



**The Abdus Salam
International Centre for Theoretical Physics**



2039-9

**Conference on 2nd Drug Development for the Third World: From
Computational Molecular Biology to Experimental Approaches**

1 - 5 June 2009

Forces and Energies

VERLINDE Christophe L.M.J.
*University of Washington Biomolecular Structure Centre
Department of Biochemistry
Health Sciences Building K-428A
1959 NE Pacific St. WA 98195-7742 Seattle
U.S.A.*

Forces and Energies

Lecture notes for Trieste

by Christophe L.M.J. Verlinde, Ph.D.

Associate Professor

University of Washington

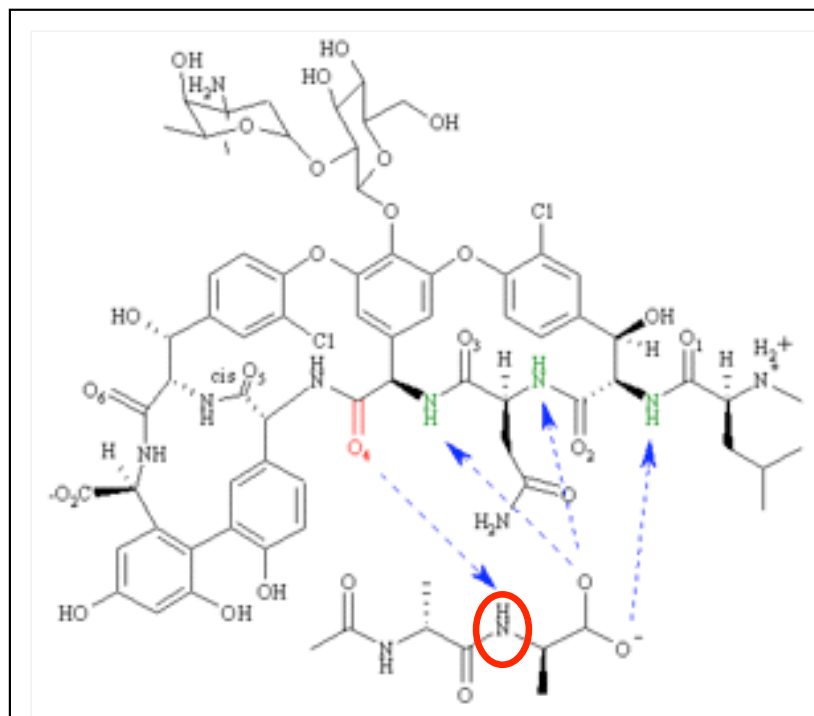
verlinde@u.washington.edu

Contents

1. Introduction
2. Structure
 1. Crystallography
 2. NMR
 3. Comparative modeling
3. Forces between molecules: electromagnetic
4. From quantum to molecular mechanics
5. Molecular mechanics: assumptions
6. Force-fields
7. Suggested reading

1. Introduction: typical drug design questions (1)

- Bacterial resistance to vancomycin:
lactate instead of D-Ala to build the cell wall

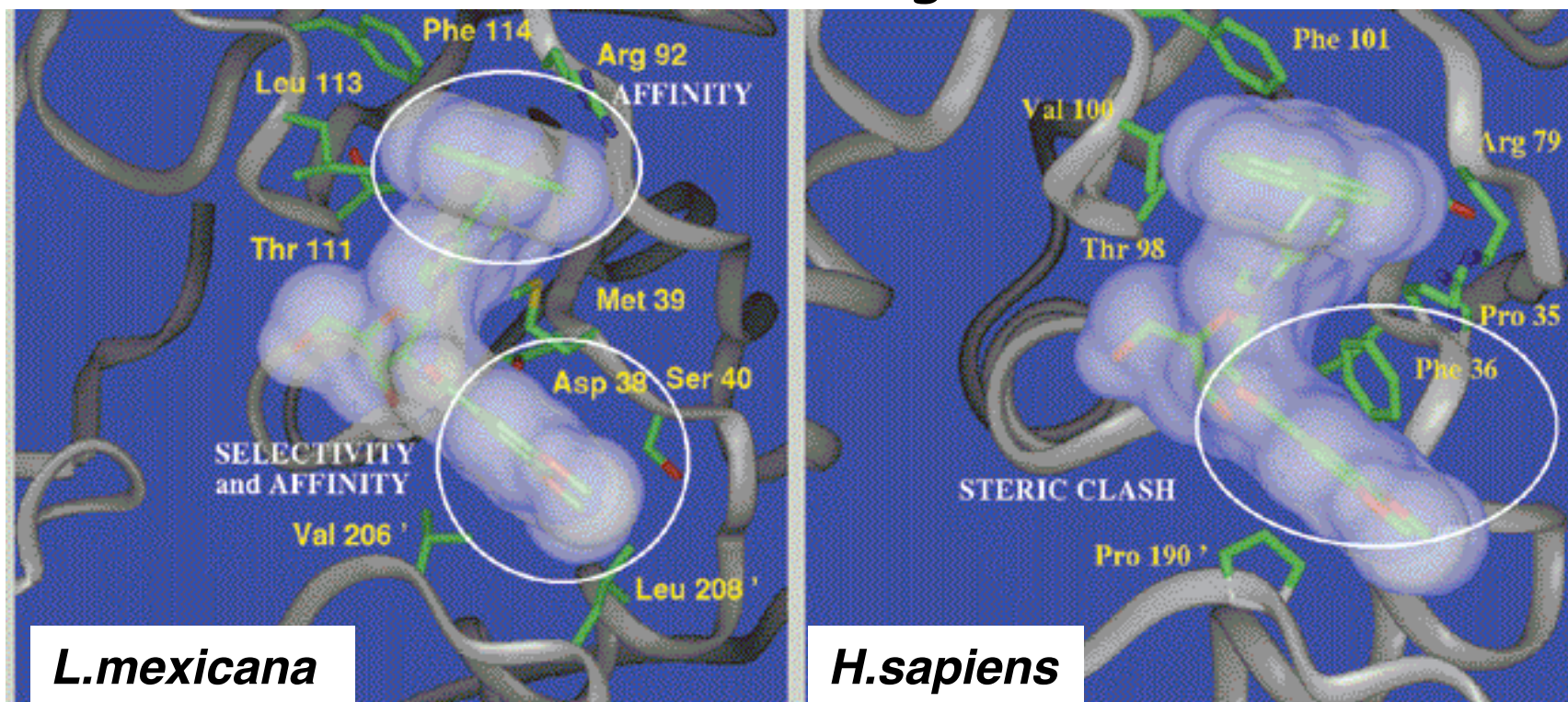


Predicted drop in affinity? (answer: 1,000 x)

Angew. Chem. Int. Ed. 42: 730-65

Introduction: typical drug design questions (2)

- Design a potent and selective inhibitor of *L.mexicana* GAPDH starting from adenosine

