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Geomagnetic reversals: implications for the core and beyond

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Geomagnetic reversals: implications for the core and beyond

ICTP Conference 'From Core to Crust'

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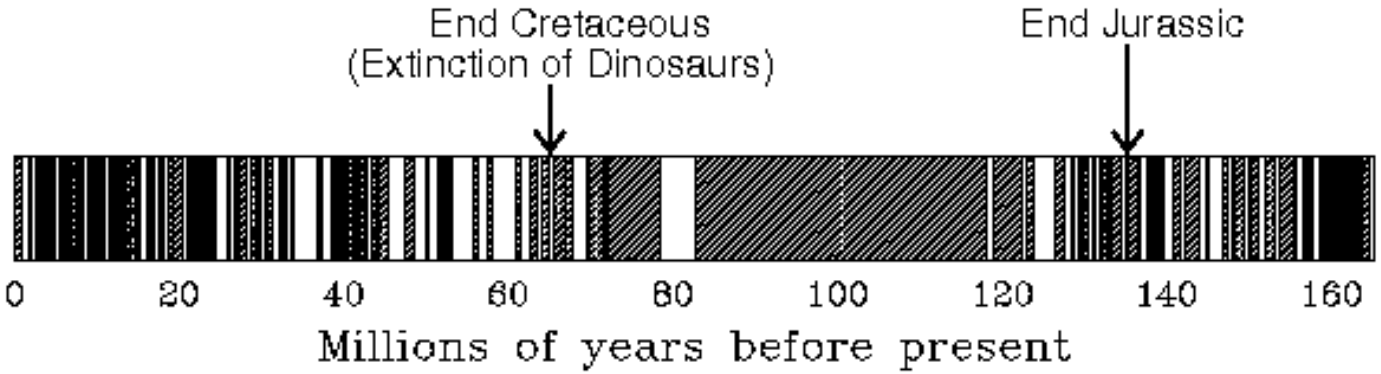
Outline

- Introduction/Motivation
- Reversal statistics/frequency
- Reversal mechanisms/diagnostics
- Conclusions

Reversals

Evidence of field reversals extends throughout the paleomagnetic record, although the record is most complete for more recent periods.



Inter-event times vary widely, from ‘subchrons’ to ‘superchrons’.

The geodynamo MHD equations

The geodynamo equations are symmetric with respect to reversals $\mathbf{B} \to -\mathbf{B}$.

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \nabla^2 \mathbf{B} \,,$$

$$Ro \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) + \mathbf{e}_z \times \mathbf{u} = -\nabla P + q\, Ra\, \Theta\, \mathbf{r}$$

$$+ E\, \nabla^2 \mathbf{u} + (\nabla \times \mathbf{B}) \times \mathbf{B} \,,$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{u} \cdot \nabla \Theta = q\, \nabla^2 \Theta + H(|\mathbf{B}|^2) \,,$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \cdot \mathbf{u} = 0 \,,$$

$$Ro = \frac{\eta}{2\Omega \mathcal{L}^2} \,, \quad q = \frac{\kappa}{\eta} \,, \quad Ra = \frac{g \alpha \beta \mathcal{L}^2}{2\Omega \kappa} \quad E = \frac{\nu}{2\Omega \mathcal{L}^2} \,.$$

Aspects of study

Different facets of reversals can be studied:

- Reversal statistics/frequency (long timescale)
- Reversal mechanisms/diagnostics (individual transitions)

Factors affecting these processes:

- Purely internal (core) effects
- CMB/Mantle effects
- Inner core growth/Earth evolution effects

Influences from outside the core

Mantle influences can arise in various ways:

- global — thermal lid, controlling the forcing parameter for convection
- spatially dependent — heterogeneous thermal b.c.s at the CMB

Inner core influences may similarly have different aspects:

- global — varying form of convection/energy budget, after IC formed
- spatially dependent — tangent cylinder effects of inner core

Geodynamo time scales

Observed phenomenon	Time scale
Secular variation	10–100 yr
Main field dynamics	1–10 kyr
Inter-reversal intervals	$>$ 100 kyr

Theoretical expectation	Time scale
Torsional oscillations/waves	10-100 yr
Convective over-turn	1 kyr
Magnetic diffusion	1–10 kyr
Mantle processes	$>$ 1 Myr
Inner core growth	$>$ 100 Myr

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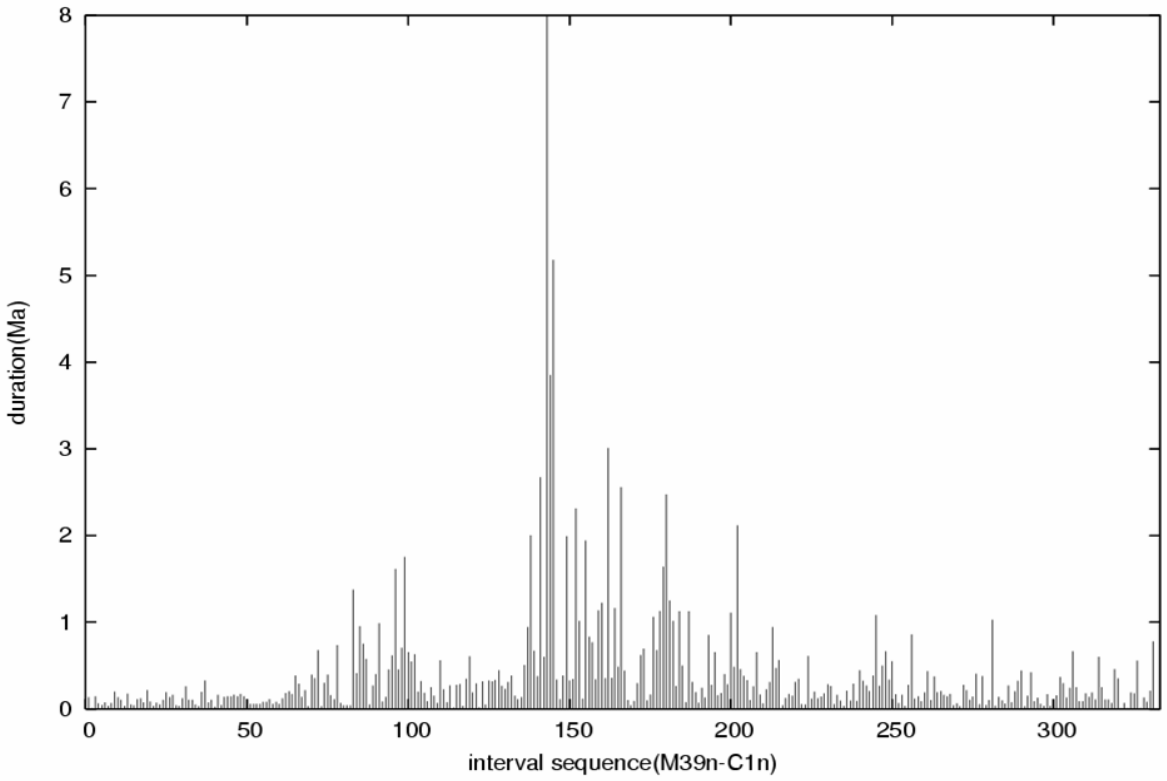
Inter-reversal durations

The dominant empirical approach has been to characterise reversals as arising from exponential or Gamma distributions, but with the mean reversal frequency varying over longer time scales.



Reversals from geodynamo calculations

Detailed MHD calculations have obtained reversals since Glatzmaier & Roberts (1995).

Sequences of multiple reversals have now been obtained by many authors, e.g. Glatzmaier et al. (1999), Kutzner & Christensen (2002), Li et al. (2002).

But the number of reversals obtained are not sufficient for serious statistical analysis of the long-term behaviour.

An alternative approach uses simpler models — toy, low order or axisymmetric — to allow comparison with long-term records, e.g. Hoyng et al. (2001), Melbourne et al. (2001), Ryan & Sarson (2007, 2008).

A low order $\alpha\omega$ model

We want to model the statistics of geomagnetic reversals on long timescales, beyond the range of detailed simulations.
We consider the toy ODE model:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\kappa S + \alpha T \ ,$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\kappa T + \omega S \ ,$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\kappa_\omega \omega + f_\omega \left[1 - \lambda_1 ST - \lambda_2 (S^2 + T^2)\right] \ .$$

For constant α, irregular reversals can already be obtained (cf. Rikitake 1958). With multiplicative noise in mind (cf. Hoyng et al. 2001), we want to couple these equations to a dynamically varying form of α.

Shell model of turbulence

Shell models provide a scalar analogue of the spectral Navier–Stokes equation.

The spectral domain is represented by N shells, of wavenumbers $k_n = k_0 2^n$, $n = 1, 2, \ldots, N$.

