Summer College on Plasma Physics

10 - 28 August 2009

Spectroscopic Plasma Diagnostics

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Technique depends on transmission of photons by materials; reflection properties; wavelength relative to mechanical or atomic spacing; photon energy relative to semiconductor band gaps & ionization potential.
\[ \hbar \omega = h \nu = E_i - E_j \] Photon energy.

\[ \lambda = \frac{2\pi c}{\omega} = \frac{c}{\nu} \]

- Spontaneous transition probability (each i-atom): \( A_{ij} \) per unit time.
- Total number of transitions \((i \rightarrow j)\) per unit volume per unit time:
  \[ A_{ij} n_i \]
  where \( n_i \) is density of i-atoms

Many different lines from each species

Many different species: Neutral +1 +2 +3 +4

OI OII OIII OIV ...
CI CII CIII CIV ...

Result: very complex spectra.
Example spectrograph

Rather old fashioned photographic plate.

Horizontal position corresponds to wavelength.

Relative intensity is important in line identification.

Rarely do we identify every single line.
The view of plasma is defined by two apertures (and a spacing between them).
Each point in one aperture (A) collects from a solid angle $\Omega$.

Number of photons detected (per unit time)

$$S = \eta \cdot \frac{A \cdot \Omega}{4\pi} \int A_{ij} n_i \, dl$$

- $\eta$ includes losses in optical components detector efficiency, etc.
- $A\Omega$ is “étendue” = light gathering power defined by optical collection system.

Assuming viewing path is narrow enough that $n_i \sim \text{const}$ across it (but not along beam).

Measure (line average) density of excited state.
Measure (impurity) **species density** by relating

Excited state $n_i$, which is the thing we actually observe,
to Ground state $n_0$ (or the total density of all states of this species).

Solving for the theoretical relationship between $n_i$ and $n_0$ requires knowledge of

- Collisional Excitation rate $n_e < \sigma_{i'j'} v > (V_{i'j'})$
- Radiative rates $(A_{i'j'})$
- Sometimes other things (Recombination Charge Exchange)

Complicated solution of multi-parameter equations governing evolution of excitation state populations. (Keep theoretical spectroscopists employed.)

There are some simple cases (e.g. Thermal Equilibrium, Coronal Equilibrium).
But these rarely apply to interesting plasmas.

More often complex, somewhat uncertain.
Spectrometers


1. Spherical Mirror(s) → Plane Wavefront
2. Grating diffracts at angle dependent on wavelength (and grating angle).
3. Focussed to exit (4) slit. (Selects '1' wavelength).

Replace exit slit with detector array → Polychromator, spectrum.

Match input slit to different plasma positions → Multiple spectra.
Dispersion of different $\lambda$.

Imaging of different slit positions.

High Resolution Crystal X-ray Spectrometer.

Fig. 1. Outline of the optics of the spectrometer used in the tokamak measurements. The wavelength dispersion is shown in A, and the vertical focusing and imaging in B. The entrance slit and the detector had Be windows to allow evacuation of the spectrometer; the slit was positioned close to a $5 \times 15$ mm$^2$ Be window on the tokamak.
Fig. 1. Vertical Bragg x-ray spectrometer in the TFTR diagnostic basement.
Spherically bent crystal

Needs no slits.

Enables simultaneous imaging of plasma and spectral resolution.
Top View

Side View

Tokamak view images to position on detectors.
Different lines correspond to different excitations and electronic configurations.

Their relative weight is different between the plasma edge (colder) and the center (hotter).
Line Width and Shift

Natural line width very small. Several broadening mechanisms arising from atom’s dynamics/environment.