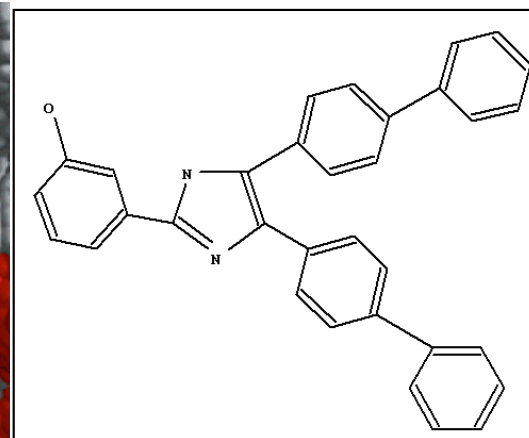
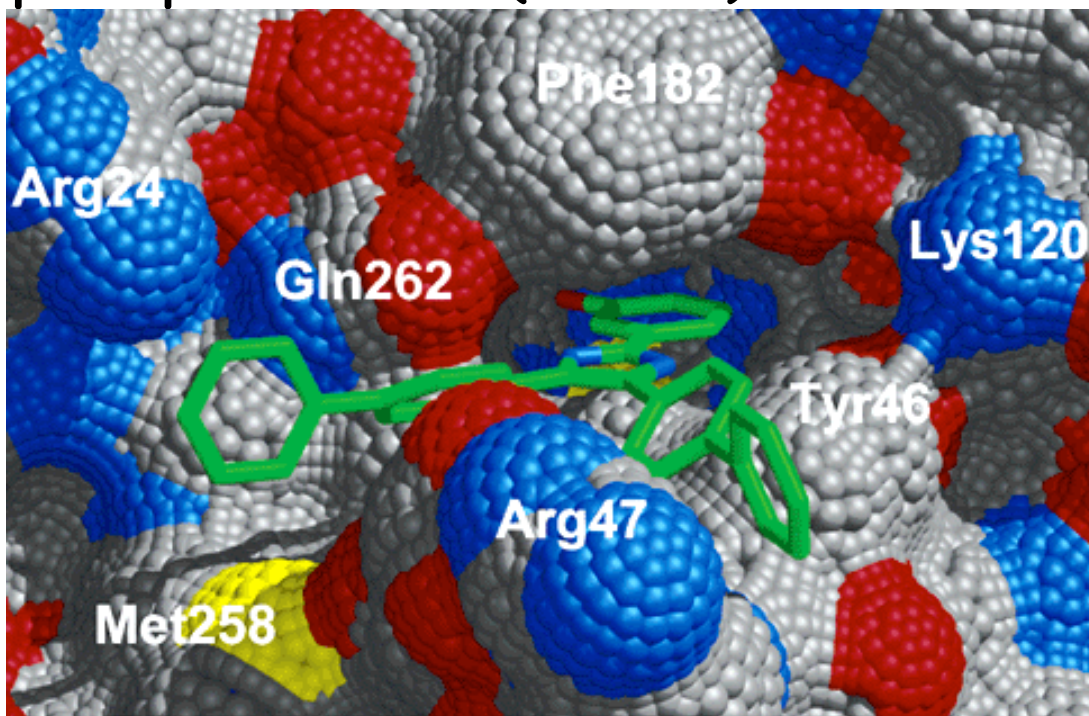


Answer: add naphthalenemethyl to N6 and 3-methoxybenzamido to C2'. Affinity $\times 10^5$

PNAS USA 96: 4273-4278

Introduction: typical drug design questions (3)

- Dock 400,000 available molecules to protein phosphatase B (PTP-B) to find new inhibitors.

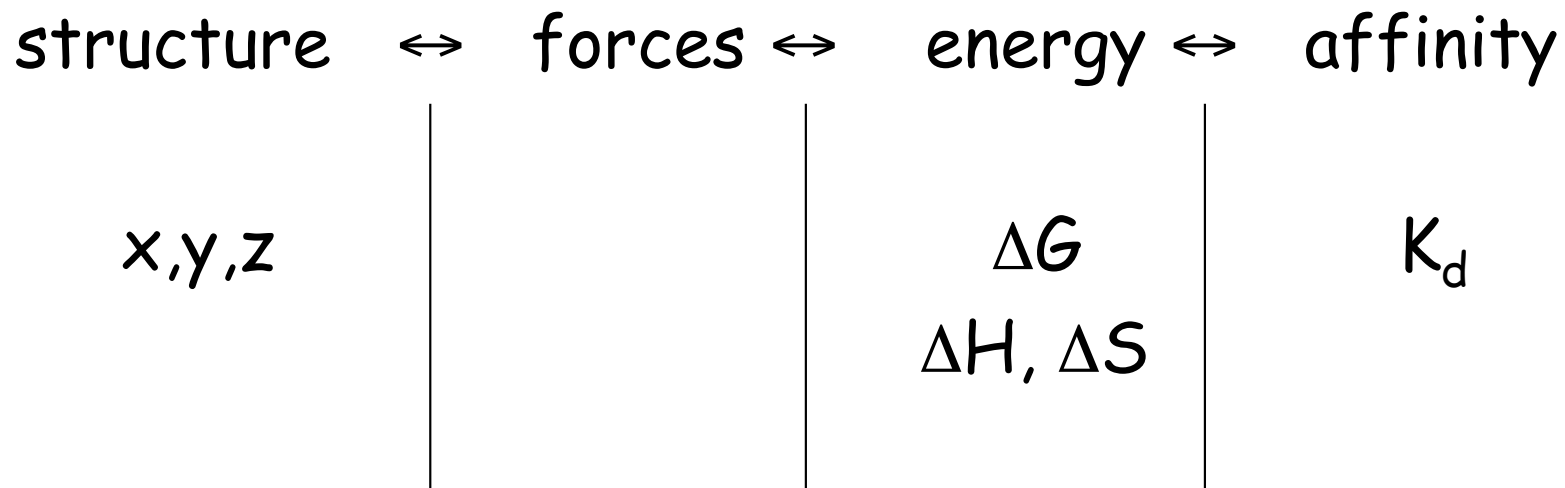


Result: 10 μM inhibitor

J.Med.Chem. 45: 2213-2221

Introduction: typical drug design questions (4)

To address the previous questions we need to examine the following relationships:



2. Structure

Three techniques produce structures for drug design: Crystallography, NMR, and Comparative modeling.

