r)$$

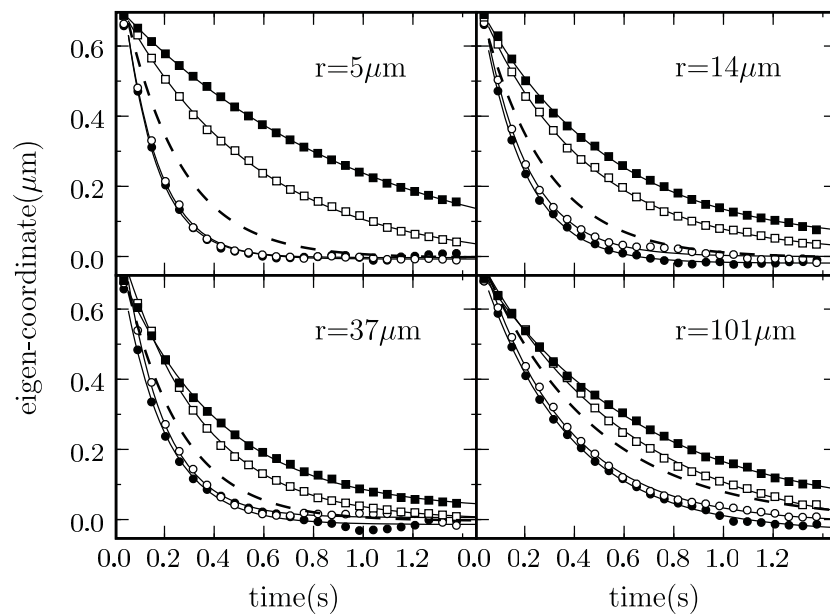
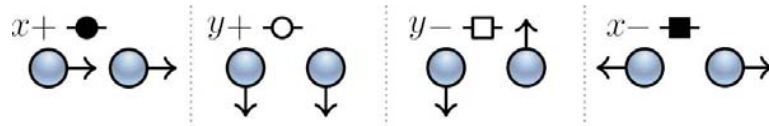
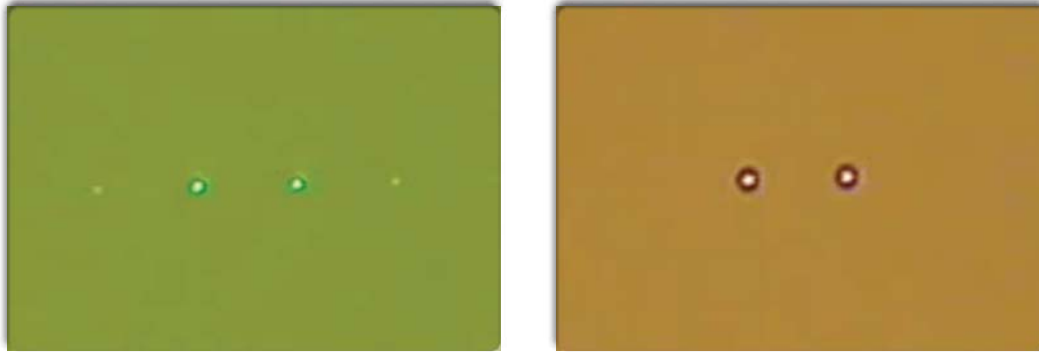
hydrodyn. inter. are extremely long ranged