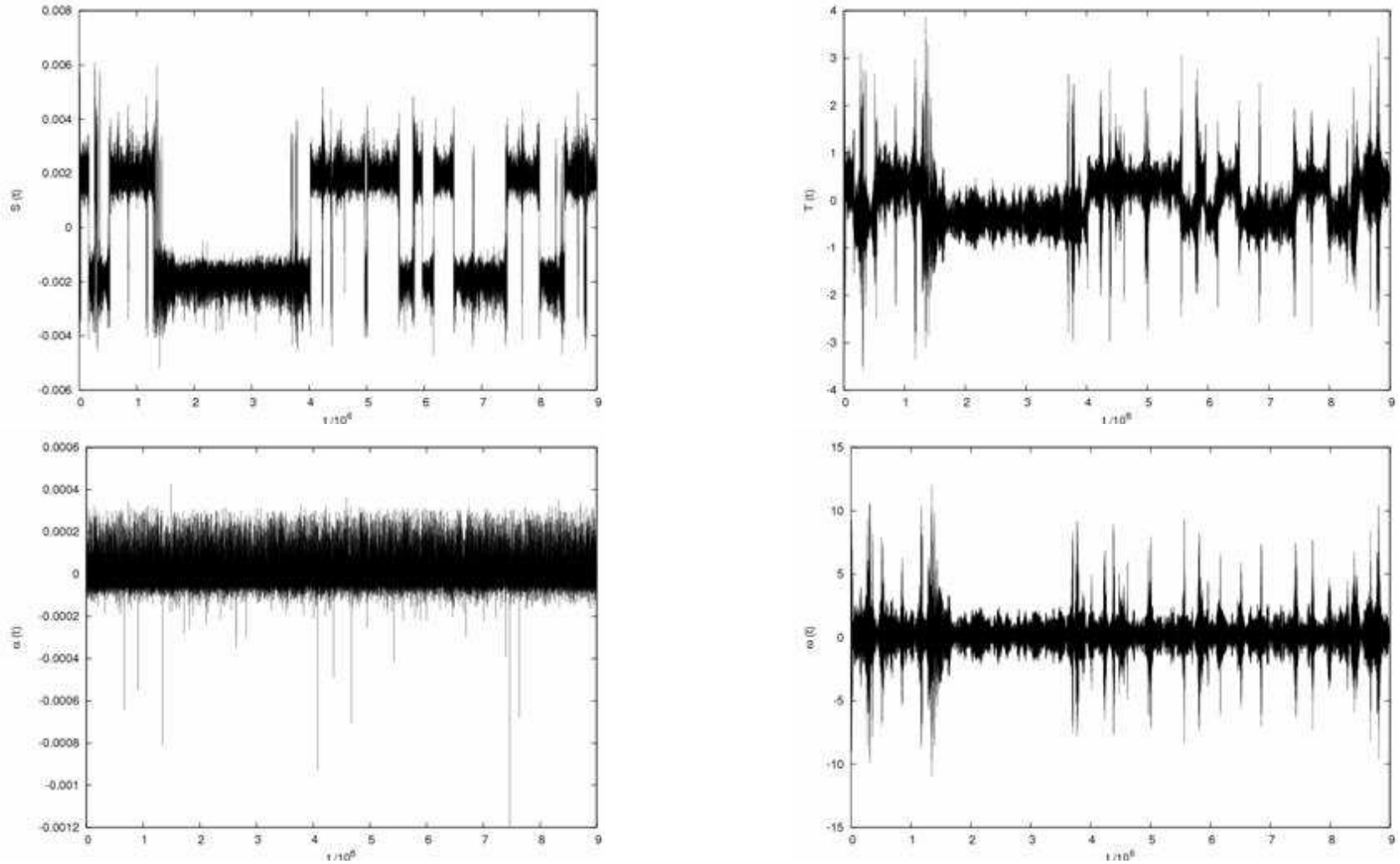
In the Gledzer–Ohkitani–Yamada (GOY) model, the complex modes u_n satisfy

$$\frac{\mathrm{d}u_n}{\mathrm{d}t} = -\nu k_n^2 u_n + f_\alpha \left[1 - \lambda_3 ST - \lambda_4 (S^2 + T^2)\right] \delta_{n,n_0}$$
$$+ik_n \left(u_{n+2}^* u_{n+1}^* - \frac{1}{4} u_{n+1}^* u_{n-1}^* + \frac{1}{8} u_{n-1}^* u_{n-2}^* \right) .$$

We base our α-effect on the shell-model helicity, H:

$$\alpha \sim -\frac{1}{3}\,\tau H\,, \qquad \Longrightarrow \qquad \alpha \sim -\frac{1}{3} \sum_{n=1}^{N} (-1)^n \left|u_n\right| .$$

Typical behaviour: S, T, α, ω



Reversal statistics

The distribution of inter-event times has often been assumed to arise from Poisson or Gamma processes.

We perform a Distribution Identification test on the ordered Gradstein & Ogg (1996; GO96) data, T_i, with reference to known probability distributions with cumulative probability function F.

The Anderson–Darling statistic, A, is

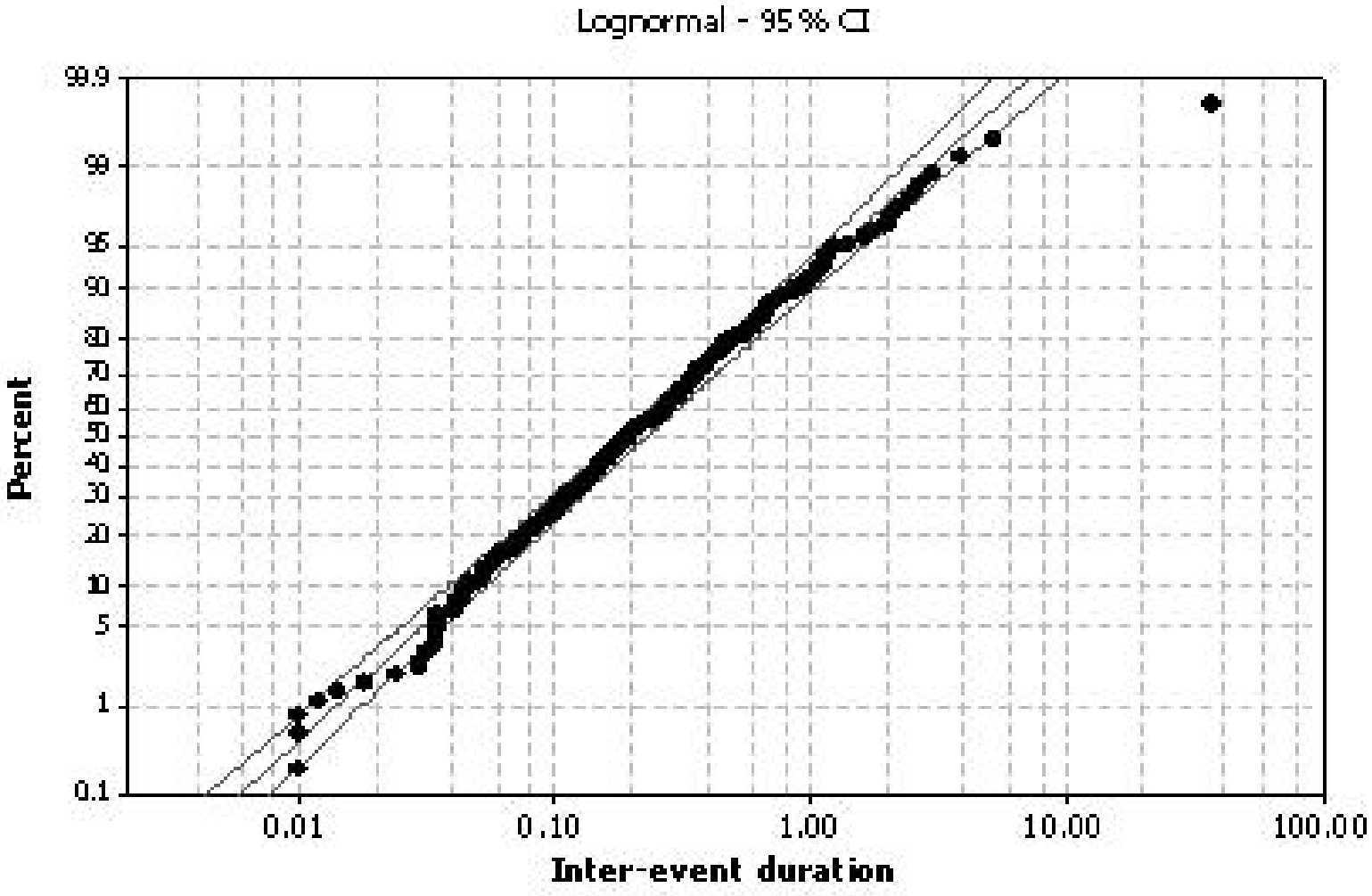
$$A = -N - \sum_{i=1}^{N} \frac{2i-1}{N} \ln F(T_i) + \ln\left(1 - F(T_{N+1-i})\right) \ .$$

A low value of A (with a high corresponding p-value) is a significant fit.