2.1 X-ray crystallography

Principle: diffraction intensities + phase info

electron density map

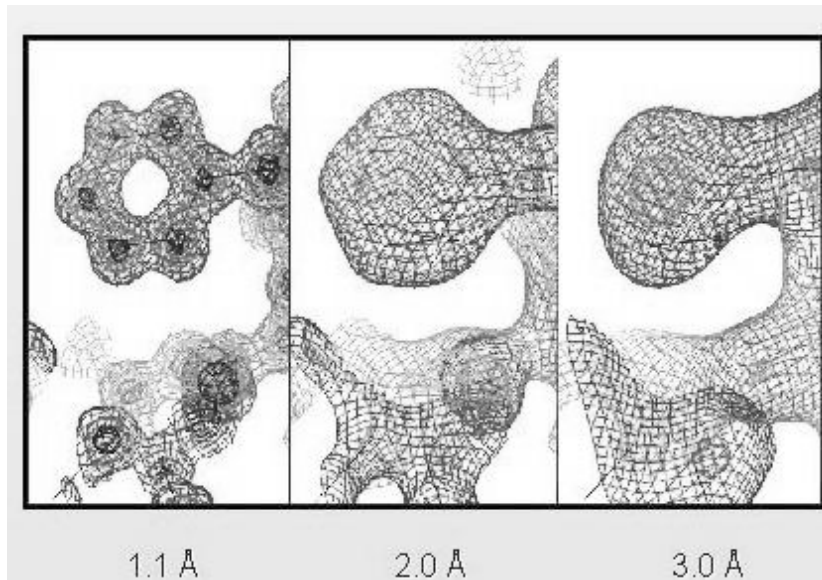
Restraints:
bond lengths
bond angles,
vdW param.,
etc.

atomic coordinates
&
'temperature factors'

X-ray crystallography (cont.)

Issues:

- Resolution: e.g. Phe at 3 levels



- R -factor (R_{work}): $R = \frac{\sum ||F_o| - |F_c||}{\sum |F_o|}$
ok if $R \leq \text{resolution}/10$ ('law of Drenth')
- Free R -factor: calculated from F 's left out
ok if $R_{free} \leq R + 0.05$

X-ray crystallography (cont.)

- Coordinate uncertainty: $\sim 0.2 \text{ \AA}$
- B-factors: spherical / ellipsoidal Gaussian model for harmonic motion

$$B = 8\pi^2 \langle u^2 \rangle$$

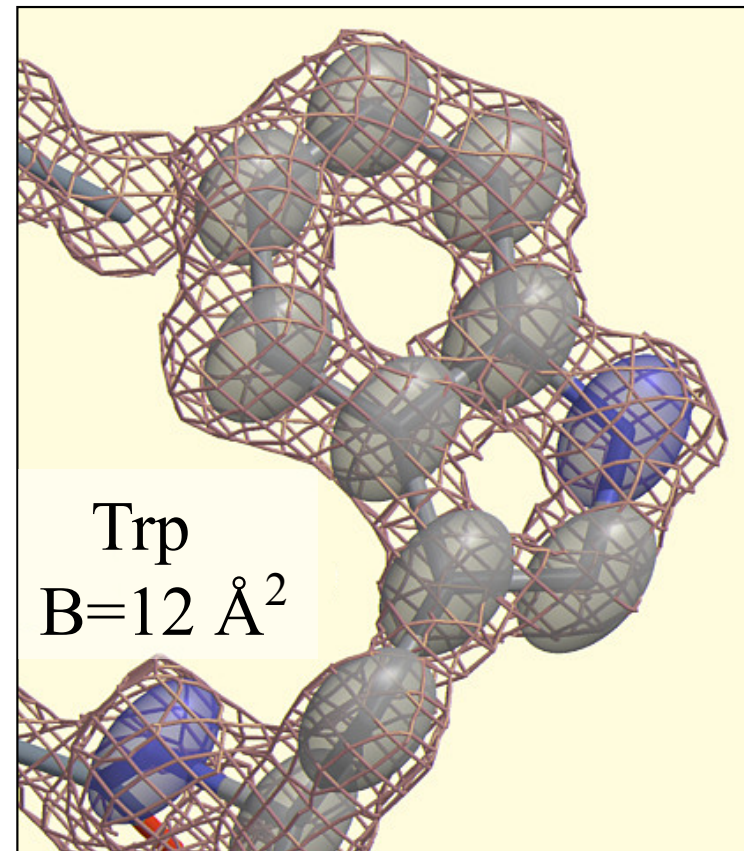
u is atomic displacement
from rest position

e.g. if $B = 10 \text{ \AA}^2$,

then $u = 0.35 \text{ \AA}$

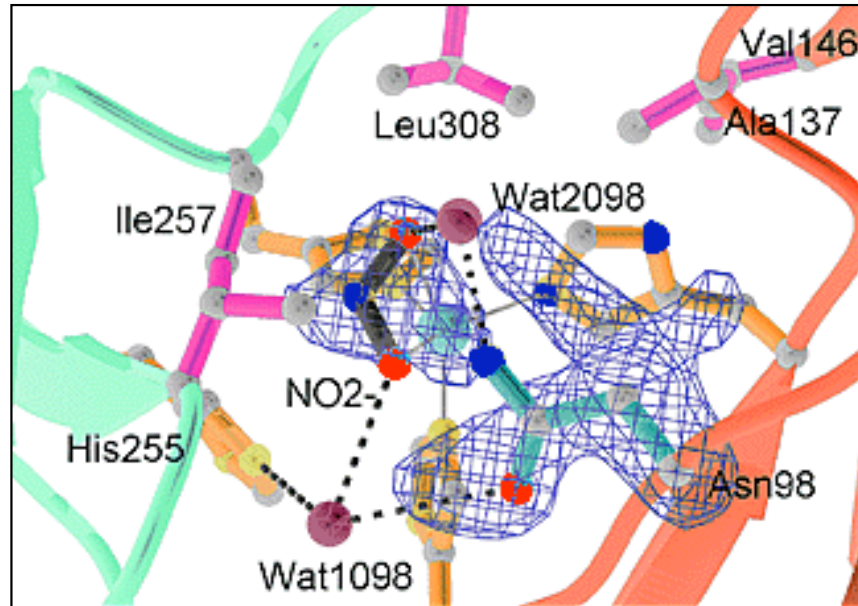
if $B = 80 \text{ \AA}^2$,

then $u = 1.00 \text{ \AA}$



X-ray crystallography (cont.)

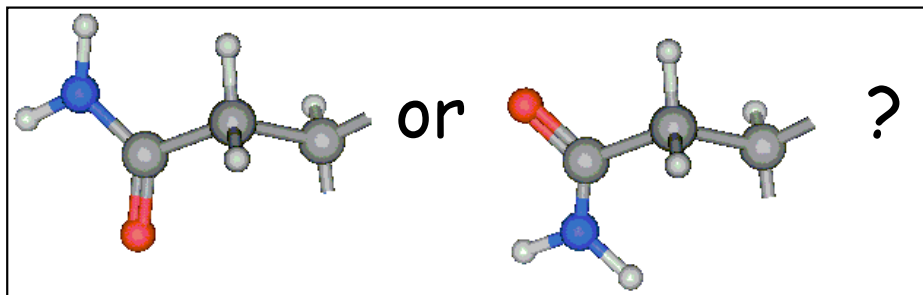
■ Alternative conformations



Nitrite reductase
at 1.0 Å resolution:
evidence for second
Asn conformation

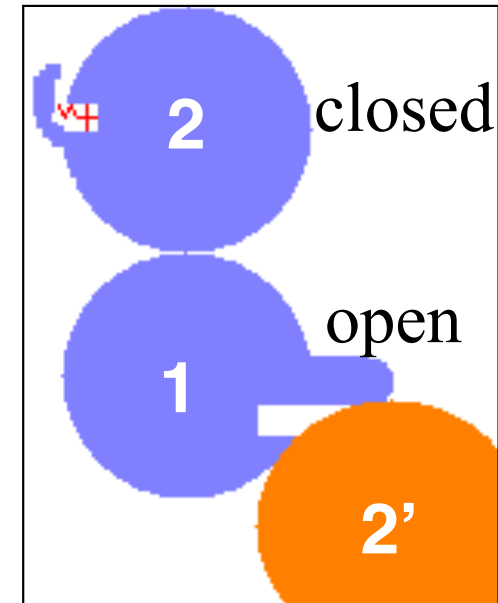
Biochem. 40: 9132-9141

- 'Flipped' amide or imidazole conformations may be needed: Asn (15%), Gln (15%), His (9%)



X-ray crystallography (cont.)

