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The Sachs-Wolfe Effect and Photon-Graviton interactions

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The Sachs-Wolfe Effect and Photon-Graviton interactions

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Motivations

•The metric space-time is a physical field (it can exchange energy with other fields);

•Microwave photons of CMB are already used to probe the early universe;

• Detection of G waves is a major challenge;

• Could photons be used as probes of G waves and G fields?

Outline

- Photon ray equations;
- Refractive index of a gravitational field;
- Gravitational red shift, a new approach;
- The Sachs-Wolf effect;
- Photons in a gravitational wave;
- Gamma-ray bursts and G waves;
- Nonlinear photon-graviton coupling

Laser experiments with an ionization front

[Dias et al. PRL (1997)]

Laser pulse
65 fs, 2.5 mJ @ 620 nm

Counter-Propagation

Co-Propagation

Simultaneous measurement:
$\beta=0.942$, $n=4.26 \times 10^{19}$ cm^{-3}

Can we find similar effects with gravitational fields?

Photon ray equations

In a gravitational field we can always use geometric optics (scaling)

Electric field:

$$E(x^\nu) = A \exp i\psi(x^\nu)$$

Wavevector components

$$k_i = \partial\psi / \partial x^i$$

$$x^i = (ct, \vec{r})$$

Photon canonical equations

$$\frac{dx^\alpha}{dt} = \frac{\partial\omega}{dk_\alpha} \qquad \frac{dk_\alpha}{dt} = -\frac{\partial\omega}{dx^\alpha}$$

$$\omega = -ck_0$$

(α=1,2,3)

Variational principle

$$\delta\psi = \delta\int k_j dx^j = \delta\int (k_\alpha dx^\alpha - \omega dt) = 0$$

Photon frequency shift

$$\frac{d\omega}{dt} = \frac{\partial\omega}{\partial t}$$

$$\omega = \omega(k_\alpha, x^\alpha, t)$$

Not to be confused with the photon proper frequency

$$\omega' = -\frac{\partial\psi}{\partial\tau} = -\frac{\partial\psi}{\partial x^0}\frac{\partial x^0}{\partial\tau} = \frac{\omega}{\sqrt{g_{00}}}$$

Proper time $\tau = \sqrt{g_{00}x^0c} = \sqrt{g_{00}}t.$

Dispersion relation in a plasma

$$k_j k^j = g^{ij} k_i k_j = \frac{\omega_p^2}{c^2}$$

Photon effective (rest) mass

$$m_{eff} = \hbar \omega_p / c.$$

$$\omega = \omega(k_\alpha, x^i)$$

$$\omega = \omega_1 + \sqrt{\nu^2 + \omega_1^2 - \omega_2^2}$$

$$Y^{ij} = g^{ij} / g^{00}$$

$$\omega_1 = Y^{0\alpha} k_\alpha c \quad , \quad \omega_2 = \sqrt{Y^{\alpha\beta} k_\alpha k_\beta c}$$

$$\nu^2 = \omega_p^2 / g^{00}$$

G field as an optical medium

$$\omega = \frac{kc}{n(k_\alpha, x^\alpha, t)}$$

Refractive index of the gravitational field

$$n(k_\alpha, x_\alpha, t) = \frac{k}{Y^{0\alpha} k_\alpha} \left[1 + \nu^2 + \sqrt{1 - \frac{Y^{\alpha\beta} k_\alpha k_\beta}{(Y^{0\alpha} k_\alpha)^2}} \right]^{-1}$$

Robertson metric

$$ds^2 = (1+2\Phi)c^2dt^2 - (1-2\Phi)a^2(t)dr^2$$

Perturbed Newtonian potential

$$\Phi \equiv \Phi(r,t)$$

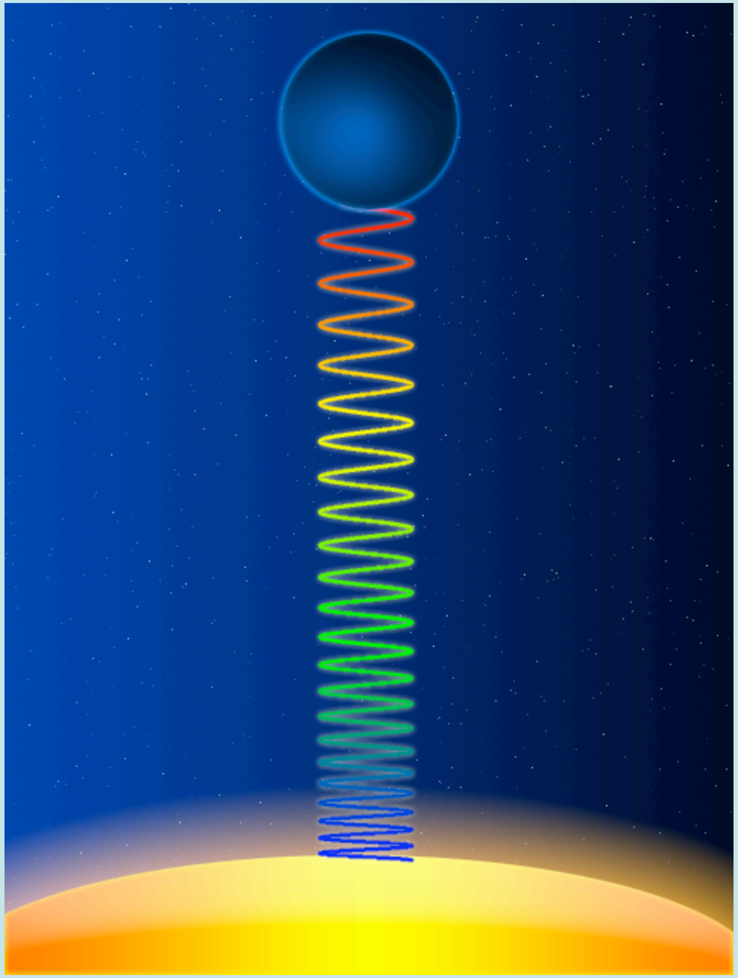
Scale function (universe expansion)