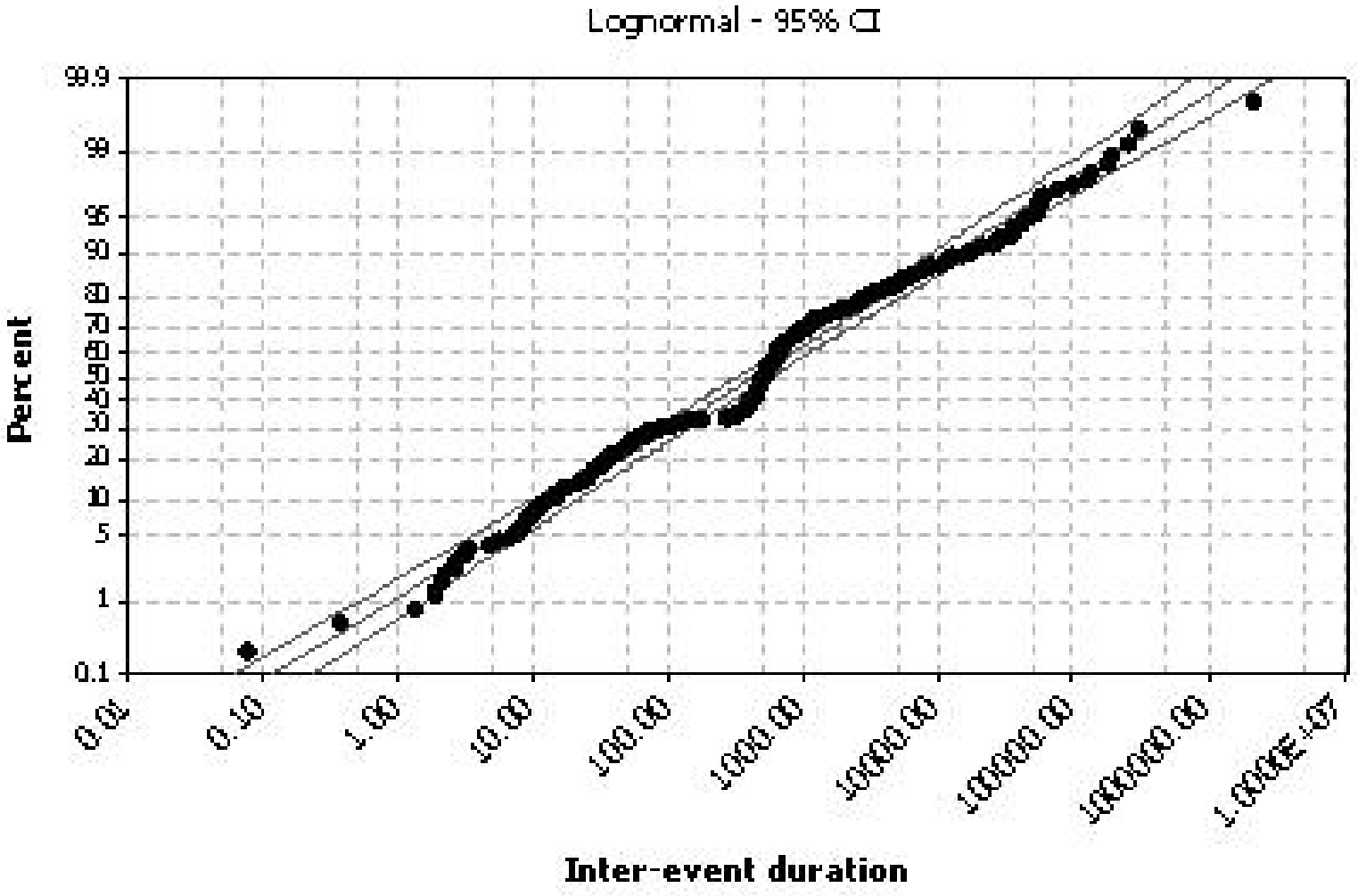
Reversal statistics

- We applied this test for 14 standard distributions, including Poisson, Gamma, lognormal and loglogistic.
- Neither Poisson nor Gamma distributions show significant fits ($A = 24$ and 14, respectively).
- Lognormal and loglogistic distributions showed the best fits ($A < 1$; as low as 0.3 for 3-parameter loglogistic). p-values for the the significance of these fits are $p > 0.2$, so the fits would not be rejected at normal significance levels.
- Similar outcomes obtained using Kolmogorov–Smirnov and χ^2 tests, using the alternative Cande & Kent (1995) reversal chronology, and extending to ca. 40 'standard' distributions.
- Other authors report that similar distributions satisfactorily fit the reversal chronology. (E.g. Carbone et al. 2006.)

Lognormal fit: GO96 data



Lognormal fit: synthetic data



Synthetic reversal statistics

- Lognormal and loglogistic distributions also give the best fits to the *synthetic* data ($A < 1$; as low as 0.5 for 3-parameter fits).
  p-values for these fits are $p > 0.1$; again the fits would not be rejected at normal significance levels.
- Synthetic reversals from *other* low order models are also best fit by lognormal and loglogistic distributions, but the fits are significantly worse ($A \approx 20$).
- These fits suggest a role for multiplicative noise, acting via the α-effect.

The role of multiplicative noise

Given a schematic equation for B,

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \alpha B \ ,$$

$B_i = B(t_i)$ will evolve, subject to $\alpha_i = \alpha(t)$, as

$$B_{i+1} = (1 + \delta t \alpha_i)\, B_i \Longrightarrow B_N = \left(\prod_{i=0}^{N} (1 + \delta t \alpha_i) \right) B_0$$

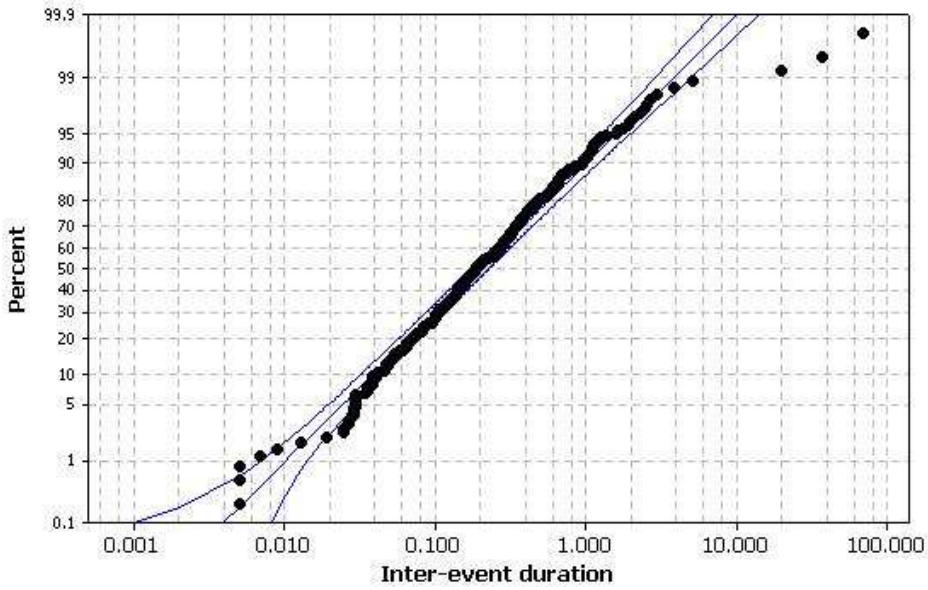
$$\Longrightarrow \ln B_N = \left(\sum_{i=0}^{N} \ln(1 + \delta t \alpha_i) \right) + \ln B_0 \approx \left(\sum_{i=0}^{N} \delta t \alpha_i \right) + \ln B_0 \ .$$

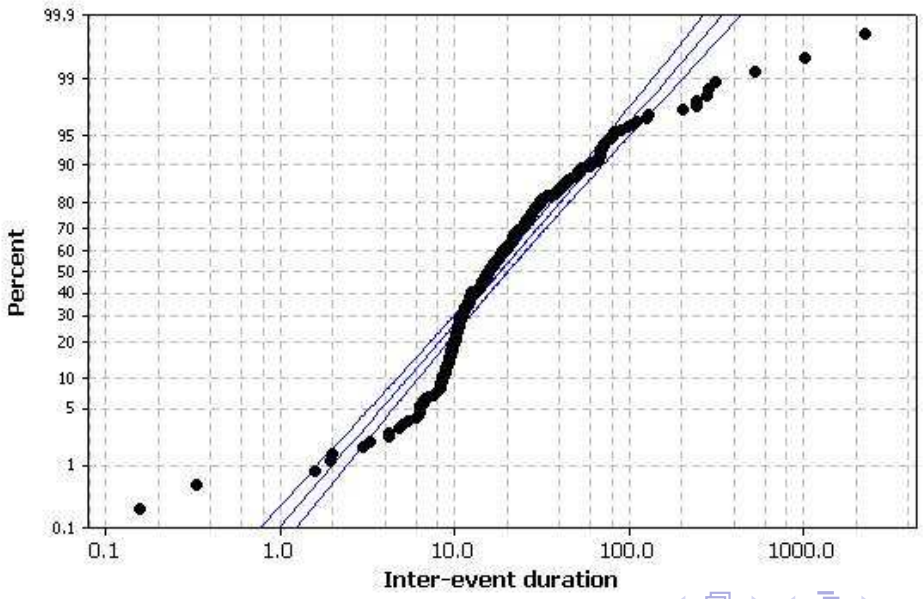
If the α_i are small, independent random perturbations, then $\ln B$ will approach the normal distribution, and B the lognormal distribution.

Significance of superchrons?

The superchron(s) arguably remain outliers to lognormal-type fits. They may be better fit by Pareto-Lévy tails, reflecting a modified underlying mechanism. (E.g. an 'amplification' mechanism, cf. Montroll & Shlesinger, 1982.)

In our model, superchrons do correspond to a different style of dynamo action. (And similar tails can be obtained for some parameter regimes.)





Implications for external influences

This work suggests that the observed record of reversals may arise entirely from internal core effects.