- Crystal waters: 30 - 70% of crystal is water
- Packing
e.g. triosephosphate isomerase
active site loop open / closed



- pH
- Hydrogen atoms: only visible at very high resolution (examine the 1.1 Å electron density map on page 8).

2.2 NMR

Principle:

nuclear spin couplings

internuclear distances

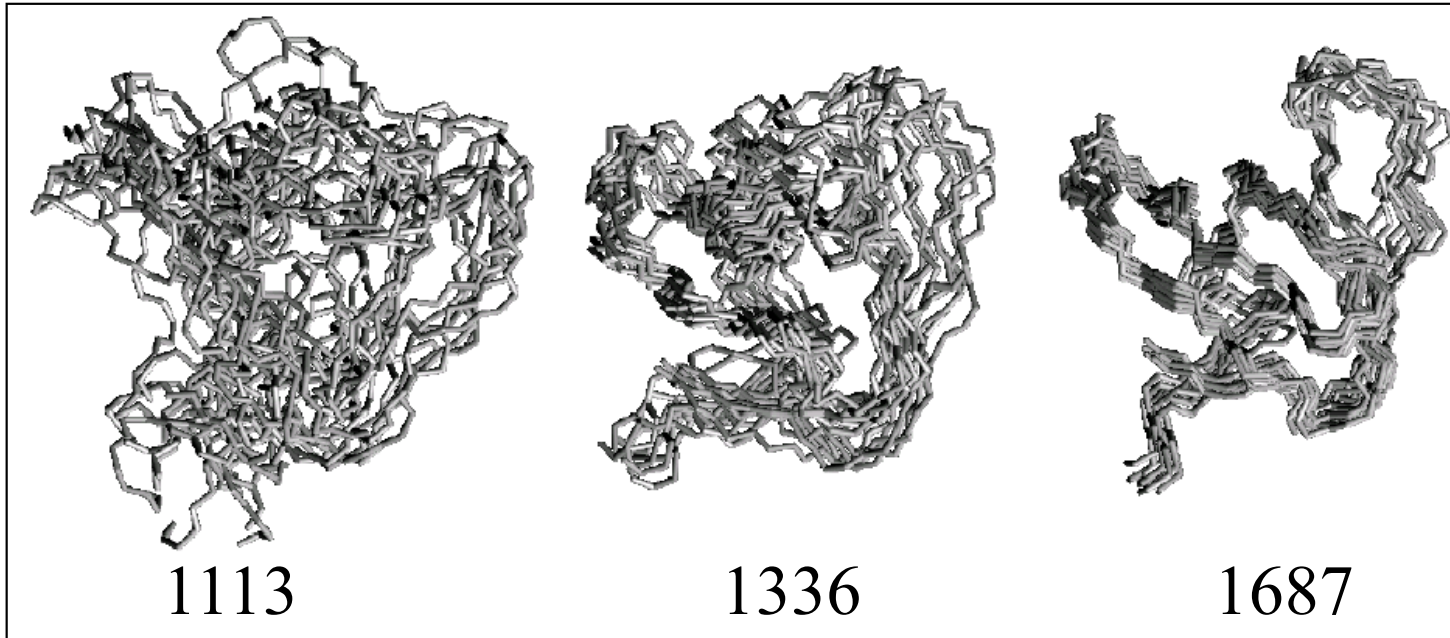
atomic coordinates

Restrictions:
bond lengths
bond angles,
vdW param.,
etc.

NMR (cont.)

Issues:

- Number of restraints



NMR ensemble: SH3 domain of Lck Tyr kinase

Biotechniques 29: 1278-1294

- All information is local

2.3 Comparative modeling

Principle:

protein 1: sequence

protein 2: sequence
& structure

sequence alignment

framework construction

PDB loops

loop construction

model refinement

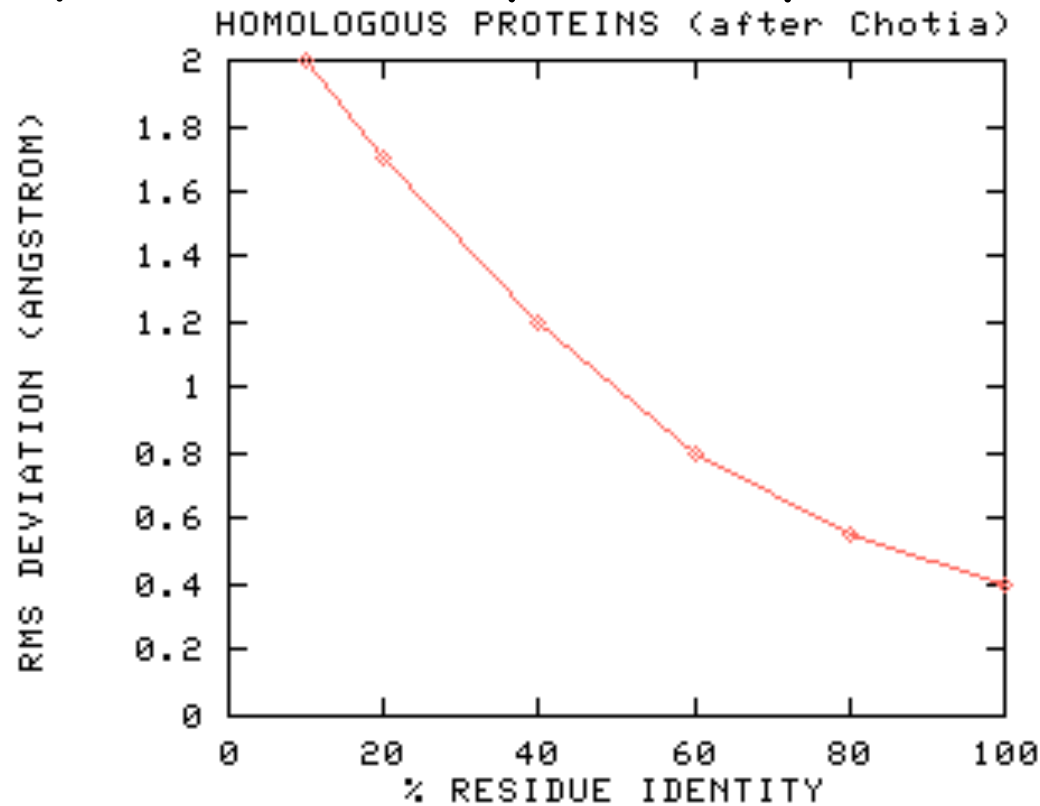
Restrictions:
bond lengths
bond angles,
vdW param.,
etc.

atomic coordinates

Comparative modeling (cont.)

