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Drift Wave Turbulence and Zonal Flows

Part I

George R. Tynan
UCSD
University of California San Diego
USA

Drift Wave Turbulence and Zonal Flows – Part I

G.R. Tynan
UCSD

This talk is made possible by the many discussions and contribution of materials and results from collaborators and colleagues:

J. Boedo, M. Burin, P.H. Diamond, R. Fonck, A. Fujisawa, O. Gurcan, C. Holland, K. Itoh, S. Itoh, G. McKee, R. Moyer, J. Yu, S. Zweben

Background and Motivation

- **Review of Drift Waves**
- **Comments on Transition to Turbulence**
- **Why Is Turbulence of Interest in Fusion?**
- What are Zonal Flows?
- Why Care About the Nonlinear Interactions Between Turbulence and Zonal Flows?

Fundamental Origins of Drift Waves & Instability

Perpendicular Ion Polarization Drift

$$\mathbf{V}_{\perp_i}^{pol} = \frac{1}{B\Omega_{C_i}} \frac{d\mathbf{E}_\perp}{dt} = \frac{-1}{B\Omega_{C_i}} \frac{d\nabla_\perp \phi}{dt}$$

$$\left[\frac{d}{dt} \approx \frac{\partial}{\partial t} + V_{ExB} \bullet \nabla_\perp \right]$$

Parallel Electron Motion Gives Boltzmann Relation

$$V_{\parallel_e} = -\frac{kT_e}{m_e \nu_e^{eff}} \left(\frac{\nabla_\parallel n_e}{n_e} - \frac{e \nabla_\parallel \phi}{kT_e} \right)$$

Charge Conservation

$$\nabla_\perp \bullet J_\perp = -\nabla_\parallel \bullet J_\parallel \quad \Rightarrow \quad \nabla_\perp \bullet e n_i V_{\perp_i}^{pol} = \nabla_\parallel \bullet e n_e V_{\parallel_e}$$

Quasi-neutrality

$$n_i \approx n_e$$

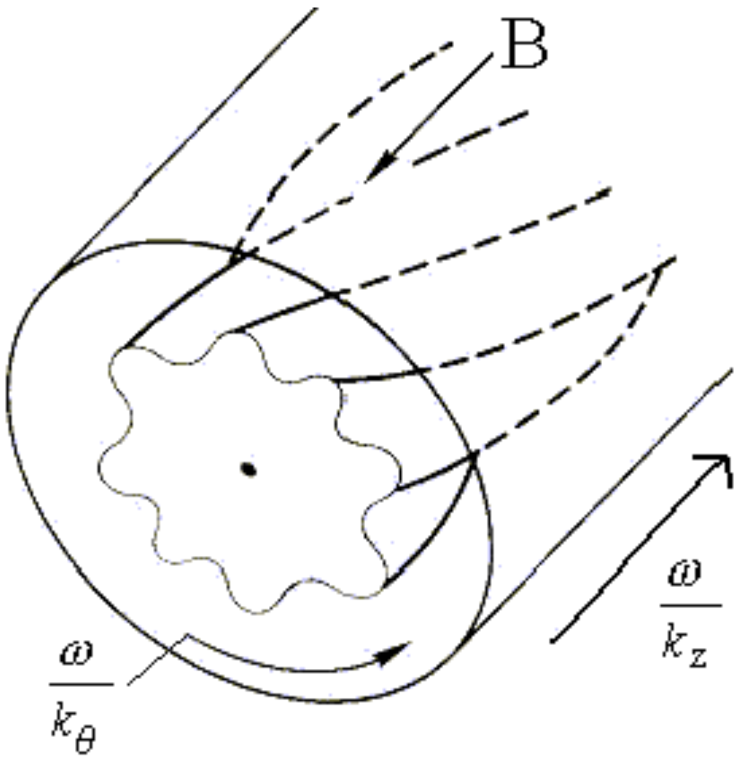
Simple Linear Collisional Drift Wave Instability

Slab Result

$$\omega_r = \frac{\omega^*}{1 + k_\perp^2 \rho_S^2}, \quad \omega^* = k_y V_{d_e}, \quad \rho_S = \frac{C_S}{\Omega_{C_i}}$$

$$\omega_i = \frac{\nu_e^{eff}}{k_\parallel^2 v_{th_e}^2} \frac{\omega^{*2}}{\left(1 + k_\perp^2 \rho_S^2\right)^3} > 0$$

Physical Picture of Drift Waves



See e.g. F.F. Chen

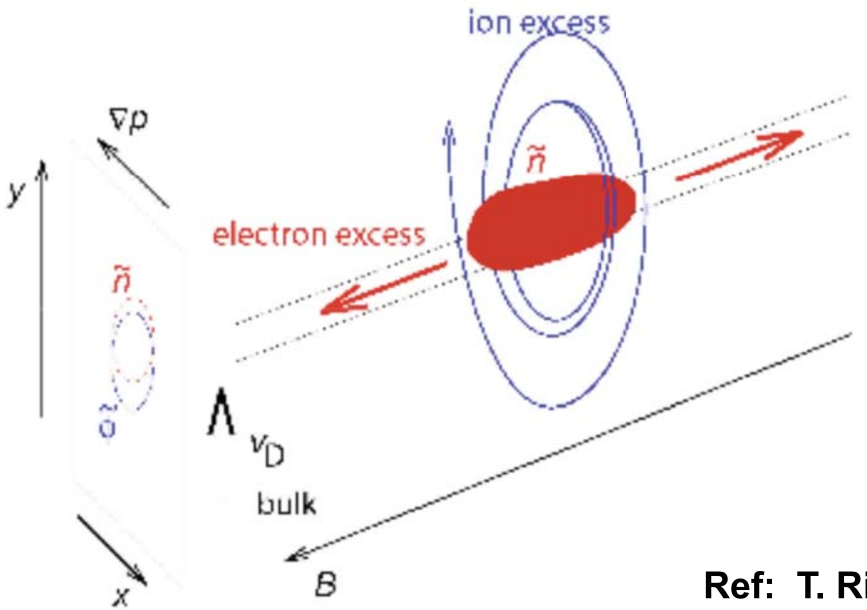
Isodensity Curve for a Single Mode

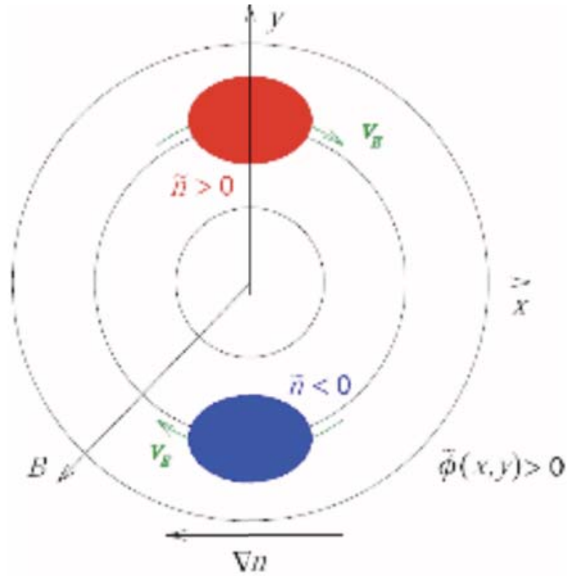
$$\lambda_z >> \lambda_\theta \text{ because } \lambda_{\text{Electron M.F.P.}} << \lambda_z << \frac{V_{\text{THRM(elec)}}}{\omega}$$

The Basic Picture of Turbulent Transport

Drift waves:

- ExB drift & density profile: $\tilde{\phi}$ excites $\tilde{n}_e$
- || electron and ⊥ ion polarisation dynamics:
  $\tilde{\phi}$ tied to $\tilde{n}_e$
- Structure propagates in *y*-direction
- Resistivity: phase shift between $\tilde{n}_e$ and $\tilde{\phi}$
- Transport across field



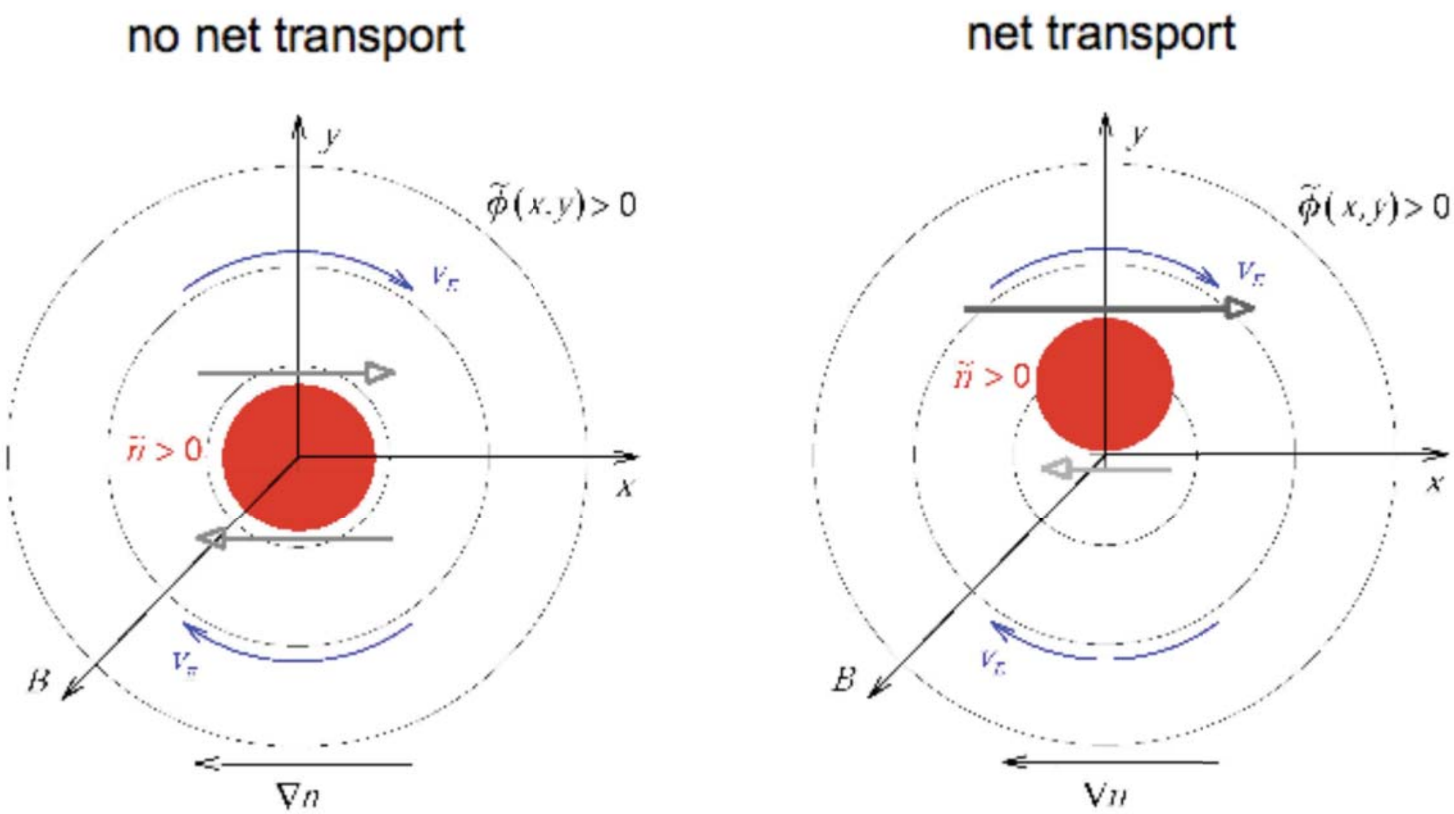


Interchange forcing:

- Compressibility ⊥ drifts due to inhomogeneous B:
  energy path between $\tilde{n}_e$ and $\tilde{\phi}$
- Bigger phase shift between $\tilde{n}_e$ and $\tilde{\phi}$
- Transport across field

Ref: T. Ribiero IPP-Garching 2005 Summer School

The Basic Picture of Turbulent Transport



Particle transport caused by gradient driven turbulence:
phase shift of pressure ahead of the electrostatic potential

Ref: T. Ribiero IPP-Garching 2005 Summer School

Characteristics of Drift Waves:

1. Electrostatic Fluctuation: $V_{TH_I} << \omega/k_z << V_{TH_E}$
2. $\omega << \Omega_{C_i}, \Omega_{C_e}$
3. $\lambda_z >> \lambda_\theta$
6. $\tilde{n}_1$ peaks near maximum ∇n_0
7. Boltzmann Relation: $\tilde{n} \propto \tilde{\phi}$
10. Propagates in Electron Diamagnetic Drift Direction
11. Growth Rate Increases with: B, ν_e
12. ω_{REAL} (theory) $\cong \omega_{REAL}$ (expt.)

From Waves to Turbulence…

- Multiple Unstable Eigenmodes Can Exist
- Linear Theory NEGLECTS Fluctuating Convective Derivative Term (Higher Order)
- What Happens When Can No Longer Neglect This Term…
- Get Exchange of Energy Across Spatio-temporal Scales…

Effect of Convective Derivative Nonlinearity…

Configuration Space…

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = ...$$

Fourier Space…

$$\frac{\partial n_k^2}{\partial t} + \sum_{\substack{k_1,k_2 \\ k=k_1+k_2}} n_k^* \mathbf{v}_{k_1} \nabla \cdot n_{k_2} = 0$$

$$\frac{\partial v_k^2}{\partial t} + \sum_{\substack{k_1,k_2 \\ k=k_1+k_2}} \mathbf{v}_k^* \cdot \left(\mathbf{v}_{k_1} \cdot \nabla\right) \mathbf{v}_{k_2} = ...$$

Different Spatio-temporal scales “interact”, or exchange energy

Flow Generation from Turbulence: Fourier Space

Usual Reynolds Stress Term in Simplified Momentum Eqn
(ala Diamond et al. PRL 1994 and others)

$$\frac{\partial \bar{u}_\theta}{\partial t} + \frac{\partial \langle \tilde{u}_r \tilde{u}_\theta \rangle}{\partial r} = -\nu_{damp} \bar{u}_\theta$$

"Radial Transport of Angular Momentum"

Consider a Zonal Flow to Have:

$$\mathbf{u} = u_\theta^Z \hat{\theta} \qquad \mathbf{k}_r^Z = k_r^Z \hat{\mathbf{r}}$$

$$\left|\mathbf{k}_r^Z\right| << \left|\mathbf{k}_1\right|, \left|\mathbf{k}_2\right|$$

$$\tau_Z \sim 1/k_r^Z u_\theta^Z >> t_{corr}$$

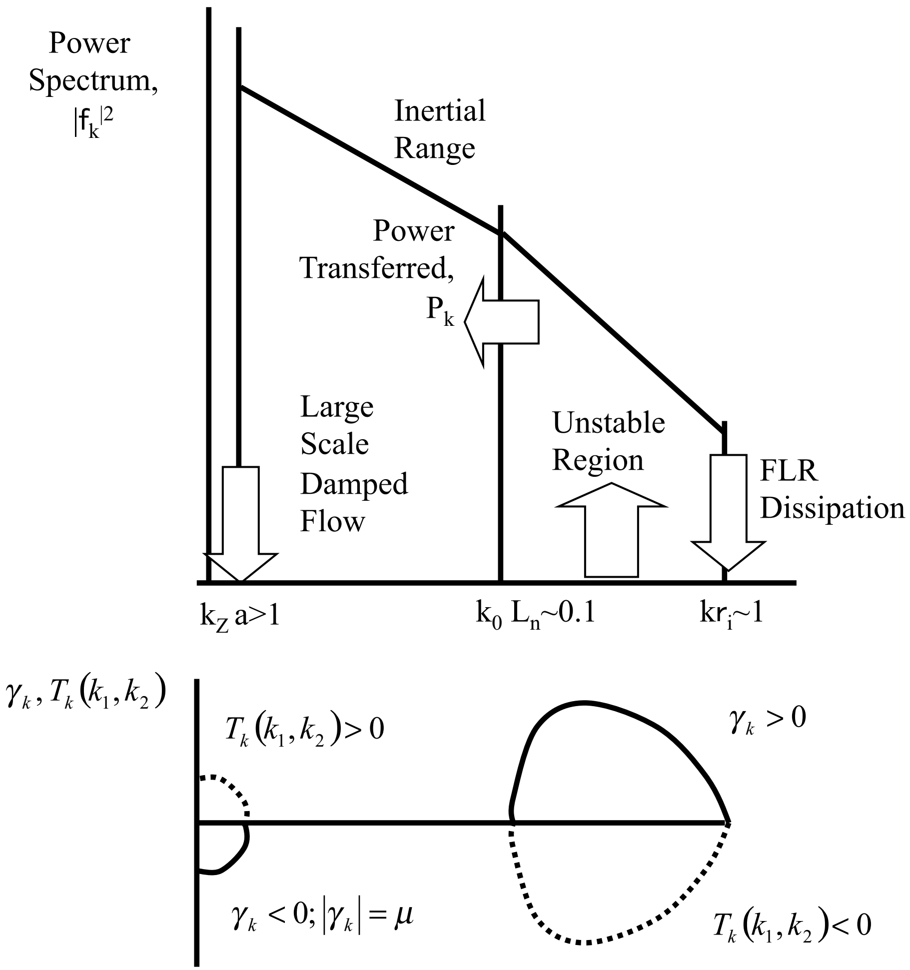
F.T., Write as KE, and Average Energy Eqn over Z-flow scales:

NL Energy Transfer

$$\frac{1}{2}\frac{\partial \left\langle u_{\theta_Z}{}^2(\mathbf{k}_Z)\right\rangle}{\partial t} - P_{k_Z}^{turb} = -\mu \left\langle u_{\theta_Z}{}^2(\mathbf{k}_Z)\right\rangle$$

where $$P_{k_Z}^{turb} = \sum_{\substack{\mathbf{k}_1\mathbf{k}_2 \\ \mathbf{k}_Z = \mathbf{k}_1 \pm \mathbf{k}_2}} \mathrm{Re}\left\langle u_{\theta_Z}{}^*(\mathbf{k}_Z)\left(\tilde{\mathbf{u}}(\mathbf{k}_1)\bullet\nabla\right)\tilde{u}_\theta(\mathbf{k}_2)\right\rangle$$

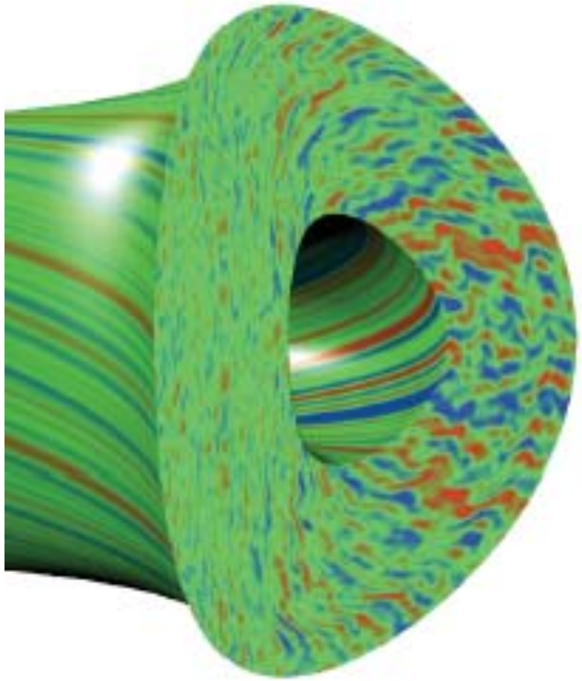
Flow Generation from Turbulence: Fourier Space



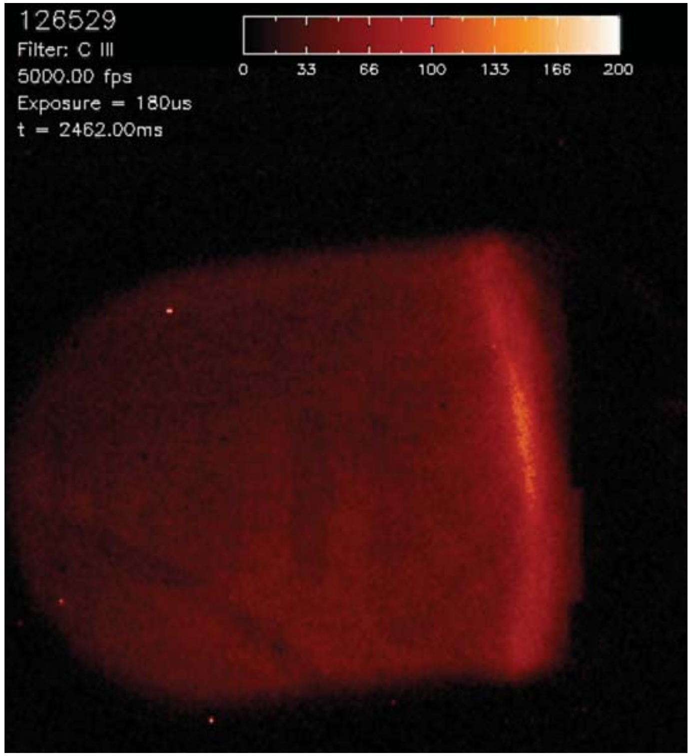
- Free Energy Source Releases Energy On One Scale
- Nonlinear Energy Transfer Moves Energy to Dissipation Region
- Shear Flows Develop Via Transfer of Energy to LARGE SCALES (small k)

