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Wave-Wave Interactions in Plasmas

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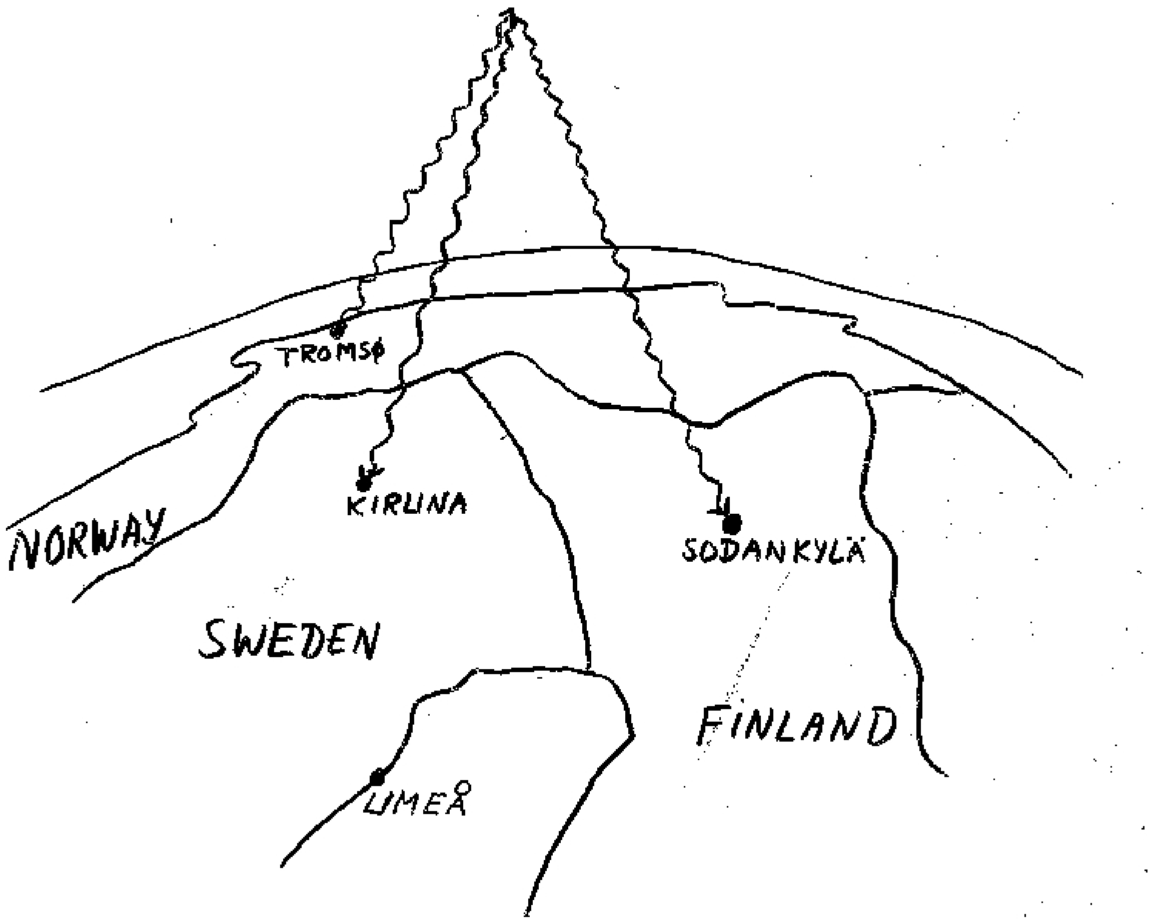
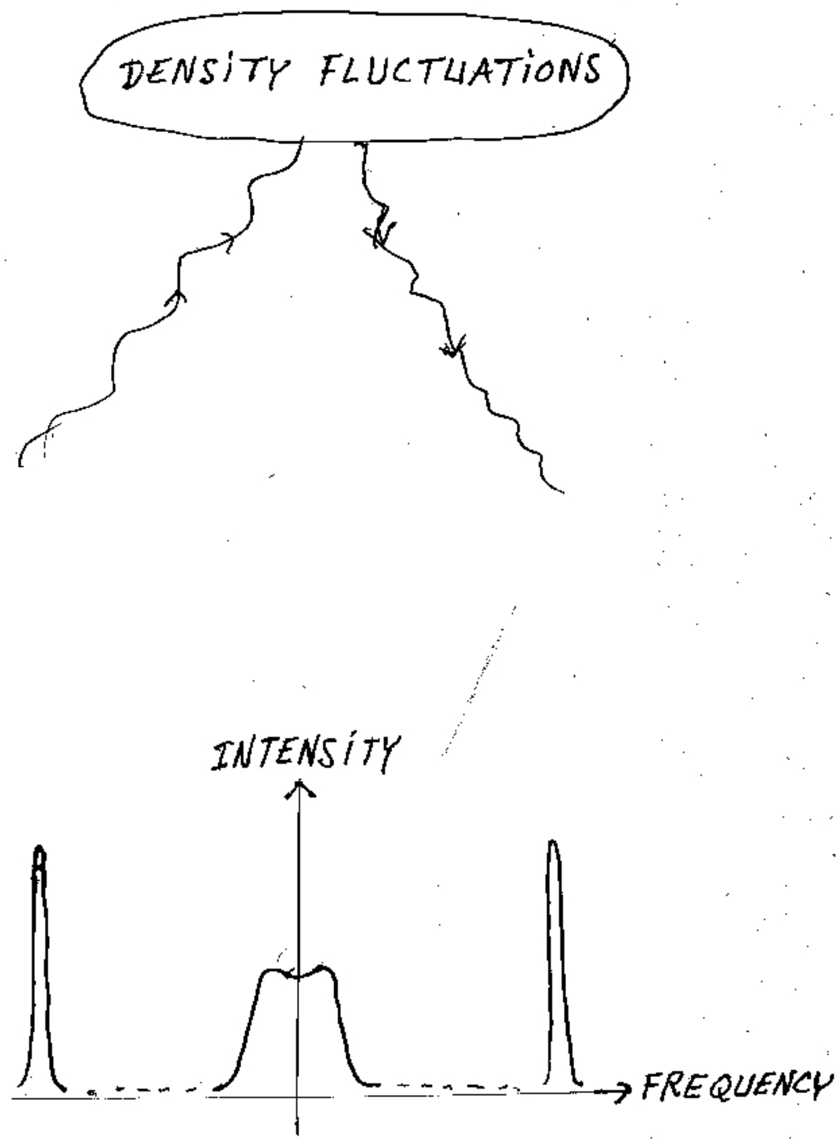
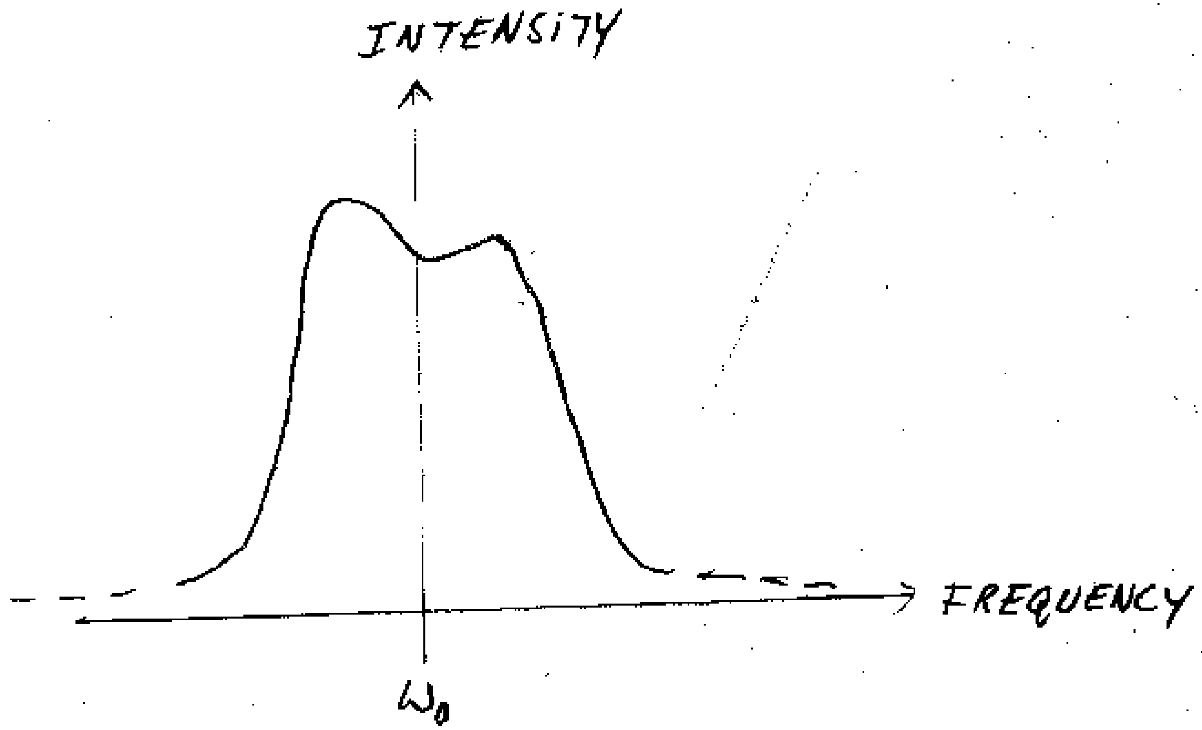