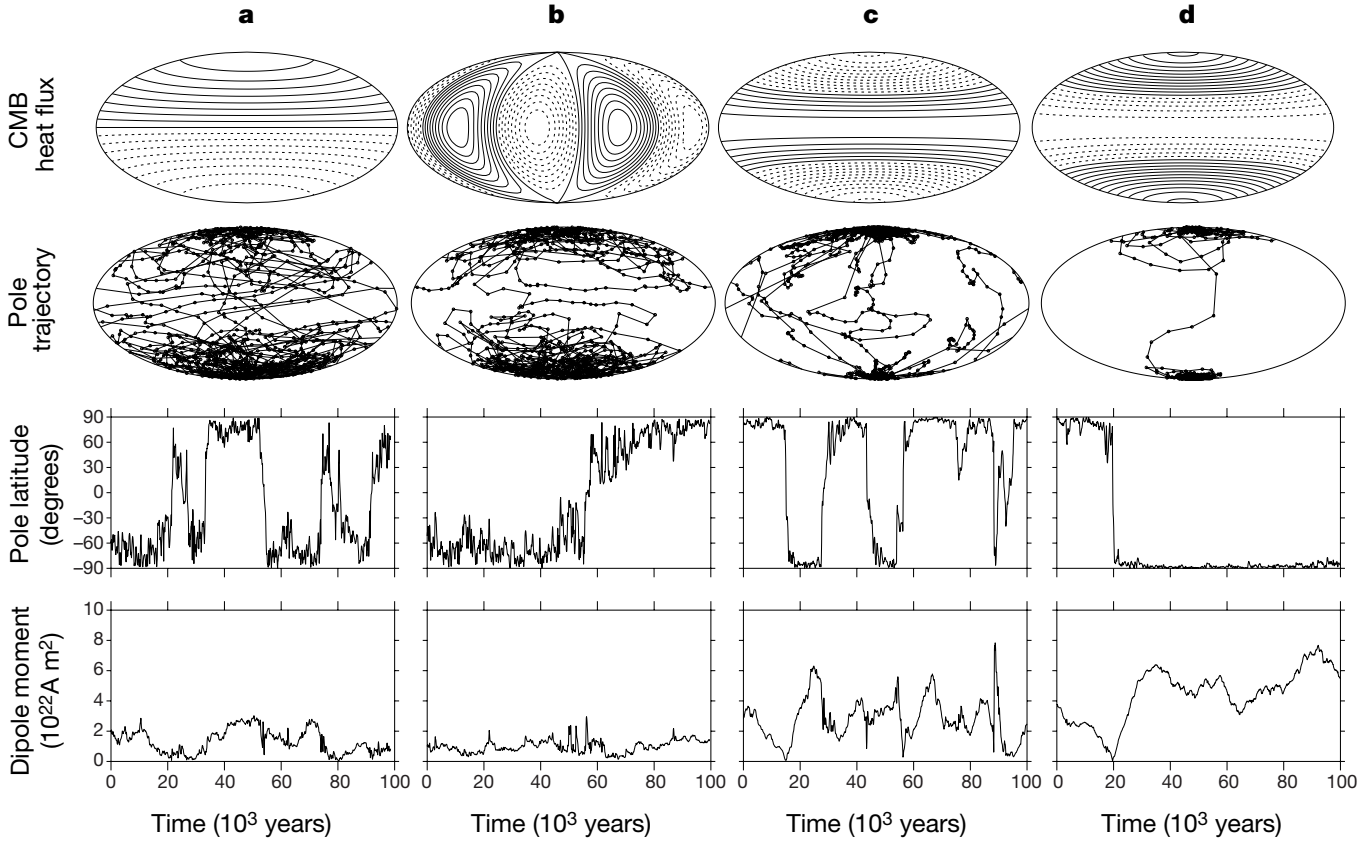
This is clearly not conclusive, however; significant analysis remains to be done. And other works highlight other interpretations of the reversal record (e.g. Jonkers 2003; Carbone et al. 2006).

Mantle developments may very plausibly affect the long-term reversal statistics.

In particular, heterogeneous structure of CMB may very plausibly produce eras with varying tendency to reverse.

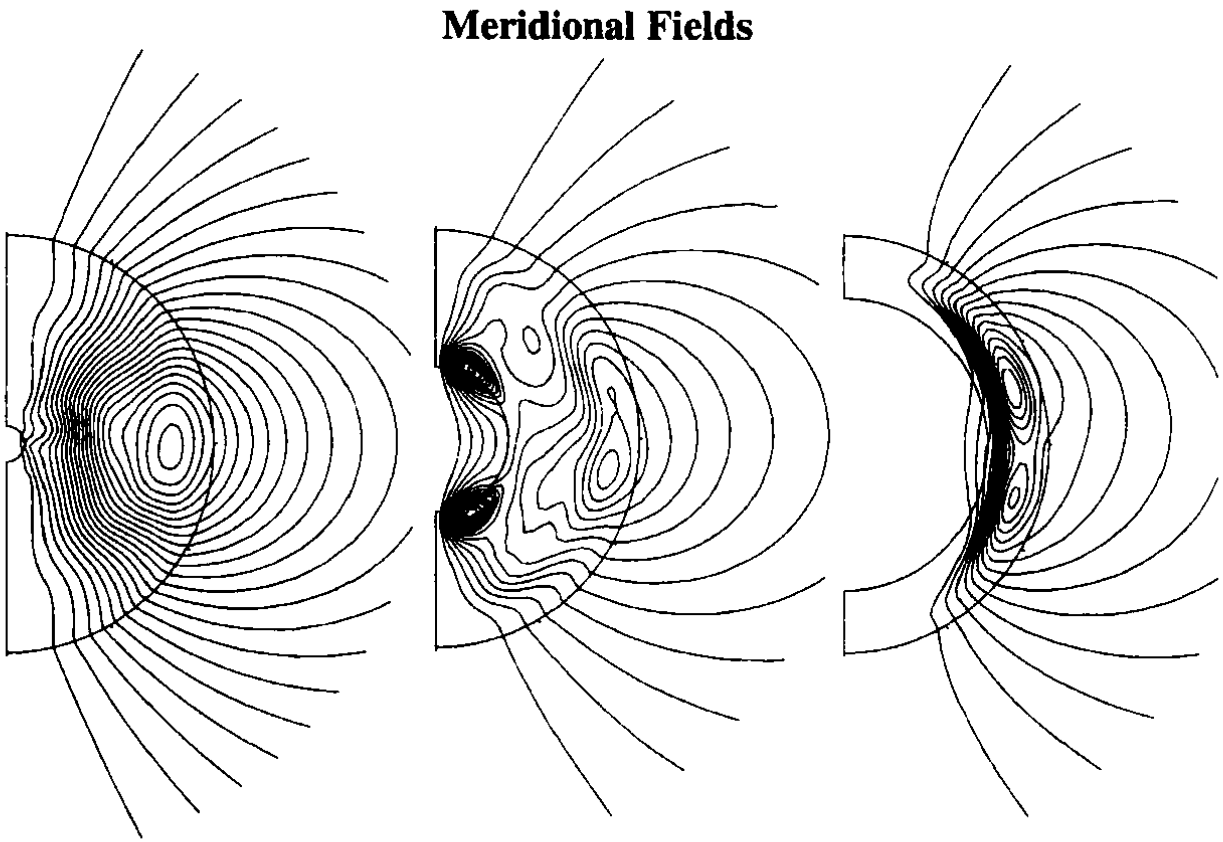
Varying CMB thermal b.c.s

Under suitable model conditions, inhomogneous thermal boundaries clearly may influence reversal frequency. E.g. Glatzmaier et al. (1999).



Effect of Inner core growth?

The effects of inner core growth are also poorly known at this time, although some investigations have been made. E.g. Roberts & Glatzmaier (2001).



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Reversal processes

The processes occurring during individual transitions can also be studied by comparing paleomagnetic data with synthetic data — again, both low order models and more detailed 'simulations' are possible.

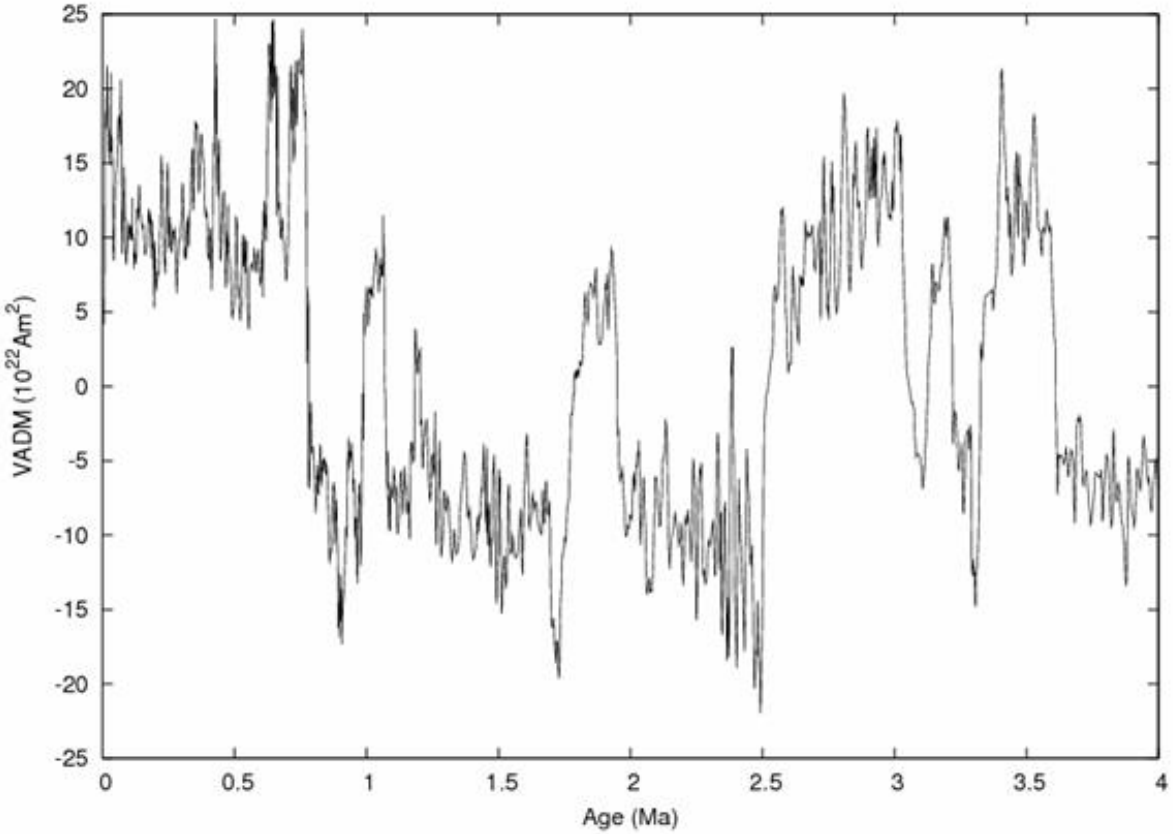
Reversals in detailed simulations are difficult to interpret. Interpretations often refer to 'kinematic' mechanisms identified in simpler contexts — but the relevance of these mechanisms to realistic dynamical solutions is not always clear.

Here, I'll initially continue to concentrate on the low order model described above.

The paleomagnetic records of reversal transitions consists of both intensity data (Virtual Dipole Moments; VDMs) and directional data (Virtual Geomagnetic Poles; VGPs).