$$a(t)$$

Gravitational red-shift

Constant frequency, ω
But proper frequency changes!

$$\frac{dk_\alpha}{dt} = -\frac{\partial \omega}{\partial x^\alpha} = \frac{kc}{n^2}\frac{\partial n}{\partial x^\alpha}$$



Gravitational red-shift (cont.)

Initial proper frequency (emitted at R)

$$\omega_e'^2 = \frac{\omega^2}{g_{00}(R)} = \omega_p^2(R) + \frac{k_e^2 c^2}{1 - 2\Phi(R)}$$

ω remains constant

Final proper frequency

$$\omega_f'^2 = \omega^2 = k_f^2 c^2 = (1 + 2\Phi(R)) \left[\omega_p^2(R) + \frac{k_e^2 c^2}{1 - 2\Phi(R)} \right]$$

Observed frequency shift

$$\Delta\omega' \simeq \omega_e' \Phi(R)$$

for $|\Phi(R)| \ll 1$,

Apparent temperature shift

$$\frac{\Delta T}{T} = \Phi(R)$$

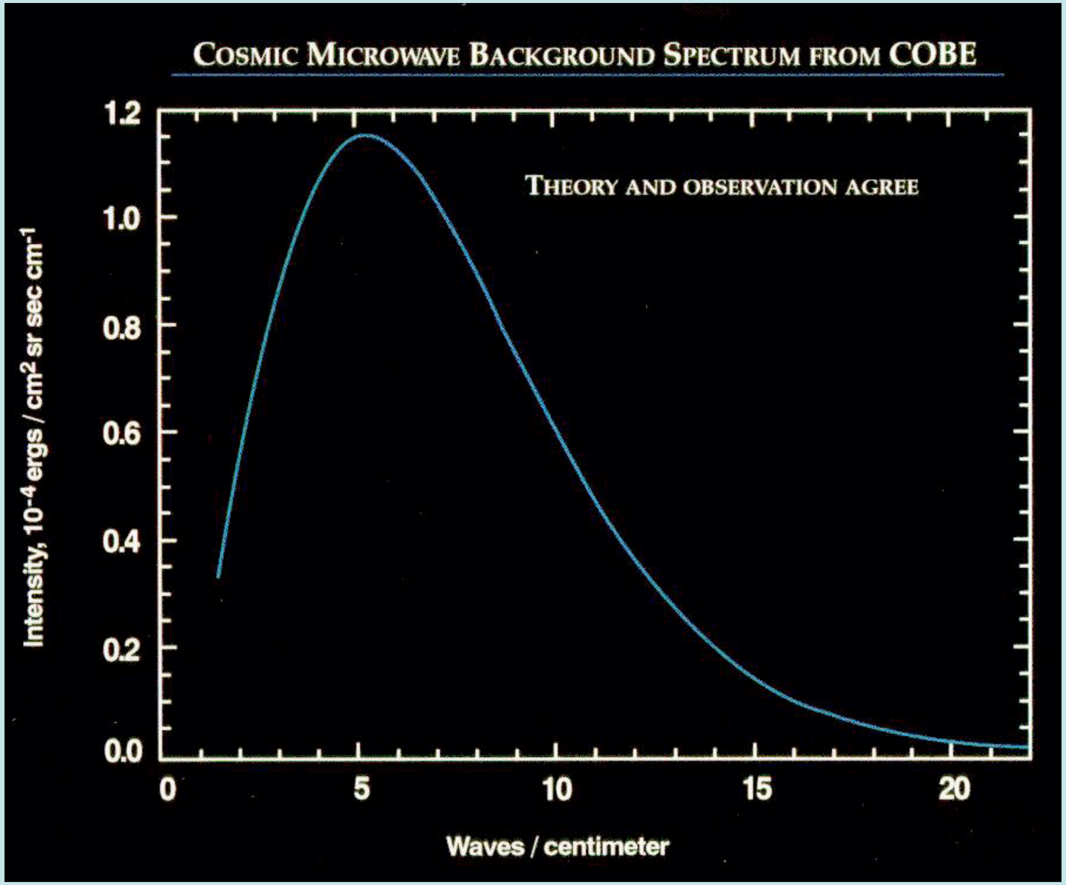
(Simple) Sachs-Wolfe effect

Cosmic Microwave Background

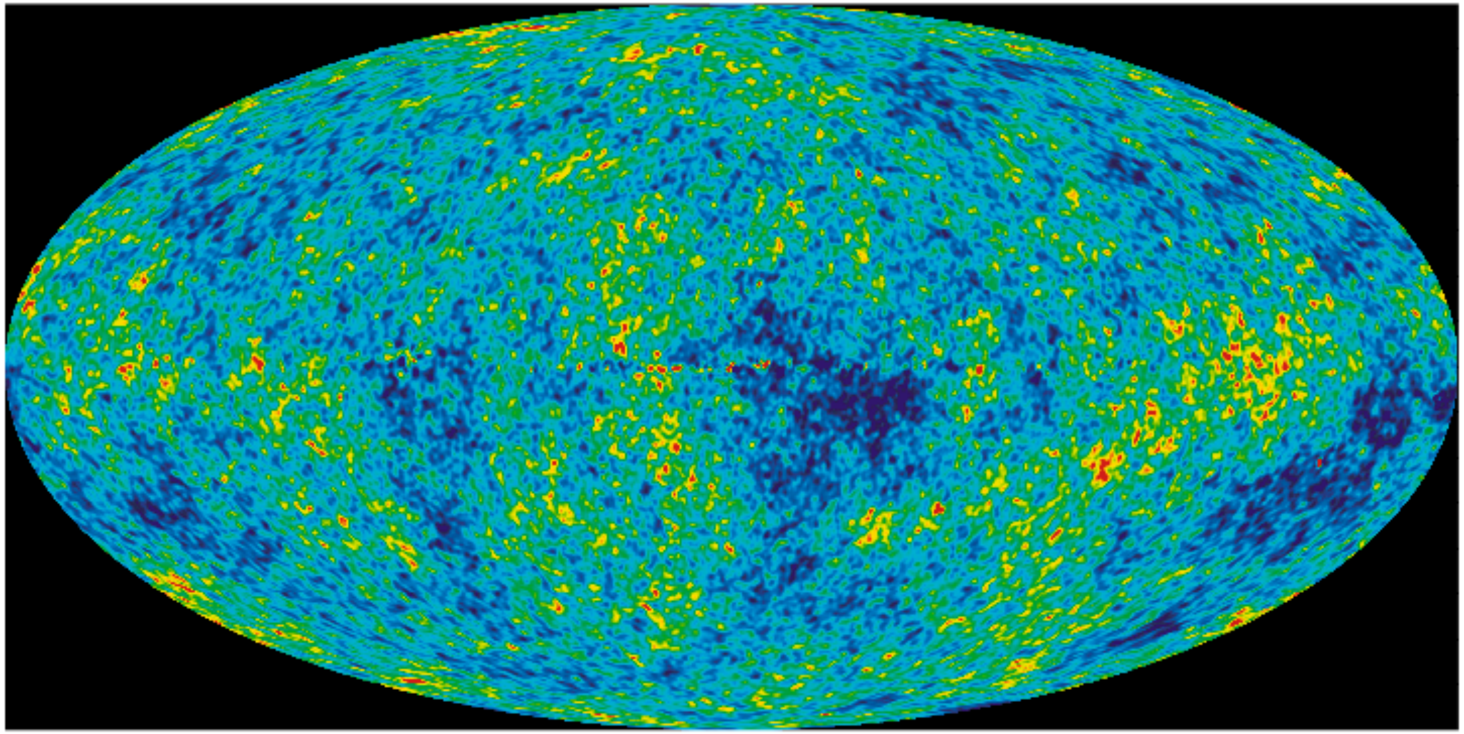
- First Predicted by G. Gamow, R. Alpher and R. Herman (1948)

- First detected by A. Penzias and R. Wilson (1965)

T = 2.725 K



Cosmic Temperature map



WMAP_2008

Cosmological red-shift (locally perturbed)

Contribution from the scale factor a(t)

Continuity equation $\dot{\rho} + 3H(\rho + p) = 0$ $\quad H = \dot{a}/a$

Equation of state $p = w\rho.$

Matter dominated universe, w = 0
Radiation dominated, w = 1/3

Apparent temperature shift (aT=const.)

$$\frac{\Delta T}{T} = -\frac{\Delta a}{a} = -\frac{2\Phi(R)}{3(1+w)}$$

Accumulated SW effect

$$\frac{\Delta T}{T} = \frac{1+3w}{3(1+w)}\,\Phi(R)$$

Matter dominated universe
(well known result)

$$\Delta T/T = \Phi(R)/3.$$

Integrated Sachs-Wolfe

$$\Phi(r,t)$$

$$\frac{dk}{dt} = -\frac{\partial \omega}{\partial r} = -\frac{2k^2c^2}{\omega(1-2\Phi)^2}\frac{\partial \Phi}{\partial r} - \frac{(1+2\Phi)}{2\omega}\frac{\partial \omega_p^2}{\partial r}$$