Images of Turbulence in Tokamaks

GYRO

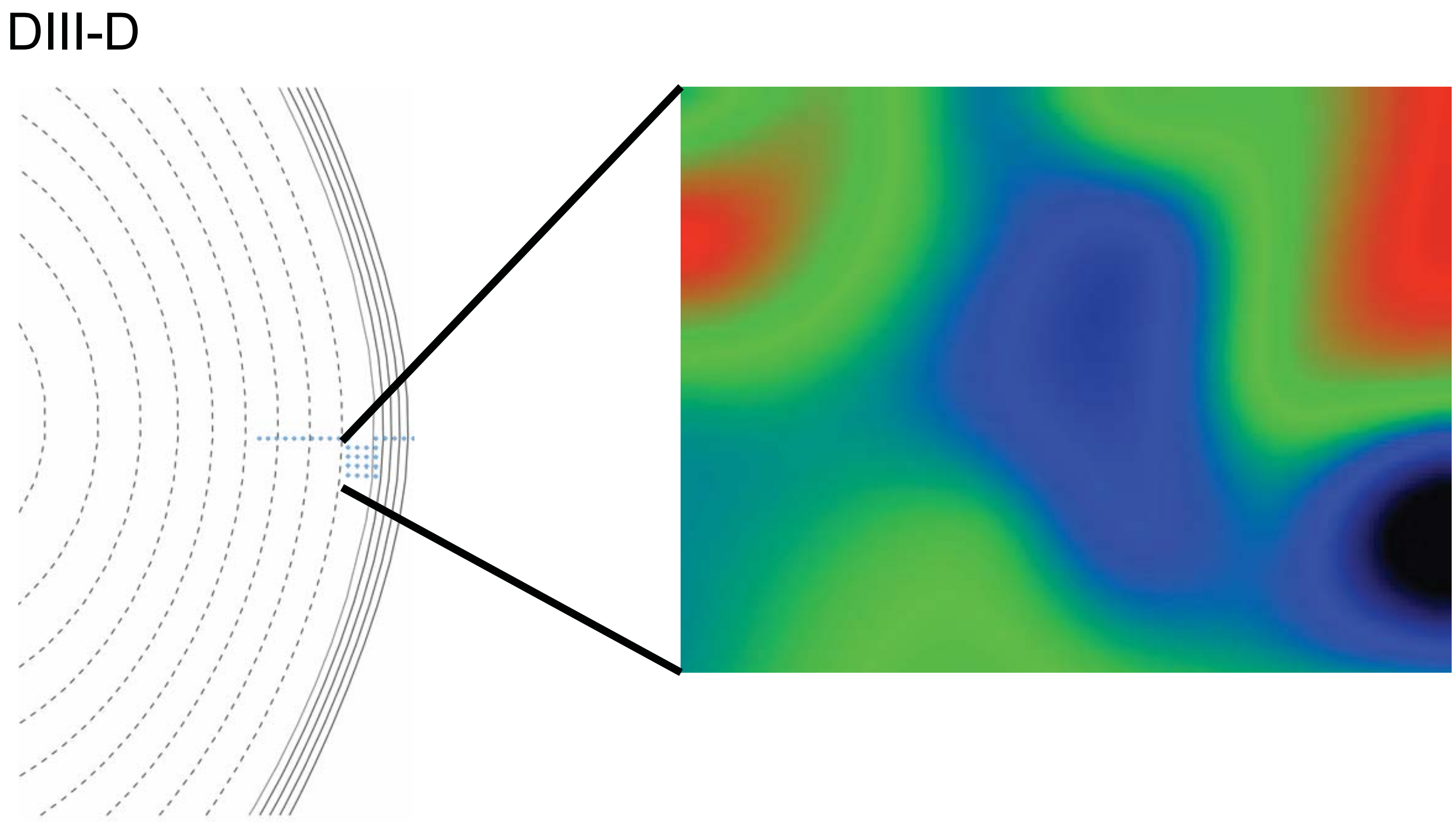


DIII-D



J. Yu   C-III Emission (UCSD)

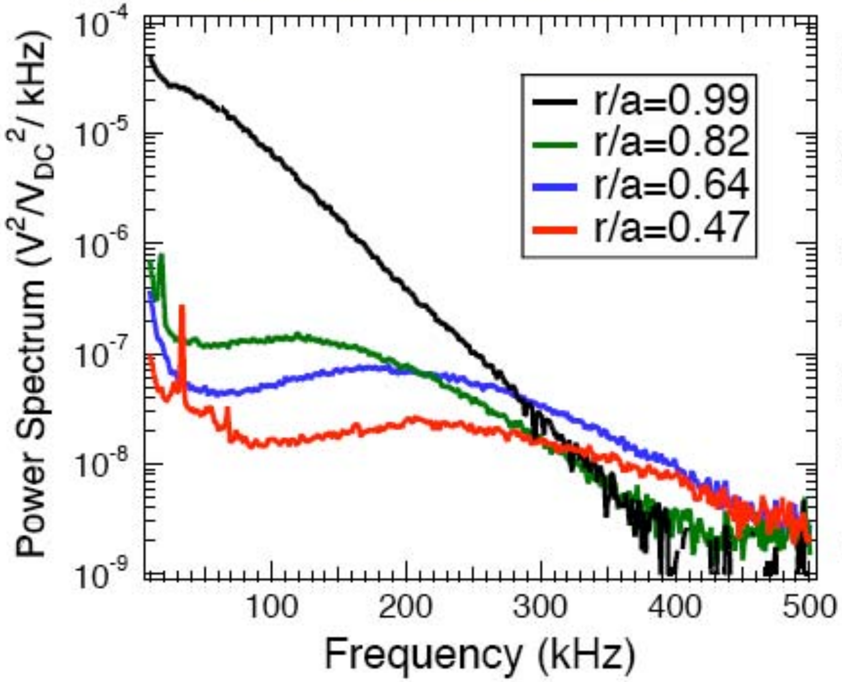
Imaging of Turbulent Density Fluctuations in the Core Region of DIII-D Tokamak



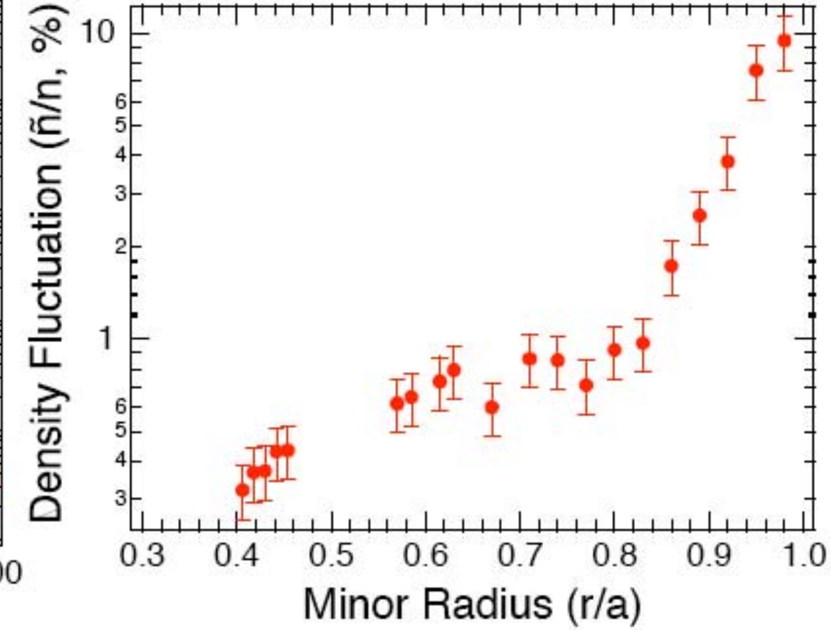
Ref: G. McKee, private communication

Fluctuation Spectra and Amplitude Vary Strongly with Radius

Fluctuation Spectra at Several Radii

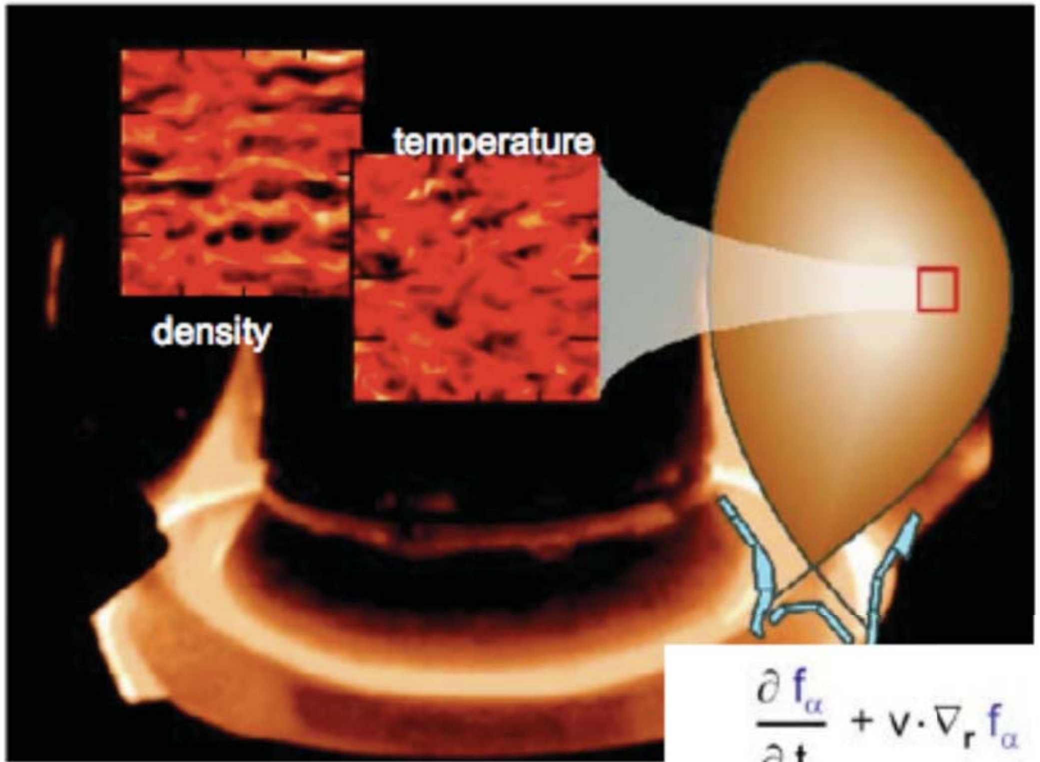


Density Fluctuation Amplitude Profile



- **Density fluctuation amplitude in L-mode discharges shows wide dynamic range across plasma radius**
- **Spectra strongly Doppler-shifted to higher frequency towards core**

Result: Turbulent Transport in Confined Plasmas



"small"-scale turbulence:

- strongly influenced by geometry
- single particle effects important which not captured by simple fluid picture

$$\frac{\partial f_\alpha}{\partial t} + v \cdot \nabla_r f_\alpha + \frac{q_\alpha}{m_\alpha}(E + v \times B)\, \nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{coll.}$$

distribution function — Electric field, Lorentz force — Influence of collisions

together with Maxwell's equations

Ref: Lackner, DEISY Talk 2005

Turbulence Leads to Cross-field Transport

$$\frac{\partial \bar{n}}{\partial t} + \nabla \cdot \tilde{\Gamma} = 0$$

$$\tilde{\Gamma} = -\frac{\langle \tilde{n} \nabla \tilde{\phi} \rangle \times \bar{\mathbf{B}}}{B^2}$$

$$m\bar{n}\frac{\partial \bar{\mathbf{v}}}{\partial t} + \nabla \cdot \overleftrightarrow{\mathbf{i}}_R + = \frac{q}{m}\left(\bar{\mathbf{E}} + \bar{\mathbf{v}} \times \bar{\mathbf{B}}\right) - \nabla\left(\bar{n}\bar{T}\right)$$

$$\overleftrightarrow{\mathbf{i}}_R \equiv \left\langle \left(-\frac{\nabla \tilde{\phi} \times \bar{\mathbf{B}}}{B^2}\right)\left(-\frac{\nabla \tilde{\phi} \times \bar{\mathbf{B}}}{B^2}\right) \right\rangle$$

$$\frac{3}{2}\frac{\partial \bar{n}\bar{T}}{\partial t} + \frac{3}{2}\frac{\partial \langle \tilde{n}\tilde{T} \rangle}{\partial t} + \nabla \cdot \left[\tilde{\mathbf{Q}}_{cond} + \tilde{\mathbf{Q}}_{conv}\right] = -\nabla \cdot \bar{\mathbf{q}}_{class} + P_{rad}$$

$$\tilde{\mathbf{Q}}_{cond} \equiv \frac{3}{2}\bar{n}\left(-\frac{\langle \tilde{T} \nabla \tilde{\phi} \rangle \times \bar{\mathbf{B}}}{B^2}\right)$$

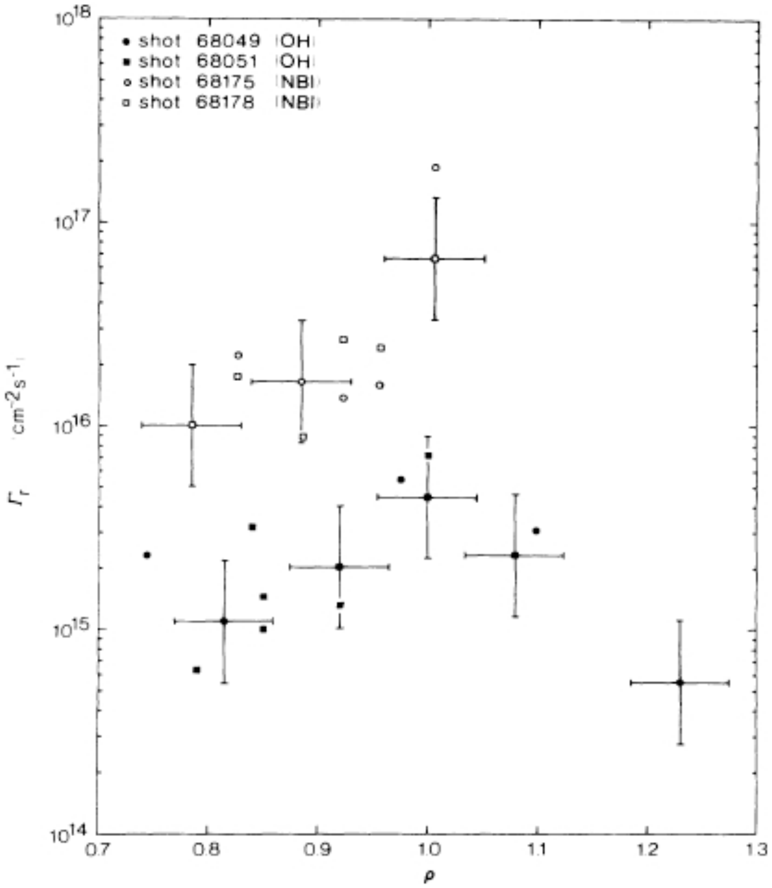
$$\tilde{\mathbf{Q}}_{conv} \equiv \frac{3}{2}\bar{T}\left(-\frac{\langle \tilde{n} \nabla \tilde{\phi} \rangle \times \bar{\mathbf{B}}}{B^2}\right)$$

Neglecting DC Convection, Magnetic Fluctuations, Parallel flow fluctuations, Viscosity, …

Assuming electrostatic ExB dynamics for velocity

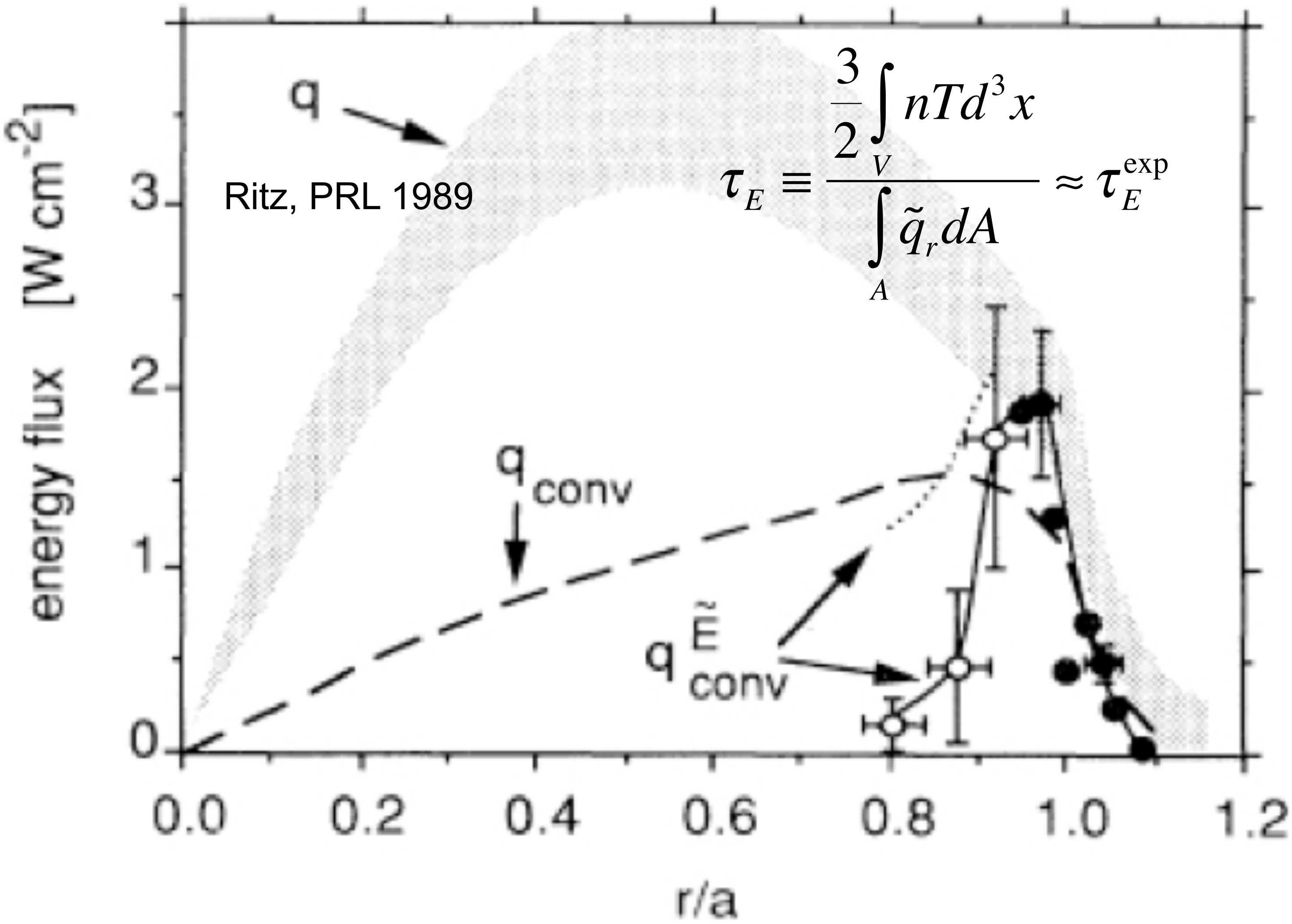
Turbulent Fluxes are (Roughly) Consistent with Global Confinement

Hallock, PRL 1987



$$\tau_p \equiv \frac{\int_V n d^3x}{\int_A \tilde{\Gamma}_r dA} \approx \tau_p^{\exp} \sim few\ m\sec$$

Turbulent Electron Heat Flux Consistent with Global Confinement (at least at edge)



$$\tau_E \equiv \frac{\frac{3}{2}\int_V nTd^3x}{\int_A \tilde{q}_r dA} \approx \tau_E^{\exp}$$

Ritz, PRL 1989

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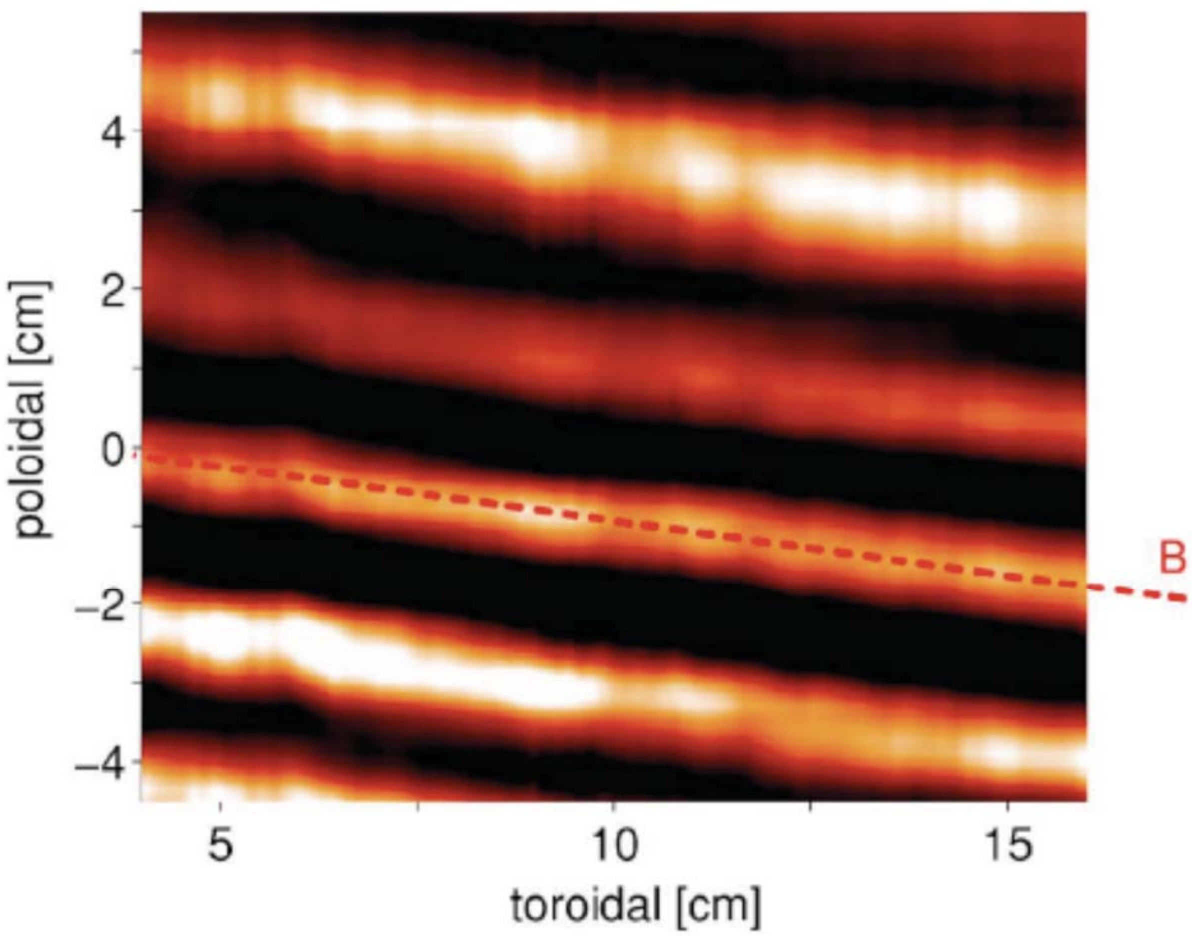
Drift Turbulence Fluctuations Elongated Along B