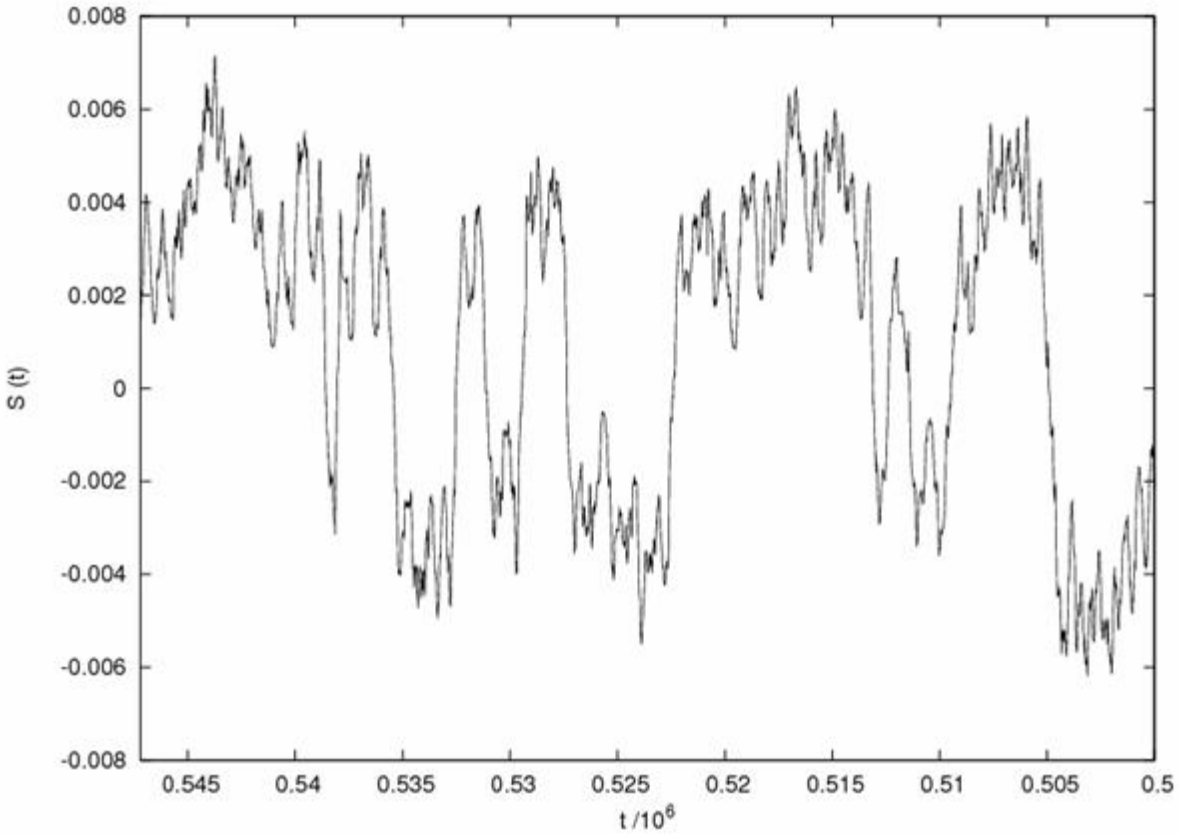
Virtual Axial Dipole Moments (VADMs)

'Saw-tooth' behaviour is often observed in paleomagnetic VADMS (Valet & Meynadier 1993; Meynadier et al. 1994).



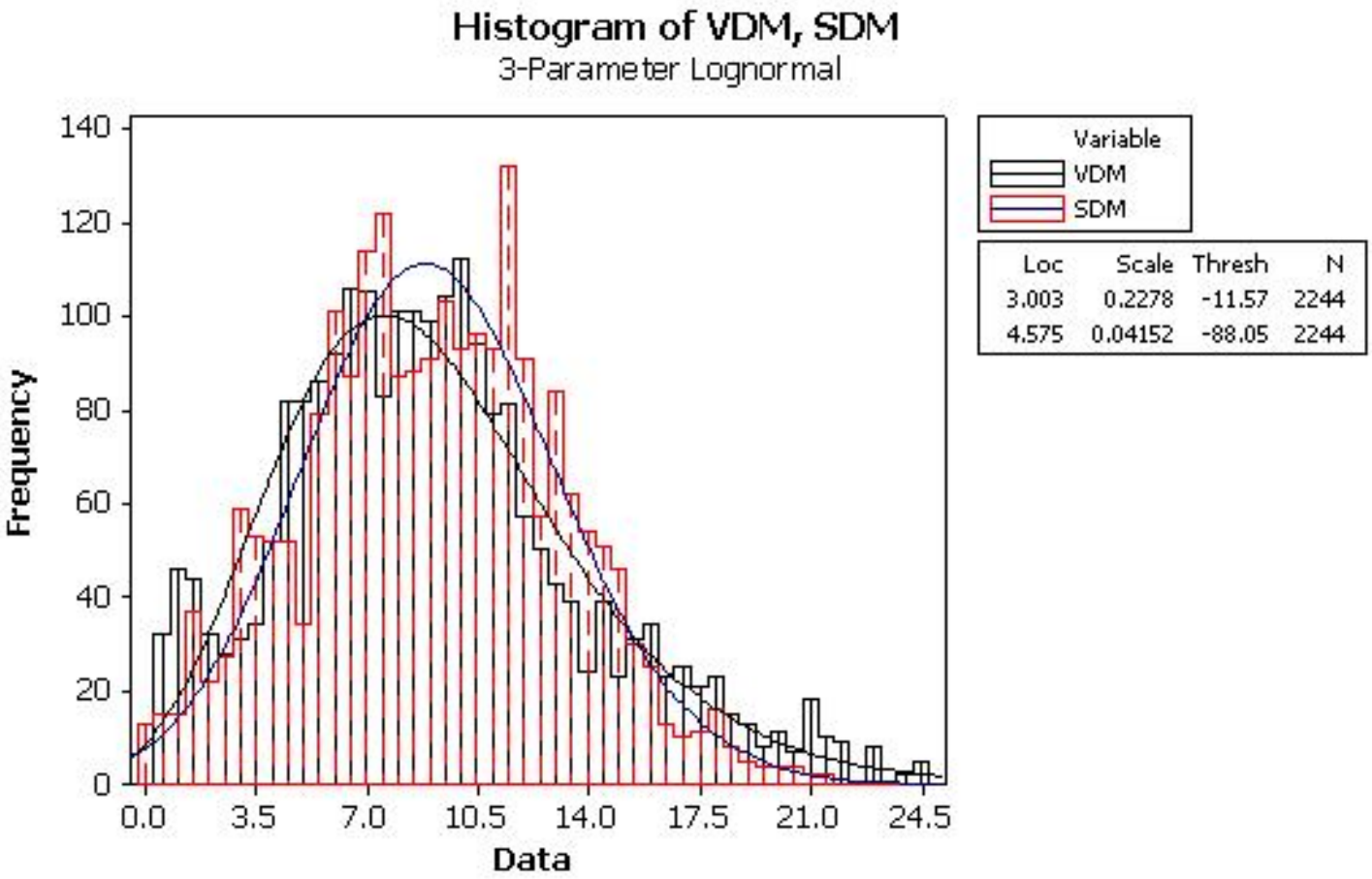
Synthetic Axial Dipole Moments (SADMs)

The fluctuations in S, in our top model, can be surprisingly similar.



Lognormal fits to VADM and SADM data

It's worth noting that both VDM and SDM amplitudes are also well fit by a lognormal distribution.



Analysis of low order model behaviour

Analysis of our (de-coupled) low order model, for $\alpha = \alpha_c$ constant, suggests mechanisms for the variations observed.
Here there are three fixed points in (S, T, ω) space:

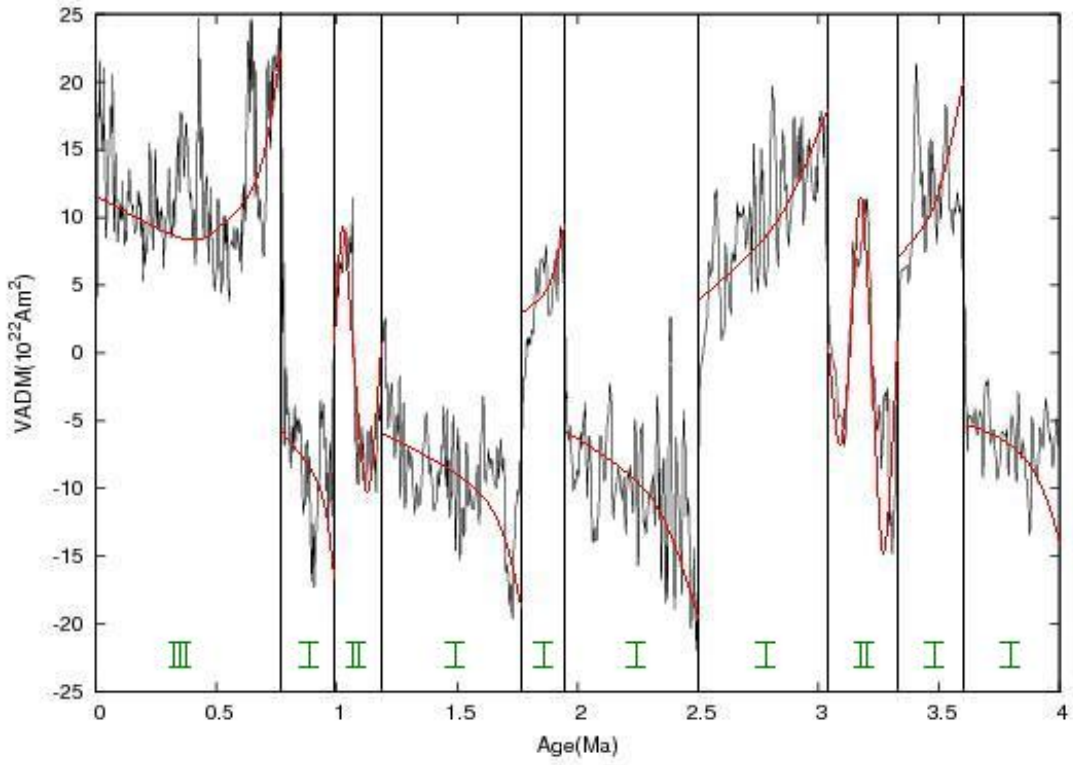
$$F^0 = (0, 0, f_\omega/\kappa_\omega) \ ,$$

$$F^\pm = (\pm\frac{\alpha_c}{\kappa}\sqrt{\frac{\kappa f_\omega \alpha_c - \kappa_\omega \kappa^3}{f_\omega \lambda_1 \alpha_c^2}}, \pm\sqrt{\frac{\kappa f_\omega \alpha_c - \kappa_\omega \kappa^3}{f_\omega \lambda_1 \alpha_c^2}}, \frac{\kappa^2}{\alpha_c}) \ .$$