Low plasma density limit

$$k(t) = k(0)\exp\left[-2c\int_{r(0)}^{r(t)} \frac{\partial \Phi(r(t'),t')}{\partial r} dt'\right]$$

$$\omega_p^2 \ll 4k^2c^2|\Phi|$$

Low frequency shift limit

$$k(t) = k(0)\left(1 - 2\int \frac{\partial \Phi}{\partial r} ds\right)$$

$$\frac{\Delta T}{T(0)} = -2\int_{r(0)}^{r(t)} \frac{\partial \Phi}{\partial r} ds$$

Mendonça, Bingham and Wang, CQG (2008)

Moving perturbations

$$\Phi(r,t) = \Phi(r - ut)$$

Dynamical invariant

$$\Omega = \omega(\eta) - uk, \text{ with } \eta = r - ut$$

Generalized Sachs-Wolfe

$$\frac{\Delta T}{T} \simeq \frac{\Phi_0}{(1-\beta)}\left[1+\beta\left(1-\frac{\omega_{p0}^2}{2\omega_T^2}\right)\right] + \frac{\beta\omega_{p0}^2}{2\omega_T^2(1-\beta)}$$

Purely vacuum perturbations

$$\frac{\Delta T}{T} \simeq \Phi_0 \frac{(1+\beta)}{(1-\beta)}$$

$$\omega_{p0} \simeq 0$$

It is not a Doppler correction!

Photons in a plane G wave

$$g^{ij} = \eta^{ij} + h^{ij}$$

Perturbed flat space time

Minkowski metric tensor

$$\eta^{\alpha\beta} = -\delta^{\alpha\beta}, \ \eta^{00} = 1$$

Weak gravitational wave

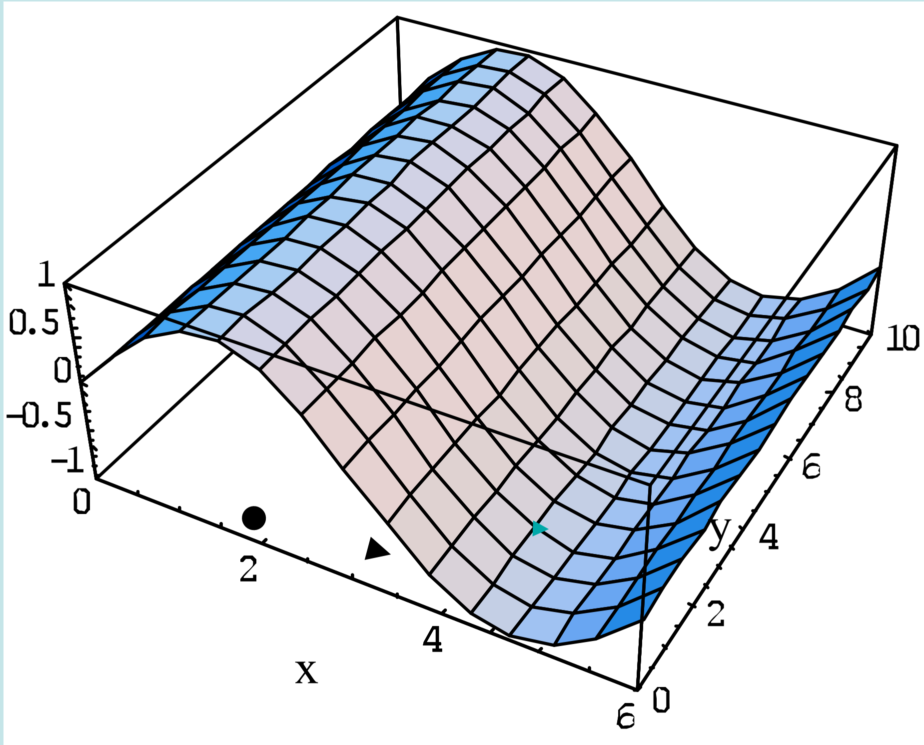
$$a = A\sin(k_0 x^0 + k_1 x^1 + \phi) = A\sin(qx - \Omega t + \phi)$$

$$h_{22} = -h_{33} = a$$

Photon dispersion relation

$$\omega = kc\left\{1 + \frac{a}{2}\left[\left(\frac{k_y}{k}\right)^2 - \left(\frac{k_z}{k}\right)^2\right]\right\}$$

