From Core to Crust: Towards an Integrated Vision of Earth’s Interior

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Elastic properties of Earth materials

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Measuring Elastic Properties of Earth Materials

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The Earth:
Internal structure and composition

from the web
Seismology: 1D structure

\[ v_P = \left[ \frac{K + 4/3G}{\rho} \right]^{1/2}, \quad v_S = \left( \frac{G}{\rho} \right)^{1/2} \]

from the web
Seismology:
3D structure, heterogeneity

\[ v_P = \left(\frac{K + 4/3G/\rho}{g_{85}}\right)^{1/2} \]
\[ v_S = \left(\frac{G/\rho}{g_{85}}\right)^{1/2} \]
\[ v_C = \left(\frac{K/\rho}{g_{85}}\right)^{1/2} \]
Mineralogical model of the deep Earth:
The role of mineral physics

\[ \langle v \rangle = \sum_i v_i \cdot F(v_i) \]

courtesy of H. Marquardt
Accessing extreme conditions:
Experimental methods

Depth (km)

Pressure (GPa)

National Ignition Facility
Diamond anvil cell
Shock wave
Large volume press

MgO Hug.

LH DAC

Estim. Geotherm

LVP

RH DAC

T (K)

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Linear elastic solid

Hooke’s law

\[ \sigma_{ij} = C_{ijkl} \varepsilon_{kl} \]
\[ \varepsilon_{ij} = S_{ijkl} \sigma_{kl} \]
\[ C_{ijkl} S_{klmn} = \delta_{im} \delta_{jn} \]

\[ \varepsilon_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i) \]
The symmetry of the elastic tensor

\( \sigma_{ij}, \varepsilon_{ij} \) symmetric \rightarrow 6 \text{ independent components}

\( c_{ij}, s_{ij} \) (in matrix form) \rightarrow 6 \times 6 \text{ components}

from Nye (1958)
Elastic moduli of isotropic media

\[ K \text{ (Bulk modulus)} = \frac{(c_{11} + 2c_{12})}{3} \]

\[ P = -3K \varepsilon_1 \]

\[ \varepsilon_1 = \varepsilon_2 = \varepsilon_3 \]

\[ G \text{ (Shear modulus)} = \frac{(c_{11} - c_{12})}{2} \]

\[ \sigma_4 = 2G \varepsilon_4 \]

\[ \epsilon_{ij} = \frac{1}{2}(\partial u_i / \partial x_j + \partial u_j / \partial x_i) \]

\[ ij \rightarrow \alpha \]
Dynamic behavior of elastic media:

Equations of motion

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} \]
Eulerian finite strain

Anisotropic case

$E_{ij} = 1/2[\delta_{ij} - (\partial X_i/\partial x_j)(\partial X_j/\partial x_i)]$

Isotropic case

$E_{ij} = -f \delta_{ij}$

\[ f = 1/2[(V_0/V)^{2/3} - 1] = 1/2[(\rho/\rho_0)^{2/3} - 1] \]
Strain dependence of $C_{ijkl}$:

Free energy

$$F_{T0} = F_0 + 1/2a_{ij}E_iE_j + 1/6a_{ijk}E_iE_jE_k + 1/24a_{ijkl}E_iE_jE_kE_l + \ldots$$

Elastic constants

$$c_{ijkl,T0} = (1 + 2f)^{7/2}\{c_{ijkl}^0 + b_1f + 1/2b_2f^2 + \ldots\} - P\Delta_{ijkl}$$

from Stixrude and Lithgow-Bertelloni (2005)
Temperature dependence of $C_{ijkl}$:

Quasi-harmonic approximation

$$F(V,T) = F_0 + F_{T0}(V,T_0) + [F_q(V,T) - F_q(V,T_0)]$$

$$F_q(V,T) = \rho_0 kT \sum_{\lambda} \ln \{1 - \exp[-h\nu_{\lambda}(E_i)/kT]\}$$

from Stixrude and Lithgow-Bertelloni (2005)
Temperature dependence of $C_{ijkl}$

\[
C_{ijkl} = C_{ijkl,T0} + \rho [c_1 \Delta U_q - c_2 \Delta (C_V T)]
\]

\[
c_1 = F(\gamma_{ij}, \eta_{ijkl}), \quad c_2 = F(\gamma_{ij})
\]

\[
\gamma_{ij} = -\partial \ln \nu \lambda / \partial E_{ij}, \quad \eta_{ijkl} = \partial \gamma_{ij} / \partial E_{kl}
\]

from Stixrude and Lithgow-Bertelloni (2005)
Aggregate properties

Voigt: uniform $\varepsilon_{ij}$ $\sigma_{ij} = \langle c_{ijkl} \rangle \varepsilon_{kl}$