1. Doppler Shift \( \Delta \nu = \nu - \nu_0 \)

<table>
<thead>
<tr>
<th>Fractional Change</th>
<th>Frequency Change</th>
<th>Wavelength Change</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\Delta \nu}{\nu} )</td>
<td>( \frac{(-)\Delta \lambda}{\lambda} \approx )</td>
<td>( \frac{v}{c} )</td>
<td></td>
</tr>
</tbody>
</table>

Distribution of velocities: \( f_i(v) \Rightarrow \) Distribution of Frequencies

Intensity as a function of frequency

\[
I_i(\nu) \propto f \left( \frac{\Delta \nu}{\nu} \right) = f_i \left( \left[ \frac{\nu}{\nu_0} - 1 \right]c \right)
\]

Shape of line directly reflects the atomic velocity distribution function along line of sight.

e.g. Maxwellian distribution

\[
f = \left( \frac{m_i}{2 \pi T_i} \right)^{\frac{1}{2}} \exp \left( -\frac{m_i v^2}{2 T_i} \right)
\]

leads to a Gaussian line shape:

\[
I_i(\lambda) \propto \exp \left( -\frac{m_i}{2 T_i} \left[ \frac{\lambda}{\lambda_0} - 1 \right]^2 c^2 \right)
\]
A convenient measure of the width of a line is Full Width Half Maximum: FWHM

for thermal Gaussian line:

\[
\frac{\Delta \nu_{1/2}}{\nu_0} = \frac{\Delta \lambda_{1/2}}{\lambda_0} = \sqrt{2 \ln 2} \frac{T_i}{m_i c^2}
\]

Measure temperature \( T_i \) from line width \( \Delta \nu_{1/2} \). Strictly this is impurity species \( n_i \) temperature, but usually relaxation/equilibrium of temperatures among ions is fast so this is ion temperature, the same for all species: impurities, bulk ions.
$T_i \approx 1200\text{eV}$ from Doppler Broadening

Widths of Ar lines (mass 40) are visibly greater than widths of Mo lines (mass $\sim 96$) because the thermal velocities are $\propto 1/m_i$. This confirms thermal.

Indicated instrumental resolution is not much smaller than line width. But can be deconvolved.
Ion flow can also be measured using line shift:

Flow velocity $V = \langle v \rangle$ is the mean over the ion/atom distribution function. Relation to line shift $\Delta \nu_o$:

$$V = \frac{\Delta \nu_o}{\nu_o} c.$$ 

Main limitations:

- Determine shift in presence of broadening
- Absolute wavelength determination
- Localization of sensitive volume
- Photon noise statistics.
Two Opposite Views

Helps resolve unambiguous absolute wavelength uncertainty.

Not always available. Absolute calibration source another possibility.
Different ionization stages (OI, OII, OIII, ...) exist from the coldest to the hottest regions.

So concentric “shells” of these different stages usually occur.

This effect gives some information about localization.

For example, the highest ionization levels occur in the center (if that’s the hottest place).

However, it remains the case that the temperature and velocity measured is an average along the line of sight, weighted by the density of the emitting species.

We'll talk later about improvements to localization.
Stark Effect

Electric Field in which the atom resides shifts energy levels: Stark Effect.

Most atoms have polarization $p \propto E$ hence energy shift $p \cdot E \propto E^2$

**quadratic shift** for most atoms’ lines.

However, hydrogen-like atoms have degeneracy that makes shift of energy levels larger and **linear** with the following formula for each level:

$$
\Delta \mathcal{E} = \sum_{k} \left( \frac{E}{Z \epsilon_0 a_0^2} \right) k \quad \text{Ry} \\
\text{Energy Shift}
$$

$$
|0, \pm 1, \pm 2 \ldots \pm (n-1) |
\text{Principal Quantum No.}
$$

$$
E = \frac{Z \epsilon_0 a_0^2}{4 \pi \epsilon_0}
\text{Electric Field c.f. field at } a_0
$$

$$
\text{Ry} = \text{Rydberg Energy (13.6eV)}
$$

$$
[a_0 = \hbar 4\pi \epsilon_0 / e^2 m_e = 5.292 \times 10^{-11} \text{m}, \text{is the Bohr radius}.]
$$

Many components whose spread is proportional to $E$.
Both upper and lower level energy shifts must be accounted for.
Stark Broadening

Atoms reside in different E-fields arising from nearby ions. We need the probability distribution of the E-field. Nearest Neighbor Approximation: take E-field to be that of just the nearest neighbor \( E \propto \frac{1}{r_{\text{NN}}^2} \), and \( \Delta \nu \propto E \).