- For $\alpha_c < 0$, F^0 is a stable spiral;
- For $0 \leq \alpha_c \leq \kappa^2 \kappa_\omega / f_\omega$, F^0 is a stable node;
- For $\kappa^2 \kappa_\omega / f_\omega < \alpha_c \leq \kappa / f_\omega (\kappa_\omega^2/4 + \kappa_\omega \kappa)$, $F^\pm$ are stable nodes (locally);
- For $\alpha_c > \kappa / f_\omega (\kappa_\omega^2/4 + \kappa_\omega \kappa)$, $F^\pm$ are stable spirals (locally);
- For $\alpha_c > \alpha_A$, $F^\pm$ are globally unstable to a chaotic attractor, allowing reversals.

Interpretation of VADM reversals?

Fluctuations in α (and hence of S, T and ω) suggest tentative identification of different types of reversals,



We need some more quantitative analysis, however.

Phase space reconstruction: SINT 2000 data

We construct the delay vectors

$$V_i = \{S_i, S_{i+\tau}, S_{i+2\tau}, ..., S_{i+(m-1)\tau}\} \ ,$$

for embedding dimension m and delay τ.

For the SINT 2000 paleointensity data (Valet et al. 2005):

- the mutual information method suggests an optimum $\tau = 13$;
- the method of false nearest neighbours suggests an embedding dimension in the range $m = 3$–7.

Phase space reconstruction: SINT 2000 data

We construct the delay vectors

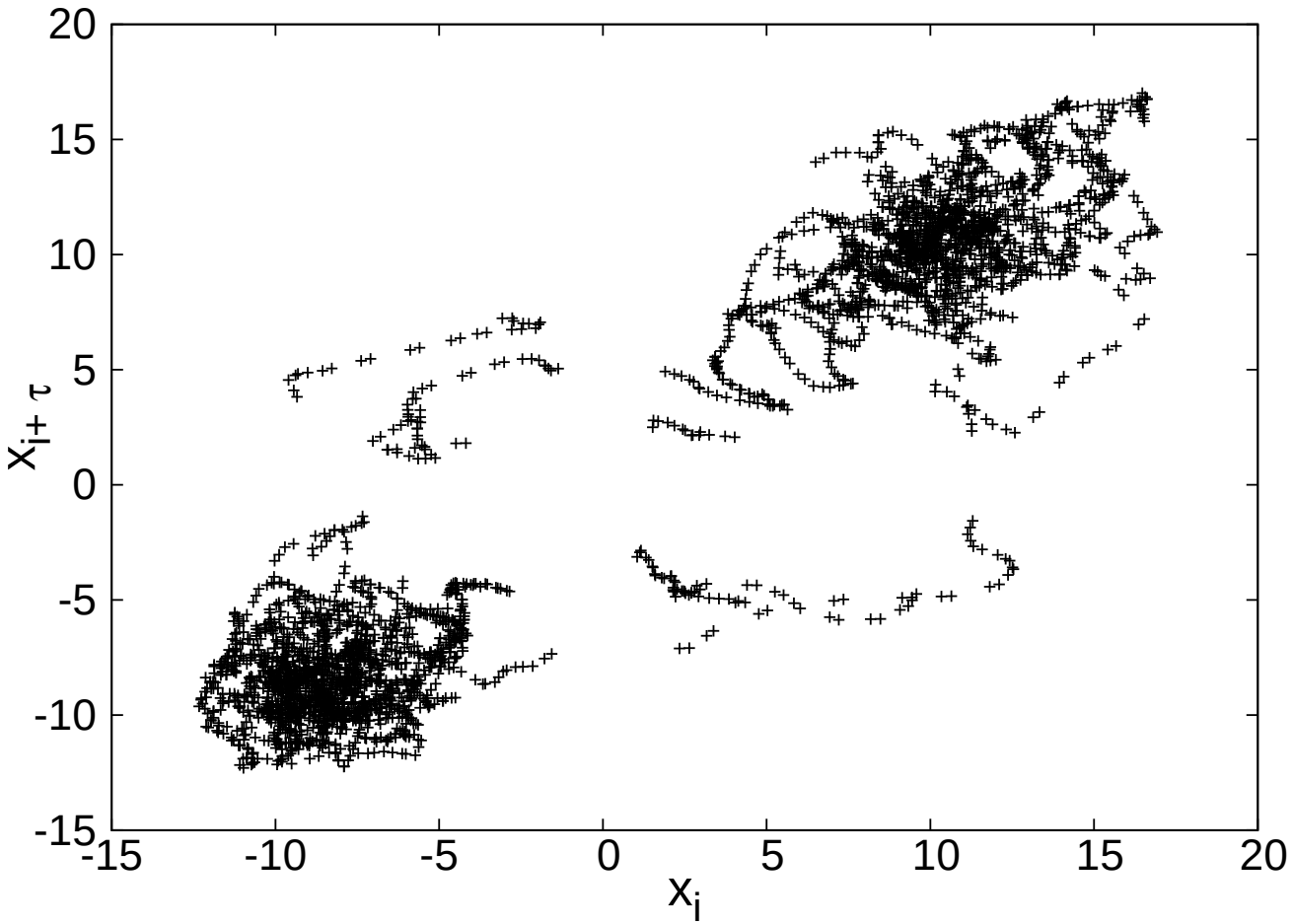
$$V_i = \{S_i, S_{i+\tau}, S_{i+2\tau}, ..., S_{i+(m-1)\tau}\} \ ,$$

for embedding dimension m and delay τ.

We can analyse these embedded vectors for characteristics of deterministic chaos:

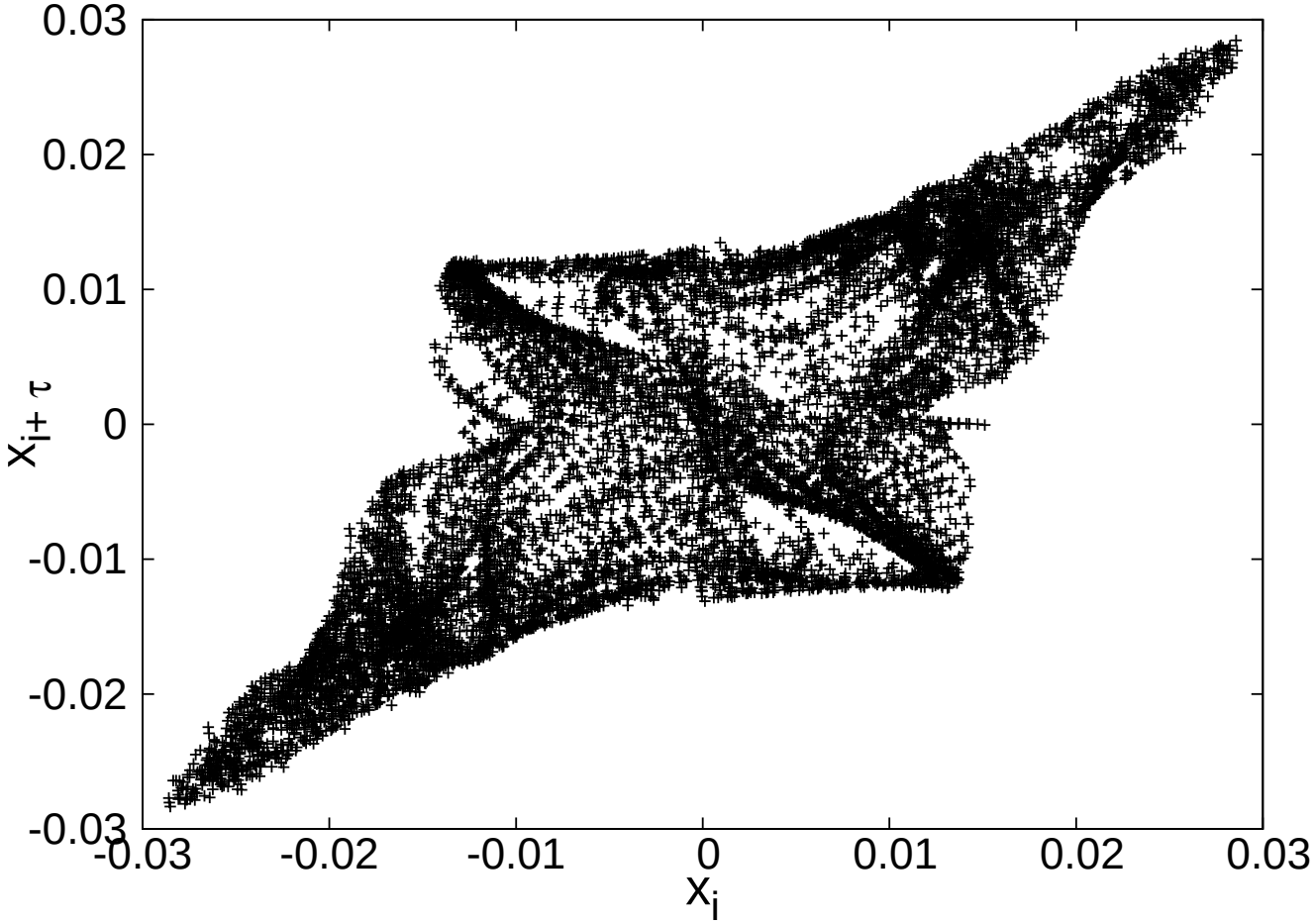
- the determinism test of Kaplan and Glass (1992) gives $\Lambda = 0.87$;
- various algorithms (Wolf et al. 1985; Rosenstein et al. 1994; Kantz 1994) give a maximum Lyapunov exponent of 11 Ma^{-1};
- the scale dependent Lyapunov exponent (SDLE) behaviour is consistent with intermittent chaos (Gao et al. 2006).

