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Neutrinos in Gauge Theories

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WHY IT IS DIFFICULT TO DISCUSS ν MASS

We agreed that we need 3 active ν_L's to have mass, in order to explain oscillations. But

(1) We do not know whether

$$-\bar{\nu}_L m_D \nu_R + h.c. \quad \text{.OR.} \quad -\bar{\nu}_L m C \bar{\nu}_L^t + h.c.$$

Since oscillations do not distinguish them (e.g., oscillations do not probe total lepton number, only lepton flavors)

(2) The seesaw with ν_R, and with crucial hypothesis "ν_R ARE HEAVY" is appealing, but not a unique possibility. E.g., add to SM a scalar that is $SU(2)_L$ Triplet and has hypercharge $= +1$ (thus, $Q = T_{3L} + Y$ gives $Q = +2, +1, 0$). We will have in the lagrangian the Yukawa type term

$$Y_{ij}\, \bar{l}_{Li}\, i\sigma_2 \cdot \vec{\sigma}\vec{\Delta} \cdot C^{-1} \cdot l_j$$

$$= Y_{ij} \left(-e_i, \nu_i\right) \begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2, & -\Delta_3 \end{pmatrix} C^{-1} \begin{pmatrix} \nu_j \\ e_j \end{pmatrix}$$

Thus, $\langle \Delta^0 \rangle = \langle \frac{\Delta_1 + i\Delta_2}{\sqrt{2}} \rangle \neq 0$ is an alternative possibility to have ν_L masses.

(3) Even assuming that the mass has Majorana character, one can have various mechanism. In the SM parlance, as clarified by Weinberg (and independently by Wilczek & Zee), one likes to consider the SM renormalizable lagrangian plus the set of gauge invariant, non-ren. operators:

$$\mathcal{L} = \mathcal{L}_{SM}^{ren.} + \frac{1}{M_{ij}} \bar{l}_i H \cdot C \cdot \bar{l}_j^t H + \dots$$

The first one, with canonical dim = 5, is the one we discussed for ν masses; however, its origin is left unspecified here.

EXERCISE Why not to write <u>else</u> $\bar{l}\vec{\sigma}H \cdot G \cdot \bar{l}\vec{\sigma}H^t$? Why not (we recall $H \equiv (H^0, H^-)$) ? Why not $\bar{l}C\epsilon\bar{l}^t \cdot H\epsilon H$ (where $\epsilon = i\sigma_2$) ? Why not $\bar{l}C\epsilon\vec{\sigma}\bar{l}^t \cdot H\epsilon\vec{\sigma}H$? [Answer: there is only 1 dim. 5 operator]

(4) The problem is that the above language does not give much clues. One way to see it is to consider the additional dim = 9 operator:

$$\frac{1}{M^5} \bar{d}_R u_R e_R C^{-1} e_R \bar{d}_R u_R + h.c.$$

It leads to $\Delta L_e = \pm 2$, thus to $0\nu2\beta$ process. Its existence shows that even if ν-mass is Majorana, it is not for granted that ν-mass contribution ~~to~~ to $0\nu2\beta$ is the leading one.

Still, one could abuse of the language
of the effective operators ~~as~~ as follows:
we could believe that the dimension-ful
parameters are "large" in comparison to EW (SM)
scale. Thus, we better think that dim=9
operators is strongly suppressed in comparison
to dim=5 operators — namely, ν-masses.

But this is not much more than prejudice.
In order to discuss the meaning of
ν-masses, we have to examine concrete
models for the physics beyond the SM,
After declaring the gauge symmetry, after fixing ~~instead~~ which new
~~... the hope ...~~
particles exist, and which do not, we
could eventually hope to connect the
physics of neutrino masses to new
observable phenomena.

In this course, we will not go this far. We
will simply examine a few promising
gauge group and analyze their particle
content.

LEPTON # IN SUPERSYMMETRY (few words)

THE DEFINITION HAS TO BE EXTENDED FROM LEPTONS TO THEIR SCALAR PARTNERS, AS IMPLIED BY:

$$m\tilde{Z}\tilde{Z} + g\tilde{\nu}^{*}\tilde{Z}\nu$$

(zino mass) ($\tilde{\nu}$-ν-gaugino interaction)

THUS, ONE CAN THINK OF LEPTON # ACTING ON THE SUPERFIELD.

