Summer College on Plasma Physics

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The Sachs-Wolfe Effect and Photon-Graviton interactions

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The Sachs-Wolfe Effect and Photon-Graviton interactions

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Motivations

• The metric space-time is a physical field (it can exchange energy with other fields);

• Microwave photons of CMB are already used to probe the early universe;

• Detection of G waves is a major challenge;

• Could photons be used as probes of G waves and G fields?
Outline

• Photon ray equations;

• Refractive index of a gravitational field;

• Gravitational red shift, a new approach;

• The Sachs-Wolf effect;

• Photons in a gravitational wave;

• Gamma-ray bursts and G waves;

• Nonlinear photon-graviton coupling
Laser experiments with an ionization front

[Diag et al. PRL (1997)]

Laser pulse
65 fs, 2.5 mJ @ 620 nm

Simultaneous measurement:
β=0.942, n=4.26 x 10^{19} cm^{-3}

Can we find similar effects with gravitational fields?
Photon ray equations

In a gravitational field we can always use geometric optics (scaling)

Electric field:

\[ E(x^\nu) = A \exp i \psi(x^\nu) \]

Wavevector components

\[ k_i = \frac{\partial \psi}{\partial x^i} \quad x^i = (ct, \vec{r}) \]

Photon canonical equations

\[ \frac{dx^\alpha}{dt} = \frac{\partial \omega}{dk_\alpha} \quad \frac{dk_\alpha}{dt} = -\frac{\partial \omega}{dx^\alpha} \]

\[ \omega = -ck_0 \]

(\(\alpha=1,2,3\))
Variational principle

\[ \delta \psi = \delta \int k_j x^j = \delta \int (k_\alpha x^\alpha - \omega dt) = 0 \]

Photon frequency shift

\[ \frac{d\omega}{dt} = \frac{\partial \omega}{\partial t} \]

\[ \omega = \omega(k_\alpha, x^\alpha, t) \]

Not to be confused with the photon proper frequency

\[ \omega' = -\frac{\partial \psi}{\partial \tau} = - \frac{\partial \psi}{\partial x^0} \frac{\partial x^0}{\partial \tau} = \frac{\omega}{\sqrt{g_{00}}} \]

Proper time

\[ \tau = \sqrt{g_{00} x^0 c} = \sqrt{g_{00} t} \]
Dispersion relation in a plasma

\[ k_j k^j = g^{ij} k_i k_j = \frac{\omega_p^2}{c^2} \]

Photon effective (rest) mass

\[ m_{\text{eff}} = \frac{\hbar \omega_p}{c} \]

\[ \omega = \omega(\mathbf{k}_\alpha, x^i) \]

\[ \omega = \omega_1 + \sqrt{\nu^2 + \omega_1^2 - \omega_2^2} \]

\[ \omega_1 = Y^{0\alpha} k_\alpha c \quad , \quad \omega_2 = \sqrt{Y^{\alpha\beta} k_\alpha k_\beta c} \]

\[ Y_{ij} = g^{ij} / g^{00} \]

\[ \nu^2 = \frac{\omega_p^2}{g^{00}} \]
G field as an optical medium

\[ \omega = \frac{kc}{n(k_{\alpha}, x_{\alpha}, t)} \]

Refractive index of the gravitational field

\[ n(k_{\alpha}, x_{\alpha}, t) = \frac{k}{Y^{0\alpha} k_{\alpha}} \left[ 1 + \nu^2 + \sqrt{1 - \frac{Y_{\alpha\beta} k_{\alpha} k_{\beta}}{(Y^{0\alpha} k_{\alpha})^2}} \right]^{-1} \]
Robertson metric

\[ ds^2 = (1 + 2\Phi)c^2 dt^2 - (1 - 2\Phi)a^2(t)dr^2 \]

Perturbed Newtonian potential

\[ \Phi \equiv \Phi(r, t) \]

Scale function (universe expansion)

\[ a(t) \]

Gravitational red-shift

Constant frequency, \( \omega \)
But proper frequency changes!

\[ \frac{dk_\alpha}{dt} = -\frac{\partial \omega}{\partial x^\alpha} = \frac{k c}{n^2} \frac{\partial n}{\partial x^\alpha} \]
Gravitational red-shift (cont.)