EISCAT (EUROPEAN INCOHERENT SCATTER FACILITY

VHF 224 MHz
UHF 933 MHz


DENSITY FLUCTUATIONS
INTENSITY
FREQUENCY


INTENSITY
FREQUENCY
ω_0

$$\omega_0^2 = \omega_{pe}^2 + k_0^2 c^2$$

$$\omega_\pm = \omega \pm \omega_0$$

$$\underset{\sim}{k}_\pm = \underset{\sim}{k} \pm \underset{\sim}{k}_0$$

$$c^2 \underset{\sim}{k}_\pm \times (\underset{\sim}{k}_\pm \times \underset{\sim}{E}_\pm) + (\omega_\pm^2 - \omega_{pe}^2) \underset{\sim}{E}_\pm \approx \omega_{pe}^2 \frac{\delta n}{n_0} \underset{\sim}{E}_{0\mp}$$

$$\varepsilon(\omega, \underset{\sim}{k})\, \delta n = -\frac{k^2 \varepsilon_0}{m_e \omega_0^2} \chi_e (1 + \chi_i) (\underset{\sim}{E}_{0+} \cdot \underset{\sim}{E}_- + \underset{\sim}{E}_{0-} \cdot \underset{\sim}{E}_+)$$

$$\varepsilon(\omega, \underset{\sim}{k}) \equiv 1 + \chi_e(\omega, \underset{\sim}{k}) + \chi_i(\omega, \underset{\sim}{k})$$

if $\omega_0 \gg \omega_{pe}$

$$\frac{?}{\chi_e} + \frac{1}{1+\chi_i} = \frac{k^2 |k_+ \times v_0|^2}{k_+^2 (k_+^2 c^2 - \omega_+^2 + \omega_{pe}^2)} + \frac{k^2 |k_- \times v_0|^2}{k_-^2 (k_-^2 c^2 - \omega_-^2 + \omega_{pe}^2)}$$

$$v_0 = \frac{q_e E_0}{m_e \omega_0}$$

if $k v_{ti} < \omega \ll k v_{te}$

$$\chi_e \approx \frac{1}{k^2 \lambda_D^2} \left(1 + i \sqrt{\frac{\pi}{2}} \frac{\omega}{k v_{te}} \right)$$

$$\chi_i \approx -\frac{\omega_{pi}^2}{\omega^2} + \frac{i \sqrt{\pi/2}}{k^2 \lambda_D^2} \frac{T_e}{T_i} \frac{\omega}{k v_{ti}} e^{-\frac{\omega^2}{2 k^2 v_{ti}^2}}$$