Issues:

- sequence identity of template



EMBO J. 5: 823-826

- loops: mostly incorrect if more than 5 residues
(CASP5 competition, 2002)

3. Forces between molecules: electromagnetic

Fundamental forces of nature:

<i>Force</i>	<i>Relative strength</i>	<i>range</i>
strong	1	10^{-15} m
electromagnetic	$7 \cdot 10^{-3}$	∞
weak	10^{-5}	10^{-17} m
gravitation	$6 \cdot 10^{-39}$	∞

Molecular contacts occur at the Ångström level (10^{-10} m). Hence, only the electromagnetic force is relevant.

Forces between molecules (cont.)

Literature abounds with interactions:

- Ionic
- Coulombic
- Charge transfer
- Dipole
- Quadrupole
- Salt bridges
- Hydrogen bonds
- Dispersion interactions
- Van der Waals
- ...

All are manifestations of the electromagnetic force !

The electromagnetic force is described by Maxwell's equations:

1. The electric flux out of a closed surface is proportional to the enclosed charge:

$$\oint E \cdot dA = q/\epsilon_0 = 4\pi kq \quad \text{or} \quad \nabla \cdot E = \rho/\epsilon_0 = 4\pi k\rho$$

2. There is no net magnetic flux out of a closed surface:

$$\oint B \cdot dA = 0 \quad \text{or} \quad \nabla \cdot B = 0$$

3. The line integral of the electric field around a closed loop equals the negative of the rate of change of the magnetic flux through the area enclosed by the loop:

$$\oint E \cdot ds = -\frac{d\Phi_B}{dt} \quad \text{or} \quad -\nabla \times E = -\frac{\partial B}{\partial t}$$

4. The line integral of the magnetic field around a closed loop is proportional to the electric current flowing through the loop:

$$\oint B \cdot ds = \mu_0 i + \frac{1}{c^2} \frac{\partial}{\partial t} \oint E \cdot dA \quad \text{or} \quad \nabla \times B = \frac{4\pi k}{c^2} J + \frac{1}{c^2} \frac{\partial E}{\partial t}$$

Maxwell's equations (cont.)

Symbols:

E = electric field	B = magnetic field
ρ = charge density	ϵ_0 = permittivity
J = current density	μ_0 = permeability
i = electric current	c = speed of light

4. From quantum to molecular mechanics

Electromagnetism at the atomic level: [Schrödinger equation](#)

For any system of electrons (...*i, j, ...*) and nuclei (...*A, B, ...*):

$$H\Psi(\dots i, j, \dots, A, B, \dots) = E\Psi(\dots i, j, \dots, A, B, \dots)$$

$$H = \frac{-h^2}{8\pi^2} \sum_A \frac{1}{m_A} \nabla_A^2 + \frac{-h^2}{8\pi^2} \sum_i \frac{1}{m_i} \nabla_i^2 - \sum_i \sum_A \frac{Z_A e^2}{r_{iA}} + \sum_i \sum_{<j} \frac{e^2}{r_{ij}} + \sum_A \sum_{<B} \frac{Z_A Z_B e^2}{r_{AB}}$$

kinetic energy
nuclei

kinetic energy
electrons

pot. E
nuclei-electr.

pot. E
electrons

pot. E
nuclei

4. From quantum to molecular mechanics

Simplification: Born-Oppenheimer approximation

Nuclei are much more massive than electrons. Therefore, nuclei are nearly fixed with respect to electron motion.

Consequence:

$$H = \frac{-\hbar^2}{8\pi^2} \sum_A \frac{1}{m_A} \nabla_A^2 + \frac{-\hbar^2}{8\pi^2} \sum_i \frac{1}{m_i} \nabla_i^2 - \sum_i \sum_A \frac{Z_A e^2}{r_{iA}} + \sum_i \sum_{<j} \frac{e^2}{r_{ij}} + \sum_A \sum_{<B} \frac{Z_A Z_B e^2}{r_{AB}}$$

0

constant

From this **simplified wave function** the **electron density** can be calculated.

From quantum to molecular mechanics (cont.)

Forces on nuclei: [Hellmann-Feynman theorem](#)

The force on any nucleus, considered fixed, in any system on nuclei and electrons is just the **classical electrostatic interaction** exerted on the nucleus by **the other nuclei** and by **the electron charge density distribution for all electrons**.

One step further: empirism

Year 2000: Very high quality quantum chemical calculations, CCSD(T):

- Feasible for ~ 15 atoms
- Relative energy error for conformations: ~ 0.2 kcal/mol

But, SBDD deals with thousands of atoms ...

Empirical methods are used: **molecular mechanics**.

5. Molecular mechanics: assumptions

1. Born-Oppenheimer approximation.
2. Hellman-Feynman theorem.
3. Electron density around nuclei can be collapsed with the nuclear charge, yielding partial atomic charges.

Issues:

- Potentials are needed to replace the electron density 'glue'. E.g. bond length potentials.
- Atomic 'ownership' of electron density in any part of space.

Partial atomic charge is no physical property !

Molecular mechanics: assumptions (cont.)

4. Quantum model can be replaced by classical force-field that reproduces molecular properties.

Issues:

- What **properties**? Geometry, relative conformational energy, heat of formation, absorption frequencies, ...
- **Partitioning:**

$$E = E_{bonds} + E_{angles} + E_{dihedrals} + E_{coulomb} + E_{vdW} + \dots$$

Complete? (e.g. cross-terms needed? polarization?)

There is no 'correct' way of partitioning: different in all ff. Consequence: swapping parameters between ff. is dicey.

Molecular mechanics: assumptions (cont.)

5. Transferability:

i.e. the validity of force-field parameters for molecules that were not used for parameterization.

Issue:

- How many atom types needed ?

6. Additivity:

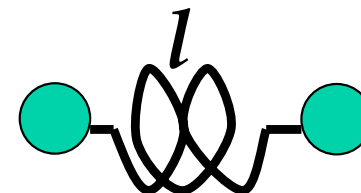
i.e. $E(A,B,C) = E(A,B) + E(A,C) + E(B,C)$

Issues:

- Polarization effects are non-additive.
e.g. with pairwise vdW potentials the dispersion interactions of crystalline Ar are underestimated by 10%.
- Entropy effects are non-additive.