Initial proper frequency (emitted at R)

\[
\omega_e^2 = \frac{\omega^2}{g_{00}(R)} = \omega_p^2(R) + \frac{k_e^2 c^2}{1 - 2\Phi(R)}
\]

\(\omega\) remains constant

Final proper frequency

\[
\omega_f^2 = \omega^2 = k_f^2 c^2 = (1 + 2\Phi(R)) \left[ \omega_p^2(R) + \frac{k_e^2 c^2}{1 - 2\Phi(R)} \right]
\]

Observed frequency shift

\[\Delta \omega' \simeq \omega_e' \Phi(R)\]

for \(|\Phi(R)| \ll 1\).

Apparent temperature shift

\[\frac{\Delta T}{T} = \Phi(R)\]

(Simple) Sachs-Wolfe effect
Cosmic Microwave Background

- First Predicted by G. Gamow, R. Alpher and R. Herman (1948)
- First detected by A. Penzias and R. Wilson (1965)

\[ T = 2.725 \text{ K} \]
Cosmic Temperature map

WMAP_2008
Cosmological red-shift (locally perturbed)

Contribution from the scale factor $a(t)$

Continuity equation

\[ \dot{\rho} + 3H(\rho + p) = 0 \]

Equation of state

\[ p = w\rho. \]

Matter dominated universe, $w = 0$
Radiation dominated, $w = 1/3$

Apparent temperature shift ($aT = \text{const.}$)

\[ \frac{\Delta T}{T} = -\frac{\Delta a}{a} = -\frac{2\Phi(R)}{3(1 + w)} \]

Accumulated SW effect

\[ \frac{\Delta T}{T} = \frac{1 + 3w}{3(1 + w)} \Phi(R) \]

Matter dominated universe (well known result)

\[ \Delta T/T = \Phi(R)/3. \]
Integrated Sachs-Wolfe

\[
\frac{dk}{dt} = -\frac{\partial \omega}{\partial r} = -\frac{2k^2c^2}{\omega(1-2\Phi)^2} \frac{\partial \Phi}{\partial r} - \frac{(1+2\Phi)}{2\omega} \frac{\partial \omega_p^2}{\partial r}
\]

Low plasma density limit

\[
k(t) = k(0) \exp \left[ -2c \int_{r(0)}^{r(t)} \frac{\partial \Phi(r(t'), t')}{\partial r} dt' \right]
\]

\[
\omega_p^2 \ll 4k^2c^2|\Phi|
\]

Low frequency shift limit

\[
k(t) = k(0) \left( 1 - 2 \int \frac{\partial \Phi}{\partial r} ds \right)
\]

\[
\frac{\Delta T}{T(0)} = -2 \int_{r(0)}^{r(t)} \frac{\partial \Phi}{\partial r} ds
\]

Mendonça, Bingham and Wang, CQG (2008)
Moving perturbations

\[ \Phi(r, t) = \Phi(r - ut) \]

Dynamical invariant

\[ \Omega = \omega(\eta) - uk, \text{ with } \eta = r - ut \]

Generalized Sachs-Wolfe

\[ \frac{\Delta T}{T} \simeq \frac{\Phi_0}{(1 - \beta)} \left[ 1 + \beta \left( 1 - \frac{\omega_{p0}^2}{2\omega_T^2} \right) \right] + \frac{\beta \omega_{p0}^2}{2\omega_T^2(1 - \beta)} \]

Purely vacuum perturbations

\[ \omega_{p0} \simeq 0 \]

It is not a Doppler correction!
Photons in a plane G wave

Perturbed flat space time

\[ g^{ij} = \eta^{ij} + h^{ij} \]

Minkowski metric tensor

\[ \eta^{\alpha\beta} = -\delta^{\alpha\beta}, \quad \eta^{00} = 1 \]

Weak gravitational wave

\[ a = A \sin(k_0 x^0 + k_1 x^1 + \phi) = A \sin(q x - \Omega t + \phi) \]

\[ h_{22} = -h_{33} = a \]

Photon dispersion relation

\[ \omega = kc \left\{ 1 + \frac{a}{2} \left[ \left( \frac{k_y}{k} \right)^2 - \left( \frac{k_z}{k} \right)^2 \right] \right\} \]
Nearly parallel photon propagation

Perpendicular photon motion

$$y(t) = c k_y \int_0^t \frac{1 + a(t')}{k(t')} \, dt'$$

Parallel photon motion

$$\frac{dx}{dt} = c \frac{k_x}{k}$$

$$\frac{dk_x}{dt} = -\frac{1}{2} \left( \frac{k_y}{k} \right)^2 \frac{\partial a}{\partial t}$$
Typical photon trajectories

\[ \frac{d\omega}{dt} = c \frac{dk_x}{dt} \]

Relevance of this numerical example:

Gravitational wave with frequency: \( \Omega = 10^4 \text{ s}^{-1} \)

and amplitude \( A = 10^{-4} \)

\( \tau = Act/2 \)
Influence of G wave amplitude

Decay of G wave (irreversibility)

Model for gamma ray bursters?