THREE-WAVE SCATTERING

$$(\underset{\sim}{k}-\underset{\sim}{k}_0)^2 c^2 - (\omega-\omega_0)^2 + \omega_p^2 \approx 0 \approx 2\omega_0(\omega-\Delta\omega)$$

$$\Delta\omega \approx \frac{\underset{\sim}{k}\cdot\underset{\sim}{k}_0 c^2}{\omega_0} - \frac{k^2c^2}{2\omega_0}$$

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} \approx \frac{k^2 v_0^2 \sin^2\varphi}{2\omega_0(\omega-\Delta\omega)}$$

FOUR-WAVE SCATTERING

MODULATIONAL INSTABILITIES

$$\omega \ll \omega_0 \qquad k \ll k_0$$

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} \simeq -\frac{k^2 v_0^2}{2\omega_0^2}\frac{(k^2c^2-\omega^2)}{\left[\left(\omega-\frac{\underset{\sim}{k}\cdot\underset{\sim}{k}_0 c^2}{\omega_0}\right)^2 - \frac{(k^2c^2-\omega^2)^2}{4\omega_0^2}\right]}$$

$$\omega_0 \gg \omega_{ce}$$

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} = k^2 \sum_{+,-} \left[\frac{|\mathbf{k}_\pm \times \mathbf{v}_0|^2}{k_\pm^2 \left(k_\pm^2 c^2 - \omega_\pm^2 + \omega_{pe}^2 - \frac{i\nu_e \omega_{pe}^2}{\omega_\pm} \right)} - \frac{|\mathbf{k}_\pm \cdot \mathbf{v}_0|^2}{k_\pm^2 \omega_\pm^2 \varepsilon(\omega_\pm, \mathbf{k}_\pm)} \right]$$

$$\chi = \frac{\omega_p^2}{k^2 v_t^2} \left\{ 1 - \sum_{n=-\infty}^{\infty} I_n(b) e^{-b} \left[\omega - \omega^* \left(1 - \frac{n\omega}{b\omega_c} \right) \right] \int_{-\infty}^{\infty} \frac{F_z \, dv_z}{\omega - k_z v_z - n\omega_c} \right\}$$

$$b = \frac{k_\perp^2 v_t^2}{\omega_c^2}$$

$$\omega^* = - \frac{k_y \varkappa v_t^2}{\omega_c}$$

$$\varkappa = - \frac{1}{n_0} \frac{\partial n_0}{\partial x}$$

$$\begin{cases} \dfrac{\partial n}{\partial t} + \nabla \cdot n \underset{\sim}{v} = 0 \\ \dfrac{\partial \underset{\sim}{v}}{\partial t} + \underset{\sim}{v} \cdot \nabla \underset{\sim}{v} = \dfrac{q}{m}\left(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B_0}\right) - \dfrac{v_t^2}{n} \nabla n - \nu \underset{\sim}{v} \end{cases}$$

$$\begin{cases} \dfrac{\partial n_1}{\partial t} + \nabla \cdot \left(n_0 \underset{\sim}{v_1} + n_1 \underset{\sim}{v_d}\right) = 0 \\ \dfrac{\partial \underset{\sim}{v_1}}{\partial t} + \underset{\sim}{v_d} \cdot \nabla \underset{\sim}{v_1} - \dfrac{q}{m}\left(\underset{\sim}{E_1} + \underset{\sim}{v_1} \times \underset{\sim}{B_0}\right) + \dfrac{v_t^2}{n_0} \nabla n_1 - \\ - \dfrac{v_t^2 n_1}{n_0^2} \nabla n_0 + \nu \underset{\sim}{v_1} = -\nabla \dfrac{\underset{\sim}{v_d}^2}{2} \end{cases}$$