BUT GAUGE INVARIANCE ALONE ADMITS SOME TERMS IN SUPERPOT:

$$LLE^c,\ LH_u,\ LQD^c$$

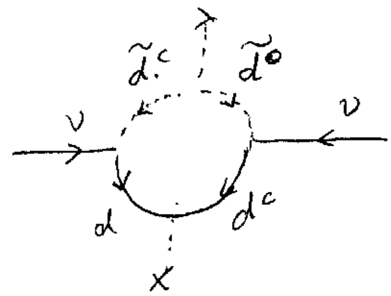
SINCE L and H_d HAVE THE SAME GAUGE PROPERTIES.

THUS, LEPTON # IS NOT AN ACCIDENTAL SYMMETRY as it was in SM.

ONE CAN IMPOSE THE LEPTON # BY HAND (or if one likes it more, one can require "matter parity" such that $L \to -L$, $Q \to -Q$, $E \to -E$, $D \to -D$, $U \to -U$ or "R-parity" where also $\theta \to -\theta$). BUT THIS MEANS THAT A MORE FAIR TERMINOLOGY WOULD REQUIRE TO CALL "MSSM" AS "MINIMAL SUPERSYMMETRIC EXTENSION OF SM WITH LEPTON # IMPOSED" (or something alike) SINCE THIS HYPOTHESIS IS AN INDEPENDENT ASSUMPTION, THAT HAS NOTHING TO DO WITH SM-GAUGE SYMMETRY.

<u>EXERCISE</u> Even in MSSM, without the above terms, one could have $\langle\tilde{\nu}\rangle \neq 0$. Can you imagine which type of problems we would meet in this case?

THERE ARE MANY OPTIONS TO GENERATE ν MASSES, AS:



MANY POSSIBLE MANIFESTATIONS IN COLLIDERS, e.g., NEUTRALINO DECAY.

(ALSO POSSIBLY PROTON DECAY, IF ALSO, $\lambda\, u^c d^c d^c$ COUPLING IS PRESENT)

ONE OF THE MAIN DOUBTS, HOWEVER, IS THAT ν MASSES "TEND" TO BE LARGE.

RIGHT-HANDED NEUTRINOS (ν_R) IN EXTENDED SM

The addition of ν_R is "suggested" by the spectrum of the SM:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} u_R \\ d_R \end{matrix} \quad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{matrix} \dots \\ e_R \end{matrix}$$

Something is missing!

Its gauge numbers are fixed: it has no color, it has no $SU(2)_L$ charge, it has no charge (thus, $Q = T_{3L} + Y \Rightarrow Y_{\nu_R} = 0$, too). It is a <u>SINGLET</u>, no SM gauge interactions.

It participates in Yukawa-type interactions and can be given Majorana mass:

$$\delta\mathcal{L} = -\,\overline{l}_{L_i}^{A}\, Y^{(\nu)}_{ij}\, \nu_{R_j} \cdot H^A + \frac{1}{2}\, \nu^t_{R_i} C^{-1} M_{ij}\, \nu_{R_j} + h.c.$$

when $l^A = \begin{pmatrix} \nu \\ e \end{pmatrix}$ AND $H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$. Consider the basis where Majorana mass matrix is diagonal

$$M_{ij} = M_i \cdot \delta_{ij}$$

Rewrite the free lagrangian of ν_R as follows

$$\mathcal{L}_{free} = i\,\overline{\nu_R}\,\hat{\partial}\,\nu_R + \frac{1}{2}\left(\nu_R C^{-1} \nu_R M + \overline{\nu_R}\, C\, \overline{\nu_R}^t\right) M$$

$$= \frac{1}{2}\left(\overline{N} \cdot \hat{\partial} \cdot N - \overline{N} N \cdot M\right) + \text{total divergence}$$

where $N = \nu_R + \nu_L^c$. (We use: $\nu_L^c \equiv C\left(\overline{\nu_R}\right)^t$)

~~EXE. prove that this can be done using [illegible] and [illegible] as appropriate to spinors.~~

RIGHT HANDED NEUTRINOS IN (UNIFIED) GAUGE GROUPS

- AMONG THE INTERESTING OPEN QUESTIONS IN THEORETICAL NEUTRINO PHYSICS, THERE ARE THOSE ON THE <u>EXISTENCE</u> OF RIGHT HANDED NEUTRINOS AND ON THE <u>NATURE</u> OF NEUTRINO MASSES.