Grulke PoP 2006 ALCATOR C-Mod



Origin of Zonal Flow Lies in 2D Dynamics

Navier-Stokes Eqn for *Rotating Fluid:*

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\bullet\nabla\right)\mathbf{v}+2\boldsymbol{\Omega}\times\mathbf{v}=-\frac{1}{\rho}\nabla p-\nabla\Phi+\nu\,\nabla^2\mathbf{v}$$

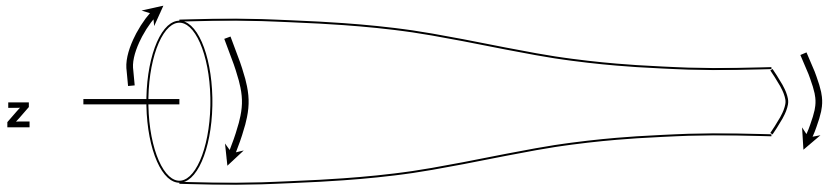
For $|\nabla\times\mathbf{v}| << |\boldsymbol{\Omega}| \quad \nabla\cdot\mathbf{v}=0 \quad \nu=0$

$$\boxed{\boldsymbol{\Omega}\cdot\nabla\mathbf{v}=0}$$

I.e. *no velocity gradient along direction of axis of rotation*

Flow Generation from Turbulence: the Vortex Merging Picture

3-D	2-D

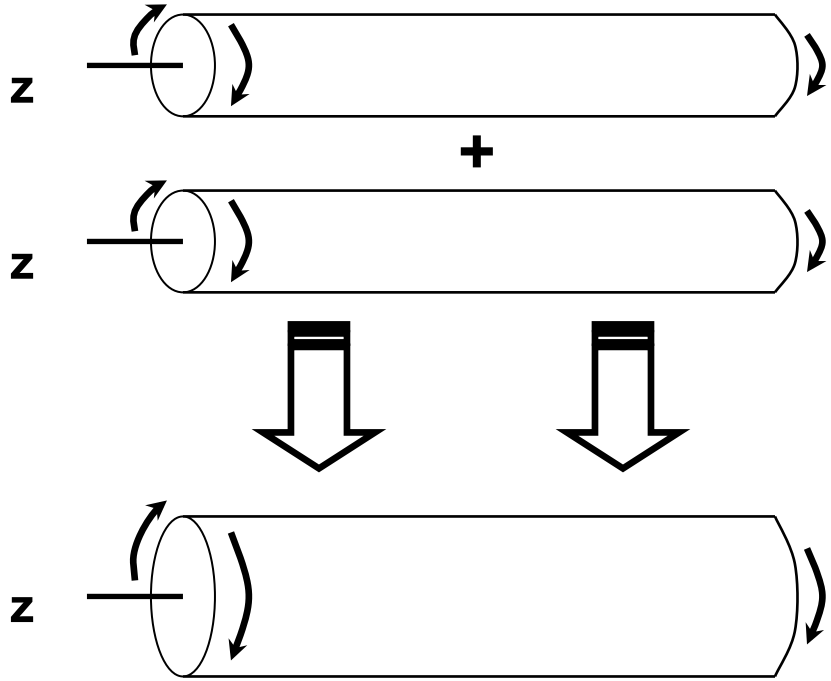


$$\frac{\partial v_z}{\partial z} \neq 0 \Rightarrow$$

==> Vortex Stretching

…Generate Flows on Smaller Scales

$$\frac{\partial v_z}{\partial z} = 0 \Rightarrow$$ Vortex Merging



2D Dynamics in Magnetized Plasmas & Rotating Fluids

Navier-Stokes Eqn for *Rotating Fluid:*

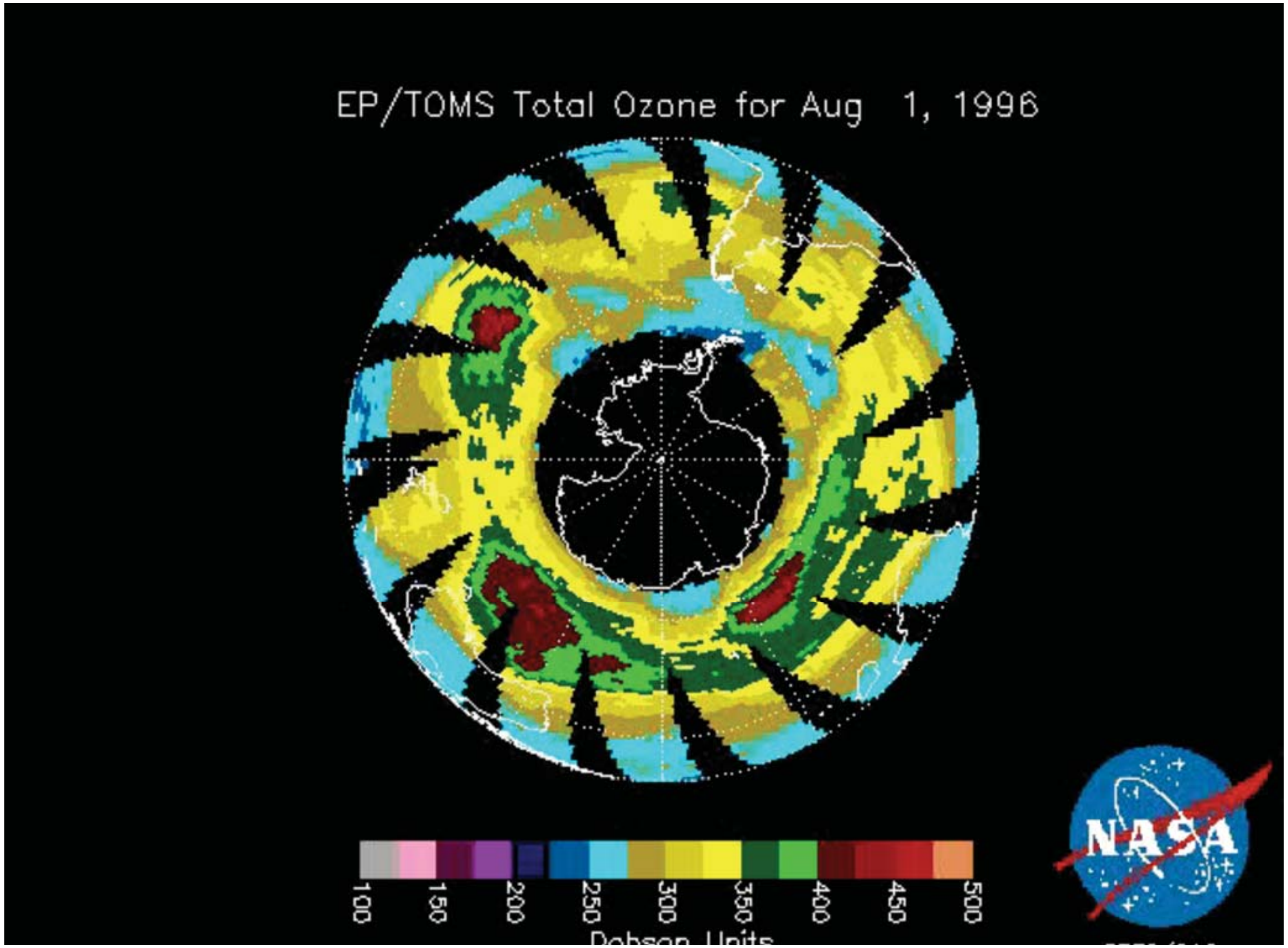
$$\left(\frac{\partial}{\partial t}+\mathbf{v}\bullet\nabla\right)\mathbf{v}+2\mathbf{\Omega}\times\mathbf{v}=-\frac{1}{\rho}\nabla p-\nabla\Phi+\nu\,\nabla^{2}\mathbf{v}$$

Momentum Eqn for *Magnetized Plasma:*

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\bullet\nabla\right)\mathbf{v}+\mathbf{B}\times\mathbf{v}=-\frac{1}{\rho}\nabla p-\nabla\Phi+\nu\,\nabla^{2}\mathbf{v}$$

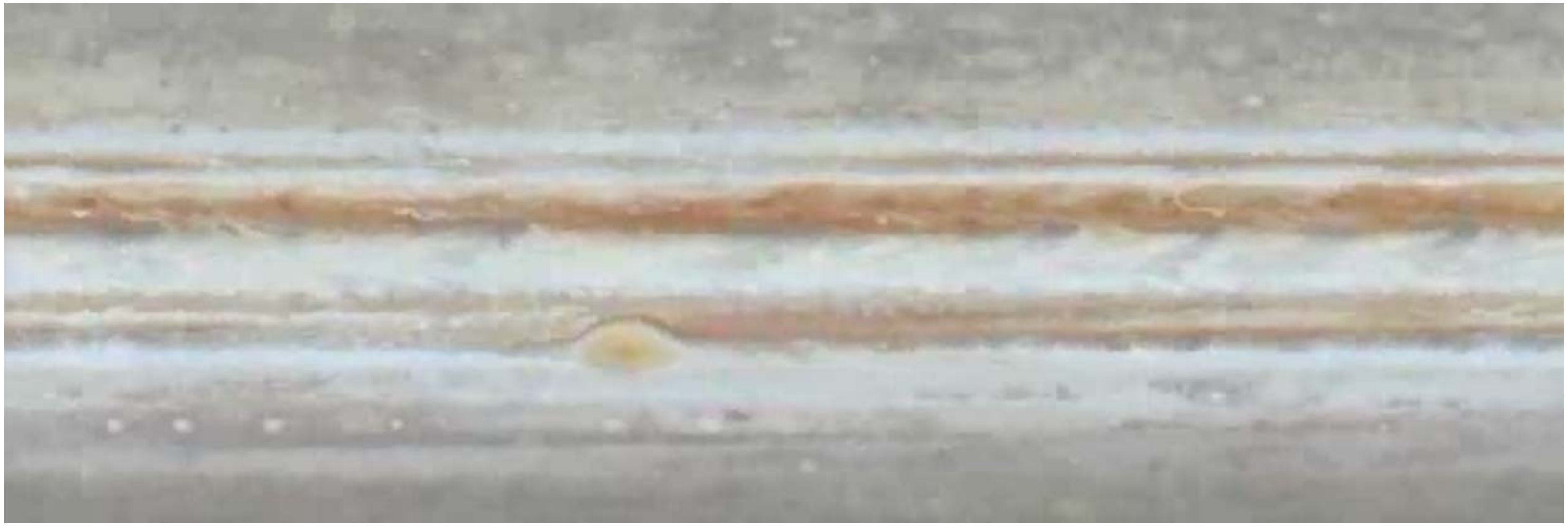
I.e. *momentum conservation same form for rotating fluid And magnetized plasma ➔> DYNAMICS ARE SAME !*