Reuss: uniform $\sigma_{ij}$ $\varepsilon_{ij} = \langle s_{ijkl} \rangle \sigma_{kl}$
Aggregate properties

Voigt: \[ M_V = \sum f_i M_i \]

Reuss: \[ 1/M_R = \sum f_i /M_i \]

\( f \): volume fraction
\( M \): elastic modulus
Aggregate properties

\[ \text{ODF} \quad f(g)dg = \frac{dV_g}{V} \]

\[ \langle c_{ijkl} \rangle = \int g_{ip} g_{jq} g_{kr} g_{ls} c'_{pqrs} f(g)dg \]

from Wenk et al. (2005)
Limitations
(\frac{\partial M}{\partial T})_P = (\frac{\partial M}{\partial T})_V + (\frac{\partial M}{\partial V})_T (\frac{\partial V}{\partial T})_P
Anelasticity

\[
M(\omega) = M_1(\omega) + iM_2(\omega)
\]

\[
Q^{-1}(\omega, \tau) = |\Delta E|/(2\pi E) = M_2(\omega)/M_1(\omega)
\]

\[
\tau_i = \tau_{i0} \exp[E^*/(kT)]
\]
Minerals of the crust

from the web
Elasticity of crustal minerals:

Impulsively stimulated light scattering

Albite (NaAlSi$_3$O$_8$)

from Brown et al. (1997, 2006)
Elasticity of crustal minerals:
Albite (NaAlSi$_3$O$_8$)

from Brown et al. (2006)
Elasticity of crustal minerals: Diopside [CaMgSiO$_3$] at high temperature

Resonant ultrasound spectroscopy

from Isaak et al. (2006)
Elasticity of crustal minerals:
Diopside [CaMgSiO$_3$] at high temperature

\[ \gamma_{Th} = \frac{VK_S \alpha_V}{C_P} \]

from Isaak et al. (2006)
The upper mantle

Cations substitutions

from the web
Elasticity of upper mantle minerals:
Brillouin scattering

\[ |\mathbf{q}| = 2k_0 \sin \left( \frac{\sigma_{\text{ext}}}{2} \right) \]

From Speziale et al. (2005)
Elasticity of upper mantle minerals:
Effect of Mg-Fe substitution in olivine [(Mg,Fe)$_2$SiO$_4$]

Brillouin scattering

from Speziale et al. (2005)
Elasticity of upper mantle minerals: Effect of Ca-Mg-Fe substitution in garnet \((\text{Mg,Fe,Ca})_3\text{Al}_2\text{Si}_3\text{O}_{12}\) from Speziale et al. (2005)
Transition zone

- Pressure/Temperature effects
- H incorporation

Ol. polymorphs + Garnet-Majorite
Elasticity of transition zone minerals:

Ultrasonic interferometry

from Angel et al. (2009)
Elasticity of transition zone minerals:

Ultrasonic interferometry in the Large Volume Press

from Li et al. (2004)
Elasticity of transition zone minerals:
Majorite garnets [(Mg,Fe)$_3$Al$_2$Si$_3$O$_{12}$ – (Mg,Fe)$_4$Si$_4$O$_{12}$]

Ultrasonic interferometry
In the Large Volume Press

from Gwanmesia et al. (2009)
Elasticity of transition zone minerals: Wadsleyite \([\beta-(\text{Mg,Fe})_2\text{SiO}_4]\)

Ultrasonic interferometry
In the Large Volume Press

from Liu et al. (2009)
Elasticity of transition zone minerals:

Single-crystal elastic anisotropy by Brillouin scattering

from Marquardt et al. (2009)
Elasticity of transition zone minerals: OH in wadsleyite [$\beta$-Mg$_2$SiO$_4$]

Brillouin scattering

from Mao et al. (2008)
Elasticity of transition zone minerals:
OH in ringwoodite \([\gamma-(\text{Mg,Fe})_2\text{SiO}_4]\)

Brillouin scattering

from Jacobsen and Smyth (2006)
Lower mantle

- Perovskite / Postperovskite
- Effect of Spin transition of Fe in (Mg,Fe)O

660 km 24 GPa

2890 km 136 GPa

Perovskites + (Mg,Fe)O

PostPerovskite + (Mg,Fe)O
Elasticity of lower mantle minerals:
Perovskite-postperovskite [(Mg,Fe)SiO₃]

Ab initio calculations

Wentzcovitch et al. (2006)  Stackhouse et al. (2005)
Elasticity of lower mantle minerals:

Sound velocity of perovskite (MgSiO₃)

Brillouin scattering

from Murakami et al. (2007)
Elasticity of lower mantle minerals:

Sound velocity of postperovskite (MgSiO$_3$)

Brillouin scattering

from Murakami et al. (2007)
Elasticity of lower mantle minerals: Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