$$|\mathbf{r}_{12}| \sim 100a \Rightarrow G/m_0 \sim 1 !!!$$

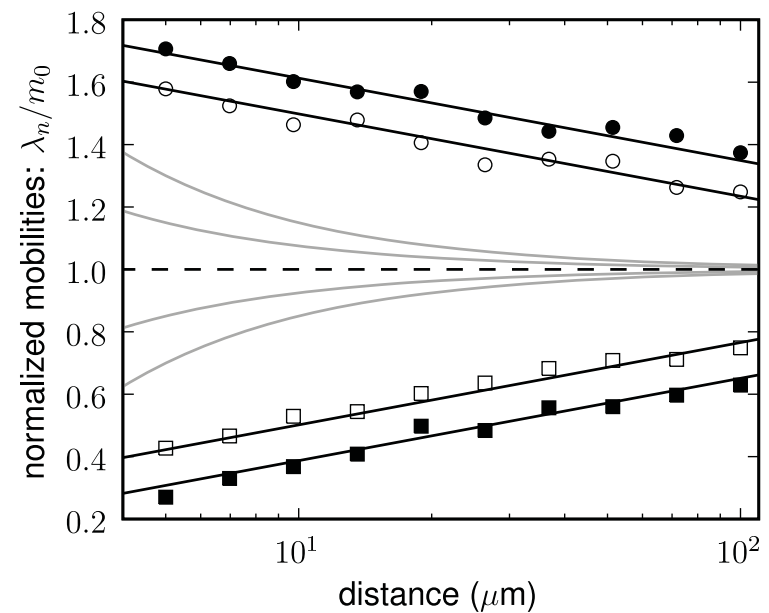
Trapping in a soap film



Hydrodynamics interactions in 2D



the power you need to transport an array of N particles doesn't depend on N

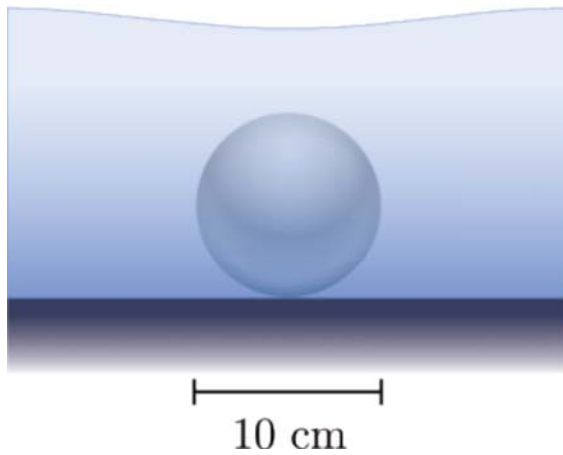


SURFACE FORCES

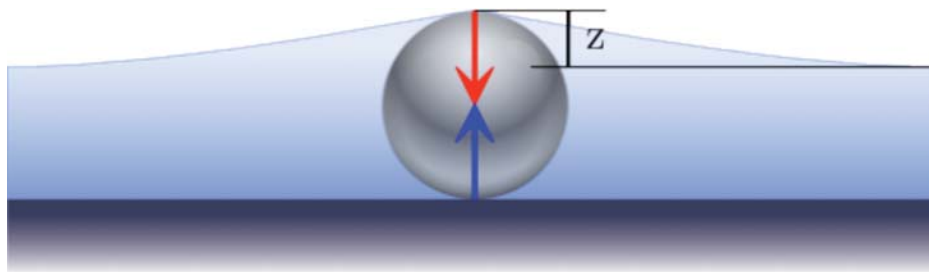
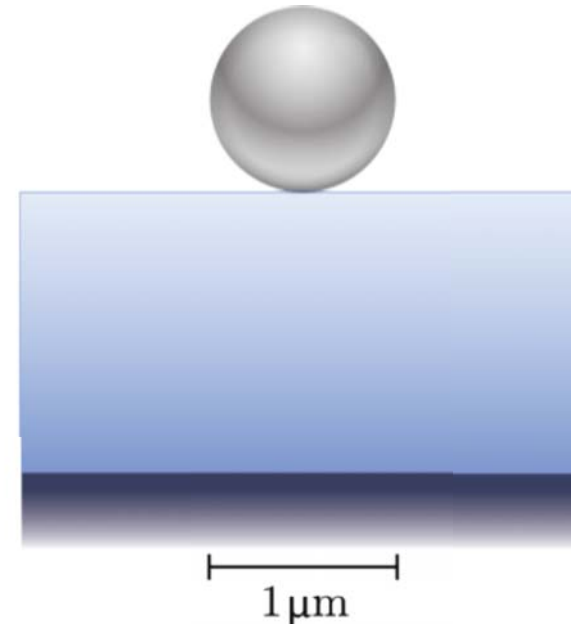
Capillary interactions

Capillary interactions

MACROSCOPIC



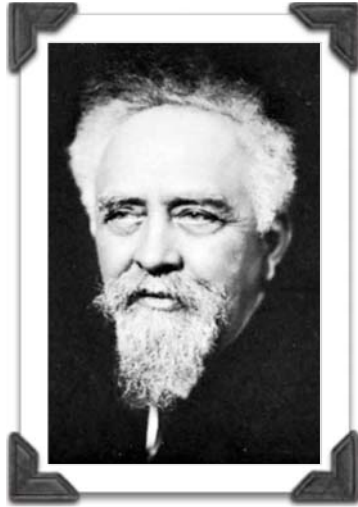
MESOSCOPIC



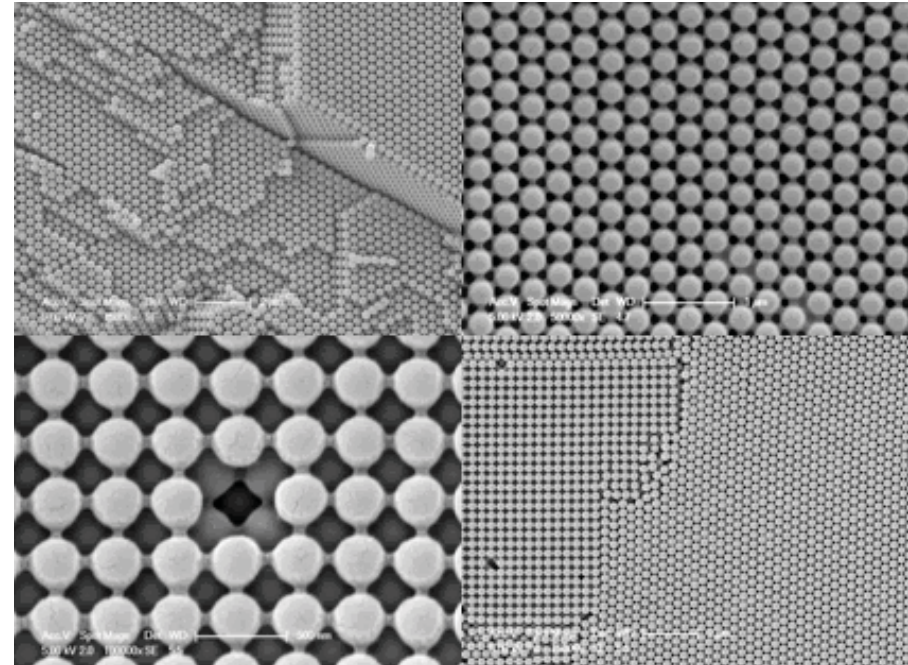
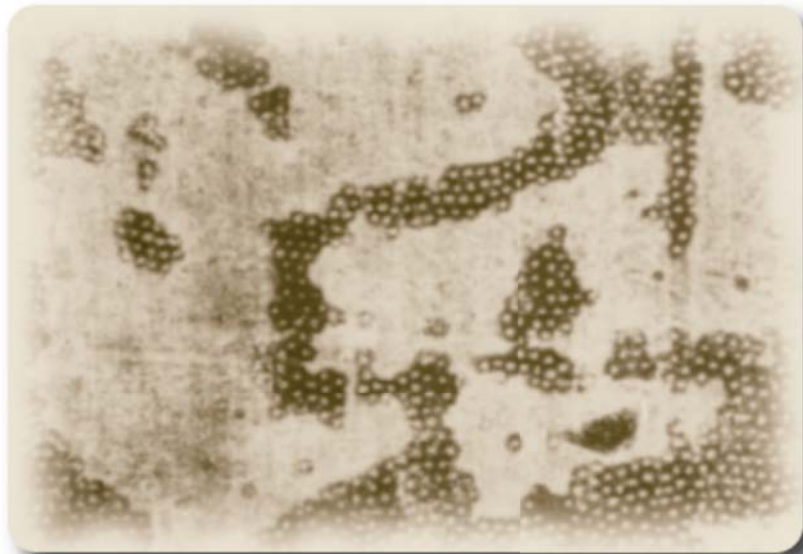
$$\rightarrow F = -kz$$