So \( I(\nu)d\nu \propto P(E)dE \propto -r_{\text{NN}}^2 dr_{\text{NN}} \propto E^{-5/2}dE \).

\[
I(\nu) \propto (\Delta \nu)^{-5/2}
\]
in wings

Cut off of 'wings' at

\[
E_0 = \frac{e}{4\pi\epsilon_0 r_0^2} \quad \text{where} \quad \frac{4}{3}\pi r_0^3 \approx \frac{1}{n_i}.
\]
(Nearest can’t be closer than ~ this.)

Cut off at frequency shift \( \Delta \nu_0 \) such that

\[
h\Delta \nu_0 = \Delta \varepsilon \simeq 3 \frac{1}{2} n(n-1) \frac{E_0}{Z\epsilon a_0^2} R_y
\]

\[
\Rightarrow \Delta \nu_0 \propto n_i^{2/3}
\]

\[
\Delta \nu_{1/2} \propto n_i^{2/3}
\]
E.g. $H_\beta$:

$$\frac{\Delta \nu \frac{1}{2}}{\nu_0} = \frac{\Delta \lambda \frac{1}{2}}{\lambda_0} \simeq 8.2 \times 10^{-5} \left( \frac{n_e}{10^{20} \text{m}^{-3}} \right)^{\frac{3}{2}}$$

(Different coefficients other n.)

Use as a density measurement at high density, low temp. x.

Example: detached cold, high-density divertor:

another example: pellet cloud.

Lumma, Terry, Lipschultz, Phys Plasmas 4, 2555 (1997)
**Zeeman Effect**

**Magnetic Field** shifts energy levels by coupling to magnetic moment.

\[
\Delta \nu = \frac{1}{2\pi} g_J \Omega / 2
\]

Landé g-factor: \( g_J = 1 \) orbital (\( \sim 2 \) spin).  
Electron gyrofrequency: \( \Omega = eB/m_e \)

\( \frac{\delta \nu}{\nu} \) is greatest for smallest \( \nu \) (longest \( \lambda \)) hence **visible** lines (c.f. ultraviolet).

Example \( B = 1 \text{T} \) \( \Delta \nu \approx 14 \text{ GHz} \).

c.f. **green light** \( \nu \approx 6 \times 10^5 \text{ GHz} \) giving \( \frac{\Delta \nu}{\nu} \approx 2.3 \times 10^{-5} \).

Compare with Doppler broadening. Doppler is bigger unless

\[ T_i \lesssim (m_i c^2 / 2 \ln 2) \times (2.3 \times 10^{-5})^2 \approx 1 \text{eV}. \text{ (Hydrogen, } B = 1 \text{T} \) }

Most hot plasma measurements are little-affected by Zeeman effect.
Zeeman Polarization

But polarization of Zeeman lines is useful: (perp. propagation)

\[ \pi \text{ (unshifted)} \rightarrow \text{Polarized } E_{\text{wave}} \parallel B \]
\[ \sigma \text{ (± shifted)} \rightarrow \text{Polarized } E_{\text{wave}} \perp B \]

Measure direction of \( B \)! If different components can be separated.

To obtain visible Zeeman Effect (separable components):
Inject a monoenergetic beam of atoms (Lithium: 670.8 nm)

Beam energetic, up to 100 keV.

But energy spread, (along line of slight) can be kept small: \(< 10\text{eV})

Several successful experiments in tokamaks \( \rightarrow B_p \rightarrow j^{(r)}_\phi , q^{(r)} \).