Zonal Flows are Common & Effect Transport in Many Systems



Zonal Flows are Common & Effect Transport in Many Systems

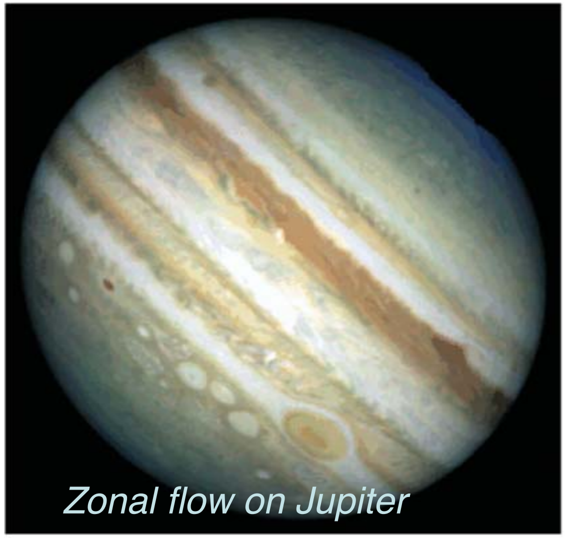
CASSINI Imaging Team, NASA



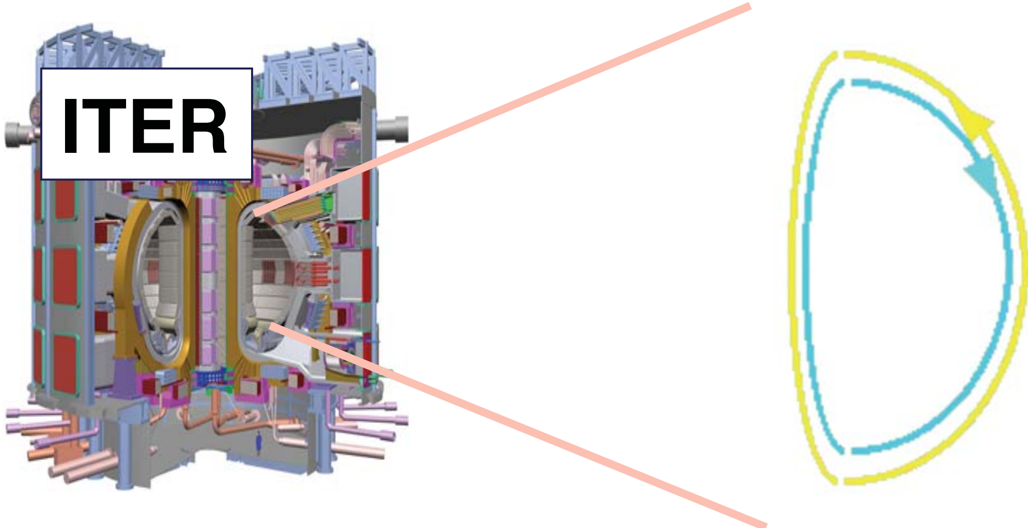
Result: Strong Similarity Between Planetary Flows & Magnetized Plasma Flows

ExB flows

$m=n=0$, k_r = finite



Zonal flow on Jupiter



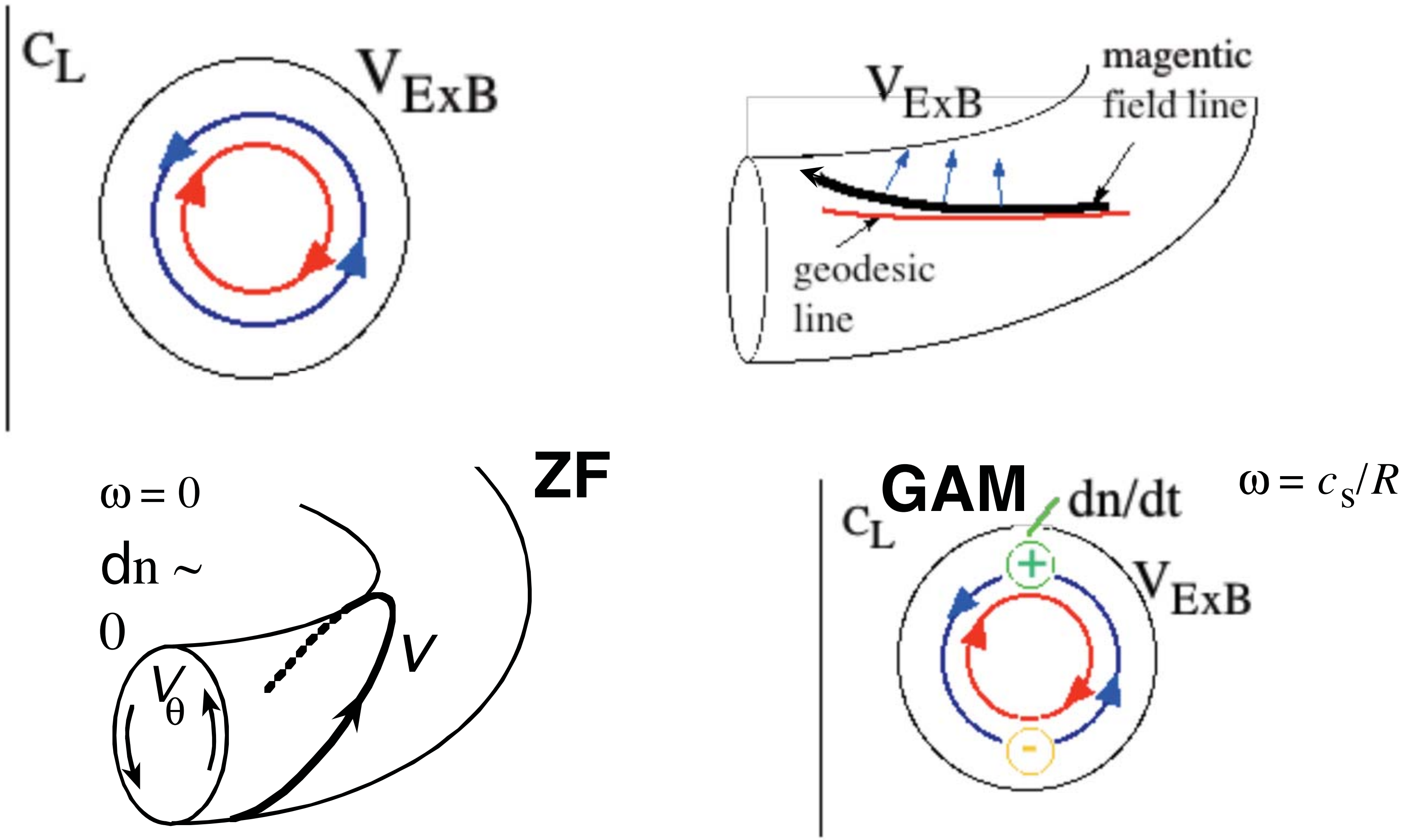
From GYRO

ZFs are "mode", but:

1. Turbulence driven
2. No linear instability
3. No direct radial transport

Ref: K. Itoh, APS 2005 Invited Talk, PoP May 2006

Large Scale Sheared Flows Can Develop



New feature: geodesic acoustic coupling (GAC)

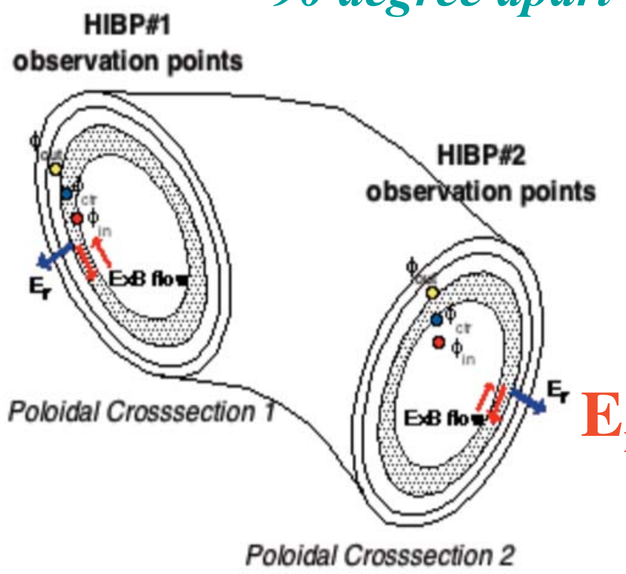
B D Scott 2003

Zonal flows really do exist !

