



**The Abdus Salam
International Centre for Theoretical Physics**



2052-54

Summer College on Plasma Physics

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KAM-theorem, QLT of plasma, Chaos and beyond

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KAM-theorem, QLT of plasma, Chaos and beyond

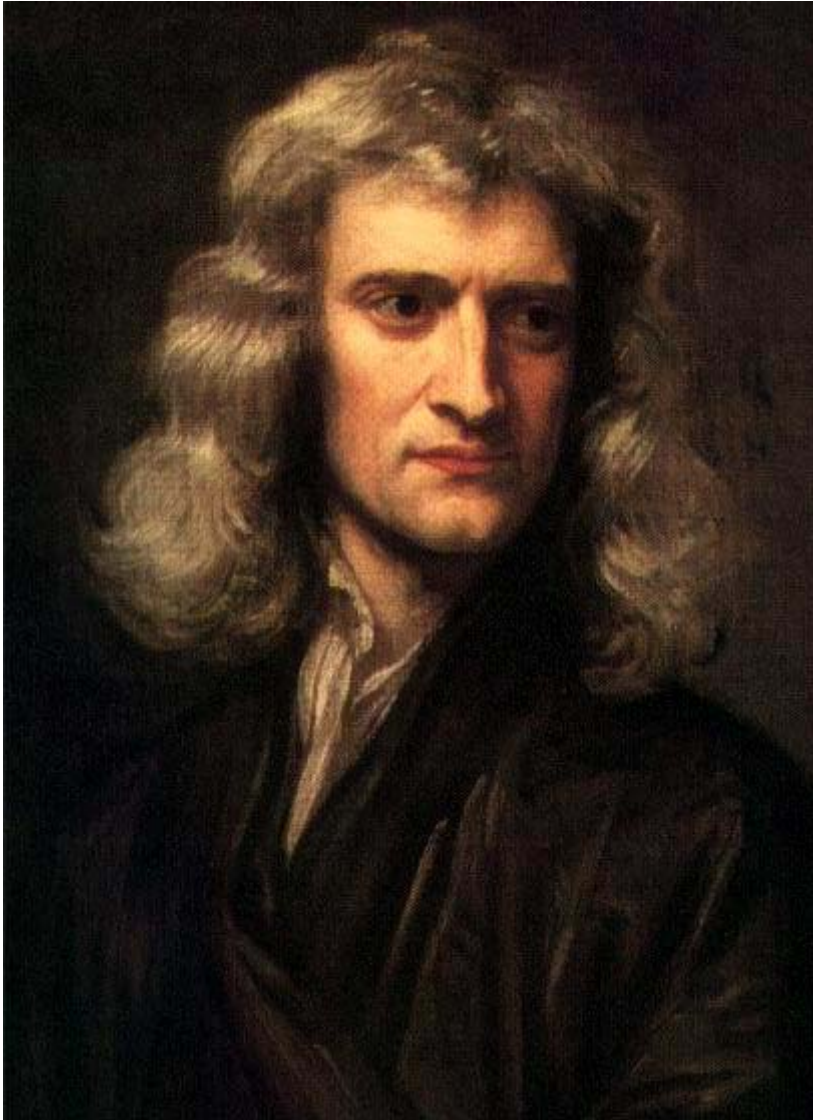
- Everything started with “Stability of Solar System”
- KAM theory
- Did it make Planetary motion more stable?
- Quasilinear Theory is opposite limit re:KAM (Landau Resonances vs. Planetary Resonances)
- Hamiltonian Chaos



**Kolmogorov: Stability of
Planetary Orbits (Lecture,
1959)**

Going beyond 2-body problem (adding planet/planet interaction)

- Newton
- Laplace
- Poincare
- Perturbation technique and extraction of secular effects
- Planetary resonances of higher orders
- KAM theorem



Philosophiæ
Naturalis
Principia
Mathematica

1687

**Newton/s conjecture - Solar System is UNSTABLE;
needs DIVINE INTERVENTIONS (how frequently?)**

Pierre-Simon Laplace

1749-1827

Mécanique céleste

Exposition du système du monde



“Je n'avais pas besoin de cette hypothèse-là”