6. Force-fields

Bond lengths



Bonds between similar atoms in a wide variety of molecules have similar lengths. Hence the idea of a **reference** bond length l_0 . Deviations from the reference cost energy:

$$E(l) = E(l_0) + \Delta l \frac{\partial E}{\partial l} + \frac{(\Delta l)^2}{2} \frac{\partial^2 E}{\partial l^2} + \dots \text{ (Taylor series)}$$

↓ ↓ ↓

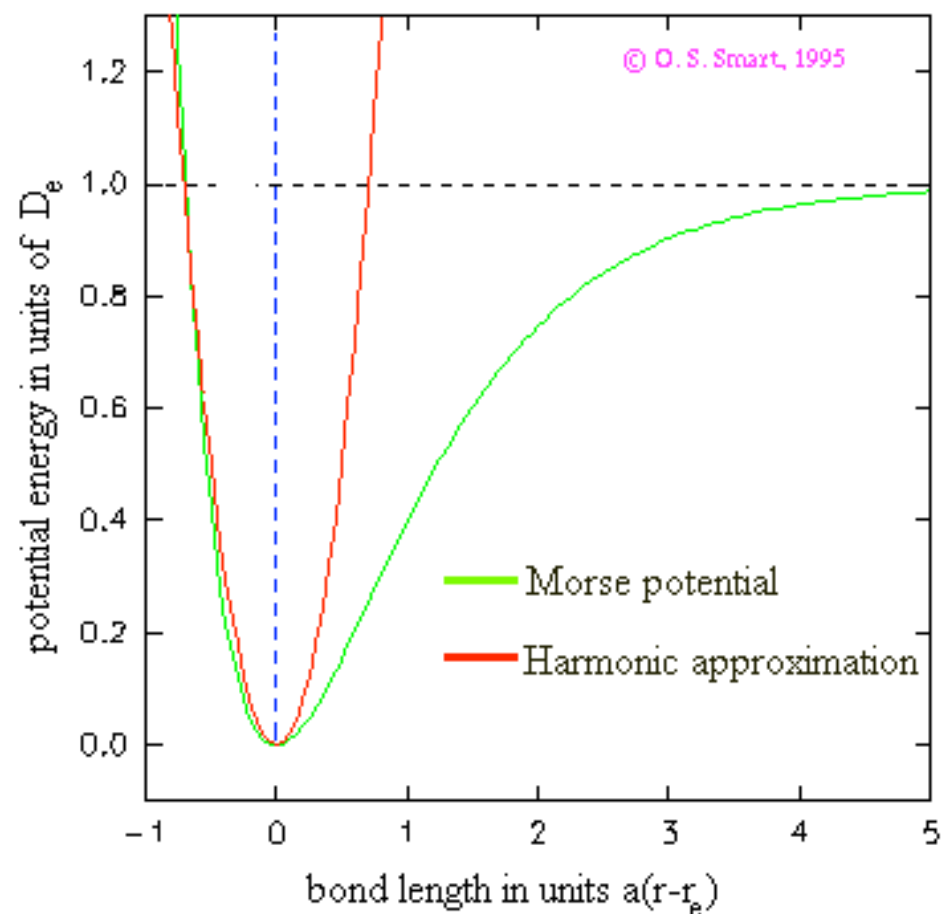
set = 0 near $l_0 = 0$ significant

$$E(l) = \frac{k_{bond}}{2} (l - l_0)^2$$

harmonic approximation

Bond lengths (cont.)

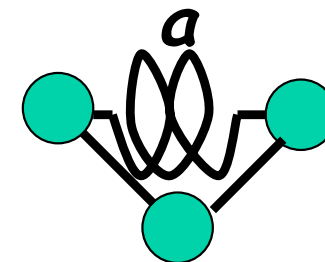
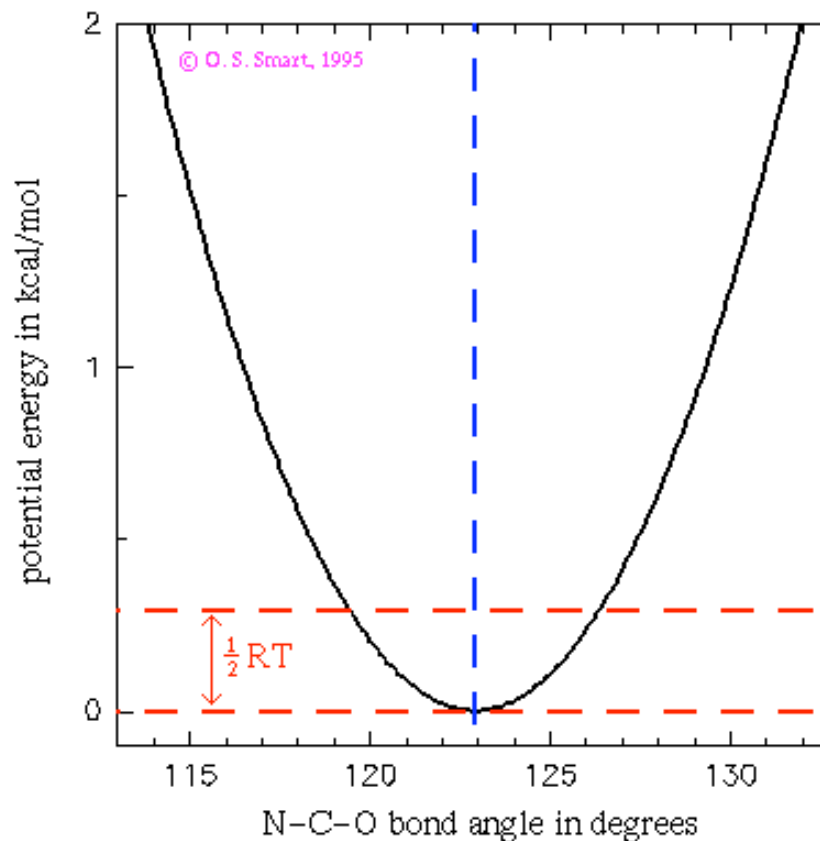
- Harmonic approximation: ok near equilibrium.
- Morse potential: closer to reality



- Typical k_{bond} : $\text{C}(\text{sp}^3)\text{-C}(\text{sp}^3)$ 300 kcal/(\AA^2 mol)

Bond angles

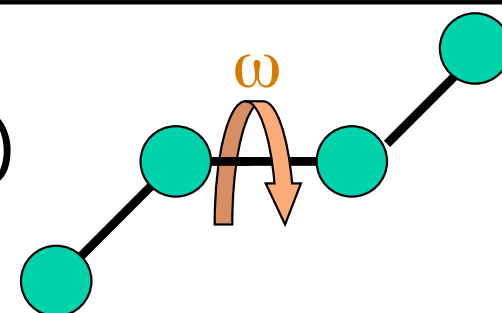
$$E(l) = \frac{k_{angle}}{2} (a - a_0)^2$$



- Typical k_{angle} : C(sp³)-C(sp³)-C(sp³) 0.01 kcal/(°² mol)
- Central atom usually participates in several angles: not independent (6 angles for C (sp³)).