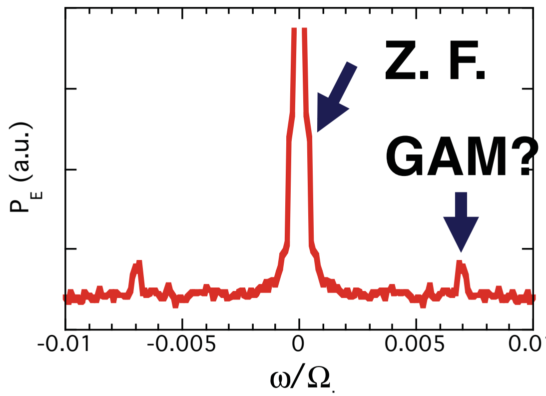
A. Fujisawa, PRL 2004

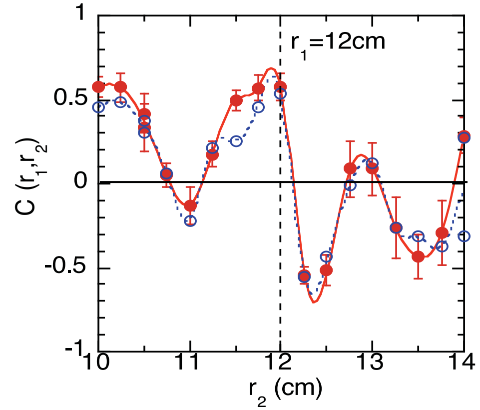
CHS Dual HIBP System

90 degree apart



$E_r(r,t)$





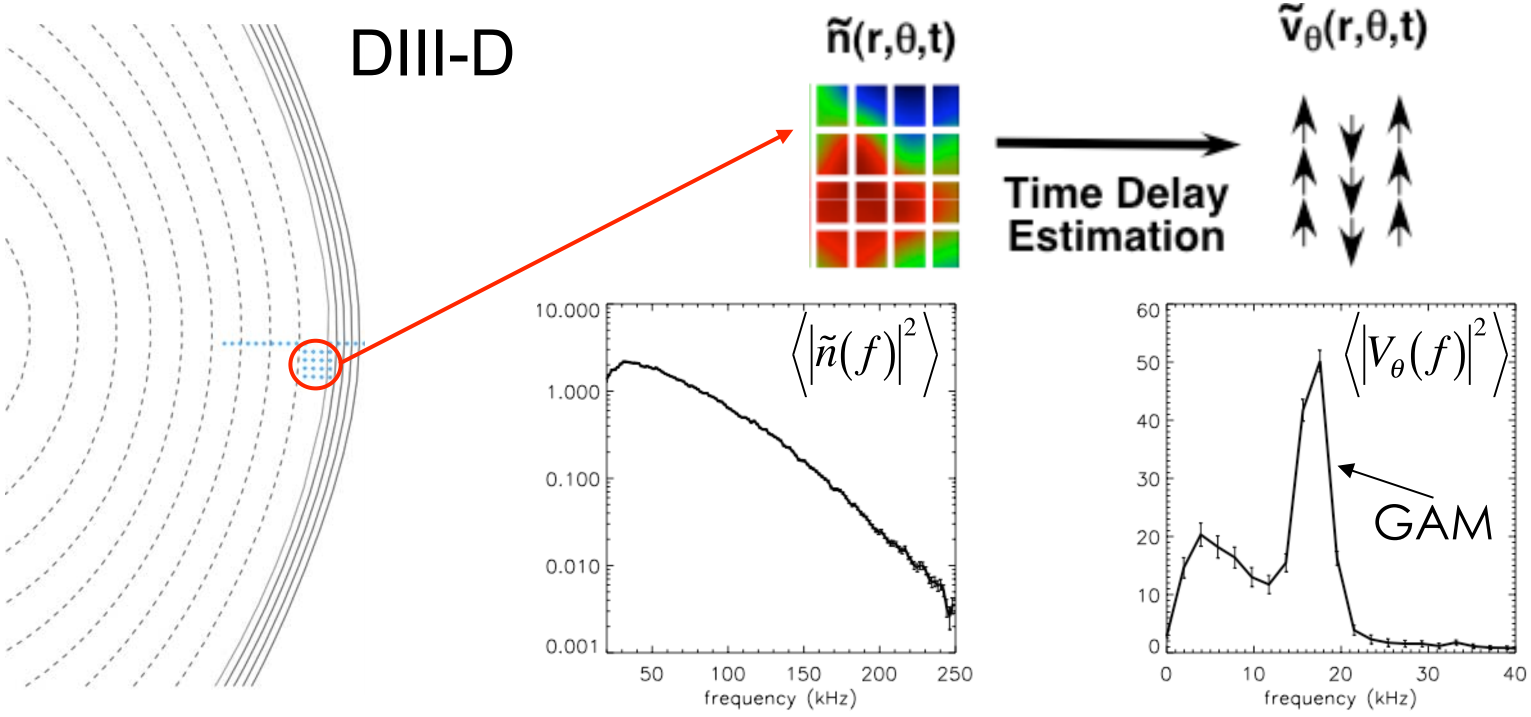
Radial distance

$E_r(r,t)$ {
High correlation on magnetic surface,
Slowly evolving in time,
Rapidly changing in radius.

GAM-ZF Observed in Edge Region

- BES measures localized, long-wavelength ($k_{\theta} r_i < 1$) density fluctuations
  - Can be radially scanned shot to shot to measure turbulence profiles
  - Recent upgrades allow for BES to measure core fluctuations

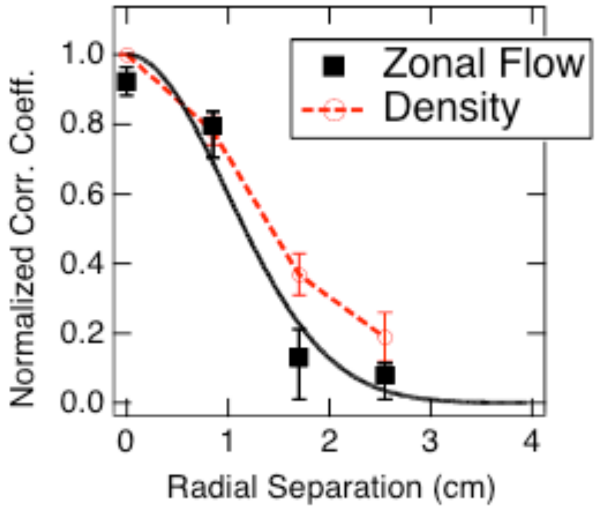
- Time-delay estimation (TDE) technique uses cross-correlations between two poloidally separated measurements to infer velocity

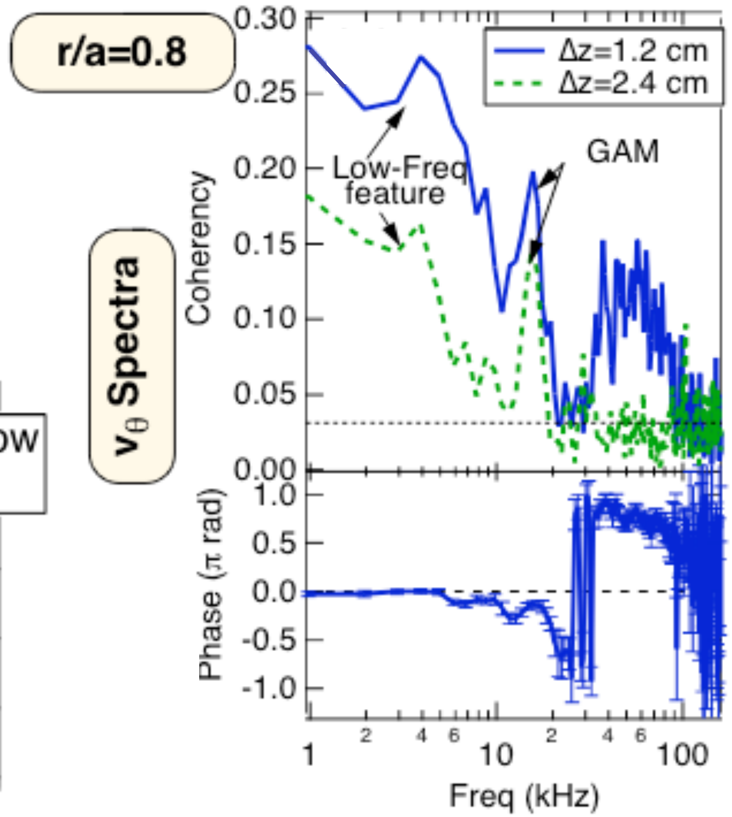


McKee PoP 2001

Measured V_q Spectra Exhibit Signatures of Both ZMF Zonal Flows and GAMs in DIII-D

- Spectra indicate broad, low-frequency structure with zero measurable poloidal phase shift
  - Consistent with low-m (m=0?)
  - Peaks at/near zero frequency
- GAM also clearly observed near 15 kHz
- ZMF zonal flow has radial correlation length comparable to underlying density fluctuations
  - Necessary for effective shearing of turbulence

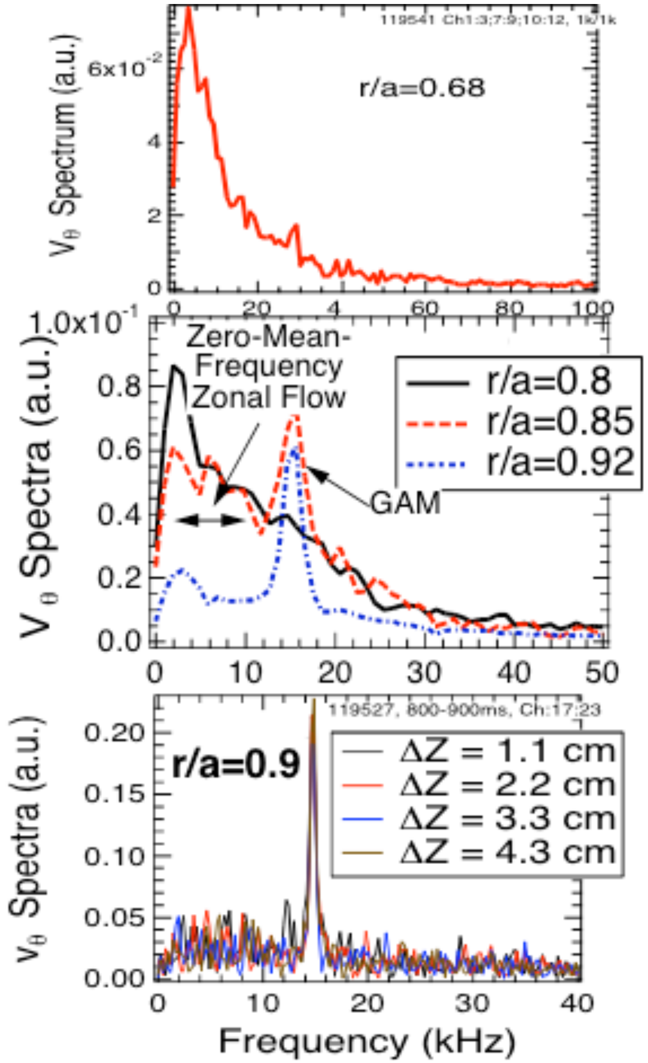




Gupta *et al*., PRL **97** 125002 (2006)

Observe Transition from ZMF-Dominated Core to GAM-Dominated Edge

- Velocity spectra show broad ZF spectrum for $r/a < 0.8$ → ZMF flow
- Superposition of broad spectrum and GAM peak near $r/a = 0.85$
- GAM dominates for $r/a > 0.9$
- Consistent with theory/simulation expectations that GAM strength increases with *q*
  - Increase in GAM strength with q_{95} also observed (McKee *et al.*, PPCF 2006)
- GAM is highly coherent, with correlation time t_{GAM} > 1 ms, two orders of magnitude larger than turbulence decorrelation t_{turb} ~ 10 ms
  - Indicates GAM is “slow” relative to edge turbulence timescales, and so can effectively interact with turbulence (Hahm *et al.*, PoP ‘99)