$$\nabla \times \underset{\sim}{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \underset{\sim}{B} = \mu_0 \sum n q \underset{\sim}{v} + \frac{1}{c^2} \frac{\partial \underset{\sim}{E}}{\partial t}$$

$$\left(\frac{1}{1+\chi_i} + \frac{1}{\chi_e}\right) n_{1e} = \frac{k^2 n_0}{2\omega_{pe}^2} v_h^2$$

$$v_h^2 = \frac{2 q_e^2}{m_e^2 \omega_0^2}\left(\underset{\sim}{E}_0^* \cdot \underset{\sim}{E}_+ + \underset{\sim}{E}_0 \cdot \underset{\sim}{E}_-\right) \qquad \omega_0 \gg \omega_{ce}$$

$$-c^2 \nabla \times \nabla \times \underset{\sim}{E}_\pm + (\omega_\pm^2 - \omega_{pe}^2)\underset{\sim}{E}_\pm = \frac{n_{1e}}{n_0}\omega_{pe}^2 \begin{cases} \underset{\sim}{E}_0 \\ \underset{\sim}{E}_0^* \end{cases}$$

$$\chi = \left\{ \frac{k^2 v_t^2}{\omega_p^2} - \frac{k^2(\omega - \underset{\sim}{k}\cdot v_d)(\omega - \underset{\sim}{k}\cdot v_d + i\nu)\left[1 - \frac{\omega_c^2}{(\omega - k\cdot v_d + i\nu)^2}\right]}{\omega_p^2 A} \right\}^{-1}$$

$$A = k^2 + \frac{\varkappa^2}{4} + \frac{q\,\underset{\sim}{k}\cdot(\underset{\sim}{\varkappa} \times B_0)}{m(\omega - \underset{\sim}{k}\cdot v_d + i\nu)} - \frac{q^2[(\underset{\sim}{k}\cdot B_0)^2 + (\underset{\sim}{\varkappa}\cdot B_0/2)^2]}{m^2(\omega - \underset{\sim}{k}\cdot v_d + i\nu)}$$

$$\underset{\sim}{\varkappa} = -\frac{\nabla n_0}{n_0}$$

$$\frac{1}{1+\chi_i} \approx \frac{k^2 v_{ti}^2 - \omega(\omega + i\nu_i)}{\omega_{pi}^2}$$

$$\frac{1}{\chi_e} \approx \frac{k^2 v_{te}^2}{\omega_{pe}^2} - \frac{i k^2 (\omega - \underset{\sim}{k}\cdot \underset{\sim}{V}_{de})\, \omega_{ce}^2}{\omega_{pe}^2 \nu_e \left[k^2 - \frac{i q_e}{m_e} \frac{\underset{\sim}{k}\cdot(\underset{\sim}{\kappa} \times \underset{\sim}{B}_0)}{\nu_e} + \frac{q_e^2}{m_e^2} \frac{(\underset{\sim}{k}\cdot \underset{\sim}{B}_0)^2}{\nu_e^2} \right]}$$

$$\underset{\sim}{\kappa} = -\frac{\nabla n_0}{n_0} \qquad |\underset{\sim}{\kappa}| \ll |\underset{\sim}{k}|$$

$$|\omega - \underset{\sim}{k}\cdot \underset{\sim}{V}_{de}| \ll \nu_e \ll \omega_{ce}$$

$$\frac{N_0^2}{c^2} = \frac{\gamma}{\omega} \frac{\nu_e}{\omega_0} \frac{\omega_{pe}^2}{\omega_0} \left| \omega \frac{\partial}{\partial \omega} \left(\frac{1}{1+\chi_i} + \frac{1}{\chi_e} \right) \right|$$