Phase portrait: SINT 2000 data



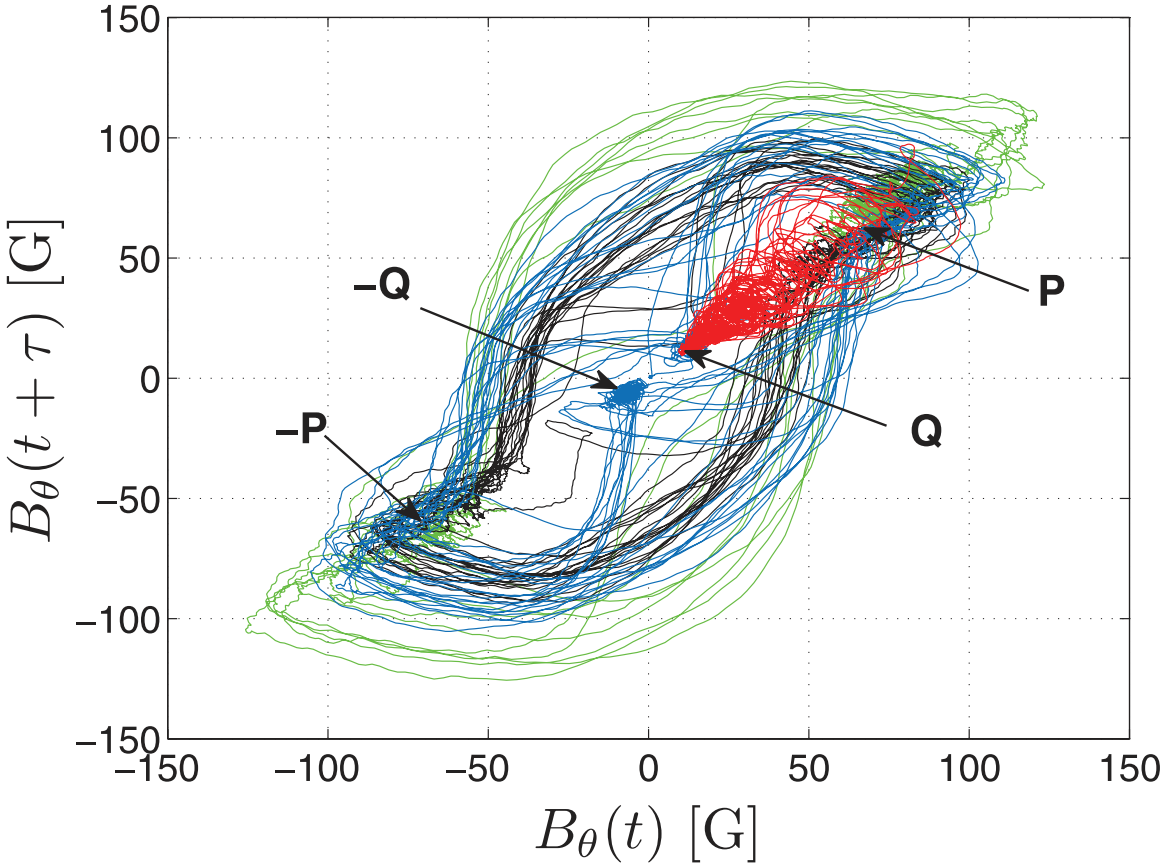
Phase portrait: Synthetic data

The embedded data-sets arguably exhibit similar attractors.



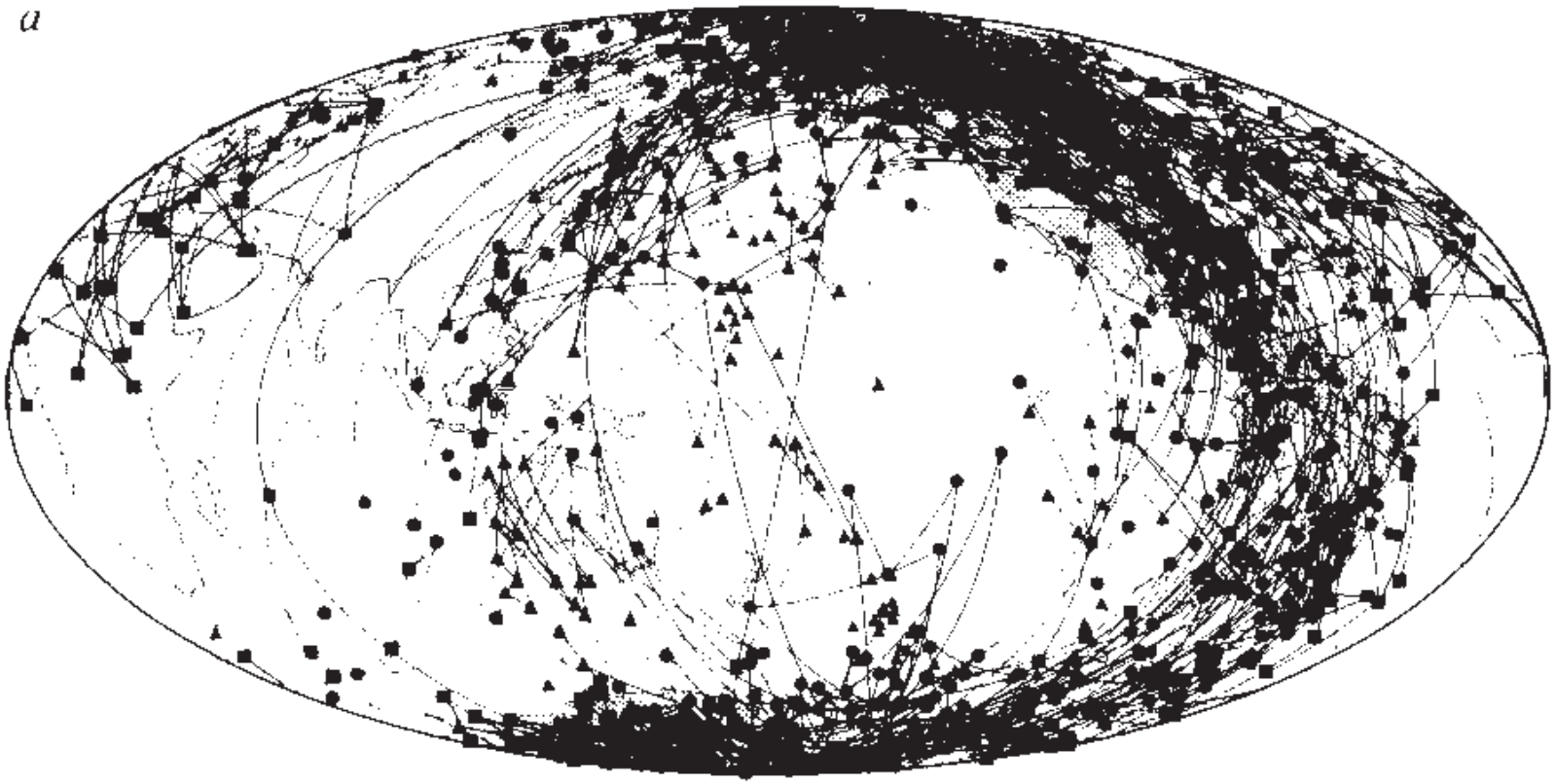
Phase portrait: experimental dynamo

Similarly simple attractors have been found from reversing data in MHD experiments. E.g. Ravelet et al. (2008).