$$k \simeq 4\pi\gamma = 1 \text{ nN/nm}$$

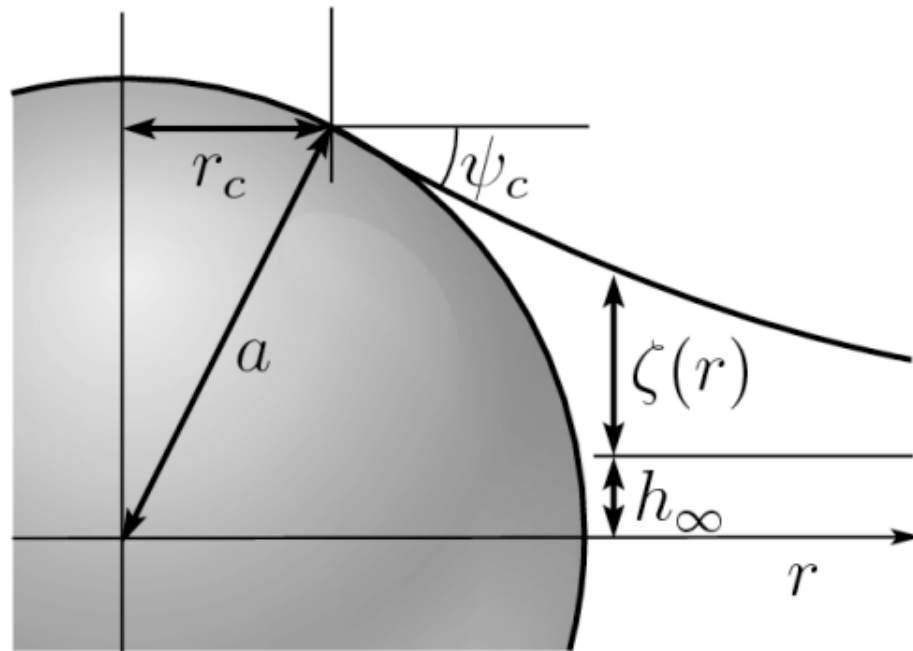
Capillary interactions



J. PERRIN J. ANN. CHIM. PHYS. (1909)



Capillary interactions



$$F(r) = 2 \cdot 2\pi r_c \cdot \psi_c \gamma \frac{\partial \zeta}{\partial r}$$

small gradient approximation:

$$\gamma \nabla^2 \zeta = \rho g \zeta$$

↓

$$\zeta(r) = -r_c \psi_c \log(qr)$$

↓

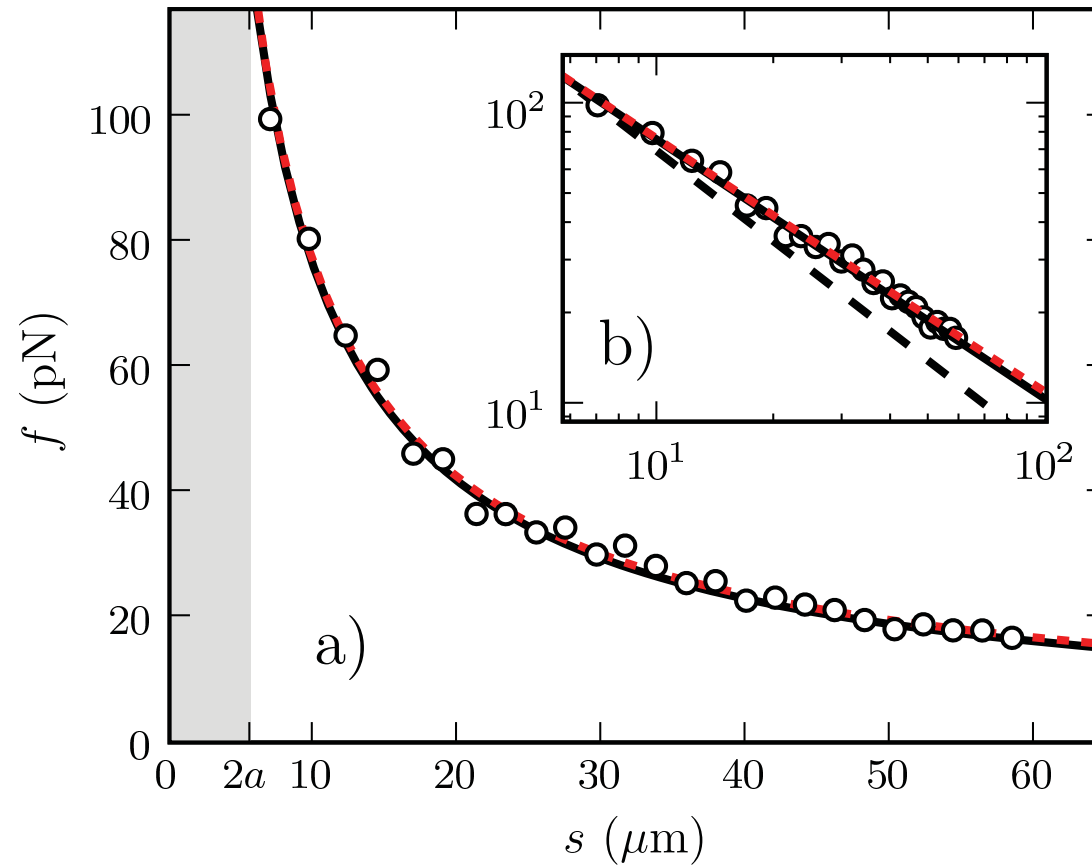
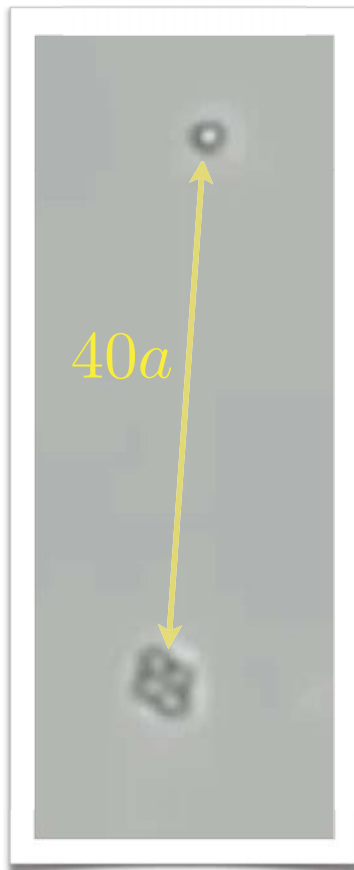
$$F(r) = 4\pi r_c^2 \psi_c^2 \gamma \frac{1}{r} \sim \frac{1}{r^\nu} \quad \nu = 0.84 \div 0.9$$