- THE PERSPECTIVES TO ANSWER THEM DIRECTLY SEEM TO BE QUITE LIMITED, AT PRESENT. ON THE OTHER HAND, THERE IS SOME HOPE TO INVESTIGATE THEM WITHIN <u>SPECIFIC</u> THEORETICAL MODELS.

- ALTHOUGH WE DO NOT HAVE A FULLY AGREED APPROACH, IT IS QUITE INTERESTING THAT SOME "UNIFIED MODELS", NAMELY, <u>GAUGE GROUPS THAT INCLUDE AND EXTEND THE STANDARD MODEL</u>, OFFER SOME CLUES TO THESE QUESTION. WE WILL TOUCH VERY BRIEFLY A FEW OF THESE MODELS, HOPING TO INTRODUCE THE INTERESTED READER TO THE VAST LITERATURE ON THE SUBJECT.

(1) LEFT-RIGHT EXTENSION OF THE SM

ONE PRINCIPAL AVENUE ARE THE LEFT-RIGHT EXTENSIONS OF THE SM, WHERE THE QUARK AND LEPTONS OF ONE FAMILY FIT INTO:

$$SU(2)_L \text{ doublets } \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad ; \quad SU(2)_R \text{ doublets } \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

THUS, THE EXISTENCE OF ν_R IS LINKED TO THE EXISTENCE OF e_R

<u>EXE</u>. The electric charge reads $Q = T_{3L} + T_{3R} + X$. Identify the U(1) charge X with a suitable combination of B and L.

② SU(5)

- The fundamental repres. of SU(5) can be decomposed (interpreted) as follows:

$$\begin{pmatrix} \frac{\lambda}{2} & 0_{2\times3} \\ 0_{3\times2} & 0_{2\times2} \end{pmatrix} \quad \begin{pmatrix} 0_{3\times3} & 0 \\ 0 & \frac{\sigma}{2} \end{pmatrix} \quad \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & 0 & \\ & & -\frac{1}{3} & & \\ & 0 & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{pmatrix} \quad \begin{pmatrix} 0 & \cdots \\ \cdots & 0 \end{pmatrix}$$

COLOR, LEFT ISOSPIN, HYPERCHARGE, OTHER STUFF.

IN FACT:

$$5 \equiv \begin{pmatrix} d_{1R} \\ d_{2R} \\ d_{3R} \\ -e_R^c \\ \nu_R^c \end{pmatrix} = \left(3, 1, -\tfrac{1}{3}\right) \oplus \left(1, 2, \tfrac{1}{2}\right)$$

WHERE WE CONSIDER THE CONJUGATE OF THE LEPTONIC DOUBLET ON SU(2) -SPACE AND ON LORENTZ -SPACE:

$$\ell^c \equiv \epsilon \cdot C\, \bar{\ell}^t = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_R^c \\ e_R^c \end{pmatrix} = \begin{pmatrix} -e_R^c \\ \nu_R^c \end{pmatrix}$$

- NEXT, WE SHOW THAT THE OTHER FERMIONS OF SM FIT INTO ANOTHER REPRESENTATION, GIVEN BY THE ASYMMETRIC PART OF $5 \cdot 5$:

$$\begin{aligned} 10 \equiv (5 \cdot 5)_{Asymm} = & \left((3\cdot3)_A, 1, -\tfrac{2}{3}\right) \\ & \oplus \left(1, (2\cdot2)_A, 1\right) \\ & \oplus \left(3, 2, \tfrac{1}{6}\right) = \end{aligned}$$

$$= \left(\bar{3}, 1, -\tfrac{2}{3}\right) \oplus (1, 1, 1) + \left(3, 2, \tfrac{1}{6}\right)$$

$$\sim u_L^c \oplus e_L^c \oplus q_L$$

- IT IS REMARKABLE THAT 5 includes d_R AND $(\ell_L)^c$ AND THAT 10 includes $(u_R)^c$, $(e_R)^c$, q_L.
BUT THERE IS NOT SPACE FOR Right-handed Neutrinos. As IN SM, this is a singlet, $1 = \nu_R$.