• Beam technology quite difficult
• Beam penetration limited by CX & ionization to

\[ \int n_e dl \sim 10^{18} \text{m}^{-2} \]
Example Zeeman B-direction measurement
Motional Stark Effect

Particle moving in B-field experiences electric field

\[ E = \mathbf{v} \wedge \mathbf{B} \]

(Lorentz transform of EM fields).

Energetic (Neutral) hydrogen beams in tokamaks experience strong E-Field.

\[ \rightarrow \text{Stark shifts.} \]

Again different Stark components have different polarizations

\[ \pi \quad E_{\text{wave}} \parallel B \]
\[ \sigma \quad E_{\text{wave}} \perp B. \]

Use to measure B direction.
Earliest measurements on tokamak PBX-M

Demonstrated value of photoelastic modulator techniques.

Extreme precision of polarization angle needed and obtained
Comparison with Zeeman

**Advantages** of Motional Stark Effect as diagnostic of B direction.

- Uses Hydrogen Beam (not e.g. Li) hence can use standard NBI heating beams and technology.
- Penetration better.
- Shift is bigger (than Zeeman) e.g., $50\text{keV} \, \text{H}^0 \, \text{in} \, 1\text{T} \rightarrow 3.1 \times 10^6 \text{Vm}^{-1}$. Then $\Delta \nu \simeq 60 \text{ nk GHz} \sim 10 \times \text{Zeeman Shift}$.
- Well separated/polarized components
- Higher Accuracy.

**Disadvantage**

-Sensitive to $E_r$.

Turn to advantage.

35 channel MSE system, using Stark polarimetry to measure both $E_r$ and $B_{pol}$
Reconstruction including $E_r$

(a) Equilibrium reconstruction $q$ profile obtained using all MSE chords and including $E_r$ (solid line) versus that obtained using only tangential MSE chords and assuming $E_r = 0$ (dashed line);

(b) $E_r$ determined from combined reconstruction with MSE data (solid line and diamonds) and determined independently from CER analysis of carbon impurities (dotted line).
Charge Exchange Spectroscopy CXS

Addresses three major problems with spectroscopy in high-temperature plasmas:

- Many (light) species are fully stripped. The ions have no electrons left, so they don’t emit line radiation.
- Localization. We really need a way to localize the emission along collection sight-line. Crossing the view with a localized source accomplishes this.
- Visible photons that transmit through glass (or quartz) and reflect from mirrors are much easier to deal with than UV or Soft X-ray.

The Idea:

Inject an energetic beam of neutral hydrogen that intersects with the line of sight.

Charge-exchange recombination collision between beam and impurity

\[ \text{H}^0 + Z^{+} \rightarrow \text{H}^{+} + (Z-1)^{+} \]


[Highly excited, and high angular momentum, states emit a cascade of lower energy (visible) photons rather than one high energy photon.]
Can use the standard neutral heating beams on present experiments.
Just need appropriate views.
Rates for production of photons by charge-exchange for various light impurities.

\[ \sim 50\text{keV} \] is also the optimal energy of positive-ion beams.

Higher energy can't be efficiently neutralized. Lower energy does not penetrate so well.
Remarkably detailed information

Profiles of carbon: temperature, density, perpendicular and toroidal velocities, from the edge of D-IIID tokamak.

[Courtesy K. Burrell 2001]

Highly influential in helping to understand the edge transport barriers.
Spectroscopic techniques give

**Impurity density** information from intensity.

**Impurity temperature and velocity** $T_i$, $V_i$ from Doppler line width, shift.

**Bulk density** $n_e = Zn_i$ in high density, low temperature plasmas from Stark Broadening.

**B-field direction** from
   - Zeeman Effect in beams
   - Motional Stark Effect in beams.

**Localized** impurity parameters in CXS with beams.

A large fraction of all the information that we have about astrophysical plasmas.