weak r dependence

Capillary interactions

VERY LONG RANGE NATURE OF CAPILLARY INTERACTIONS

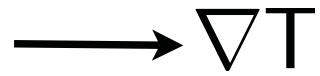
R. DI LEONARDO, et al. PRL (2008)



SURFACE FORCES

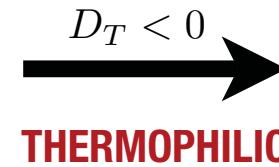
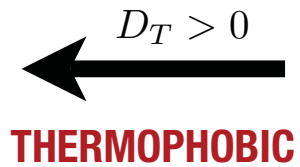
Thermophoresis

Thermophoresis



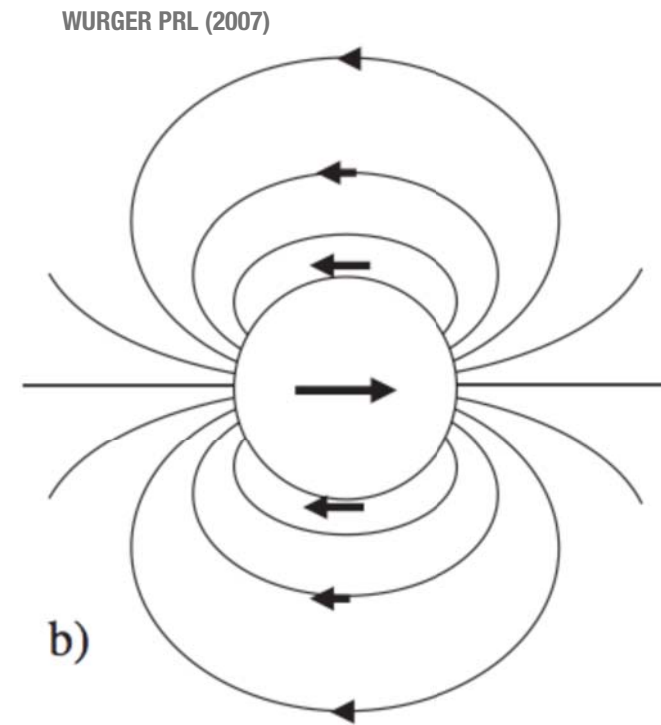
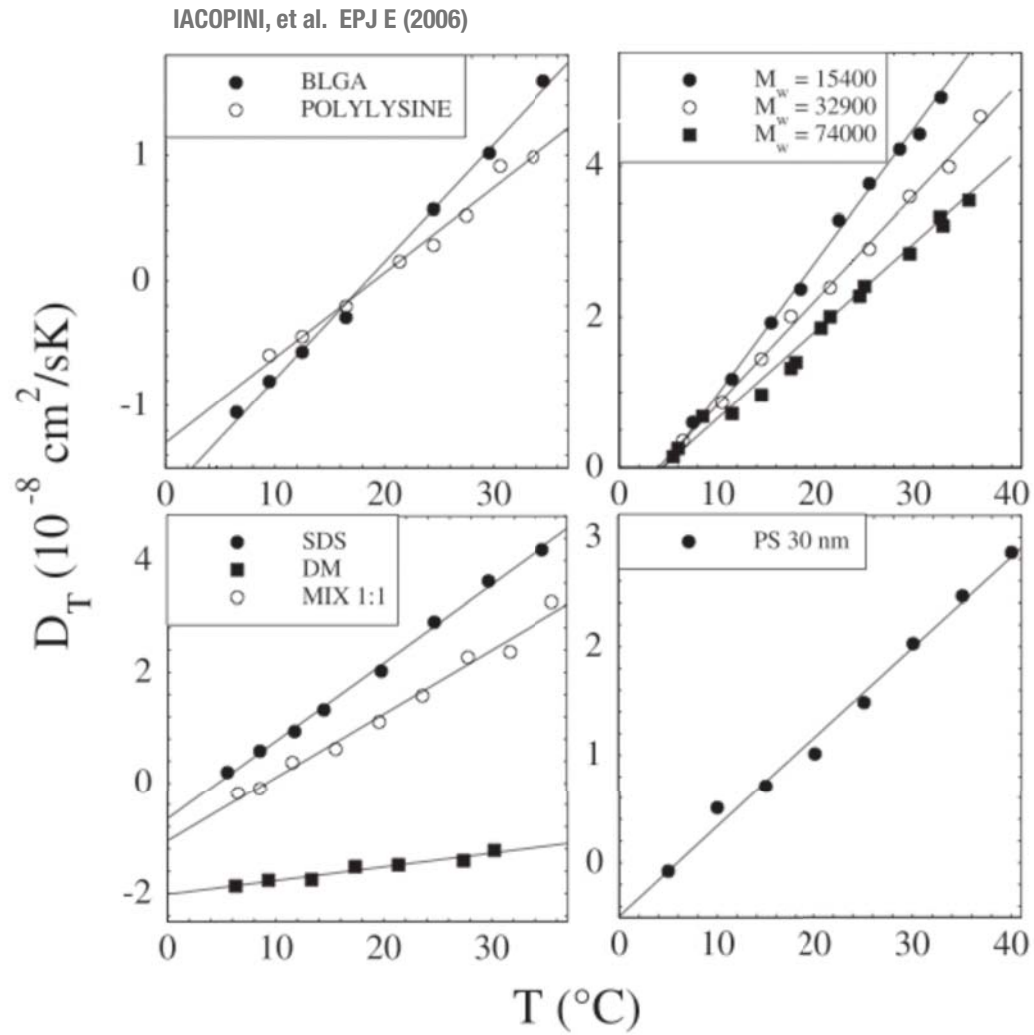
DRIFT
VELOCITY $U = -D_T \nabla T$