$$\begin{cases} \dfrac{1}{1+\chi_i} \approx -\dfrac{i\omega\nu_i}{\omega_{pi}^2} \\[2ex] \dfrac{1}{\chi_e} \approx -\dfrac{i(\omega - \underset{\sim}{k}\cdot\underset{\sim}{v}_{de})\,\omega_{ce}^2}{\omega_{pe}^2\nu_e} \end{cases}$$

$$\omega \simeq \frac{\underset{\sim}{k}\cdot\underset{\sim}{v}_{de}}{1+\psi}$$

$$\psi = \left|\frac{\nu_i\nu_e}{\omega_{ci}\omega_{ce}}\right|$$

$$\frac{\partial n}{\partial t} + \nabla \cdot n \underset{\sim}{v} = 0$$

$$\frac{\partial \underset{\sim}{v}}{\partial t} + \underset{\sim}{v} \cdot \nabla \underset{\sim}{v} = \frac{q}{m}\left(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B}\right) - \frac{\nabla n T}{m n} - \nu \underset{\sim}{v}$$

$$\frac{3}{2}\left(\frac{\partial T}{\partial t} + \underset{\sim}{v} \cdot \nabla T\right) + T \nabla \cdot \underset{\sim}{v} = \frac{\kappa_\parallel}{n_0} \nabla_\parallel^2 T + \frac{\kappa_\perp}{n_0} \nabla_\perp^2 T$$

$$- \frac{3}{2\tau}\left(T - T_g\right) + m \nu \underset{\sim}{v}^2$$

$$\nabla \times \underset{\sim}{E} = - \frac{\partial \underset{\sim}{B}}{\partial t}$$

$$\nabla \times \underset{\sim}{B} = \mu_0 \sum n q \underset{\sim}{v} + \frac{1}{c^2} \frac{\partial \underset{\sim}{E}}{\partial t}$$

$$\frac{1}{1+\chi_i} + \frac{1}{\chi_e} = -\left(1 + \frac{4i}{3\tilde{\omega}}\nu_e\right)\frac{k^2 v_{ts}^2}{\omega_{pe}^2}$$

$$v_{ts}^2 = -\omega_{pe}^2 \sum_{+,-} \left\{ \frac{|\underset{\sim}{k}_\pm \times \underset{\sim}{v}_0|^2}{k_\pm^2 \left(k_\pm^2 c^2 - \omega_\pm^2 + \omega_{pe}^2 + \frac{i\nu_e \omega_{pe}^2}{\omega_0}\right)} - \frac{|\underset{\sim}{k}_\pm \cdot \underset{\sim}{v}_0|^2}{k_\pm^2 \omega_\pm^2 \varepsilon(\omega_\pm, k_\pm)} \right\}$$

$$\tilde{\omega} = \omega - \underset{\sim}{k} \cdot \underset{\sim}{v}_{de} + \frac{i}{\tau} + \frac{2i}{3n_0}\left(k_z^2 R_{ze} + k_\perp^2 R_{\perp e}\right)$$

$$I(\omega, \underset{\sim}{k}) = \frac{|\mathcal{E}(\omega, \underset{\sim}{k})|^2}{|\mathcal{E}_{nl}(\omega, \underset{\sim}{k})|^2} I_0(\omega, \underset{\sim}{k}) +$$

$$+ \frac{|V_+|^2 I_0(\omega+\omega_0, \underset{\sim}{k}+\underset{\sim}{k}_0) + |V_-|^2 I_0(\omega-\omega_0, \underset{\sim}{k}-\underset{\sim}{k}_0)}{|\mathcal{E}_{nl}(\omega, \underset{\sim}{k})|^2}$$