Nuclear resonant X-ray inelastic scattering

from Sturhahn et al. (2009)
Elasticity of lower mantle minerals:
Low sound velocity in Fe-rich postperovskite \([(Mg_{0.6}Fe_{0.4})SiO_3]\)

\[3/v_D^3 = 1/v_P^3 + 2/v_S^3\]

from Mao et al. (2006)
Elasticity of lower mantle minerals:
Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

Impulsively stimulated light scattering

from Crowhurst et al. (2008)
Elasticity of lower mantle minerals:
Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

Nuclear resonant X-ray inelastic scattering

\[ \frac{3}{v_D^3} = \frac{1}{v_P^3} + \frac{2}{v_S^3} \]

from Lin et al. (2006)
Elasticity of lower mantle minerals:
Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

(Mg$_{0.9}$Fe$_{0.1}$)O to 80 GPa

Brillouin scattering

from Marquardt et al. (2009)
Shear elastic anisotropy

Single-crystal MgO at 81 GPa: (measured at APS)

Marquardt et al. (subm.)
Velocity anisotropy (measurements): 

(Mg_{0.9}Fe_{0.1})O to 81 GPa

Marquardt et al. (2009)
Elasticity of lower mantle minerals: Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

(Mg$_{0.9}$Fe$_{0.1}$)O to 80 GPa

from Marquardt et al. (2009)
Elasticity of lower mantle minerals: Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O] from Marquardt et al. (2009)
Elasticity of lower mantle minerals:
Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

Tsuchiya et al. (2006)

from Marquardt et al. (2009)
Elasticity of lower mantle minerals: Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

Model composition
80% (Mg,Fe)SiO$_3$
20% (Mg,Fe)O

from Cammarano et al. (submitted 2009)
Outer core

- Sound velocity of liquid Fe at core conditions

Liquid Fe + alloying elements
Elasticity of the outer core

Shock wave velocity measurements

\[
\rho_0 U_S = \rho_1 (U_S - u_P)
\]

\[
P = \rho_0 U_S u_P
\]

\[
E_1 - E_0 = \frac{1}{2} \rho_0 U_S u_P^2
\]

from Jeanloz (1989)
Elasticity of the outer core
Computational and experimental constraints on sound velocity of liquid Fe

from Laio et al. (2000)
from Alfè et al. (2002)
Elasticity of the outer core
New experimental results on sound velocity of liquid Fe

Shock wave velocity measurements

from Nguyen et al. (2004)
- Sound velocity of $\epsilon$-Fe
- Elastic anisotropy of $\epsilon$-Fe

Uchida et al. (2001)

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Elasticity of Fe in the inner core

Momentum resolved inelastic X-ray scattering

from Antonangeli et al. (2004)
Elasticity of Fe in the inner core

Momentum resolved inelastic X-ray scattering

from Fiquet et al. (2001)
Elasticity of Fe in the inner core

Momentum resolved inelastic X-ray scattering

from Fiquet et al. (2001); Antonangeli et al. (2004)
Elasticity of single-crystal hcp Co:
Structural proxy for ε-Fe

Momentum resolved inelastic X-ray scattering

from Antonangeli et al. (2004)
Elasticity of Fe in the inner core

Impulsively stimulated light scattering

from Crowhurst et al. (2004)
Elasticity of Fe in the inner core

High P-T average velocity for isotropic aggregates

Nuclear resonant X-ray inelastic scattering

from Lin et al. (2005)
Elasticity of Fe in the inner core
Seismic anisotropy of the inner core

from Ishii et al. (2002)
Elasticity of Fe in the inner core

Radial X-ray diffraction

\[ d_m(hkl) + d_P(hkl) \left[ 1 + (1-\cos^2 \phi) Q(hkl) \right] \]

\[ Q(hkl) = F(\sigma_{11} - \sigma_{33}, s_{ijkl}) \]

from Wenk et al. (2004)
Elasticity of Fe in the inner core

Single-crystal anisotropy of $\varepsilon$-Fe: a combined approach

$P = 52$ GPa

Radial X-ray diffraction
Momentum resolved inelastic X-ray scattering
Nuclear resonant X-ray inelastic scattering

Mao et al. (2008)

Steinle-Neumann et al. (2001)
Elasticity of the inner core
More complex picture from seismology

from Ishii and Dziewonski (2003)
New perspectives
(my personal view)
Experiments at simultaneous high-pressure and high-temperature

\[ P = 49 \text{ GPa}; \ T = 2000 \text{ C} \]

\[(\text{Mg}_{0.9}\text{Fe}_{0.1})\text{O at 12 GPa}\]

from Murakami et al. (2009)

Marquardt et al. (unpubl.)
A enormous “thank you” to Hauke (my first student and good friend)