Nearly parallel photon propagation



Perpendicular photon motion

$$y(t) = ck_y \int^t \frac{1 + a(t')}{k(t')} dt'$$

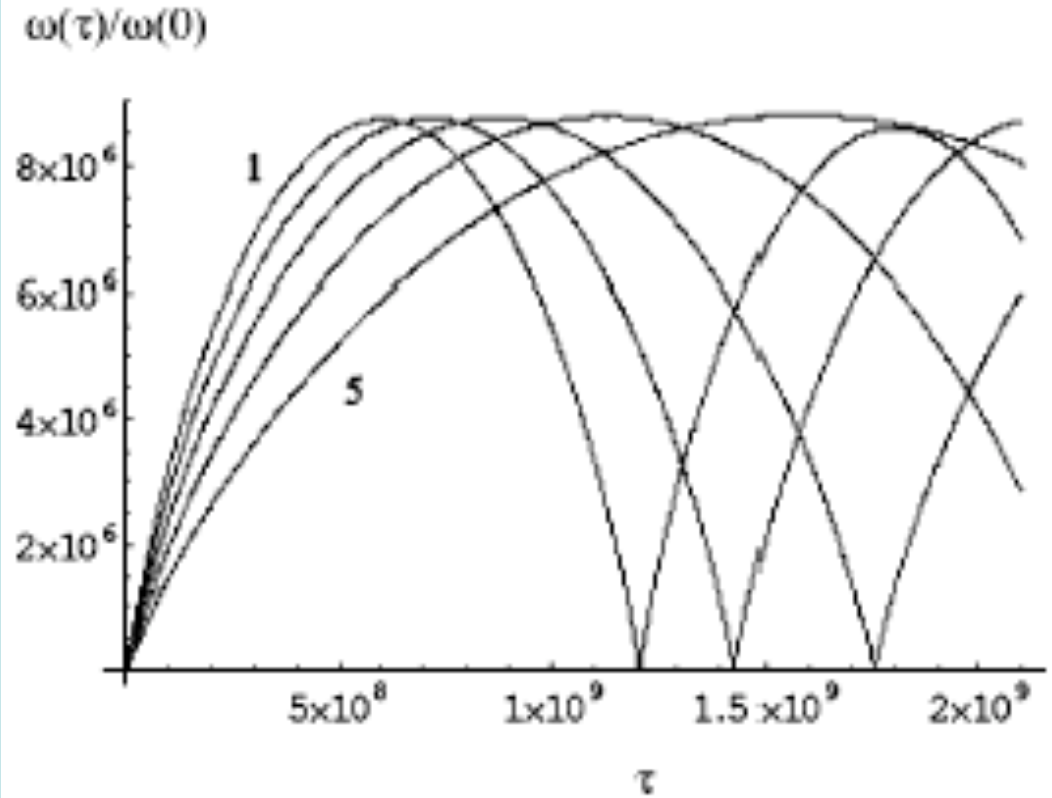
Parallel photon motion

$$\frac{dx}{dt} = c\frac{k_x}{k}$$

$$\frac{dk_x}{dt} = -\frac{1}{2}\left(\frac{k_y}{k}\right)^2 \frac{\partial a}{\partial t}$$

Typical photon trajectories

$$\frac{d\omega}{dt} = c\frac{dk_x}{dt}$$

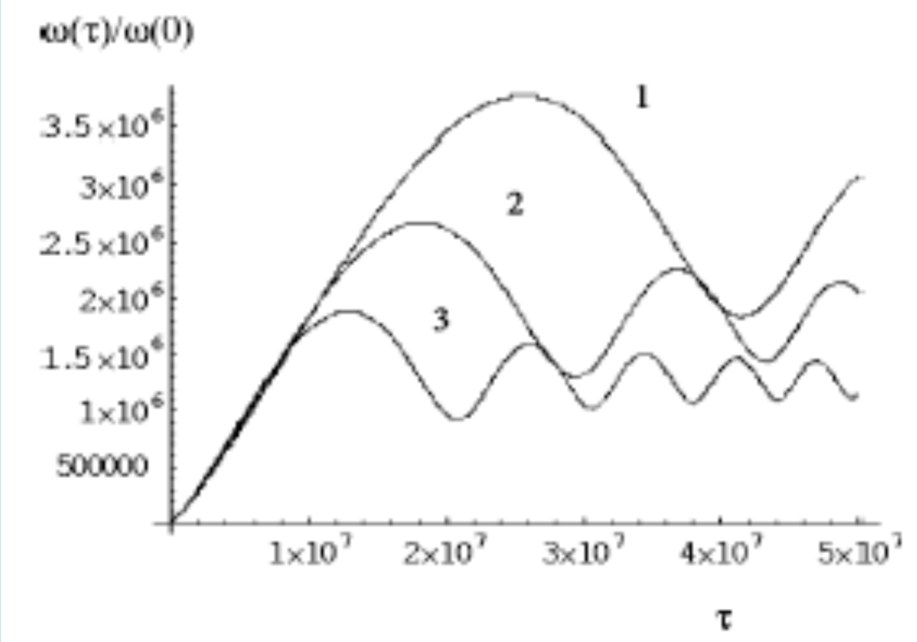


- Relevance of this numerical example:

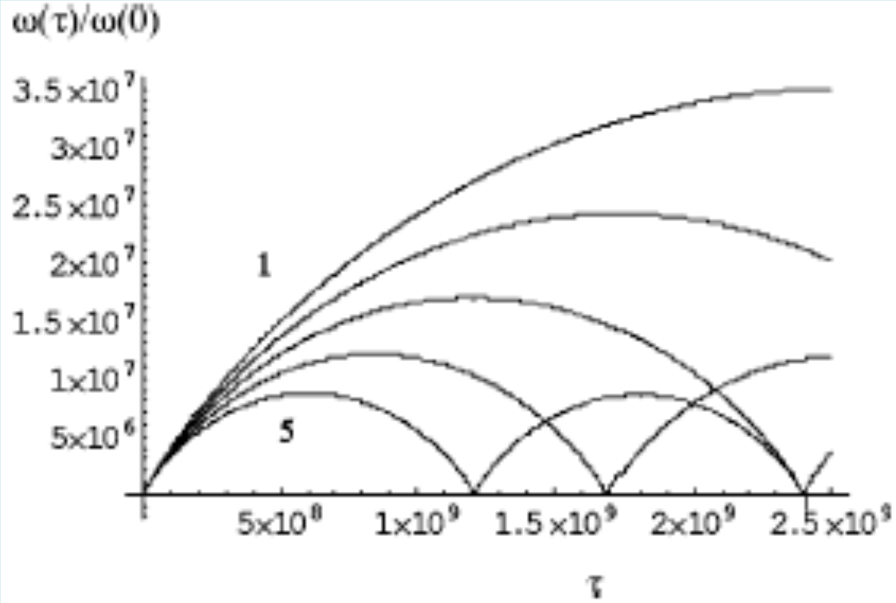
  Gravitational wave with frequency: $\Omega=10^4\ s^{-1}$ and amplitude $A=10^{-4}$