Torsion angles (aka dihedral angles)

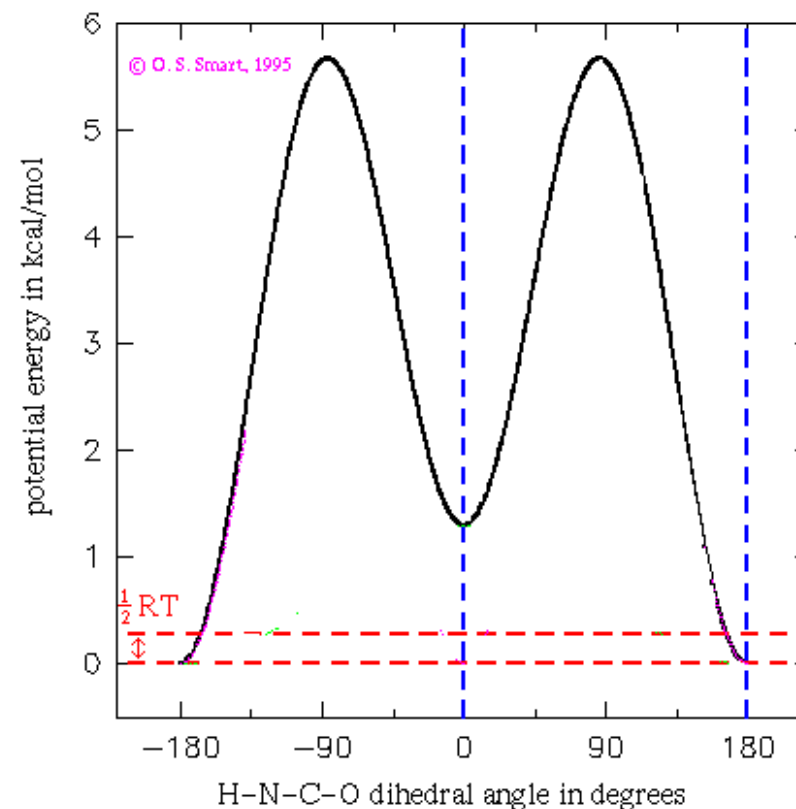
Non-bonded interactions (Coulomb and vdW) are insufficient to reproduce energies of conformers and torsion barriers.



Why? Does not include hyperconjugation, etc.

$$E_{\text{torsion}} = \sum_{n=0}^N \frac{V_n}{2} [1 + \cos(n\omega - \omega_0)]$$

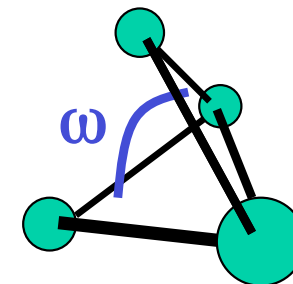
- n : multiplicity, the number of minima
- ω_0 : phase factor, the angular shift of the cos function



Improper torsions

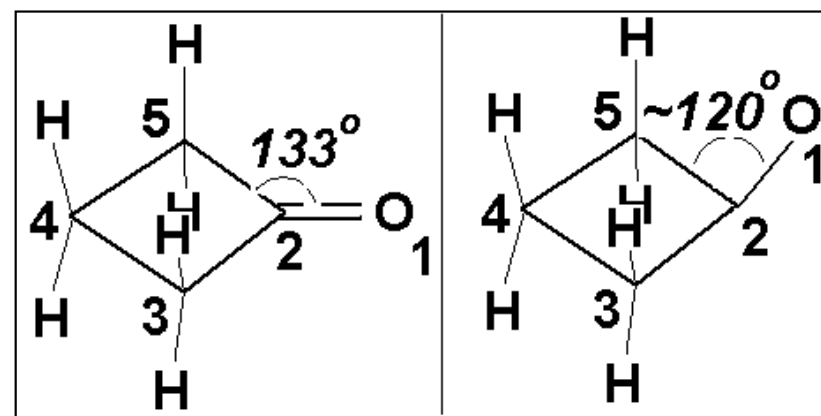
Often correct hybridization geometry is not maintained, especially in MD simulations:

- planarity
- chirality



Example: cyclobutanone

- $\langle CCO = 133^\circ$, but $a_0 = 120^\circ$
- the slightest non-planarity will lead to one of the $\langle CCO \rightarrow 120^\circ$
- solution: impose improper dihedral $\omega_{1-5-3-3-2} = 0$



$$E_{impr} = k_{impr} (1 - \cos 2\omega)$$

Electrostatics

Maxwell's law for the electrostatic field: $\oint E \cdot dA = q/\epsilon$

- In vacuum: $\epsilon = \epsilon_0$
- In a homogeneous dielectric: $\epsilon = \epsilon_0 \epsilon_r$
 ϵ_r is called the dielectric constant or relative permittivity
- Special case: two interacting point charges separated by distance r_{12}

$$E_{ES} = \frac{q_1 q_2}{4\pi\epsilon r_{12}} \quad \text{aka Coulomb's law}$$

if q_1, q_2 in units of $|e^-|$, then $E_{ES} = -332 \frac{q_1 q_2}{\epsilon r_{12}} \text{ kcal/mol}$

- Problems:
 - **biology occurs in non-homogeneous dielectric media !**
 - **partial atomic charges**

Electrostatics (cont.)

Electrostatics in non-homogeneous dielectric media will be discussed in a later lecture on solvation.

Various schemes to derive partial atomic charges are in use. In order of increasing quality:

- Gasteiger-Marsili charges: based on electronegativity equilibration.
- Mulliken population analysis: squaring of the wave function.
- Bader charges: volume integration of the squared wave function.
- CHELP, CHELPG, RESP: charges fitted to reproduce the electrostatic potential derived from the wave function.

the only acceptable charges

van der Waals

▪ Repulsion

At short distances atoms repel each other. Why?

- Pauli exclusion principle: no two electrons can have the same quantum numbers.
- Consequence: electrons get excited to higher levels.
- Result: electron density between nuclei decreases, leading to nuclear repulsion.

$$E_{rep} = \frac{C_{rep}}{r^{12}}$$

There is no theoretical justification for the r^{-12} form of this potential.

van der Waals (cont.)

■ Attraction

- Between all atoms and molecules there is an attractive force due to temporarily induced changes in electron density (dipole-dipole, dipole-quadrupole, quadrupole-quadrupole, ...).

$$E_{London} = -C_{disp}/r^6 - C_{disp}/r^8 - C_{disp}/r^{10} - \dots$$

Higher order terms are usually absorbed in the 1st term.

- When polar molecules approach each other their permanent electrostatic field induces dipoles.

$$E_{pol,ind} = -C_{pol,ind}/r^6$$

- Rotational polarization involves orienting dipoles.

$$E_{pol,ori} = -C_{pol,ori}/r^6$$

van der Waals (cont.)