COLD

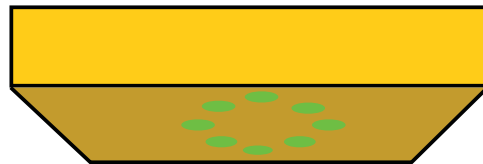
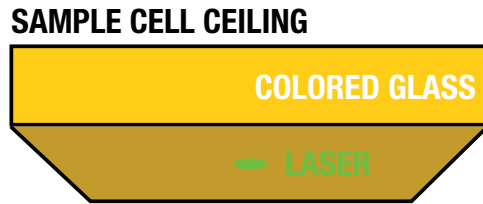


HOT

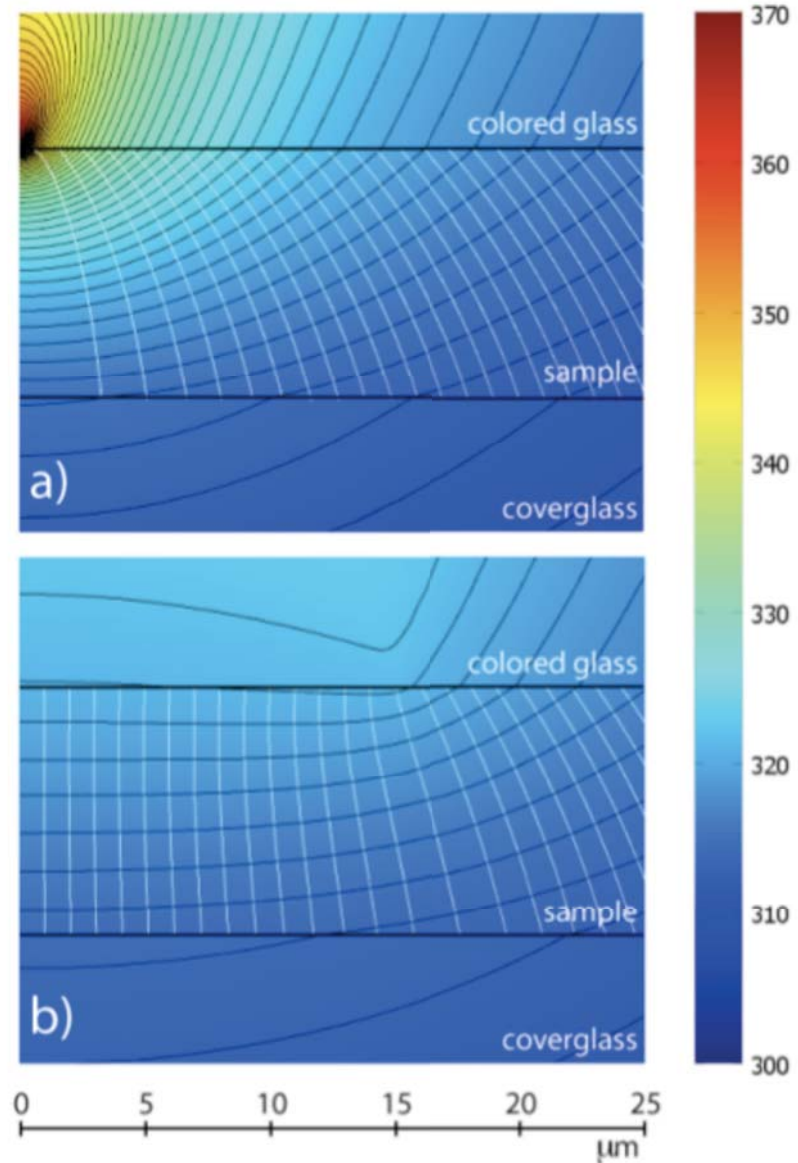
Thermophoresis



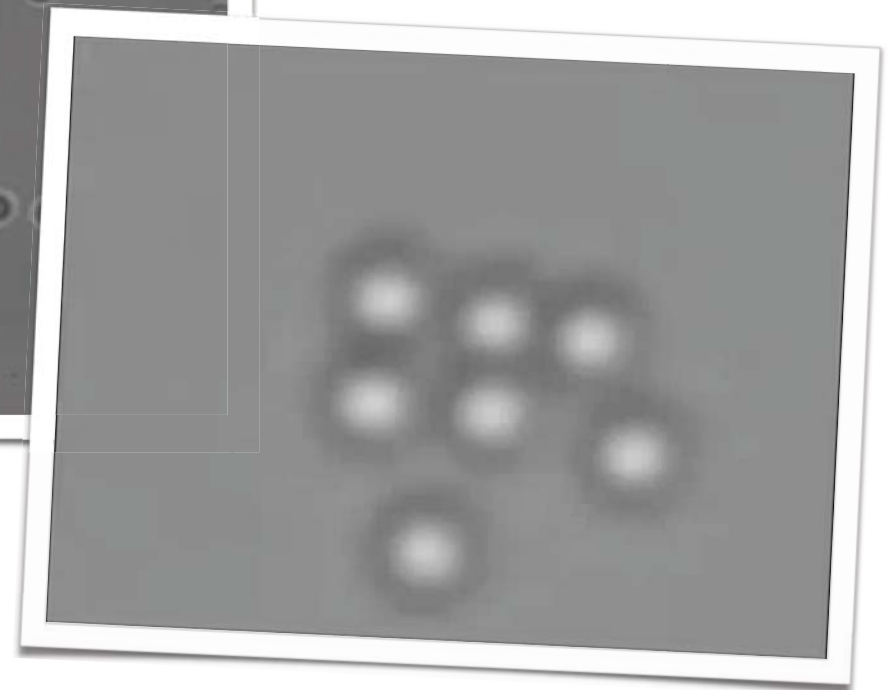