$\tau=Act/2$

Decay of G wave (irreversibility)



Influence of G wave amplitude



Model for gamma ray bursters?

Mendonça and L.Drury, PRD (2002)

SN explosions and GR bursts:
Coupling between G waves and photons (and neutrinos)

Spherical gravitational
Wave burst

Accelerated escaping photons

[Acceleration of neutrinos could also be relevant to
Energetic Cosmic rays - not affected by the GZK limit]

Photons in a spherical G wave

Space-time metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ and $h_{\mu\nu} \to 0$ at infinity.

Photon equations of motion

$$\frac{dx^\gamma}{dt} = 1 + h^{0\gamma} - \frac{1}{k} h^{\gamma\beta} k_\beta + \frac{1}{2k^3} h^{\alpha\beta} k_\alpha k_\beta k^\gamma$$

$$\frac{dk_\gamma}{dt} = -(\partial_\gamma h^{0\alpha}) k_\alpha + \frac{1}{2}\left[-(\partial_\gamma h^{00}) + (\partial_\gamma h^{\alpha\beta}) \frac{k_\alpha k_\beta}{k^2}\right]$$

Quadrupole moment

$$Q_{\alpha\beta} = \int \rho(x) x^\alpha x^\beta d^3x$$

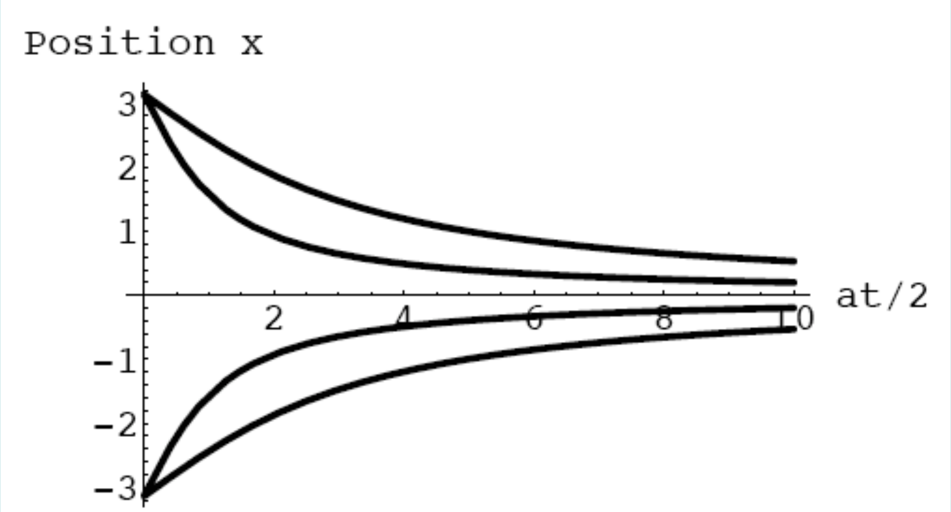
$$\bar{h}_{\alpha\beta}(t) = \frac{2}{r}\frac{d^2}{dt^2} Q_{\alpha\beta}(t-r) + O(1/r^2)$$

Wang, Mendonça, Bingham (2009?)

Photon up-shift and bunching

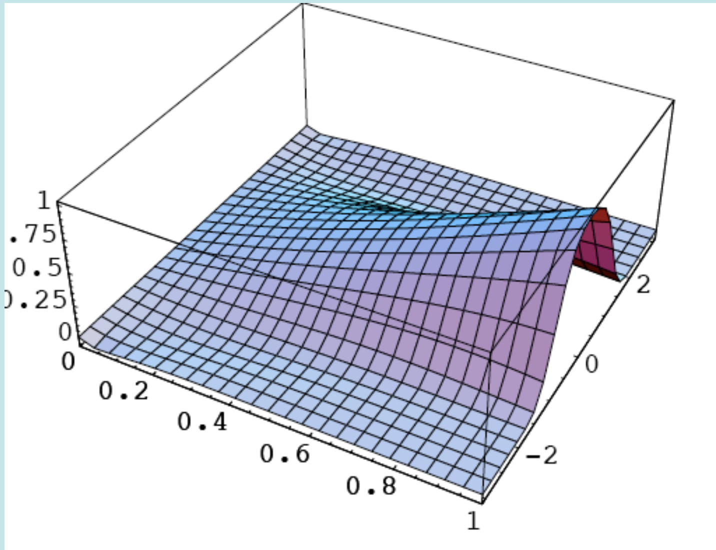
$$\frac{dr}{dt} = 1 - \frac{1}{2} h^{rr}$$