- Sum: Lennard-Jones potential

$$E_{LJ} = C_{rep} / r^{12} - C_{attr} / r^6$$

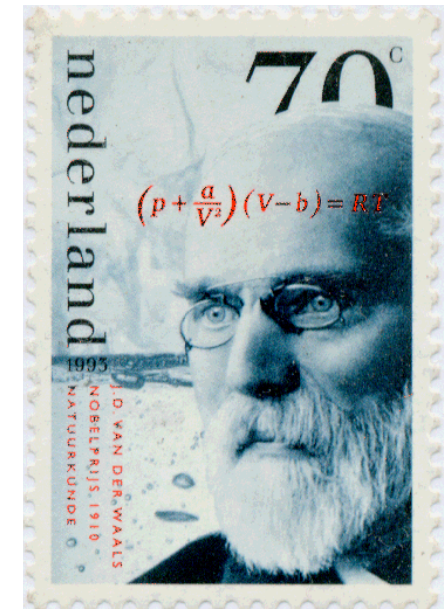
Often recast in terms of:

- R , distance where E is a minimum
- ϵ , the E of the minimum

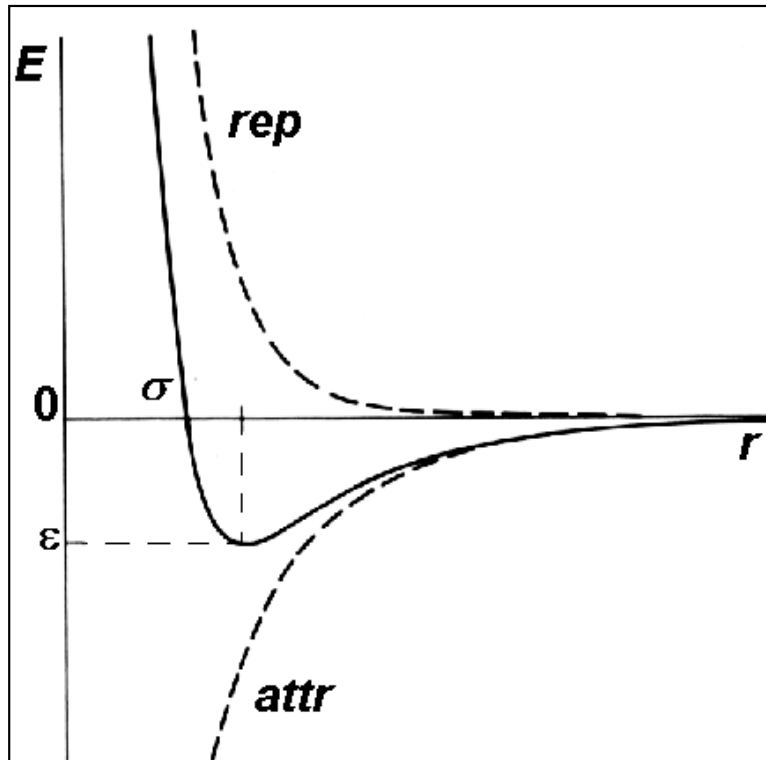
$$E_{LJ} = \epsilon \left[\left(\frac{R}{r} \right)^{12} - 2 \left(\frac{R}{r} \right)^6 \right]$$

Or in terms of σ , the collision diameter, where E is zero:

$$E_{LJ} = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - 2 \left(\frac{\sigma}{r} \right)^6 \right]$$



van der Waals (cont.)



Common mixing rules (Lorentz-Berthelot):

$$\sigma_{AB} = \frac{1}{2} (\sigma_{AA} + \sigma_{BB})$$

$$\epsilon_{AB} = (\epsilon_{AA} \epsilon_{BB})^{0.5}$$

van der Waals (cont.)

- In most force-fields 1-3 vdW interactions are ignored.
- In many force-fields 1-4 vdW are scaled down.
- Several force-fields replace the E_{LJ} in case of hydrogen bonds with:

$$E_{LJ} = \left[C_{rep} / r^{12} - C_{attr} / r^{10} \right] \cos^4 \theta$$

where θ is the donor-H...acceptor angle.

7. Suggested readings

General:

- Andrew R. Leach. "Molecular modelling: Principles and applications". Addison-Wesley Longman, Harlow, 1996.
- Jacob N. Israelachvili. "Intermolecular and surface forces". 2nd ed., Acad. Press., London, San Diego, 1991.

Examples from introduction:

- Vancomycin: Hubbard B.K., Walsh C.T.(2003). Vancomycin assembly: nature's way. *Angew. Chem. Int. Ed.* 42: 730-65.
- GAPDH: Aronov A.M., Buckner F.S., Van Voorhis W.C., Verlinde C.L.M.J., Opperdoes F.R., Hol W.G.J., Gelb M.H. (1999). Structure-based design of sub-micromolar, biologically active inhibitors of trypanosomatid glyceraldehyde- 3-phosphate dehydrogenase. *PNAS USA* 96: 4273-4278.
- PTP-B: Doman T.N., McGovern S.L., Witherbee B.J., Kasten T.P., Kurumbail R., Stallings W.C., Connolly D.T., Shoichet B.K. (2002). Molecular docking and high-throughput screening for novel inhibitors of protein tyrosine phosphatase-1B. *J.Med.Chem.* 45: 2213-2221.

Suggested readings (cont.)

Structures:

- X-ray crystallography:
 - Davis A.M, Teague S.J., Kleywegt G.J.(2003). Application and Limitations of X-ray Crystallographic Data in Structure-Based Ligand and Drug Design. *Angew. Chem. Int. Ed.* 42: 2718-2736.
 - Kleywegt G.J., Henrick K., Dodson E.J., van Aalten D.M. (2003). Pound-wise but penny-foolish: How well do micromolecules fare in macro-molecular refinement? *Structure* 11: 1051-1059.
 - McDonald I.K. and Thornton J.M. (1995) The application of hydrogen bonding analysis in X-ray crystallography to help orientate asparagine, glutamine and histidine side chains. *Protein Eng.* 8: 217-224.
- NMR:
 - Wider G. (2000). Structure Determination of Biological Macromolecules in Solution Using NMR spectroscopy *BioTechniques* 29: 1278-1294.

Suggested readings (cont.)

Structures (cont.):

- Comparative modeling:
 - Ring, C.S., Cohen F.E. (1993). Modeling protein structures: construction and their applications. *FASEB J.* 7: 783-790.
 - Chothia C, Lesk AM. (1986). The relation between the divergence of sequence and structure in proteins. *EMBO J.* 5: 823-826.
 - Tramontano A, Morea V. (2003). Assessment of homology-based predictions in CASP5. *Proteins* 53 Suppl 6: 352-368.

Quantum chemistry:

- Barden C.J., Schaeffer H.F.(2000). Quantum chemistry in the 21st century. *Pure and Applied Chem.*72: 1405-1423.
- Pople J.A. (1999). Quantum chemical models (Noble lecture). *Angew. Chem. Int. Ed.* 38: 1894-1902.

Suggested readings (cont.)

Additivity

- Dill, K.A. (1997). Additivity principles in biochemistry. *J. Biol. Chem.* 272: 701-714.

Force-fields

- See: Leach, Chapter 3 "Empirical force-field models".