$$H = H_0(I) + \varepsilon V(I, \mathcal{G})$$

$$\omega(I) = \frac{\partial H_0}{\partial I}$$

$$\Delta I \approx \varepsilon^{\frac{1}{2}}$$



**Strength of
Planet/Planet
interaction**

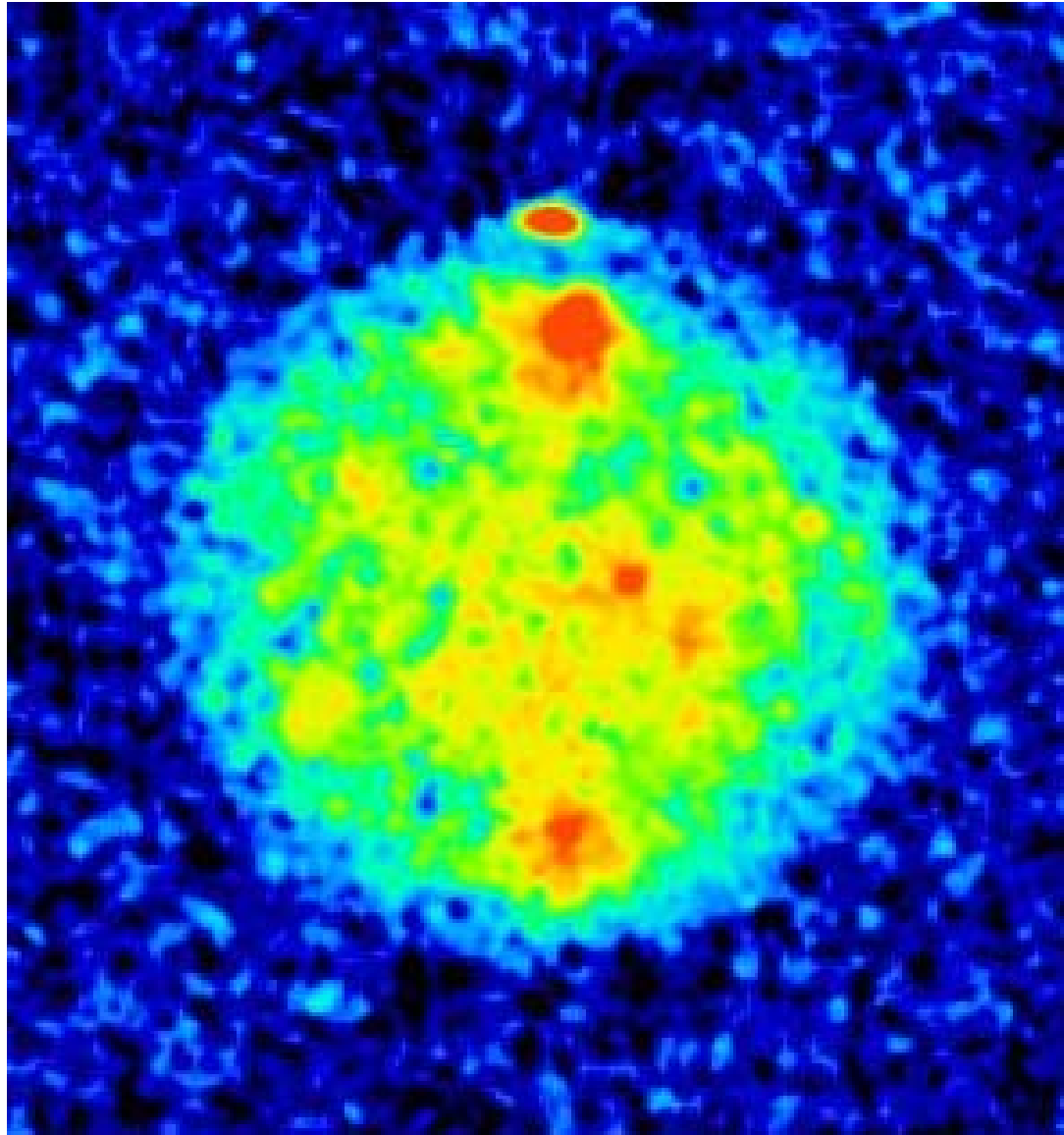
**Width of
resonant
region**

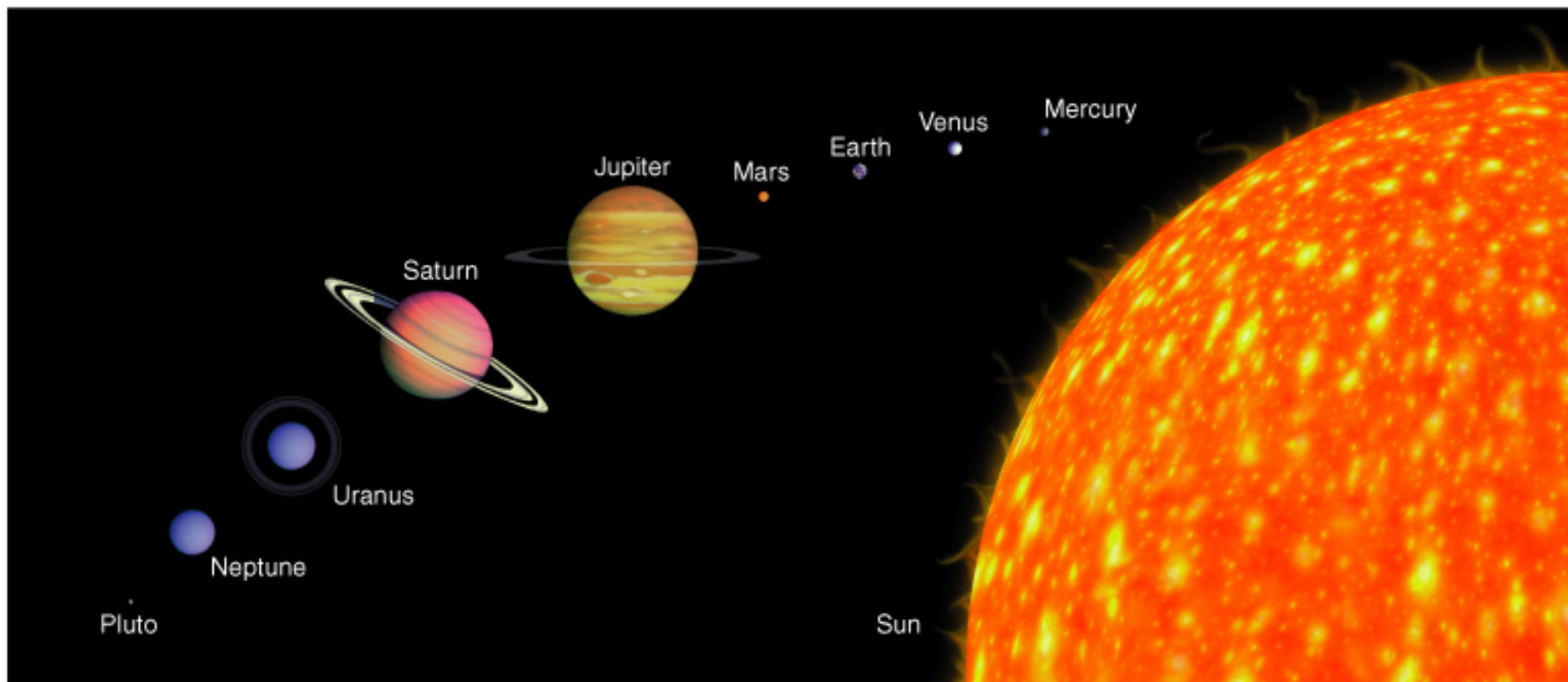
Where do we stay today ?