$$\frac{dk}{dt} = k \frac{\partial}{\partial r} h^{rr}$$



Photon momentum

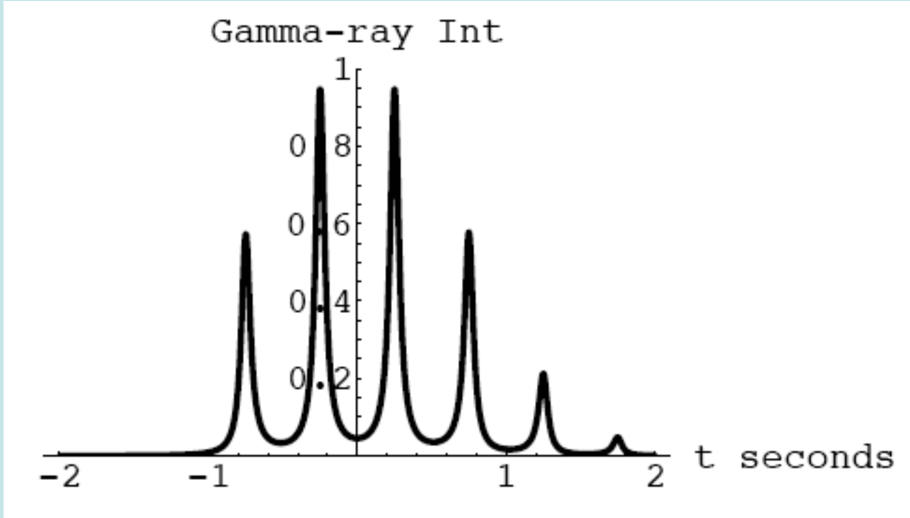
$$k(r) = k_0 \left(\frac{r}{r_0} \right)^{\pm Q}$$



Example:
k_0 = 1 eV, r_0 = 150 Km,
k = 1 MeV, r = 1 AU

Expected Gamma-ray signals

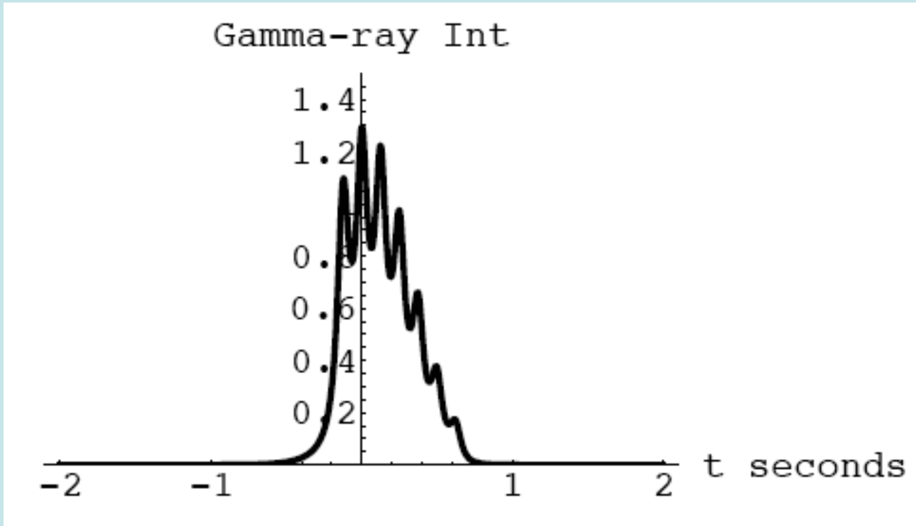
Complete and incomplete bunching inside Gwave packet



Question:

- **Is it gauge dependent?**
- **Is there any conformal field?**

$$G^{ij} = A\, g^{ij}$$



Observed Gamma-ray bursts

nature Vol 444|21/28 December 2006|doi:10.1038/nature05376

LETTERS

A new γ-ray burst classification scheme from GRB 060614

N. Gehrels[1], J. P. Norris[1], S. D. Barthelmy[1], J. Granot[2], Y. Kaneko[3], C. Kouveliotou[4], C. B. Markwardt[1,5], P. Mészáros[6,7], E. Nakar[8], J. A. Nousek[6], P. T. O'Brien[9], M. Page[10], D. M. Palmer[11], A. M. Parsons[1], P. W. A. Roming[6], T. Sakamoto[1,12], C. L. Sarazin[13], P. Schady[6,10], M. Stamatikos[1,12] & S. E. Woosley[14]