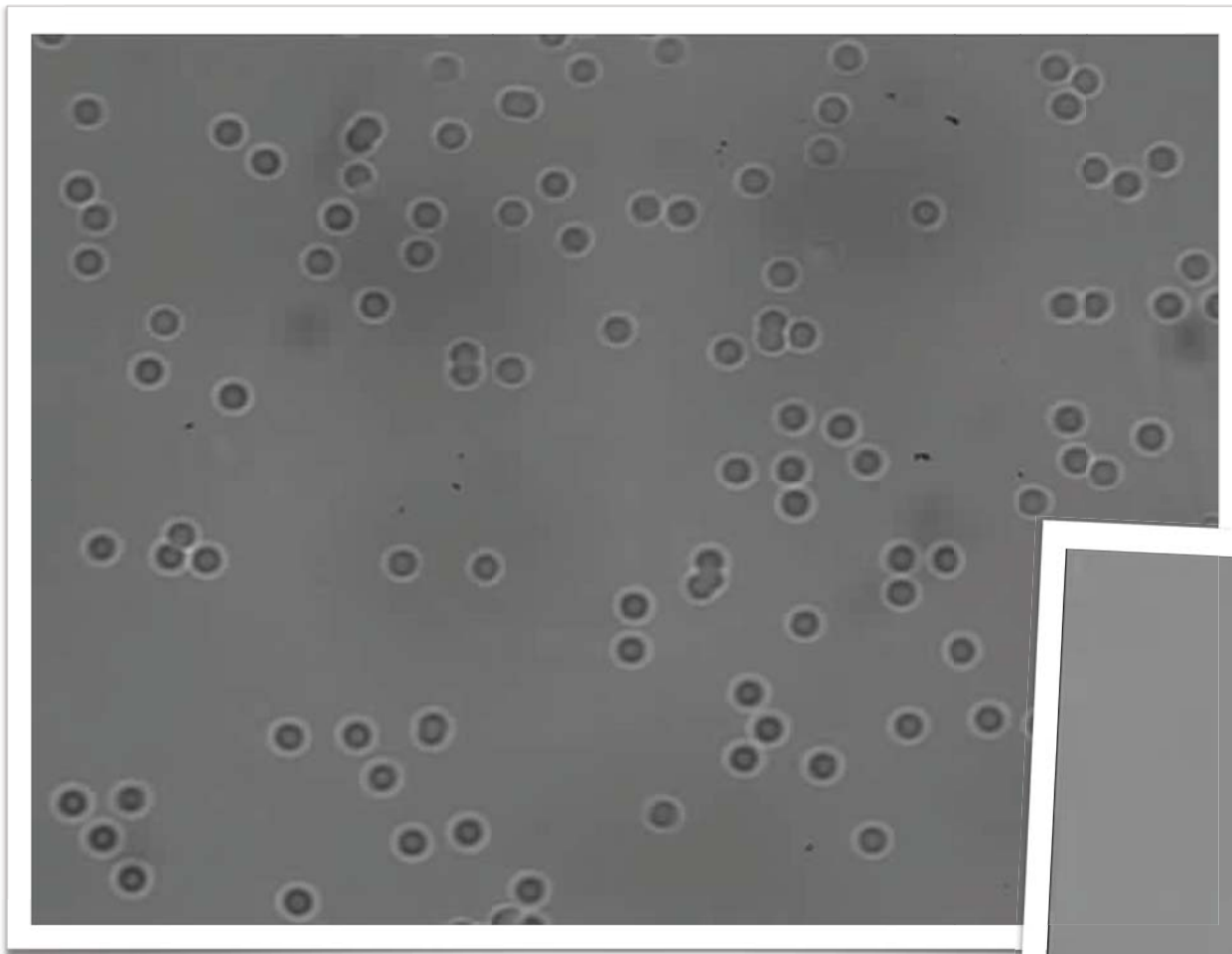
Holographically generated T gradients



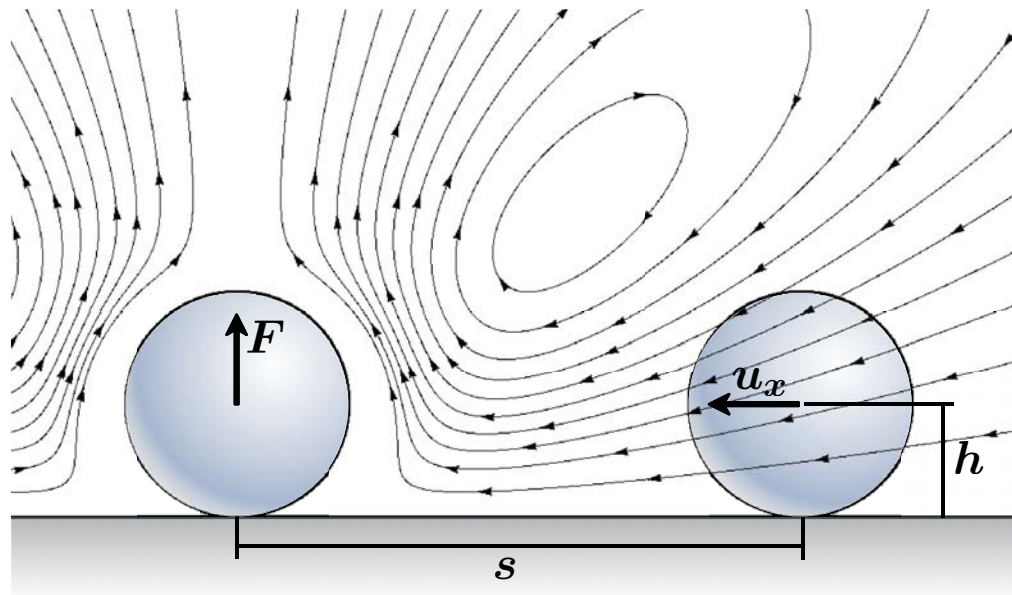
Di LEONARDO et al. LANGMUIR (2009)



Colloidal attraction!



