Generalized B-splines: a possible tool in Isogeometric Analysis

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In many application problems governed by partial differential equations, such as solids, structures and fluids the standard analysis methods, Finite Elements Methods (FEM), are based on crude approximations of the involved geometry. On the other hand, the geometric approximation inherent in the mesh can lead to accuracy problems.

Therefore in recent years an analysis framework based on functions capable of representing exact geometry was developed, giving rise to Isogeometric Analysis, where the term *isogeometric* is due to the fact that the solution space for dependent variables is represented in terms of the same functions which represent the geometry. Isogeometric Analysis so far developed is based on NURBS basis functions, and has revealed to be an effective tool as in this way it is possible to fit exact geometries at the coarsest level of discretization and eliminate geometry errors from the very beginning.

Nevertheless rational representations present several drawbacks. For instance rational curves require additional parameters (weights) whose selection is often not clear, the derivative of a degree-p rational curve is of degree $2p$. Exact integration of rational curves is hard and requires (whenever possible) non rational forms. Moreover the rational model cannot encompass transcendental curves: many of them (helix, cycloid, ...) are of interest in applications.

Therefore our attention has been focused on overcoming the problems of NURBS standards, analyzing Isogeometric Analysis schemes with different bases, but still equipped with refinement properties.
In particular, we consider piecewise functions whose sections belong to spaces of the form

$$< 1, t, \cdots, t^{p-3}, u(t), v(t) >$$

with $u(t) = \cos(\omega t),\ v(t) = \sin(\omega t)$, or $u(t) = \cosh(\omega t),\ v(t) = \sinh(\omega t)$, which allow exact representation of conic sections and provide spaces closed with respect to differentiation, so that seem extremely well suited for Isogeometric Analysis.

The above mentioned spaces possess all the interesting properties of classical polynomial splines. They admit a representation in term of functions which are a natural extension of polynomial B-splines and classical algorithms as degree elevation, knot insertion, differentiation formulas,etc... can be explicitly rephrased for them. So that they are properly referred to as *generalized B-splines.*
In this talk we present some applications of generalized B-splines in the context of Isogeometric Analysis. In particular we emphasize that Isogeometric Analysis can be seen as a superset of FEM, and the efficient solution of obtained linear systems is a crucial point whatever is the space of functions we deal with.

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