(3) SO(10)

SO(10) seems very big at first sight. HOWEVER, ORTHOGONAL GROUPS IN EVEN SPACE DIM, SO(2n) ARE INTERESTING SINCE THEY EXTEND UNITARY GROUPS U(n):

$$z \to U \cdot z \iff z' + i z'' \to (U' + i U'')(z' + i z'')$$

$U \in U(n)$

COMPLEX VECTOR IN n-dim

REAL PART, IMAGINARY PART

$$\iff \begin{pmatrix} z' \\ z'' \end{pmatrix} \to \begin{pmatrix} U' & -U'' \\ U'' & U' \end{pmatrix} \begin{pmatrix} z' \\ z'' \end{pmatrix}$$

REAL VECTOR IN 2n-DIM

ORTHOGONAL

WE MADE AN IMPORTANT OBSERVATION: $U(n) \subset SO(2n)$.

- THUS, WE CAN NOTE THAT $SU(5) \subset SO(10)$
- ALSO WE CAN NOTE $SO(6) \times SO(4) \subset SO(10)$, AND $SU(3)_c \subset SO(6)$, $SU(2)_L \subset SO(4)$; THUS IT LOOKS THAT WE HAVE WIDE SPACE TO INCLUDE SM INTO $SO(10)$.
- THE DETAILS OF THE CORRESPONDENCE ARE INTERESTING. WE WILL DESCRIBE SOME ASPECTS OF THE ELEGANT UNDERLYING FORMALISM, WITHOUT ENTERING HOWEVER INTO ALL MATHEMATICAL DETAILS. ~~WE~~ ~~MATHEMATICAL STATEMENTS (that~~ ~~BEGIN WITH TWO MATHEMATICAL~~ ~~we will not prove here)~~

IN ORDER TO PROCEED, WE NEED A FEW MATH THEOREMS. (DO NOT WORRY OF PROOFS OR DETAILS NOW, WE TRY TO LEARN BY EXAMPLES).

(a) (DEFINITION OF "SPINORS")

DENOTE $+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ AND $- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

WE WILL CONSIDER THE 2^n VECTORS IN THE TENSOR SPACE AS

$$++-+--\ldots = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ldots$$

NAMED "SPINORS".

(b) (REPRESENTATION OF $SO(2n)$ ON "SPINORIAL" SPACE)

$SO(2n)$ IS REPRESENTED ON THE 2^{n-1} DIMENSIONAL SPACE OF "SPINORS" WITH EVEN (OR ODD) # OF MINUS SIGNS.

(c) (REPRESENTATION OF $U(n)$)

$U(n)$ IS REPRESENTED ON THE SUBSPACES WITH FIXED # OF MINUS SIGN.

$\boxed{1^{st} \text{ example}, \quad SO(4) \text{ namely } n=2}$

The $2^n = 4$ representation breaks into even and odd representations of $SO(4)$:

$++$	$\emptyset$ minus signs	SINGLET OF $SU(2)$
$--$	2 minus signs	"
$-+$	} 1 minus signs	DOUBLET OF $SU(2)$
$+-$		

The physical interpretation is:

$++$	SINGLET OF $SU(2)_L$	$\rightarrow (u_R)^c$		$(\nu_R)^c$
$--$	"	$\rightarrow (d_R)^c$	OR POSSIBLY	$(e_R)^c$
$-+$	} DOUBLET OF $SU(2)_L$	$\rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L$		$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$
$+-$				

2nd example, SO(6) namely $n=3$

THE EVEN AND ODD $2^{n-1}=4$ DIM. REPRESENTATIONS ARE

$+++$	singlet of SU(3)	$++-$, $+-+$, $-++$	triplet of SU(3)
$--+$, $-+-$, $+--$	triplet of SU(3) ;	$---$	singlet of SU(3)

THE PHYSICAL INTERPRETATION OF SINGLETS is "LEPTONS" (OR ANTILEPTONS) THE ONE OF TRIPLETS IS "QUARKS" (OR ANTIQUARKS).