A novel colloidal interaction

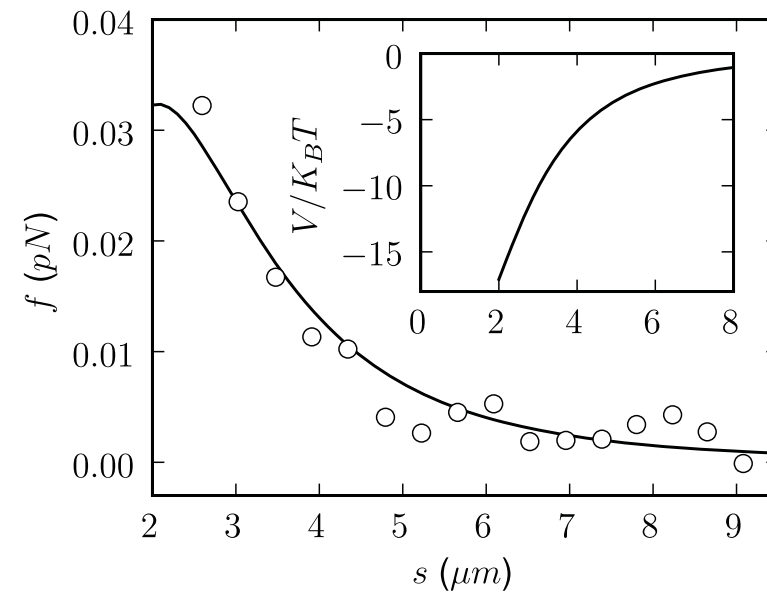


COLLOIDAL ATTRACTION INDUCED BY TEMPERATURE GRADIENTS

R. DI LEONARDO, et al. Langmuir (2009)

WEINERT & BRAUN PRL (2008)

$$\frac{F}{K_B T/a} = -18\lambda S_T \nabla T \frac{a^5}{s^4}$$



NOISY ENVIRONMENT

Brownian parametric oscillator

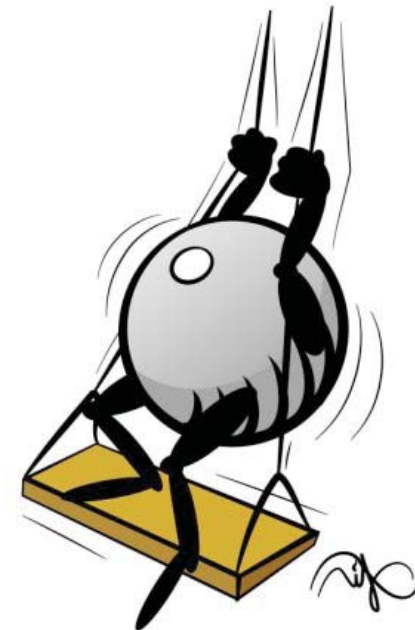
The Brownian parametric oscillator

PARAMETRIC RESONANCE



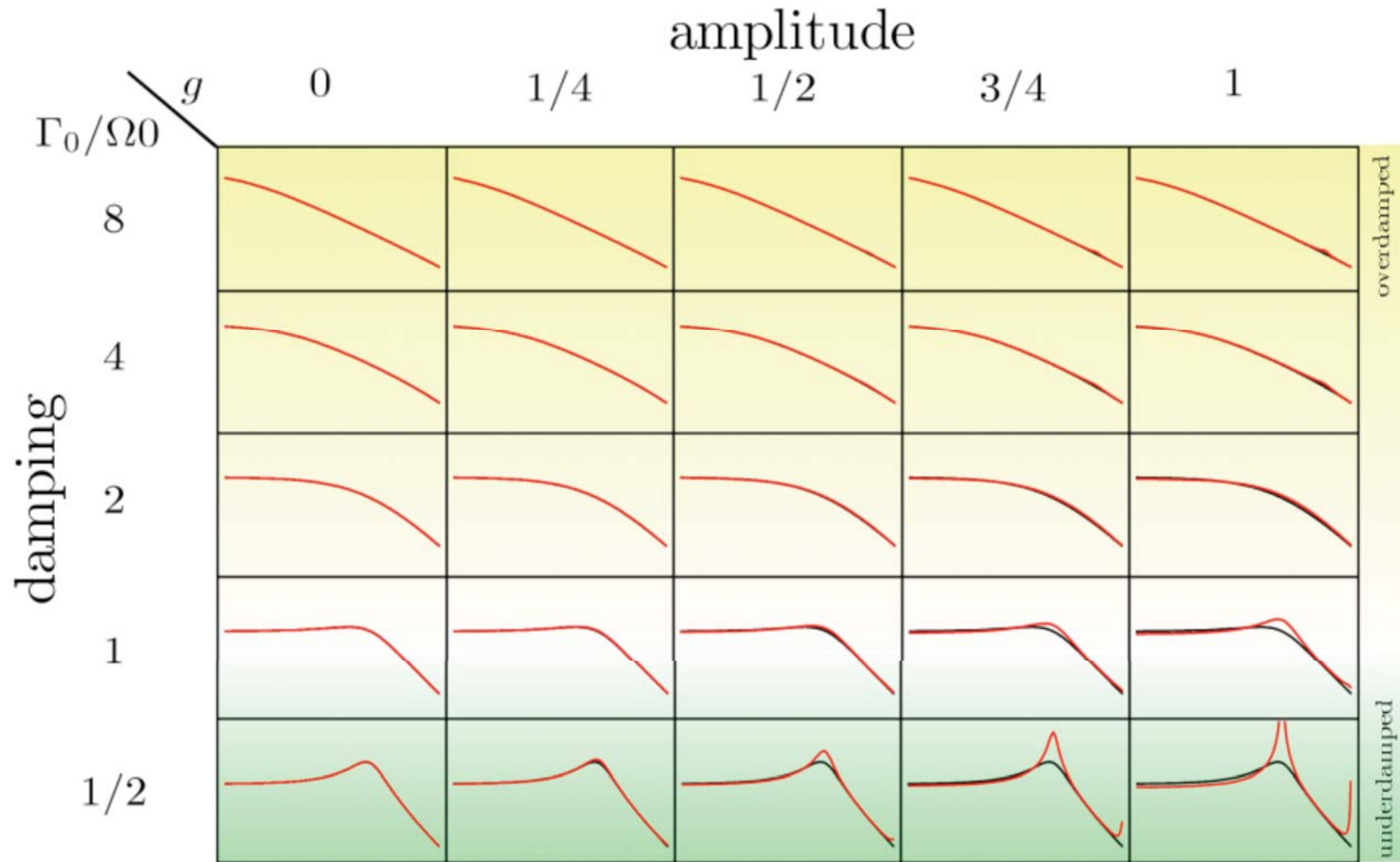
$$\ddot{x}(t) + \Omega_0^2(t)x(t) + \Gamma_0\dot{x}(t) = 0$$