SN explosions and GR bursts: Coupling between G waves and photons (and neutrinos)

Spherical gravitational Wave burst

Accelerated escaping photons

[Acceleration of neutrinos could also be relevant to Energetic Cosmic rays - not affected by the GZK limit]
Photons in a spherical G wave

**Space-time metric**

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

\[ \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \text{ and } h_{\mu\nu} \to 0 \text{ at infinity.} \]

**Photon equations of motion**

\[ \frac{dx^\gamma}{dt} = 1 + h^{0\gamma} + \frac{1}{k} h^{\gamma\beta} k_\beta + \frac{1}{2k^2} h^{\alpha\beta} k_\alpha k_\beta k^\gamma \]

\[ \frac{dk_\gamma}{dt} = - (\partial_\gamma h^{0\alpha}) k_\alpha + \frac{1}{2} \left[ - (\partial_\gamma h^{00}) + (\partial_\gamma h^{\alpha\beta}) \frac{k_\alpha k_\beta}{k^2} \right] \]

**Quadrupole moment**

\[ Q_{\alpha\beta} = \int \rho(x) x^\alpha x^\beta d^3x \]

\[ \bar{h}_{\alpha\beta}(t) = \frac{2}{r \, dt^2} Q_{\alpha\beta}(t - r) + O(1/r^2) \]

Wang, Mendonça, Bingham (2009?)
Photon up-shift and bunching

\[ \frac{dr}{dt} = 1 - \frac{1}{2} h^{rr} \]

\[ \frac{dk}{dt} = k \frac{\partial}{\partial r} h^{rr} \]

Photon momentum

Example:
\[ k_0 = 1 \text{ eV}, \ r_0 = 150 \text{ Km}, \]
\[ k = 1 \text{ MeV}, \ r = 1 \text{ AU} \]
Expected Gamma-ray signals

Complete and incomplete bunching inside Gwave packet

Question:
- Is it gauge dependent?
- Is there any conformal field?

\[ G^{ij} = A \ g^{ij} \]
Are we already detecting G waves?

Observed Gamma-ray bursts

A new γ-ray burst classification scheme from GRB 060614


Figure 1 | The light curve of GRB 060614 as observed with the BAT

Figure 2 | Spectral lag as a function of peak luminosity showing GRB 060614 in the region of short-duration GRBs. The lags and peak
Photon-graviton interactions

Maxwell’s equations in a gravitational field

\[ F_{ik}^{;i} = -\frac{4\pi}{c} J^i \]

\[ F_{ik} = \partial_i A^k - \partial_k A^i \]

In vacuum (\( J^i = 0 \))

\[ \partial_k g^{ij} g^{km} F_{jm} = -\frac{F_{ik}}{2g} \partial_k g \]

Einstein equation (in the weak field approximation: \( g_{ik} = \eta_{ik} + h_{ik} \))

\[ \partial^l \partial_l h_{ij} = -16\pi G T_{ij} \]

E.m. energy momentum tensor

\[ T_{ij} = \frac{1}{4\pi} \left( -F_{il} F_{kj}^l + \frac{1}{4} g_{ik} F_{lm} F^{lm} \right) \]
Nonlinear wave coupling

Wave perturbations

Mode coupled equations

\[ 2ik^{l} \partial_{l} A = Wh^{*} A' \exp(i\phi) \]
\[ 2ik'^{l} \partial_{l} A' = W'hA \exp(-i\phi) \]
\[ 2iq^{l} \partial_{l} h = W_{g} A A' \exp(-i\phi) \]

Phase matching

\[ \phi = (k'^{l} - q_{l} - k_{l})x^{l} \approx 0 \]

Important question: allowed geometric configurations

No coupling for co-propagation, \( W = 0 \)

Coupling exists for counter-propagation, \( W \neq 0 \)

\[ A_{j} = e_{j} A \exp(ik_{l}x^{l}) \]
\[ h_{ij} = \epsilon_{ij} h \exp(iq_{l}x^{l}) \]
Conclusions

- Photons can be energized by Gravitational fields and waves;
- Geometric optics can be efficiently used;
- The gravitational field behaves as an optical medium;
- A generalized Sachs-Wolfe effect (including dynamical and plasma corrections) can be obtained;
- Gravitational waves can energize photons;
- Possible explanation for gamma-ray bursts (??);
- Gravitational waves are probably being observed;
- Nonlinear photon-graviton interactions are possible.