3rd example, full SO(10) namely $n=5$

SO(6)	SO(4)	CHARGE	PHYSICAL IDENTIFICATION
$+++$	$++$	0	$(\nu_R)^c$
$+++$	$--$	$+1$	$(e_R)^c$
$--+$	$++$	$-2/3$	$(u_R)^c$
$-+-$	$++$	"	
$+--$	$++$	"	
$--+$	$--$	$+1/3$	$(d_R)^c$
$-+-$	$--$	"	
$+--$	$--$	"	
$++-$	$-+$	$+2/3$	u_L
$+-+$	$-+$	"	
$-++$	$-+$	"	
$++-$	$+-$	$-1/3$	d_L
$+-+$	$+-$	"	
$-++$	$+-$	"	
$---$	$-+$	0	ν_L
$---$	$+-$	-1	e_L

$$\text{CHARGE} = \frac{1}{6}\left(1^{st}\text{ sign} + 2^{nd}\text{ sign} + 3^{rd}\text{ sign}\right) - \frac{1}{2}\left(4^{th}\text{ sign}\right)$$

EXE.: Assign the above 16 spinors to SU(5) representations.

EXE.: SHOW THAT $\frac{1}{6}(1^{st}\text{sign} + 2^{nd}\text{sign} + 3^{rd}\text{sign})$ is A LINEAR COMBINATION OF B AND L.

WHAT IS MISSING...

USUALLY, THE MOST DIFFICULT PART OF A UNIFIED GROUP IS THE ONE CONCERNING HIGGS FIELDS.

INDEED, ONE NEEDS TO PRESCRIBE SOME HIGGS FIELDS TO BREAK THE GAUGE SYMMETRY AND (USUALLY OTHER) TO GIVE RISE TO FERMION MASSES.

FURTHERMORE, THE SCALAR POTENTIAL ARE OFTEN DIFFICULT TO BE ANALYZED.

WE DO NOT DISCUSS THESE IMPORTANT TOPIC HERE, AND LEAVE THE MOTIVATED STUDENT TO THE STUDY OF THE LITERATURE AND TO HIS/HER RESEARCH WORK ON UNIFIED GROUPS: GOOD LUCK!

EXERCISE: Construct a model with few higgs fields based on SO_{10} that obeys the experimental constraints. Derive the prediction for the proton decay rate.
[hint: if you solve this, please come to me that I have a few additional questions]

APPENDIX:

CONTENT OF THE COURSE

1. NEUTRINOS AND THEIR MANIFESTATIONS

WEAK INTERACTIONS – FERMI LAGRANGIAN – (V–A) STRUCTURE
MISSING ENERGY – NEUTRINO MASS? – LEPTON NUMBERS

2. FLAVOR OSCILLATIONS AND LEPTONIC MIXING

FIELDS & STATES – GENERAL FORMULAE – 2 FLAVOR CASE
INTERPRETATION OF THE DATA – SUMMARY OF PARAMETERS
OSCILLATIONS IN MATTER

3. MAJORANA NEUTRINOS

CHARGE CONJUGATION – MAJORANA SPINOR – MAJORANA MASS
$0\nu 2\beta$ & ν MASSES – SEESAW – LEPTONIC ASYMMETRY

4. NEUTRINOS IN GAUGE THEORIES

DIFFICULTIES TO DISCUSS THE MEANING OF ν MASS – EFFECTIVE OPERATORS – "SUSY" –
FERMIONS IN Left-right, SU_5 and SO_{10} gauge groups.

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Please feel free to write me at:
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I will be particularly happy to discuss solutions of exercises, to receive corrections
.... and in general to talk of physics!

— Thanks, Francesco. —