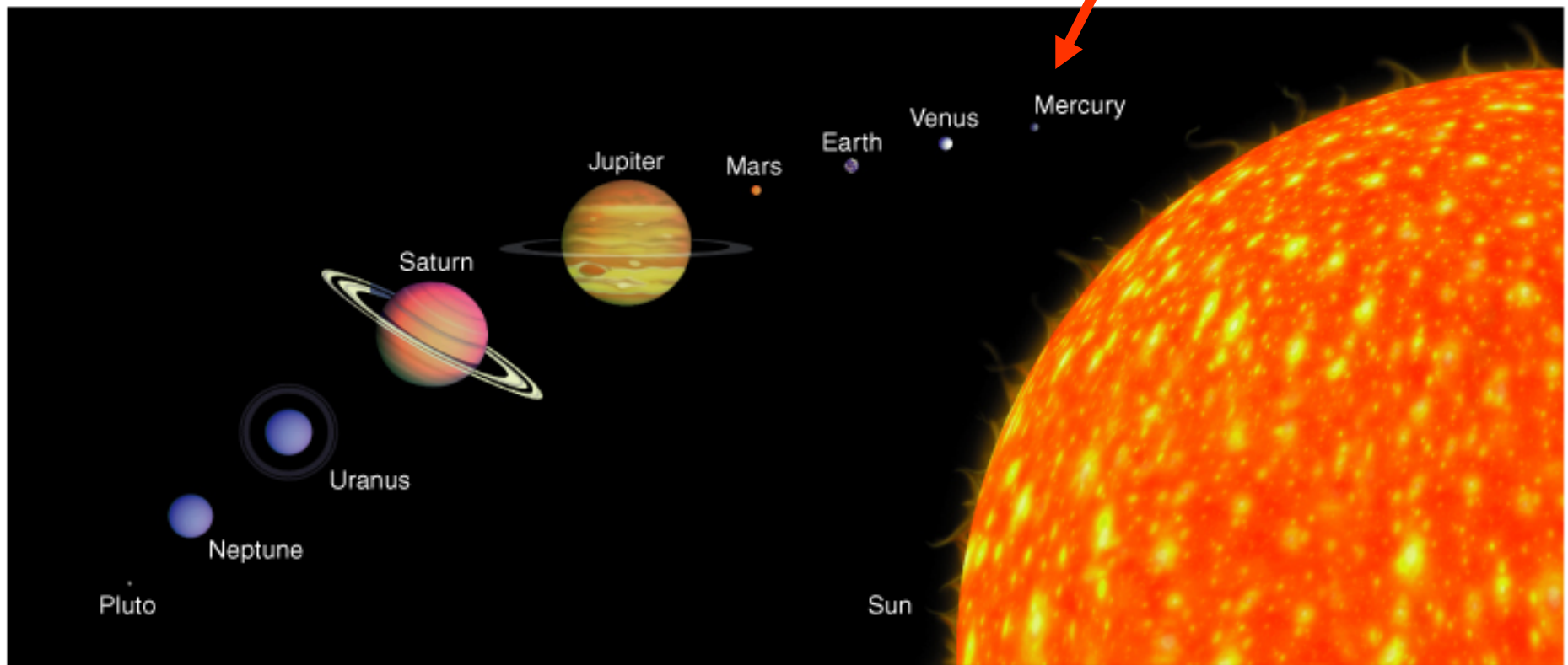
- KAM –theorem is not applicable to Solar System (actual planets too heavy)
- Search of secular effects with sophisticated computer simulation
- Effects of multi-dimensionality: tidal forces, spin-orbit interaction

Modern view

- Endangered species (planets) identified: Pluto, Mercury; time scale for cataclysmic outcome 100 – 800 Mln years
- Should we be afraid ?
- IAU and Pluto
- Discovery of frozen water on the poles of Mercury







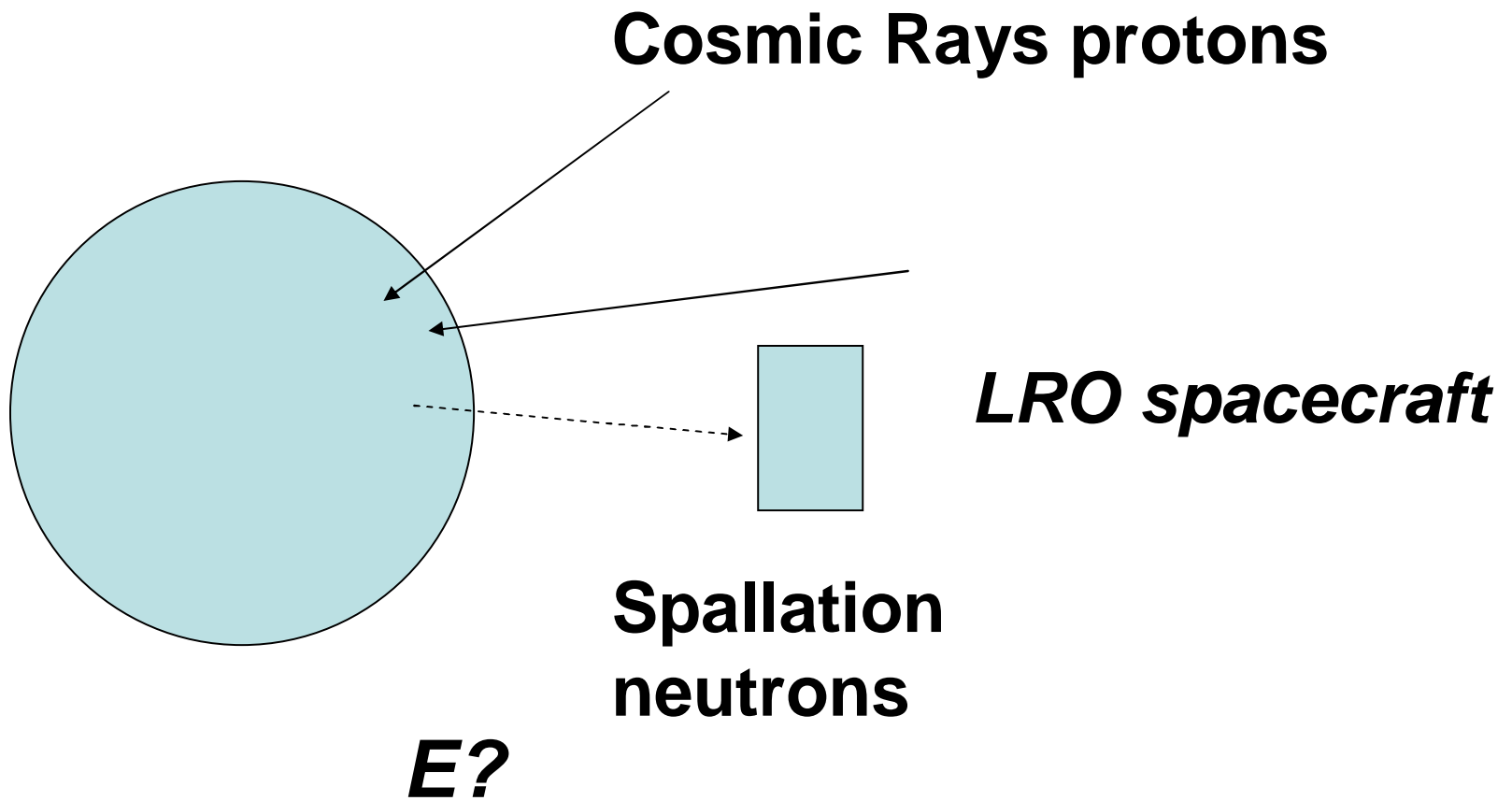
IKI –Institute of Space Research, Moscow;