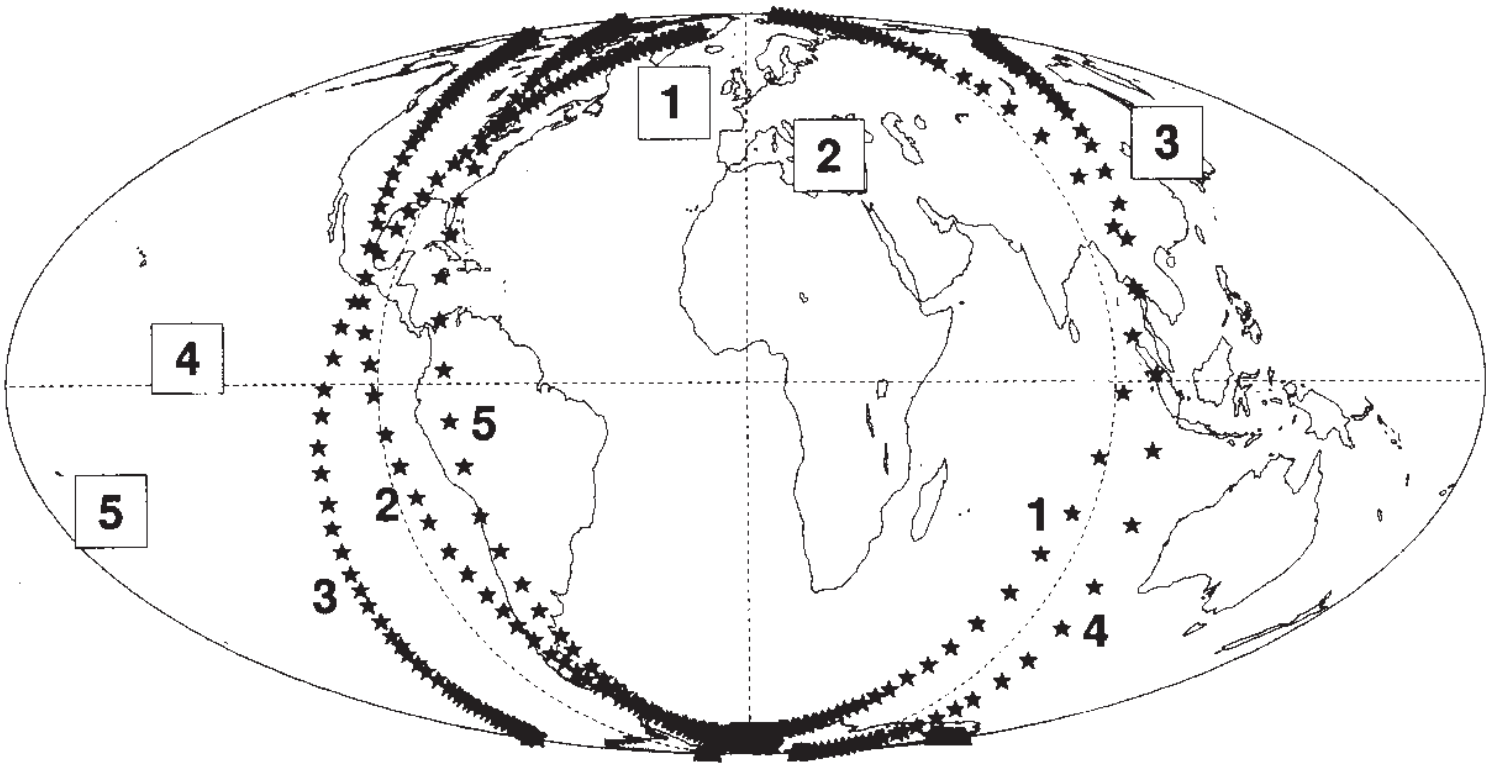
Evidence for external influences?

Some studies suggest that VGP transition paths follow preferred routes, linked to CMB structures (e.g. Laj et al. 1991).



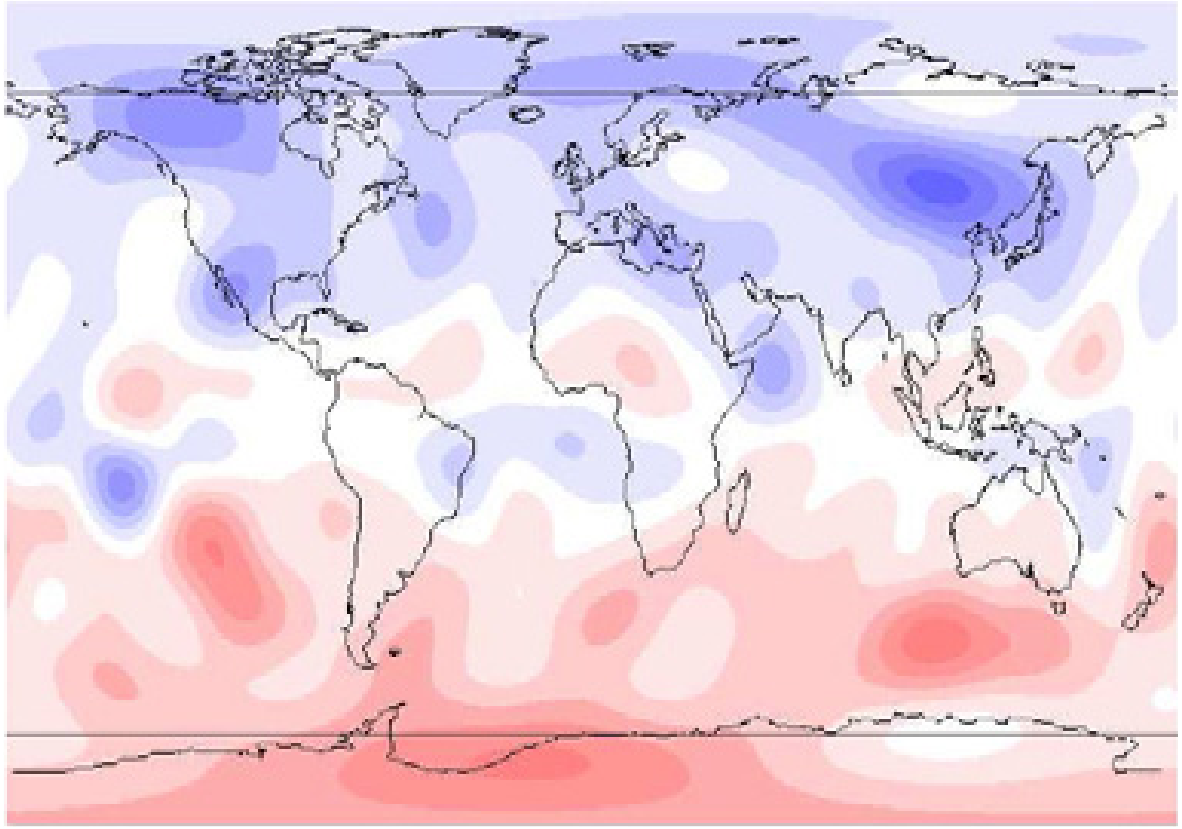
Link between VGPs and CMB flux patches

Such paths can be expected from reversals which involve latitudinally migrating flux patches ('dynamo waves'). E.g. Gubbins & Sarson (1994).



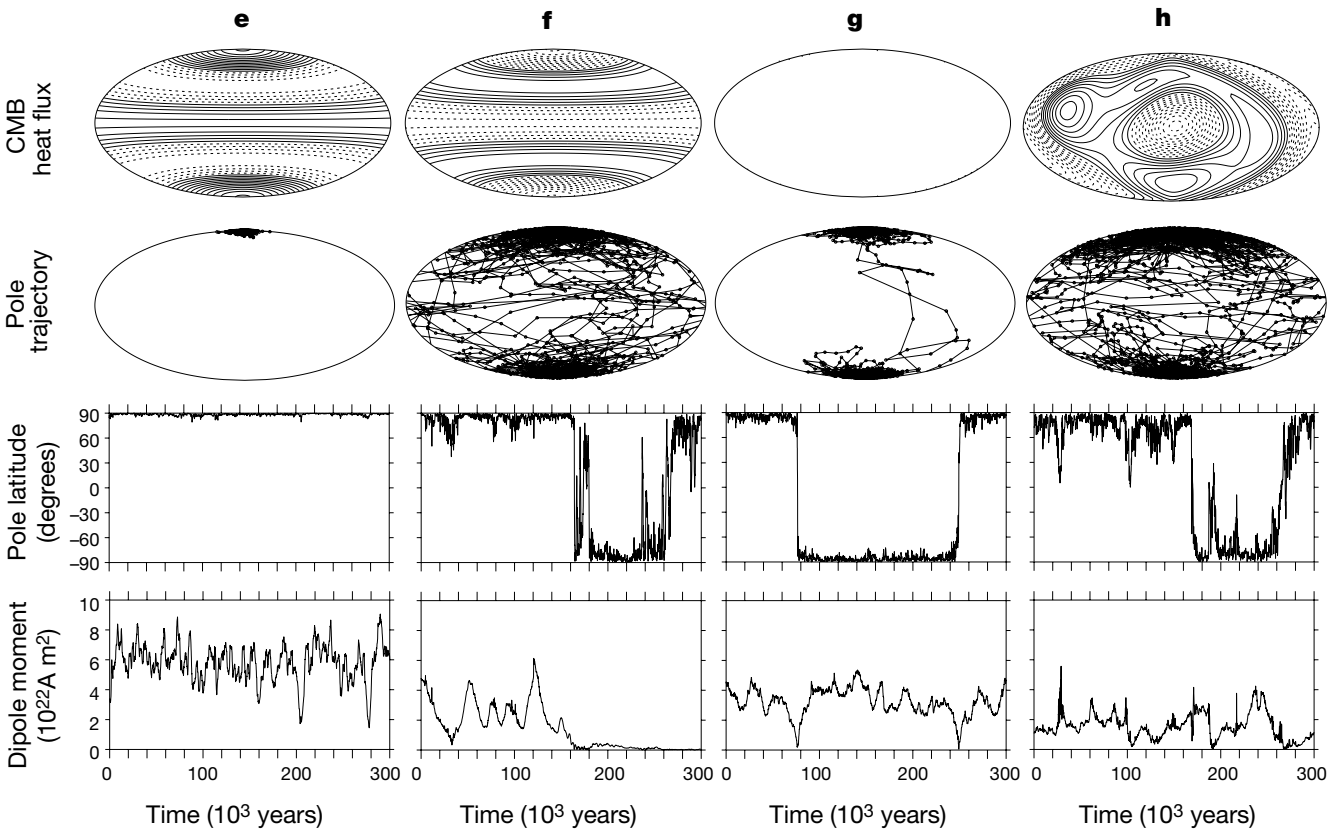
Heterogeneous thermal boundary conditions

And heterogeneous thermal boundary conditions may plausibly lead to preferred latitudes for such flux and flow features (e.g. Willis et al. 2007).



Heterogeneous thermal boundary conditions

In many models, this does lead to some clustering of VGP paths. (e.g. Glatzmaier et al. 1999).



The magnitude of these effects are very unclear, however.

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Conclusions

Many details of the processes behind geomagnetic reversals (as with the geodynamo in general) remain unclear, with rival interpretations remaining possible.

At present, the case for control from the mantle has not been definitively made.

The situation can only improve with more observations, and with a better understanding of the processes occurring in numerical models.

Research on reversal processes may ultimately help constrain our understanding of other parts of the Earth's deep interior (or vice versa).