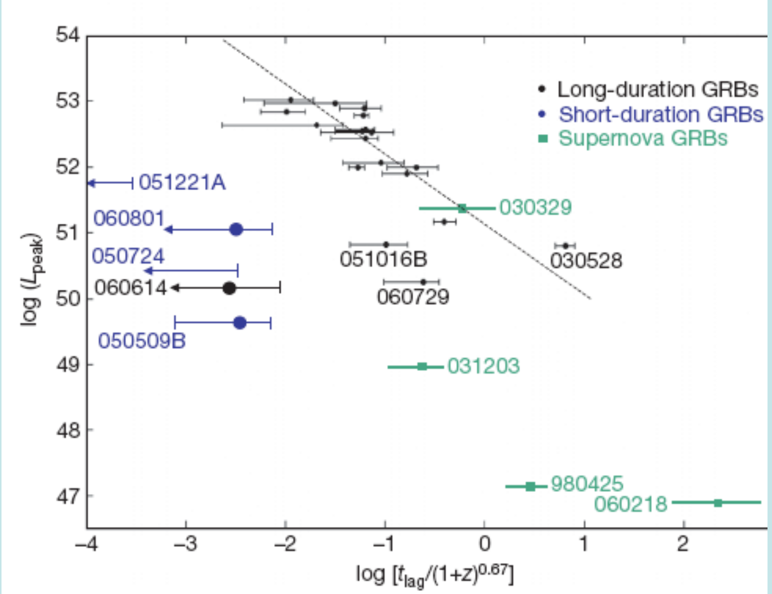


Figure 2 | Spectral lag as a function of peak luminosity showing GRB 060614 in the region of short-duration GRBs. The lags and peak

Are we already detecting G waves?

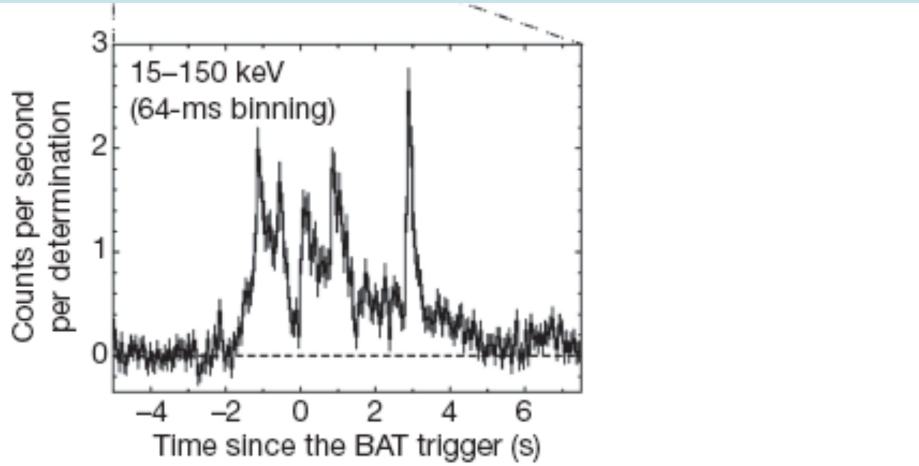


Figure 1 | The light curve of GRB 060614 as observed with the BAT.

Photon-graviton interactions

Maxwell's equations in a gravitational field

$$F^{ik}_{;i} = -\frac{4\pi}{c} J^i$$

$$F_{ik} = \partial_i A^k - \partial_k A^i$$

Electromagnetic field tensor

In vacuum ($J^i = 0$)

$$\partial_k g^{ij} g^{km} F_{jm} = -\frac{F^{ik}}{2g} \partial_k g$$

Einstein equation (in the weak field approximation: $g_{ik} = \eta_{ik} + h_{ik}$)

$$\partial^l \partial_l h_{ij} = -16\pi G T_{ij}$$

E.m. energy momentum tensor

$$T_{ij} = \frac{1}{4\pi}\left(-F_{il}F^{l}_{k} + \frac{1}{4} g_{ik} F_{lm} F^{lm}\right)$$

Nonlinear wave coupling

Wave perturbations

$$A_j = e_j A \exp(i k_l x^l)$$
$$h_{ij} = \varepsilon_{ij} h \exp(i q_l x^l)$$

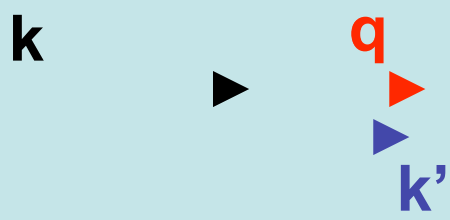
Mode coupled equations

$$2ik^l \partial_l A = W h^* A' \exp(i\phi)$$
$$2ik'^l \partial_l A' = W' h A \exp(-i\phi)$$
$$2iq^l \partial_l h = W_G A A' \exp(-i\phi)$$

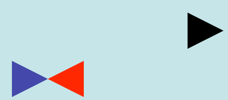
Phase matching

$$\phi = \left(k'_l - q_l - k_l\right) x^l \approx 0$$

Important question: allowed geometric configurations



No coupling for co-propagation, W = 0



Coupling exists for counter-propagation, W ≠0

Conclusions

- Photons can be energized by Gravitational fields and waves;
- Geometric optics can be efficiently used;
- The gravitational field behaves as an optical medium;
- A generalized Sachs-Wolfe effect (including dynamical and plasma corrections) can be obtained;
- Gravitational waves can energize photons;
- Possible explanation for gamma-ray bursts (?);
- Gravitational waves are probably being observed;
- Nonlinear photon-graviton interactions are possible.