Image of the Nucleus of Halley's Comet by Vega S/C camera



(C) IKF

Smoothed



What is Physics?

(What is role of Science? Religion?)

- ***Traditional approach' based on definition of Physics as the scientific discipline..***

- **“The Game of Eternity” as a metaphor:**

Its “rules” are introduced by ?? (“Designer”, Creator, Nature) from the outset .. And “Reality Show” was launched according to this rules..Big Bang!

No more “Divine Interventions”(?)

We (the humans) are trying to decipher that rules (the Laws of Nature)

Two greatest components (mysteries of this game)

- Mystery of Universe:

The Task is to construct the scenario with time line of its evolution (from Big Bang to “Self-Organization” in multitude of forms of celestial objects

- Mystery of Life:

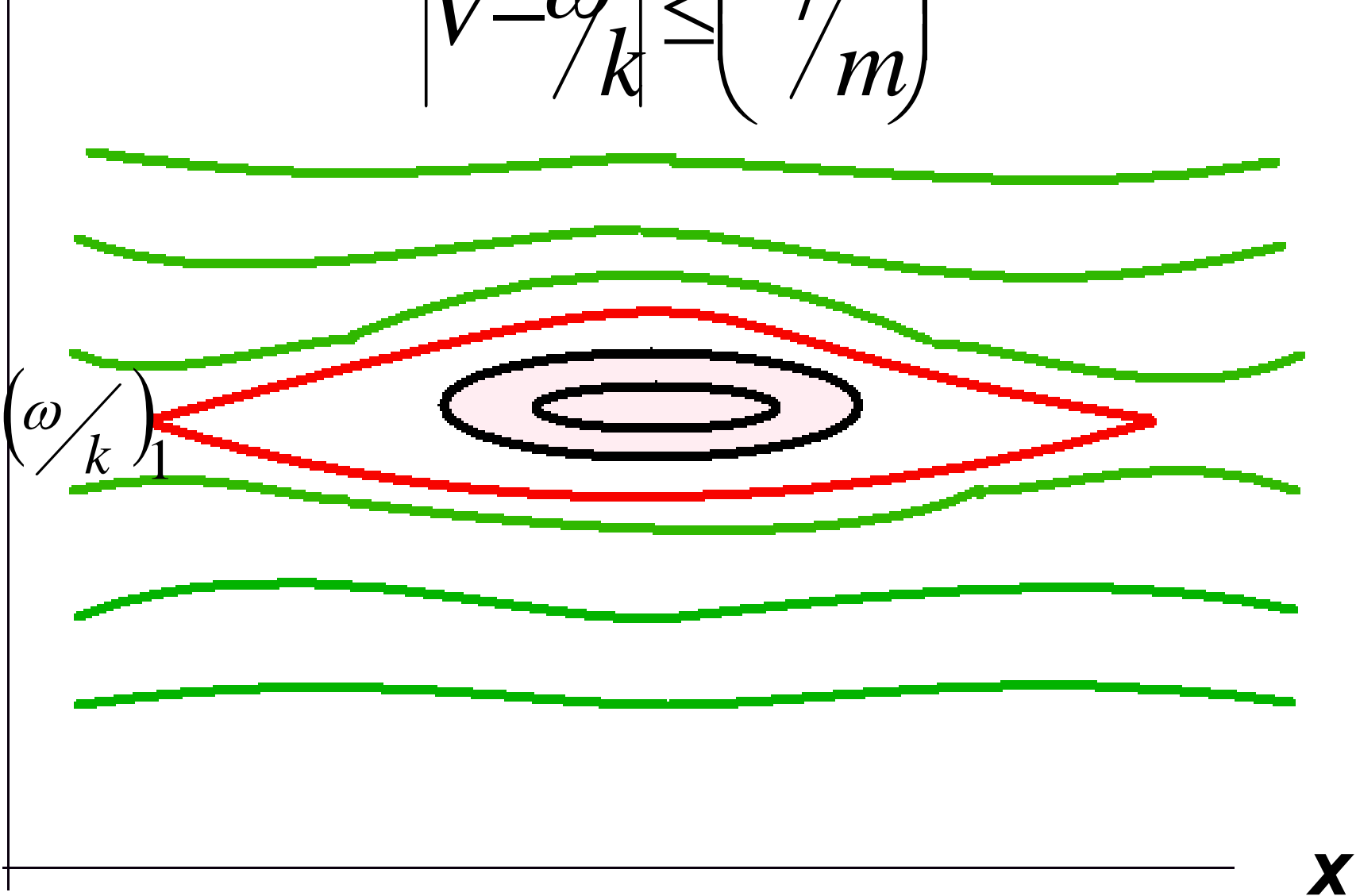
The equivalent task is to understand
The Origin of Life