THE BROWNIAN PARAMETRIC OSCILLATOR



$$\ddot{x}(t) + \Omega_0^2(t)x(t) + \Gamma_0\dot{x}(t) = \xi(t)$$

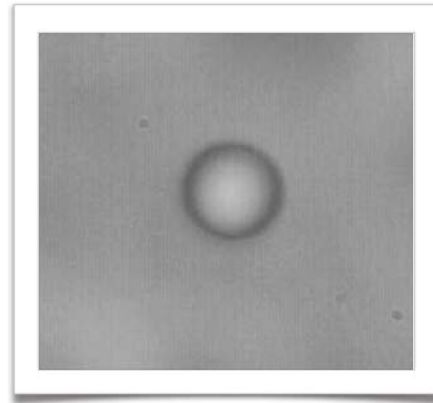
The Brownian parametric oscillator



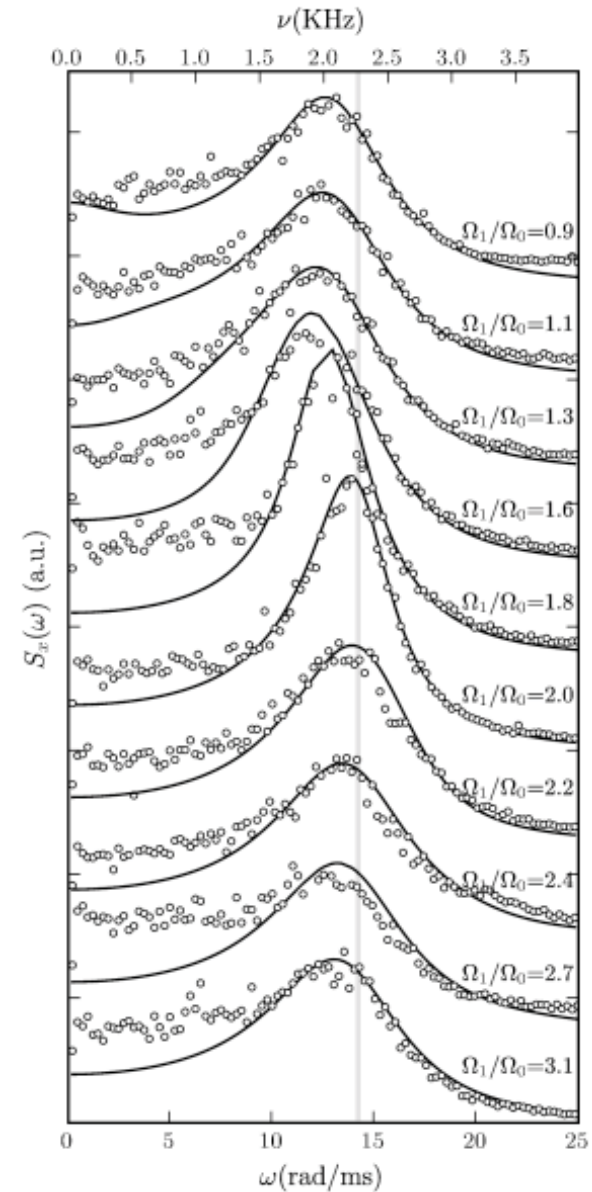
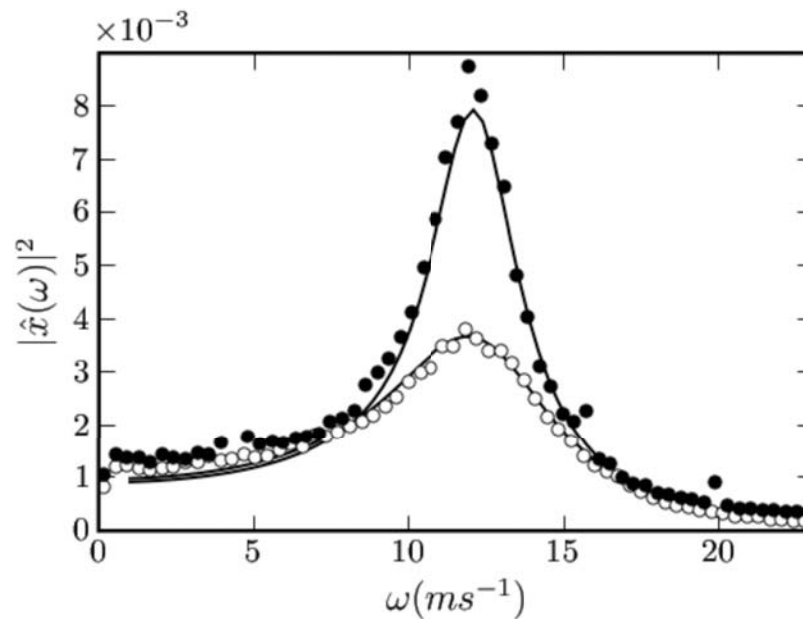
The Brownian parametric oscillator

PARAMETRIC EXCITATION OF OPTICALLY TRAPPED AEROSOLS

R. DI LEONARDO, et al. PRL (2007)



WATER DROPLET (2000 fps)



Collaborations

S. BIANCHI, G. BOLOGNESI, G. RUOCCO,

F. IANNI, F. SAGLIMBENI

CNR-INFM, CRS-SOFT, Dip. Fisica "La Sapienza" Roma

J. LEACH, S. KEEN, A. YAO, M. PADGETT

University of Glasgow, UK