$$\mathcal{E}_{nl}(\omega, \underset{\sim}{k}) = \mathcal{E}(\omega, \underset{\sim}{k}) - \frac{|C_+|^2}{D(\omega+\omega_0, \underset{\sim}{k}+\underset{\sim}{k}_0)} + \frac{|C_-|^2}{D(\omega-\omega_0, \underset{\sim}{k}-\underset{\sim}{k}_0)}$$

$$D(\omega, \underset{\sim}{k}) = \left(1 - \frac{\omega_p^2}{\omega^2} - \frac{k^2 v_t^2}{\omega^2}\right)(k^2c^2 - \omega^2 + \omega_p^2)^2 -$$

$$- \frac{\omega_c^2}{\omega^2}(k^2c^2 - \omega^2)\left[(k^2c^2 - \omega^2)\left(1 - \frac{\omega_p^2}{\omega^2} - \frac{k_z^2 v_t^2}{\omega^2}\right) + \frac{k_\perp^2 c^2 \omega_p^2}{\omega^2}\right]$$

ULTRACOLD QUANTUM PLASMAS

2006

$$\left(\partial_t^2 - c^2\nabla^2 + \omega_{pe}^2\right) \underset{\sim}{E} + \omega_{pe}^2 \frac{\delta n}{n_0} \underset{\sim}{E} = 0$$

$$\left(\partial_t^2 + \frac{\hbar^2}{4 m_e m_i}\nabla^4 - \frac{m_e}{m_i} v_{Fe}^2 \nabla^2\right)\delta n = \frac{n_0 e^2}{2 m_e m_i \omega_0^2} \nabla^2 |\underset{\sim}{E}|^2$$

$$|E_0|^2 = \frac{\omega_0^2}{c^2} |A_0|^2$$

$$\omega^2 - \frac{\hbar^2 k^4}{4 m_e m_i} = \frac{\omega_{pe}^2 e^2 k^2}{2 m_e m_i c^2} |A_0|^2 \left(\frac{1}{D_+} + \frac{1}{D_-} \right)$$

$$D_\pm = \omega_\pm^2 - k_\pm^2 c^2 - \omega_{pe}^2$$

$$\gamma_B = \frac{\omega_{pe}}{2} \frac{e|A_0|}{m_e c} \left(\frac{m_e}{m_i}\right)^{1/4} \left(\frac{m_e}{\hbar \omega_0}\right)^{1/2} \qquad \text{if } D_- \approx 0$$

$$\omega << \Omega_B \equiv \frac{\hbar k^2}{2\sqrt{m_e m_i}}$$

$$\omega = \underset{\sim}{k} \cdot \underset{\sim}{v}_g \pm \left[\delta^2 - \frac{\delta \omega_{pe}^2 e^2 k^2 |A_0|^2}{2 \omega_0 \Omega_B^2 m_e m_i c^2} \right]^{1/2}$$

$$\underset{\sim}{v}_g = \frac{c^2 \underset{\sim}{k}_0}{\omega_0} \qquad \delta = k_0^2 c^2 / 2\omega_0$$

TWO PUMPS

1. $\omega_1 - \omega_2 = \omega_p$

2. $2\omega_1 - \omega_2 = \omega_3$

CIRCULARLY POLARIZED WAVES

$$\omega \simeq \frac{\omega_{pi}^2}{\omega_{Ei}}$$

$$\omega_{pi}^2 = \frac{e^2 n_0}{\varepsilon_0 m_{oi}}$$

1979

$$\omega \approx \frac{k^2 c^2}{\omega_{pi}^2}\,\omega_{Ei}$$

$$\omega_{Ei} = \frac{e E_0}{c\, m_{oi}}$$

$$\omega \approx \frac{e}{\pi (90\,\varepsilon_0 \hbar c)^{1/2}} \frac{k^2 c^2}{(\omega_p^2 + k^2 c^2)^{1/2}} \frac{E_0}{E_{crit}}$$

$$E_{crit} = \frac{m_e^2 c^3}{e \hbar} \simeq 10^{18}\ V/m$$

2005

Thank you for listening!