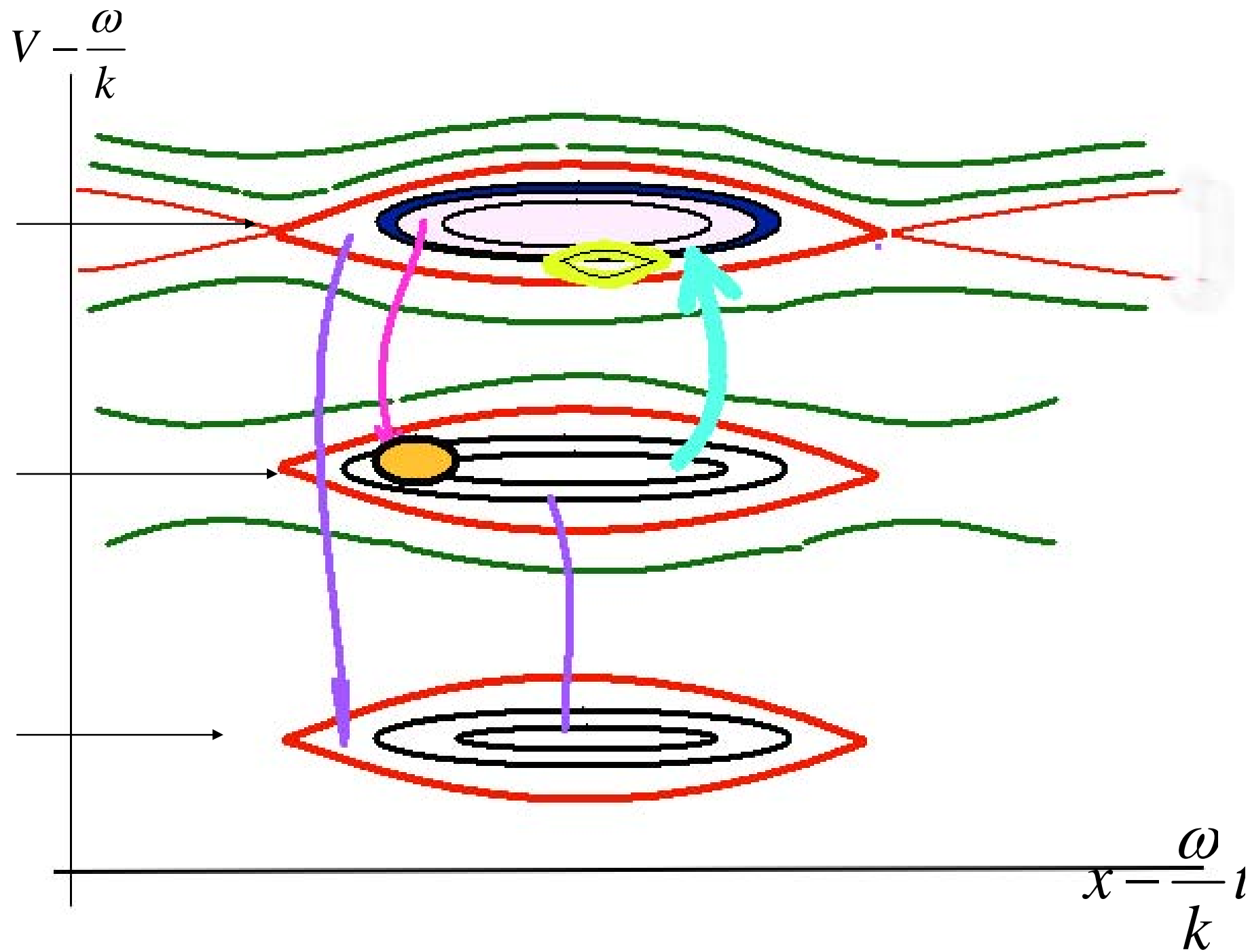
Back to KAM theorem

$$m \frac{dV}{dt} = e \sum E_i \exp i(\omega_i - kv)t$$

v

$$\left| \frac{V - \omega}{k} \right| \leq \left(\frac{e\phi}{m} \right)^{1/2}$$





$$\left(\frac{e\varphi}{m} \right)^{1/2} \text{ much less than } \left(\frac{\omega}{k} \right)_{n+1} - \left(\frac{\omega}{k} \right)_n$$

This limit corresponds to KAM (Kolmogorov-Arnold-Mozer) case.

KAM-Theorem :

***As applied to our case of Charged Particle –
Wave Packet Interaction –***

“Particle preserves its orbit “

$$\left(\frac{e\phi}{m} \right)^{1/2} \text{ greater than } \left(\frac{\omega}{k} \right)_{n+1} - \left(\frac{\omega}{k} \right)_n$$

That means - overlapping of neighboring resonances

Repercussions:

- "collectivization" of particles between neighboring waves;

- particles moving from one resonance to another – "random walk"? And if yes

- what is **Diffusion Coefficient**?(in velocity space)

$$m \frac{dV}{dt} = e \sum E_i \exp i(\omega_i - kv)t$$

$$V = \frac{e}{m} \sum E_i \exp i(\omega - kv)t \Big/ i(\omega - kv)$$

$$\mathbf{V} \times d\mathbf{V}/dt =$$

$$\frac{e^2}{m^2} \sum \sum EE^* \exp i(\omega_i - \omega_j - k_i v + k_j v)t \Big/ i(\omega - kv)$$

$$V^2 \propto Dt$$

$$\mathbf{D} = \pi e^2 / m^2 \sum |E|^2 \delta(kv - \omega)$$

$$\sum_k = \frac{1}{2\pi} \int dk$$

***Repercussions: Quasilinear Theory,
Plateau Formation,***

***Beam + Plasma Instability Saturation
etc.***

Instability: Velocity Anisotropy
(“Cyclotron Instability” of Alfvén waves)

$$\omega + kv_z = \omega_B \quad (\text{Cyclotron resonance})$$
$$\gamma \propto \int d\mathbf{v} \left[\left(1 - kv_{\perp}/\omega\right) \frac{\partial f}{\partial v_{\perp}} + kv_{\perp}/\omega \frac{\partial f}{\partial v_z} \right]$$

(Sagdeev & Shafranov, 1960)

Now should be applied
at cyclotron resonance

for accelerated particles (CR protons)

$$\hat{D}_{QL} f =$$

$$(e/M)^2 \sum |E|^2 \delta(\omega - \omega^l) \left[(1 - kv_z/\omega) \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp + kv_z/\omega \frac{\partial}{\partial v_z} \right]$$

$$\times \left[(1 - kv_z/\omega) \frac{\partial f}{\partial v_\perp} + kv_z/\omega \frac{\partial f}{\partial v_z} \right]$$

$$\omega^l = \omega_B + kv_z$$

(Vedenov, Velikhov, Sagdeev, 1961)

$$\hat{D}_{QL} f =$$

$$(e/M)^2 \sum |E|^2 \delta(\omega - \omega') \left[(1 - kv_z/\omega) \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp + kv_z/\omega \frac{\partial}{\partial v_z} \right]$$

$$\times \left[(1 - kv_z/\omega) \frac{\partial f}{\partial v_\perp} + kv_z/\omega \frac{\partial f}{\partial v_z} \right]$$