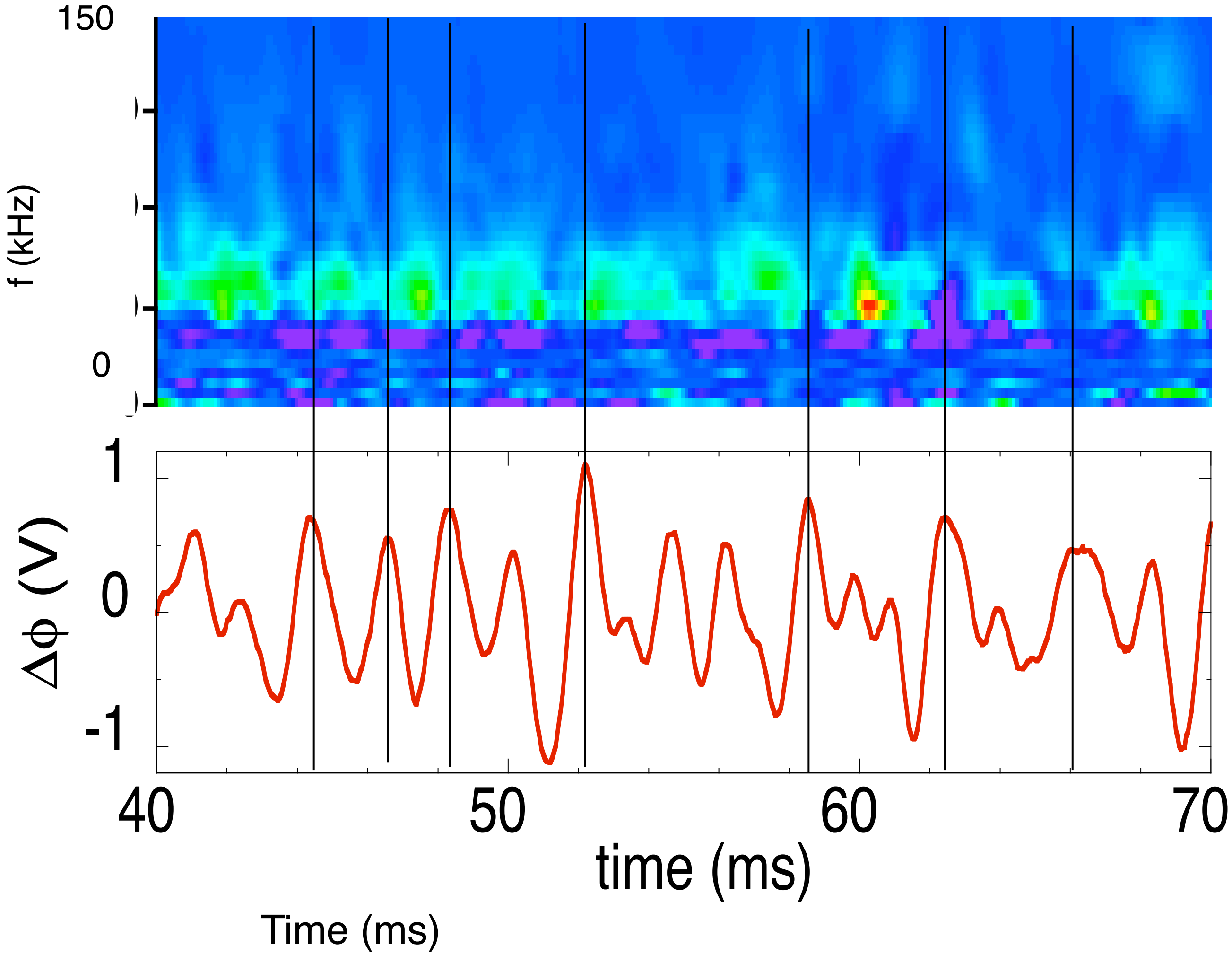
McKee IAEA 2006

Regulation of particle transport by ZF

Fujisawa, PPCF 2006   HIBP on CHS

$$\Gamma(\omega) = \frac{1}{B}\left\langle \tilde{E}_{\theta,\omega}\, \tilde{n}_{\omega} \right\rangle$$

ZF



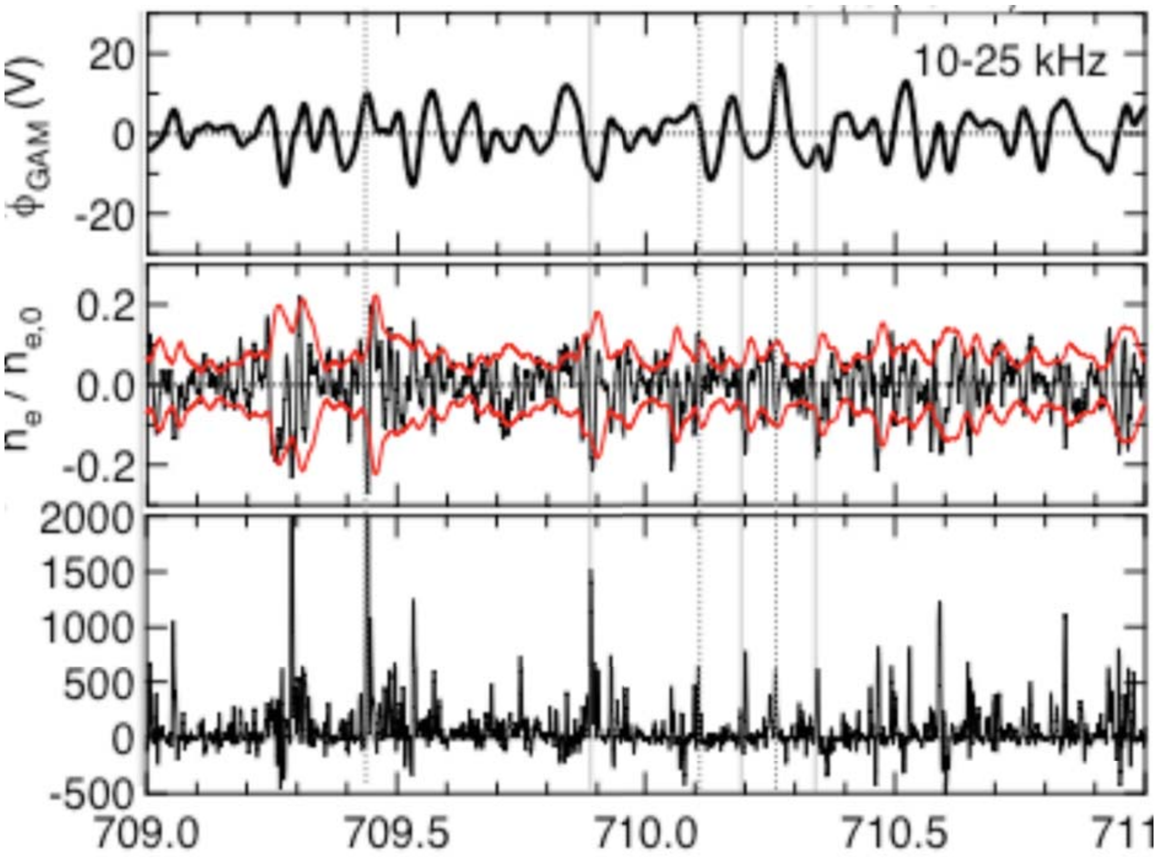
Regulation of transport by ZF

HIBP on JFT-2M   Ido, NF 2006

GAMs

DW

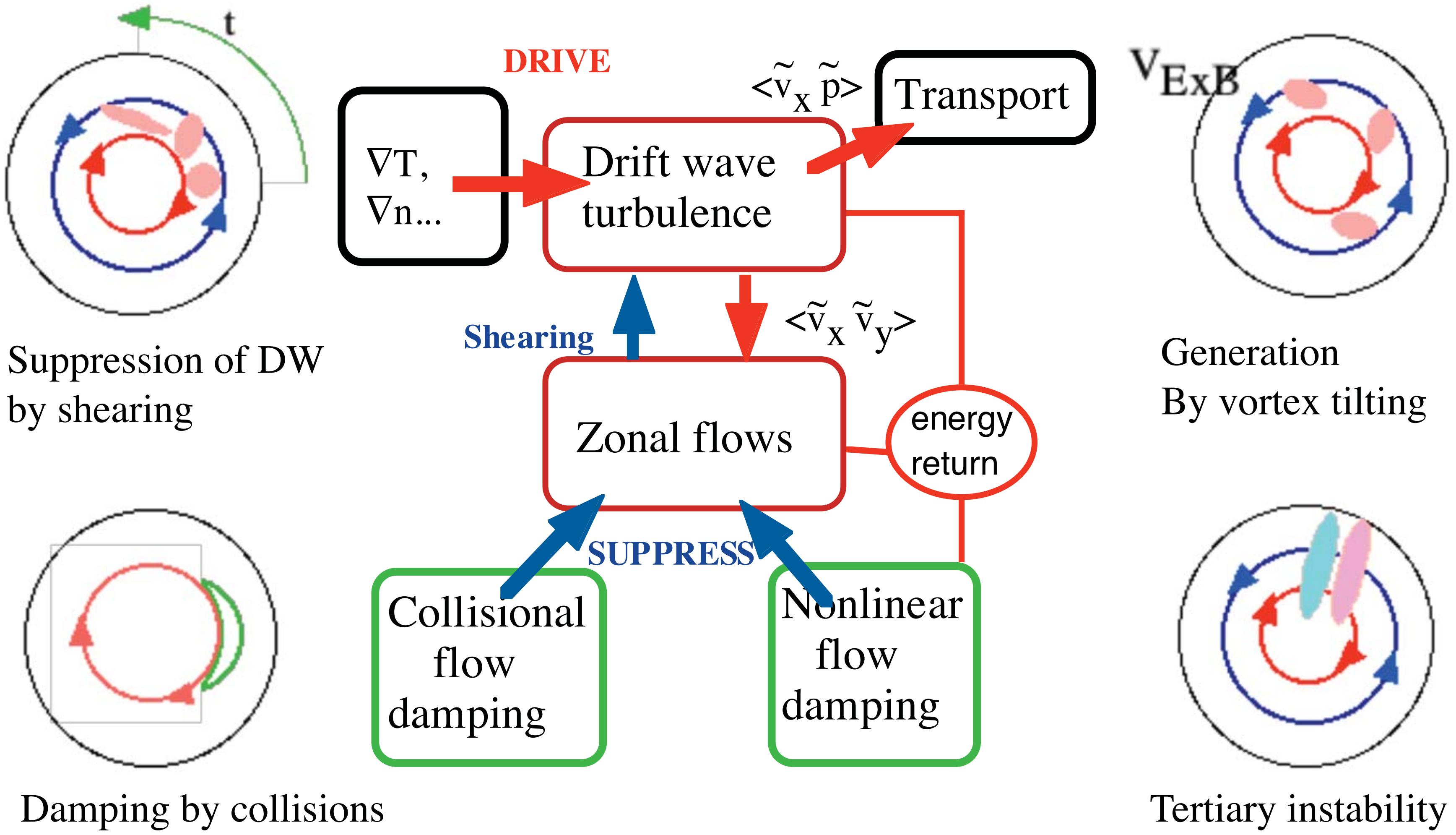
$\Gamma(t)$



Time (ms)

Schematic of NONLINEAR drift turbulence-zonal Flow interactions

Ref: Itoh APS 2005



SUMMARY of Part I

- Reviewed Drift Wave Picture
- Discussed How Waves ➔ Turbulence
- Introduced Zonal Flows & Summarized Interactions with Drift Turbulence

In Part II…

- Look at Basic DW Experiments
- Transition to Turbulence from DWs
- Onset of Nonlinear Energy Transfer
- Development of Zonal Flows & Back Reaction on Turbulence
- Impact on Flux vs Gradient Relations