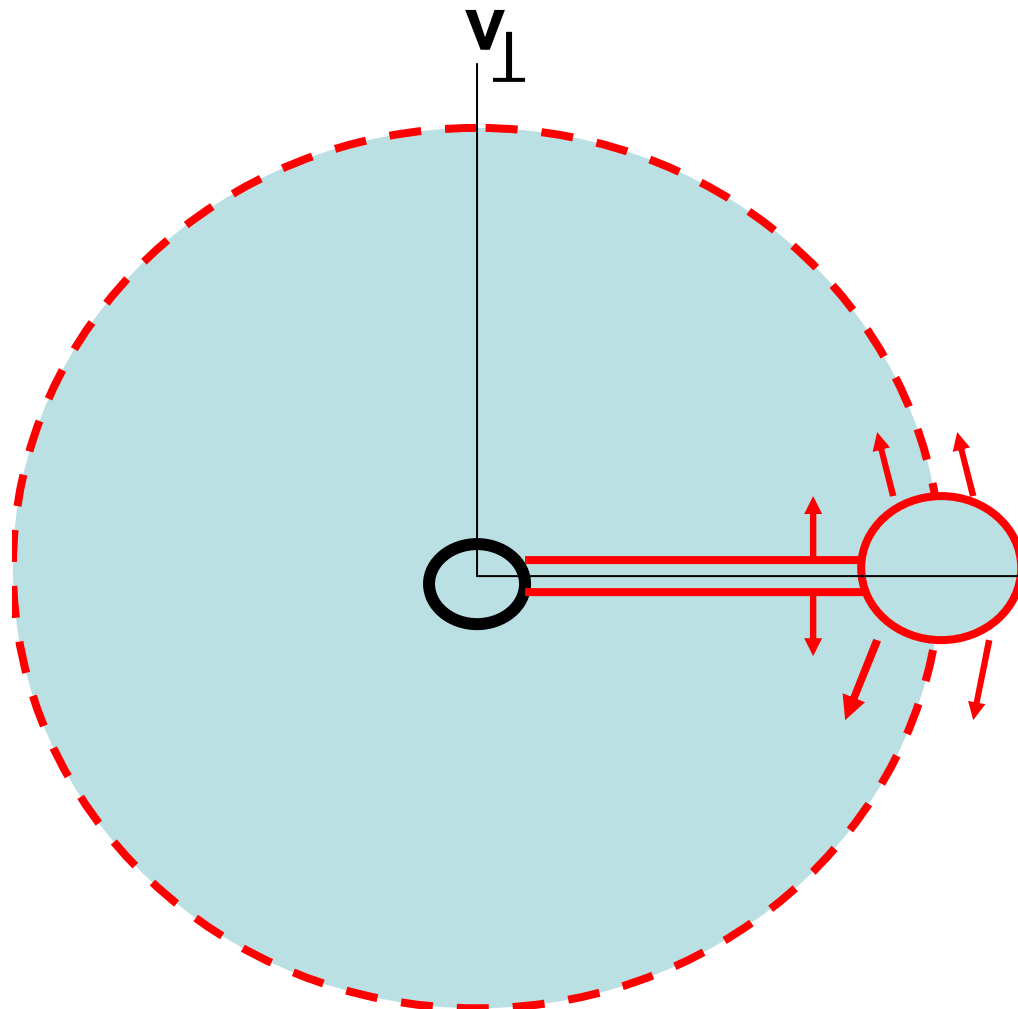
$$\frac{1}{\omega_B} \frac{\partial}{\partial \mathcal{G}} \left(\sum |\mathbf{B}|^2 \delta(\omega - \omega') \frac{\partial}{\partial \mathcal{G}} f \right)$$

(Kennel, Petcheck, 1966)

Feedback on particles: Quasilinear Theory of Particles/Cyclotron Waves Interaction



plateau



$$\frac{v_{\perp}^2 + v_{\parallel}^2}{2} - \frac{\omega v_{\parallel}}{k} = \text{const}$$

If $\beta \gg 1$

Simplified approach

- Spatial Diffusion approximation is valid:

-QL estimate of $v_{\text{eff}} \approx \omega_B \frac{(\delta B)^2}{B^2}$

$$L_{\text{eff}} \approx \frac{c}{v_{\text{eff}}} ; \quad |$$

Magnetic field lines diffusion



**Marshall
ROSENBLUTH**

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$



**George
ZASLAVSKY**

Z plays role of time;

**dB – role of wave
amplitude**

Web Map (Zaslavsky Map)

$$V' = V * \cos(Q) - (U + K * \sin(V + 2 * \pi * F * N)) * \sin(Q)$$

$$U' = V * \sin(Q) + (U + K * \sin(V + 2 * \pi * F * N)) * \cos(Q)$$

$$Q = 2 * \pi / A$$

$$U' = U \cos(Q) - (U + K \sin(U + 2 \times \text{PI} \times F \times N)) \times \sin(Q)$$

$$U' = U \sin(Q) + (U + K \sin(U + 2 \times \text{PI} \times F \times N)) \times \cos(Q)$$

$$Q = 2 \times \text{PI} / A$$

— PARAMETERS OF MAP —

K .130000E+00

A .411000E+03

F .250000E+00

THE CHANGING OF ALL VALUES
IS FOLLOWED BY START MAP FROM THE BEGINNING

----- PLOTTING PARAMETERS -----

STEP FOR PLOTTING= 1

START VARIABLES BY VALUE

START VARIABLES BY CURSOR

COLOR 15

COLOR INCREMENT= 0

START 'SHIFT X' = .0000000E+00

START 'SHIFT Y' = .0000000E+00

START 'SIZE X' = 2*PI* .1200000E+02

START 'SIZE Y' = 2*PI* .899062E+01

PUT ZOOM

CURRENT 'SHIFT X' = .0000000E+00

CURRENT 'SHIFT Y' = .0000000E+00

CURRENT 'SIZE X' = 2*PI* .1200000E+02

CURRENT 'SIZE Y' = 2*PI* .899062E+01

SCREEN PROPORTIONAL= YES

TORUS

LENGTH OF LINE= 0.0000000000000000

--- MAP TYPES ---

WEB WAVE MAP

STANDARD

RELAT. BASIN

INHOMOGENEOUS

WEB 2-WAVE

DIFFUS 2-WAVE

2-K

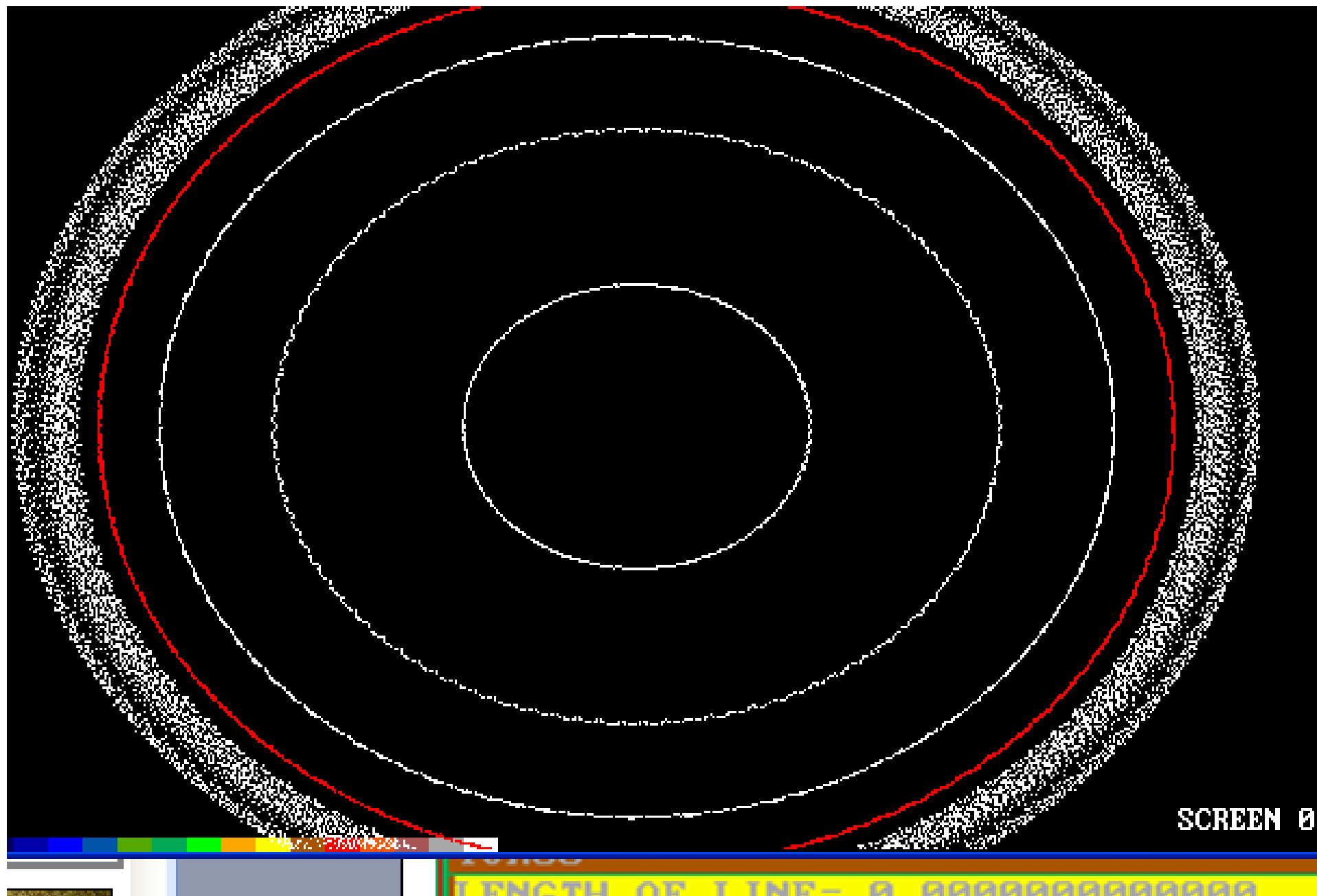
SUPERBALL

BAR

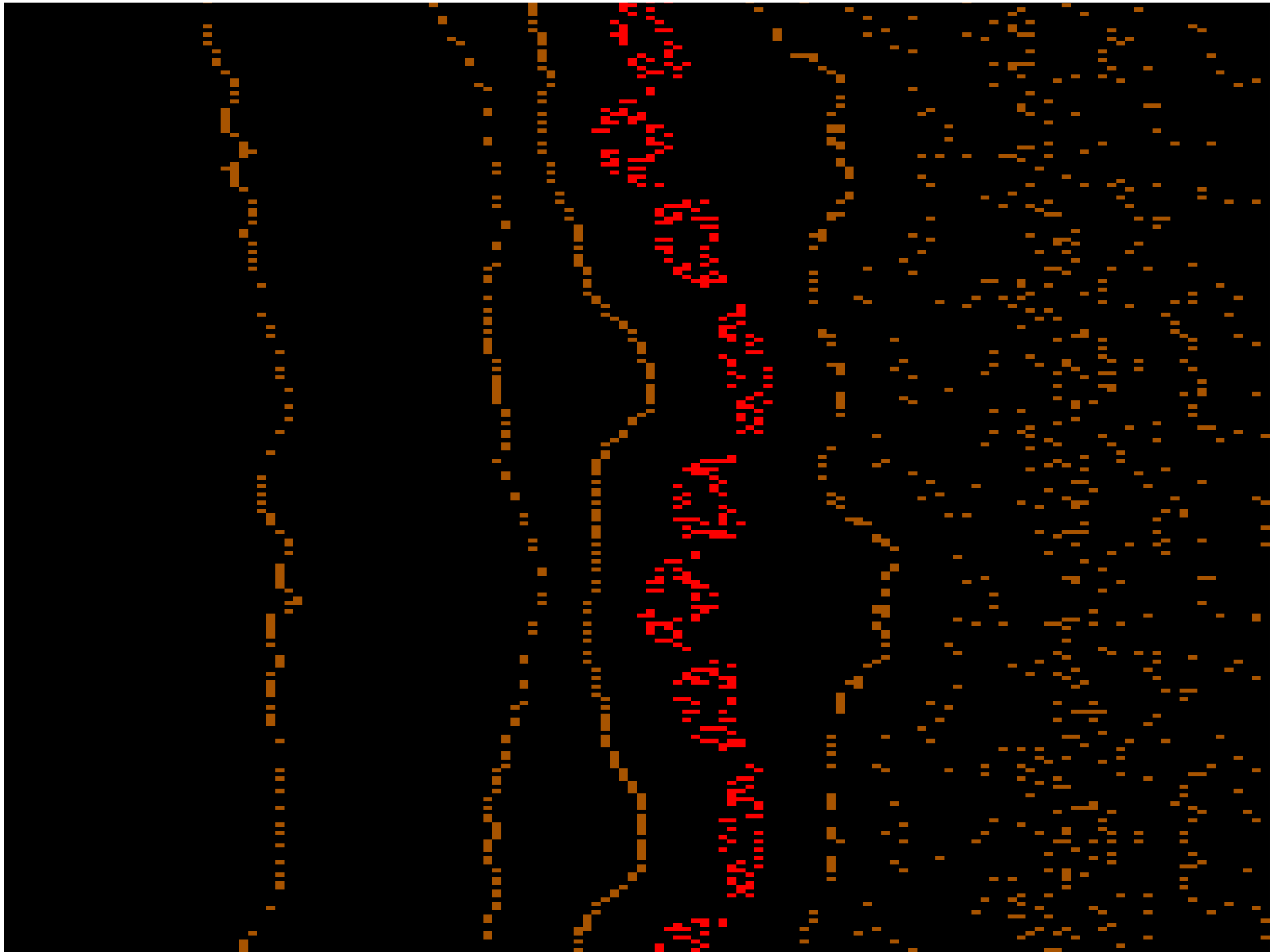
ANTI_STANDARD

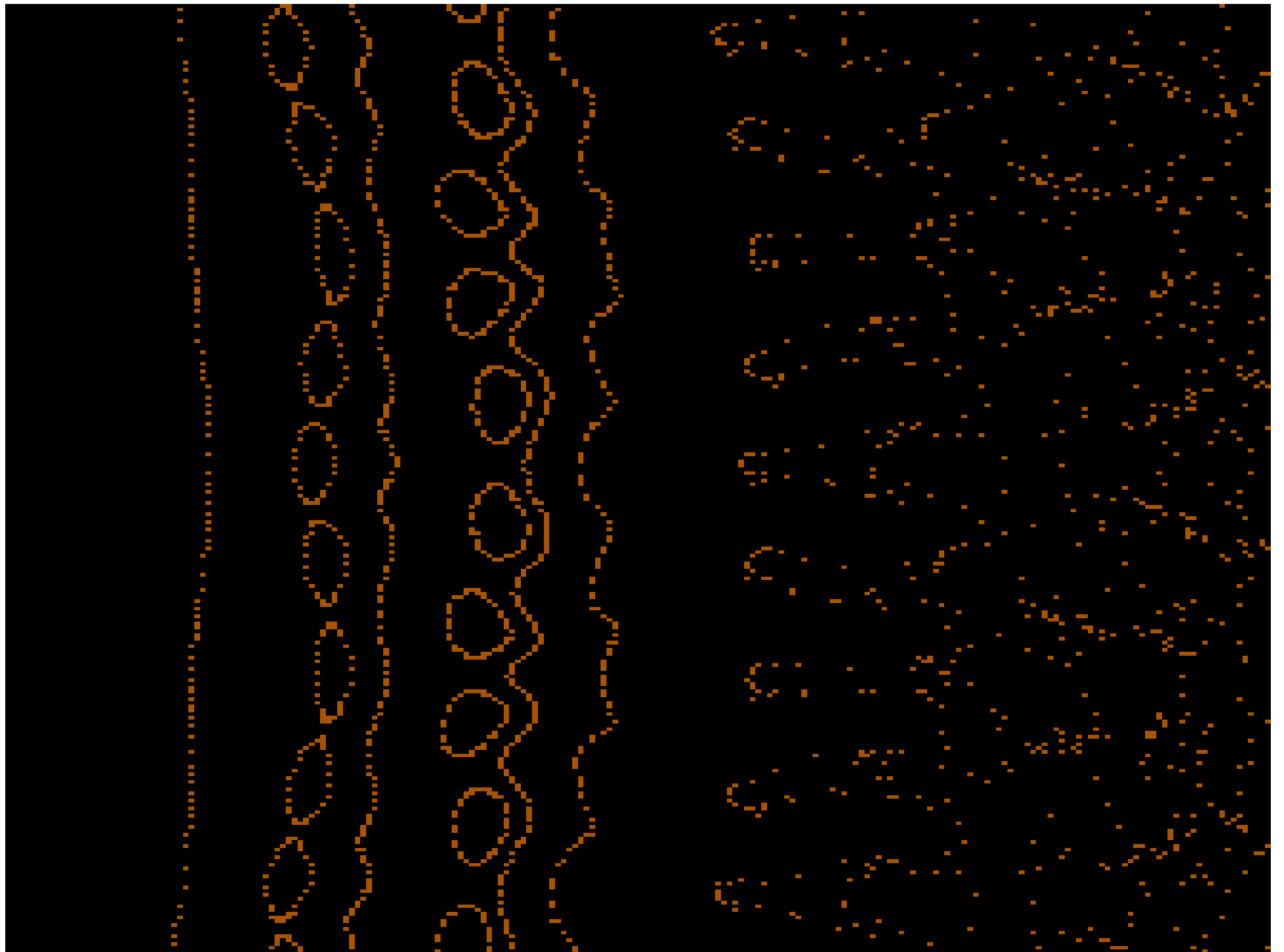
STANDARD*2

STANDARD*N



“Minimal Chaos”





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***Where we are now in
these games?***